

The Inverse Problem: Ocean Tides Derived from Earth Tide Observations

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Summary. Evidence has been accumulated to the point that open ocean tides can indeed be mapped by solving the inverse problem using land-based and island-based tidal gravity observations, supplemented by shore and island ocean tide-gauge observations and a few deep ocean-bottom observations. Past efforts have been towards a better understanding of open ocean tides both through numerical integration of Laplace's tidal equations and through direct observations of tides in the open oceans. The co-amplitude and co-phase tidal charts principally of the tidal constituent M_2 calculated by numerical integration of Laplace's tidal equations generally fail to agree among themselves, and with the tidal observations on mid-ocean islands. The problem of open ocean tides remains open.

During the last few years, we have indirectly mapped both the M_2 and O_1 ocean tides in the northeastern Pacific Ocean, based on the inverse scheme of "Linear Programming." A comparison between the inversion results of the M_2 and O_1 ocean tides and the three ocean-bottom observations made by Scripps, which were not included in the inversion scheme, gives an excellent agreement and assures that the proposed technique of indirect mapping of open ocean tides by means of tidal gravity observations can contribute significantly to attack the classic problem of open ocean tides.

Ever since we proposed the possibility of indirectly mapping ocean tides by means of land- and island-based tidal gravity measurements (Kuo et al., 1970), skepticism has been raised principally by physical oceanographers, concerning its actual feasibility, typically such as the recent one by Zetler (1978):

"It has been demonstrated that land-based earth tides observed on a gravimeter near an ocean are modified by the ocean tidal loading. It is more controversial, however, whether it is possible to map the open ocean tides by solving the inverse problem using land-based gravity measurements, shore constraints, and a few ocean-bottom stations. Certainly it can be done with an infinite of stations of perfect precision; the number and precision of land and ocean measurements necessary to achieve required accuracy have not yet been determined."

Evidence has been accumulated to the point that open ocean tides can indeed be mapped by solving the inverse problem using land- and island-based tidal gravity measurements, coupled with shore and island ocean tidal measurements and a few ocean-bottom measurements. There is definitely no need to have an infinite of stations of perfect precision. The degree of precision of tidal gravity measurements, from our experience, must be 1% or better.

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which is perfectly achievable.

Much effort has been directed toward a better understanding of open ocean tides, both through numerical integration of Laplace's tidal equations, (Pekeris and Accad, 1969; Zahel, 1970; Hendershott and Munk, 1970; and others) and through direct measurements of tides in the deep oceans (Nowroozi et al., 1969; Filloux, 1971, Munk et al., 1970; and others). The co-amplitude and co-phase tidal charts principally of the tidal constituent M_2 calculated by numerical integration of Laplace's tidal equations are apparently quite sensitive to the boundaries of the ocean basins and the law of friction, and generally fail to give close agreement with the tidal observations on mid-ocean islands. Moreover, the agreement among various tidal charts is still poor as shown in Figures 1 and 2, comparing the tidal charts for example, the M_2 and O_1 in the northeastern Pacific Ocean.

During the last few years, we have indirectly mapped both the M_2 and O_1 ocean tides in the northeastern Pacific Ocean, based on the inversion scheme of "Linear Programming." A comparison between the inversion results of the M_2 and O_1 ocean tides and of the three ocean-bottom observations made by Munk et al. (1970) and Irish et al. (1971), which were not included in the inversion scheme, gives an excellent agreement.

The basic data used in the inversion are principally from the tidal gravity observations made on North America and on the islands in the northeastern Pacific Ocean and from the tide gauge observations on the coasts and islands. A total of 17 tidal gravity stations and 62 coastal and island tide gauge stations was used for the northeastern Pacific Ocean. The distributions of the tidal gravity stations and the tide gauge stations of the contiguous continents and islands of the northeastern Pacific used in the inversion are shown in Figure 3.

The M_2 and O_1 worldwide tide maps of Tiron et al. (1967), designated as Tiron M_2 Map and Tiron O_1 Map, respectively, as shown in Figures 4 and 5, were adopted as the starting models and were digitized at 2° by 2° spacings. As a matter of fact, Kuo and Jachens (1977) have shown that a starting model is not of crucial importance in the inversion. For oceanic regions within 2.5° of the tidal gravity observational stations, these maps were modified to conform with coastal observations. The predicted tidal gravity effects arising solely from ocean tides were calculated by numerical convolution of Tiron M_2 and O_1 Maps with linear combination of a mass loading Green's function for an oceanic crust model (Farrell, 1972) and, in addition, of a Newtonian attraction Green's function for a density coating layer on the surface of a sphere, which is not accounted for in the formulation of the loading Green's function by Farrell (1972).

A fourth order two-dimensional polynomial was selected as the highest order polynomial surface

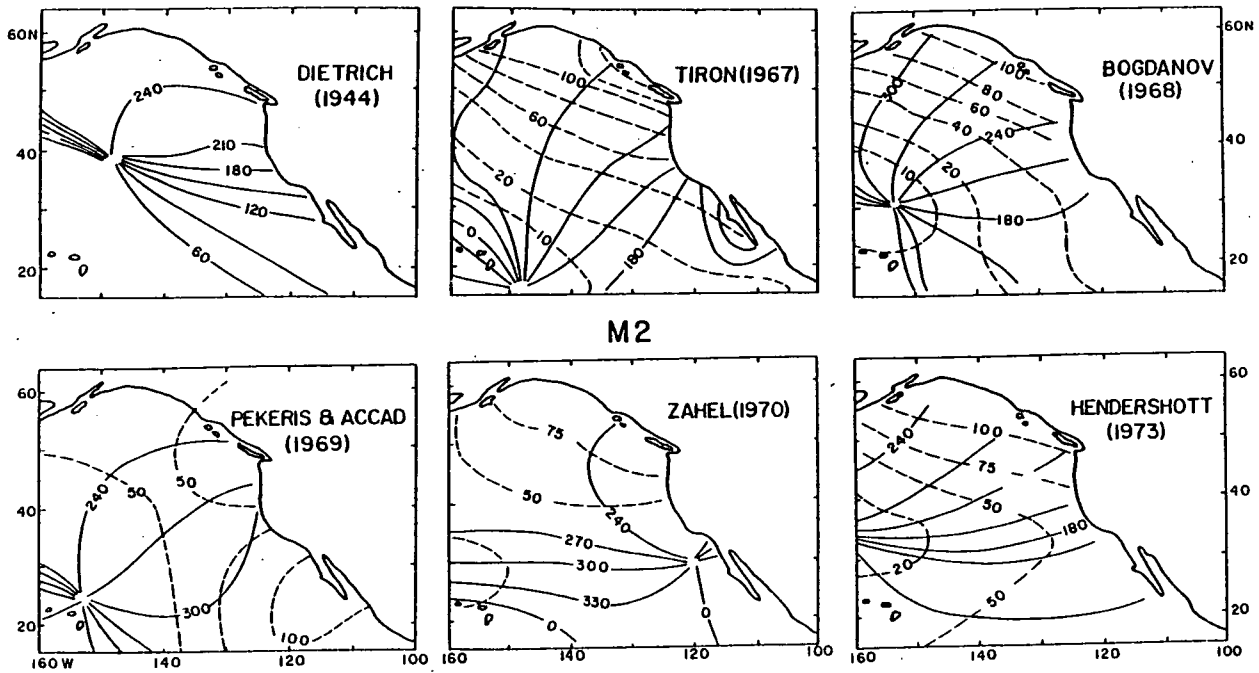


Figure 1. The M_2 ocean tidal constituent as derived by empirical (Dietrich, 1944) and Laplace tidal equations by various authors.

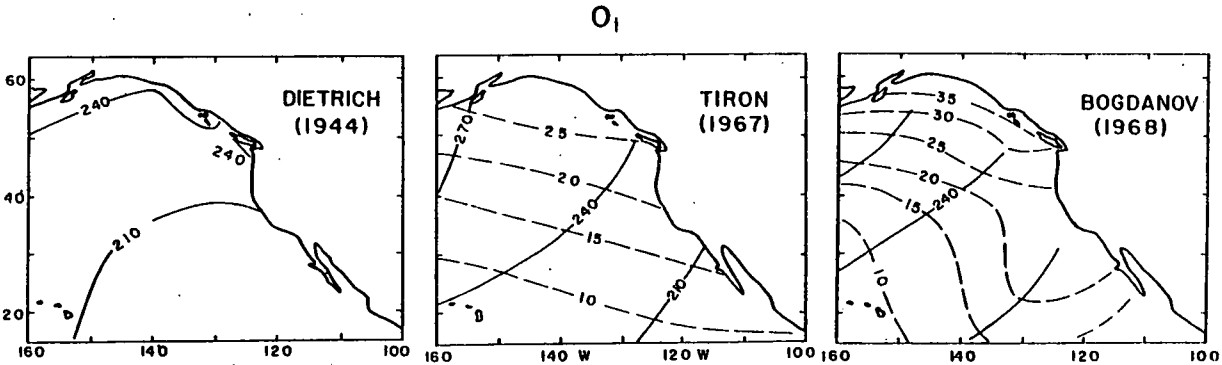


Figure 2. The O_1 ocean tidal constituent as derived by empirical (Dietrich, 1977) and Laplace tidal equations by various authors.

correction applied to Tiron M_2 and O_1 Maps. The limited distribution of the available tidal gravity data does not warrant a surface of order higher than fourth.

The inversion scheme is based on "Linear Programming." The reason of choosing this Linear Programming Inversion is very simple. Linear programming is concerned with the optimum operation of interdependent variables, that is the minimization of a linear objective function, whose variables satisfy a system of linear inequalities (Danzig, 1977). Unlike the other numerical solutions of minimization problems by iterative procedures, the solution to a linear programming problem, if exists, is unique and gives a true minimum for the entire system, i.e. a global minimum. Thus,

linear programming procedure can be ideally used to seek the optimal inverse solution subject to constraints imposed by the tidal gravity and ocean tide gauge observations.

Figures 6 and 7 give the new M_2 and O_1 tide maps for the northeastern Pacific Ocean, bounded on the west by 160°W on the south by 15°N , and on the north and east by North America, resulting from the application of the linear programming inversion. The most prominent feature of the new M_2 map is the existence of a single amphidromic system in the region, centered at approximately 26°N latitude, 137°W longitude and rotates counterclockwise. The amplitude contours display a low amplitude trough of less than 20 cm. The amplitude increases smoothly north of the trough attaining magnitudes

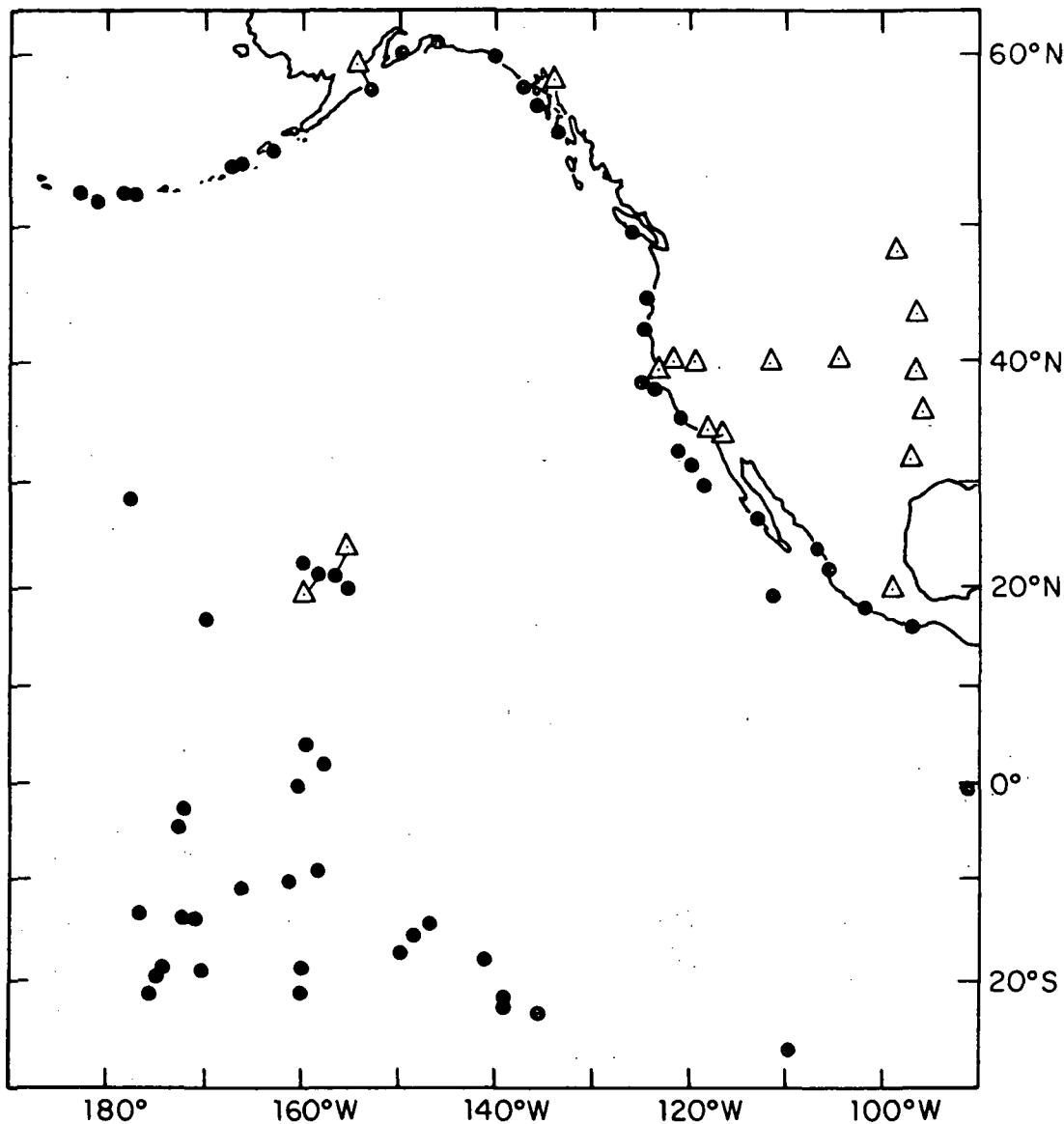


Figure 3. Distribution of tidal gravity, tide gauge and ocean-bottom tide stations in the north-eastern Pacific Ocean.

greater than 120 cm near the head of the Gulf of Alaska. The prominent feature of the new O_1 map is the modification of both the coamplitude and cophase lines from that of the Tiron O_1 Map. The new O_1 map shows a delay of the cophase lines, compared with the cophase lines of the Tiron O_1 Map. Except that the coamplitude line of 25 cm remains nearly the same as Tiron O_1 Map, all the other coamplitude lines of 10, 15, and 20 cm are curved somewhat to conform with the west coast of North America.

The goodness of the inversion results must be critically tested to insure the validity of the inversion procedure. It can be best accomplished by comparing the inversion results with observations. The observations from the three ocean-bottom sites, Kathy, Filloux, and Josie II, shown as solid circles in Figure 8 were not included as the basic data in the inversion procedure, and are far re-

moved from possible local perturbation introduced by islands or submarine topographic features. The comparison between the values obtained by the various tidal charts, the inversion procedure and these corresponding observed values is given in Tables I and II.

The agreement between the inversion results and the observations at all three sites is better than 6 cm in amplitude and better than 5° in phase for M_2 . Although there are no available observed data of O_1 for the station Josie II, the agreement for O_1 at the stations, Kathy and Filloux is better than 1 cm in amplitude and 1° in phase. The inversion results appear to be quite insensitive to uncertainties in knowledge of worldwide ocean tidal distributions and to possible biases in tidal gravimeter calibration. The agreement at Kathy and Filloux for both M_2 and O_1 is embarrassingly good, probably somewhat fortuitous, since the starting

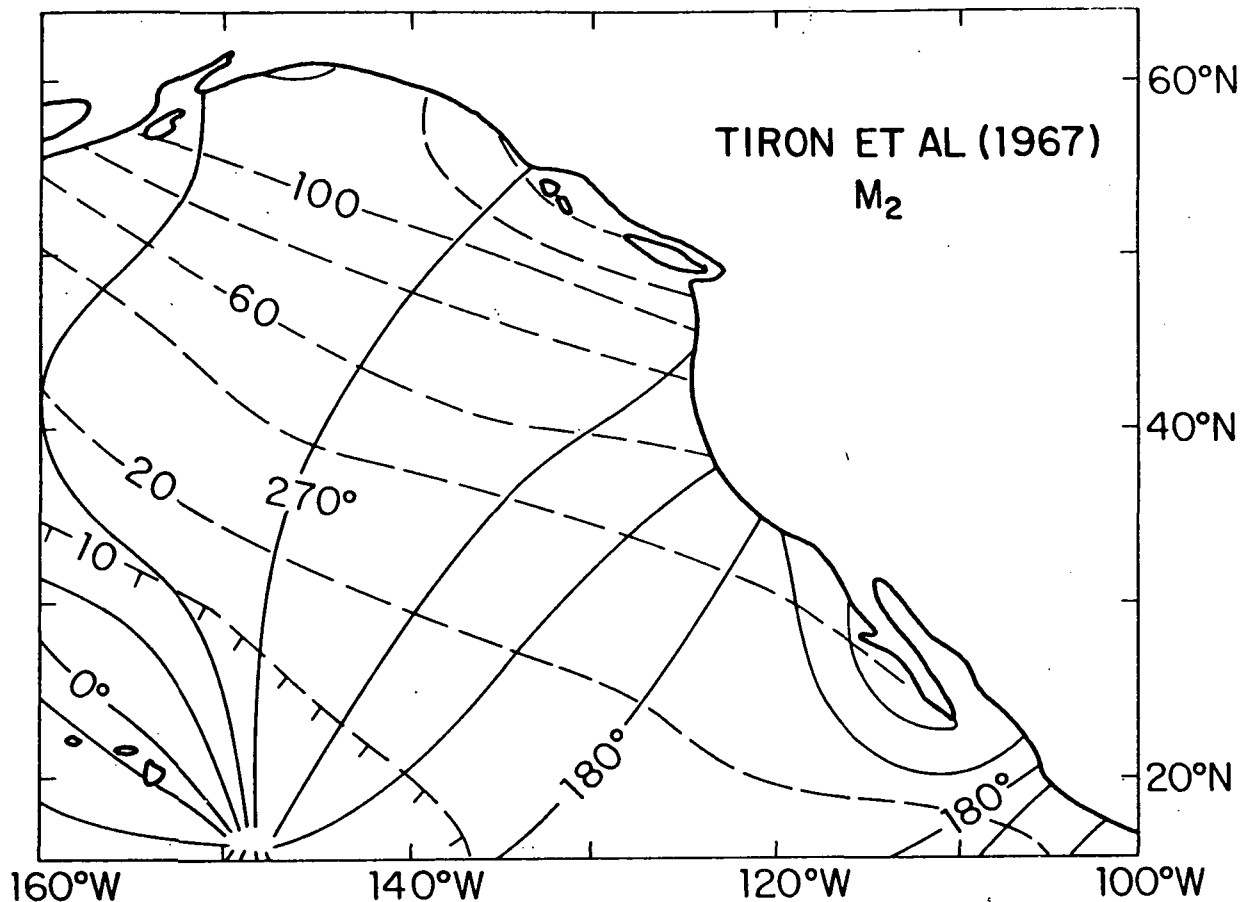


Figure 4. Tiron et al., (1967), Original M_2 Tidal Map.

TABLE I
COMPARISONS OF THE INVERSED RESULTS OF M_2 AND O_1
WITH THE THREE OCEAN-BOTTOM TIDAL OBS. (MUNK ET AL., 1970) AND AVAILABLE
TIDAL CHARTS AT THE STATIONS, KATHY, FILLOUX AND
JOSIE II.

STATION		OBSERVED (O) A(cm)/G(deg)	AVAILABLE COTIDAL CHARTS A/G						INVERSION A/G
			D	T	B	P&A	Z	H*	
Kathy 124°25.8'W 27°45.0'N	M_2	28.6/128.0	-/115	31/180	41/172	75/305	35/305	60/160	28.3/132.9
	O_1	17.5/199.0	-/190	15/219	21/208	-	-	-	18.0/199.0
Filloux 129°01.1'W 24°46.9'N	M_2	18.8/107.0	-/92	20/186	35/172	52/300	32/320	55/157	17.8/104.5
	O_1	15.6/201.3	-/190	12/220	22/210	-	-	-	15.5/200.0
Josie II 144°59.7'W 34°00.3'N	M_2	26.6/267.0	-/75	27/261	15/232	25/262	35/270	30/197	20.0/270.0
	O_1	Not determined	-/210	15/245	25/235	-	-	-	17.0/224.0

*D = Dietrich, 1977; T = Tiron et al., 1967; B = Bogdanov et al., 1968

P&A = Pekeris & Accad, 1969; Z = Zahel, 1970; H = Hendershott, 1973.

O = Observed (Munk et al., 1970)

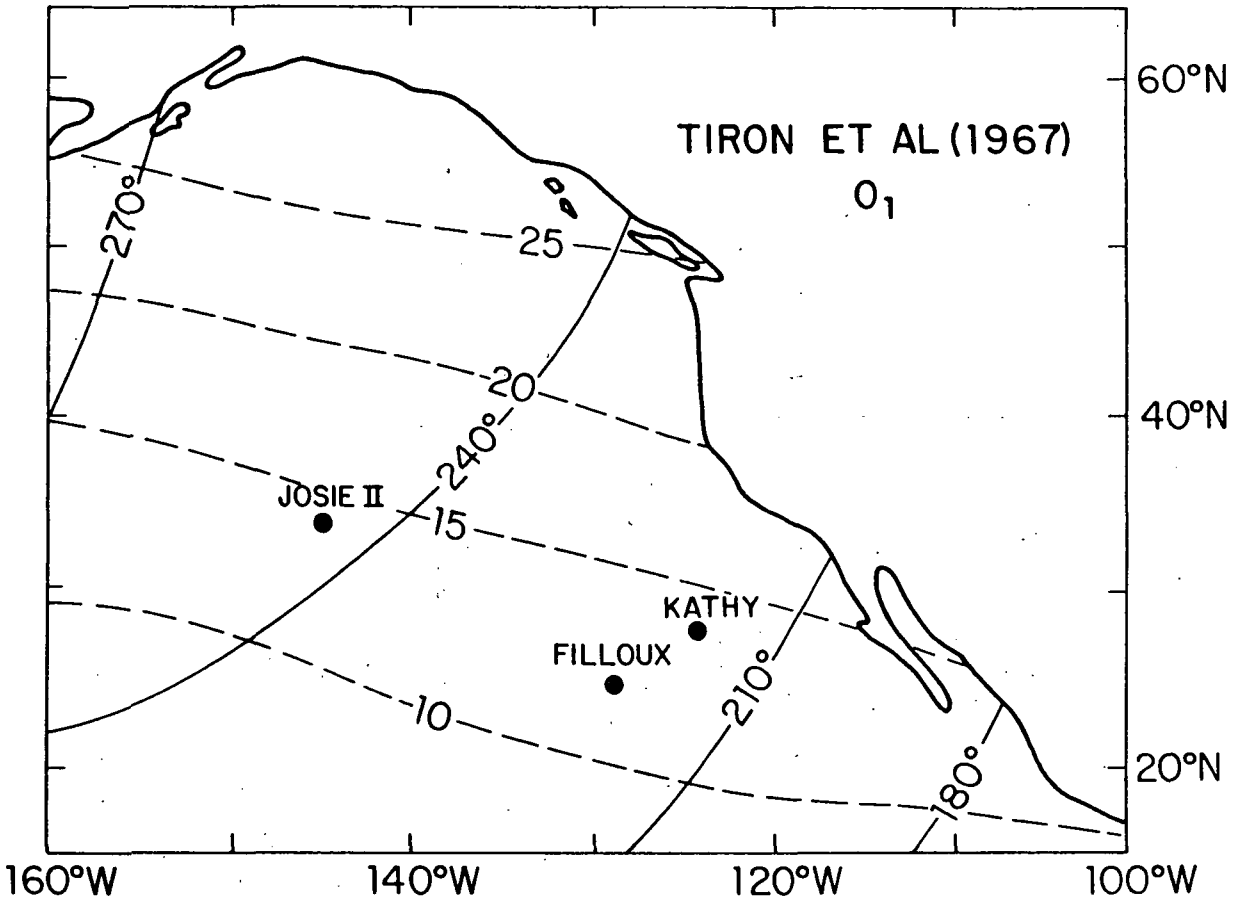


Figure 5. Tiron et al., (1967), Original O_1 Tidal Map.

TABLE II
DIFFERENCES

STATION		D-O	T-O	B-O	P&A - O	Z-O	H-O	INV-O
Kathy	M_2	-/-13°	+2.4/+52	+12.4/+44	+46.4/+177	+6.4/+177	+31.4/+32	-0.3/+4.9
	O_1	-/-9	-2.4/+20	+3.5/+9	-	-	-	+0.5/0
Filloux	M_2	-/-15	+1.2/+79	+16.2/+65	+33.2/+193	+13.2/+213	+36.2/+50	-1.0/-2.5
	O_1	-/-11.3	-3.6/+18.7	6.4/+8.7	-	-	-	-0.1/-1.3
Josie II	M_2	-/-192	0.4/-6	-11.6/-35	-1.6/-5	+8.4/+3	+3.4/-70	-6.6/+3.0
	O_1	Not Observed	-	-	-	-	-	-

model could not be read reliably to one degree and one cm.

The crucial difference between the present procedure and those of past workers is that land-based and island-based tidal gravity observations

were used as the bases for interpolating between widely spaced direct ocean tidal observations rather than the traditional empirical procedures based on the numerical integration Laplace tidal equations. For comparison, Figure 9 shows a map

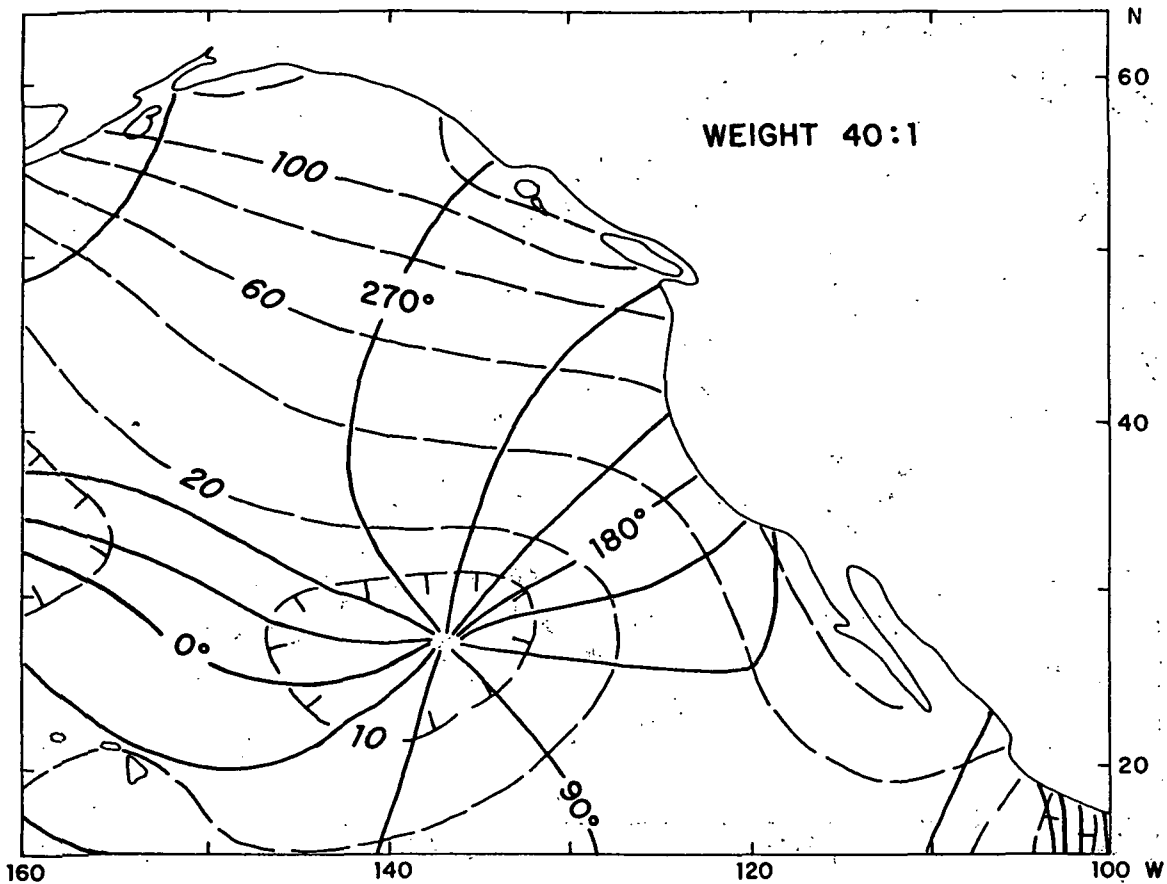


Figure 6. M_2 obtained from inversion for the north-eastern Pacific Ocean.

derived by Munk et al. (1970) based on ocean-bottom tidal measurements and empirical inference.

The inversion results of the M_2 and O_1 tides in the northeastern Pacific Ocean assure that earth tidal gravity can contribute significantly to attack one of the remaining classic geodynamic problems, - the open ocean tides. The worldwide open ocean tides can be mapped by means of solving the inverse problem of ocean tides for a series of tidal gravity observations, complemented by coastal, shelf, and island tidal gauge observations and a limited number of ocean-bottom tidal observations.

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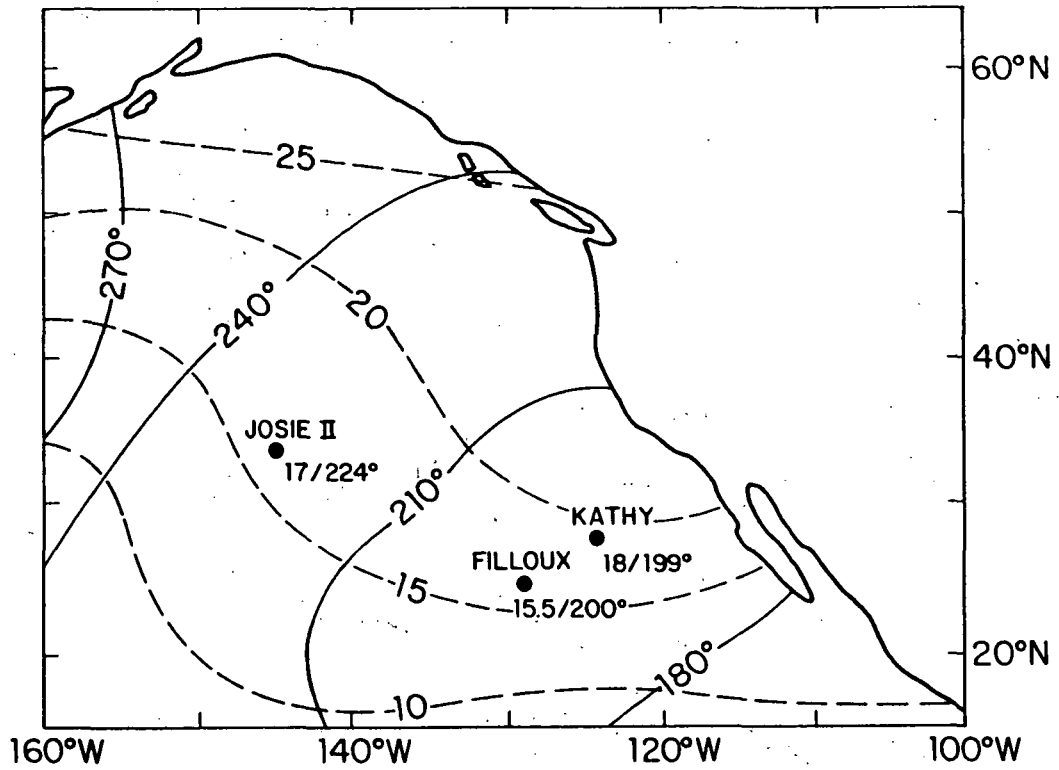


Figure 7. O_1 obtained from inversion for the north-eastern Pacific Ocean.

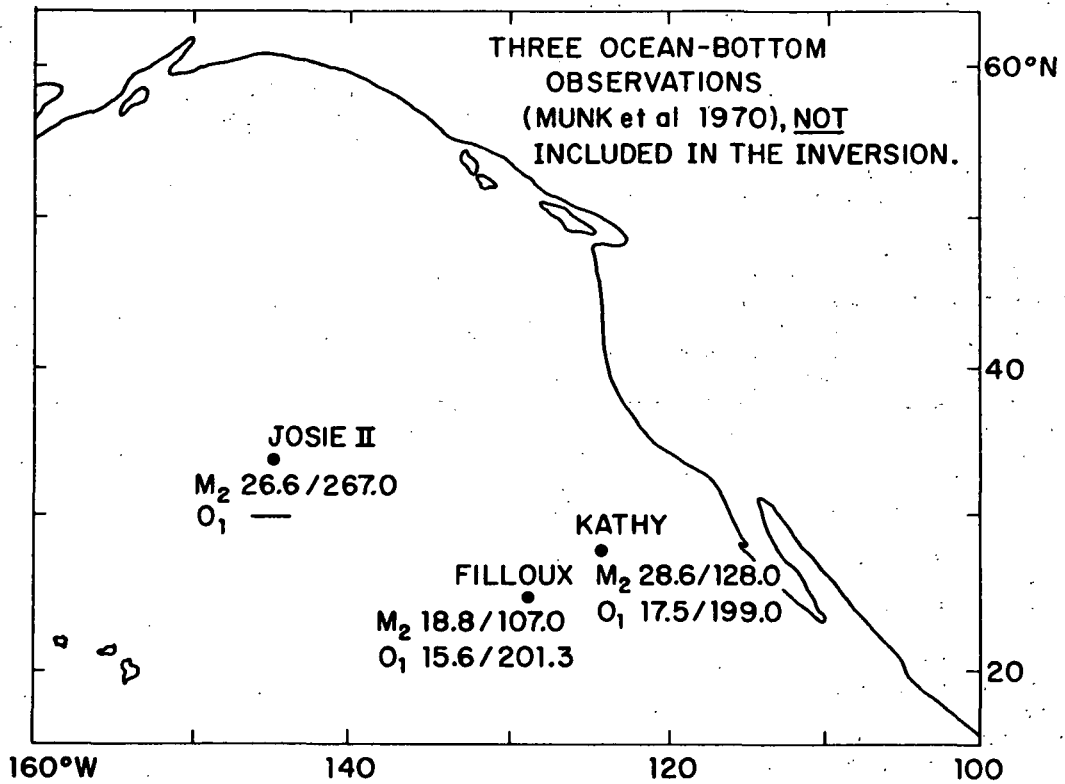


Figure 8. Three ocean-bottom observations (Munk et.al. 1970), not included in the inversion.

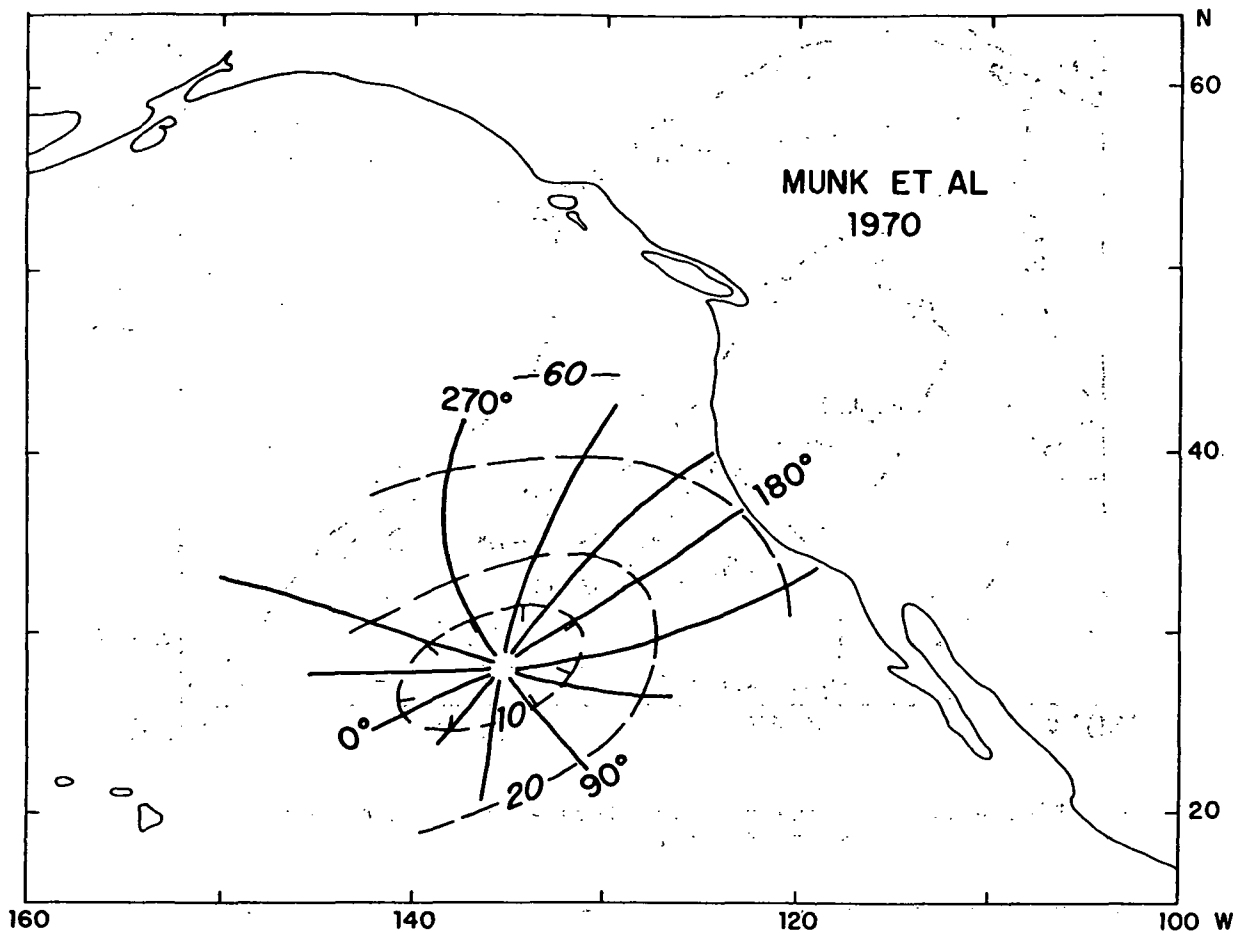


Figure 9. The M_2 ocean tide as derived by Munk et al. (1970) based on ocean-bottom measurements.

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