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ABSTRACT

Calculations based on two specific pulsar models show that in either case the 511 keV gamma-ray line from the galactic center can be explained as the result of the annihilation of positrons produced by pulsars.

I. INTRODUCTION

Leventhal, MacCallum and Stang (1978) have recently reported the detection of 511 keV positron annihilation radiation from the direction of the galactic center. Their observed flux level of $(1.22 \pm 0.22) \times 10^{-3}$ photons $cm^{-2} s^{-1}$ corresponds to an annihilation rate $10^{43.5} s^{-1}$ if the radiation originates at the distance of the galactic center and if positron annihilation produces a continuum from the ortho state as well as the 511 keV line from the para state. The authors note that current pulsar models generate large positron fluxes and that "the large number of radio pulsars within our field of view could in principle be injecting an adequate number of positrons into the interstellar medium to account for the feature." They also note more speculative sources of the positrons such as evaporating primordial black holes, a massive black hole at the galactic center, and various matter-antimatter symmetric cosmologies.*

The aim of this article is to examine in a little more detail their suggestion that pulsars can yield an adequate supply of positrons to account for the observed line. Since any more detailed estimate must necessarily be model-dependent, the following estimates will be based on the "polar-cap light-cylinder" (PCLC) model (Sturrock, 1971a) and a more recent modification, the "polar-cap force-balance" (PCFB) model (Roberts and Sturrock, 1972a,b, 1973).

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* Steigman (1979) has kindly pointed out to me that current evidence weighs against the last possibility. Matter-antimatter annihilation would lead to comparable number fluxes of 511 keV photons (from $e^- - e^+$ annihilation) and gamma rays of energy $\gtrsim 100$ MeV (from nucleon-antinucleon annihilation), whereas current evidence (Leventhal et al., 1978; Fichtel et al., 1975) indicates that the former exceeds the latter by more than order magnitude.

II. POSITRON FLUX IN THE PCLC MODEL

The total number of electrons and positrons leaving each polar cap is given by equation (5.6) of Sturrock (1971). Hence the same formula gives the total number of positrons leaving both polar caps:

$$J_{e+} = 10^{-41.4}\ B^{3}\ R^{13/2}\ P^{-9/2}, \tag{2.1}$$

where R (cm) is the radius of the neutron star, B (gauss) is the mean field strength of the surface and P (s) is the period.

The total number of positrons emitted during the "lifetime" of the pulsar is given by

$$N_{e+} = \int_{0}^{\infty} J_{e+}(t)dt\,. \tag{2.2}$$

We see from equation (2.1) that this may be reexpressed as

$$N_{e+} = J_{e+}(0) \int_{0}^{\infty} (P/P_{o})^{-m}dt \tag{2.3}$$

where (for the PCLC model) $m = 9/2$ and P_o is the value of P at $t = 0$.

There are certain assumptions involved in setting the upper limit in the integral of equation (2.2) as ∞ and in assuming that, in equation (2.1), P is the only quantity which changes in time. Pulsars "turn off" due either to cessation of pair production (Sturrock, 1971b) or to magnetic field decay (Manchester and Taylor, 1977,* p. 164). However, the timescale

* This book is referred to subsequently as "MT."

for turn-off appears to be of order 10^6 years, whereas we will be considering pulsars similar to the Crab pulsar, which initially have characteristic ages of order 10^3 years.

In terms of the familiar "braking index" n,

$$\dot{P} \propto P^{2-n} \tag{2.4}$$

which may be expressed as

$$P^{n-2} \dot{P} = P_o^{n-1} \tau_o^{-1} \quad . \tag{2.5}$$

where τ_o is the initial value of the characteristic "age" $\tau(s)$ defined by

$$\dot{P} = P \tau^{-1} \quad . \tag{2.6}$$

Hence

$$\frac{P}{P_o} = \left[1 + (n-1) \tau_o^{-1} \tau\right]^{\frac{1}{n-1}} . \tag{2.7}$$

By substituting equation (2.7) into equation (2.3), we arrive at the following estimate for the total number of positrons produced by a pulsar:

$$N_{e^+} = \frac{1}{m-n+1} J_{e^+}(0) \tau_o \quad . \tag{2.8}$$

For the braking index, we adopt the value $n \approx 2.5$ found by Groth (1975) for the Crab pulsar. (The changes in our estimates which would be made by adopting $n = 3$ would be negligible.) Hence, for the present model, (2.1) and (2.8) lead to

$$N_{e^+} = 10^{-41.9} B^3 R^{13/2} P_o^{-9/2} \tau_o \quad . \tag{2.9}$$

By calculating the magnetic torque and by estimating the torque also from the characteristic age τ, we may obtain (Sturrock, 1971b) the following expression for the magnetic field strength

$$B = 10^{15.1} I^{1/2} R^{-3} P \tau^{-1/2} \quad , \tag{2.10}$$

where $I(g\ cm^2)$ is the moment of inertia. We may now eliminate B from equation (2.9) by means of equation (2.10) to obtain instead

$$N_{e+} = 10^{3.4} I^{3/2} R^{-5/2} P_o^{-3/2} \tau_o^{-1/2} \quad . \tag{2.11}$$

In terms of the neutron-star model of Baym, Pethick and Sutherland (1971), we may conveniently express the principal parameters of a neutron star as follows (Sturrock, Baker and Turk, 1976):

$$M = 10^{33.45} \mu, \ I = 10^{44.79} \mu, \ R = 10^{5.85} \mu^{-1/2} \quad , \tag{2.12}$$

where the maximum mass is given by $\mu = 1$. Then equation (2.11) takes the form

$$N_{e+} = 10^{56.0} \mu^{11/4} P_o^{-3/2} \tau_o^{-1/2} \quad . \tag{2.13}$$

III. PCFB MODEL

It is possible to repeat the calculations of Sturrock (1971a) in terms of the PCFB model. Then equation (2.1) is replaced by

$$J_{e+} = 10^{-10.0} M^{-5/6} B^3 R^{13/2} P^{-11/3} \quad . \tag{3.1}$$

Since now $m = 11/3$, equation (2.9) is replaced by

$$N_{e+} = 10^{10.3} M^{-5/6} B^3 R^{13/2} P_o^{-11/3} \tau_o \quad . \tag{3.2}$$

On noting that, for the present model,

$$B = 10^{2.5} M^{1/3} I^{1/2} R^{-3} P^{2/3} \tau^{-1/2} \quad , \tag{3.3}$$

and substituting this expression in equation (3.2), we obtain the following expression for the total number of positrons produced by a pulsar according to the PCFB model:

$$N_{e+} = 10^{-2.8} M^{1/6} I^{3/2} R^{-5/2} P_o^{-5/3} \tau_o^{-1/2} \quad . \tag{3.4}$$

On using the paramaterization of equation (2.12), this formula becomes

$$N_{e+} = 10^{55.3} \mu^{35/12} P_o^{-5/3} \tau_o^{-1/2} \quad . \tag{3.5}$$

IV DISCUSSION

Estimates will be based upon the Crab pulsar, since this is the one for which we have most information. At the present time, $P = 10^{-1.48}$ and $\tau = 10^{10.90}$. However, on noting that the Crab pulsar was "born" 925 y ($10^{10.47}$s) ago, we find [by using equation (2.7) to work backwards] that $P_o = 10^{-1.71}$ and, since $\tau \propto P^{n-1}$, $\tau_o = 10^{10.54}$.

The total power budget of a pulsar is given by

$$S_T = (2\pi)^2 \ I \ P^{-2} \ \tau^{-1} \tag{4.1}$$

which may be expressed approximately as

$$S_T = 10^{46.4} \ \mu \ P^{-2} \ \tau^{-1} \quad . \tag{4.2}$$

Hence the estimated power budget of the Crab nebula 10^{38} erg s^{-1} (MT, p.60) indicates that $\mu \approx 10^{-0.5}$. We may now estimate the total number of positrons ejected by a pulsar such as the Crab by using the above estimate of the mass, the above estimates of the initial values of P and τ, and equations (2.13) and (3.5). These estimates are found to be $10^{51.9}$ for the PCLC model and $10^{51.4}$ for the PCFB model. In view of the uncertainty in our estimate of the mass (not to mention uncertainties in the models), this difference is insignificant. We may therefore ignore the differences between the two models from this point on and adopt the mean value $10^{51.7}$ for the total number of positrons produced by a pulsar such as the Crab.

On referring back to the annihilation rate $10^{43.5}$ s^{-1} in the galactic center inferred by Leventhal et al. (1978), we see that the positrons can be supplied by pulsars if the birthrate of pulsars in the galactic center is $10^{-8.2}$ s^{-1} or $10^{-0.7}$ y^{-1}.

Manchester and Taylor (MT, p.138) estimate the (area) density of "visible" pulsars in the solar neighborhood to be $90 kpc^{-2}$ or about $10^{-4.0}$ pc^{-2}. The distribution of pulsars in the period-pulse-width diagram (MT, p. 18) indicates that, if the typical pulsar beam is a pencil beam of circular cross-section, the half-angle ψ of the beam is 5° or $10^{-1.1}$ radian. Since on the polar-cap model there are two such beams and each beam will sweep out a solid angle of $2\pi . 2\psi$ (for the orthogonal case), we may infer that we see a fraction $10^{-0.8}$ of all pulsars. Hence the density of pulsars is inferred to be $10^{-3.2}$ pc^{-2}. The average age of pulsars is estimated on kinematic considerations (MT, p.162) to be about 10^6 y. This leads to $10^{-9.2}$ pc^{-2} y^{-1} as an estimate of the birth rate of pulsars.

A current estimate of the mass density in the solar neighborhood (Whitley, 1977) is 10^{-1} $M_\odot pc^{-3}$. The mass distribution with respect to the galactic plane (Allen, 1973) has an equivalent thickness of 350 pc or $10^{2.5}$ pc, so that the mass density in the solar neighborhood is $10^{1.5}$ $M_\odot pc^{-2}$. Hence we infer that the mean birthrate of pulsars, referred to mass rather than area, is $10^{-10.7}$ $M_\odot^{-1}$ y^{-1}.

The entrance aperture of the Leventhal et al. instrument has an FWHM width of 15°. Adopting this as a sharp edge to the window, we see that the instrument accepts all emission originating closer than 1.3 kpc from the galactic center. Extrapolation of the data presented by Oort (1977) indicates that the total mass within this circle is $10^{10.3}$ $M_\odot$. Hence the pulsar birthrate in this region should be about $10^{-0.4}$ y^{-1}.

The above estimate of the birthrate in the region of the galactic center viewed by the Leventhal instrument differs only by a factor of 2 from what is required to explain their 511 keV gamma-ray flux as resulting from positrons produced by pulsars. Although the closeness of the agreement should not be taken too seriously, since the calculation of the required birthrate is sensitive to the assumed pulsar magnetic field strength, rotation rate and mass, it seems fair to conclude that, within the limitations of current knowledge, the intensity of the 511 keV gamma-ray line from the galactic center can be explained as the result of annihilation of positrons produced by pulsars.

The above conclusion is necessarily model-dependent. For comparison, we may note that the model of the Crab pulsar developed by Cheng and Ruderman (1977) leads to a flux of secondary positrons of about $10^{36} s^{-1}$. [According to the parameters of that model, it appears that photons radiated (by the synchrotron mechanism) from the secondary particles will have too low energy to lead to additional stages of pair production.] This flux is smaller by a factor of about 10^5 than the flux produced on either the PCLC or PCFB model, indicating that one could not explain the 511 keV line from the galactic center as the result of positron production by pulsars on the basis of the Cheng-Ruderman model.

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* Referred to in text as "MT."