

Problems and Advances in Monitoring Horizontal Strain

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Abstract. After a brief description of the modern instrumentation used in Geodesy for the detection of the deformations of the crust of the Earth, follow a list of problems and a discussion concerning the needs for the survey of the physical quantities of interest in geodesy, geology, geophysics and engineering such as the strain invariants, the optimal network of baselines and the accuracy. An analytic method is also given for the computation of the effect of a source of dilatation in a spherical Earth.

Introduction.

From the observation of the stress drop in the earth's crust caused by earthquakes in several regions of the world we have learned that the most frequent stress drops are those at low level (Caputo, Console, (1978)); but we have also learned that stress drops of less than 0.4 bar are almost never found in earthquakes recorded with usual seismographs. The corresponding strain released amounts to several parts of 10^{-6} . This is a quantity which is measurable with instruments of various type, including portable geodetic instruments, with an accuracy better than 10%. This in turn implies that we can try to monitor the accumulation of strain which precedes the earthquakes. The same accuracy is sufficient for monitoring the deformation of large structures such as dams, bridges, buildings, tunnels, mineshafts and others. To determine this accumulation of strain one should observe directly in the rock. This in turn can be achieved with several techniques some of which are of geodetic interest. Other geodetic techniques are of lesser accuracy but are of interest because they can be applied in some circumstances when the more accurate techniques are not possible because of logistic difficulties; they will be discussed in a separate presentation.

In the following we shall first describe the geodetic techniques used in the past, and discuss for future needs. Some advances will also be presented.

Geodetic measurements.

The basic concept in the geodetic measurement of horizontal deformation is that the distances between selected points are measured at different times and then compared. The instrumentation is based on electromagnetic waves which are used in two ways.

One way is to observe the motion of the interference fringes of a monochromatic

light source caused by the relative motion of two bench marks connected with the light source and a reflector. This method allows to monitor continuously slow variations of distances with a resolution of the order of 10^{-10} . The distance between the bench marks is limited by the fact that the light beam should stay in the vacuum; distances of almost 1 km have been used. There should not be basic difficulties in making this instrument portable or to operate it between bench marks connected to the rock such as in natural caves in mine shafts, railway of road tunnels, or in tunnels drilled for this purpose. Since the instrument does not give any absolute measurement of distance but it monitors its variation the need to make it portable is for saving time in assembling it and to make its use easier. A good example for this type of instruments is that of Wyatt and Berger (1978). A method of much lesser accuracy, at most one part in 10^{-7} , but of much easier and faster use, is that of measuring distances between several points using laser instruments with one or more colors. To increase the accuracy of the instruments with one color Slater and Huggett (1976) introduced the technique of using a radial set of baselines, of almost instantaneous measurements and of comparing the ratios of the lengths of the baselines at different times to detect relative variations of length. The method reduces the errors due to the variation of the atmospheric parameters and is applicable with the hypothesis that the long term changes in the index of refraction over the baselines are proportional to the index of refraction at each line; the proportionality coefficient must be the same for all the lines in the array during the specific survey. The method of the ratio is useful because it requires to record the atmospheric parameters during the survey only at one end point of the baseline. The precision of the method is about one or two parts in 10^{-7} .

This method has been successfully used by measuring the atmospheric parameters at both end point of all baselines for very long baselines (of the order of 100 km); the precision of the method is reported of 2 parts in 10^{-7} (Carter and Vincenty (1978)). Another method to minimize the errors due to the non uniform and constant atmosphere has been introduced by Parm (1973, 1975); he used base lines over flat terrain and made the observations only when the wind was almost parallel to the baseline.

These methods do not give the absolute measure of the baseline with the same accuracy obtained measuring the average atmospheric parameters along the baseline by means of an aircraft (Savage (1975) and Savage and Prescott (1973)) although the va-

riations of the length of the baseline is obtained perhaps with greater accuracy.

However the best results in measuring the variations of the length of baseline and its absolute length are obtained with a multiwave length geodimeter (Hugget et al. (1977), Slater and Hugget (1976)) with an accuracy of 10^{-7} in the variation of the length and in the absolute value of the length of the baseline. It seems therefore that the accuracy of the instrumentation would allow to obtain the necessary accuracy on the surface of the earth.

The computation of the strain invariants.

Problems which are still partly or completely unsolved are those connected with logistics and cost. But the most important problem to solve is the meaning of the information which is now being collected on the surface of the earth in connection with what is happening below it and which we really want to know. Strictly speaking the problem which I should discuss here is two dimensional but in reality the problem is intrinsically threedimensional and one should never forget it. Therefore I shall try to set it in its proper space, the partition line between horizontal and vertical deformations will be set later where and how it will seem most proper.

The stress accumulated in the Earth's crust probably has uniform distribution over fairly large scale at least in the tectonic processes. This circumstance allows to observe all the components of the stress and therefore the stress invariants even when the baselines of the trilateration are large.

If one succeeds in measuring the displacement vectors

$\bar{u}_0(u_{10}, u_{20}, u_{30}), \bar{u}_1(u_{11}, u_{21}, u_{31}), \bar{u}_2(u_{12}, u_{22}, u_{32}), \bar{u}_3(u_{13}, u_{23}, u_{33}), P_j(x_{1j}, x_{2j}, x_{3j}), j=0,3,$
 $(x_{ij}$ are cartesian orthogonal coordinates with the x_3 axis normal to the surface of the Earth), assuming that the displacement is a linear function of the coordinates, one may write the following system of 9 equations

$$u_{ji} - u_{jo} = \frac{du_i}{dx_k} (x_{ki} - x_{ko}), i, j, k=1,2,3 \quad (1)$$

in the 9 unknown $\frac{du_i}{dx_j}$ which will give the components of the strain and therefore the invariants. It may be noted that on the surface of the Earth one may reasonably assume

$$\frac{du_3}{dx_1} = -\frac{du_1}{dx_3}, \frac{du_3}{dx_2} = -\frac{du_2}{dx_3}, \frac{du_3}{dx_3} = \frac{si}{si-1} \left(\frac{du_1}{dx_1} + \frac{du_2}{dx_2} \right)$$

where si is the Poisson ratio. It is obvious that if one observes only horizontal displacement and one estimates the invari-

ants with the hypothesis that $\frac{du_1}{dx_3} = \frac{du_2}{dx_3} = 0$, with few exceptional cases (e.g. see Caputo 1978), the results could be misleading. The cavity effects and the topographic and thermal effects (Harrison 1976, Harrison and Herbst 1977) give classic examples of wrong interpretation of strain observed in wrong places or better they give examples of wrong extrapolations of data, as it would be in our case.

Since we can measure displacements on the surface of the Earth the question is: how may we observe the displacement vector in points under the surface of the Earth? An estimate of the vertical component $\frac{du_3}{dx_3}$ at depth could be made by measuring directly the dilatation d and then subtracting the surface components computed with measurements on the surface of the earth. The Sacks Everston (Sacks et al. 1971) dilatometer is a good example of an instrument which can measure dilatation with an accuracy of 10^{-7} . But the additional knowledge of d does not allow to compute the maximum shear strain and its direction which are of the greatest importance. The real problem is to estimate

$\frac{du_3}{dx_2}, \frac{du_3}{dx_1}, \frac{du_2}{dx_3}$ and $\frac{du_1}{dx_3}$, levellings allow to estimate the last two and therefore to retrieve the first two, but only on the surface.

Operating in a mine shaft could be an experiment to try, however, in this case, the displacement at the end of the baseline could be determined only by means of an open polygon with an accuracy which would hardly be acceptable and in any way would be by far inferior to that of the surface measurements. The use of a road or railway tunnel could allow to determine the displacement vector with a closed polygon, but in most cases the tunnel would not be located in a useful position. The problem is open and it is one of the most important.

A problem which I think we should also consider is that of the most suitable distribution of the baselines or of the bench marks for determining the distribution of strains on the surface of the Earth in the case of the hypothesis $\frac{du_1}{dx_3} = \frac{du_2}{dx_3} = 0$. In this case the system (1) is reduced to 2 linear systems of 2 equations each namely

$$\begin{aligned} u_{11} - u_{10} &= \frac{du_1}{dx_1} (x_{11} - x_{10}) + \frac{du_1}{dx_2} (x_{21} - x_{20}) \\ u_{12} - u_{10} &= \frac{du_1}{dx_1} (x_{12} - x_{10}) + \frac{du_1}{dx_2} (x_{22} - x_{20}) \\ u_{21} - u_{20} &= \frac{du_2}{dx_1} (x_{12} - x_{10}) + \frac{du_2}{dx_2} (x_{21} - x_{20}) \\ u_{22} - u_{20} &= \frac{du_2}{dx_1} (x_{12} - x_{10}) + \frac{du_2}{dx_2} (x_{22} - x_{20}) \end{aligned} \quad (2)$$

The matrix of the coefficients in both systems is

$$\begin{matrix} x_{11} - x_{10} & x_{21} - x_{20} \\ x_{12} - x_{10} & x_{22} - x_{20} \end{matrix} \quad (3)$$

which should be different from zero. Therefore the three points P_0, P_1, P_2 should not be in a straight line.

Accuracy of the determination of strain.

Given the m.s.e.s $(x_1) = s(x_2)$ in the determination of the coordinates of the points of the network in all the surveys, which in turn give the displacement vectors, the m.s.e. of the invariants, are in the first approximation

$$s(d) = \frac{s}{\sin z} 2 \left(\frac{1}{d_1^2} + \frac{1}{d_2^2} \right)^{\frac{1}{2}} = 2\bar{s}$$

$$s(a) = \bar{s} \quad (4)$$

$$s(S) = 2\sqrt{2} \bar{s}$$

Where S is the maximum shear strain, a is its direction, z is the angle $P_1 P_0 P_2$ and d_1 and d_2 are the length of $P_0 P_1$ and $P_0 P_2$. If the strain field is uniform then the best $s(a)$ and $s(S)$ are obtained for $z = \pi/2$ and for d_1 and d_2 very large. This implies that when one computes the strain invariants one loses at least a factor 3 in the accuracy.

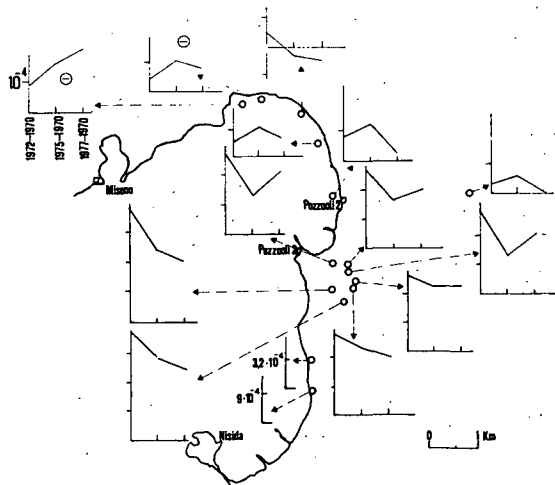


Fig. 1a - Dilatation as function of time in the period 1970-1977 in the Phlegrean Fields near Naples. Each diagram gives the dilatation in the point connected by the dashed arrow. In the top left diagram are indicated the units of dilatation and the time interval in which it has been accumulated.

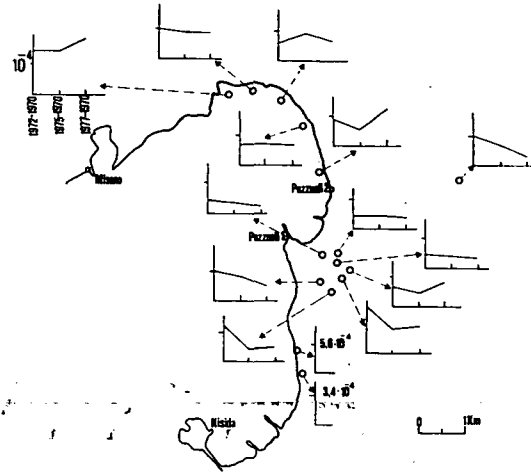


Fig. 1b - Dilatation as function of time in the period 1970-1977 in the Phlegrean Fields near Naples. Each diagram gives the maximum shear in the point connected by the dashed arrow. In the top left diagram are indicated the units of dilatation and the time interval in which it has been accumulated.

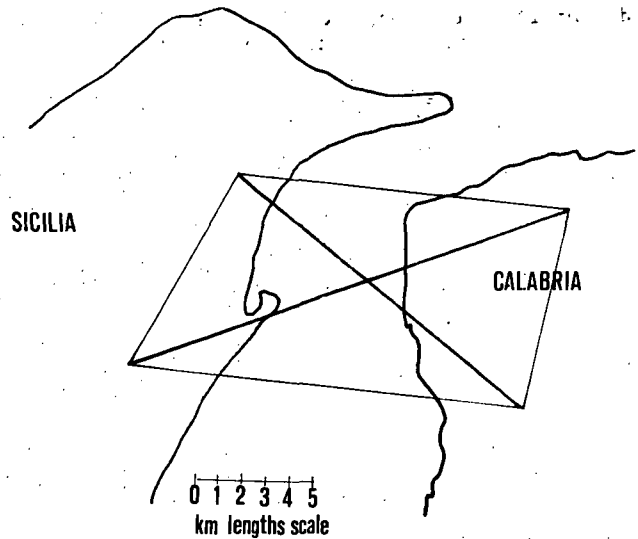


Fig. 1c - Network of baselines for the measurement of displacements across the Messina Straits.

When the strain field is not uniform formulae (4) set a limit to the accuracy of the determination of the invariants. It is the indetermination principle of the strain fields observations, which is expressed by

$$l \epsilon > \bar{s}$$

where ϵ is the strain observed in the path l . An almost complete example of a determination of a strain field is that of the Phlegrean Fields near Naples (Caputo 1978) shown in Fig. 1a, 1b.

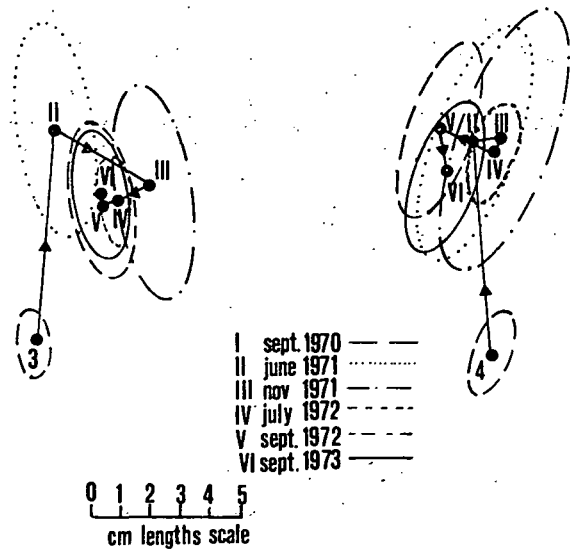


Fig. 1d - Displacement reported in the two points located North in the network of Fig. 1 .

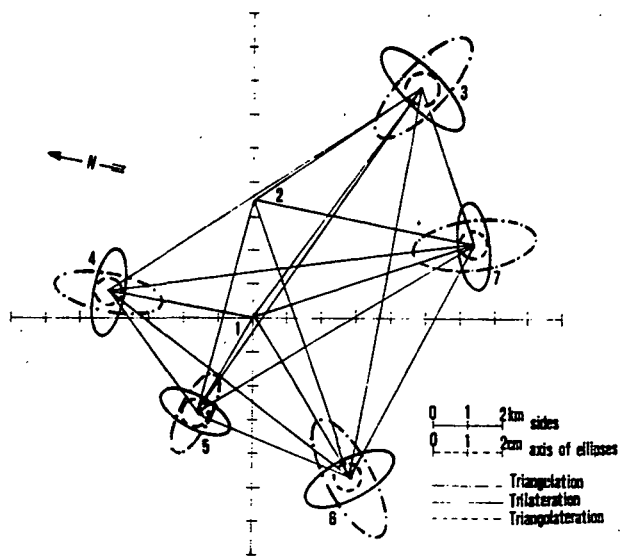


Fig. 1e - Network of baselines for the measurement of strain in the Ancona area.

The accuracy which one may achieve with standard portable instrumentation and procedures in a triangular network with 5 km baselines is $s \approx 1$ cm and one obtains $s(d) = 6 \cdot 10^{-6}$; if one uses the mekometer then $s \approx 2$ mm and the baselines are 2 km therefore $s(d) = 3 \cdot 10^{-6}$; by using a multiwavelength geodimeter with baselines of 5 km $s(d)$ would be $3 \cdot 10^{-7}$.

It is desirable to develop numerical methods and network designs in order to improve the accuracy in the determination of the invariants.

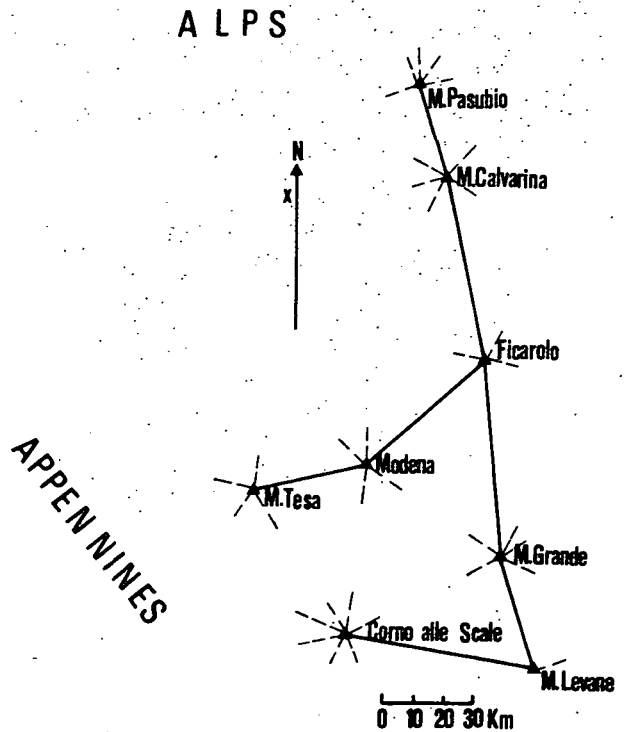


Fig. 1f - Network of baselines for the measurements of displacements between the Appennines and the Alps.

With reference to the network design let us consider the limiting case of a network of infinite size in two dimension. If the distribution of the strain is unknown obviously one should consider the most symmetric distribution of bench marks. According to the principle of the Pflastersatz the most simple distributions of points which have properties of symmetry are the three indicated in the Fig.2

When one considers the baselines needed to determine the invariants of strain one sees that the networks (c) of Fig.2 is not convenient because of the length of the baselines which should also be $\sqrt{3} l$ (with l the minimum distance between points).

Since the two networks of points (a) and (b) have no condition equation the ratio of the numbers of measured baselines to the number of points where the strain is measured is the same. Also, to the first order $s(d) = \frac{1}{\sqrt{3}}$ for the network (b), $\Delta s(d) = \frac{1}{\sqrt{3}}$ for (a); that is they are almost the same. However the distribution of the points where the invariants are measured is better in (a), because in (b) some points have the invariants which are direct interpolation of others.

We may also note that it may be needed to reinforce the strength of the networks which we have considered here and for which, it is immediately seen that the condition equations are non existing. If the baselines are short enough one may consider to measure some of the angles or some

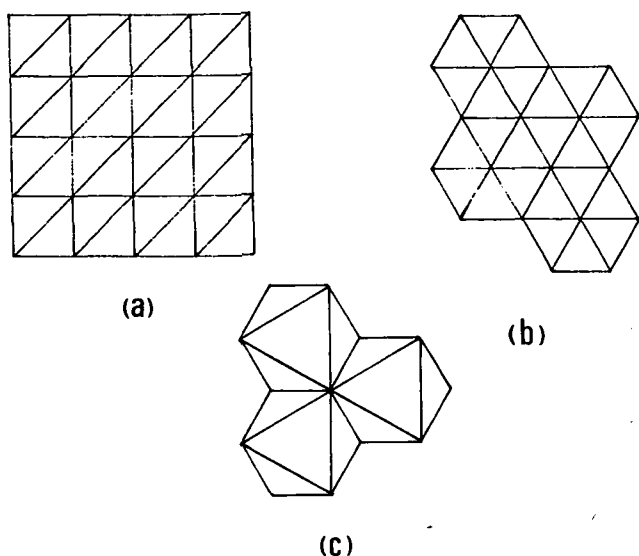


Fig. 2 - Possible networks of baselines formed with regular polygons.

more baselines between the existing points. For instance in the network which we used across the Messina Straits (Caputo et al. (1974)), when we measured only the baselines we had only one condition equation; we had more condition equations when (at times) we measured also the angles; the decision to adopt this procedure was due to logistic problems. In the case of the network of the Ancona area (Baldi et al. (1976)) we decided instead to add more points to the network and we had 9 condition equation. The method of adding more baselines is often preferred to the measurement of angles because one obtains the strains in a larger number of points and therefore obtain a higher resolution in the distribution of strain; also with the modern instrumentation, in networks with baselines of 10 km or longer, the measurement of angles is almost one order of magnitude less accurate. Before closing I must call the attention to the set of problems which I raised and which we should eventually discuss with the many more which probably will be raised after my presentation. These problems are

- a) the need to have a three-dimensional description of the strain fields
- b) the need to have available a multi-wave length geodimeter
- c) the need to monitor strain on long baselines with an interferometer easy to install and to operate
- d) the need to measure on bench marks which are rigidly connected to the medium to observe
- e) to establish networks which are optimal for the final objective
- f) to select a test ground where to make an experiment to satisfy all these requirements and to do the experiment. (This is a proposal for those who are interested).

- g) to develop mathematical models for the displacement fields caused by internal perturbation of the Earth elastic field.
- h) the use of theodolites, in conjunction with electromagnetic distance equipment with accuracy \bar{s} , is of use only if the length of the baseline l is $l < 8 \cdot 10^5 \bar{s}$ in which case its error would be $2 \bar{s}$.
- i) preference to interferometers when investigating slow rate accumulation of strain.
- j) need to adjust the observed displacement fields with the Navier equations.

The displacement field caused by a source of dilatation in a spherical Earth.

It has been established that prior to some earthquakes there is a deformation of the Earth's crust measurable on its surface. According to the dilatancy model in some cases this deformations caused by a dilatation in a three-dimensional portion of the Earth's crust. It is therefore needed to have a three-dimensional geometric elastic model of this phenomenon for the interpretation of the geodetic data.

A solution for a flat Earth model and a point source of dilatation has been given by Sacks et al (1975); we shall extend here this solution to a spherical Earth model S_0 of radius r_0 and a point source of dilatation P_1 at a distance h from the center of the Earth.

The method consists in finding first the field \bar{u} generated by the dilatation source in P_1 in an infinite space assuming the origin of the polar coordinate system in P_1 ; then we shall move the origin of the coordinates in the center O of S_0 and finally add to \bar{u} a vector \bar{v} which renders the component of the stress applied to S_0 nil on S_0 as it naturally is. In a more sophisticated solution one may render \bar{v} such to balance the weight caused by the topography.

We shall assume that the body forces caused by the source and its perturbation are negligible; in the first approximation this is acceptable when the source dimensions and the dilatation are limited to realistic cases. We assume also that the gravity field of the Earth has negligible effect with respect to the other fields playing the major role in this phenomenon. Solutions taking into account all those effects would obviously be more accurate; the solution presented here is considered only a first approximation of the spherical Earth.

The displacement caused in the space by a generic source of dilatation in P with the condition of convergence at infinity is

$$u_r = dV r^{-2}, u_z = 0 \quad (5)$$

where dV is the change in volume of any volume which includes P_1 . It is immediately verified that the dilatation of this displacement field is nil at all points different from P_1 .

The formulæ for the mentioned transformation of coordinates, with reference to the figure 3 are

$$r' = [r^2 + h^2 - 2rh \cos z]^{\frac{1}{2}} = \frac{h \sin z}{\sin a} \quad (6)$$

and substituting in (1) we obtain

$$u_r = dV [r^2 + h^2 - 2rh \cos z]^{-\frac{1}{2}} \cos a \quad (7)$$

$$u_z = dV [r^2 + h^2 - 2rh \cos z]^{-\frac{1}{2}} \sin a$$

where $\sin a$ and $\cos a$ are obtained from (6):

$$\sin a = h \sin z [r^2 + h^2 - 2rh \cos z]^{-\frac{1}{2}} \quad (8)$$

$$\cos a = \left[1 - \frac{h^2 \sin^2 z}{r^2 + h^2 - 2rh \cos z} \right]^{\frac{1}{2}}$$

$$u_r = \frac{dV(r - h \cos z)}{[r^2 + h^2 - 2rh \cos z]^{\frac{3}{2}}} \quad (9)$$

$$u_z = \frac{dV h \sin z}{[r^2 + h^2 - 2rh \cos z]^{\frac{3}{2}}}$$

We shall compute now the solution \bar{v} of the Navier equations which is to be added to \bar{u} in order to obtain the final solution $\bar{u} + \bar{v}$ such that the stress components t_r applied to S_0 are nil on S_0 . According to the formulæ of Caputo (1961) the general solutions of the Navier equations in the axisymmetrical case and homogeneous Earth of this case are

$$v_r = \sum_1^{\infty} n (A_{1,n} r^{n+1} + A_{3,n} r^{n-1}) \frac{2n+1}{2} P_n(\cos z) + A_{1,0} r \quad (10)$$

$$v_z = \sum_1^{\infty} n (B_{1,n} r^{n+1} + A_{3,n} r^{n-1}) \frac{2n+1}{2n(n+1)} \frac{dP_n}{dz}$$

$$B_{3,n} = \frac{A_{3,n}}{n}$$

$$B_{1,0} = -\frac{3g+5}{2}, \quad g = \frac{1}{m}$$

1 and m are the elastic parameters.

The components of the stress of the solution \bar{v} are

$$\begin{aligned} t_{rr} = & \sum_2^{\infty} n \left\{ [(n+3)1+2(n+1)m] A_{1,n}^{-1} B_{1,n}^{-1} r^{n+1} \right. \\ & \left. + [(n-1)1+2(n-1)m] A_{3,n}^{-1} B_{3,n}^{-1} r^{n-2} \right\} P_n \\ & + (3l A_{1,0} + 2m B_{1,0}) P_1 \quad (11) \\ t_{rz} = & m \sum_1^{\infty} n \left[(n B_{1,n} + A_{1,n}) r^n + [(n-2) B_{3,n} + \right. \\ & \left. + A_{3,n}] r^{n-2} \right] \frac{dP_n}{dz} \end{aligned}$$

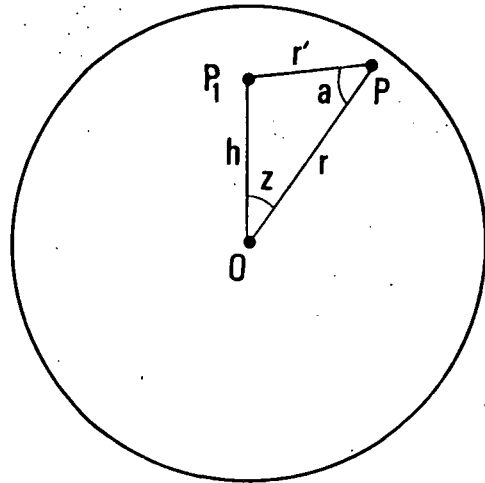


Fig. 3 - Coordinate system used in formulæ (5) through (12).

If we require that the components of the stress t_r of the solution $\bar{u} + \bar{v}$ applied to S_0 are nil on S_0 , the conditions are

$$\begin{aligned} & \sum_2^{\infty} n \left\{ [(n+3)1+2m(n+1)] \bar{A}_{1,n}^{-1} \bar{B}_{1,n}^{-1} r_0^{n+1} \right. \\ & \left. + (n-1)(1+2m) \bar{A}_{3,n}^{-1} \bar{B}_{3,n}^{-1} r_0^{n-2} \right\} P_n \\ & + 3l \bar{A}_{1,0} + 2m \bar{B}_{1,0} + \frac{1}{r_0} \left[\frac{d}{dr} (r^2 u_r) \right]_{r=r_0} + \quad (12) \\ & + \frac{1}{r_0 \sin z} \frac{d}{dz} (u_z \sin z)_{r=r_0} + 2m \left[\frac{d u_r}{dr} \right]_{r=r_0} = 0 \\ & \sum_1^{\infty} n \left[(\bar{A}_{1,n} + n \bar{B}_{1,n}) r_0^n + (\bar{A}_{3,n} + (n-2) \bar{B}_{3,n}) r_0^{n-2} \right] x \end{aligned}$$

$$x \frac{dP_n}{dz} + \left[\frac{du_z}{dr} - \frac{u_z}{r} + \frac{1}{r} \frac{du_r}{dz} \right]_{r=r_0} = 0$$

where $\bar{A}_{i,n}$ and $\bar{B}_{i,n}$ are the unknown.

An expansion of u_r and u_z , appearing in (12), in series of Legendre polynomials gives finally the values of $\bar{A}_{i,n}$ and $\bar{B}_{i,n}$ to substitute in (10) to obtain the final solution.

The extension of this result to a prestressed, Earth is almost obvious. What we really need is to prepare tables and figures with the computations of the effects of a dilatation in P_1 according to the formulae and to observe the phenomena on the real Earth.

The extension of the results of this section to a layered Earth is also trivial (Caputo 1961).

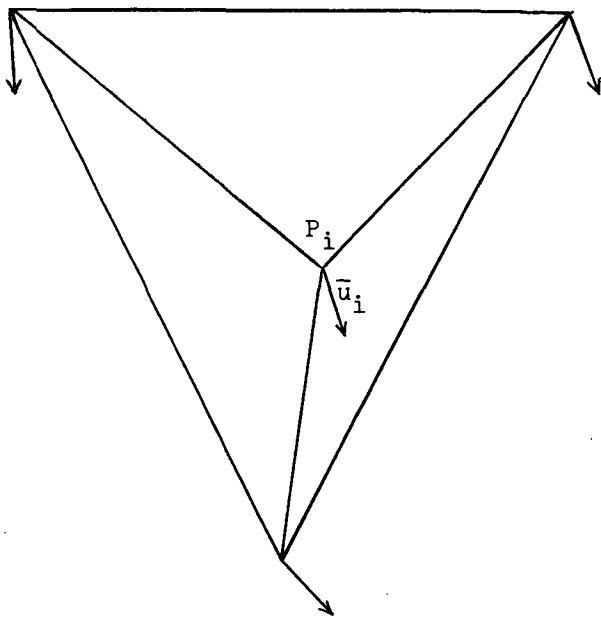


Fig. 4 - The four points P_i should not belong to a single plane.

References.

- Baldi P., Caputo M., Postpischl D., Boschi E., Unguendoli M., Geodetic network for crustal movements in the Ancona area, Boll.Geod.Sc.Affini, XXXV, 4, 494-507, 1976.
- Berger J. A note on thermoelastic strains and tilts J.Geophys.Res., 80, 274-277, 1975.
- Caputo M., Strain and other geodetic observations in the Phlegrean Fields near Naples in the last 2000 years. Geophys. J.R.astr.Soc. In press.
- Caputo M., Deformation of a layered earth by an axially symmetric surface mass distribution J.Geophys.Res., 66,5,1479-1483,1961.

- Caputo M., Console R., Model and observed statistics of California earthquakes parameters Annali di Geofisica, in press 1977.
- Caputo M., Folloni G., Modern views regarding slow horizontal movements of the soil and feasibility of measuring them, Geologia Applicata e Idrogeologia, X, I, 289-307, 1975.
- Caputo M., Folloni G., Pieri L., Unguendoli M., Geodimetric control across the Straits of Messina, Geophys.J.R. astr. Soc., 38, 1-8, 1974.
- Carter W.E., and Vincenty T., Survey of the McDonald Observatory radial line scheme by relative lateration techniques, Technical Report NOS 73 NGS-8, National Oceanic and Atmospheric Administration, 1978.
- Hugget, G.R., Slater L.E., and Langbein J., Fault slip episodes near Hollister, California: initial results using a multiwavelength distance-measuring instrument, J.Geophys.Res., 82, 3361-3368 (1977).
- Harrison J.C., Cavity and topographic effects in tilts and strain measurements, J.Geophys.Res., 81, 2, 319-328, 1976.
- Harrison J.C., Herbst K., Thermoelastic strains and tilts revisited, preprint 1977.
- Parm.T., Observing procedure and meteorological factors for electronic distance measurements, paper presented at International Symposium on Electronic Distance Measurements, Int.Ass.of Geod., Lagos, 1975.
- Parm T., High precision traverse for scale determination of satellite and stellar triangulation and for controlling the first order triangulation, paper presented at General Assembly, Int.Ass. of Geod., Grenoble, 1975.
- Sacks I.S., Suyehiro S., Everston D.W., and Yamakishi Y., Sacks-Everston strainmeter, its installation in Japan and some preliminary results concerning strain steps. Pap.in Meteor. and Geophys., 22, 3-4, 195-208, 1971.
- Sacks I.S., Snoke J.A., Yamakishi Y., and Suyehiro S., Borehole Strainmeters. Long-term stability and sensitivity to dilatancy, Carnegie Inst.Wash. Year Book, 74, 1975.
- Savage J.C., and Prescott W.H., Precision of geodolite distance measurements for determining fault movements, J.Geophys. Res., 78,6001-6008, 1973.
- Savage J.C. and Prescott W.H., Geodimeter measurements of strain during the Southern California uplift, Preprint, 1978.
- Slater L.E., Can the ratios of single wavelength EDM data improve the resolution of small changes in baseline length? A comparison with multiwavelength EDM data. Preprint, 1978.
- Slater L.E. and Huggett G.R., A multiwavelength distance-measuring instrument for geophysical experiments, J.Geophys.Res., 81, 6299-6306, 1976.
- Wyatt F., Berger J., Correlation of short based tilt measurements and long base strain measurements. Abstract, EOS, 59, 4, 331, 1978.