N79 21508

Body Tides on an Elliptical Rotating Earth

John M. Wahr
CIRES, University of Colorado
Boulder, Colorado 80309

Abstract. The complete tidal response of an elliptical, rotating, elastic Earth is found to contain small displacements which do not fit into the conventional Love number framework. Corresponding observable tidal quantities (gravity, tilt, strain, Eulerian potential, etc.) are modified by the addition of small (≤1%) latitude dependent terms.

Introduction

Traditional body tide calculations assume a spherically symmetric non-rotating Earth, with surface motion at a given frequency described by a small set of 'Love numbers', principally h, k and ℓ. Simple linear combinations of these numbers are found to compare with experiment.

The spherically symmetric case has been extended by others (Jeffreys and Vicente, 1957a, 1957b; Molodensky, 1961; Shen and Mansinha, 1976; Sasao *et al.* 1978) to include effects of non-sphericity on the diurnal tides. Particular attention has been focused on the dynamical effects of a rotating fluid core with an accompanying elliptical core-mantle boundary. The mantle has been repeatedly modelled as a spherically symmetric, non-rotating elastic shell, and the fluid core assumed to be spherically stratified. In each of these expanded treatments, the computed tidal response consists of the conventional body tide, represented at the free surface by the Love numbers h, k and ℓ, together with a toroidal component describing the Earth's nutational behavior. In this way the diurnal Love numbers are modified for the most important effects of rotation and ellipticity. The nutations, traditionally computed for a rigid Earth (Woolard, 1953; Kihoshita, 1977) are also improved by including spherically stratified elastic behavior throughout the Earth.

Qualitative Effects of Rotation and Ellipticity

The work described below extends these results by also including the effects of rotation in the inner core and mantle, together with elliptically stratified, realistic material properties throughout the Earth. Numerical investigations suggest a computed surface solution accuracy of at least three parts in a thousand, the order of the Earth's ellipticity. All important tidal bands are considered.

It is found that for the complete rotating, elliptical problem, surface motion is too complex to be represented by the familiar Love numbers: h, k and ℓ. Even the inclusion of nutational motion into the diurnal solution and length of day changes into the long period solution is not sufficient. Other small (≤1%) unmodelled motions remain. These new terms, which will not be detailed here, can be handled by defining more Love numbers. The utility of this procedure, however, is not always maximal. It is, of course, physical observables which are the ultimate computational goal. For an elliptical, rotating mantle the relations between an expanded Love number set and these observables can be disturbingly complex. Consequently, it is often more useful to present results for the observational quantities, directly.

As an example, consider the tidal gravity signal as measured by a standard gravimeter. For a second degree semi-diurnal tide, it has the form

$$\text{Gravity} = -\frac{2}{r_o}\, g\, V_{n=2}(\omega)\left[\delta_2(\omega)\, Y_{n=2}^{m=2}(\theta,\phi) + \delta_4(\omega)\, Y_4^2(\theta,\phi)\right]\cdot e^{i\omega t} \tag{1}$$

where ω is the perturbing frequency, Y_n^m a surface spherical harmonic, r_o the Earth's mean radius (6371 km), g the computed gravity acceleration at the Earth's equator assuming a spherical density distribution ($g = 979.8259\ \text{cm/sec}^2$), $V_n(\omega)$ the appropriate term in the luni-solar potential as observed at the Earth's equator, and $\delta_2(\omega)$ and $\delta_4(\omega)$ are gravimetric factors which reflect the dynamical behavior of the Earth at the frequency, ω. (For a spherical Earth $\delta_4 = 0$).

It is convenient to think of eq. (1) as representing a latitude dependent gravimetric factor:

$$\delta = \delta_2 + \delta_4 \frac{Y_4^2(\theta,\phi)}{Y_2^2(\theta,\phi)} \tag{2}$$

Similar latitude dependent phenomenon are present in all tidal bands and for all observable quantities (i.e.), surface gravity, tilt, strain, latitude and longitude variations, tidal perturbations in inertial space gravity, inertial space station displacements, etc. They are found at about the one per cent level or less. For example, computations of the M_2 semi-diurnal gravimetric factors shown in eq. (1) give, for Earth model PEM-C (Dziewonski, *et al.*, 1975):

spherical case:	$\delta_2 = 1.159$	$\delta_4 = 0$
rotating, elliptical case:	$\delta_2 = 1.160$	$\delta_4 = -.005$

Acknowledgements. This work is supported under NASA grant NSG 7319.

References

Dziewonski, A. M., A. L. Hales, and E. R. Lapwood, Parametrically simple Earth models consistent with geophysical data, Physics of the Earth and Planetary Interiors, 10, 12-48, 1975.

Jeffreys, H. and R. O. Vicente, The theory of nutation and the variation of latitude, Mon. Not. R. Astr. Soc., 117, 142-161, 1957a.

Jeffreys, H. and R. O. Vicente, The theory of nutation and the variation of latitude: the Roche Model Core, Mon. Not. R. Astr. Soc., 117, 162-173, 1957b.

Proc. of the 9th GEOP Conference, *An International Symposium on the Applications of Geodesy to Geodynamics, October 2-5, 1978*, Dept. of Geodetic Science Rept. No. 280, The Ohio State Univ., Columbus, Ohio 43210.

Kinoshita, H., Theory of the rotation of the rigid Earth, Celestial Mech., 15, 277-326, 1977.
Molodensky, M. S., The theory of nutation and diurnal Earth tides, Comm. Obs. R. Belgique, 288, 25-56, 1961.
Sasao, T., S. Okubu and M. Saito, A simple theory on dynamical effects of stratified fluid core upon nutational motion of the earth, to be published in the Proceedings of IAU Symposium No. 78 "Nutation and the Earth's Rotation", Kiev, May 1977), in press. 1978.
Shen, P-Y. and L. Mansinha, Oscillation, nutation and wobble of an elliptical rotating Earth with liquid outer core, Geophys. J. R. Astr. Soc., 46, 467-496, 1976.
Woolard, E. W., Theory of the rotation of the earth around its center of mass. Astronomical Papers for the American Ephemeris and Nautical Almanac (Gov't. Printing Office, Washington, DC, 1953), Vol. XV,Pt.1, 1953.

Observation of the Nearly Diurnal Resonance of the Earth Using a Laser Strainmeter

Judah Levine *
Time and Frequency Division, National Bureau of Standards
Boulder, Colorado 80302

Abstract. We have used two years of strain tide data to study the response of the Earth to the diurnal and semidiurnal tidal excitations. Our results show that there is significant structure in the response of the earth to tidal excitations near one cycle/sidereal day. This structure agrees with the resonance behavior predicted from the calculations of the forced elastic-gravitational response of an elliptical, rotating earth with a liquid outer core. The data can also be used to test for possible preferred frames and spatial anisotropies. We find that upper bounds on the parameterized post-Newtonian (PPN) parameters which characterize these effects are $\alpha_2 \le 0.007$ and $\zeta_w \le 0.005$.

Introduction

We have analyzed approximately two years of strain-tide data obtained using the 30-meter laser strainmeter we have previously described (Levine and Hall, 1972).

Data Acquisition

The data were obtained using a 30-meter long laser strainmeter located in the Poorman Mine, an unworked gold mine located approximately 8 km west of Boulder, Colorado at latitude 40.03°N, and longitude 254.67°E.

The heart of the strainmeter is an evacuated 30-meter Fabry-Perot interferometer located along the length of the tunnel. The axis of the interferometer is 7° west of North.

The interferometer is illuminated by a 3.39-μm helium-neon laser. A servo loop piezoelectrically tunes the laser to keep its wavelength coincident with one of the transmission maxima of the long interferometer. The frequency of the laser, f, is therefore related to the length of the interferometer, L, by

$$f = \frac{nc}{2L}$$

where n is an integer and c is the velocity of light. Thus

$$\frac{\Delta f}{f} = -\frac{\Delta L}{L} \quad .$$

A second 3.39-μm laser is stabilized using saturated absorption in methane. The beat frequency between the two lasers is extracted for further processing. Then

$$\frac{\Delta f_{beat}}{f} = \frac{\Delta L}{L}$$

or

$$\Delta f_{beat} = 8.85 \times 10^{13} \frac{\Delta L}{L}$$

so that a measurement of the fluctuations in the beat frequency provides a direct measurement of the fractional change in the length of the long path. The relationship between beat frequency and strain has no adjustable constants or calibration factors.

The beat frequency is digitally recorded 10 times/hour along with other information including local barometric pressure, etc. The digital data are bandpass filtered using a symmetric convolution filter and then decimated to one sample every two hours for comparison with theory.

Data Analysis

We have used all of the components published by Cartwright and Edden (1973) in our analysis. For each component with frequency f_k we construct a time series of the form

$$a_k \cos(2\pi f_k t + \phi_k + \alpha_k) Y_n^m(\theta,\varphi) T(n,m,\theta,\varphi,\theta_s)$$

where a_k is the amplitude given by Cartwright and Edden, ϕ_k is the phase of the component, Y_n^m is the spherical harmonic computed at the station co-latitude θ and East Longitude φ. The quantity T is a function converting potential to strain along the axis of the strainmeter θ_s (Levine and Harrison, 1976). The quantity α_k is -90° if (n+m) is odd and is zero otherwise (Cartwright and Tayler, 1971).

The theoretical series were further modified by a function to correct for local topography, local crustal inhomogeneities, cavity effects and ocean loads (Levine and Harrison, 1976). We assumed initially that these effects do not vary rapidly with frequency, so that all of the diurnal components have the same correction as O_1, and that all of the semidiurnal components have the same correction as M_2.

The terms are then grouped by frequency. Each frequency group contains all of the terms (regardless of parentage) which differ by less than one cycle/year from each other.

This process produces 48 time series. We fit these series to our data by the least-squares method using an adjustable amplitude and an adjustable phase for each cycle/year group.

The results of this process are shown in Fig. 1 for the diurnal amplitude, Fig. 2 for the diurnal phase, and Fig. 3 for the semidiurnal amplitude and phase. The error bars represent one standard deviation and are obtained from estimates of the

*Fellow, Joint Institute for Laboratory Astrophysics of the National Bureau of Standards and the University of Colorado.

Proc. of the 9th GEOP Conference, *An International Symposium on the Applications of Geodesy to Geodynamics, October 2-5, 1978*, Dept. of Geodetic Science Rept. No. 280. The Ohio State Univ., Columbus, Ohio 43210.

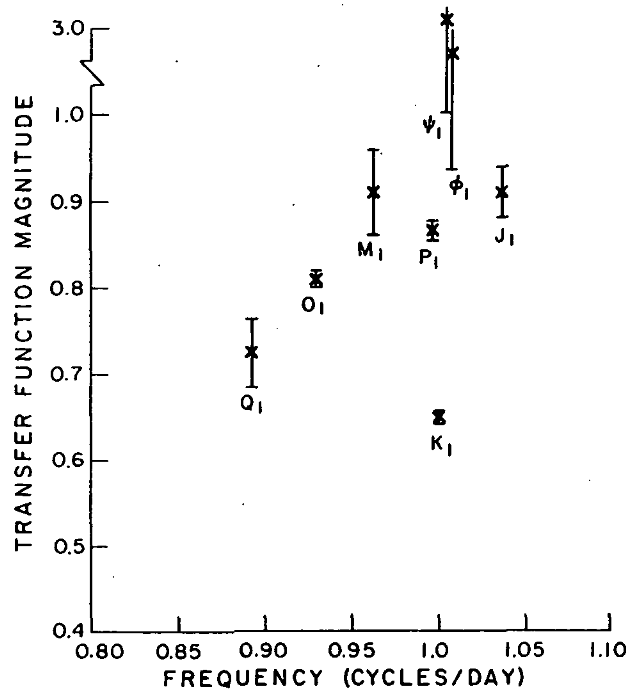

Fig. 1. Normalized transfer function for the diurnal tides. For clarity only the major components are plotted. The error bars are one standard deviation.

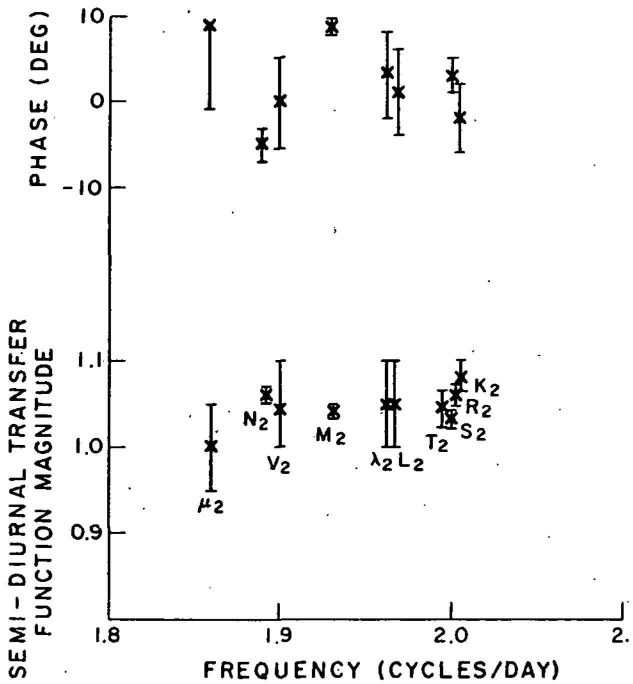

Fig. 3. Normalized transfer function amplitude (lower curve) and phase (upper curve) for the semidiurnal tides. For clarity only the major components are plotted. The error bars are one standard deviation.

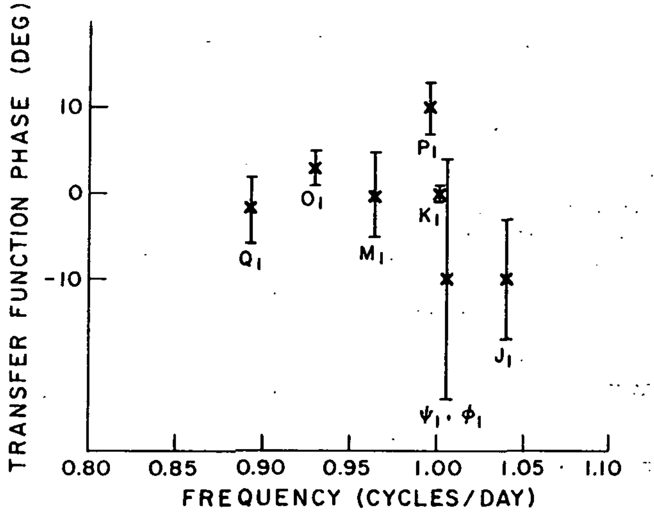

Fig. 2. Normalized transfer function phase for the diurnal tides. For clarity only the major components are plotted. The error bars are one standard deviation.

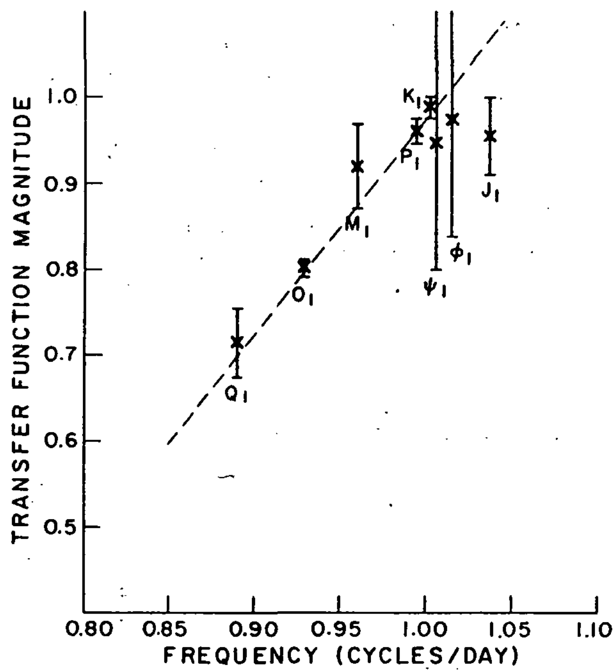

Fig. 4. Normalized transfer function amplitude for the diurnal tides when the frequency-dependent Love numbers are inserted into the fitting function. The dashed line is obtained by fitting a straight line to the transfer function amplitudes weighted by their respective uncertainties.

noise.

The amplitude of the diurnal transfer function shows a statistically significant structure. There appears to be a significant decrease in the transfer function for the lower frequency diurnal components. We attribute this to a slow change in the contribution of the ocean load to our observed data.

More significantly there is a dip in the transfer function near one cycle/day (we have not plotted the S_1 amplitude at one cycle/day since it is heavily contaminated by thermoelastic processes). This dip is consistent with the effects of the nearly diurnal resonance associated with the liquid core. The consistency can be shown more clearly by inserting the resonance directly into the fitting function. This may be done using the frequency dependent Love numbers published by Molodensky (1961) and Shen and Mansinha (1976). These Love numbers produce changes in the transfer function at all of the diurnal frequencies but, except near one cycle/day, the new transfer function lies within one standard deviation of the old one. The modified transfer function is shown in Fig. 4. As can be seen from the figure the resonance models account for the dip in the transfer function near one cycle/day. Unfortunately, the data cannot be used to infer the fine structure near the resonance. Furthermore, the insertions of realistic estimates for the energy loss in the nearly diurnal band may affect the shapes predicted by all of the models somewhat. It is unlikely we will be able to test any of these effects using our data. The strongest test for the resonance obtainable from our data comes from the components P_1 and K_1 which are really on the tail of the resonance function. Our measurements at ϕ_1 and ψ_1 (where the resonance has the largest effect) have error bars whose size probably does not permit them to be used in a quantitative comparison between theory and experiment or in attempts to differentiate among the various calculations of the effect of the resonance.

These results may be compared with the analysis of Warburton and Goodkind (1977) using data from a cryogenic gravimeter, with the gravimeter data analysis of Abours and Lecolazet (1978) and with the analyses of Lecolazet and Melchior (1977). Their results are generally in agreement with ours, confirming the general shape of the resonance. None of the analyses is able to make a quantitative comparison with theory because of the relatively poor signal-to-noise ratio in the measurements of the crucial components ϕ_1 and ψ_1.

The semidiurnal transfer function amplitude shows far less structure, and the agreement between experiment and theory is quite good. There is no evidence of anomaly at S_2 (two cycles/day) confirming that the anomaly at S_1 is almost certainly of thermoelastic origin.

We may place upper limits on the various PPN parameters by calculating the magnitudes of the anomalous tidal components in terms of the PPN parameters α_2 and ζ_w. In this way we conclude $\alpha_2 \leq 0.007$ and $\zeta \leq 0.005$.

Conclusions

These results confirm the general correctness of the various published earth models. From the point of view of the current discussion they are also significant in that they illustrate the sort of measurements that can be performed with laser strainmeters. These measurements show that we can make meaningful measurements of diurnal strain changes at the 10^{-10} level, and that almost continuous operation for several years is possible.

From the geodetic point of view, the limitation on the utility of laser strainmeters arises from their sensitivity to local effects, especially to spurious motions near the piers.

Spurious motions of the piers at the level of millimeters will even play a significant role in geodetic measurements made over much longer baselines. Motions of this magnitude represent fractional changes of parts in 10^8 even over 50 km baselines, so that such effects will make significant contributions to the error budget of any geodetic instrument now in operation or under construction.

It is important to compare the data obtained with laser strainmeters with measurements obtained using electromagnetic distance measuring equipment operating over parallel baselines. If this comparison shows that the two provide a consistent picture of the regional strain field, then laser strainmeters may prove useful in measuring regional strain at fixed observatories. In this service they can provide significantly higher sensitivity than any other technique.

References

Abours, S. and R. Lecolazet, New results about the dynamical effects of the liquid outer core as observed in Strasbourg, Proc. 8th Int. Conf. on Earth Tides, 1978 (in press).

Berger, J. and J. Levine, The spectrum of the earth noise from 10^{-8} to 10^{+2} Hz, J. Geophys. Res., 79, 1210-14, 1974.

Cartwright, D.E. and A.C. Edden, Corrected tables of tidal harmonics, Geophys. J. Roy. Astr. Soc., 33, 253-2640, 1973.

Cartwright, D.E. and R.J. Tayler, New computations of the tide-generating potential, Geophys. J. Roy. Astr. Soc., 23, 45-74, 1971.

Lecolazet, R. and P. Melchior, Experimental determination of the dynamical effects of the liquid core of the earth, Ann. Geophys., 33, 11-22, 1977.

Levine, J. and J.L. Hall, Design and operation of a methane absorption stabilized laser strainmeter, J. Geophys. Res., 77, 2595-2609, 1972.

Levine, J. and J.C. Harrison, Earth tide strain measurements in the Poorman Mine near Boulder, Colorado, J. Geophys. Res., 81, 2543-55, 1976.

Molodensky, M.S., The theory of nutations and diurnal earth tides, Comm. Obs. R. Belgique, 188, 25-56, 1961.

Shen, Po-Yu and L. Mansinha, Oscillation, nutation and wobble of an elliptical rotating earth with liquid outer core, Geophys. J. Roy. Astr. Soc., 46, 467-496, 1976.

Warburton, R.J. and J.M. Goodkind, Detailed gravity tide spectrum between 1 and 4 cycles per day, Geophys. J. Roy. Astr. Soc., 52, 117-130, 1977.