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# A Drag-Free Lo-Lo Satellite System for Improved Gravity Field Measurements

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Abstract. Since the force of gravity is inversely proportional to the square of the distance between two masses, it is obvious that the effect of small gravitational anomalies on the motion of an orbiting satellite will increase as the satellite altitude decreases. However, at very low altitudes, the effect of atmospheric drag results in drastically reduced orbit lifetimes and considerable uncertainty in satellite motions. The concept suggested herein employs a DISturbance COmpensation System (DISCOS) on each of a pair of satellites at very low altitudes to provide refined measurements of the earth's gravitational field. The DISCOS maintains the satellites in orbit and essentially eliminates motion uncertainties due mostly to drag and to a lesser extent from solar radiation pressure. By a closed-loop measurement of the relative range-rate between the two low satellites, one can determine the earth's gravitational field with a considerably greater accuracy than could be obtained by tracking a single satellite.

## Introduction

Since the advent of artificial satellites of the earth, the science of geodesy has advanced with remarkable rapidity. Without employing orbiting satellites, our knowledge of the earth's gravity field could not be determined to nearly the level to which it is now known. This improved knowledge of geodesy is accomplished essentially by studying the long period motions of orbiting satellites, particularly satellites at quite moderate altitudes. The lower the altitude of the satellite, the more pronounced the gravitational effect of the earth's gravity field, particularly the higher order harmonics. Although it is obviously desirable to go to still lower altitudes, the uncertainty of the along track force caused by air drag results in a severe degradation of the accuracy to which the satellite motion can be measured.

To determine the earth's gravity field, with greater precision it is necessary to make increasingly more accurate measurements of the satellite's motion. A basic limitation in the use of a single satellite to determine the earth's gravity field is that it is very difficult to measure the minute difference in velocity of the low orbiting satellite due to small gravitational effects in the presence of the overwhelming 7 kms/sec of satellite orbit velocity.

Suggested herein is the concept of providing a drag-free capability to each one of a pair of satellites at very low altitudes. The principal advantages of such a system are that 1) A drag free system allows the satellite to go to extremely low altitudes where they are most sensitive to gravitational anomalies and meaningful velocity measurements can be accomplished because the satellite is free of the effects of atmospheric drag; 2) Precise measurements of the relative velocities of two satellites in identical orbits is vastly simpler than the measurement of the absolute velocity of a single satellite. This is because it is less difficult to directly measure a small quantity rather than to make a measurement of a small quantity in the presence of a very large quantity.

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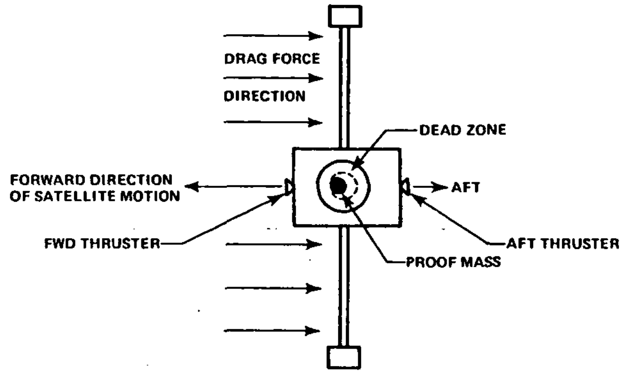


**Fig. 1 Illustration of DISCOS as used on the Triad Satellite**

## DISCOS Concept and Performance

A DISturbance COmpensation System (DISCOS) has been shown to perform satisfactorily in orbit on the TRIAD satellite. An illustration of a single axis of the three-axis DISCOS system that was used on TRIAD is shown in Figure 1. The TRIAD satellite was extended in the vertical direction to achieve gravity-gradient stabilization. As such, it presented a considerable cross-sectional area on which the atmosphere acted to create drag. At the center of the satellite was a dense spherical mass called the proof mass which was contained within a spherical cavity that shielded the proof mass from the effect of external disturbance forces such as solar radiation pressure and air drag. Contained within the spherical cavity at the center of the satellite was an imaginary spherical cavity known as the dead zone. When the effect of drag was sufficient to push the satellite backward until the dead zone touched the surface of the proof mass, the aft thruster fired thereby causing the satellite to move forward relative to the proof mass. By this means, the proof mass was allowed to fly in an orbit determined purely by gravitational forces, and the satellite, because of the DISCOS system, was constrained to follow the motion of the proof mass. In Figure 2 is shown a particular 12 minute period of data collected from the TRIAD satellite. In this figure is shown the along-track position on the proof mass expressed in millimeters as a function of time from the start of a pass of the satellite. During the time of this pass, the principal disturbance force on the satellite was drag, so that at approximately one minute of time, the integrated drag force was sufficient to cause the proof mass to touch the imaginary dead zone. This is a consequence of 1.0 mm of motion of the proof mass from the geometric center of the cavity. At that point, which is shown in Figure 2 as the first of three "Aft Thruster Fires" indications, the impulse of the aft thruster causes a velocity of the satellite relative to the proof mass such that proof mass has an apparent motion back towards the geometric center of the cavity. However, due to the fact that the drag forces are continuing at this time, the proof mass never reaches the center of the cavity. Four minutes after the first of the thruster firings, the proof mass again touches the dead zone. The parabolic shape of the displacement curve of the proof mass as a function of time is an indication of an essentially constant drag force during this four minute period. From the times of 5 minutes to approximately 11 minutes, the satellite is again pushed away from the proof mass but this time the different shape of the essentially parabolic curve indicates that the drag force at that time was smaller. Finally we see at 11 minutes the third of the firings of the aft thruster. This data is typical of what occurs in a DISCOS system as the firing of the thrusters are used to keep the proof mass centered in the satellite's spherical cavity.

## Application of DISCOS to Satellites at Very Low Altitude

The TRIAD satellite containing the first three-axis DISCOS flew at an altitude of approximately 700 km and used three pounds of cold gas (Freon) propellant to achieve 18 months of drag free operation. If one is to maintain a spacecraft at a much lower altitude or if one is to maintain drag free operation for a longer period of time more total impulse from the propellant is required. This can be accomplished either by providing a greater quantity of propellant and/or by providing a much higher specific impulse as compared with cold gas. In Figure 3 is shown the concept of a satellite which could be one of a pair used in a drag free lo-lo system for obtaining more refined measurements of the earth's gravity field. The satellite shown in Figure 3 has a DISCOS system at its center mass and two very large tanks containing propellant (e.g., hydrazine) located symmetrically on each side. It is estimated that for a satellite of this size, 2000 kilograms of hydrazine could be readily provided. Figure 3 also shows the concept of simple angular momentum flywheel that could be used to stabilize the satellite in roll while aerodynamic forces could be used to stabilize the satellite in pitch and yaw. A radar

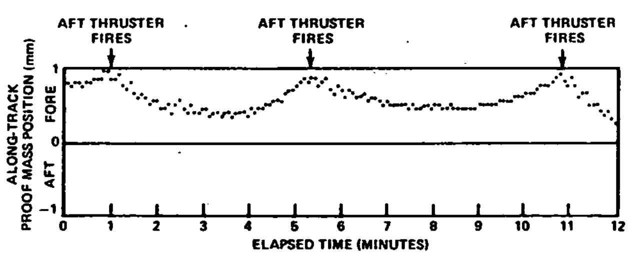


Fig. 2 DISCOS proof mass motion.

altimeter is shown which might provide improved data on ocean surface topography which of course could contribute materially to our knowledge of the earth's gravity field and ocean dynamics. Furthermore, this concept shows a vector and scalar magnetometers which are indicative of another class of measurement which is best accomplished with satellites at very low altitudes.

In Figure 4 is shown the propellant usage for such a satellite in the altitude region between 100 and 250 kms. The propellant usage in Figure 4 is expressed in kilograms/month for a satellite using hydrazine as the DISCOS system propellant and having a frontal area of 1 square meter. Two curves are shown in Figure 4; one for a solar minimum and the other for a solar maximum. If a satellite such as that shown in Figure 3 can contain 2000 kilograms of hydrazine, it will provide approximately 2 months of drag-free operation at a 125 km and approximately 20 months operation (at a solar minimum) at an altitude of 150 km.

A consideration which makes the satellite concept as shown in Figure 3 reasonable from a cost standpoint is the advent of the Shuttle as a launch vehicle. The Shuttle is particularly well suited to launch satellites that are extremely heavy but that require very low altitude for their operation. If longer times in orbit are required, the Shuttle astronauts could rendezvous with the pair of drag-free satellites and refuel these spacecraft in-orbit. Although there would be times when it is desirable to fly the satellite at altitudes as low as 125 to 150 kilometers, most of the time a considerable improvement in the knowledge of the earth's gravity field would result from flying the satellites at altitudes in the region of 250 kilometers. At such an altitude, even at the solar maximum, 2000 kilograms of hydrazine would provide approximately 200 months of in-orbit operation without refueling. This mission duration time would be satisfactory for significantly improving the accuracy to which we know of the earth's gravitational field.

Another effect that must be considered in allowing spacecraft to fly at extremely low altitudes is the aerodynamic heating that results from friction of the high speed spacecraft with the earth's atmosphere. In Figure 5 is shown the temperature at the stagnation point which would be obtained at the leading surface of a satellite that was flying at altitudes between 100 and 250 kilometers. The assumptions here are that the front surface has an infra-red emmisivity of 0.8 and there is no energy input from the sun. From these curves (which show solar maximum and solar minimum), it can be seen that at an altitude of 125 kilometers, the skin temperature at the stagnation point under the conditions described above is below 300°C. Such a temperature could readily be withstood even by aluminum. When one rises to an altitude of 150 kilometers, Figure 5 shows that the temperature of the front surface is not very much different from room temperature and therefore would not compromise the design of a spacecraft. In essence, Figure 5 shows us that there is no significant temperature barrier to operating a single satellite or a pair of satellites at extremely low altitudes; i.e., above 125 km.

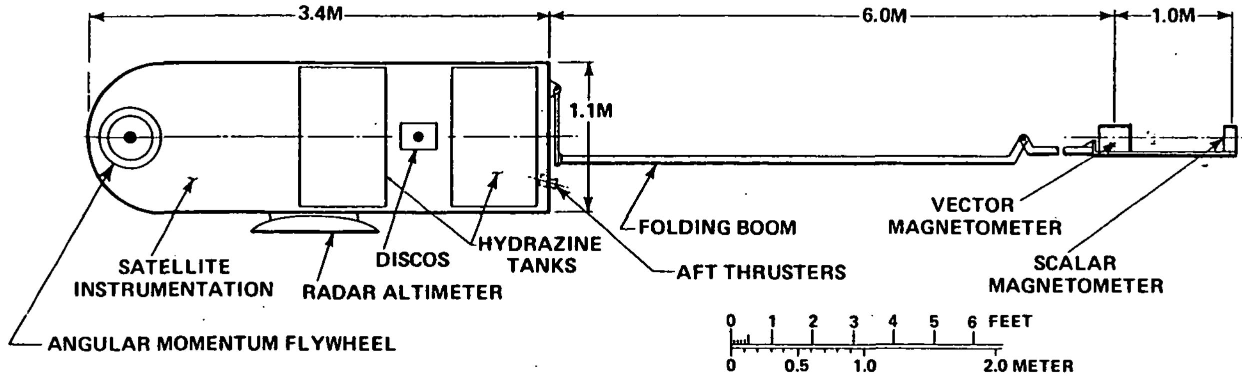


Fig. 3 Geomapsat conceptual design.

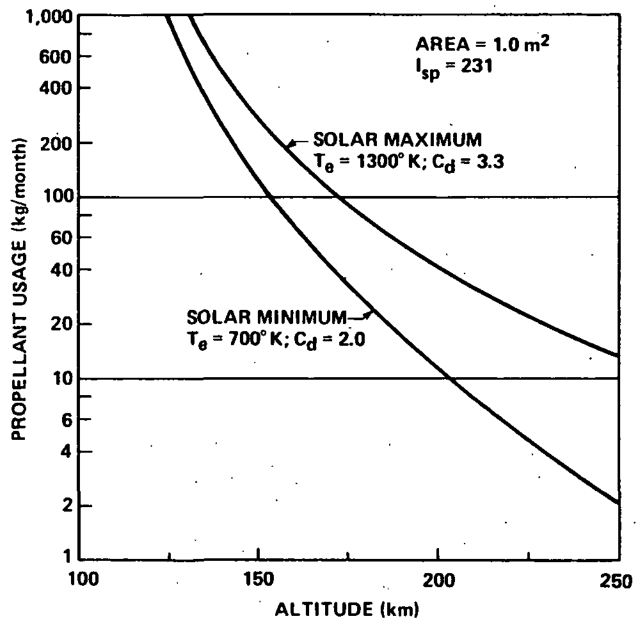


Fig. 4 Propellant usage for satellites at very low altitude.

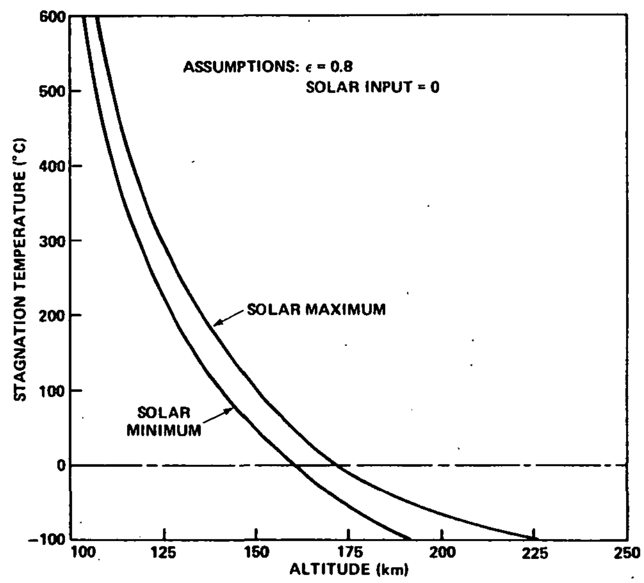


Fig. 5 Effect of altitude on stagnation temperature.

Analysis of Lo-Lo System of Satellites

In this section, an approximate expression is developed for the relationship between the gravitational environment and the relative range-rate between the satellites of a lo-lo system. Through judicious approximations an analytic formulation can be obtained. This is useful to enhance conceptual understanding and to provide a basis for evaluating and interpreting the more detailed numerical simulation which follow.

Assume that the two satellites in Figure 6 are in identical circular orbits but separated in phase by the angle $\alpha$ and have mean anomalies $M_i$ where i = 1 or 2 which identifies each one of the two spacecraft. Let T be the kinetic energy per unit mass and U the gravitational energy per unit mass. If the rotation of the gravitational field is neglected it is possible to introduce the principle of conservation of energy

$$T + U = K \tag{1}$$

where K is a constant. Let $L_i$, $H_i$, $C_i$ be the spatial perturbations of the spacecraft from the cir-

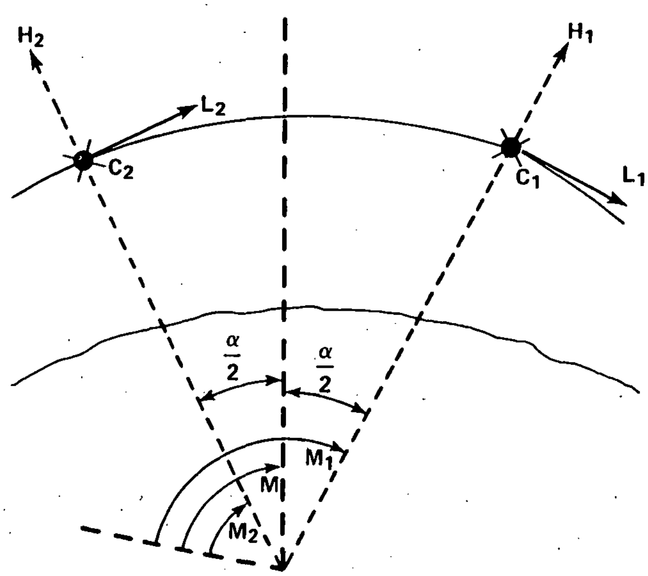


Fig. 6 Geometry of lo-lo system of satellites and definition of $L_i$, $C_i$, $H_i$ coordinate systems, i = 1 to 2.

cular reference orbit consistent with the reference systems in the figure. Then

$$T = \tfrac{1}{2}\left[(v_o + \dot{L}_i)^2 + \dot{C}_i^{\,2} + \dot{H}_i^{\,2}\right] \tag{2}$$

where $v_o$ is the nominal circular velocity. Let

$$U = V - \frac{k}{a} \tag{3}$$

where V is the perturbing potential per unit mass, k is the gravitational constant and a the radius to the satellite. Substituting Eqs. (2) and (3), Eq. (1) gives

$$\tfrac{1}{2}v_o^{\,2} + v_o \dot{L}_i + V_i - \frac{k}{r} + \left[\tfrac{1}{2}\left(\dot{L}_i^{\,2} + \dot{H}_i^{\,2} + \dot{C}_i^{\,2}\right)\right] = K \tag{4}$$

The bracketed term is of second order since $\dot{L}_i$, $H_i$, $C_i << v_o$ and can be neglected.

For closed circular motion

$$K = -\frac{k}{2a} \tag{5}$$

and

$$v_o = \left(\frac{k}{a}\right)^{\frac{1}{2}} \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (4) give

$$\dot{L}_i = -v_o^{-1}\, V(M_i)\ . \tag{7}$$

The relative range-rate between the two spacecraft in the same plane is

$$\dot{\rho} = (\dot{H}_1 + \dot{H}_2)\sin\frac{\alpha}{2} + (\dot{L}_1 - \dot{L}_2)\cos\frac{\alpha}{2}\ . \tag{8}$$

For small separation distances, i.e., $\alpha$ small, it is appropriate the range-rate be

$$\dot{\rho} \approx \dot{L}_1 - \dot{L}_2 \tag{9}$$

which using Eq. (7) becomes

$$\dot{\rho} = -v_o^{-1}\,(V(M_1) - V(M_2)). \tag{10}$$

Under the assumptions stated, this equation demonstrates that the relative range-rate is a measure of the difference in the perturbing potential. This result has been previously derived by Wolff [1969] and Comfort [1973]. Now, let the perturbing potential be expressed by

$$V = \sum_{\ell=2}^{\infty} V_\ell\ . \tag{11}$$

For a near circular orbit Kaula [1966] gives

$$V_\ell = \frac{k}{R}\left(\frac{R}{a}\right)^{\ell+1} \sum_{n=o}^{p} \sum_{p=o}^{\ell} F_{\ell mp}\, S_{\ell mp}(\omega, M, \Omega, \theta) \tag{12}$$

where $F_{\ell mp}$ is soley a function of the inclination and

$$S_{\ell mp} = \begin{bmatrix} C_{\ell m} \\ -S_{\ell m} \end{bmatrix}_{(\ell-m)\text{odd}}^{(\ell-m)\text{even}} \text{Cos}\left[(\ell-2p)(M+\omega) + m(\Omega-\theta)\right]$$
$$\begin{bmatrix} S_{\ell m} \\ C_{\ell m} \end{bmatrix}_{(\ell-m)\text{odd}}^{(\ell-m)\text{even}} \text{Sin}\left[(\ell-2p)(M+\omega) + m(\Omega-\theta)\right] \tag{13}$$

where R is a scaling distance, $\omega$ the argument of perigee, $\Omega$ the right ascension of the ascending note, $\theta$ the Greenwich sidereal time and $\ell$ and m the order and degree respectively. For a polar satellite $(\Omega-\theta)$ is constant since the rotation of the earth has been neglected here. Consequently $V_\ell$ is a harmonic function with arguments $(\ell-2p)(M+\omega)$.

Partitioning the potential in components $V_\ell$ permits Eq. (10) to be rewritten as

$$\dot{\rho} = \sum_{\ell=2}^{\infty} \dot{\rho}_\ell \tag{14}$$

where

$$\dot{\rho}_\ell = -v_o^{-1}\left[V_\ell(M_1) - V_\ell(M_2)\right] \tag{15}$$

For the lo-lo configuration

$$M_1 = M + \frac{\alpha}{2}\ , \quad M_2 = M - \frac{\alpha}{2} \tag{16}$$

so that

$$S_{\ell mp}(M_1) - S_{\ell mp}(M_2) = 2\sin\frac{(\ell-2p)\alpha}{2}\,\bar{S}_{\ell mp}(M,\omega,\Omega,\theta) \tag{17}$$

where $\bar{S}_{\ell mp}$ is the derivative of $S_{\ell mp}$ with respect to its argument and $\omega = \omega_1 = \omega_2$, $\Omega = \Omega_1 = \Omega_2$. Substituting Eqs. (12) and (17) into Eq. (15) gives

$$\dot{\rho}_\ell = -v_o^{-1}\left(\frac{k}{R}\right)\left(\frac{R}{a}\right)^{\ell+1} \sum_{m=o}^{p} \sum_{p=o}^{\ell} \left[2\sin\frac{(\ell-2p)\alpha}{2}\right] F_{\ell mp}\bar{S}_{\ell mp} \tag{18}$$

The bracketed term represents a function dependent on the separation of the two spacecraft that modulates the amplitude of the range-rate. This term demonstrates the tuning capability of the lo-lo configuration. For a particular value of $\ell$ say $\ell_o$, Eq. (18) shows that frequencies $\ell_o$, $\ell_o-2$, $\ell_o-4$, $\cdots$ times $(M+\omega)$ exist where contributions to the range-rate with frequency $\ell_o(M+\omega)$ results from coefficients $C_{\ell m}$ and $S_{\ell m}$ such that

$$\ell = \ell_o + 2p \quad p = 0,1,2,\cdots \tag{19}$$

In this context, Eq. (19) shows that by selecting a separation

$$\alpha = \frac{\pi(2n+1)}{\ell_o} \qquad n = 0,1,2,\cdots \tag{20}$$

the contributions to the range-rate from coefficients $C_{\ell m}$ and $S_{\ell m}$

$$\ell = \ell_o + 2p \qquad p = 0,1,2,\cdots \tag{21}$$

will be doubled. Conversely, all coefficients such that

$$\ell = 2m(\ell_o + 2p) \qquad n = 0,1,2,\cdots \tag{22}$$

will contribute nothing to the signal. This tuning capability is illustrated in Figure 7. Another way to conceptualize tuning is to consider the separation to be such that the variations in the gravitational field are either negatively or positively correlated. Enhancement in the signal, will occur with the former and destruction with the latter.

An estimate of the amplitude of the range-rate for a specific gravity anomaly can be obtained as follows. The gravity anomaly $\Delta g$ at the spacecraft can be written in terms of the disturbing potential V as

$$\Delta g = -\left.\frac{\partial V}{\partial r}\right|_{r=a} - 2\,\frac{V}{a} \tag{23}$$

Substituting Eqs. (11) and (12) for V gives

$$\Delta g = \frac{k}{R^2} \sum_{\ell=2}^{\infty} (\ell-1)\left(\frac{R}{a}\right)^{\ell+2} \sum_{m=o}^{\ell} \sum_{p=o}^{\ell} F_{\ell mp}\, S_{\ell mp} \tag{24}$$

Let $\Delta g_\ell$ be defined by

$$\Delta g = \sum_{\ell=2}^{\infty} \Delta g_\ell \tag{25}$$

where

$$\Delta g_\ell = (\ell-1)\, \frac{k}{R^2}\left(\frac{R}{a}\right)^{\ell+2} \sum_{m=o}^{\ell} \sum_{p=o}^{\ell} F_{\ell mp}\, S_{\ell mp} \tag{26}$$

Comparing Eqs. (12) and (26), gives

$$V_\ell = \frac{a}{\ell-1}\, \Delta g_\ell \tag{27}$$

Since the interest is in recovering a gravity anomaly whose magnitude is specified at the surface, let

$$\overline{\Delta g}_\ell = \left.\Delta g_\ell\right|_{a=R} \tag{28}$$

From Eq. (26), it follows that

$$\Delta g_\ell = \left(\frac{R}{a}\right)^{\ell+2} \overline{\Delta g}_\ell \tag{29}$$

so that Eq. (27) can be rewritten as

$$V_\ell = \frac{R}{\ell-1}\left(\frac{R}{a}\right)^{\ell+1} \overline{\Delta g}_\ell \tag{30}$$

Substituting this into Eq. (15) gives

$$\dot{\rho}_\ell = -v_o^{-1}\, \frac{R}{\ell-1}\left(\frac{R}{a}\right)^{\ell+1}\left[\overline{\Delta g}_\ell(M_1+\omega) - \overline{\Delta g}_\ell(M_2+\omega)\right] \tag{31}$$

as the expression for range-rate. The maximum amplitude for $\dot{\rho}_\ell$ will occur at the spacing given by Eq. (20) where

$$\left.\dot{\rho}_\ell\right|_{max} = -v_o^{-1}\, \frac{2R}{\ell-1}\left(\frac{R}{a}\right)^{\ell+1} \overline{\Delta g}_\ell\,(M+\omega + \frac{\pi}{2}). \tag{32}$$

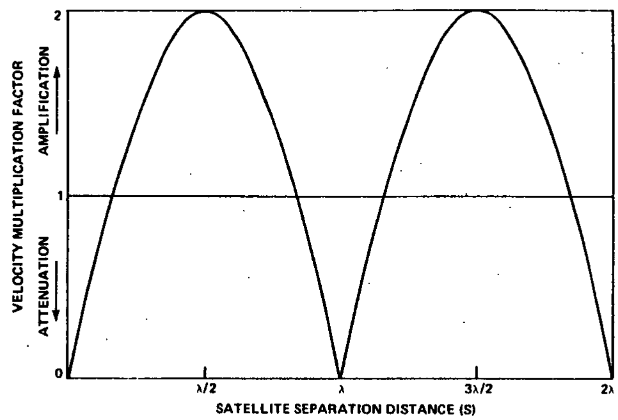


**Fig. 7 Effect of separation distance on velocity amplification.**

This equation has been used as the basis to determine the accuracy in the range-rate measurement that is necessary to recover a specified gravity anomaly. Results are presented in Figure 8 where the curves represent the amplitude and wavelength of the gravity anomalies that produce a one way range-rate of the specified instrument accuracy. At the longer wavelengths a small angle approximation for optimum separation becomes less valid. A recent study by Goldfinger [1978] have shown that for harmonics of order two even at the optimal spacing the error is less than a factor of 2.

Corruption of the signal by errors in the knowledge of the initial conditions has been discussed by Pisacane [1978]. There, a computer simulation study show that the effects of the initial condition errors can be effectively eliminated by high pass filtering.

Velocity Measurement Concept

The relative velocity measuring instrumentation concept is indicated in Figure 9. The basic method can be described by thinking of the second satellite as a pure transponder whereby the tone ($Nf_1$) transmitted by the first satellite is simply sent back with a doppler offset resulting from the relative motion between the satellites. The same doppler effect would also be experienced on the return trip resulting in a received frequency of $Nf_1(1 - \dot{\rho}/c)^2$ at the first satellite; where $\dot{\rho}$

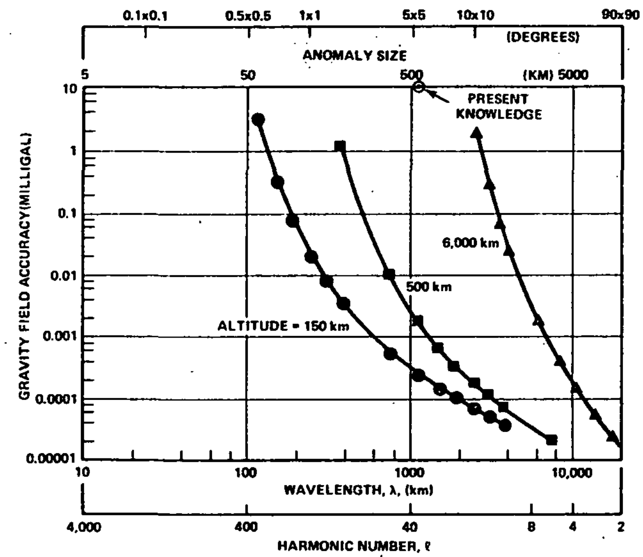


Fig. 8 Anticipated accuracy for recovery of the gravity field based on relative velocity measurement accuracy of $10^{-4}$ mm/sec.

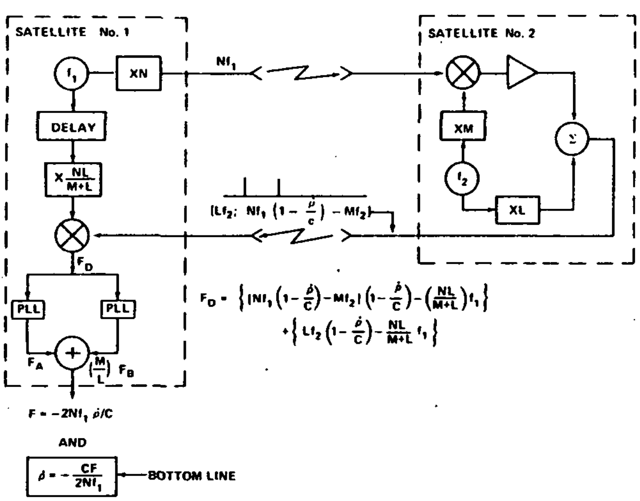


Fig. 9 System for determining relative velocity of a lo-lo satellite pair.

| | | |
|---|---|---|
| FREQUENCY | = | 100 GHz |
| ANTENNA BEAM WIDTH | = | 2° x 10° |
| ANTENNA SIZE | = | 18mm x 90mm (0.7" x 3.5") |
| NOISE FIGURE | = | 10 dB |
| BANDWIDTH | = | 1.0 Hz |
| TRANSMITTED POWER | = | 2.0 mW |
| RANGE | = | 112 km |
| SIGNAL-TO-NOISE RATIO | = | 45 dB |
| INTEGRATION TIME | = | 5.0 SECONDS |
| 1σ PHASE JITTER | = | 0.06 DEGREES |
| 1σ VELOCITY NOISE | = | $10^{-4}$ mm/sec |

Fig. 10 An example of lo-lo closed-loop link parameters.

represents an average velocity over the measurement interval. If the return signal is then differenced with the original transmission frequency, the velocity component, approximately $2Nf_1 \dot{\rho}/c$, will be separated for measurement. The actual implementation incorporates a frequency translation in the second satellite to avoid oscillation difficulties. However, the implementation provides for removal of both oscillator offset errors thereby providing the same final result.

If both oscillators were properly characterized as having constant but unknown frequencies, i.e., having only bias errors, the technique just discussed would fully remove all oscillator errors. When time varying errors are included, their effect can in concept be removed by matching the delays into the differencing circuits. The delay shown in the figure between the $f_1$ oscillator and the return mixer is intended to carry out this function. Removal of time varying effects of the $f_2$ oscillator requires that the two delays between the $f_2$ oscillator and the summing circuit in the second satellite be matched. Assuming that complete oscillator errors are removed by the implementation, the remaining error sources are due to phase variations in uncommon circuits, propagation effects and thermal noise limitations in the link.

Assuming all other effects can be made negligible, the signal-to-noise performance shown in Figure 10, can be realized with quite reasonable antenna dimensions and transmission powers. While the system is not fundamentally limited to this precision, a high degree of circuit phase stability will be required to achieve the indicated performance. Prior to further evaluation of the instrumentation limitations, the $10^{-4}$ mm/sec noise value is being considered as the practical measurement limit.

## Conclusions

The advent of the shuttle launch system makes it possible to launch a pair of satellite containing several thousand kilograms of propellant into a low altitude orbit.

These large quantities of propellant can be used in a DISCOS system to maintain a satellite in orbit and to free the satellite from the otherwise overwhelming disturbance effects of atmospheric drag.

A pair of such satellites at very low altitudes can greatly refine our knowledge of the earth's gravitational field.

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