Implications of Cavity, Topographic and Geologic Influences on Tilt and Strain Observations

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The existence of cavity effects was pointed out by King and Bilham in 1973, about a year after our first GEOP conference on earth and ocean tides. The principle involved is illustrated by figure 1 which shows the cross-section of a circular tunnel with tilmeters installed at A and B. In the lower part of this figure the tunnel is shown deformed by horizontal compressive stress and two effects are obvious. Firstly, the tilmeters suffer a very local strain-induced tilt which is not really what we mean by "tilt" at all and prompts the question "what is tilt anyway"; and secondly, the strain across the tunnel is three times as great as it would have been had the tunnel not been present. Clearly one cannot install a tilmeter or strainmeter in an underground cavity and expect to measure a tilt or strain representative of the surrounding rock without first understanding the effect of the cavity. Changes in strain are likely to be associated with tilting in all cases of interest, be one concerned with earth-tides, build-up of tectonic strain in an earthquake region, or teleseismic and near-field tilts and strains following earthquakes. This discussion of cavity, topographic and geologic effects is equally valid for all these applications.

The concept of scale is very important. With earthtides and many other problems we are dealing with large scale deformations and considering macroscopically simple earth models, such as, for example, radially symmetric spherical earths. However, we are observing these deformations on a microscopically complicated earth, in cavities, often close to irregular topography and in the presence of local variations in elastic properties due to geological inhomogeneities. As none of these irregularities introduces net forces or couples, the influence of each is restricted to its immediate vicinity (St. Venant's principle) and its effect on the overall deformation of the earth is small. Unfortunately practical reasons often force us to measure in just those sites where the perturbations are large, and the traditional sites for tilt and strain observations - disused mines and tunnels - are often particularly badly affected. It is, however, usually the large scale "homogeneous" deformation which is of interest. The large scale strain obeys the large scale (plane or spherical) free surface boundary conditions and therefore has only 3 independent components. The homogeneous tilt is the tilt of the free horizontal surface and the boundary conditions ensure that this is the same as the tilt of a vertical line element; however, the tilt of an inclined line element is affected by strain even in the homogeneous case.

A measurement of strain is a measurement of change in distance per unit distance (d11) between two points on the earth's surface or on the interior of a cavity inside the earth. As long as we are in the linear regime of elastic behaviour and small deformations, this strain will be a linear combination of the three components of the homogeneous strain. An ellipsoidal or infinitely long cylindrical cavity strains uniformly and we can define the entire strain tensor for the cavity; an irregular cavity deforms in an irregular manner and the best we can do is to define linear strain between two specified points. An observed tilt is similarly the sum of the homogeneous tilt plus strain-coupled tilts from each component of homogeneous strain.

Interpretation of an observed tilt or strain thus requires that the appropriate coupling coefficients be determined and this is normally done by numerical calculations using finite element modeling. (Levine and Harrison, 1976; Berger and Beaumont, 1976; Emter et al., 1977). This modeling is greatly facilitated if there is a clear separation of scales, so that the geology is on a large scale relative to the topography and the topography on a large scale relative to the cavity. Then the effects may be computed separately and the total estimated by a series of matrix multiplications. Otherwise all effects must be included in the one finite element model and the calculation may become very unwieldy.

Ellipsoidal and cylindrical cavities were treated analytically by Harrison (1976). Strain is measured correctly along the axis of an infinitely long tunnel; for a tunnel of finite length a circular cross-section is optimal and for such a tunnel a length/diameter ratio greater than 20:1 results in less than 1% strain error. Narrow cavities are extremely compressible in the direction of their short dimension; a fine example of this effect comes from the Schiltach Observatory in the Black Forest where a x58 strain magnification has been observed across a narrow cleft, in good agreement with finite element calculations which predict x53 (Emter et al., 1977). The floor of an infinitely long tunnel shows no tilt coupling in the direction of the tunnel axis, and the sides of a vertical borehole tilt as if the hole were not present.

Finite element calculations have given insight into the behaviour of more complex cavities. The walls of a tunnel of square cross-section bend outwards as a result of cross-tunnel tension, while the floor remains flat (figure 2). Cracks (figure 3) or geometrical irregularities (figure 4) however induce local tilts and the cracks, behaving as narrow cavities, exhibit very large strains.

The cavity effects have very important implications as tilt and strain have usually been measured in underground cavities to avoid meteor-
Fig. 1. A circular tunnel is deformed by horizontal strain. Tiltmeters at A and B record strain-coupled tilts and the horizontal strain across the tunnel is three times as large as they would have been had the cavity not been present.

Fig. 2. Deformation of a square tunnel by horizontal tension. Only the bottom right corner of the tunnel is shown; the solid line represents the undeformed tunnel and the dashed line the deformed corner.

Fig. 3. Deformation of the tunnel of Figure 2 showing the effect of a crack.

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...ological perturbations. These cavities were excavated with explosives resulting in a fractured aureole surrounding the cavity in addition to cracks and joints which may be present naturally. Tilts and strains must be measured over baselines at least as long as the thickness of this fractured aureole. Strain measurements have been made over long baselines, typically 30m or more, along the axes of tunnels. They are not much affected by the cavity effects that one knows about and corrections are normally small. However, these observations may be affected by fractures and material inhomogeneities of which one is not aware, and differences in tidal strains measured with end to end strainmeters along tunnels in Great Britain suggest some such effect (Evans et al., in press).

For tilt the situation is entirely different. Tilt measurements have traditionally been made with short base (<1m) instruments in geometrically complex situations. Finite element modeling would be very difficult and is, in practice, impossible because the fracture pattern in the vicinity of the tiltmeter is unknown. Thus all earth-tide tilt observations made with short base instruments are essentially worthless, and the observational techniques must be completely revised. One exception is when the tilt is primarily a loading tilt from a nearby ocean; in this case the load induced tilt is much larger than both the load induced and body-tide strain, and the strain-coupled tilt is correspondingly less important.

Topographic effects are also more important than anyone had expected. Finite element calculations for a hill and valley situation with 45° slopes, figure 5, shows that the local strain in some places is actually of opposite sign to the homogeneous while in others, it is 3 times as large; up to 3.7 time if one considers strains measured across the valley. Even a valley with slopes of 1 in 10 produces a 36% modification of strain and tilt-strain coupling coefficients of the same order of magnitude.
As might be expected from the preceding discussion, material inhomogeneities ("geologic effects") also produce strain perturbations and tilt-strain coupling. Three examples are shown in Figures 6, 7 and 8: the first, the effects of a sediment-granite contact crudely modeling the Front Range-plains boundary near Boulder, Colorado; the second computations by Beaumont and Berger (1974) showing the effects of an assumed change in elastic constants due to dilatancy on the tilt and strain earth tides; and the third (unpublished) showing computations by the author of the effects of an assumed partially molten zone beneath Yellowstone. These effects are large — modifications of the homogeneous tilt and strain tides are of the order of many tens percent, up to 100%, they are localized to the vicinity of the inhomogeneity; and the tilt anomalies appear to be larger but more restricted in area than those in strain.

A general agreement between tilt and strain tide observations and the predictions of these cavity, topographic and geologic influences is now well established. In general it is not possible to correct tilt observations made with short base tiltmeters with useful accuracy, because of the difficulty of correcting for the very small scale geometric and material inhomogeneities which have important influences on such measurements. All such measurements, that is all but a very few of body tide tilt observations made to date, are therefore worthless considering the accuracies required to contribute useful information about the earth. The strain situation is generally better because strain measurements have been made with long (30m - 100m) instruments along the axes of tunnels, which happens to be the correct technique from the point of view of cavity effects. Corrections for the cavity are therefore small and probably realistic. Levine and Harrison (1975) and Berger and Beaumont (1976) have corrected earth tide strain observations for the topographic and geologic effects. These corrections generally improve agreement between theory and observation and this is particularly obvious when they are large (20-30%). Nevertheless only about half of the observations are in agreement with theory within the observational accuracy; this may be because the ocean loads are not accurately known but there is also the possibility that the modeling calculations are
Fig. 6. Strain-strain (solid line), tilt-strain (horizontal element-dashed line) and tilt-strain (vertical element-dotted line) coupling factors for an idealized cross-section near Boulder, Colorado. Basement (unshaded) properties are \( V_p = 5.8 \) km/s, \( V_s = 1.80 \) km/s.

not as accurate as the finite element computations because unknown but significant faults, joints, elastic inhomogeneities, etc. have not been included.

In summary tilt and strain tide observations are importantly (pathologically at the 100%, typically at the few 10s% level) affected by cavities, topography and geological inhomogeneities; gravity observations are practically unaffected. It is important that tilt and strain be observed with long base instruments because small irregularities, too small to model realistically, can have important local effects. On the other hand we do not really know what we mean by "long" because we do not know on what, if any scale, rock behaves as a continuous homogeneous medium; earth tide observations can help us answer this question, and different areas probably behave in different ways as a result of differing rock types and tectonic histories. The traditional earth tide observatory, and abandoned mine or tunnel is a very poor place to measure body tides because of complicated cavities, topography and geology. Instead the ideal site for observing the body tide is in flat terrain with horizontally layered, mechanically homogeneous geology. Strain will be measured with long surface or trench mounted laser strain meters and tilt with long, surface or trench mounted liquid levels, or with borehole tiltmeters. Horizontal geological discontinuities can produce large perturbations of the tilt and strain tides and these perturbations, using the known homogeneous tidal strains and tilts, can be used to explore local structure in favorable cases and, through possible time
Fig. 8. Modification of tides due to partially molten zone beneath Yellowstone National Park on the assumption of a 10% reduction in $V_p$. Models I and II are for the body shown extending to 100 km with, in I material properties corresponding to flat inclusions and in II, properties corresponding to round inclusions. Model III has the material properties of I but extends to 200 km depth. (a) radial displacement and N-S tilt and (b) E-W and N-S strain.

variations of tidal admittances, in earthquake prediction. Ocean load uncertainties currently preclude the use of tidal observations for determining whole earth structure or body tide energy dissipation, but it does seem possible that they can be utilized for exploration on a smaller scale - at least this is a possibility that deserves further investigation.

References

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