Models for Extracting Vertical Crustal Movements from Leveling Data

Sandford H. Holdahl
National Geodetic Survey, National Ocean Survey/NOAA
Rockville, Maryland 20852

Abstract. Various adjustment strategies are now being used in North America to obtain vertical crustal movements from repeated leveling. The more successful models utilize polynomials or multiquadric analysis to describe elevation change with a velocity surface. Other features permit determination of nonlinear motions, motions associated with earthquakes or episodes, and vertical motions of blocks where boundaries are prespecified. The preferred models for estimating crustal motions permit the use of detached segments of releveling to govern the shape of a velocity surface and allow for input from nonleveling sources such as tide gages and paired lake gages. Some models for extracting vertical crustal movements from releveling data are also excellent for adjusting leveling networks, and permit mixing old and new data in areas exhibiting vertical motion. The new adjustment techniques are more general than older static models and will undoubtedly be used routinely in the future as the constitution of level networks becomes mainly relevelings.

Introduction

In the United States, most leveling surveys have been performed to support individual engineering or mapping projects. Until recent years the timing and arrangement of the surveys were rarely influenced by the geodesist's need to detect verical crustal movement. The most prominent factors influencing the network development were the availability of cooperative funding from local government and the desire to eliminate what were regarded as weaknesses in the network. Generally, the development of most national networks geographically follows the development of a nation, and "ideal" plans for quickly establishing a network of strategically spaced lines are rarely implemented.

Because of the manner in which most national networks evolve and are maintained, the geodesist is challenged to find ways of detecting crustal movements from scattered releveling over an original network which is also not time-homogeneous. The detection of vertical crustal movements is important to the earth scientist, but for the geodesist it is also necessary to model such movements when adjusting networks of leveled height differences of different dates. The geodesist then wants to find models which bring consistency between the observations, the detected movements or velocities, and the heights being published.

Various methods for determining and predicting vertical crustal movements have been used in North America. They are described in the following pages. Each of the methods works well in particular circumstances; two of the methods are general enough to be used frequently in height computations. Figure 1 through 4 schematically illustrate characteristics of leveling networks. Various line types indicate that observations were obtained in different years.

Methods and Models

Method 1

Occasionally the distribution of original and repeated levelings is almost ideal for a small area (see Figure 1). Two levelings covering the study area, each accomplished within a short time period and adequately separated from each other in time, may be adjusted independently. After the adjustments, movements are calculated by comparing the two sets of adjusted heights. To make the comparison, a movement is assumed to be known for one of the common points; usually the movement at that point is taken to be zero and the computed movements are considered relative. If one of the points is a tide gage, absolute movement at that point can be inferred from the tide gage record. Absolute movements at other points can be calculated by adding in a constant when making the comparison. Velocities are obtained by dividing the movements by the time elapsed between epochs.

Method 1 is worth mentioning because it does not involve complicated mathematical models and thus avoids the need to develop special computer programs. However, the network in Figure 1 is handled equally well by the more general methods described next.

Method 2

Often this method is more applicable for crustal movement determination than Method 1 because data requirements are less restrictive. The original and repeat observations existing in an area will generally not be separated by a constant time interval. By forming velocity difference observations from repeat levelings, the data are effectively made homogeneous. Since velocity observations are independent of date, velocity misclosures should theoretically equal zero if leveling is perfect and the assumption of constant movement is correct.

The velocity difference, Δv, between points connected by releveling is computed according to:

$$\Delta v = \frac{\Delta h_2 - \Delta h_1}{\Delta t} \quad (1)$$

where Δh_1 and Δh_2 are the old and new observed height differences respectively, and Δt is the time elapsed between levelings.

The variance of the velocity difference is computed using equation (2),

$$m_{\Delta v}^2 = (m_1^2 + m_2^2)S \,/\, \Delta t^2 \quad (2)$$

Proc. of the 9th GEOP Conference, *An International Symposium on the Applications of Geodesy to Geodynamics, October 2-5, 1978*. Dept. of Geodetic Science Rept. No. 280. The Ohio State Univ., Columbus, Ohio 43210.

FIGURE 1

Complete Releveling - Two Leveling Epochs

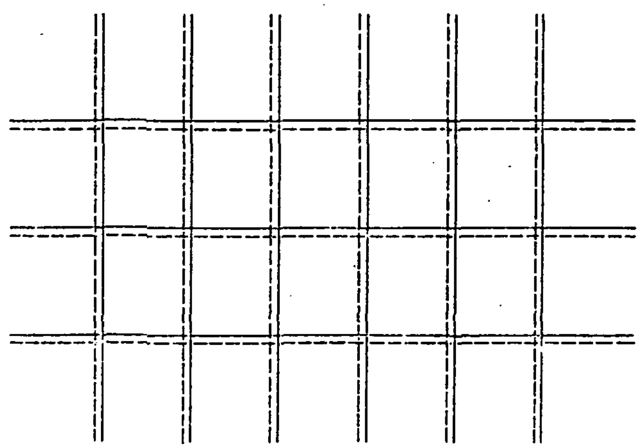


FIGURE 3

Multiple Relevelings, Single Levelings

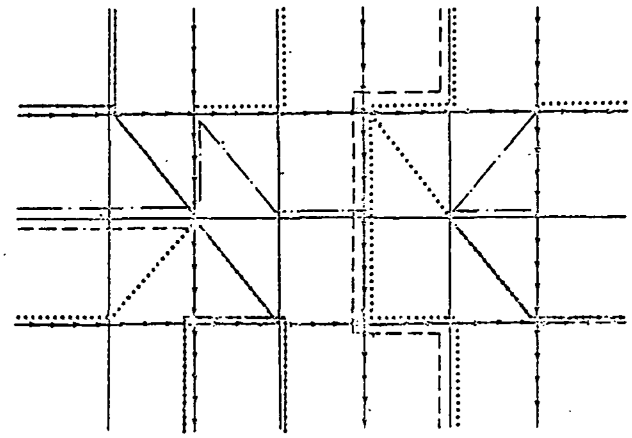


FIGURE 2

Scattered Relevelings

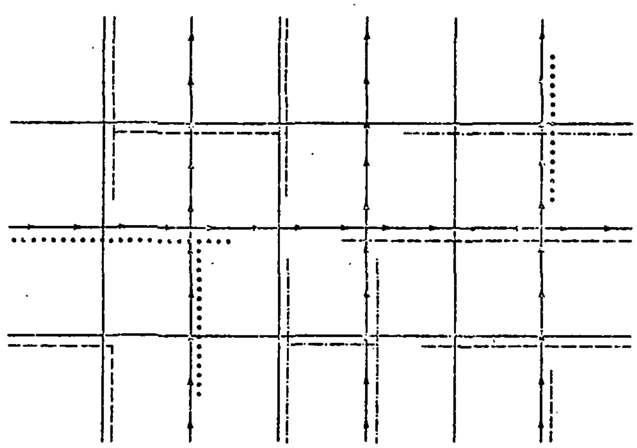
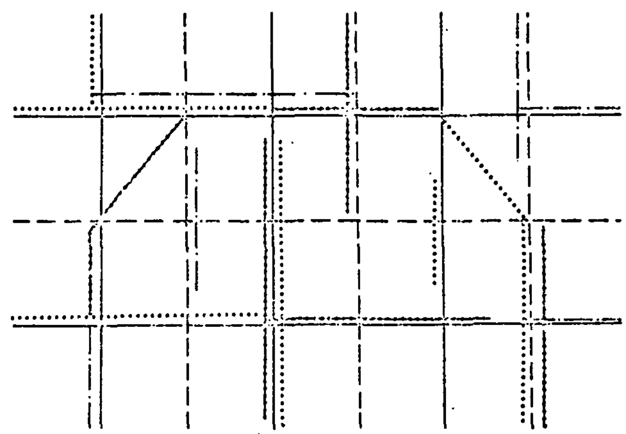


FIGURE 4

Multiple Relevelings, Single Levelings, and Detached Relevelings



where m_1^2 and m_2^2 are the a priori unit variances of the old and new levelings, and S is the distance between link terminals.

The velocity differences forming a network may be disconnected but not by long distances. Constraints requiring velocities in the same small locality to be identical can hold the segments together. These constraints are reasonable only if the geographical separation of observations is not great or the variation in velocity occurs very gradually over the unconnected area. Weighted constraints that allow some motion between neighboring junctions may also be used to hold disconnected segments together, but selecting a weight may be like guesswork.

Method 3

Method 3, which fits a velocity surface through a field of velocity differences, is described in detail by Vanicek and Cristodulides [1974]. The primary advantage of surface fitting is its usefulness on networks of scattered relevelings of the type shown in Figure 2.

The velocity surface V can be expressed by a generalized two-dimensional polynomial:

$$V(x,y) = \sum_{i=1}^{k} c_i f_i(x,y) \qquad (3)$$

where f_i are arbitrarily chosen, linearly independent functions of the coordinates x and y, and c_i are the coefficients which best fit the observations. A velocity difference can be written as:

$$\Delta V(x,y) = \sum_{i=1}^{k} c_i \Delta f_i(x,y) \qquad (4)$$

where $\Delta f_i(x,y) = f_i(x_B,x_B) - f_i(x_A,x_A)$, for a pair of connected bench marks, A and B. Because of simplicity, the two dimensional-algebraic functions $x^i y^j$, i, j = 0, 1, 2, . . ., m are frequently used for the f_i.

In the above type of adjustment, the unknowns are the coefficients c_i, and the observations are velocity differences computed from levelings using equation 1. The origin of the coordinate system is usually taken to be a point near the center of the study area. Once the coefficients are known, velocities for desired points are calculated using equation 3.

Although method 3 works well on the network shown in Figure 2, it would not fully utilize the measurements shown in Figure 4. To do so would require a nondiagonal weight matrix or preprocessing to obtain weighted mean velocity differences when there is more than one releveling. Neither does the model utilize the information found in circuit misclosures. The main advantage of Method 3 is that it minimizes the number of unknowns in the solution.

Method 4

This technique uses simple polynomials to describe height variations at selected bench marks in the study area. If implemented in its most basic form, the observations are differences of elevation rather than velocity differences. It is assumed that elevation differences are connected as shown in Figure 3.

At the onset, we pick a starting or reference time, t_o. Then, for example, the height of a bench mark A at time t_i is written as follows:

$$h_{a,i} = h_{a,o} + a_1(t_i - t_o) + a(t_i - t_o)^2 + \ldots \quad (5)$$

where $h_{a,o}$ is the elevation of bench mark A at the reference time. The observation equation for Method 4 is:

$$r_{b-a,i} = h_{a,i} - h_{b,i} - \Delta h_{b-a,i} \quad (6)$$

where $\Delta h_{b-a,i}$ is the observed difference of elevation between bench marks A and B at time t_i.

Ordinarily the data redundancy will not permit the use of polynomials higher than degree 3; there is not much advantage in a higher degree even if permitted by the data. When the degree of the polynomial (equation 5) is 2, then a_1 is the velocity of elevation change at time t_o and a_2 the acceleration. At a time other than t_o the instantaneous velocity at bench mark A is calculated according to

$$V_{a,i} = a_1 + 2a_2(t_i - t_o) \quad (7)$$

When the degree of the polynomial (equation 5) is of degree 1, a_1 is a constant velocity.

The above method has some very nice advantages:

(1) If there are three or more relevelings over the same segments, all can be put into the adjustment without resorting to a nondiagonal weight matrix. Single levelings, in appropriate locations, can also add strength to the solution.

(2) Velocity and velocity difference observations are easily introduced to the adjustment. Velocities which have been inferred from tide gage records are entered as weighted parameters; velocity difference observations, computed from pairs of lake level gage records may be entered as differences between the Δt coefficients corresponding to the two points on the sides of the lake.

(3) The solution produces a homogeneous set of heights which correspond to a selected point in time, t_o.

(4) Each point polynomial may have its own degree, the degree being limited only by common sense and the number of excess observation of different date contacting the point and connecting it to the network.

Regarding item (4), it is occasionally difficult to decide how many unknowns can be solved for at each point. Unusual configurations of observations may cause one to guess incorrectly. Therefore, for large complex networks, it is helpful to have a preprocessing program or subroutine to determine solvability.

Method 5

This is a combination of Methods 3 and 4. For any bench mark A in the study area, we can give the following expression for its height at time t_i

$$h_{a,i} = h_{a,o} + V(x_a,y_a)(t_i - t_o) \quad (8)$$

where, for example,

$$V(x_a,y_a) = c_o + c_1 x_a + c_2 y_a + c_3 x_a y_a + c_4 x_a^2 + \ldots \quad (9)$$

The unknowns in the adjustment are the height at each point corresponding to time t_o, and the coefficients c_k, k = 1,2,3,...m which define the velocity surface. If u is the number of unknown junction heights, then the total number of unknowns is u + m. The observation equation is then written as in equation (6). Note that the constant term of equation (9) drops out. The constant term is the absolute velocity of height at the origin of the network. If known, this can be conveniently specified and its uncertainty propagated into computed velocities.

Method 5 has the important advantages of Methods 3 and 4. Height differences are adjusted rather than velocity differences; therefore, no processsing is required to convert leveling observations to velocity differences. Method 5 is preferable to Method 3 when the number of unknowns in the adjustment does not tax the computer.

The choice of whether to use Method 4 or Method 5 will depend on the configuration of relevelings and the extent of the geodesist's foreknowledge of the movement pattern in the study area. In general, Method 4 is more sensitive, but the relevelings must be interconnected; otherwise, each independent subnetwork must have its own initialization in height and velocity.

The solutions produced by Methods 4 and 5 are conceptually different. Method 5 gives a solution that requires all bench marks in a relatively small locality to take on the same velocity, because velocity is a function of position. Method 4 does not naturally provide for this local consistency, but it can be forced by the addition of appropriate constraints between the velocity unknowns of points in the same locality.

Generally, the weighted sum of squared residuals (V'PV) from Method 4 will be less than the corresponding sum from Method 5. Method 5 produces a smoothly fluctuating velocity surface, whereas Method 4 accommodates the observations with any number of bumps and dips having whatever amplitudes are required to minimize corrections to the observations. It is probably misleading to argue which method is best from this point of view because both produce results of high value and the comparison of the two solutions may be of most interest. The larger separations of the two solutions can be regarded as local velocity anomalies. These should be examined closely as they may be indications of local accumulations of systematic errors in the leveling data.

Polynomial expressions for velocity surfaces may produce problems with computer graphics because the fitted surfaces quickly taken on extreme values outside the data area. This may ruin the scale of three-dimensional plots, produce error messages, or use excessive computer time for contouring. To avoid these problems, multiquadric (MQ) analysis has been used as an alternative to polynomials in crustal movement investigations [Holdahl and Hardy, 1977].

Method 6

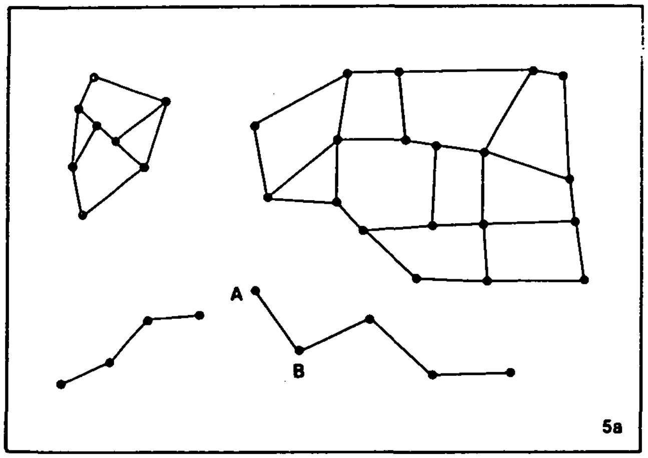
This is essentially the same as Method 5 except that we employ MQ analysis, and replace equation (9) with

$$V(x_a,y_a) = \sum_{j=1}^{k} c_j Q \left[x_a,y_a,x_j,y_j,D\right] \quad (10)$$

The C_j are undetermined coefficients; Q is a quadric kernel function; the x_j, y_j are the positions of nodal points; and D is a geometric parameter which may or may not be needed depending on the quadric form. Nodal points should be located where there are solvable point velocities or tilt information. If the hyperboloid is selected as the quadric form, we then have the following expression for velocity of elevation change:

$$V(x_a,y_a) = \sum_{j=1}^{k} c_j \left[(x_a - x_j)^2 + (y_a - y_j)^2 + D^2\right]^{\frac{1}{2}}. \quad (11)$$

Substituting (11) into (8) gives us a model for leveling adjustments, which advantageously produces automated graphic displays of the velocity surface and velocity error surface without



destruction of scale by an extreme value calculated at an uncontrolled edge of a rectangular study area.

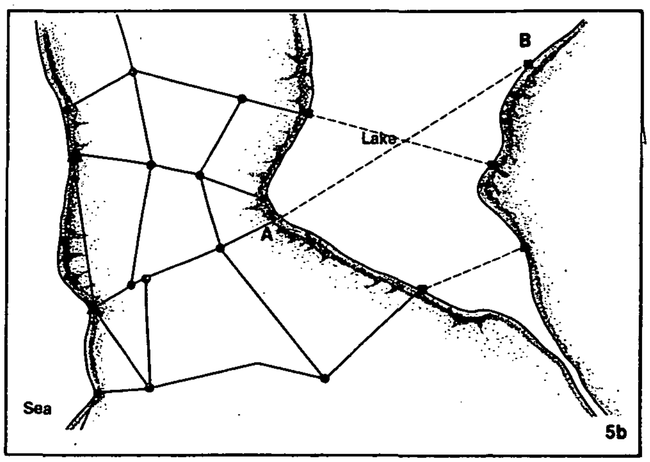
In Method 6, nodal points should be placed at each location that has a solvable point velocity, as in Method 4. Additional nodal points can be added wherever detached tilt information is located.

Scattered or detached relevelings (see Figure 5a) cannot contribute to the determination of the absolute position of a velocity surface unless they are individually initialized by a measured or assumed velocity. However, detached relevelings can be used to aid in determining the local shape of a velocity surface. This was mentioned previously as being the original motivation for developing Method 3. Methods 5 and 6 also permit the use of detached relevelings. In Method 5, the velocity difference observation equation resulting from a detached releveling between points A and B would be written as

$$r(x,y) = \sum_{i=1}^{nx} \sum_{j=1}^{ny} c_{ij} \left(x_b^i y_b^j - x_a^i y_a^j\right) - \Delta V_{b-a}. \quad (12)$$

In Method 6,

$$r(x,y) = \sum_{j=1}^{k} c_j \left[Q(x_b,y_b,x_j,y_j,D) - Q(x_a,y_a,x_j,y_j,D)\right] - \Delta V_{b-a} \quad (13)$$



where ΔV_{b-a} is obtained from the observed height differences, as in equation (1). These same equations can be used to incorporate relative velocities between pairs of water level gages on a lake (see Figure 5b). In the latter case, ΔV_{b-a} would be the slope of a line fitted through a plot of differences between readings from gages at A and B obtained over a period of years.

For two or more relevelings over a detached segment, equations (12) and (13) are not appropriate. In Models 5 and6, it is better to use the usual height difference observation equations, and add another observation (fictitious or otherwise) that specifies a height at t_o for one of the points on the detached segment. This eliminates concern for correlation of velocity difference observations. In setting up crustal movement studies, it has been convenient to have only one fictitious fixed height, which may be called "mean sea level." It has a height of 0.0 meter at the reference time, t_o, and other heights at t_o are provided to the adjustment as fictitious height difference measurements from it with an appropriate weight. These fictitious height difference measurements are, in fact, contraints. No special coding is required if they are treated as measurements.

Method 7

Vanicek revised his original model (Method 3) to consider episodic and nonlinear movements. In the expanded model the "observations" are relative movements obtained by comparing repeated measurements of height differences, whereas in Method 3 they are velocity differences.

The mathematical formulation for the movement surface, u is

$$u(x,y,t) = \sum_{k=1}^{n_t} c_{ok}T_k(t) + \sum_{i=0}^{n_x}\sum_{j=0}^{n_y}\sum_{k=1}^{n_t} c_{ijk}x^i y^j T_k(t) \quad i+j\neq 0$$

$$= \underline{T}(t)\underline{c}_o + X(x,y,t)\underline{c} \qquad (14)$$

where

$$T_k(t) = t^k \qquad k=1,n_p$$

$$T_k(t) = \begin{cases} 0 & t<b_k \\ (t-b_k)/(e_k-b_k) & b_k\leq t\leq e_k \\ 1 & t>e_k \end{cases} \qquad k=n_p+1,n_p+n_e$$

and $\underline{c}$ is the n-vector of unknown coefficients c_{ijk} for the previously chosen values n_x,n_y of maximum power in x and y.

In these formulas x and y are local horizontal Cartesian coordinates calculated from latitude and longitude through the following transformation equations:

$$x= (\phi-\phi_o)R \text{ ; } y = (\lambda-\lambda_o)R\cos\phi_o \qquad (15)$$

where (ϕ_o,λ_o) is the centroid of all bench marks and R is the mean radius of the Earth. Time, t, is reckoned from a stipulated date, t_o, for which $u(x,y,t_o)$ is everywhere equal to zero. In addition, b_k,e_k, for $k=n_p+1,n_p+n_e$, are the beginning and ending dates of n_e movement episodes so that $n_t=n_p+n_e$. We note that the episodic movements are treated as linear within the duration of the episode.

Observation equations for m releveled segments can now be written:

$$\underline{\Delta h}(x_1,y_1,x_2,y_2,t_2) - \underline{\Delta h}(x_1,y_1,x_2,y_2,t_1)$$

$$= \underline{d}(x_1,y_1,x_2,y_2,t_1,t_2)$$

$$= \sum_{i=0}^{n_x}\sum_{j=0}^{n_y}\sum_{k=1}^{n_t} c_{ijk}(x_2^i y_2^j - x_1^i y_1^j)\left[T_k(t_2)-T_k(t_1)\right]$$

$$+ \underline{r}(x_1,y_1,x_2,y_2,t_1,t_2) \qquad i+j\neq 0$$

$$= \underline{B}(x_1,y_1,x_2,y_2,t_1,t_2)c+r \qquad (16)$$

where $\underline{d}$ denotes the m-vector of the differences of leveled height differences, Δh, and $\underline{r}$ is the residual vector. If

$$m > n = (n_x \cdot n_y + n_x + n_y)n_t \qquad (17)$$

we can find the solution, $\underline{c}$, through the method of least squares. The normal equations are solved, in the computer program, through orthogonalization.

The shift coefficients, $\underline{c}_o$, cannot be determined from the releveled segments alone. Movement $u^*(x,y,t)$ of at least one, but generally n_g tide gages (x,y) must be determined from sea level records at n_d dates to allow for evaluating the shift coefficients. The following $n_g \cdot n_d$ observation equations can be then formulated:

$$u^*(x_i,y_i,t_j) = \underline{T}(t_j)\underline{c}_o + \underline{X}(x_i,y_i,t_j)\underline{c} + \underline{r}^*,$$

$$i=1,n_g;j=1,n_d \qquad (13)$$

If $n_g \cdot n_d > n_t$, then the equations (18) may be solved for $\underline{c}_o$, again using the method of least squares.

For each tide gage the uplift u^* must be determined so as to satisfy the following condition:

$$u^*(x,y,t_o) = 0. \qquad (19)$$

As with Method 3, the advantage is primarily the reduced number of unknowns. However, the same disadvantages remain: (1) observations may be correlated and (2) information contained in circuit misclosures is not utilized.

The advantage of being able to estimate episodic and nonlinear movements is very attractive, but these same features can easily be incorporated in Methods 5 and 6 without concern for correlation of observations. Equation (8), for expressing height of point A at time t_i, can be modified to include terms corresponding to the elevation change associated with earthquakes that occurred between the times when levelings were accomplished within the study area:

$$h_{a,i} = h_{a,o} + V(x_a,y_a)(t_i-t_o) + \sum_{j=1}^{ne} u_j(x_a,y_a,x_j,y_j,t_j,d_j) \qquad (20)$$

where ne is the number of events or episodes, t_j the time of an earthquake, d_j the depth of the earthquake in units identical to x and y, and (x_j,y_j) the location of an event. A logical choice for the function u, suggested by R. Snay, is one where episodic elevation change decays with distance from the event:

$$u(x,y,x_j,y_j,t_j,d_j) = 0, \qquad \text{if } t_i < t_j$$

$$= \sum_{j=1}^{ne} a_j \left[(x-x_j)^2+(y-y_j)^2+d_j^2\right]^{-\frac{1}{2}}, \qquad \text{if } t_i > t_j. \quad (21)$$

The coefficients, a_j, are to be solved for in the adjustment. A nice advantage is that only one unknown is introduced for each earthquake or episode. If Figure 5c, three events are illustrated, and the contribution of those events can be evaluated at any time or location following the adjustment. If episodic motions are not modeled in the adjustment, observations which are suspected of having been affected by earthquakes must be removed prior to adjustment. Removal of observations should be the last alternative, and is difficult to justify except when the leveling is suspect or when insufficient relevelings exist to permit solving for episodic motion.

Another form of flexibility involves modeling of vertical block motions characterized by discontinuities of movement at fault lines, Methods 5 and 6 can be modified to accomplish this, As in Figure 5d, we can divide a study area into three blocks, P, Q, R, and express, for example, the height of a point A, at time, t_i, on block P, as follows:

$$h_{a,i} = h_{a,o} + V_P(x_a,y_a)(t_i-t_o) \qquad (22)$$

where V_P describes the velocity surface of block P. The height difference between points A and B, where B is on block Q, is given by

$$\Delta h_{b-a,i} = h_{b,o} - h_{a,o} + [V_Q(x_b,y_b) - V_P(x_a,y_a)]\,(t_i-t_o) \qquad (23)$$

Essentially, every point is located on one of the blocks, and each block has its own velocity surface. Equation (22) can be supplemented with terms which permit episodic or nonlinear vertical motions within selected blocks.

By permitting the model to solve for block motion, episodic motions, and accelerations we greatly increase its flexibility. But there is the concern that almost any kind of blunder or systematic error may be modeled as crustal movement. With an inflexible model the opposite is true, i.e., unusual movements will be forced to occur at constant rates and be partially absorbed by large corrections to the observations. The ideal adjustment model, then, is one that is very flexible and provides the geodesist with the possibility of describing vertical movement of any type; and the best strategy for the adjustment is to use only as much of that flexibility as is prudent after considering the seismicity, geology, and engineering activity in the area.

Solvability

Networks of relevelings can become complicated in the sense that casual observation of the net may not reveal which unknowns are solvable. An algorithm has been developed by Allen Pope of the National Geodetic Survey, that uses the observation equations to resolve such questions.

If we use Method 4, the solvability algorithm will resolve exactly which heights and point velocities are solvable. When Methods 5, 6, and 7 are used, the solvability algorithm will show that all coefficients of the velocity surface are unsolvable if one attempts to solve for too many. It has been very helpful to use the solvability algorithm as an analysis tool by first formulating the leveling adjustment using the observation equations according to Method 4, i.e., pretending to solve for the reference-time heights and velocities at all junctions; and secondly formulating the problem the way it would actually be adjusted, using Method 6. When solvability fails using Method 6, the user can identify the cause by reviewing the output of solvability as applied to Method 4. It tells which points have solvable velocities. The number of coefficients that may be used to describe the velocity surface is equal to the number of solvable point velocities, plus the number of paired junctions which do not have solvable velocities but have relative velocity information in between. These latter pieces of floating tilt information must be counted by looking at the network diagram.

Systematic Errors

Certain leveling errors are time-dependent. Therefore, an attempt should be made to eliminate them prior to adjustment. Without elimination, their influence is modeled as vertical movement. Where short time intervals or slow velocities are involved, the error due to uncorrected systematic leveling errors may be larger than the real movements.

One of the errors most damaging to leveling is caused by refraction. During normal daylight working hours the line of sight is bent upwards. The uphill sight will bend more than the downhill sight because the density of air changes most rapidly near the ground, the hotter air being nearest the ground. The amount of bending is proportional to the square of the sight length, the leveled height difference, and the vertical temperature change, Δt, between heights of 50 and 250 cm. Δt is dependent on time of day, season local turbidity of the atmosphere, cloud cover, the direction of leveling, and the slope of the leveling path.

Few countries have applied the refraction correction that was developed by T. J. Kukkamaki in 1937. The measurement of Δt, for input to the formula, has been considered an awkward task for a leveling team. The correction itself has long been considered too small, probably because most of the documented experience with measuring Δt comes from northern countries where the sun exerts a lesser influence because of its lower declination. In the lower latitudes of the United States large temperature gradients have been observed 5 to 10 times as great as the average values observed in England. Further, experiments and microclimate theory support the idea that refraction error is less on the north face of a mountain than on the south face. This produces a north-south accumulation of error when leveling on undulating terrain [Holdahl 1978]. It was thought that only the leveling in countries with large mountains suffered significantly from refraction error. This is incorrect because terrain that merely undulates in the north and south direction can yield a large accumulation of error if leveling extends for several hundred kilometers.

Refraction error affects computations of crustal movements in several ways.

(1) Two levelings accomplished in distinctly different seasons (i.e., seasons with different declinations of the sun) will usually yield an apparent relative elevation change.
(2) Single levelings, that are not corrected for refraction, but are permitted as observations in some adjustment models, will contribute to circuit misclosures in a way that cannot be distinguished from a contribution to the same circuit misclosure caused by a real crustal movement.
(3) A releveling accomplished with a maximum sight length specification, which significantly differs from the specification used in the original survey, will normally yield an apparent crustal movement if all other conditions are equal.

Because it has rarely been measured, it is necessary to estimate Δt if a refraction correction is to be applied to old leveling data. A method based on historic measurements of solar radiation is being developed to accomplish this [Holdahl 1978]. This method is untested at this time. Until some method is shown to be corrective, it is doubtful that high reliability can be associated with conclusions derived from large networks of releveling in areas of undulating terrain and high levels of solar radiation.

Another leveling error, which might be termed systematic, results from neglect of gravity anomalies. Some of the above-described methods for calculating crustal motions are not vulnerable to this error (Methods 1,2,3, and 7). Methods 4,5, and 6 would, however, be adversely influenced by gravity anomalies if all the following conditions exist:

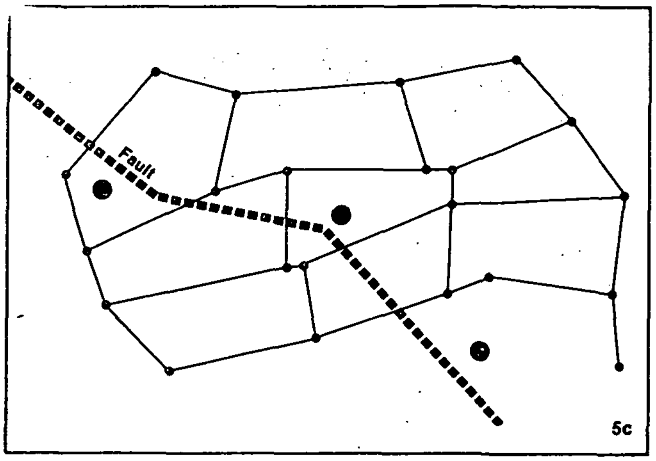
(1) the height differences were not provided in geopotential units;
(2) the gravity anomaly is large and height rapidly changes over the study area;
(3) single levelings, i.e., segments or links which have not been releveled, are used in the network adjustment.

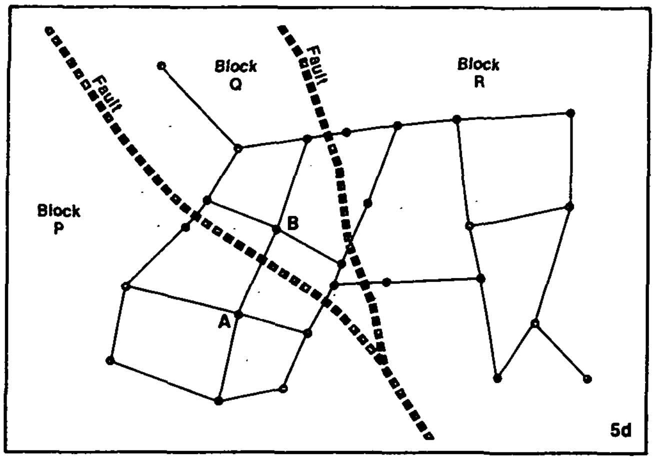
Misclosures of leveled circuits are theoretically equal to zero only when the height differences are computed in geopotential units, thus taking into consideration the variation of true gravity along the level lines. For small study areas, where variation in the gravity anomaly is small or relevelings exist over the entire net, the use of geopotential units is unimportant if only the velocities of elevation change are desired. The heights, rather than the velocities, are most sensitive to gravity anomalies. The gravity effect tends to cancel in the velocity determination when releveling is complete. Networks with a significant percentage of unreleveled segments should usually be adjusted using geopotential units because the cancellation may be incomplete or nonexistent.

Smaller types of known systematic errors exist and their influence is dealt with by corrections to the observed height differences. These include the

(1) Astronomic correction: accounts for deviation of the vertical due to positions of the Sun and Moon
(2) Rod correction: accounts for minor scale error due to deviation of calibrated rod length from nominal rod length,
(3) Rod temperature correction: accounts for contraction and expansion of the rod length due to heat,
(4) Collimation correction: accounts for nonparallelism of the telescope and an equipotential surface passing through its center,
(5) Orthometric correction: corrects for convergence of normal equipotential surfaces. Satisfactory only when gravity anomalies are near zero.

Unfortunately, there seem to be some systematic errors which are unknown or poorly understood. This is evident from sea slope determinations that have been accomplished at different times in the last several decades on the California coast. These determinations show a spread that is too wide to be attributed to random leveling errors. The most recent sea slope determination agrees well with estimates obtained by oceanographic techniques but the disagreement of present and former determinations is still a mystery. The geodesist should use height-velocity adjustment models to filter data and calculate motions, always being alert for measurements that may be contaminated by systematic error accumulation. At the same time the user must be aware that any model has "built-in" assumptions and constraints which restrict the ways in which motion may be resolved, while nature produces an endless variety of ways to exhibit motion.





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Detection of Accelerated Crustal Movements Based on Terrestrial Techniques

Robert O. Castle
U.S. Geological Survey
Menlo Park, California 94025

Summary. Reported accelerations in the vertical displacement field disclosed through repeated levelings are unassailable where the magnitudes of the measured displacements that define these accelerations overwhelm any conceivable survey error and the nature of the measured movement is fully consistent with its predicted occurrence. Specific examples include those accelerations associated with seismic slip events and massive withdrawals of underground fluids. Aseismic accelerations based on progressively smaller vertical displacements are increasingly equivocally defined; nonetheless, the reality of sharply defined aseismic accelerations associated with modest vertical displacements is independently confirmed by both continuous sea-level measurements [Wyss, 1977] and lake-level records [Wilson and Wood, 1978]. Although replication of observed elevations during the periods that both preceded and followed aseismic uplift in southern California [Savage and Prescott, 1979] is excellent evidence of the existence of major vertical accelerations, unambiguous data sets of this sort rarely occur in the geodetic record. Moreover, because the detection of vertical accelerations commonly depends on the results of a single leveling, validation of these accelerations may require detailed assessment of the accuracy of the critical survey. Where the interval between levelings is significantly greater than the inferred period of the acceleration, characterizations of accelerations based solely on the results of repeated level surveys becomes significantly less meaningful. However, four-dimensional modeling techniques that depend only on the existence of continuous and/or discontinuous releveled segments [Vaníček et al., 1979], rather than continuous line or network relevelings, may permit the detection and relatively unambiguous representation of otherwise unrecognizable crustal accelerations.

Continuing efforts directed toward the recognition of accelerations in the vertical displacement field should be based ideally on the results of repeated level surveys coupled with those additional measurements, such as continuous sea-level records, repeated gravity surveys and so forth, that provide temporal constraints on any measured vertical movements. In addition, repeated levelings designed to detect vertical accelerations should be tailored to the need. If, for example, the purpose is the best possible characterization of the vertical movement history athwart an active fault, the most useful program probably will continue to consist of frequently repeated surveys along the same line referred to a control point well removed from the deformational field associated with continuing fault movement. On the other hand, where a generalized representation of regionally developed accelerations is desired, four-dimensional surface fitting that utilizes segmented relevelings randomly distributed in both space and time probably is the most cost-effective approach for meeting this goal. While surface-fitting techniques tend to subdue short wavelength features, they are especially well suited to the depiction of those accelerations accompanying artificially induced subsidence and broadly defined tectonic deformation such as that recognized in southern California.

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