

# Observation of the Nearly Diurnal Resonance of the Earth Using a Laser Strainmeter

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**Abstract.** We have used two years of strain tide data to study the response of the Earth to the diurnal and semidiurnal tidal excitations. Our results show that there is significant structure in the response of the earth to tidal excitations near one cycle/sidereal day. This structure agrees with the resonance behavior predicted from the calculations of the forced elastic-gravitational response of an elliptical, rotating earth with a liquid outer core. The data can also be used to test for possible preferred frames and spatial anisotropies. We find that upper bounds on the parameterized post-Newtonian (PPN) parameters which characterize these effects are  $\alpha_2 \leq 0.007$  and  $\zeta_w \leq 0.005$ .

## Introduction

We have analyzed approximately two years of strain-tide data obtained using the 30-meter laser strainmeter we have previously described (Levine and Hall, 1972).

## Data Acquisition

The data were obtained using a 30-meter long laser strainmeter located in the Poorman Mine, an unworked gold mine located approximately 8 km west of Boulder, Colorado at latitude  $40.03^\circ\text{N}$ , and longitude  $254.67^\circ\text{E}$ .

The heart of the strainmeter is an evacuated 30-meter Fabry-Perot interferometer located along the length of the tunnel. The axis of the interferometer is  $7^\circ$  west of North.

The interferometer is illuminated by a  $3.39\text{-}\mu\text{m}$  helium-neon laser. A servo loop piezoelectrically tunes the laser to keep its wavelength coincident with one of the transmission maxima of the long interferometer. The frequency of the laser,  $f$ , is therefore related to the length of the interferometer,  $L$ , by

$$f = \frac{nc}{2L}$$

where  $n$  is an integer and  $c$  is the velocity of light. Thus

$$\frac{\Delta f}{f} = - \frac{\Delta L}{L}$$

A second  $3.39\text{-}\mu\text{m}$  laser is stabilized using saturated absorption in methane. The beat frequency between the two lasers is extracted for further processing. Then

$$\frac{\Delta f_{\text{beat}}}{f} = \frac{\Delta L}{L}$$

or

$$\Delta f_{\text{beat}} = 8.85 \times 10^{13} \frac{\Delta L}{L}$$

so that a measurement of the fluctuations in the beat frequency provides a direct measurement of the fractional change in the length of the long path. The relationship between beat frequency and strain has no adjustable constants or calibration factors.

The beat frequency is digitally recorded 10 times/hour along with other information including local barometric pressure, etc. The digital data are bandpass filtered using a symmetric convolution filter and then decimated to one sample every two hours for comparison with theory.

## Data Analysis

We have used all of the components published by Cartwright and Edden (1973) in our analysis. For each component with frequency  $f_k$  we construct a time series of the form

$$a_k \cos(2\pi f_k t + \phi_k + \alpha_k) Y_n^m(\theta, \varphi) T(n, m, \theta, \varphi, \theta_s)$$

where  $a_k$  is the amplitude given by Cartwright and Edden,  $\phi_k$  is the phase of the component,  $Y_n^m$  is the spherical harmonic computed at the station co-latitude  $\theta$  and East Longitude  $\varphi$ . The quantity  $T$  is a function converting potential to strain along the axis of the strainmeter  $\theta_s$  (Levine and Harrison, 1976). The quantity  $\alpha_k$  is  $-90^\circ$  if  $(n+m)$  is odd and is zero otherwise (Cartwright and Tayler, 1971).

The theoretical series were further modified by a function to correct for local topography, local crustal inhomogeneities, cavity effects and ocean loads (Levine and Harrison, 1976). We assumed initially that these effects do not vary rapidly with frequency, so that all of the diurnal components have the same correction as  $O_1$ , and that all of the semidiurnal components have the same correction as  $M_2$ .

The terms are then grouped by frequency. Each frequency group contains all of the terms (regardless of parentage) which differ by less than one cycle/year from each other.

This process produces 48 time series. We fit these series to our data by the least-squares method using an adjustable amplitude and an adjustable phase for each cycle/year group.

The results of this process are shown in Fig. 1 for the diurnal amplitude, Fig. 2 for the diurnal phase, and Fig. 3 for the semidiurnal amplitude and phase. The error bars represent one standard deviation and are obtained from estimates of the

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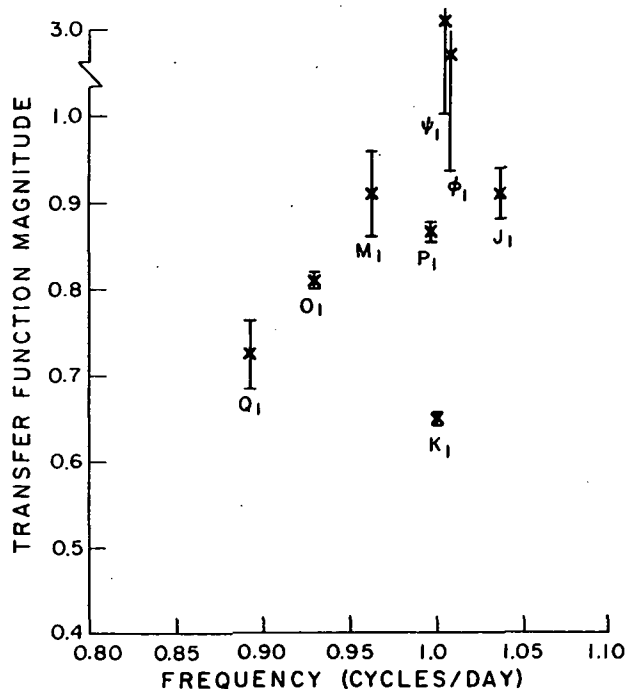


Fig. 1. Normalized transfer function for the diurnal tides. For clarity only the major components are plotted. The error bars are one standard deviation.

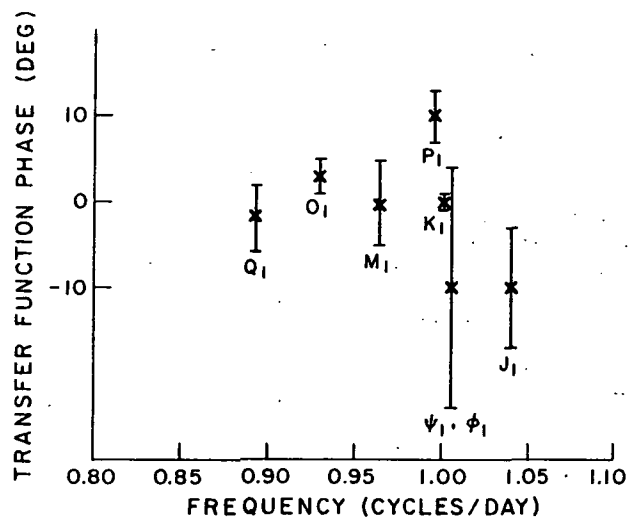


Fig. 2. Normalized transfer function phase for the diurnal tides. For clarity only the major components are plotted. The error bars are one standard deviation.

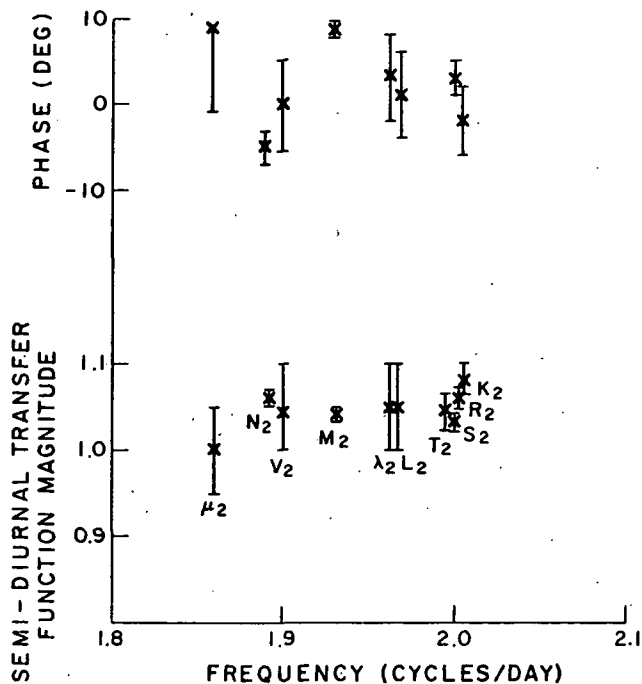


Fig. 3. Normalized transfer function amplitude (lower curve) and phase (upper curve) for the semidiurnal tides. For clarity only the major components are plotted. The error bars are one standard deviation.

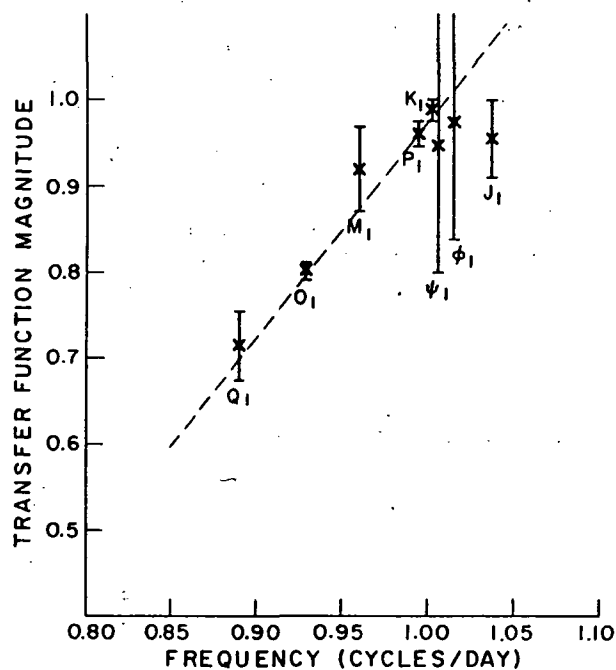


Fig. 4. Normalized transfer function amplitude for the diurnal tides when the frequency-dependent Love numbers are inserted into the fitting function. The dashed line is obtained by fitting a straight line to the transfer function amplitudes weighted by their respective uncertainties.

noise.

The amplitude of the diurnal transfer function shows a statistically significant structure. There appears to be a significant decrease in the transfer function for the lower frequency diurnal components. We attribute this to a slow change in the contribution of the ocean load to our observed data.

More significantly there is a dip in the transfer function near one cycle/day (we have not plotted the  $S_1$  amplitude at one cycle/day since it is heavily contaminated by thermoelastic processes). This dip is consistent with the effects of the nearly diurnal resonance associated with the liquid core. The consistency can be shown more clearly by inserting the resonance directly into the fitting function. This may be done using the frequency dependent Love numbers published by Molodensky (1961) and Shen and Mansinha (1976). These Love numbers produce changes in the transfer function at all of the diurnal frequencies but, except near one cycle/day, the new transfer function lies within one standard deviation of the old one. The modified transfer function is shown in Fig. 4. As can be seen from the figure the resonance models account for the dip in the transfer function near one cycle/day. Unfortunately, the data cannot be used to infer the fine structure near the resonance. Furthermore, the insertions of realistic estimates for the energy loss in the nearly diurnal band may affect the shapes predicted by all of the models somewhat. It is unlikely we will be able to test any of these effects using our data. The strongest test for the resonance obtainable from our data comes from the components  $P_1$  and  $K_1$  which are really on the tail of the resonance function. Our measurements at  $\phi_1$  and  $\psi_1$  (where the resonance has the largest effect) have error bars whose size probably does not permit them to be used in a quantitative comparison between theory and experiment or in attempts to differentiate among the various calculations of the effect of the resonance.

These results may be compared with the analysis of Warburton and Goodkind (1977) using data from a cryogenic gravimeter, with the gravimeter data analysis of Abours and Lecolazet (1978) and with the analyses of Lecolazet and Melchior (1977). Their results are generally in agreement with ours, confirming the general shape of the resonance. None of the analyses is able to make a quantitative comparison with theory because of the relatively poor signal-to-noise ratio in the measurements of the crucial components  $\phi_1$  and  $\psi_1$ .

The semidiurnal transfer function amplitude shows far less structure, and the agreement between experiment and theory is quite good. There is no evidence of anomaly at  $S_2$  (two cycles/day) confirming that the anomaly at  $S_1$  is almost certainly of thermoelastic origin.

We may place upper limits on the various PPN parameters by calculating the magnitudes of the anomalous tidal components in terms of the PPN parameters  $\alpha_2$  and  $\zeta_w$ . In this way we conclude  $\alpha_2 \leq 0.007$  and  $\zeta \leq 0.005$ .

## Conclusions

These results confirm the general correctness of the various published earth models. From the point of view of the current discussion they are also significant in that they illustrate the sort of measurements that can be performed with laser strainmeters. These measurements show that we can make meaningful measurements of diurnal strain changes at the  $10^{-10}$  level, and that almost continuous operation for several years is possible.

From the geodetic point of view, the limitation on the utility of laser strainmeters arises from their sensitivity to local effects, especially to spurious motions near the piers.

Spurious motions of the piers at the level of millimeters will even play a significant role in geodetic measurements made over much longer baselines. Motions of this magnitude represent fractional changes of parts in  $10^8$  even over 50 km baselines, so that such effects will make significant contributions to the error budget of any geodetic instrument now in operation or under construction.

It is important to compare the data obtained with laser strainmeters with measurements obtained using electromagnetic distance measuring equipment operating over parallel baselines. If this comparison shows that the two provide a consistent picture of the regional strain field, then laser strainmeters may prove useful in measuring regional strain at fixed observatories. In this service they can provide significantly higher sensitivity than any other technique.

## References

- Abours, S. and R. Lecolazet, New results about the dynamical effects of the liquid outer core as observed in Strasbourg, Proc. 8th Int. Conf. on Earth Tides, 1978 (in press).
- Berger, J. and J. Levine, The spectrum of the earth noise from  $10^{-8}$  to  $10^{+2}$  Hz, *J. Geophys. Res.*, **79**, 1210-14, 1974.
- Cartwright, D.E. and A.C. Edden, Corrected tables of tidal harmonics, *Geophys. J. Roy. Astr. Soc.*, **33**, 253-2640, 1973.
- Cartwright, D.E. and R.J. Tayler, New computations of the tide-generating potential, *Geophys. J. Roy. Astr. Soc.*, **23**, 45-74, 1971.
- Lecolazet, R. and P. Melchior, Experimental determination of the dynamical effects of the liquid core of the earth, *Ann. Geophys.*, **33**, 11-22, 1977.
- Levine, J. and J.L. Hall, Design and operation of a methane absorption stabilized laser strainmeter, *J. Geophys. Res.*, **77**, 2595-2609, 1972.
- Levine, J. and J.C. Harrison, Earth tide strain measurements in the Poorman Mine near Boulder, Colorado, *J. Geophys. Res.*, **81**, 2543-55, 1976.
- Molodensky, M.S., The theory of nutations and diurnal earth tides, *Comm. Obs. R. Belgique*, **188**, 25-56, 1961.

Shen, Po-Yu and L. Mansinha, Oscillation, nutation and wobble of an elliptical rotating earth with liquid outer core, Geophys. J. Roy. Astr. Soc., 46, 467-496, 1976.

Warburton, R.J. and J.M. Goodkind, Detailed gravity tide spectrum between 1 and 4 cycles per day, Geophys. J. Roy. Astr. Soc., 52, 117-130, 1977.