Tidal Friction in the Solid Earth

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Abstract. The earth's imperfectly elastic response to body and loading tidal forces is discussed using complex Love numbers and complex mass loading coefficients. Exact analytical expressions have been derived relating the energy dissipation within an inhomogeneous, compressible solid earth to the surface values of these complex characteristic numbers, thus relating the global dissipation function Q to the phase shifts in the potential, gravity, tilt, strain and displacement tides.

Integration of a global ocean tidal model shows that energy dissipated in the solid earth due to ocean loading is at least 10 % of that dissipated in the body tide; however both body and loading tide together do not account for more than a few percent of the astronomically observed dissipation.

The commonly used relation $Q^{-1} = -\tan\varphi$ where φ is the observed phase lag only applies when selfgravitation and hydrostatic prestress are ignored. It, therefore, is not applicable to the earth and in fact there is no unique relation between the global Q and the tidal phase shifts, this relation being very dependent on the distribution of Q with depth. Determinations of the global Q from satellite observations may be in error by 70 %, and calculations on the basis of seismic Q-models predict phase shifts in the gravity tide of only a few thousands of a degree in place of the currently predicted tenths.

Unlike in the body tide case, dissipation in the loading tide is sensitive to properties of the asthenosphere, and phase shifts in the M_2 loading tides in displacement, gravity and tilt may be as high as several degrees for loads near ocean ridges and subduction zones.

Rate of Tidal Dissipation in the Solid Earth

Dissipation of body and loading tidal energy within the solid earth may be determined from its complex Love numbers and complex mass loading coefficients, respectively [Zschau, 1979 a, b] . Such the expressions for the dissipated energy turn out to be fairly simple,

$$\Delta E_n = -\frac{2\times n+1}{4\,GR} K_n \iint_S \psi_n^2 \, dS \qquad (1)$$

for the body tide, and

$$\Delta E_n = \frac{2\times n+1}{4\,GR} (H_n^* - K_n^*) \iint_S \psi_n^{*2} \, dS \qquad (2)$$

for the loading tide, where ΔE_n is the energy dissipated during one cycle of harmonic loading, K_n and H_n^*, K_n^* are the imaginary parts of the surface Love numbers and mass load coefficients, respectively, ψ_n is the amplitude of the body force potential, ψ_n^* is the amplitude of the surface load potential, R is the earth's radius and G is the gravitational constant. n describes the degree of the expansion into spherical harmonics. The integration is taken over the surface of the earth. The expressions above have been obtained without approximating the real earth by an incompressible and homogeneous one as was necessary in former calculation, for instance by Munk & Mac Donald [1960] .

Using these formulas, and assuming the mantle Q structure LMS as obtained from the observation of the earth's free oscillation [see Smith, 1972] , a body tide solid dissipation rate of

$$3.19 \times 10^{17} \text{ erg/s},$$

i.e. about 1 % of the astronomically observed dissipation rate, has been obtained. The corresponding computation for the loading tide dissipation rate within the earth's crust and mantle, carried out on the basis of a global M_2 ocean tide model [Hendershott, 1972] , yields a minimum value of roughly 10 % of the body tide solid dissipation rate, i.e.

$$3.21 \times 10^{16} \text{ erg/s}.$$

This value has been determined from the low degree harmonics of the ocean tide distribution up to n=25, and, therefore, does not represent the high amplitude tides in the shelf areas. The latter may contribute significantly to the total dissipation, because the dissipated energy is proportional to the square of the marine tidal amplitude. On the other hand, the elastic strain energy stored in the mantle as well as the energy dissipated in the mantle decreases with increasing degree n of the spherical harmonic loading for n > 5 as may be seen from Fig. 1. This suggests that solid earth dissipation in the shelf areas does not change the total dissipation rate drastically. Anyway, both body tide - and loading tide dissipation together most probably do not account for more than a few percent of the astronomically observed dissipation rate.

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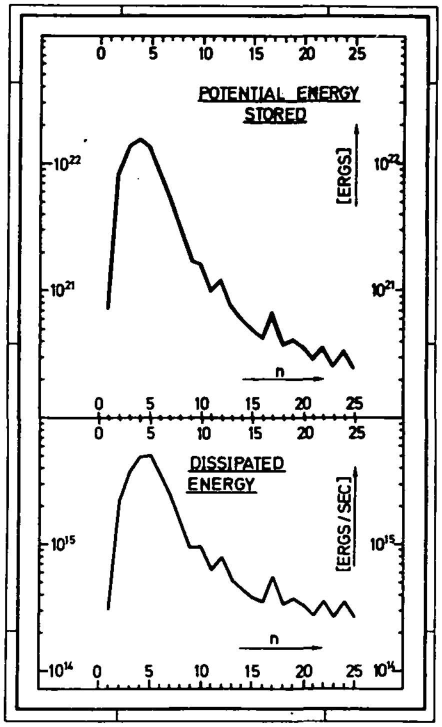

Fig. 1. Loading tide shear strain energy stored and dissipated within the solid earth [from Zschau, 1979b]. n is the degree of the spherical harmonic expansion of Hendershott's global M_2 ocean tide model [Hendershott, 1972]. Results are valid for the Gutenberg Earth with the free oscillation Q structure LMS for the mantle and the surface wave Q structure for the crust.

Body Tide Phase Shifts and the Earth's Dissipation Function Q

In Zschau [1979b] the earth's global dissipation function Q has been related to the phase shifts in the potential, gravity, tilt, strain and displacement body tides. The following expressions valid for constant Q distribution within the earth have been obtained:

Tangential displacement: $\tan\varphi = L/l = -0.897\ Q^{-1}$

Potential variation due to tidal deformation: $\tan\varphi = K/k = -0.612\ Q^{-1}$

Radial displacement: $\tan\varphi = H/h = -0.555\ Q^{-1}$

Surface areal strain: $\tan\varphi = \frac{H-3L}{h-3l} = -0.318\ Q^{-1}$

Tilt: $\tan\varphi = \frac{K-H}{1+k-h} = +0.217\ Q^{-1}$

gravity variation: $\tan\varphi = \frac{H-\frac{3}{2}K}{1+h-\frac{3}{2}k} = -0.0508\ Q^{-1}$

(h,H), (k,K), and (l,L) complex Love numbers; the index n has been omitted here.

	CONVENTIONAL	CORRECT	
OBSERVED DELAY ANGLES	$1/\tan\alpha_{h-\frac{3}{2}k}$	Q_{gl} (CONSTANT Q)	Q_{gl} (LOW Q ZONE)
RECORDING GRAVIMETERS 0.1°	79	28	5.5
RECORDING GRAVIMETERS 0.2°	39	14	<3
SATELLITE OBSERVATIONS 0.5°	$1/\tan\alpha_k$ 60	36	34

TABLE 1. Conventional and correct global Q values (Q_{gl}) corresponding to tidal delay angles of gravity and satellite observations.

The above equations show that the commonly used relation

$$Q^{-1} = -\tan\varphi \tag{3}$$

where φ is the observed phase shift is not applicable in the earth tide case. For the tidal gravity variation this had already been pointed out by Slichter [1960] who instead of (3) introduced the formula

$$Q^{-1} = -\frac{\delta}{\delta-1}\ \tan\varphi \tag{4}$$

where δ is the gravimetric δ-factor, and φ is the gravity phase shift. With $\delta = 1.16$ this yields

$$\tan\varphi = -0.138\ Q^{-1} \tag{5}$$

which is still too large by more than a factor of 2 as is obvious from the comparison of this formula with the corresponding one given above. One may show that Slichter's formula is equivalent to

$$Q^{-1} = \frac{H-\frac{3}{2}K}{h-\frac{3}{2}k} \tag{6}$$

which does not correspond to the basic definition of Q^{-1} as the strain energy dissipated during one cycle of loading over 2π the peak energy stored in the system. The general expression for the earth's global dissipation function Q in terms of its complex Love numbers is, however,

$$Q^{-1} = \frac{Kh-H\times(1+k)}{h\times(1+k-h)} \quad \text{[Zschau, 1979b]} \tag{7}$$

This equation is valid for an incompressible body with homogeneous density. None of the above phase shifts is equivalent to this expression, hence in general

$$Q^{-1} \neq -\tan\varphi \tag{8}$$

As shown in Zschau [1979b] the reason for this is that in the case of tidal deformations, selfgravitation and hydrostatic prestress cannot be ignored.

Besides the fact that (3) is not applicable in the tidal case, there, furthermore, is no unique relation between the global Q and the tidal phase shifts, this relation being very dependent on the distribution of Q with depth. For instance, a low Q zone in the upper mantle such as given in model LMS [see Smith, 1972] may alter the ratio between observed tidal phase shifts and the global (average) Q up to a factor of 5.

Table 1 gives some examples for the errors involved when not taking account of the above aspects: Let the phase delay of the body tide gravity be 0.1° with respect to the external forces. Proceeding in the conventional manner, i.e. using Slichter's formula with $\delta = 1.16$ gives the wrong global body tide Q = 79. In the case of constant Q within the mantle, we find the correct value to be Q = 28. If we assume a low Q asthenosphere, i.e. let the real Q distribution within the earth differ by only a constant factor from the free oscillation Q model LMS, we find that the global Q has to be chosen as low as 5.5 to correspond to the gravity phase delay of 0.1°. This is less than 7 % of the value Q = 79, obtained by the conventional method. For the same reason it turns out that the body tide gravity phase delay due to friction within the solid earth amounts more likely to a few thousandth of a degree than to a few tenth of a degree as expected so far. We, therefore, suggest that the average O1 gravity phase shift of -0.2° as observed for Europe may not be attributed to imperfect elasticity in the mantle like it is proposed by Melchior et al. [1976], but rather to the indirect effect of the O_1 tide in the oceans. There seems to be no chance at all at the moment to get information on the mantle Q from body tide gravity investigations.

The delay angle of the potential bulge due to tidal deformation of the earth has been determined from the orbits of artificial satellites to be 0.5° [Lambeck et al., 1974] . Lambeck et al. relate this delay angle to a mantle Q of 60 which one obtains by using formula (3). The correct Q corresponding to this delay angle is 36, if the Gutenberg earth and constant Q values within the mantle are adopted. For the LMS equivalent model, we have calculated the global Q which corresponds to the delay angle of 0.5° to be 34. This shows that the delay angle of the tidal potential bulge is less sensitive to the geometry of the Q distribution within the earth than the phase delay of the tidal gravity at the deformed surface as measured by a gravimeter. The Q values of 34 and 36 are much lower than the lowest limit of the upper mantle seismic Q.

The usage of the exact theoretical relationship between the bulge of the tidal potential and the body's global Q may also be important for other planets as for instance for Mars. From observations of the secular acceleration of the Mars satellite Phobos, Smith and Born [1976] deduced the phase angle of the potential bulge due to the body tide of Mars. They related this phase angle to the global Q of Mars by the simple formula (3), and found a global Q between 50 and 150. However, as one cannot neglect selfgravitation and the hydrostatic prestress for Mars, equation (3) is not applicable, and, therefore, the Q between 50 and 150 is probably too high for Mars, provided that the observed phase angle of the tidal bulge is true.

Similar considerations as above may also be important for the determination of the lunar global Q from observations of its physical librations [see Yoder, 1978] .

The Effect of Imperfect Mantle Elasticity on Loading Tides

Unlike in the body tide case, also high degree harmonics of the load are important in the loading tide case. It is found that for the free oscillation Q model LMS, the loading tides of harmonic degrees slightly less than 100 are strongly effected by the low Q asthenosphere, the global loading tide Qs being up to nearly 7 times smaller than the global body tide Q (see Fig. 2). Correspondingly, the loading tide phase shifts due to imperfections in the elasticity of the mantle are by more than one order of magnitude higher than those of the body tide. Near ocean ridges and near subduction zones the M_2 loading tide phase shifts may even be as high as a several degrees for the displacements as well as for gravity and tilt, provided the Maxwell constitutive law is valid. From the computation of phase shift Green's functions it is obvious that these maximum phase shifts occur at about 80 to 100 km distance from the load, this distance depending on the depth of the assumed low viscous asthenosphere (see Fig. 3). Our numerical results suggest that loading tide investigations could become an effective tool for studying the upper mantle viscosity in regions where viscosities lower than 10^{19} Poise may be expected.

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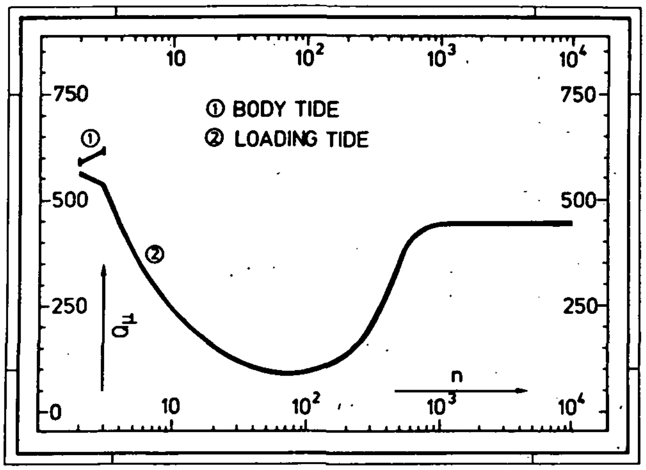

Fig. 2. Body tide Q and loading tide Q for different degrees n of the spherical harmonic expansions of the tides [from Zschau, 1979b]. Results are valid for the Gutenberg Earth with the LMS Q structure from free oscillation data for the mantle, and the MM 8 Q structure from surface wave data for the crust. The low Q values for n slightly lower than 100 in the loading tide case, are due to the low Q asthenosphere in model LMS. The high Q values for higher n and for lower n are due to the high Q values in the crust and in the lower mantle, respectively.

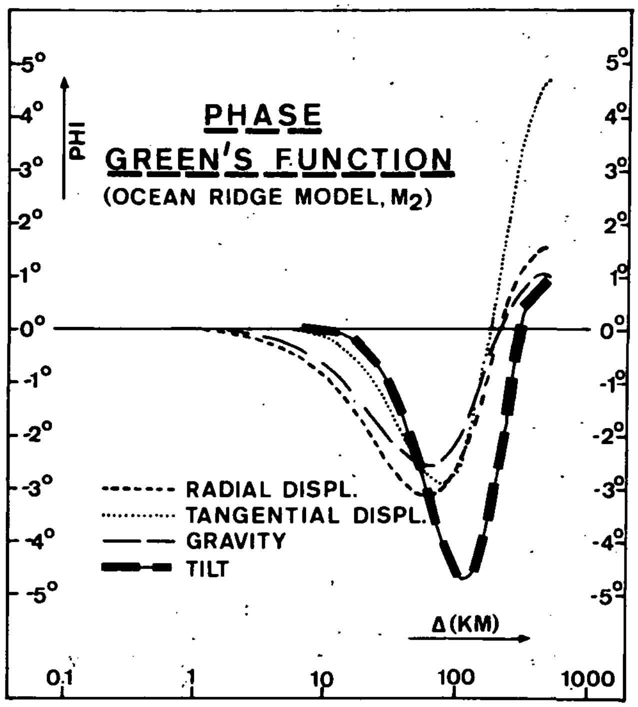

Fig. 3. M_2 phase Green's functions for an ocean ridge model [from Zschau, 1979a,b]. They represent the phase shifts between the viscoelastic earth's responses to a varying point load, and the responses of the corresponding elastic earth. The high phase shifts at distances of about 100 km from the load are due to the low viscous asthenosphere. Δ : distance from the point load, PHI: phase shift.

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