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Viscosity and Thermal Conductivity of Model Jupiter Atmospheres

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SUMMARY

The viscosity and thermal conductivity coefficients are estimated for three models of the atmosphere of Jupiter: a Heavy Model consisting of 22% helium and 78% hydrogen, a Nominal Model consisting of 11% helium and 89% hydrogen, and a Light Model consisting of pure hydrogen. The effect of trace elements is neglected. Linearized approximations are used for the transport coefficients of the mixtures; these are found to be in almost constant ratio to the values for pure hydrogen, independent of temperature. Short basic language programs for computing the coefficients are listed.

INTRODUCTION

The Jovian atmosphere is presently thought to consist mainly of molecular hydrogen, H_2, perhaps mixed with helium, He, and with traces of methane, CH_4, and ammonia, NH_3. The amount of He seems to be the most uncertain; estimates range from about 11% by volume for the Nominal Jovian Atmosphere Model, to 22% for the Heavy Models, and to 0.0% for the Light Models. The traces of methane are thought to be about 0.1% and of ammonia about 0.02%; these are significant amounts as far as radiation absorption and emissivity are concerned, but are trivial amounts as far as thermodynamic and transport properties of the atmosphere are concerned. The temperature range of interest is from 100 to 1000 K. The thermodynamic properties needed for Jupiter probe design have been previously calculated and tabulated for this range (J. Sperans, Ames Research Center, NASA, private communication). Recently, a need has appeared for estimates of viscosity and thermal conductivity of the Jovian atmosphere, in connection with the subsonic motion of bodies through that atmosphere. The present report gives estimates of these quantities based on linearized formulas for the transport properties of mixtures and neglecting the effect of traces of CH_4 and NH_3.

VISCOSITY OF MODEL JUPITER ATMOSPHERES

Viscosity of Mixtures

According to rigorous kinetic theory, the coefficient of viscosity of a multicomponent gas mixture should be expressed as a ratio of determinants.

However, Hirschfelder et al. (ref. 1) show that the off-diagonal terms are small compared to the trace elements of these matrices and that the major contribution to the viscosity of the mixture can be expressed as

$$\eta = \sum_i \frac{x_i^2}{H_{ii}} + \ . \ . \ . \tag{1}$$

where x_i is the mol fraction of the component i and the matrix trace elements H_{ii} are

$$H_{ii} = \frac{x_i^2}{\eta_{ii}} + \sum_{k \neq i} \frac{2x_i x_k}{\eta_{ik}} \frac{m_i m_k}{(m_i + m_k)^2} \left(\frac{5}{3A_{ik}} + \frac{m_k}{m_i}\right) \tag{2}$$

The coefficients η_{ik} are defined as

$$\eta_{ik} = \frac{5}{16Q_{ik}} \left(2\pi kT \frac{m_i m_k}{m_i + m_k}\right)^{1/2} \tag{3}$$

where m_i is the molecular weight of component i and Q_{ik} is the Chapman-Enskog cross section for momentum transfer in collisions between particles i and k. The coefficient A_{ik} is the ratio of Q_{ik} and the cross sections appropriate for mass diffusion; it is somewhat greater than unity because deeper penetration of the interparticle potential is required to influence particle flux than for momentum transfer. In fact, if A_{ik} is assumed equal to 5/3, the off-diagonal elements of the determinants vanish exactly (ref. 1); although this is somewhat too large a value, the effect on H_{ii} from using this approximation is not serious, because terms with $m_k << m_i$ contribute little to the sum in equation (2). With this simplification, the elements H_{ii} become

$$H_{ii} = 2x_i \sum_k \frac{x_k}{\eta_{ik}} \left(1 + \frac{m_i}{m_k}\right)^{-1} \tag{4}$$

and equation (1) can be expressed in the simple linear form

$$\eta = \frac{5\pi}{32} \sum_i \rho_i \bar{u}_i \lambda_i \tag{5}$$

if the effective mean free path λ_i is defined

$$\lambda_i^{-1} = \sqrt{2}n \sum_k x_k Q_{ik} \left(\frac{1 + m_i/m_k}{2}\right)^{-1/2} \tag{6}$$

Hansen (ref. 2) shows that this value of λ_i can be given a simple physical interpretation as the mean distance traveled by a molecule of type i

before it loses its excess momentum relative to the surrounding gas medium in a series of head-on elastic conditions, since the fraction of excess momentum transferred in a single collision between particles i and k is

$$f_{ik} = \frac{2}{1 + m_i/m_k} \tag{7}$$

and the fraction of excess momentum transferred per unit time is the product of f_{ik} and the collision rate θ_{ik} summed over all k:

$$\sum_k f_{ik}\theta_{ik} = \sqrt{2}n\bar{u}_i \sum_k \frac{x_k Q_{ik}}{\left(\frac{1 + m_i/m_k}{2}\right)} \tag{8}$$

Multiplying the reciprocal of equation (8) by the mean velocity $\bar{u}_i$ of particle i leads to the expression of equation (6). Equation (5) can thus be expressed

$$\eta = \sum_i \eta_i\left(\frac{\lambda_i}{\lambda_i^0}\right) \tag{9}$$

where the η_i values are the viscosities of the pure components of the mixture, and λ_i^0 is the mean free path in a pure gas of species i at the density ρ_i. Thus,

$$\frac{\lambda_i^0}{\lambda_i} = 1 + \sum_{k \neq i} \frac{x_k}{x_i}\frac{Q_{ik}}{Q_{ii}}\left(\frac{1 + m_i/m_k}{2}\right)^{-1/2} \tag{10}$$

Buddenberg and Wilke (ref. 3) empirically find that agreement with experimental measurements of viscosities of many gas mixtures is improved if each term in the summation of equation (10) is multiplied by 0.69; presumably, this correction accounts for the deviation of A_{ik} from the 5/3 value assumed. Accordingly, the formula used for the calculations of the viscosity of the Jupiter atmosphere is

$$\frac{\lambda_i^0}{\lambda_i} = 1 + 0.69\sum_{k \neq i} \frac{x_k}{x_i}\frac{Q_{ik}}{Q_{ii}}\left(\frac{1 + m_i/m_k}{2}\right)^{-1/2} \tag{10a}$$

The collision cross sections Q_{ii} are most expeditiously determined by fitting the known viscosities of the pure species with the theoretical formula:

$$\eta_i = \frac{5\pi}{32}\rho_i u_i \lambda_i^0 = \frac{5\pi}{32}\left(\frac{2m_i kT}{\pi}\right)^{1/2} Q_{ii}^{-1} \tag{11}$$

A convenient functional form used for Q_{ii} in making this fit is

$$Q_{ii}(T) = Q_0\left(\frac{T}{T_0}\right)^n\left(\frac{1 + C/T_0}{1 + C/T}\right) \tag{12}$$

In equations (5) and (6) it is assumed that there is no dependence of viscosity on density (usually a good approximation except at very high densities where the collisions can no longer be considered as simple two-body collisions), so that the entire temperature dependence of the coefficient of viscosity is due to the $T^{1/2}$ dependence of the mean molecular velocity and to the decrease in cross-section size as the collision energy increases.

The cross sections Q_{ik} are determined from the pure species cross sections, assuming that the relation between them is the same as for hard spheres,

$$Q_{ik} = \left(\frac{Q_{ii}^{1/2} + Q_{kk}^{1/2}}{2}\right)^2 \tag{13}$$

namely, the collision diameter for the unlike collision pair is the average of the collision diameters for the pure specie collisions. This hard-sphere type approximation can be expected to be quite good when the colliding species both have closed-shell electronic structures, which is the case for both H_2 and He, the species of concern for the Jovian atmosphere.

Viscosity of Pure Molecular Hydrogen

The coefficient of viscosity of pure H_2 is reproduced reasonably well over the temperature range of 100 to 1000 K by the formula

$$\eta = \frac{5\pi}{32}\left(\frac{2mkT}{\pi}\right)^{1/2} Q^{-1} = 90.6\left(\frac{T}{300}\right)^{0.6658}(1 + 4.0/T)^{-1}\mu P \tag{14}$$

where m is the mass of the H_2 molecule. Accordingly, the momentum transfer cross section Q for H_2 - H_2 collisions is taken to be

$$Q = 32.2 \times 10^{-16}\left(\frac{300}{T}\right)^{0.1658}\left(1 + \frac{4.0}{T}\right)\text{cm}^2 \tag{15}$$

The deviations of equation (14) from the H_2 coefficients of viscosity given by Watson (ref. 4), by Kestin (ref. 5), by Hilsenrath et al. (ref. 6),

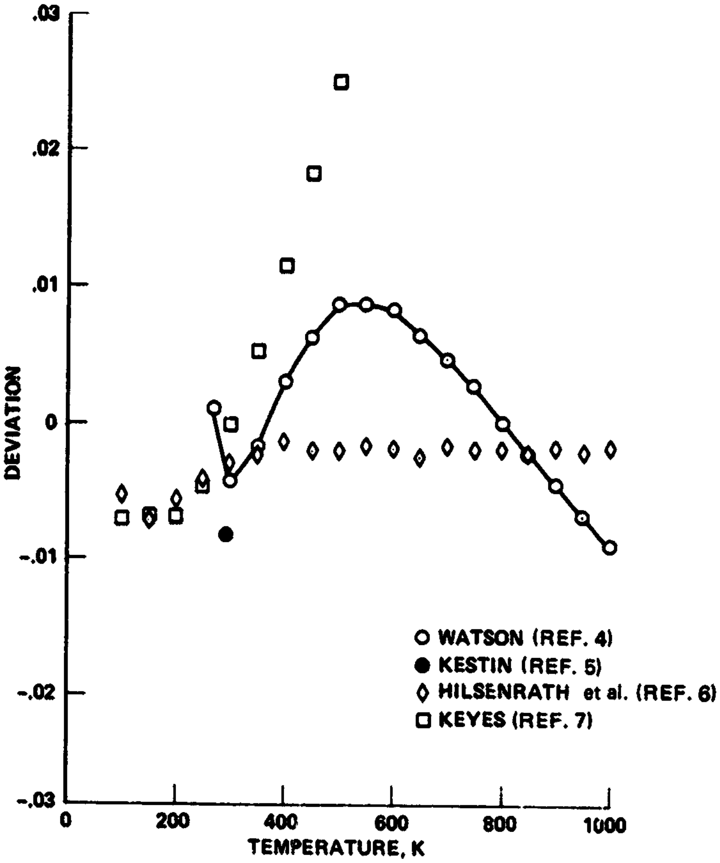

Figure 1.- Deviations of equation (14) from published viscosities of pure H_2.

and by Keyes (ref. 7) are shown in figure 1. The parameters of equation (14) were chosen to give the best fit to Hilsenrath's values from 100 to 300 K and to Watson's values from 300 to 1000 K. The latter are expected to be the most reliable in that range since they have been carefully fitted to the most recent and most carefully calibrated experimental data. Hilsenrath's values are based on collision integrals calculated using a Lennard-Jones 6-12 potential, with the force parameters of the potential fitted to older viscosity measurements; these are expected to be the best values in the low-temperature range. It can be seen that the deviations of equation (14) are generally well within 1% of the best values. Even the old formula given by Keyes for the range of 100 to 500 K agrees within 3%,

$$\eta = \frac{6.23T^{1/2}}{1 + \frac{70.8}{T} e^{-39.8/T}} \mu P \qquad (16)$$

and the value documented by Kestin at 293 K falls within the 1% range. In view of uncertainties in the mixture formulas, equation (14) is considered adequately precise.

Viscosity of Pure Helium

The coefficient of viscosity of pure He is reproduced reasonably well over the temperature range of 100 to 1000 K by the formula

$$\eta = \frac{5\pi}{32}\left(\frac{2mkT}{\pi}\right)^{1/2} Q^{-1} = 191.6\left(\frac{T}{300}\right)^{0.7176}\left(1 - \frac{11.4}{T}\right)^{-1} \mu P \qquad (17)$$

where m is the mass of the He atom. Accordingly, the momentum transfer cross section Q for He-He collisions is taken to be

$$Q = 21.5\times10^{-16}\left(\frac{300}{T}\right)^{0.2176}\left(1 - \frac{11.4}{T}\right)\mathrm{cm}^2 \qquad (18)$$

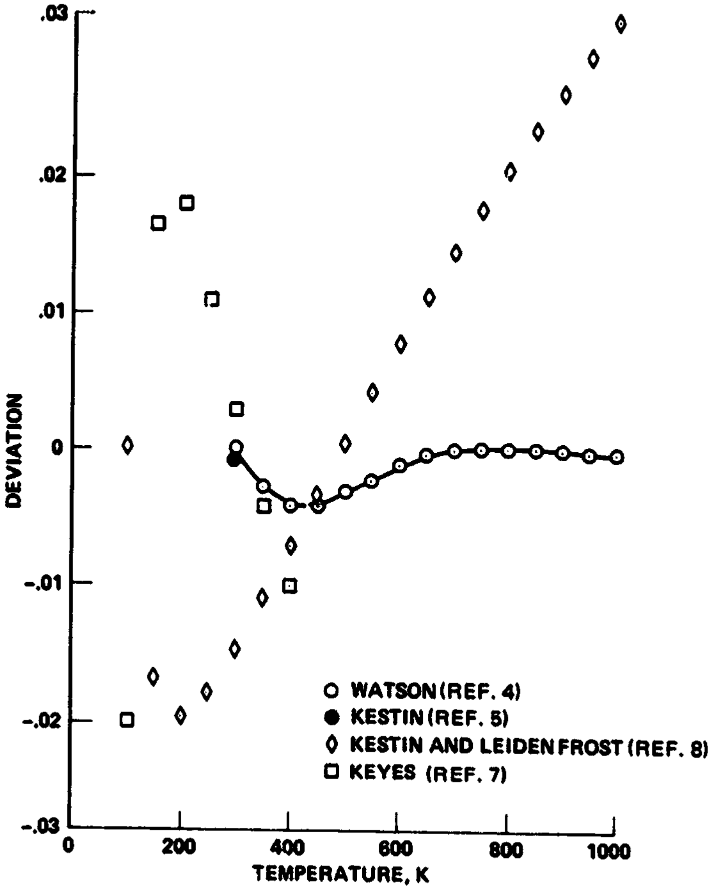

Figure 2.- Deviations of equation (17) from published viscosities of pure He.

The deviations of equation (17) from the He coefficients of viscosity given by Watson (ref. 4), by Kestin (ref. 5), by Kestin and Leidenfrost (ref. 8), and by Keyes (ref. 7) are shown in figure 2. Again the values of Watson are considered to be the best in the range from 300 + 1000 K. The formula has been fitted to these values and to the 100 K value of Kestin and Leidenfrost. The deviations from Watson's values are generally less than 0.5%; in the low-temperature range, where the formula bisects the values of Kestin and Leidenfrost and of the Keyes equation designed for the range of 104 to 373 K,

$$\eta = \frac{18.05T^{1/2}}{1 + \frac{253}{T}\,e^{-115.1/T}}\ \mu\mathrm{P} \qquad (19)$$

deviations are generally less than 2%. Again, the uncertainties are consistent with the uncertainties in the mixture formulas.

Viscosity of Jovian Atmospheres

The viscosity of model Jovian atmosphere mixtures is shown as a function of temperature in table 1 for the temperature range from 100 to 1000 K. The first column is the temperature; the second column is the viscosity of pure H_2 (which corresponds to both the Cool Light Model and Warm Light Model of the Jovian Atmosphere); the third column is the viscosity of an 89% H_2 - 11% He mixture (which corresponds to the Nominal Jovian Atmosphere Model); and the final column is the viscosity of a 78% H_2 - 22% He mixture (which corresponds to both the Cool Heavy Model and the Warm Heavy Model of the Jovian Atmosphere). The Cool and Warm Models have only a different altitude dependence of temperature. The viscosity of the Nominal Model Atmosphere (with 11% He) is almost exactly 25% greater than the viscosity of pure H_2. The ratio of the two is almost independent of temperature. Similarly, the viscosity of the Heavy Atmosphere Models (with 22% He) is almost exactly 47% greater than the viscosity of pure H_2.

TABLE 1.- VISCOSITY OF JUPITER MODEL ATMOSPHERES

Temperature, K	Light atm.,[a] μP	Nominal atm.,[b] μP	Heavy atm.,[c] μP
100	41.92	52.98	62.77
150	55.62	69.92	82.46
200	67.8	85.08	100.17
250	78.97	99.02	116.52
300	89.4	112.08	131.87
350	99.25	124.44	146.42
400	108.64	136.23	160.32
450	117.63	147.54	173.68
500	126.29	158.46	186.57
550	134.66	169.02	199.06
600	142.78	179.27	211.19
650	150.67	189.24	223.01
700	158.36	198.97	234.54
750	165.86	208.47	245.81
800	173.2	217.77	256.84
850	180.39	226.88	267.65
900	187.44	235.81	278.27
950	194.35	244.59	288.7
1000	201.15	253.22	298.95

[a] Viscosity of pure H_2.

[b] Viscosity of 89% H_2 - 11% He mixture.

[c] Viscosity of 78% H_2 - 22% He mixture.

Table 2 shows the viscosity of the Jupiter Nominal Atmosphere as a function of altitude and the ratio of the mixture viscosity to the pure H_2 viscosity.

TABLE 2.- VISCOSITY OF JUPITER NOMINAL ATMOSPHERE

Altitude, km	Temperature, K	Viscosity, μP	Viscosity ratio, mix/H_2
-270.79	681.77	195.5	1.256
-246.93	637.33	186.79	1.256
-224.62	595.74	178.45	1.255
-203.74	556.83	170.48	1.255
-184.23	520.44	162.85	1.255
-165.98	486.39	155.56	1.254
-148.91	454.55	148.59	1.254
-132.96	424.78	141.93	1.254
-118.03	396.93	135.55	1.254
-104.09	370.89	129.46	1.254
-91.06	346.51	123.62	1.254
-78.67	323.66	118.03	1.253
-67.49	302.25	112.68	1.253
-56.86	282.17	107.54	1.253
-46.94	263.31	102.6	1.254
-37.66	245.59	97.86	1.254
-29.03	228.92	93.29	1.254
-20.97	213.21	88.88	1.254
-13.48	198.4	84.63	1.255
-6.49	184.43	80.53	1.255
0	171.23	76.55	1.256
6.02	158.77	72.71	1.256
11.6	147	68.98	1.257
16.75	134.3	64.85	1.258
21.43	121.6	60.59	1.26
25.79	116.65	58.9	1.261
29.97	111.7	57.17	1.261
34.04	110.9	56.89	1.262
38.09	110.1	56.61	1.262
42.16	112.35	57.4	1.261
46.32	114.6	58.18	1.261
50.57	117.29	59.12	1.261
54.91	119.98	60.04	1.26
59.36	122.66	60.96	1.26
63.9	125.35	61.87	1.259

TABLE 2.- CONTINUED

Altitude, km	Temperature, K	Viscosity, μP	Viscosity ratio, mix/H_2
68.54	128.04	62.77	1.259
73.29	130.73	63.67	1.259
78.13	133.41	64.56	1.258
83.07	136.1	65.45	1.258
88.1	138.1	66.1	1.258
93.2	140.09	66.75	1.258
98.37	142.09	67.4	1.258
103.62	144.09	68.05	1.257
108.94	146.08	68.69	1.257
114.34	148.08	69.33	1.257
119.81	150.08	69.97	1.257
125.36	152.08	70.6	1.257
130.98	154.07	71.23	1.257
136.67	156.07	71.86	1.257
142.44	158.07	72.49	1.256
148.28	160.06	73.11	1.256
154.2	162.06	73.73	1.256
160.19	164.06	74.35	1.256
166.25	166.05	74.97	1.256
172.39	168.05	75.58	1.256
178.61	170.05	76.19	1.256
184.9	172.04	76.8	1.256
191.26	174.04	77.41	1.256
197.7	176.04	78.01	1.255
204.21	178.03	78.61	1.255
210.8	180.03	79.21	1.255
217.46	182.03	79.81	1.255
224.2	184.03	80.41	1.255
231.01	186.02	81	1.255
237.89	188.02	81.59	1.255
244.85	190.02	82.18	1.255
251.89	192.01	82.77	1.255
259	194.01	83.35	1.255
266.19	196.01	83.94	1.255
273.45	198	84.52	1.255

TABLE 2.- CONTINUED

Altitude, km	Temperature, K	Viscosity, μP	Viscosity ratio, mix/H_2
280.78	200	85.1	1.255
288.16	200	85.1	1.255
295.53	200	85.1	1.255
302.91	200	85.1	1.255
310.29	200	85.1	1.255
317.67	200	85.1	1.255
325.05	200	85.1	1.255
332.44	200	85.1	1.255
339.82	200	85.1	1.255
347.21	200	85.1	1.255
354.6	200	85.1	1.255
362.39	221.67	91.27	1.254
370.98	243.33	97.24	1.254
380.37	265	103.05	1.254
390.57	286.67	108.7	1.253
401.58	308.33	114.21	1.253
413.39	330	119.6	1.253
426	351.67	124.87	1.254
439.42	373.33	130.03	1.254
453.65	395	135.1	1.254
468.69	416.67	140.08	1.254
484.54	438.33	144.98	1.254
501.19	460	149.8	1.254
518.66	481.67	154.54	1.254
536.95	503.33	159.21	1.255
556.04	525	163.82	1.255
575.96	546.67	168.37	1.255
596.69	568.33	172.85	1.255
618.24	590	177.29	1.255
640.6	611.67	181.67	1.256
663.79	633.33	185.99	1.256
687.8	655	190.27	1.256
712.64	676.67	194.51	1.256
738.3	698.33	198.7	1.256
764.79	720	202.85	1.256

TABLE 2.- CONCLUDED

Altitude, km	Temperature, K	Viscosity, μP	Viscosity ratio, mix/H_2
792.11	741.67	206.96	1.257
820.27	763.33	211.02	1.257
849.25	785	215.06	1.257
879.07	806.67	219.05	1.257
909.73	828.33	223.01	1.257
941.23	850	226.94	1.258

THERMAL CONDUCTIVITY OF JUPITER ATMOSPHERES

Thermal Conductivity of Mixtures

Theoretical expressions for thermal conductivity of gases and of gas mixtures have not been verified with the same accuracy as for viscosities. This occurs partly because a number of intricate corrections are involved in making measurements, and also because the heat transfer associated with internal modes of molecular energy is difficult to treat with the same rigor as that associated with the kinetic modes of energy. In the simple Eucken approximation, the internal degrees of freedom are assumed to have a Maxwell distribution that is totally independent of the molecule's velocity. This is certainly a good approximation for the stiff vibrational modes [illegible] H_2 molecule at the temperatures of interest, where there is [illegible] little coupling between vibrational and kinetic modes during a collision perturbation; however, the same may not be exactly true for the rotational modes of the H_2 molecule. Nevertheless, the Eucken approximation seems to agree just as well with experimental data as the more elaborate theoretical models that have been devised; consequently, it will be used here in view of its simplicity and the fact that the comparisons between experiment and the theoretical model seem adequate to the order of accuracy being considered.

The coefficient of thermal conductivity of a pure gas can be expressed with the Eucken approximation

$$\kappa = \frac{5\pi}{32} \frac{\rho u \lambda}{M} \left(\frac{5}{2} C_v + C_v^*\right) \tag{20}$$

where M is the gram molecular weight, C_v is the specific heat per mole of the gas associated with its kinetic energy, and C_v^* is the specific heat associated with internal molecular energy. For a Maxwell-Boltzmann distribution of velocities, the kinetic specific heat is just 3/2 times the gas constant R

$$C_v = \frac{3R}{2} \tag{21}$$

The internal specific heat is zero for He and for H_2 it is the sum of the rotational and vibrational contributions:

$$C_v^* \text{ (He)} = 0 \tag{22a}$$

$$C_v^* \text{ (H}_2\text{)} = R\left[1 + \left(\frac{\theta_v/2T}{\sinh \theta_v/2T}\right)^2\right] \tag{22b}$$

In equations (22a) and (22b) it has been assumed that no electronic energy is excited for either He or H_2 (which is a good assumption below 1000 K) and that the rotational states of H_2 are fully excited while the vibrational modes are at the equilibrium excitation for a harmonic oscillator. Since the characteristic vibrational temperature for H_2 is large ($\theta_v = 6159$ K) the latter assumption is very good below 1000 K; however, the characteristic rotational temperature for H_2 is rather large ($\theta_r = 85.42$ K), so the approximation of equation (22b) can be expected to hold some error at the low-temperature end of the range near 100 K.

The thermal conductivity of the gas mixture is now expressed

$$\kappa = \frac{5\pi}{32}\sum_i \frac{\rho_i \bar{u}_i \lambda_i^0}{m_i}\left(\frac{\lambda_i}{\lambda_i^0}\right)\left(\frac{5}{2}\, C_i + C_i^*\right) \tag{23}$$

Strictly speaking, the ratio of the effective mean free path for energy transfer (λ_i) to the mean free path in the pure ith component with density ρ_i (λ_i^0) should be somewhat different for the translational and internal degrees of freedom. The difference is not large, however, if the molecular masses are about equal, which is the case for the present gases of interest. The fraction of excess kinetic energy, associated with the motion between the line of centers at impact, transferred between particle i and k in elastic collision is

$$f_{ik} = \frac{4m_i/m_k}{1 + m_i/m_k} \tag{24}$$

Accordingly, the excess kinetic energy transferred per unit time is the product of the collision rate θ_{ik} and the fraction f_{ik} summed over all k

$$\sum_k f_{ik}\theta_{ik} = \sqrt{2}\, n\bar{u}_i \sum_k \frac{x_k \dfrac{m_i}{m_k} Q_{ik}}{\left(\dfrac{1 + m_i/m_k}{2}\right)^2} \tag{25}$$

and the ratio λ_i^0/λ_i can be expressed

$$\frac{\lambda_i^0}{\lambda_i} = 1 + \sum_{k \neq i} \frac{x_k}{x_i} \frac{Q_{ik}}{Q_{ii}} \frac{m_i/m_k}{\left(\frac{1 + m_i/m_k}{2}\right)^{3/2}} \tag{26}$$

In this approximation, the thermal conductivity of the mixture is given by a simple linear expression analogous to the linearized formula for the viscosities of mixtures

$$\kappa = \sum_i \kappa_i \left(\frac{\lambda_i}{\lambda_i^0}\right) \tag{27}$$

Undoubtedly, an empirical correction term, such as that provided by Buddenberg and Wilke for the viscosity of mixtures, could account for some difference in the cross sections for diffusion and for energy transfer and thus improve the theoretical model somewhat; however, such a correction has not been established at present. Since the molecular masses are nearly equal in this case, the correction could only be a modest one, of the order of a few percent at best. Such a correction takes on major importance only in ionized gases where the very lightweight electrons carry a large fraction of the energy transfer.

Thermal Conductivity of Pure Molecular Hydrogen

The coefficient of thermal conductivity of pure H_2 is reproduced reasonably well over the temperature range of 100 to 1000 K by the formula

$$\kappa = \frac{5\pi}{32}\left(\frac{2mkT}{\pi}\right)^{1/2}\left[\frac{19}{4} + \left(\frac{\theta_v/2T}{\sinh \theta_v/2T}\right)^2\right]\frac{R}{mQ}$$

$$= 1.10\times10^{-4} \frac{(T/300)^{0.6983}}{\left(1 + \frac{49.4}{T}\right)} \left[\frac{19}{4} + \left(\frac{3080/T}{\sinh \frac{3080}{T}}\right)^2\right] \text{cal/cm deg sec} \tag{28}$$

where again m is the mass of the H_2 molecule. Accordingly, the energy transfer cross section Q, for H_2 - H_2 collisions, is taken to be

$$Q = 26.1\times10^{-16} \left(\frac{300}{T}\right)^{0.1983}\left(1 + \frac{49.4}{T}\right)\text{cm}^2 \tag{29}$$

The deviation of equation (28) from the H_2 coefficients of thermal conductivity given by Hilsenrath et al. (ref. 6) is shown in figure 3. The deviations, generally less than 1%, are adequate for present purposes. No

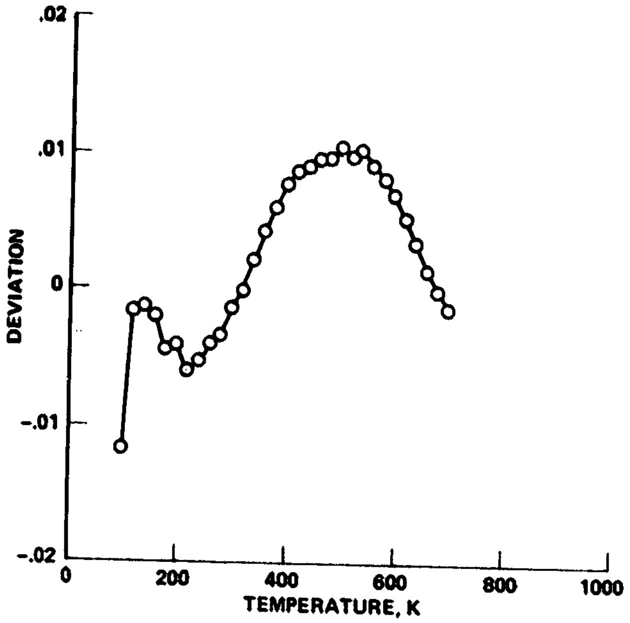

Figure 3.- Deviations of equation (28) from published thermal conductivities of pure H_2

doubt, some of the deviation near 100 K is due to the approximate treatment of the equilibrium rotational excitation.

Thermal Conductivity of Pure Helium

The coefficient of thermal conductivity of pure He is reproduced reasonably well over the temperature range of 100 to 1000 K by the formula

$$\kappa = \frac{5\pi}{32}\left(\frac{2mkT}{\pi}\right)^{1/2}\frac{15}{4}\,\frac{R}{mQ} = 3.42\times10^{-4}\,\frac{(T/300)^{0.7412}}{(1 - 13.7/T)}\ \text{cal/cm deg sec} \qquad (30)$$

where now m is the mass of the He atom. Accordingly, the energy transfer cross section Q, for He - He collisions, is taken to be

$$Q = 5.96\times10^{-16}\left(\frac{300}{T}\right)^{0.2412}\times\left(1 - \frac{13.7}{T}\right)\text{cm}^2 \qquad (31)$$

The deviations of equation (30) from the He coefficients of thermal conductivity given by Powell, Ho, and Liley (ref. 9) are shown in figure 4. The deviations are generally within 2%. Some deviations from coefficients given by Powell (ref. 10) are also shown, with about the same result. These deviations are about double the deviations previously encountered for the simple cross-section expressions, but are still well within the uncertainties of the mixture formulas used for thermal conductivities.

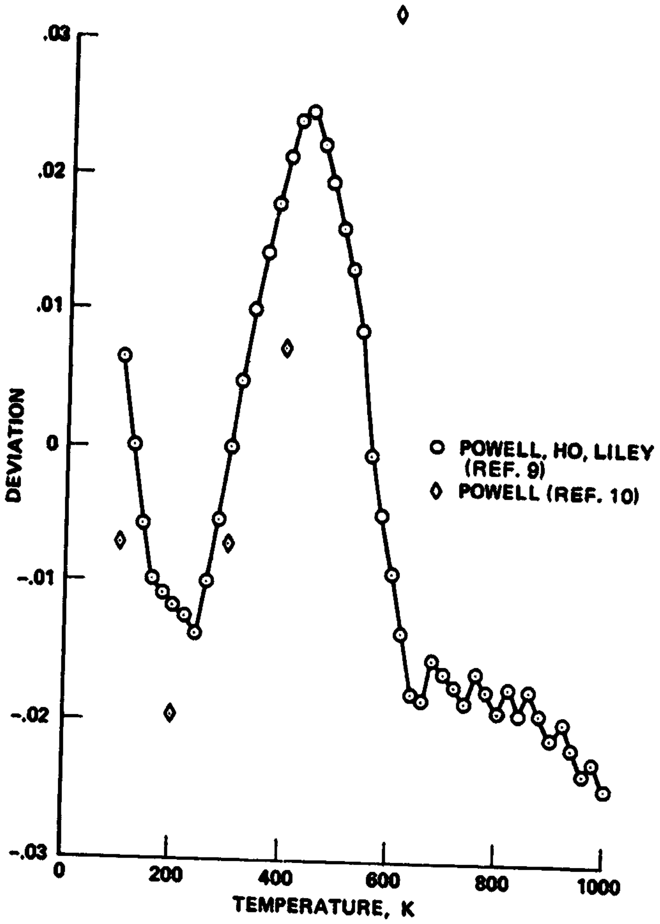

Figure 4.- Deviations of equation (30) from published thermal conductivities of pure He

Thermal Conductivity of Jovian Atmospheres

The thermal conductivity of model Jovian atmosphere mixtures is shown as a function of temperature in table 3 for the temperature range from 100 to 1000 K. Again, the first column is the temperature; the second column is the thermal conductivity of pure H_2 (which corresponds to both the Cool Light Model and the Warm Light Model of the Jovian Atmosphere), the third column is the conductivity of an 89% H_2 - 11% He mixture (which corresponds to the Nominal Jovian Atmosphere Model); and the final column is the conductivity of a 78% H_2 - 22% He mixture (which corresponds to both the Cool Heavy Model and Warm Heavy Model of the Jovian Atmosphere). The conductivity of the Nominal Model Atmosphere (with 11% He) is almost constantly 2% less than for pure H_2, independent of temperature. Similarly, the conductivity of the Heavy Atmosphere Models (with 22% He) is almost constantly 4% less than for pure H_2.

TABLE 3.- THERMAL CONDUCTIVITY OF JUPITER MODEL ATMOSPHERES

Temperature, K	Thermal conductivity, mcal/cm-deg-sec		
	Light atm.[a]	Nominal atm.[b]	Heavy atm.[c]
100	0.1623	0.161	0.1597
150	.2422	.2395	.2365
200	.3156	.3115	.307
250	.3841	.3787	.3727
300	.4486	.4419	.4345
350	.5099	.5021	.4933
400	.5685	.5596	.5496
450	.6249	.6149	.6038
500	.6794	.6684	.6561
550	.7323	.7203	.7069
600	.7839	.771	.7565
650	.8344	.8205	.805
700	.884	.8693	.8527
750	.9331	.9174	.8998
800	.9816	.965	.9464
850	1.0299	1.0123	.9926
900	1.0779	1.0594	1.0387
950	1.1258	1.1064	1.0845
1000	1.1737	1.1533	1.1303

[a] Thermal conductivity of pure H_2.

[b] Thermal conductivity of 89% H_2 - 11% He mixture.

[c] Thermal conductivity of 78% H_2 - 28% He mixture.

Table 4 shows the thermal conductivity of the Jupiter Nominal Atmosphere as a function of altitude and the ratio of the mixture conductivity to the pure H_2 conductivity.

TABLE 4.- THERMAL CONDUCTIVITY OF JUPITER NOMINAL ATMOSPHERE

Altitude, km	Temperature, K	Conductivity, mcal/cm-deg sec	Conductivity ratio, mix/H_2
-270.79	681.77	0.8516	0.9833
-246.93	637.33	.8081	.9834
-224.62	595.74	.7667	.9835
-203.74	556.83	.7273	.9836
-184.23	520.44	.6898	.9837
-165.98	486.39	.654	.9839
-148.91	454.55	.6199	.984
-132.96	424.78	.5873	.9842
-118.03	396.93	.5561	.9843
-104.09	370.89	.5264	.9845
- 91.06	346.51	.498	.9847
- 78.87	323.66	.4707	.9849
- 67.49	302.25	.4447	.9851
- 56.86	282.17	.4198	.9854
- 46.94	263.31	.3959	.9856
- 37.66	245.59	.3729	.9859
- 29.03	228.92	.3509	.9862
- 20.97	213.21	.3297	.9866
- 13.48	198.4	.3093	.987
- 6.49	184.43	.2897	.9874
0	171.23	.2707	.9878
6.02	158.77	.2525	.9883
11.6	147	.2349	.9888
16.75	134.3	.2156	.9895
21.43	121.6	.1958	.9903
25.79	116.65	.188	.9906
29.97	111.7	.18	.991
34.04	110.9	.1788	.991
38.09	110.1	.1775	.9911
42.16	112.35	.1811	.9909
46.32	114.6	.1847	.9908
50.57	117.29	.189	.9906
54.91	119.98	.1932	.9904
59.36	122.66	.1975	.9902
63.9	125.35	.2017	.99

TABLE 4.- CONTINUED

Altitude, km	Temperature, K	Conductivity, mcal/cm-deg sec	Conductivity ratio, mix/H_2
68.54	128.04	0.2059	0.9898
73.29	130.73	.2101	.9897
78.13	133.41	.2142	.9895
83.07	136.1	.2184	.9894
88.1	138.1	.2214	.9893
93.2	140.09	.2245	.9892
98.37	142.09	.2275	.9891
103.62	144.09	.2306	.989
108.94	146.08	.2336	.9889
114.34	148.08	.2366	.9888
119.81	150.08	.2396	.9887
125.36	152.08	.2426	.9886
130.98	154.07	.2455	.9885
136.67	156.07	.2485	.9884
142.44	158.07	.2515	.9883
148.28	160.06	.2544	.9882
154.2	162.06	.2574	.9882
160.19	164.06	.2603	.9881
166.25	166.05	.2632	.988
172.39	168.05	.2661	.9879
178.61	170.05	.269	.9879
184.9	172.04	.2719	.9878
191.26	174.04	.2748	.9877
197.7	176.04	.2777	.9876
204.21	178.03	.2805	.9876
210.8	180.03	.2834	.9875
217.46	182.03	.2863	.9874
224.2	184.03	.2891	.9874
231.01	186.02	.2919	.9873
237.89	188.02	.2948	.9873
244.85	190.02	.2976	.9872
251.89	192.01	.3004	.9871
259	194.01	.3032	.9871
266.19	196.01	.306	.987
273.45	198	.3087	.987

TABLE 4.- CONTINUED

Altitude, km	Temperature, K	Conductivity, mcal/cm-deg sec	Conductivity ratio, mix/H_2
280.78	200	0.3115	0.9869
288.16	200	.3115	.9869
295.53	200	.3115	.9869
302.91	200	.3115	.9869
310.29	200	.3115	.9869
317.67	200	.3115	.9869
325.05	200	.3115	.9869
332.44	200	.3115	.9869
339.82	200	.3115	.9869
347.21	200	.3115	.9869
354.6	200	.3115	.9869
362.39	221.67	.3412	.9864
370.98	243.33	.37	.986
380.37	265	.398	.9856
390.57	286.67	.4254	.9853
401.58	308.33	.4522	.9851
413.39	330	.4784	.9848
426	351.67	.504	.9846
439.42	373.33	.5292	.9845
453.65	395	.5539	.9843
468.69	416.67	.5783	.9842
484.54	438.33	.6022	.9841
501.19	460	.6258	.984
518.66	481.67	.649	.9839
536.95	503.33	.6719	.9838
556.04	525	.6946	.9837
575.96	546.67	.7169	.9837
596.69	568.33	.739	.9836
618.24	590	.7609	.9835
640.6	611.67	.7826	.9835
663.79	633.33	.8041	.9834
687.8	655	.8255	.9834
712.64	676.67	.8466	.9833
738.3	698.33	.8677	.9833
764.79	720	.8886	.9832

TABLE 4.- CONCLUDED

Altitude, km	Temperature, K	Conductivity, mcal/cm-deg sec	Conductivity ratio, mix/H_2
792.11	741.67	0.9094	0.9832
820.27	763.33	.9301	.9831
849.25	785	.9508	.9831
879.07	806.67	.9714	.983
909.73	828.33	.9919	.983
941.23	850	1.0123	.9829

CONCLUDING REMARKS

The approximations used in estimating the viscosity of Jovian atmosphere mixtures are believed reliable within a few percent. The approximations used in estimating the thermal conductivities of Jovian atmosphere mixtures hold uncertainties of the order of 10 to 15%. In both cases, the mixture transport coefficients are almost a constant factor times the value for pure H_2, independent of temperature. This permits a quick, easy method of evaluating the appropriate values for the mixtures.

The transport coefficients included here are appropriate only for subsonic flow fields in the Jovian atmosphere. At high supersonic speeds, the H_2 dissociates and both the He and H atoms are electronically excited and ionized in flow following strong shock waves and in boundary-layer flow. In this case, the dissociation and ionization reactions promote very large peaks in the thermal conductivity over the range of pressures and temperatures where the reaction rates are significant. The ionization of the species also decreases viscosity by an order of magnitude or more, because the long-range coulomb forces prevent long mean-free-path momentum transfer in the gas. The characteristic dissociation temperature of H_2 (that is, the ratio of dissociation energy $\mathcal{D}$ to the Boltzmann constant k) is 51,965 K, and the characteristic electronic excitation and ionization temperatures are several times larger. These reactions will not become important until temperatures are of the order of 0.1 the characteristic temperature. In other words, the mixture formulas used here could be applied to temperatures of several thousand degrees. However, in such a case, the simple formulas used for the viscosities and thermal conductivities of the pure H_2 and He would have to be modified; they were designed only to apply in the range from 100 to 1000 K.

Tables 5 and 6 are printouts of the BASIC language programs that have been used to calculate the coefficients of viscosity and thermal conductivity of H_2 - He mixtures. In the future, we may well find that other mixtures are appropriate for the Jovian atmosphere, in which case the appropriate values of the mole fractions x_1 (of H_2) and x_2 (of He) can be introduced as input to

TABLE 5.- BASIC PROGRAM FOR VISCOSITY OF JOVIAN MODEL ATMOSPHERES

```
10 PRINT "TABLE I - VISCOSITY OF JUPITER MODEL ATMOSPHERES, MICROPOISE"
20 PRINT
30 PRINT "T,DEG K","LIGHT ATM","NOMINAL ATM","HEAVY ATM"
40 PRINT
50 X1=.89          ! Mol fraction of H2 in nominal atmosphere model
60 X2=.11          ! Mol fraction of He in nominal atmosphere model
70 Y1=.78          ! Mol fraction of H2 in heavy atmosphere model
80 Y2=.22          ! Mol fraction of He in heavy atmosphere model
90 T=1E2           ! Initial temperature, deg K
100 FOR I=1 TO 19
140 N1=90.6*(T/300)^.6658
142 N1=N1/(1+4/T)                      !  Viscosity of H2, micropoise
150 N2=191.6*(T/300)^.7176
152 N2=N2/(1-11.4/T)                   !  Viscosity of He, micropoise
160 Q1=32.3*(1+4/T)*(300/T)^.1658      ! Cross section H2-H2, Ang2
170 Q2=21.5*(1-11.4/T)*(300/T)^.2176   ! Cross section He-He, Ang2
180 Q3=(Q1^.5+Q2^.5)/2
190 Q3=Q3*Q3                           ! Cross section H2-He, Ang2
200 L1=1+.7967*(X2/X1)*(Q3/Q1)      !  Ratio L(pure)/L(mix) for H2
210 L2=1+.5634*(X1/X2)*(Q3/Q2)      !  Ratio L(pure)/L(mix) for He
220 N=N1/L1+N2/L2   ! Viscosity of nominal atmosphere, micropoise
230 L3=1+.7967*(Y2/Y1)*(Q3/Q1)      !  Ratio L(pure)/L(mix) for H2
240 L4=1+.5634*(Y1/Y2)*(Q3/Q2)      !  Ratio L(pure)/L(mix) for He
250 N3=N1/L3+N2/L4    ! Viscosity of heavy atmosphere, micropoise
260 PRINT T,1E-2*INT(1E2*N1),1E-2*INT(1E2*N),1E-2*INT(1E2*N3)
270 T=T+50                        ! Next value of temperature, deg K
280 NEXT I
999 END
```

TABLE 6.- BASIC PROGRAM FOR CONDUCTIVITY OF JOVIAN MODEL ATMOSPHERES

```
10 PRINT "TABLE III - THERMAL CONDUCTIVITY OF JUPITER MODEL ATMOSPHERES"
11 PRINT "                          MILLICAL/CM-DEG-SEC"
20 PRINT
30 PRINT "T,DEG K","LIGHT ATM","NOMINAL ATM","HEAVY ATM"
40 PRINT
50 X1=.89  ! Mol fraction of H2 in nominal atmosphere model
60 X2=.11  ! Mol fraction of He in nominal atmosphere model
70 Y1=.78  ! Mol fraction of H2 in heavy atmosphere model
80 Y2=.22  ! Mol fraction of He in heavy atmosphere model
90 T=100   ! Initial temperature, deg K
100 FOR I=1 TO 19
140 K1=.11*(T/300)^.6983/(1+49.4/T)
142 V=3079.5/T
144 V=2*V/(EXP(V)-EXP(-V))
146 K1=K1*(4.75+V*V)  ! Conductivity of H2, millical/cm-deg-sec
150 K2=.3418*(T/300)^.7412/(1-13.74/T) ! Conductivity of He,mcal/cm-deg-sec
160 Q1=26.1*(1+49.4/T)*(300/T)^.1983   ! Cross section H2-H2, Ang2
170 Q2=5.96*(1-13.74/T)*(300/T)^.2412  ! Cross section He-He, Ang2
180 Q3=(Q1^.5+Q2^.5)/2
190 Q3=Q3*Q3                           ! Cross section H2-He, Ang2
200 L1=1+.7698*(X2/X1)*(Q3/Q1)   ! Ratio L(pure)/L(mix) for H2
210 L2=1+1.0887*(X1/X2)*(Q3/Q2)  ! Ratio L(pure)/L(mix) for He
220 K=K1/L1+K2/L2    ! Thermal conductivity nominal atm,mcal/cm-deg-sec
230 L3=1+.7698*(Y2/Y1)*(Q3/Q1)   ! Ratio L(pure)/L(mix) for H2
240 L4=1+1.0887*(Y1/Y2)*(Q3/Q2)  ! Ratio L(pure)/L(mix) for He
250 K3=K1/L3+K2/L4   ! Thermal conductivity heavy atm, mcal/cm-deg-sec
260 PRINT T,1E-4*INT(1E4*K1),1E-4*INT(1E4*K),1E-4*INT(1E4*K3)
270 T=T+50     ! Next value of temperature, deg K
280 NEXT I
999 END
```

the program. If sizable fractions of other gases were to be discovered in the Jovian atmosphere, they would have to be included, of course.

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REFERENCES

1. Hirschfelder, Joseph O.; Curtiss, Charles F.; and Bird, R. Byron: Molecular Theory of Gases and Liquids. John Wiley and Sons, New York, 1954, p. 532.
2. Hansen, C. Frederick: Interpretation of Linear Approximations for the Viscosity of Gas Mixtures. Phys. Fluids, vol. 4, 1961, pp. 926-927.
3. Buddenberg, J. W.; and Wilke, C. R.: Calculation of Gas-Mixture Viscosities. Ind. & Eng. Chem., vol. 41, 1949, pp. 1345-1347.
4. Watson, J. T. R.: Viscosity of Gases in Metric Units, Dept. of Trade and Industry, National Engineering Laboratory, Her Majesty's Stationery Office, Edinburgh, 1972.
5. Kestin, J.: Viscosity of Gases. Ch. 2u, American Institute of Physics Handbook, Dwight E. Gray ed., Second ed., McGraw Hill Book Co., 1963.
6. Hilsenrath, Joseph; Beckett, C. W.; Benedict, L. F.; Hoge, H. J.; Masi, J. F.; Nuttall, R. L.; Touloukian, Y. S.; and Woolley, H. W.: Tables of Thermodynamic and Transport Properties of Air, Argon, Carbon Dioxide, Carbon Monoxide, Hydrogen, Nitrogen, Oxygen, and Steam. Pergamon Press, 1960. (Also NBS Circular 564, 1955).
7. Keyes, Frederick G.: The Heat Conductivity, Viscosity, Specific Heat, and Prandtl Number for Thirteen Gases. Project SQUID, MIT Tech Rept. 37, 1952.
8. Kestin, J. and Leidenfrost, W.: The Viscosity of Helium. Physica, vol. 25, 1959, pp. 537-555.
9. Powell, R. W.; Ho, C. Y.; and Liley, P. E.: Thermal Conductivity of Selected Materials. NSRDS - NBS Report 8, Nov. 25, 1966.
10. Powell, R. L.: Thermal Conductivity, Ch. 4g. American Institute of Physics Handbook, Dwight E. Gray Ed., Second ed., McGraw Hill Book Co., 1963.