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Thermal Power Systems
Small Power Systems Applications Project

Decision Analysis for Evaluating and Ranking Small Solar Thermal Power System Technologies

Volume I - A Brief Introduction to Multiattribute Decision Analysis

SPSA

Prepared for
U.S. Department of Energy
by
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

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Volume I - A Brief Introduction to Multiattribute Decision Analysis

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June 1, 1978

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Abstract

Multiattribute decision analysis is a methodology for providing information to a decision maker for comparing and selecting between complex alternatives. This Report presents a brief introduction to the principal concepts of the Keeney and Raiffa approach to multiattribute decision analysis. The Report presents the concepts of decision alternatives, outcomes, objectives, attributes and their states, attribute utility functions, and the necessary independence properties for the attribute states to be aggregated into a numerical representation of the preferences of the decision maker for the outcomes and the decision alternatives.

FOREWORD

This Report has been prepared in response to the many requests for an introductory level explanation of the multiattribute decision analysis methods used in the evaluation of technology alternatives for Small Power Systems Applications (SPSA). It is intended to focus on the methodology rather than on the context of the illustrative examples. Hence, simple examples are employed here. These simple examples are not drawn from Small Power Systems Applications due to the rapidly evolving and changing criteria and attribute scales to be used in that context. It is believed that the use of examples likely to stimulate extensive discussion would distract the reader from the introductory level explanation.

This Report is intended to serve as an introduction to the Report on the "Criteria and Methodology for Ranking Small Power Systems Applications Technology Alternatives." Thus, this introduction is Volume I of a three-volume effort. The second volume contains a detailed description of the criteria, their attributes, and the decision analysis process developed to evaluate and rank technology alternatives for small (1-10 MWe) solar thermal power systems applications. The third, and final volume in this series will cover the interviews and subsequent analysis for evaluating and ranking the SPSA technology alternatives. Volume III is presently scheduled for completion in May, 1979.

The immediate purpose of this document is to aid people in the understanding and use of the second and third volumes at the Jet Propulsion Laboratory, the Department of Energy and cooperating organizations such as Battelle-Pacific Northwest Laboratories and the Solar Energy Research Institute. An additional use of this introductory Report is to explain multiattribute decision analysis to individuals facing other problems that may be amenable to this methodology.

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1. Introduction

Multiattribute decision analysis is a methodology for providing information to a decision maker for comparing and selecting between complex alternatives. The methodology of multiattribute decision analysis is derived from the techniques of operations research, statistics, economics, mathematics, and psychology. Thus, researchers from a wide range of disciplines have participated in the development of multiattribute decision analysis. The first books and papers on the subject appeared in the late 1960's [1-4]. The most practical, extensive, and complete presentation of an approach to multiattribute decision analysis is given in the 1976 work of Keeney and Raiffa [5]. While several approaches to multiattribute decision analysis have been developed [6-17], the principal method described in this Report corresponds to that of Keeney and Raiffa [5].

2. The Decision Analysis Paradigm

The paradigm for any decision analysis is depicted in Figure 1. A problem or a requirement exists. A set of alternative actions, called "alternatives," are available. The decision maker must select one, and only one, alternative. Examples of alternatives might relate to a corporate policy decision, a capital budgeting decision, selection of a system design, or a personnel decision. The selected alternative is an input to a system model which is a set of descriptions or mathematical equations relating the selected alternative to the resultant "outcome."

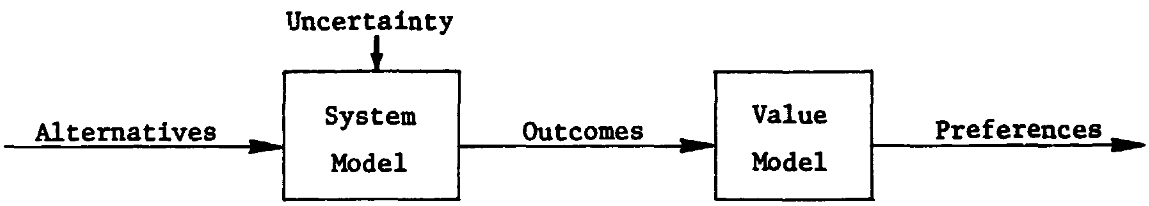


Figure 1. The Decision Analysis Paradigm

When no uncertainty exists in the system model, then the selection of a specific alternative results in a prescribed outcome. For example, the investment of $1,000 in a Federally insured savings account at 6% interest compounded annually will result in an outcome for certain of $1,060 after one year. If uncertainty exists, then the selection of a specific alternative could result in any one of several outcomes. Investing in the stock market would be a prime example. The alternative would be the investment portfolio selected. The system model would include all of the economic forces and uncertainties that act on the stock market. The outcome would be the value of the investment portfolio at the time it was sold plus any dividends that had accrued. This is an extreme example of a system model dominated by uncertainty, because no valid system model of the actions of the stock market presently exists [18, 19].

The problems to which decision analysis can be most usefully applied lie somewhere between these two extremes. For example, an advanced energy system is to be developed. Certain engineering principles and experience with prototype designs give a good indication of how the system will perform. However, some uncertainties will still exist such as the cost of the energy system in mass production or the reliability in commercial operation.

In the decision analysis paradigm, the outcomes of the system model provide the input to the decision maker's value model. The output of the value model states the decision maker's preferences, either in terms of a rank ordering of the outcomes or a numerical scale that measures strength of preference as well as rank ordering.

3. The Decision Maker

Any decision analysis assumes that there is a single decision maker or a group of people that can be expected to reach a consensus of opinion. It is the preferences of this person, or of this group, that will be used in the decision model. This requires that the decision analysts interact with the decision maker or his representatives in formulating the value model. Where consensus is required of a group of people, the consensus must be obtained by means external to the decision analysis. There presently exists no analytical means of combining disparate preferences into a single or "true" preference, nor may there ever be [5, 20-23]. There exist axiomatic formulations for obtaining group preferences, based on concepts of rationality, fairness, and efficiency, but they ultimately all contain one or more logical assumptions that can be seriously questioned. Even the selection of a group decision model is in itself a group decision. An extensive discussion of a group decision analysis where the members of the group had disparate preferences is that of Dyer and Miles [24-26].

4. Objectives and Attributes

The multiattribute aspect of decision analysis appears because, for most complicated problems or complex systems, the outcomes must be evaluated in terms of several objectives (also called "goals" or "criteria"). Objectives of a decision analysis are stated in terms of properties, either desirable or undesirable, that determine the decision maker's preferences for the outcomes. For the design of an automobile, several objectives might be: (1) to minimize cost, (2) to minimize fuel consumption, (3) to minimize air pollution, and (4) to maximize safety. The purpose of the value model is to take the outcomes of the system model, determine the degree to which the outcomes satisfy each of the objectives, and then make the necessary trade-offs between the objectives to arrive at a ranking for the outcomes that correctly expresses the preferences of the decision maker.

The value model is developed in terms of a hierarchy of objectives and sub-objectives, as shown in Figure 2 for the design of an automobile. In order to quantify the value model, a unit of measurement must be assigned to the lowest members of the objectives hierarchy. These measurement units are called "attributes," and they are scaled in convenient units to measure the degree to which the associated objective is satisfied. In Figure 2, six attributes are used to quantify the value model, and the outcomes of the associated system model would be expressed as a six-component vector, with the components corresponding to the six attributes, i.e., $x = (x_1,x_2,x_3,x_4,x_5,x_6)$, where x_i is the ith attribute of the value model. A specific occurrence of an attribute is called a "state" of the attribute. An "attribute state" for the objective "minimize fuel consumption" might be x_3 = 35 miles per gallon.

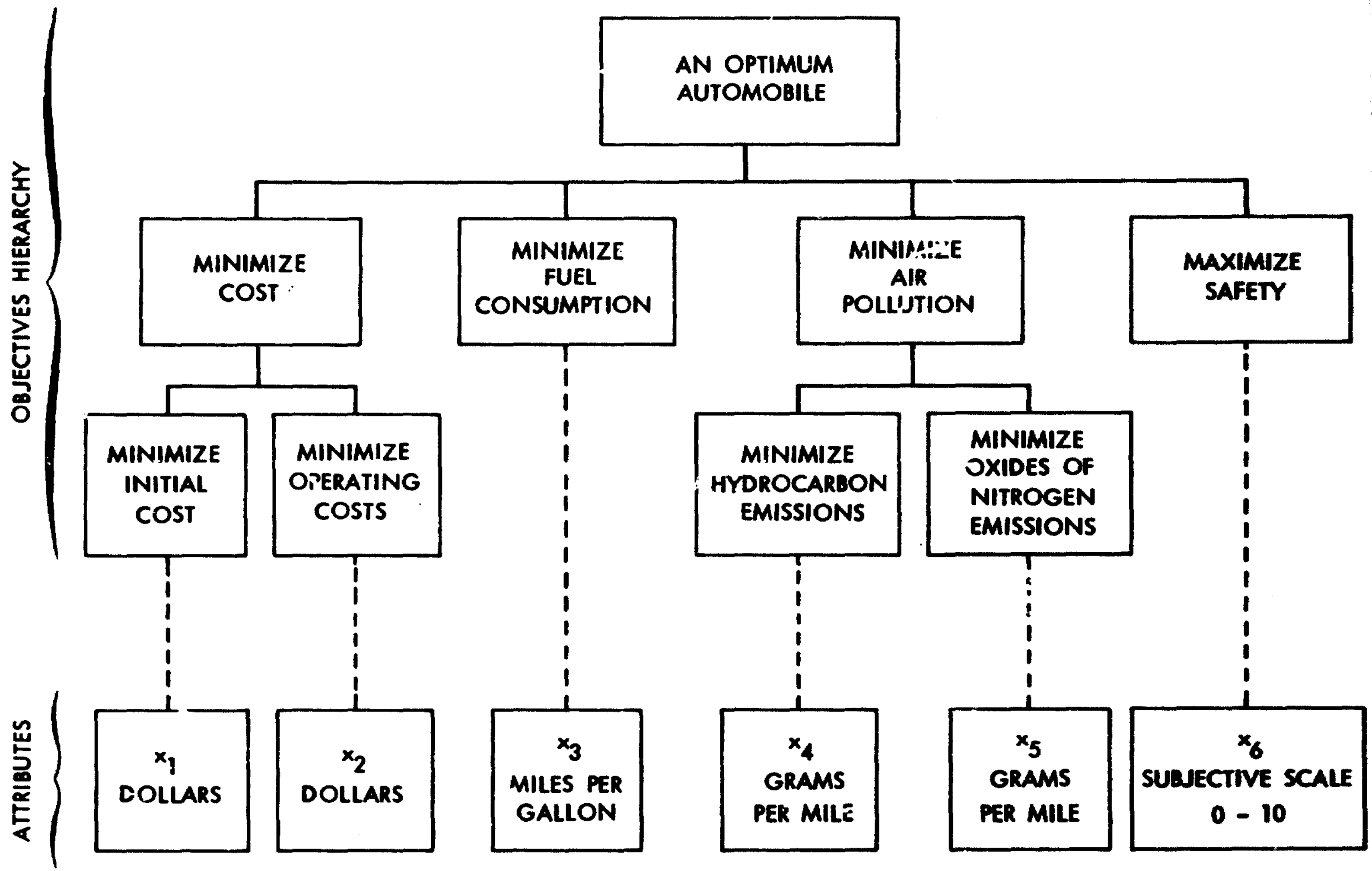


Figure 2. Value Model Hierarchy for Objectives and Attributes for Automobile Design

5. Aggregating Objectives into a Value Model

Once attributes have been assigned to all the objectives and attribute states have been determined for all the outcomes, it is then necessary to aggregate the attribute states of each outcome through trade-offs into a single unit of measurement that will correctly represent the preference ordering of the decision maker for the outcomes.

One common method for aggregating the attribute states is the "willingness to pay" or "pricing out" technique [5]. One attribute is singled out as the measurement attribute, preferably an attribute measured in money. Then, one at a time, each of the other attributes are changed to a reference state, with the money state of the measurement attribute adjusted by means of an assessed trade-off or rate of substitution to compensate for the change in the other attribute. Through this process, all the outcomes are expressed in terms of n attributes, n-1 of which all have been adjusted to the same reference attribute state. The preference ranking of the decision maker is then expressed by the state assigned to the measurement attribute. Unfortunately, easy as this method is to apply, it is valid with constant rates of substitution only when two conditions are satisfied:

1. The trade-offs for the measurement attribute and any other attribute are independent of the states of the other n-2 attributes.
2. The trade-offs for the measurement attribute and any other attribute do not depend on the states of either the measurement attribute or the other attribute. (This literally is what "constant rate of substitution" means.)

For two-attribute outcomes, $x = (x_1,x_2)$, this requires that the trade-off curves be straight lines (A trade-off curve is a line through a set of outcomes for which the decision maker is "indifferent" or "equally prefers.") Thus, this "pricing out" technique is extremely difficult to apply to objectives whose attributes exhibit a diminishing return to scale. For outcomes represented by more than two attributes, similar arguments apply.

For the Keeney and Raiffa formulation [5] that is to be described in the remainder of this report, the attribute states are quantified on a numerical scale that represents the preferences of the decision maker for the various states that the attribute can assume. The function that transforms an attribute state into a numerical representation of attribute preference is called an attribute utility function. It is represented by the expression $u_i(x_i)$, the attribute utility function value for the ith attribute in the attribute state x_i. The proper algebraic expression combining the attribute utility functions is called an outcome utility function $u(x)$, with $x = (x_1,\ldots,x_n)$ being the n-attribute outcome. An outcome utility function is a numerical representation of the decision maker's preferences for the outcomes. The use of the word "utility" to represent preference or value has a rich and venerable history in economics [27-31]. It is convenient to measure attribute utility functions on a scale from $u_i = 0.0$ to $u_i = 1.0$, where $u_i = 0.0$ corresponds to the least-preferred ith attribute state that occurs among the outcomes under consideration, and $u_i = 1.0$ corresponds to the most-preferred state. An example of an attribute utility function for the attribute x_4 of Figure 2 (grams per mile of hydrocarbon emissions) is shown in Figure 3.

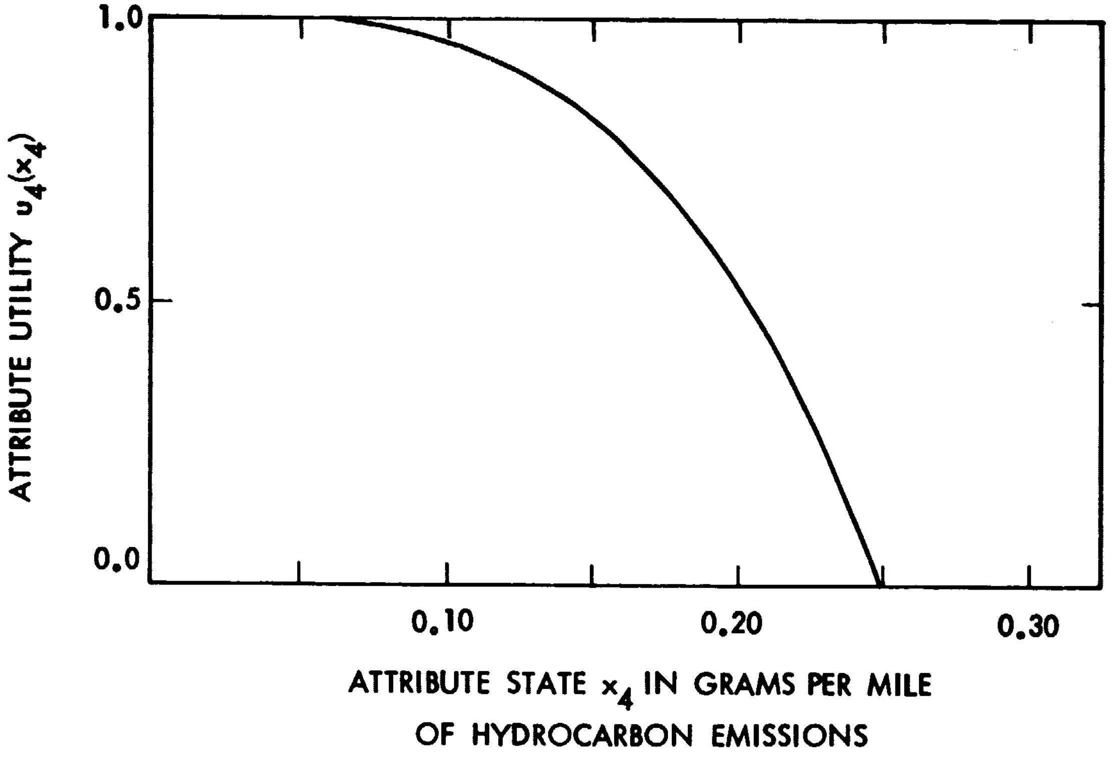


Figure 3. Typical Attribute Utility Function

Since $u_4(x_4 = 0.2) = 0.5$ in Figure 3, one could interpret this as meaning that the decision maker has the same increase in preference in decreasing hydrocarbon emissions from 0.25 grams/mile to 0.20 grams/mile as decreasing hydrocarbon emissions from 0.20 grams/mile to 0.05 grams/mile. Other interpretations are possible.

If all the other attributes of an outcome are held at constant states, it is theoretically possible to construct an attribute utility function as shown in Figure 3. There are several techniques for determining the form of the attribute utility function. These include direct magnitude estimation, preference difference assessments (either rank ordering of preference differences or equal preference interval scaling), ratio scaling, and the lottery method [12,32]. The lottery method will be used in this Report because of its theoretical consistency, its widespread discussion in the literature, and the fact that it provides a straighforward way of handling uncertainty.

If the other attributes of an outcome are not held at constant states, but instead are allowed to vary, then the concept of an attribute utility function has no theoretical or practical validity unless certain independence conditions are satisfied. The most obvious independence condition is that the form of the attribute utility function should not change if the other attributes are held at constant, but different, attribute states. This independence condition allows the concept of an attribute utility function to be meaningful. The name given to this type of attribute independence is "utility independence."

If the attributes satisfy utility independence, then it would seem to naturally follow that the utility of the outcomes would be computed as a weighted sum of the attribute utilities, e.g., where $x = (x_1, x_2)$,

$$u(x) = k_1 u_1(x_1) + k_2 u_2(x_2) \quad .$$

Unfortunately, utility independence of the attributes alone does not suffice mathematically to insure that the outcome utilities can be correctly computed as the weighted sum of the attribute utilities. For the two-attribute case, $x = (x_1, x_2)$, the correct formula, assuming utility independence, is [5,33]:

$$u(x) = k_1 u_1(x_1) + k_2 u_2(x_2) + (1 - k_1 - k_2)\ u_1(x_1)\ u_2(x_2) \quad .$$

The correct mathematical derivation shows that a multiplicative term $u_1(x_1)\ u_2(x_2)$ with a weighting factor of $(1 - k_1 - k_2)$ appears.

A simple example can illustrate why the weighted sum can yield incorrect orderings of outcome preference. Consider the selection of an aircraft with the two attributes of concern being x_1 = payload weight and x_2 = aircraft range. A weighted sum could incorrectly give preferential ranking to an aircraft with a very large payload weight, but whose range was less than the majority of the routes being considered. In the two-attribute formula just given, assigning small values to k_1 and k_2 results in a formula approximately of the form:

$$u(x) \approx u_1(x_1)\ u_2(x_2) \quad .$$

It is in this multiplicative term that the requirement x_1 and x_2 have preferred states is correctly modeled.

Keeney has developed a practical algebraic expression for combining the attribute utility functions to obtain an outcome utility function. Rather than testing each attribute for utility independence, Keeney has shown that it is only necessary to test one attribute for utility independence (call it the reference attribute), and then to verify that the pairwise trade-offs of the reference attribute versus each of the other n-1 attributes are independent of the states of the remaining n-2 attributes. This pairwise trade-off independence is called "preferential independence." These n independence conditions then lead to an algebraic expression for the outcome utility function of the form:

$$u(x) = \frac{1}{k}\left\{\prod_{i=1}^{n}\left[1 + kk_i u_i(x_i)\right] - 1\right\}$$

where:

$u(x)$ = the outcome utility function.

$u_i(x_i)$ = the attribute utility function of the ith attribute.

k_i = a scaling constant that determines the "weight" or "importance" of the ith attribute.

k = an algebraic function of the k_i's, which scales $u(x)$ from $u = 0.0$ to $u = 1.0$.

For the details of the proof, the reader is referred to Keeney [34] and Keeney and Raiffa [5].

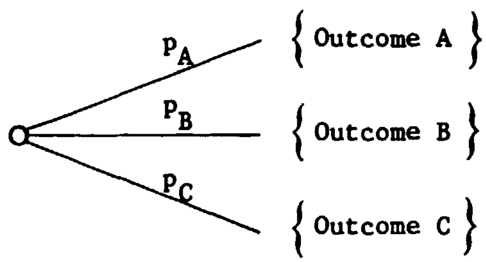
The n scaling constants, k_i, determine the relative importance of the associated attributes. The k_i's range in value from 0.0 to 1.0, with the larger values associated with the more important attributes.

When the necessary independence conditions are violated, it is possible to divide the attribute state ranges into intervals over which the independence conditions are approximately valid, or the set of attributes can be redefined so the independence conditions are valid. It is important to distinguish these independence conditions from the "technical dependence" of variables which define the system model. This latter type of dependence arises very naturally in engineering systems, and only restricts the set of feasible systems of the system model. In the aircraft example cited earlier in this Report, the engineering constraints on the design of the aircraft result in a technical dependence (or functional relationship) between aircraft range and payload weight. It must be stressed that this "technical dependence" has no effect on the value model and is not to be confused with the utility and preferential independence conditions required of the Keeney and Raiffa formulation of the value model.

6. The Lottery Method

The lottery method, first introduced by von Neumann and Morgenstern [35], serves three purposes in the Keeney and Raiffa formulation [5] of the multiattribute decision problem. It permits the construction of outcome or attribute utility functions, given the preferences of the decision maker for various outcome or attribute states; it permits the determination of the attribute scaling constants; and it permits uncertainty to be treated in a straightforward manner such that utility function values can be assigned to alternatives with uncertain outcomes.

A lottery is a gamble between a set of outcomes in which a single outcome will result. Associated with each outcome is a probability of the occurrence of that outcome. Lotteries are diagrammed as follows:



The diagram is to be interpreted as requiring that Outcome A will occur with probability p_A, Outcome B will occur with probability p_B, and Outcome C will occur with probability p_C. Only one outcome can result (mutually exclusive) and one of the outcomes will result for certain, i.e., $p_A + p_B + p_C = 1.0$ (collectively exhaustive). The probabilities may not be at all easy to assess, especially if the probability of the occurrence of an outcome is extremely small, or is based entirely on subjective judgment with little supporting data [36]. There are also a number of psychological biases that enter into the estimation of probabilities [37].

Ultimately, if the value model is to represent the decision maker's preferences, the probabilities used in the decision analysis must correspond to the judgment of the decision maker.

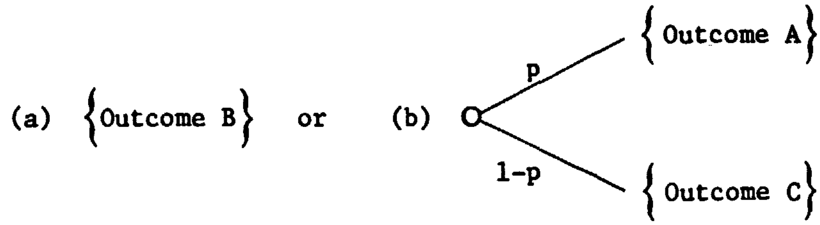
The most common use of the lottery method is for handling uncertainty and it is for this purpose that von Neumann and Morgenstern [35] introduced the method. Lotteries are used to represent the uncertainty that the decision maker may have with respect to the outcomes that could result from the selection of a specific alternative. Von Neumann and Morgenstern assumed that lotteries and outcomes could be rank ordered in preference. They proved that the correct numerical representation of the decision maker's preference for a lottery is simply the expected value of the utilities of the outcomes of the lotteries, i.e., for a lottery that yields Outcome A with probability p and Outcome C with probability 1-p, the correct numerical assignment of utility to the lottery is just:

$$u(\text{Lottery}) = p\, u(A) + (1-p)\, u(C) \quad .$$

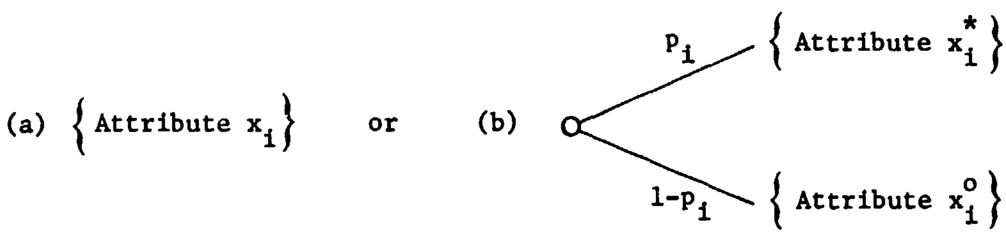
Several other axiomatic formulations and proofs of the decision criterion of expected utility have appeared in the literature [38-43]. Elementary proofs are given in Hadley [44] and Luce and Raiffa [45]. See also other References [46-50].

Lotteries are used in the following manner to establish outcome and attribute utility functions. The decision maker (or his representative) is first asked to rank order in preference three or more outcomes (or attribute states). Let us assume that Outcome A is preferred to Outcome B, which is in turn preferred to Outcome C. Then the decision maker is asked

for a probability p such that he is indifferent between receiving Outcome B for certain, or a lottery which yields Outcome A with probability p and Outcome C with probability 1-p. In terms of the following diagram, the decision maker is asked to assign a probability p to the following lottery such that he is indifferent between receiving (a) or (b):



If one arbitrarily assigns a utility of u(A) = 1.0 and u(C) = 0.0 (this is not absolutely necessary, but simplifes the arithmetic). The utility of Outcome B will just be u(b) = p. When the utility function of the ith attribute is being assessed, the most-preferred attribute state is arbitrarily assigned a utility of $u_i = 1.0$ and the least-preferred state is assigned a utility of $u_i = 0.0$. Then the attribute utility function is determined by the same lottery technique as used for the outcomes. If x_i^* is defined to be the most-preferred state of the ith attribute, and x_i^o is defined to be the least-preferred state, then the decision maker is asked to assign a probability p_i to the following lottery such that he is indifferent between receiving (a) or (b):



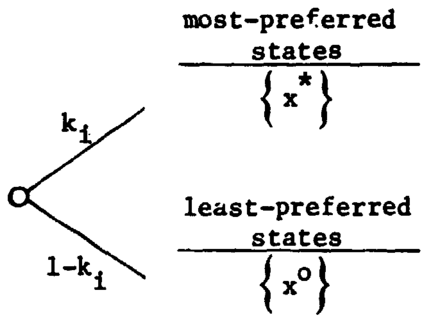
In the attribute utility function of Figure 3 in Section 5, p_i would equal 0.5 for x_i = 0.20 grams/mile, i.e., $u_i(x_i$ = 0.20 grams/mile) = 0.5.

The attribute scaling constants, k_i, are assessed in a similar manner using the lottery method. The scaling constant k_i measures the importance of the ith attribute, or the degree to which the ith attribute can influence the decision maker's preferences between the outcomes. Several artificial outcomes are constructed (they may not be technically feasible, but it is still possible to assign a preference to them in principle). The first outcome is constructed by placing all of the attributes at their most-preferred states (define this outcome to be x^*). The second outcome is constructed by placing all of the attributes at their least-preferred states (define this outcome to be x^o). Then construct an outcome in which all of the attributes are placed at their least-preferred states *except* the ith attribute, which is placed at its most-preferred state. Then the decision maker is asked to assign a probability k_i such that he is indifferent between (a) and (b):

(a)

Attribute	State
x_i	most-preferred
$x_j \neq x_i$	least-preferred

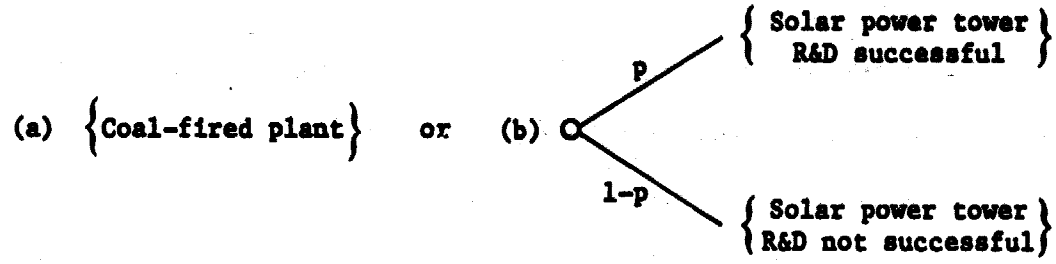
or (b)



The probability k_i is the attribute scaling constant for the ith attribute. This procedure is discussed in detail in Keeney and Raiffa [5].

Selecting a specific decision alternative results in a prescribed outcome, when no uncertainty exists in the system model. It is then possible to speak of decision alternatives and outcomes interchangeably because one implies the other. When uncertainty in the system model exists, specification of a decision alternative does not specify a unique outcome. Instead it specifies a lottery over a set of outcomes. Since the decision maker cannot have the most-preferred outcome for certain, the best that can be done is to select the alternative that results in the most-preferred lottery. Thus, a decision alternative can be assigned a utility, namely the expected utility of its corresponding lottery, and the most-preferred alternative is that for which the utility is maximized.

As a final example for this Report, consider the case of the chief executive officer of an electric utility company who must decide on the next unit of expansion for baseload electrical capacity. Let us suppose that after much engineering, environmental, and regulatory analysis that the decision alternatives are either (a) a coal-fired plant with stack scrubbers or (b) a solar central power tower. Where an initial capital investment is concerned, the alternative (a) of a coal-fired plant involves no uncertainty. The decision alternative and the outcome are one and the same. However, with the alternative (b) of a solar central power tower, the R&D program in progress could significantly influence the cost-competitiveness of the solar central power tower. Thus, if the chief executive officer has to make the decision prior the completion of the R&D program, he is faced with a decision to choose between alternative (a) which yields an outcome for certain, and alternative (b) which is a lottery over the outcomes of the R&D program. The decision analysis diagram would be:



The decision would be made based on a value model that includes as objectives such factors as annualized cost, environmental effects, coal availability, <u>and</u> the probability p that the R&D program would be successful.

7. Summary

Multiattribute decision analysis is a technique for aiding the decision maker in comparing and selecting between complex alternatives. The complexity may arise for several reasons: (1) the system model may be inherently complex, (2) the value model of the decision maker may contain many objectives, and trade-offs need to be made between these objectives, and (3) uncertainties may exist that prevent the selection of a specific alternative from uniquely determining the outcome. Multiattribute decision analysis has the analytical capability to handle all of these complexities with a practical and theoretically consistent approach.

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