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Comparison of Surface and Satellite Gravity Data

Richard H. Rapp
Department of Geodetic Science, The Ohio State University
Columbus, Ohio 43210

Abstract. Satellite derived potential coefficients (GEM 9) are compared to terrestrial gravity data by degree in terms of coefficient differences and in terms of mean anomaly differences. We found the root mean square undulation difference (to degree 20) was ±9 m and the anomaly difference was ±7 mgals with GEM 9 commission errors of ±1.7 m and ±3.8 mgals. The standard deviations of the GEM 9 implied undulations increased from ±4 cm at degree 2 to ±53 cm at degree 20. The corresponding values implied by a recent (June 1978) terrestrial 5° field were ±2.53 m and ±0.38 m (at degree 20).

Comparisons of 5° equal area and 1° x 1° blocks showed discrepancies of ±11 and ±25 mgals respectively when using the GEM 9 coefficients to degree 20. Comparisons between Geos-3 altimeter derived anomalies and 1° x 1° terrestrial data showed that ±6-8 mgals is a reasonable accuracy estimate for the altimeter derived anomalies. Limited comparisons have also been made with anomalies derived from satellite to satellite tracking data indicating an accuracy of about ±6 mgals for the recovery of 5° equal area blocks.

Introduction

The determination of the gravity field of the earth has been one of the classic goals of geodesy. The uses of the gravity field in geodesy originally related to geoid undulation and deflection of the vertical computation. Later applications arose in trajectory and orbit computations. Now we see needs for the global gravity field for better understanding the process in the earth's interior.

Initially gravity measurements were made with pendulums and then gravimeters which made accurate relative measurements. Even with rapid progress in equipment and techniques there are gaps in the earth's terrestrial gravity coverage.

The use of satellites to determine potential coefficients improved the situation with regard to the long wavelength behavior of the gravity field. Specifically, gravity anomalies can be derived from these potential coefficients. In early computations, a comparison of anomalies derived from potential coefficients with terrestrial mean anomalies was made to evaluate various potential coefficient sets derived from the analysis of satellite orbits (Kaula, 1966). Such procedures not only gave some indications of which potential coefficient solutions might be more reliable, but they also gave some confidence that there was some agreement between the satellite derived anomalies and the terrestrial anomalies.

These anomaly comparisons have continued for a number of different purposes (Lambeck, 1971, Rapp, 1972, 1975). This paper is an attempt to look at the current situation in several different ways.

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The Terrestrial Data

For our comparisons we will be using a recently (June, 1978) updated set of 1° x 1° mean gravity anomalies. This updating started from a set of 35011 anomalies supplied by the Defense Mapping Agency Aerospace Center in St. Louis. We updated this set of anomalies by adding, replacing or deleting 11933 values. These values were obtained from various sources such as recently published maps or data sent by various organizations for our use. The final data set contained 39405 1° x 1° anomalies some of which had been estimated thru geophysical correlation techniques. The location of these anomalies is shown in Figure 1.

In the update that we performed we would often find anomaly estimates from two sources that were widely different. As an example I show in Table 1 1° x 1° anomaly estimates for three blocks from different sources.

Clearly the differences are not small. Nor are such discrepancies unusual. However the number (on the order of 100-200) of such discrepancies are small on a percentage basis. Thus there are a number of areas where we have a poorly defined 1° x 1° mean anomalies.

In summary we will work in our comparisons with 39405 1° x 1° mean anomalies where the root mean square standard deviation is ±16 mgals but where some standard deviations may be as large as ±81 mgals.

Table 1. Location of Larger Discrepancies Between Terrestrial Data Sources

$\varphi°$	$\lambda°$	Source A	Source B	Difference
27	85	-15±10 mgals	-147±9 mgals	-132
62	216	-44±15 mgals	106±19 mgals	150
-5	134	-55±11 mgals	72±22 mgals	127

Anomaly Computations from Potential Coefficients

The usual procedure to compute gravity anomalies from fully normalized potential coefficients $(\bar{C}_{\ell m}, \bar{S}_{\ell m})$ is (Rapp, 1977a)

$$\Delta g = \frac{kM}{r^2}\sum_{\ell=2}^{\infty}(\ell-1)\left(\frac{R_B}{r}\right)^{\ell}\sum_{m=0}^{\ell}(\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda)\,\bar{P}_{\ell m}(\sin\bar{\varphi}) \quad (1)$$

where R_B is the radius of the Bjerhammar sphere which is somewhat imbedded within the earth and r is the geocentric distance to the point in question. In

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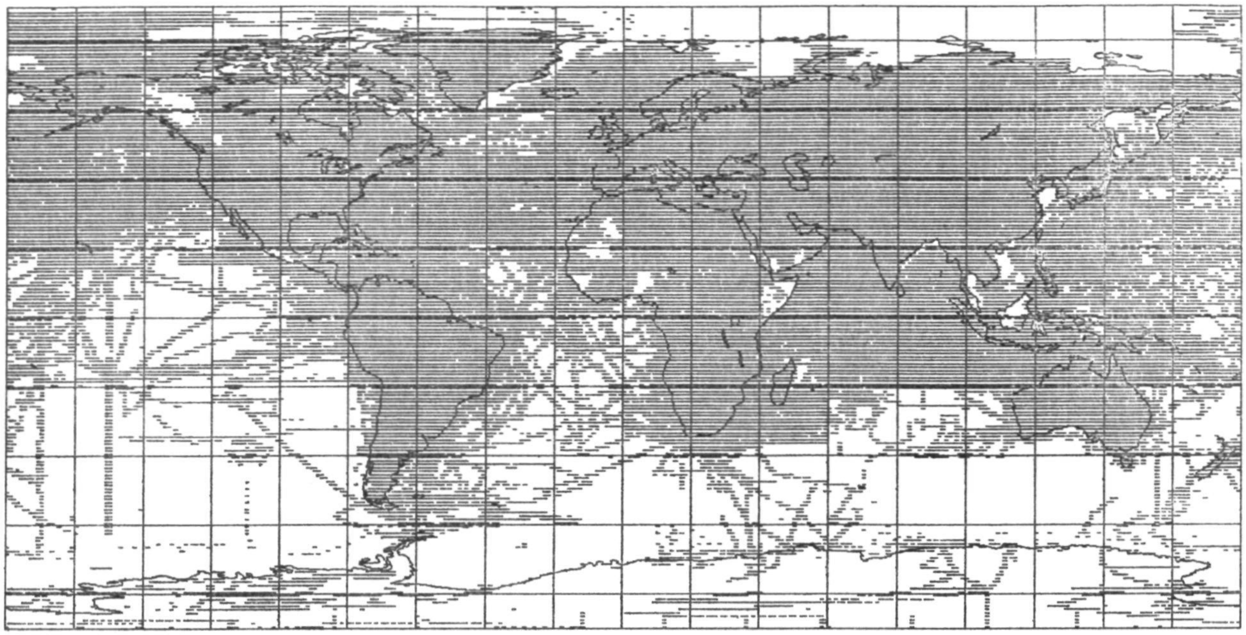

Figure 1. Location of the 39405 $1^\circ \times 1^\circ$ Anomalies of the June 1978 Data Tape.

practice R_e is usually taken to be that equatorial radius (or scale factor) used in the determination of the potential coefficients. Equation (1) will yield the anomalies with respect to an ellipsoid of defined flattening implying a set of reference $\overline{C}_{\ell,0}$ coefficients (ℓ even) which is subtracted from the original $\overline{C}_{\ell,0}$ values. Such anomalies are given with respect to an ellipsoid with no atmospheric mass. Most terrestrial anomalies have been computed with respect to a gravity formula in which the mass of the atmosphere is included. For consistency purposes 0.87 mgals should be subtracted from the anomaly obtained from (1).

Another problem continues to appear that is related to the convergence of the equation (1) at the surface of the earth. Various solutions to this problem have been discussed. Recently Moritz (1978) has argued that although the series diverges at the surface of the earth, a practical convergence can be expected when using a finite set of coefficients. Arnold (1978) recently claims to have proven that the spherical harmonic expansion does converge on the surface. Sjöberg (1977) has given some simple examples demonstrating divergence. To avoid the question I suggested (Rapp, 1977b) that anomalies could be evaluated using (1) on a sphere enclosing all the masses of the earth. Then these anomalies could be downward continued (by collocation, for example) to the terrestrial surface. Numerical tests indicated better agreement (±1 mgal for 5° anomalies) with the terrestrial data when this approach was used than when a direct evaluation on the surface was used. Clearly more study is needed in this area. For this paper all anomaly evaluations have been carried out at the surface of the earth ignoring the convergence problem.

Potential Coefficients from Terrestrial Anomalies

We should note here that potential coefficients can also be determined from a global estimate of the terrestrial gravity field (Rapp, 1977a). Using the $1^\circ \times 1^\circ$ data set previously discussed we computed a set of 1654 5° equal area anomalies using procedures described in Rapp (1978). These anomalies were used to generate potential coefficients to degree 20 using equation (6) of Rapp (1977a). These coefficients will be compared to satellite derived coefficients in a later section.

Comparison Quantities

We can compare the satellite and terrestrial data in several ways. The most obvious is the computation of anomalies from potential coefficients using (1) and the comparison with terrestrial data in various size blocks. The comparison can be made by computing the root mean square difference between the satellite and terrestrial data. This difference will be caused by three factors: 1) errors in the terrestrial data; 2) errors in the potential coefficients; and 3) errors caused by the neglect of higher degree terms in the spherical harmonic expansion. Kaula (1966) has described methods to separate these terms.

We can also compare the potential coefficients from the satellite and terrestrial results. This comparison is instructive to consider, by degree, the differences in terms of anomalies and undulations. The mean square undulation difference would be given by:

$$\delta N_\ell^2 = R^2 \sum_{m=0}^{\ell} (\Delta\overline{C}_{\ell m}^{\,2} + \Delta\overline{S}_{\ell m}^{\,2}) \qquad (2)$$

where R is a mean earth radius. The anomaly dif-

Table 2. Comparison of Terrestrial Anomalies to Anomalies Implied by GEM 9

	Block Size	
ℓ (max)	5°	1°
12	±10 mgals	-
20	±11 mgals	±25 mgals

ference by degree would be given by:

$$\delta g_\ell^2 = \gamma^2 (\ell - 1)^2 \sum_{m=0}^{\ell} (\Delta \bar{C}_{\ell m}^2 + \Delta \bar{S}_{\ell m}^2) \quad (3)$$

where γ is a mean value of gravity over the earth. The total difference between the two sets of coefficients could be expressed as:

$$\delta N^2 = \sum_{\ell=2}^{\ell_{max}} \delta N_\ell^2 \quad (4)$$

$$\delta g^2 = \sum_{\ell=2}^{\ell_{max}} \delta g_\ell^2 \quad (5)$$

Similar equations can be written for the accuracy of the various quanties given the standard deviations of the potential coefficients.

Results Using the GEM 9 Potential Coefficients

To implement the comparisons described in the previous section we will use the GEM 9 (Lerch, et als., 1977) potential coefficients. This coefficient set is complete to degree 20 with some higher order terms. It is based solely on satellite data.

Table 2 shows the root mean square difference between the anomalies computed from the potential coefficients and the terrestrial data. The 5° comparisons were made using 1062 blocks whose standard deviations were less than ±6 mgals. The 1° x 1° comparisons were made using 16579 blocks whose standard deviations were less than ±16 mgals.

In Figures 2 (for geoid undulations) and 3 (for anomalies) information is given, by degree, for the following quantities:

1. Root mean square value implied by the GEM 9 coefficients;
2. Root mean square difference between the GEM 9 coefficients and the coefficients implied by the 5° block terrestrial data;
3. The standard deviations computed from the accuracies of the terrestrial coefficients and the GEM 9 coefficients.

From Figure 1 we see that the GEM 9 undulation has a standard deviation of ±46 cm at degree 12 while the terrestrial standard deviation is ±50 cm and the differences at that degree is ±80 cm. At the higher degrees the differences and the standard deviations approach the magnitude of the undulation at that degree . Similar comments can be made for the anomaly information in Figure 3.

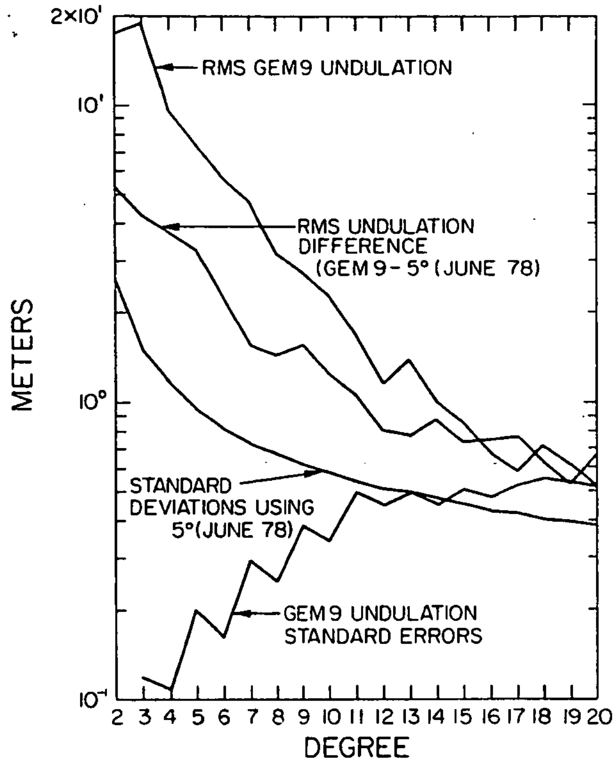

Figure 2. GEM 9 and 5° Terrestrial Implied Potential Coefficient Comparisons in Terms of Geoid Undulations.

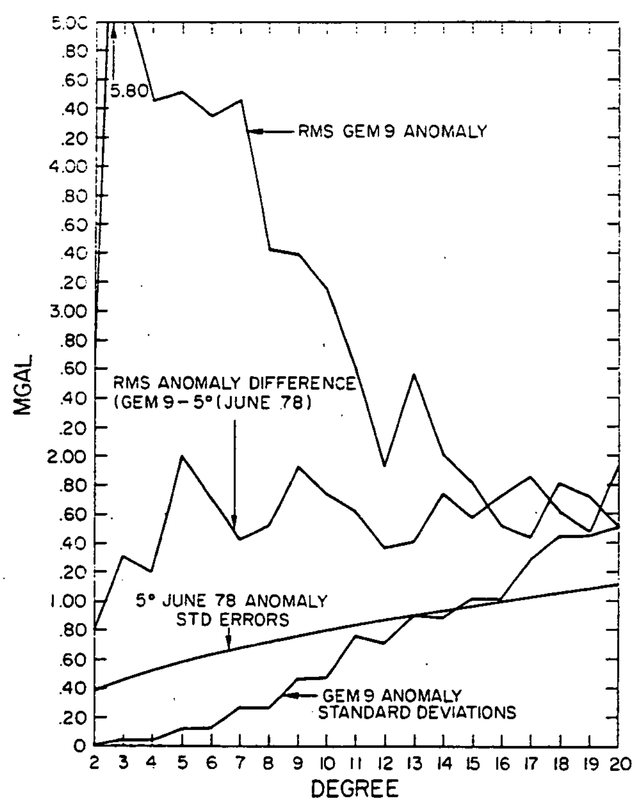

Figure 3. GEM 9 and 5° Terrestrial Implied Potential Coefficients in Terms of Gravity Anomalies.

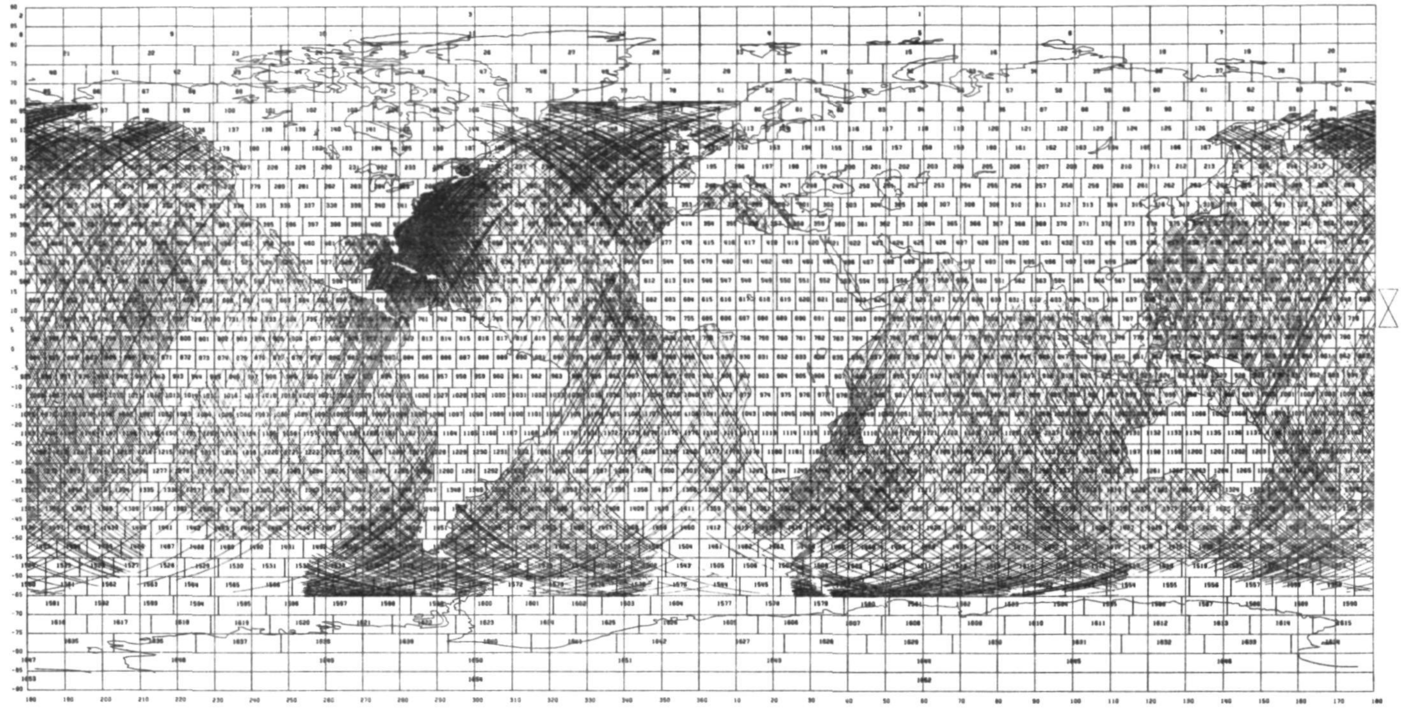

Figure 4. Location of Edited Geos-3 Altimeter Data and 5° Equal Area Blocks.

Table 3 shows comparisons between the two potential coefficient sets when all coefficients between degree 2 and 20 are considered.

Geos-3 Altimeter Results and Comparisons

The Geos-3 altimeter data has greatly improved our knowledge of gravity (and undulations) at sea in 5° and 1° x 1° mean anomalies. The location of edited Geos-3 data available at The Ohio State University is shown in Figure 4 along with the location of the 5° equal area blocks. Anomalies and undulations have been computed from this data using the procedures described in Rapp (1977d, 1979). From this data we have now computed 29478 1° x 1° anomalies and undulations. Of these there are 27465 1° x 1° anomalies that have standard deviations ≤ ±15 mgals. The location of these anomalies is shown in Figure 5.

The 1° x 1° anomalies derived from the Geos-3 satellite altimeter data have been compared to the terrestrial data in two data sets. The first set lies off the East Coast of the United States in an area where the altimeter data is dense and the terrestrial data is of above average reliablity. The second set comprises the whole altimeter derived anomalies compared to the available terrestrial data subject to the following accuracy limitation (which also applies to the first data set): Comparisons between the two anomalies are only made if the terrestrial anomaly standard deviation is ≤ 25 mgals and the altimeter derived anomaly standard deviation is ≤ 15 mgals. The results are given in Table 4. By including the comparison with the GEM 9 anomalies (using the potential coefficients to $\ell = 20$) we can see the improvement the altimeter results have given over the GEM 9 anomaly field. The accuracy estimates (of about ±8 mgals) for the altimeter derived anomalies apears consistent with the RMS anomaly differences.

In some cases we have found very large discrepancies between the altimeter derived anomalies and the terrestrial data. Specifically we found 17 differences greater than 100 mgals and 203 differences greater than 50 mgals. I give in Table 5, 5 blocks where the differences are large. A number of these cases occur in areas where the anomaly field is changing quite rapidly and only one ship track is available thru a block.

Table 3. Root Mean Square Difference and Commission Errors of the GEM 9 and 5° Terrestrial Implied Potential Coefficients

	Undulation	Anomaly
RMS Difference	±9.1 m	±7.0 mgals
GEM 9 Commission Errors	±1.7 m	±3.8 mgals
5° Terrestrial Comm. Errors	±3.9 m	±3.5 mgals

Table 4. Comparison of Altimeter Derived Anomalies and Terrestrial Anomalies in 1° x 1° Blocks

	Set 1	Set 2
RMS Diff. (GEM 9 - Terr.)	±32 mgals	±25 mgals
RMS Diff. (Alt. - Terr.)	±11 mgals	±15 mgals
RMS Terr. Std. Dev.	±11 mgals	±15 mgals
RMS Alt. Std. Dev.	± 7 mgals	± 8 mgals
Number of Comparisons	659	16579

Table 5. Information Related to Large $1^\circ \times 1^\circ$ Anomaly Differences Between Terrestrial and Altimeter Derived Values

φ°	λ°	Δg_{ALT}	Δg_{TERR}	Difference
7	153	8± 7 mgals	-258±17 mgals	266 mgals
47	153	-15±10 mgals	-202±23 mgals	187 mgals
56	162	-52± 9 mgals	-188±21 mgals	136 mgals
46	171	-46± 9 mgals	85±13 mgals	-131 mgals
37	213	-14± 8 mgals	86±18 mgals	-102 mgals

It seems clear that there is generally good agreement between the terrestrial and altimeter $1^\circ \times 1^\circ$ anomalies consistent with an accuracy estimate of ±8 mgals for the altimeter data. The large discrepancies discussed above indicate areas where more detailed information is needed on the anomaly field.

Computations have also been made in computing 5° equal area mean anomalies and undulations. The predicted accuracy of the 5° anomalies is on the order of ±3 mgals which is consistent with comparisons made with terrestrial data.

Satellite to Satellite Tracking Results

A recent data type for the recovery of gravity anomalies is that of satellite to satellite tracking. Such data is currently available only in limited areas and only experimental types of results have been obtained.

One experiment has involved the tracking of the Apollo spacecraft by the ATS-6 satellite. Since the Apollo vehicle was at an altitude of only about 230 km the range rate signal could be strongly perturbed by local anomalies. The analysis of this data and a description of the experiment is found in Vonbun et al. (1977). Using limited data they were able to recover some $5^\circ \times 5^\circ$ mean free-air anomalies to an accuracy of about ±7 mgals based on a comparison with ground truth.

Another experiment involving the ATS-6 satellite has used Geos-3. A description of this experiment and some data analysis may be found in Marsh et al.(1977) The analysis of some of this data for various size mean anomaly blocks has recently been described by Hajela (1978). In this study 5° equal area anomalies were recovered to an accuracy of about ±6 mgals.

At this point we do not have sufficient data to significantly improve our surface gravity field from satellite to satellite tracking data. However test results on anomaly recovery are sufficiently promising that more such data should be sought.

Conclusions

This paper has been a brief survey of ways in which satellite derived gravity data compares with surface data. These comparisons have been performed using block means (such as 5° equal area and $1^\circ \times 1^\circ$) and in terms of potential coefficients. We found that the differences between the terrestrial data and GEM 9 potential coefficients was ±11 mgals for 5° blocks and ±25 mgals for $1^\circ \times 1^\circ$ blocks. Much of this difference is caused by the fact that the GEM 9 set is complete to degree 20 only.

Comparisons with potential coefficients derived from the terrestrial 5° data, showed poor agreement at the lower degree. Overall there was a ±9.1 m undulation difference and a ±7.0 mgal anomaly difference. The standard deviation, by degree of the undulation or anomaly difference, was found to approach the actual magnitude of the quantity near degree 18. The standard deviation at a given degree, for the undulation,

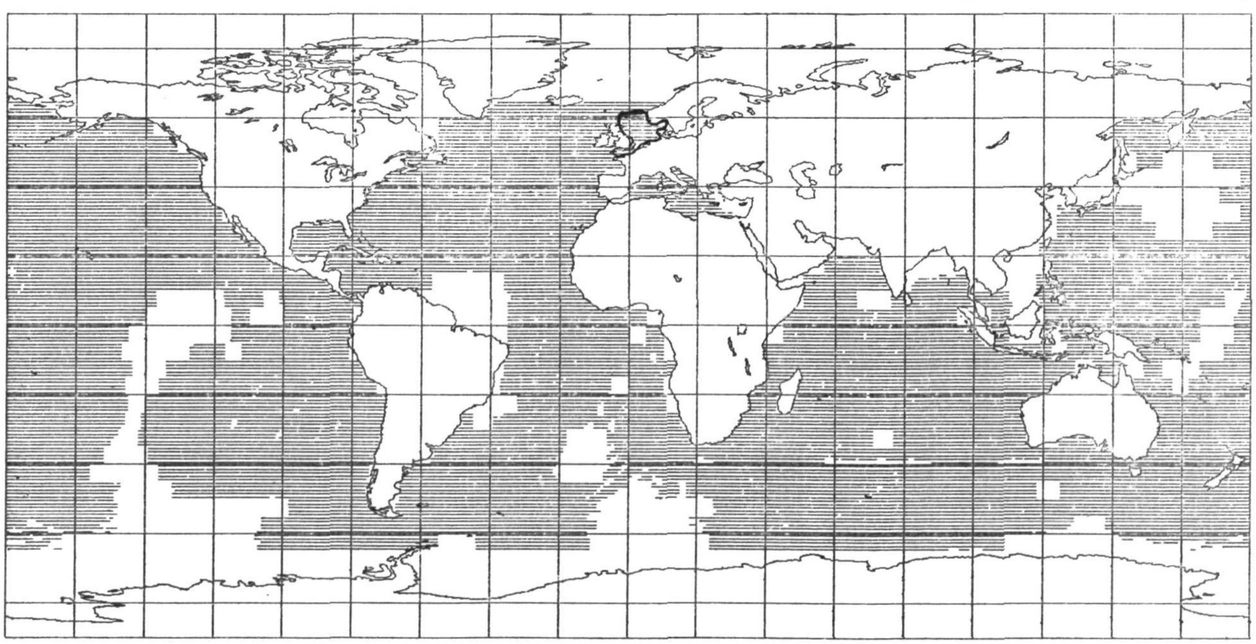

Figure 5. Location of 27465 $1^\circ \times 1^\circ$ Anomalies Computed from Geos-3 Altimeter Data.

was smaller for the GEM 9 coefficients than for the terrestrial derived coefficients up to degree 13. Figures 2 and 3 summarized the differences found.

Comparisons of the Geos-3 altimeter derived 1° anomalies and the surface data indicated the predicted standard deviations of about ±7 mgals were reasonable. However, some large discrepancies exist between the altimeter derived and terrestrial anomalies ranging up to 266 mgals. The use of the altimeter anomalies may be a way to detect bad 1° x 1° anomaly estimates.

Finally satellite to satellite tracking results were briefly discussed noting the achievement of encouraging results with additional data needed.

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