NASA TECHNICAL MEMORANDUM NASA TM-75384

APPROXIMATION THEORY FOR BOUNDARY LAYER SUCTION THROUGH INDIVIDUAL SLITS

A. WALZ

Translation of "Naherungstheorie fur Grenzschichtabsaugung durch Einzelschiltze", Deutsche Versuchsanstalt fur Luft- und Raumfahrt, Institut fur Angewandte Mathematik und Mechanik, Freiburg, West Germany, DVL-184, June 1962

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MAY 1979

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-75384		
4. Title and Subtitle: APPROXIMATION THEORY FOR BOUNDARY LAYER SUCTION THROUGH INDIVIDUAL SLITS		5. Report Date: May 1979
		6. Performing Organization Code
7. Author(s): A. Walz, Institute for Applied Mathematics and Mechanics, German Research Institute for Aviation.		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address: Leo Kanner Associates, Redwood City, California 94063		11. Contract or Grant No.: NASw 3199
		13. Type of Report and Period Covered: Translation
12. Sponsoring Agency Name and Address: National Aeronautics and Space Administration, Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of Naeherungstheorie fuer Grenzschichtabsaugung durch Einzelschlitze", Deutsche Versuchsanstalt fuer Luft- und Raumfahrt, Institut fuer Angewahdte Mathematik und Mechanik, Freiburg, West Germany, DVL-184, June 1962.

16. Abstract

The basic concepts of influencing boundary layers are summarized, especially the prevention of flow detachment and the reduction of frictional resistance. A methematical analysis of suction through a slit is presented, two parameters, for thickness and for shape of the boundary layer, being introduced to specify the flow's velocity profile behind the slit. An approximation of the shape parameter produces a useful formula, which can be used to determine the most favorable position of the slit. An aerodynamic example is given.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified		

Précis

The application of boundary layer suction is as a rule concerned with single slits, while the majority of the theories deal with the case of continuous suction. In this report, a simple theory for suction by single slits is developed. The theory is connected with the calculation procedure for laminar and turbulent boundary layers, described in DVL Reports 84(1959) and 136(1960).

In this theory there are only two unknowns for the velocity profile, a thickness parameter and a shape parameter. The basic concept of suction theory now consists in ascertaining the values of these two parameters behind the suction slit from the parameter values in front of the slit, since the arriving velocity profile with the given suction quantity is "clipped" according to the continuity condition and the parameter values for the remaining profile are specified in an appropriate manner. The calculation then continues behind the slit with this parameter value.

The theory is fully developed in this report at first only for turbulent boundary layers on the basis of velocity profiles according to the common and simple exponential law. As early as 1943 the author had presented an incomplete form of this theory in a limited-access report of the Aerodynamical Research Institute, Göttingen, and compared it with measurements. Several essential findings of this older report have been reproduced in the current report.

Contents

I. Introduction. p.1

II. The Fundamental Possibilities for Influencing the Boundary Layer. p.2

II.1. Measures to Prevent Stream Detachment. p.2

II. 2. Measures to Reduce the Frictional Resistance (Maintenance of the Boundary Layer's Laminarity. p.4

III. Approximation Theory for Suction through a slit. p.8

III.1 A General Statement of the Problem. p.8

III.2 A More Precise Statement of the Problem. p.9

III.3 The Influence of the Suction Sink on the External Velocity Distribution. p.10

III.4 Alterations of the Boundary Layer Parameters by Suction. p.14

III.4.1 The Fundamental Physical Concept. p.14

III.4.2 Determination of the Boundary Layer Parameters Altered by Suction. p.16

III.4.2.1 A Brief Summary of the Boundary Layer Calculation Procedure for Constant Material Values (Incompressible Flow). p.16

III.4.2.2. Derivation of the Connection between the Suction Number C_Q and the Boundary Layer Parameters. p.19

IV. The Application of Suction Theory to Turbulent Boundary Layers. p.22

IV.1 Usage Formulas on the Basis of Power Profiles. p.22

IV.2 A General Appraisal of the Suction Effect. p.25

V. An Example from Aerodynamics: Ascertainment of the Most Favorable Disposition of the Suction Slit for Increases of $c_{a\ max}$ on NACA-profile 23015. p.28

VI. Summary. p.31

Illustrations. p.33

References. p.44

APPROXIMATION THEORY FOR BOUNDARY LAYER SUCTION THROUGH INDIVIDUAL SLITS

A. Walz

I. Introduction. /5*

The viscosity of fluids has effects that are almost always undesirable in flow technology. Among these are, e.g., the resistance of bodies in the flow to friction and pressure, the pressure losses in pipelines, and the detachment of the flow with its subsequent disadvantages. For the order of Reynold's numbers prevalent in flow technology, the viscosity is practically restricted in its action to a very thin boundary layer. Thus the above unfavorable phenomena are also to be referred to processes in the boundary layer, for large Reynold's numbers.

L. PRANDTL [1] was the first to show (1904) the connection between the processes in the boundary layer and the outer flow of potential. From then onwards, the way was clear for all investigations that attempted to choose the contours of objects in a flow or the cross sectional curve of channels in such a way that the boundary layer processes, in respect of flow resistance and detachment phenomena, occur in an optimal manner. Some of this work led to the development of airfoil profiles with small resistance (the laminar profile) and at the same time a high maximum lift [2,3,4,5]. Recently, R. EPPLER [6] has treated this problem with the modern methods of potential theory and boundary layer theory. F. X. Wortman [7] has made ex-

*Numbers in the margin indicate pagination in the foreign text.

tensive measurements in regard to this problem. But also in this connection we should mention all the mathematically profound and experimentally difficult works that explore the conditions for stability of the laminar boundary layer (cf. the newest findings of H. Görtler's research group, Freiburg Institute, West Germany, and for a bibliography see, e.g., DVL Report 8).

Let us now assume that we have exhausted all possibilities to constitute an optimal pressure, working on the boundary layer from without. It is now to be asked whether there are yet other physical or technical possibilities for intervention in the boundary layer processes. In critically evaluating these possibilities, it is seen that a distinction should be made between the measures for preventing flow detachment and those for reducing the frictional resistance. This will be illustrated with the help of figures 1 through 7.

II. The Fundamental Possibilities for Influencing the Boundary Layer. /6

II. 1. Measures to Prevent Stream Detachment.

The manner in which a boundary layer (assumed to be laminar) affects a pressure gradient dp/dx in the flow direction should be immediately apparent from fig. 1. In the case of $dp/dx = 0$ (flow along a level plate), we find something like the indicated curve of the velocity u with the distance y from the wall (Blasius profile [9]). The particles of fluid close to the wall still have a slight kinetic energy, by which they become "playthings" of external pressure gradients dp/dx, which are nonetheless active or "impressed" in the neighborhood of the wall. If $dp/dx < 0$, then the particles near the wall are accelerated. Velocity profiles that are more full than the Blasius profile are the result. In the case of a positive pressure gradient ($dp/dx > 0$), kinetic energy is expended in the formation of the static pressure field. The velocity close to the wall decreases even further than for the Blasius profile. The result is velocity profiles with a turning point so that, in the neighborhood of the wall and for sufficiently large dp/dx, there may occur an inversion of the velocity direction, with consequent flow detachment.

To avoid flow detachment in this case we can use, for example, a wall stage with adjacent suction channel, as shown schematically in fig. 2. The "sick" flow of the boundary layer near the wall disappears now in the suction channel (naturally one must provide for a lower pressure in the channel with respect to the outer pressure), and a new, "healthy" boundary layer flow begins behind the suction point. It has a fuller velocity profile, able to overcome further pressure rises.

A comparison of the velocity profiles before and after the suction point shows immediately that, while the suction has prevented detachment of the flow, it leads to a greater wall friction behind the suction point, due to the greater velocity. We thus cannot expect to reduce the frictional resistance by suction alone. The points later discussed in connection with figs. 8 and 9 have some bearing here. We now consider still other possibilities for preventing flow detachment.

It is technologically more simple to produce a suction by a slit in the surface, as sketched in fig. 3. The result is practically the same as the case in fig. 2: the energy-poor /7
boundary-layer flow beneath the dot-and-dash streamline is engulfed in the suction slit. Behind this slit, a fuller and energy-rich velocity profile is again available to confront new

pressure increases without danger of detachment.

Fig. 4 represents the case of a continuous suction, such as by a porous wall. It is the object of much research (see the bibliography in [10]). We shall content ourselves here with a reference to the bibliography. Another possible way to increase the kinetic energy of the boundary layer near the wall is by <u>blowing</u>, tangential to the surface and in the flow direction, as shown in fig. 5. By this means, the velocity profile behind the blowing point becomes more full. But at the same time, the thickness of the boundary layer can grow, which is not the case for the preceding measures.

Finally, it is possible to influence the boundary layer in prevention of flow detachment by moving the wall in the flow direction. But this measure has till now hardly been applied in practice, apparently due to the mechanical difficulties. (Example: the Flettner Rotor for producing a circulation flow, see [10], p. 247).

To illustrate what advantages can result to flow technology by the prevention of flow detachment, the classical experiment of L. PRANDTL [11] on a short diffuser with two suction slits should be recalled, figs. 6 and 7. Also the successful attempts of the AVA Göttingen to increase the maximum lift of airfoil profiles should be consulted (for a summary of which see, e.g., [12].

II.2. <u>Measures to Reduce the Frictional Resistance (Maintaintenance of the Boundary Layer's Laminarity)</u>.

For the measures depicted in figs. 1-5 we can say in summary that, near the wall, the velocity within the boundary layer is increased. In this way, the flow is prevented from detachment in regions of pressure rise. But these measures, as already indicated, at first appear to be ill-suited to reducing the frictional resistance, since the shearing stress τ_w of the wall increases, after Newton's theorem, with the velocity gradients perpendicular to the wall (y-direction). /8

$$\tau_w = \left(\mu \frac{\partial u}{\partial y}\right)_{y=0} \tag{1}$$

u = the velocity component parallel to the wall (x-direction)
μ = the viscosity coefficient

But reducing the thickness of the boundary layer and making the velocity profile more full can have an additional important effect, about which we have not yet spoken: the laminar condition of the boundary layer can be stabilized or--in other words--can be prevented from entering the turbulent condition at turning points. The consequence of this is illustrated in figs. 8a, 8b, and 9. If the laminar and the turbulent velocity profile are compared, figs. 8a and 8b, with the thickness of the boundary layer being equal in both cases, $\delta_l = \delta_t$, and the velocity u_δ at the edge of the boundary layer also being equal for the two, it is seen that the velocity gradient $(\partial u/\partial y)_{y=0}$ of a turbulent boundary layer is much greater than that of a laminar, since the turbulent exchange movements bring kinetic energy into the wall proximity.[1] In this way the frictional resistance can be much greater for a turbulent, than a laminar boundary layer. If we consider, for example, the total frictional coefficient C_F of a level plate (W = total resistance per unit of width, l = length

---

[1] If it is intended to prevent flow detachment, then the conversion of the boundary layer from the laminar to the turbulent condition is to be regarded as a desirable process, which can have as favorable an effect as suction. Thus, for flow processes with a small Reynold's number, artificial turbulence, such as produced by a trip wire, is a simple measure to prevent detachment, often used in practice.

of the plate, ρ = density of the fluid)

$$C_F = \frac{W}{\frac{\rho}{2} u_\delta^2 l} = 2 \int_0^1 \frac{\tau_w}{\rho u_\delta^2} \, d\left(\frac{x}{l}\right) = 2 \int_0^1 c_f \, d\left(\frac{x}{1}\right) \qquad (2)$$

with

$$c_f = \frac{\tau_w}{\rho u_\delta^2} \quad = \text{the local coefficient of friction,} \qquad (3)$$

then we find C_F and the Reynold's number /9

$$R_1 = \frac{\rho u_\infty l}{\mu} \qquad (4)$$

connected by the curve $C_F(R_1)$, which is very different for a laminar and a turbulent boundary layer (fig. 9). When the Reynold's numbers are greater than 10^7, $(C_F)_{turb}$ can be almost one power of ten greater than $(C_F)_{lam}$. For $R_1 > 10^7$, the difference between the two can become greater still.

If special measures are not taken to maintain the laminarity of the boundary layer, there occurs a transition at about $R_x \approx 10^6$ (x = path length) from the laminar to the turbulent condition, for flow along a smooth and even plate. This is a consequence of instability of the laminar boundary layer. Now it is stated in the stability theory of a laminar boundary layer that the layer can remain laminar, even at $R_x > 10^6$, if

a) the thickness of the boundary layer is reduced by suction,

b) the velocity profile is made more full than the Blasius profile of the level plate.

However both of these are conditions that can be fulfilled by suction of the boundary layer. Apparently it is very important not to suck off more fluid than is absolutely necessary to maintain the stability of the laminar layer, since both measures, a) and b), increase the wall shearing stress of the laminar boundary layer. Furthermore, a certain amount of power must be expended in transporting the suctioned material, which increases with the quantity transported. Since the maintenance of boundary layer laminarity can improve the frictional coefficient by several orders of magnitude (especially for the case of large Reynold's numbers R_1), these unfavorable consequences of suction play only a minor role in comparison to the theoretically minimal quantity of suction. It is also apparent that the theoretically minimal quantity of suction is attained for the case of continuous suction (fig. 4), which is also a measure for preventing flow detachment. The technical realization of this case is naturally very difficult, since porous surfaces are easily occluded.

As H. HOLSTEIN [14] was the first to show, and as W. PFENNINGER [15] has recently demonstrated with measurements, the optimal case of continuous suction can be approximated with a rather large number of slits, succeeding each other in the flow direction. The technique of suction through single slits is also promising for aircraft design, as seen from both past[2] and recent experiences. /10

W. PFENNINGER [16] has briefly reported on new experimental results of suction by single slits, during which it was possible to maintain the laminar boundary layer on airfoils up to very high Reynold's numbers, $R_1 \approx 3 \cdot 10^7$. The coefficient of profile resistance C_F (with the power expenditure for the suction taken into account) was $C_F \approx 10^{-3}$. (For a turbulent boundary layer, C_F was nearly 6 times as great). The results, discussed in [16], as to the amount of suction that should be employed, are also in good accordance with the theoretical cal-

[2] Cf. the bibliography on the tests in the AVA Göttingen and the ETH Zürich, [12] to [15].

culations for the stability problem.

According to a report by H. R. Head [17], results have lately been achieved in England that are equally favorable.

Thus we can consider the maintenance of boundary layer laminarity by suction to be a special modern problem in flow mechanics.

III. Approximation Theory for Suction through a Slit.

III.1. A General Statement of the Problem.

It is a great deal more easy to implement suction with single slits, than continuous suction through porous materials (danger of occlusion). Yet for the theoretical treatment of this problem, we find a totally different picture, since we can easily specify exact solutions of the boundary layer equations, when dealing with the continuous suction of a laminar boundary layer (cf., e.g., B. H. Schlichting [10]); but the theoretical treatment of single-slit suction is complicated (cf., e.g., W. Wuest [18, 19], W. Rheinboldt [20]).

In this report we shall describe a simple theory for suction with single slits. It can be applied to laminar, as well as turbulent boundary layers. This theory follows very closely the approximation theory that was presented in DVL Reports 84 and 136. The theory enables the design engineer to calculate, with sufficient accuracy, the amount and power of suction needed, for example, to raise the maximum lift coefficient of a given airfoil profile by a certain amount, or to prevent flow detach- /1
ment in a given short diffuser. In applying this theory to laminar boundary layers, we must also take into consideration the

elementary findings of stability theory (cf., e.g., [10,19]).

It is even possible to use this approximation theory to investigate the case of continuous suction as a boundary case with a great number of single slits arranged in succession. Still, a calculation of this sort has not yet been attempted.

As long ago as 1943 the author had presented an imperfectly developed form of this theory for suction through single slits in a ZWB Report of the AVA Göttingen (Nr. 1775)[21], not generally available, and correlated it with measurements. In order to calculate the turbulent boundary layer before and behind the suction site, the then-common procedure of E. GRUSCHWITZ [22] was used. It has been shown that, for the correlative measurements in the region of Reynold's numbers that was employed, this procedure is only slightly less exact than the newer procedures, such as those of E. TRUCKENBRODT [23] and A. WALZ [24]. In this report we shall again present the essential results of the previous calculations and correlations with measurement, but shall give only the improved version of the theory. Work is now being done to confirm the former results with this improved theory, as well as to calculate the case of suction of laminar boundary layers.

III.2. A More Precise Statement of the Problem.

We shall consider a two-dimensional, stationary, incompressible flow along a solid surface, e.g., along the contour of an airfoil profile. Let us assume that an infinitely long suction slit runs transverse to the principal direction of the flow. The breadth of this slit is taken to be negligibly small in comparison to the depth l of the airfoil, or

$$\frac{s}{l} \ll 1 \qquad (1)$$

If, now, a lower pressure is used to suction flow material into the interior of the airfoil, this suction flow has two diverse effects:

1. a direct effect on the boundary layer flow, since its material is disappearing into the suction sink;

2. an indirect effect on the boundary layer, since a sink flow is superimposed on the potential flow outside the boundary layer. The velocity distribution at the edge of the boundary layer, altered by the action of the sink, causes the boundary layer before and behind the suction slit to develop differently, than if the sink were not there. /12

A theory for suction must thus comprise, in addition to the main problem of boundary layer theory, also a pure problem of potential theory, due to the sink effect. In the boundary layer calculations, the potential theory distribution of velocity at the edge of the boundary layer must be known in advance. For this reason we choose to deal at first with this supplementary, potential theory problem of the sink effect. It will be seen that this effect, even for very small suctions, is considerable.

III.3. The Influence of the Suction Sink on the External Velocity Distribution.

We shall now calculate the additional velocity that a two-dimensional, pointlike sink (with linear extension along the airfoil span) produces on the surface of an airfoil profile. For simplicity we now assume that the profile depth is infinitely great and, consequently, that the airfoil surface, upon which the sink is situated, is infinitely extended. If Q is the suction quantity that disappears in a unit of time across the wingspread b of the airfoil and into the sink, then the potential theory additional velocity Δu_δ at a distance a from the sink midpoint is, in the familiar manner,

$$\Delta u_\delta = \frac{Q}{\pi ab} \tag{2}$$

where $2ab\pi$ is a cylinder surface with the sink line as axis.

It is helpful to make the additional velocity Δu_δ nondimensional with the undisturbed velocity of the approach flow u_∞, and also to introduce in eq. (2) an airfoil surface (now taken to be finite):

$$F = bl \tag{3}$$

where l is the depth of the profile. We now obtain:

$$\frac{\Delta u_\delta}{u_\infty} = \frac{Q}{u_\infty lb} \frac{l}{\pi a} = \frac{Q}{u_\infty F} \frac{l}{\pi a} = c_Q \frac{l}{\pi a} \tag{4}$$

We designate /13

$$c_Q = \frac{Q}{u_\infty F} \tag{5}$$

as the "suction figure". Thus the suction quantity Q is related to an imaginary quantity, $u_\infty F$, supposed to occur during flow with a velocity u_∞ along the airfoil surface F in unit time.

Relation (4) can also be used for a finite breadth s of the suction slit, if the midpoint of the sink is placed in the middle of the suction slit. The calculation in this case proceeds only to the edge of the slit, so that the singularity $\Delta u_\delta \to \infty$ for $a \to 0$ at the sink midpoint causes no trouble in practice.

F. EHLERS [25] (AVA Göttingen) has proved, in an unpublished work of 1944, that the simple relation (4) for the additional velocity, resulting from the effect of the sink, must be corrected if a finite wing depth l is assumed. In this case we must assume a sink on a circular periphery and, in order to determine the additional velocity on a given profile, and include the sink in the conformal representation of the circle (z-plane) on the profile contour (ξ-plane). According to Ehlers, the additional velocity is more exactly expressed by

$$\frac{\Delta u_\delta}{u_\infty} = \frac{c_Q}{\pi R/l} \left|\frac{dz}{d\xi}\right| \left(\operatorname{ctg} \frac{\varphi - \varphi_A}{2} - \operatorname{ctg} \frac{\varphi_A}{2}\right) \tag{6}$$

than by (4). Here, alluding to [25] and [26], R is the radius of the image circle $\left|\frac{dz}{d\xi}\right|$ is the absolute value of the differential quotient of the image function $\xi = f(z)$ that represents the circle (z-plane) on the profile plane (ξ-plane), and ϕ is the angle on the image circle, with which the individual profile points are co-ordinated: $\phi = 0$ corresponds to the rear edge of the profile, $\phi = \phi_A$ to the suction site on the profile.

The total velocity U_δ of potential theory, which is decisive for the development of boundary layers, is, from a linear superposition of the main and the additional flow:

$$\frac{U_\delta}{u_\infty} = \frac{u_\delta}{u_\infty} + \frac{\Delta u_\delta}{u_\infty} \tag{7}$$

The theoretical investigations of Ehlers have also produced the interesting result that the sink flow also influences the circulation about the profile, i.e., the lift A. This result is physically explained by the fact that the sink flow, as well as the main flow about the profile, is subject to the Kutta-Zhukovskiy flow-off condition at the rear edge. From the side of the suction site that is closer to the rear edge, less material flows to the sink, since the sharp rear edge hinders a flow from the underside of the profile around to the slit. The sink flow thus contributes to the circulation about the airfoil /14 profile.

The lift coefficient c_a is defined, as is known, by:

$$c_a = \frac{A}{\frac{\rho}{2} u_\infty^2 F} \tag{7a}$$

According to Ehlers, the lift coefficient is enhanced by

$$\Delta c_a = 2 c_Q \operatorname{ctg} \frac{\varphi_A}{2} \tag{8}$$

due to the effect of the suction sink on the circulation. This effect of a sink flow, especially the accelerated increase in c_a as the suction sink approaches the rear edge $(\varphi_A \to 0)$, has been qualitatively confirmed by the measurements of B. REGENSCHEIT [26]. For $\phi_A = 0$, (8) produces the value $\Delta c_a = \infty$, which is no longer physically logical. Evidently formulas (8) and also (6) can only be used if ϕ_A and $\phi - \phi_A$ are substantially different from zero. In practice, this difficulty in the use of the Ehlers formula does not occur, since there are always finite widths s of the suction slit. Thus, in the case of formula (8), the distance of the sink midpoint from the rear edge is always s/2 and the corresponding arc ϕ_A on the image circle (due to the large value of $|dz/d\xi|$ in the neighborhood of the rear edge) is always relatively large. The additional velocity Δu_δ of eq. (6) is only of interest for boundary layer calculations up to the edge of the suction slit, or to the distances $\pm s/2$ from the sink midpoint. Here also the infinity site is avoided. /15

Fig. 10 shows a schematic representation of streamlines in the region of a suction slit. An essential feature of this pattern is the backwater line (dot-and-dash) which, for a properly chosen suction quantity, ends at the rear edge of the suction slit. The flow material between this line and the profile surface disappears into the suction slit. Fig. 11 shows (qualitatively) the corresponding velocity curve at the edge of the boundary layer, including the additional velocity due to the effect of the sink. It can be seen that the sink creates an acceleration of the total velocity U_δ upstream and a retardation downstream. For boundary layer calculations, at any rate, only the velocity distribution above the solid profile surface in front of point I and behind point II in fig. 11 is of interest. This is the region that can be used with sufficient exactness in formula (6). In the region I,II = s directly above the suction slit, there are positive additional velocities in the forward half and negative ones in the rear half, so that the integral of the additional velocities between I and II is equivalent to zero. Thus, for the development of the boundary layer directly above the slit, i.e. between I and II, the suction sink is, to a first approximation, without meaning. Since there is no wall friction above the suction slit, we can ignore the finite (and presumably very small, according to eq. (1)) breadth s of the suction slit for the boundary layer calculations, and take as a basis the velocity curve $U_\delta(x)$ shown schematically in fig. 12, with a discontinuity at the suction site.

This discontinuity presents no difficulties for the calculations. It is the case, for boundary layer computations, that the influences of the acceleration before the slit and of the retardation behind are somewhat compensated in their effect on the development of the boundary layer profile between I and II, when the velocity discontinuity is assumed.

III.4. Alterations of the Boundary Layer Parameters by Suction.

III.4.1. The Fundamental Physical Concept.

It is now important to ascertain the direct alteration of the velocity profile due to a given suction quantity. We can assume that only a portion of the boundary layer material, as a rule less than half, is removed. For this case, the streamline curve in fig. 10 can also be used. A more vigorous suction, according to experimental experience, provides no advantageous effects, and when the suction is to reduce the resistance it is only a disadvantage, as we have already discussed. /16

Fig. 10 already enables us to evaluate how the velocity profile is altered by suction:

A certain velocity profile arrives at the forward edge of the suction slit (point I). The backwater line divides this profile into an inner (near the wall) and an outer region. Behind the suction slit, the inner region of the velocity profile has disappeared. The outer region now forms the initial profile for the development of the boundary layer behind the suction slit. The velocity profile, "clipped" by suction, glides almost without friction along the backwater line above the slit and across to the rear edge of the latter. In this way, the velocity at the "division surface" is reduced to zero when the backwater point reaches the rear edge of the slit II. Thus the more full velocity profile II behind the slit is derived from the velocity profile I in front of the slit. At the same time, the total thickness of the boundary layer has been reduced from δ_I to δ_{II}. The exact shape of the velocity profile II is in any case unknown. It must therefore be clarified whether and how the velocity profile II for a given suction quantity and for a given velocity profile I can be characterized so unequivocally that the boundary layer calculation can be continued behind the suction slit. We shall limit ourselves here to a boundary layer calculation with an approximation procedure, as has been described, for example, in DVL Reports 84 and 136. In this procedure, the velocity profile of the boundary layer is established by two characteristic magnitudes: a thickness parameter and a shape parameter. Our problem is solved if it is possible to calculate the numerical values of these two parameters for the velocity profile II from the numerical values of the parameters of the velocity profile I, for given suction quantities.

III.4.2. Determination of the Boundary Layer Parameters Altered by Suction.

III.4.2.1. A Brief Summary of the Boundary Layer Calculation Procedure for Constant Material Values (Incompressible Flow).

The above-mentioned approximation theory for boundary layer calculation [24] basically works with integral conditions /17
for impulse and energy within the boundary layer. These two integral conditions are presented in the form of two ordinary differential equations for two characteristic magnitudes of the boundary layer. As already indicated, it is convenient to choose a thickness parameter and a shape parameter as the characteristic magnitudes, and the velocity profile u(x,y) is then generally described by an expression in the form:

$$\frac{u(x,y)}{u_\delta(x)} = f\left[\frac{y}{\delta(x)}, H(x)\right] \tag{9}$$

Here $\delta(x)$ is the thickness parameter (the total thickness of the boundary layer) and H(x) is the shape parameter. The function f, which is different for a laminar and a turbulent boundary layer, is assumed to be known in reference to special exact solutions or measurements. The definition of the shape parameter can be chosen at will.

In this approximation theory with integral conditions, the following three integral expressions play a special role in the case of incompressible flow (to which we limit ourselves here):

$$\delta_1 = \int_0^\delta \left(1 - \frac{u}{u_\delta}\right) dy \; ; \qquad \delta_2 = \int_0^\delta \frac{u}{u_\delta}\left(1 - \frac{u}{u_\delta}\right) dy \; ;$$

Displacement Thickness Thickness of Impulse Loss

$$\delta_3 = \int_0^\delta \frac{u}{u_\delta}\left[1 - \left(\frac{u}{u_\delta}\right)^2\right] dy$$

Thickness of Energy Loss (10,11,12)

From physical and formal reasons it is expedient to introduce as the thickness parameter of the boundary layer not the "total thickness δ", which is difficult to define, but the magnitudes:

$$Z = \delta_2 \left(R_{\delta_2}\right)^n \quad \text{with} \quad R_{\delta_2} = \frac{s\, u_\delta\, \delta_2}{\mu}$$

n=1 for a laminar
n=0.268 for a turbulent boundary layer (13,14)

and to introduce as shape parameter:

$$H = \frac{\delta_3}{\delta_2} \tag{15}$$

These differential equations (with ' indicating the derivative with regard to x) now appear as: /18

$$Z' + Z\,\frac{u_\delta'}{u_\delta}\,F_1(H) - F_2(H) = 0 \tag{16}$$

$$H' + H\,\frac{u_\delta'}{u_\delta}\,F_3(H) - \frac{F_4\left(H, R_{\delta_2}{}^{n-N}\right)}{Z} = 0 \;. \tag{17}$$

with

$$F_1 = (2+n) + (1+n)\frac{\delta_1}{\delta_2}(H)\, ; \quad F_2 = (1+n)\,\alpha(H) \tag{18,19}$$

$$F_3 = 1 - \frac{\delta_1}{\delta_2}(H)\, ; \qquad F_4 = 2\beta(H) R_{\delta_2}^{\,n-N} - H\alpha(H) \tag{20,21}$$

$\alpha(H)$ = partial function of the local frictional coefficient, dependent on the shape parameter H

$\beta(H)$ = portion of the dissipation function dependent on the shape parameter H (for a turbulent boundary layer, $\beta(H)\,\alpha\beta$ =0.0056)

laminar boundary layer $\quad n = N = 1 \qquad (22)$

turbulent boundary layer $\quad n = 0.268,\ N = 0.168 \mid n - N = 0.1 \qquad (23)$

$u_\delta(x)$ is the velocity distribution, to be assumed as given, at the edge of the boundary layer; in the case treated here, it must also include the additional velocity of the suction sink.

In DVL Report 136 /‾24/ analytical expressions for the universal functions F_1 to F_4 for laminar and turbulent boundary layers (for incompressible and compressible flow, with and without heat transfer) as well as solution methods for the system of equations (16) and (17) have been given, so that no further exposition is needed here.

We must now answer the question as to how the parameters Z and H during incompressible flow are altered under the influence of an unsteady suction by a slit. It should also be mentioned that there are no fundamental difficulties in generalizing this theory for compressible flow.

III.4.2.2. Derivation of the Connection between the Suction Number C_Q and the Boundary Layer Parameters. /19

We have already established in section III.4.1. that the shape of the velocity profile II behind the suction slit is not at first completely known. But since, within the framework of the approximation theory sketched in section III.4.2.1., integrals of the form (10), (11), and (12) on the velocity profile are essentially concerned, it is possible to represent these integral expressions at the site II, i.e. behind the suction, by the difference of two integral expressions. Starting from the physical representation of the flow curve in the region of the suction slit, as shown in fig. 11a, the following are valid:

$$(\delta_1)_{II} = (\delta_1)_I - (\delta_1)_Q \tag{24}$$

$$(\delta_2)_{II} = (\delta_2)_I - (\delta_2)_Q \tag{25}$$

$$(\delta_3)_{II} = (\delta_3)_I - (\delta_3)_Q \tag{26}$$

with

$$(\delta_1)_Q = \int_0^{y_Q} \left(1 - \frac{u}{u_\delta}\right) dy \tag{27}$$

$$(\delta_2)_Q = \int_0^{y_Q} \frac{u}{u} \left(1 - \frac{u}{u_\delta}\right) dy \tag{28}$$

$$(\delta_3)_Q = \int_0^{y_Q} \frac{u}{u_\delta} \left[1 - \left(\frac{u}{u_\delta}\right)^2\right] dy \tag{29}$$

Here y_Q is the distance from the wall to which the velocity profile I, arriving at the suction slit (fig. 11a), must be "clipped" in order that the given suction quantity per unit of time

$$Q = c_Q\, u_\infty\, b l \tag{30}$$

disappears into the suction slit. With (30),

$$Q = c_Q\, u_\infty\, b l = b u_\delta \int_0^{y_Q} \frac{u}{u_\delta}\, dy = b u_\delta\, \delta_Q \tag{31}$$

must be valid. Thus the "thickness of suction" δ_Q is defined as: /20

$$\delta_Q = \int_0^{y_Q} \frac{u}{u_\delta}\, dy \tag{32}$$

From (31) and (32) it also follows that

$$\frac{\delta_Q}{l} = c_Q \frac{u_\infty}{u_\delta} \tag{32a}$$

As we shall later show in detail, there is no difficulty in calculating the integral expressions (27) to (29), and thereby the values $(\delta_1)_{II}$, $(\delta_2)_{II}$, $(\delta_3)_{II}$ in (24) to (26) for a given group of laminar or turbulent velocity profiles according to eq. (9) as functions of the suction number c_Q, the shape parameter H, and the local velocity ratio u_δ/u_∞ .

In this way, from eqs. (13) and (14), the thickness parameter Z_{II} can also be immediately determined as the initial value of the computation behind the suction slit.

It appears at first somewhat more difficult to ascertain the shape parameter H_{II} of the velocity profile, altered by suction, from the magnitudes $(\delta_1)_{II}$, $(\delta_2)_{II}$, $(\delta_3)_{II}$, i.e., to again classify this somewhat "truncated" velocity profile in the form category of expression (9).

For a single-parameter family of curves, which we have taken as a basis in the approximation theory with the general expression (9), the relationships δ_1/δ_2 and $\delta_3/\delta_2 = H$ (see eq. (15)) are specifically correlated. Thus δ_1/δ_2 is a specific function of the chosen shape parameter H. For the velocity profiles altered by suction we can no longer assume that they may be accurately classified in the original family. We must much rather expect that the relationships $(\delta_1)_{II}/(\delta_2)_{II}$ and

$$\frac{(\delta_3)_{II}}{(\delta_2)_{II}} = H_{II} \tag{33}$$

of the profiles altered by suction can no longer be specifically coordinated to each other. If we nonetheless assume the expression (9) of the original velocity profiles to be valid, then we /21
obtain, from $(\delta_1)_{II}/(\delta_2)_{II}$ and $(\delta_3)_{II}/(\delta_2)_{II}$, two rather different H_{II}-values: $H_{II_{12}}$ and $H_{II_{32}}$. It is now our concern to arrange, at least approximately, the velocity profile altered by suction in the original[1] family of single-parameter velocity profiles, i.e., to find in this family a profile that is most similar to that altered by suction. A physically reasonable prescription for this arrangement of the "clipped" velocity profiles in the original family of curves is now, evidently, to form an arithmetic mean $\bar{H}_{II}$ from the two generally different values $H_{II_{12}}$ and $H_{II_{32}}$,

$$\bar{H}_{II} = \frac{H_{II_{12}} + H_{II_{32}}}{2} \tag{34}$$

and to continue the calculation behind the suction site II with this value. This principle for the determination of H_{II} is the more dependable as the values of $H_{II_{12}}$ and $H_{II_{32}}$ are less different from each other.

As we shall later establish, this prerequisite for the dependability of the approximation theory is fulfilled in a useful manner.

IV. The Application of Suction Theory to Turbulent Boundary Layers.

IV.1. Usage Formulas on the Basis of Power Profiles.

We shall now occupy ourselves at first with the suction of turbulent boundary layers. For the velocity profiles in this case (at any rate for the calculation of the integral expressions (24) to (29)), we can take as a basis the simple exponential expression

(35)

with

$$0 < k < 1 \qquad \text{or more exact} \quad 0.1 < k < 0.7 \tag{36}$$

On this basis, the integral expressions (10) to (12) can be im- /22
mediately and continuously evaluated. These are the expressions that are valid for the velocity profile in front of the suction site, and which we must thus distinguish with the index I.

From (10), (11), (12) with (35) it follows that

$$(\frac{\delta_1}{\delta})_I = \frac{k_I}{1+k_I} \; ; \; (\frac{\delta_2}{\delta})_I = \frac{k_I}{(1+k_I)(1+2k_I)} \; ;$$

$$(\frac{\delta_3}{\delta})_I = \frac{2k_I}{(1+k_I)(1+3k_I)} \tag{37,38,39}$$

From which is obtained

$$(\frac{\delta_2}{\delta_2})_I = 1 + 2k_I \tag{40}$$

and, with (15),

$$(\frac{\delta_3}{\delta_2})_I = H_I = 2\,\frac{1+2k_I}{1+3k_I} \quad \text{or} \quad k_I = \frac{2-H_I}{3H_I-4} \tag{41,42}$$

The integral expressions (24) to (29) assume the form, with (35),

$$\frac{(\delta_1)_Q}{\delta_I} = \frac{y_Q}{\delta_I} - \frac{(y_Q/\delta_I)^{1+k_I}}{1+k_I} \tag{43}$$

$$\frac{(\delta_2)}{\delta_I} = \frac{(y_Q/\delta_I)^{1+k_I}}{1+k_I} - \frac{(y_Q/\delta_I)}{1+2k_I} \tag{44}$$

$$\frac{(\delta_3)}{\delta_I} = \frac{(y_Q/\delta_I)^{1+k_I}}{1+k_I} - \frac{(y_Q/\delta_I)^{1+3k_I}}{1+3k_I} \tag{45}$$

From this it follows, for the magnitudes behind the suction site (distinguished by the index II), that

$$\frac{(\delta_1)_{II}}{\delta_I} = \frac{k_I}{1+k_I} - \frac{y_Q}{\delta_I} + \frac{(y_Q/\delta_I)^{1+k_I}}{1+k} \tag{46}$$

$$\frac{(\delta_2)_{II}}{\delta_I} = \frac{k_I}{(1+k_I)(1+2k_I)} - \frac{(y_Q/\delta_I)^{1+k_I}}{1+k_I} + \frac{(y_Q/\delta_I)^{1+2k_I}}{1+2k_I} \tag{47}$$

$$\frac{(\delta_3)_{II}}{\delta_I} = \frac{2k_I}{(1+k_I)(1+3k_I)} - \frac{(y_Q/\delta_I)^{1+k_I}}{1+k_I} + \frac{(y_Q/\delta_I)^{1+3k_I}}{1+3k_I} \tag{48}$$

For the thickness of suction δ_Q/δ, with (32), it is now valid that

$$\frac{\delta_Q}{\delta_I} = \int_0^{y_Q/\delta_I} \frac{u}{u_\delta}\, d\left(\frac{y}{\delta_I}\right) = \frac{(y_Q/\delta_I)^{1+k_I}}{1+k_I} \tag{49}$$

From which the magnitudes, needed in (46) to (48), with (32a) and (38) yield

$$\frac{y_Q}{\delta_I} = \left[(1+k_I)\frac{\delta_Q}{\delta_I}\right]^{\frac{1}{1+k_I}} = \left[(1+k_I)\frac{c_Q}{(u_\delta/u_\infty)_I}\frac{(\delta_2/\delta)_I}{(\delta_2/l)_I}\right]^{\frac{1}{1+k_I}}$$

$$= \left[\frac{c_Q}{(u_\delta/u_\infty)_I}\frac{k_I}{1+2k_I}\frac{1}{(\delta_2/l)_I}\right]^{\frac{1}{1+k_I}} \tag{50}$$

All the magnitudes appearing in (50) are known: $k_I(H_I)$ follows from (41); $(\delta_2/l)_I$ from Z_I according to eqs. (13) and (14); and u_δ/u_∞ and c_Q are given in the original statement of the problem.

We can now, to check the methods, described in section III.4.2.2, for the determination of the shape parameter H_{II} behind the suction slit, easily form the relations $(\delta_1/\delta_2)_{II}$ and $(\delta_3/\delta_2)_{II}$. The result is shown in fig. 13. H_{II} with $k_I(H_I)$ as parameter is diagrammed above the "suction distance" y_Q/δ_I (which is connected to the suction number c_Q, according to (50)). On the one hand, with (47) and (48), H_{II} follows directly from /24 the definition (33):

$$\frac{(\delta_3)_{II}}{(\delta_2)_{II}} = H_{II_{32}}$$

On the other hand, one can calculate H_{II} from $(\delta_1/\delta_2)_{II}$ by way of relations (40) and (41), namely as:

$$H_{II_{12}} = 2\,\frac{1+2\,k_{II}}{1+3\,k_{II}} \tag{51}$$

with

$$k_{II} = \frac{1}{2}\left[\left(\frac{\delta_1}{\delta_2}\right)_{II} - 1\right] \tag{51a}$$

As can be seen from fig. 13, the values $H_{II_{32}}$ and $H_{II_{12}}$ are only slightly different, so that we should regard the value $\bar{H}_{II}$, obtained by arithmetic averaging according to eq. (24), as a usable initial value for the shape parameter behind the suction site.

This analysis is immediately valid for turbulent boundary layers. For laminar boundary layers a corresponding verification is still wanting.

The value Z_{II}, which in addition to H_{II} is used to continue the boundary layer calculation behind the suction site, follows unequivocally from eqs. (47), (13), and (14).

IV.2. A General Appraisal of the Suction Effect.

With the simple derived relationships, we can now make a quick survey of the effect of a definite suction quantity on the boundary layer. For this purpose, we begin with fig. 13, from which it is immediately apparent to what relative distance from the wall, y_Q/δ_I, the velocity profile with shape parameter $H_I(k_I)$, arriving at the suction slit, must be "clipped" in order for a velocity profile with a prescribed value H_{II} to arise behind the suction slit. The relative suction distance y_Q/δ_1 and the suction number c_Q are connected to each other by eq. (50). The other magnitudes appearing in eq. (50)--u_δ/u_∞, $k(H_I)$, and $(\delta_2/l)_I$--are known with the calculation of the potential flow and the boundary layer up to the suction site I. The relations (41) and (42) provide the connection between k and H. /25

We shall discuss the application of eq. (50), in combination with fig. 13, by an example. For this example we shall assume that the turbulent boundary layer in front of the suction slit has passed through a region of pressure rise of such intensity that, without suction at this location, flow detachment would immediately ensue. According to [1], detachment of the turbulent boundary layer can be expected when the shape parameter H is somewhat less than the value 1.57[1]:

$$H_{det.} < 1.57 \tag{52}$$

To this value of H_{det} there corresponds, according to eq. (42), the value

$$k_{det.} > 0.606 \tag{53}$$

We should now determine the suction quantity that reduces the shape parameter $H_I = 1.57$ ($k_I = 0.606$) to the value $H_{II} =$

[1] In many instances flow detachment is first observed when $H < 1.53$. Such cases apparently prevail when a pressure drop, following a pressure rise, suctions away the dead water region, formed by the detachment, in the flow direction.

1.80 (k_{II} = 01.43). This value of H_{II} characterizes a velocity profile such as that appearing in a flow without pressure gradients and with large Reynold's numbers (cf. [1], DVL Report 136, p. 51). To achieve such a suction effect, one must "clip" the velocity profile to y_Q/δ_I = 0.53, according to fig. 13. The appropriate suction number c_Q is, from eq. (50):

$$c_Q = \left(\frac{y_Q}{\delta_I}\right)^{1+k_I} \cdot \frac{1+2k_I}{k_I}\left(\frac{u_\delta}{u_\infty}\,\frac{\delta_2}{l}\right) = 0{,}53^{1{,}606} \cdot \frac{2{,}212}{0{,}606}\left(\frac{u_\delta}{u_\infty}\,\frac{\delta_2}{l}\right)_I \tag{54}$$

$$= 1{,}32\left(\frac{u_\delta}{u_\infty}\,\frac{\delta_2}{l}\right)_I$$

The suction quantity Q then follows from eq. (5).

Thus c_Q is proportional to the thickness of impulse loss δ_2/l (related to the wing depth l). For large Reynold's numbers, as can be assumed in the region of validity of boundary layer theory, δ_2/l is a very small number (almost one power of ten smaller than the ratio δ/l). Thus, c_Q is also very small. For Reynold's numbers $R_l = \rho u_\infty l/\mu$ of magnitude 10^6, c_Q in the chosen example has the magnitude of 10^{-2}, according to calculations and measurements.[2] /26

We shall now evaluate the sink effect for c_Q = 0.01, using the simple formula (4). For c_Q = 0.01, the following additional velocities $\Delta u_\delta/u_\infty$ obtain, dependent on the distance a/l from the sink midpoint on the profile surface:

---

[2] It should be mentioned here that, when using suction to maintain the laminarity of the boundary layer, even smaller values of c_Q come into consideration. In the experimental findings of Pfenninger [16]$_4$, cited at the close of section II, c_Q has the magnitude of 10^{-4}.

a/l	0	0,01	0,02	0,05	0,10
$\Delta\left(\frac{u_\delta}{u_\infty}\right)$	∞	0,318	0,159	0,064	0,032

It is seen that the additional velocities at c_Q = 0.01 are already so large that they must be taken into account in the calculation of the boundary layer.

V. An Example from Aerodynamics: Ascertainment of the Most Favorable Disposition of the Suction Slit for Increases of $c_{a\ max}$ on NACA-profile 23015.

If it is intended to suppress flow detachment by suction, then the question arises as to the disposition of the suction slit so that the suction quantity for a given velocity curve $u\,\delta(x)$ becomes a minimum.

To begin, we can make the following rough predictions about the slit disposition that can bring about a minimum suction quantity:

The suction slit should be disposed downstream at the latest where flow detachment would ensue without suction (with utilization of the sink effect, which somewhat reduces the pressure rise, also slightly behind this position). But the slit should not lie to much in front of the detachment site, since a boundary layer, made thinner by suction, grows more quickly than a thick one, due to the increased friction. /27

The most expedient disposition of the suction slit can now be clarified by the developed theory in combination with a calculation for pressure distribution and boundary layer.

As an example for such a calculation, in 1942 in the Aerodynamical Research Institute, Göttingen, an airfoil profile of the type NACA 23015 was chosen (ZWB Report 1775) [21]. To check the theory, in a water channel of the AVA Göttingen measurements of the pressure distribution were carried out for this profile, along with flow observations by the spread of a dye. From the measurement of the pressure distribution $p(x)/\frac{\rho u_\infty^2}{2}$, the velocity distribution $u_\delta(x)/u_\infty$, necessary as a basis for the boundary layer calculation, was obtained from Bernouilli's Principle. The flow observations enabled recognition of both the turning point and the detachment point of the turbulent boundary layer, with good accuracy. In this way, the prerequisites needed to compare measurements with boundary layer calculations (with suction) were produced.

At that time the integral condition of the impulse, eq. (16), for laminar and turbulent boundary layers, was already available for the boundary layer calculation.[1] The "wall binding" served as the second equation for a laminar boundary layer (see, e.g., [21], eq. (11a)). For a turbulent boundary layer, as already mentioned at the beginning, the Gruschwitz differential equation for the shape parameter was used [21] (cf. eq. (38)):

$$\eta = 1 - \left(\frac{u}{u_\delta}\right)^2_{y=\delta_2} = 1 - \left(\frac{\delta_2}{\delta}\right)^{2k} = 1 - \left[\frac{k}{(1+k)(1+2k)}\right]^{2k} \qquad (55)$$

[1] For a turbulent boundary layer, the empirical law for the shearing stress of the wall was not yet so accurately known as is today possible, after the works, e.g., of Ludwieg-Tillmann [29]. In calculating the detachment point of turbulent boundary layers, this uncertainty was not really significant, since in this case the frictional member is subordinate to the pressure member in the pressure equation.

Eq. (42) provides the connection with the magnitude H, used by us. K. WIEGHARDT [27] has proved that this equation, with constants that have been empirically determined from a limited number of measurements, can be interpreted as an integral condition for the energy. The equation has shown itself to be very /28 useful in the region of Reynold's numbers for which the constants had been determined. As a rule, its results are only slightly worse than the new energy equation (17), which has a better physical foundation (cf., e.g., J. ACKERET [28]).

Thus the comparison between calculation and measurement, carried out in the cited internal report, is essentially still valid. We therefore reproduce several pictures from this report, which reveal the essential findings (figs. 14 to 19). It can be established that the boundary layer calculation, with consideration of the suction influence, renders rather well the practically attainable $c_{a\ max}$ values as a function of the suction number c_Q and the angle of attack α. The theory also predicts well the most favorable disposition of the suction slit (in the present profile example, $x/l = 0.2$), as seen in figs. 21 and 22.

A check with the somewhat improved theory, presented in this report, is in preparation. The improvements, as already mentioned, concern the integral condition for the energy, eq. (17), and furthermore the determination of the shape parameter H_{II} behind the slit. In the earlier calculations, only the relation $(\delta_1)_{II}/(\delta_2)_{II}$ was used to determine the shape parameter of that time, $\eta(H)$. According to the improvement of the theory presented here, the shape parameter $(H)_{II}$ of the velocity profile can be determined more certainly as a mean from the relations $(\delta_1)_{II}/(\delta_2)_{II}$ and $(\delta_3)_{II}/(\delta_2)_{II}$, using eq. (34). As seen in fig. 13, this mean $\bar{H}$ does not differ essentially from its two components. This fact also indicates that nothing decisive will be altered when the former result of the comparison between measurement and calculation is subjected to verification.

The result of such a verification, as well as the application of the here-depicted theory in the case of a laminar boundary layer (the problem of maintaining the laminarity of the boundary layer by numerous, successive, single slits; see, e.g., the measurements of Pfenninger [15,16]) shall be reported in due time.[2]

VI. Summary. /29

The majority of the presently known theories for the suction of the flow's boundary layer concern the physically optimal case of the continuous distribution of the suction quantity over the surface, which is at the same time mathematically more easy and more exact. In practice, at any rate, to approximate the physically optimal case of the suction effect, reasons of design dictate a rather large number of successive single slits, but also and frequently only a single slit, if, e.g., flow detachment is to be avoided. An exact treatment of this case is difficult. On the contrary, an approximate solution, especially in connection with the calculation procedures for laminar and turbulent boundary layers, described in DVL Reports 84 and 136, can be relatively easily produced. The author had already developed an imperfect form of this suction theory in 1943. The most important results of this report, at the time enjoying only

[2]During the printing of this report, W. Wuest, AVA Göttingen, has programmed the approximation theory, developed here, for a turbulent boundary layer. Systematic calculations with many successive single slits have produced a satisfactory accord with experiment.

limited publication, have been again presented in the current report. Comparisons with measurements have demonstrated the fundamental validity of this simple theory, above all for turbulent boundary layers. Today this theory can also be checked in the case of suction to maintain the laminarity of the boundary layer, in combination with the stability theory and the measurements of W. PFENNINGER. Works in this direction are being prepared.

For suction by single slits, the external velocity field of potential theory, i.e. the starting basis for the boundary layer calculation, is influenced by the suction sink. Simple evaluation formulas are adduced for this influence.

Alfred Walz, University Lecturer and
Doctor of Engineering, Freiburg, FRG.

Figures

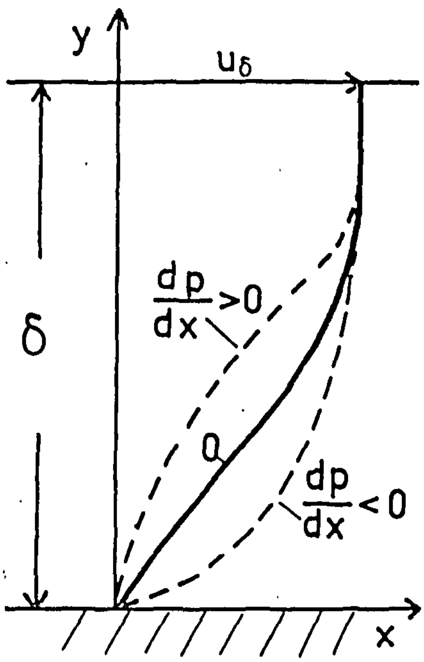


Figure 1

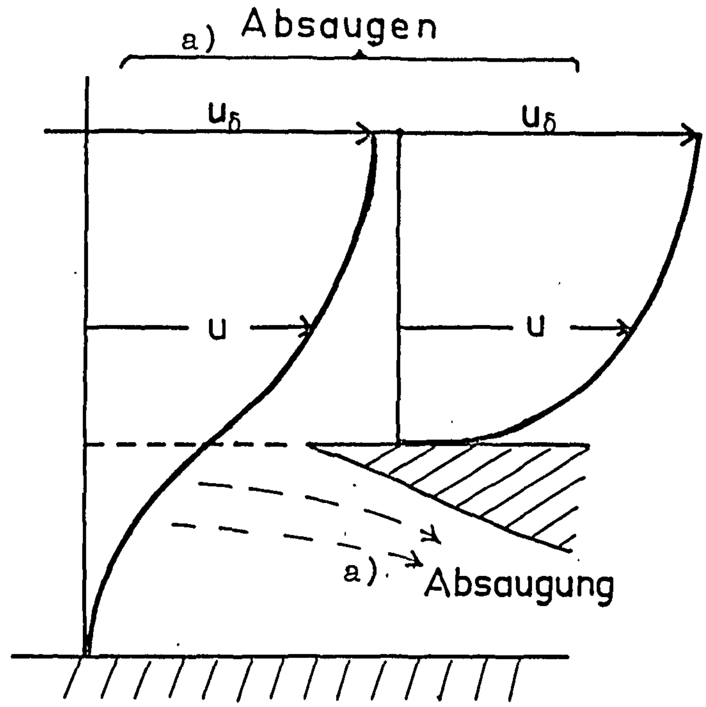
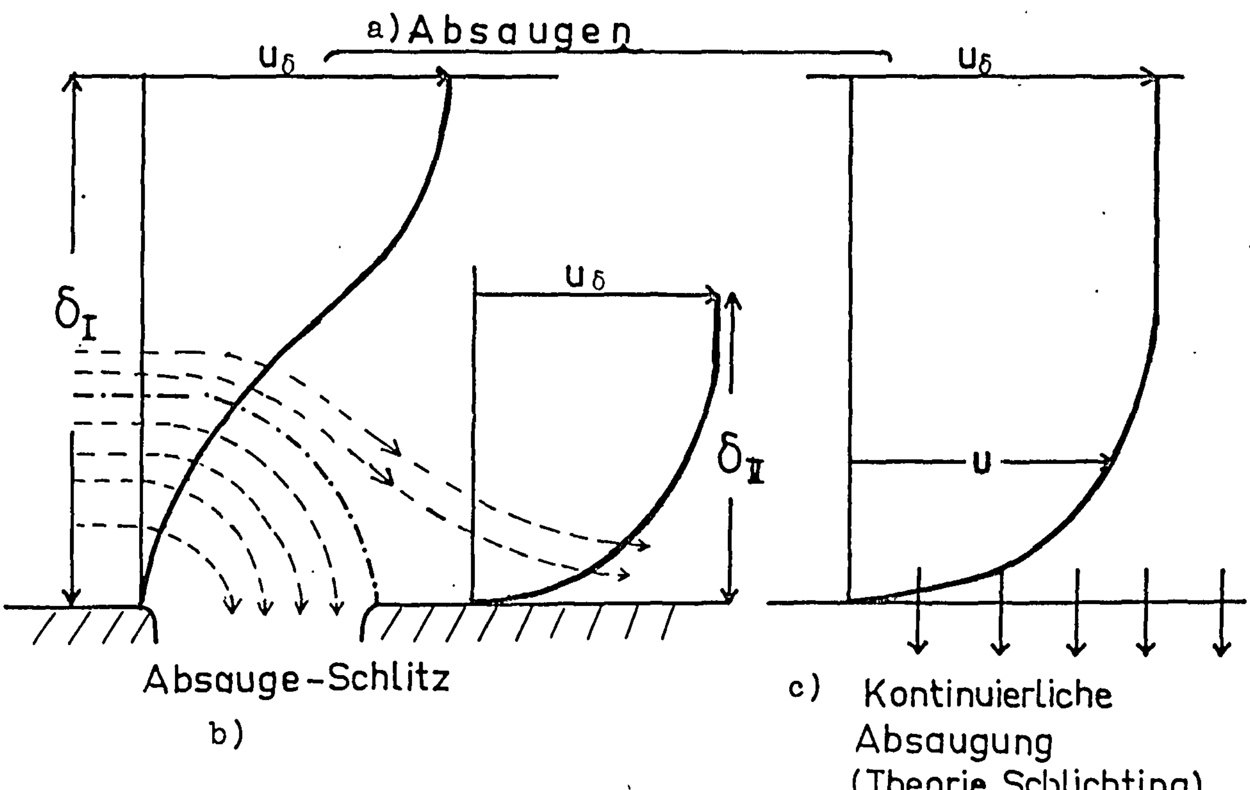


Figure 2. Key: a) suction.



Figures 3 and 4. Key: a) suction; b) suction slit; c) continuous suction (Schlichting Theory)

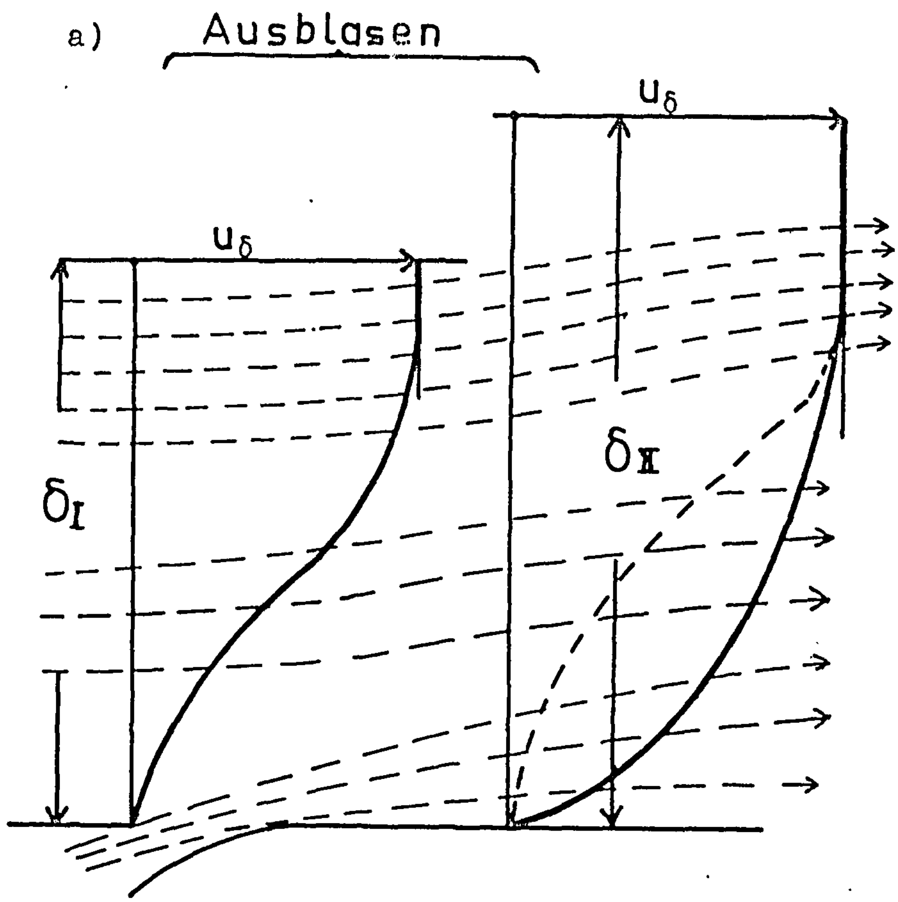


Figure 5. Key: a) blowing.

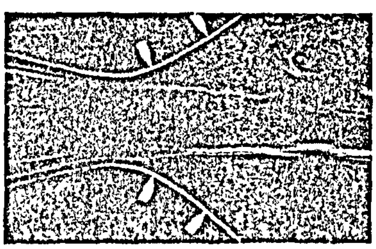


Figure 6. Ordinary flow in an abruptly broadened channel.

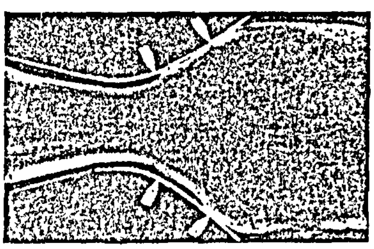
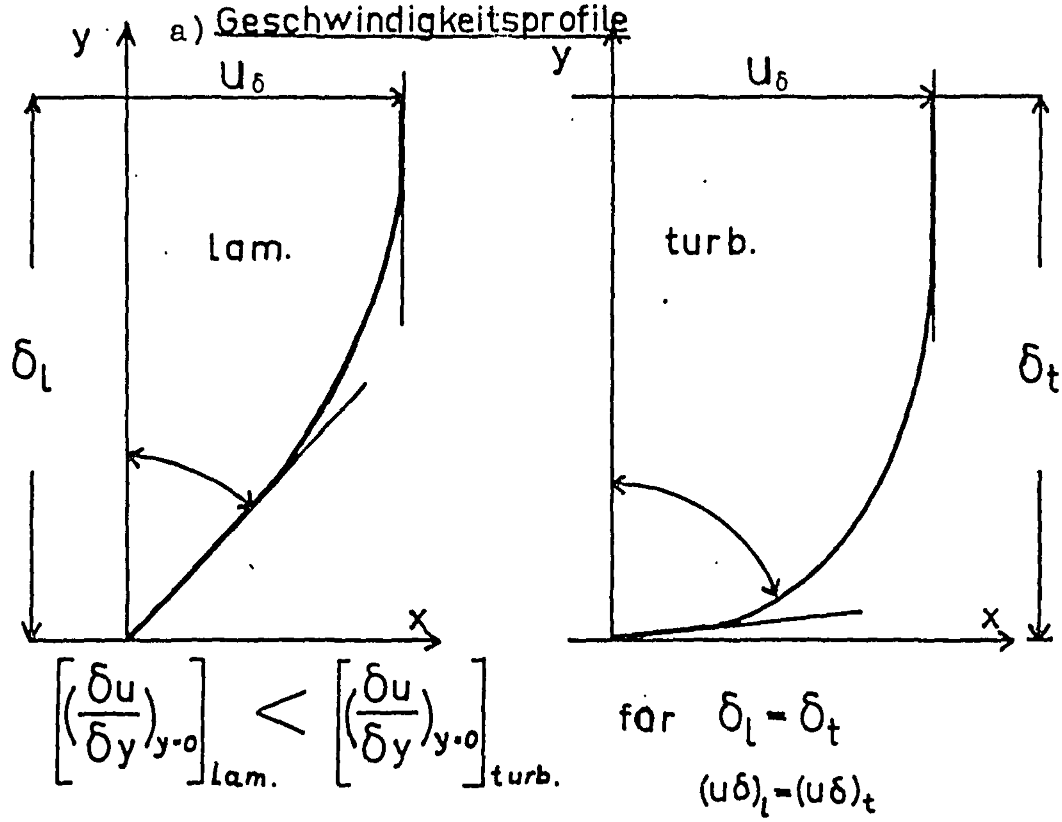


Figure 7. Flow in an abruptly broadened channel with suction at the walls. The white markers indicate the sites of the (invisible) suction slits.



Figures 8a and 8b. Key: a) velocity profiles

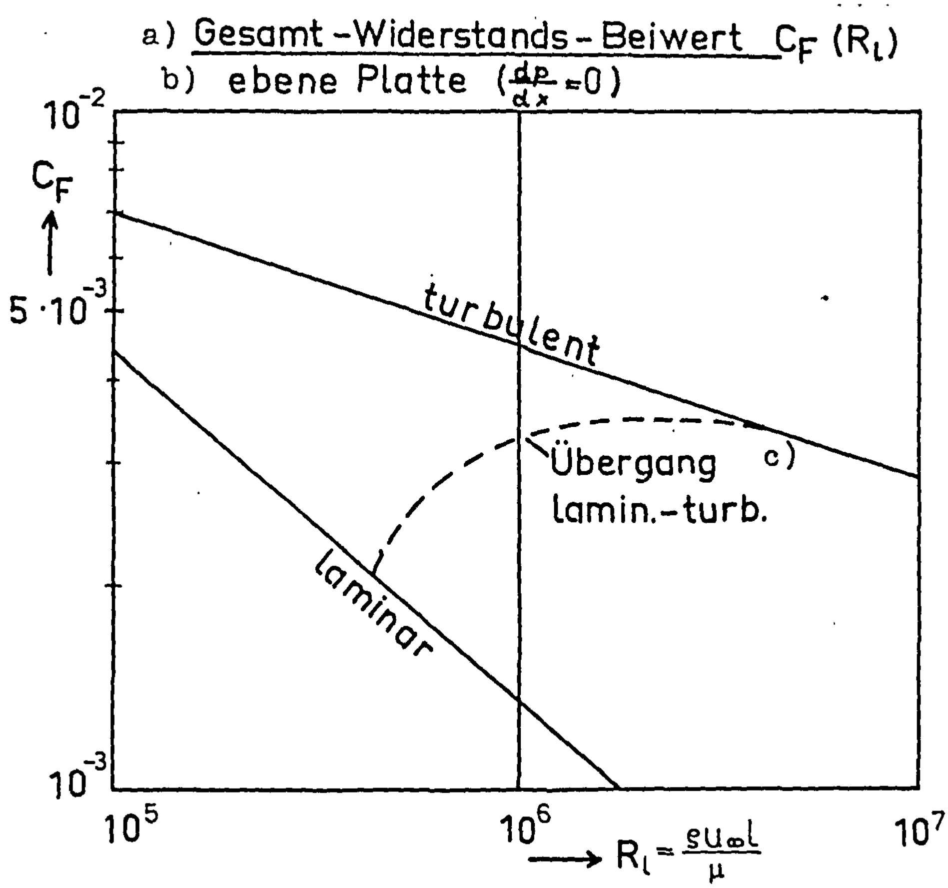
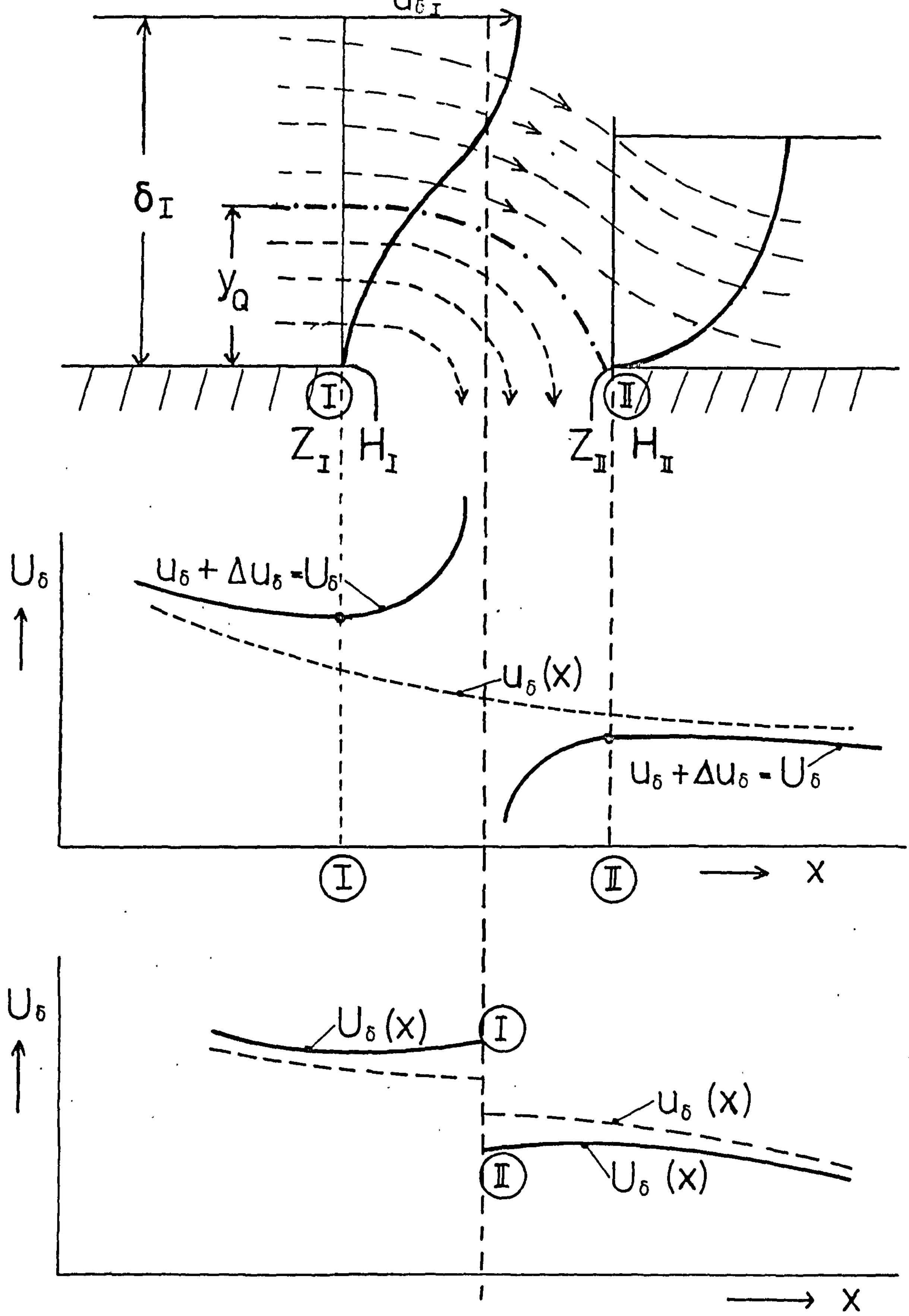


Figure 9. Key: a) total resistance coefficient; b) even plate; c) transition lam-turb.



Figures 10, 11, and 12.

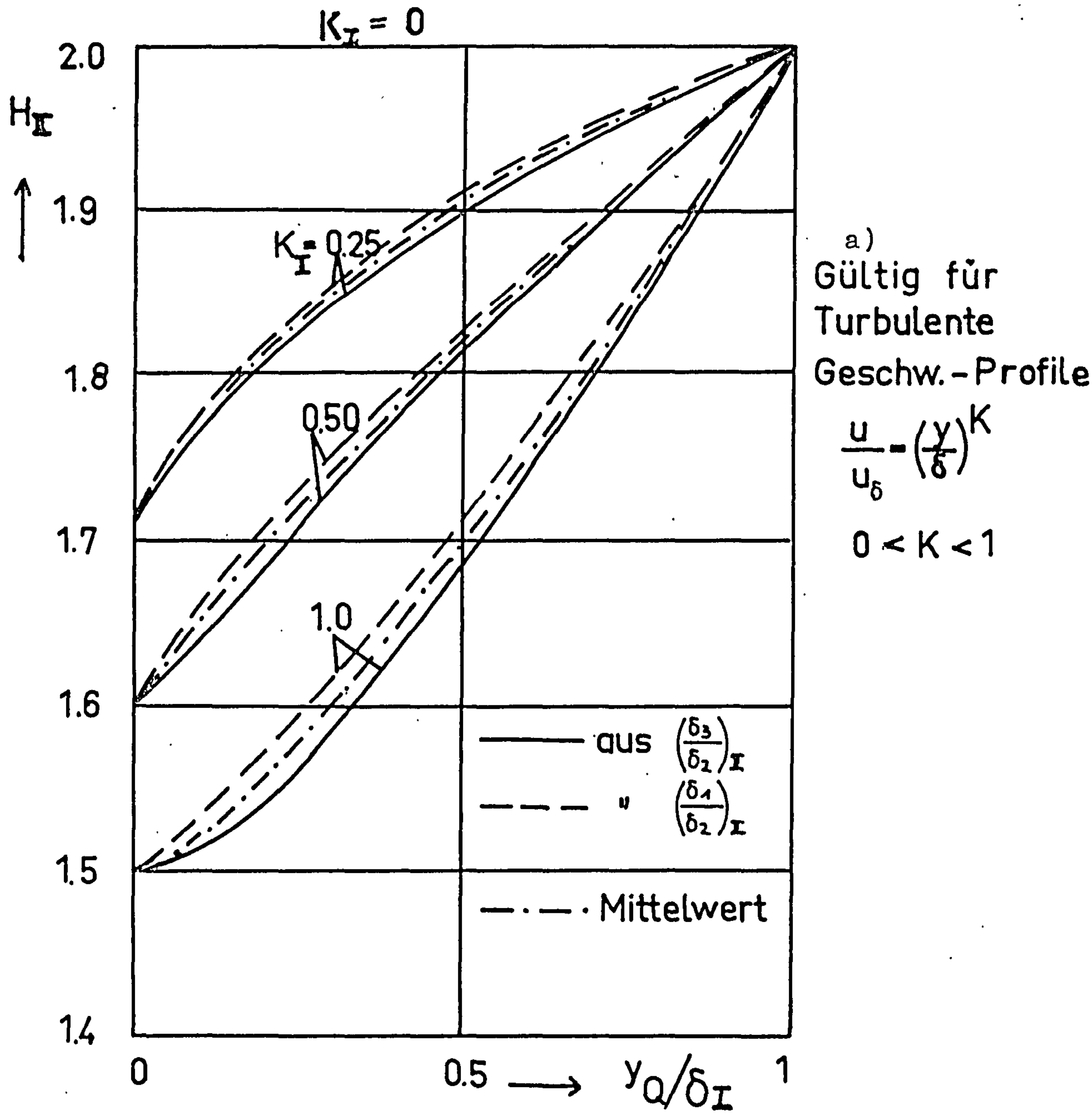
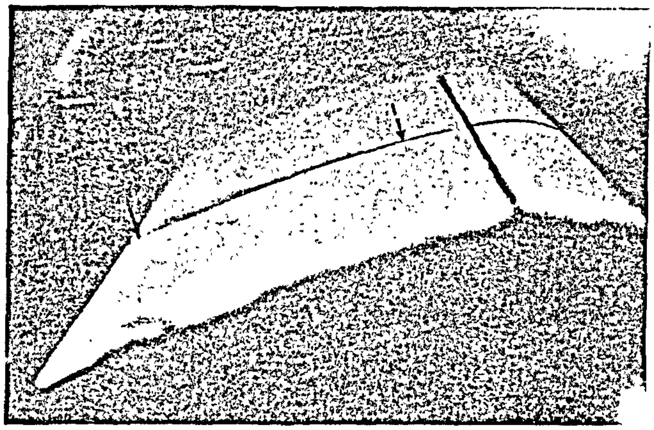
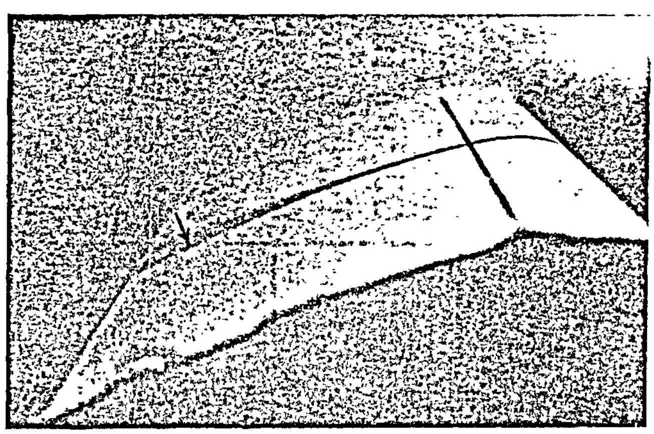


Figure 13. Key: a) valid for turbulent velocity profiles.



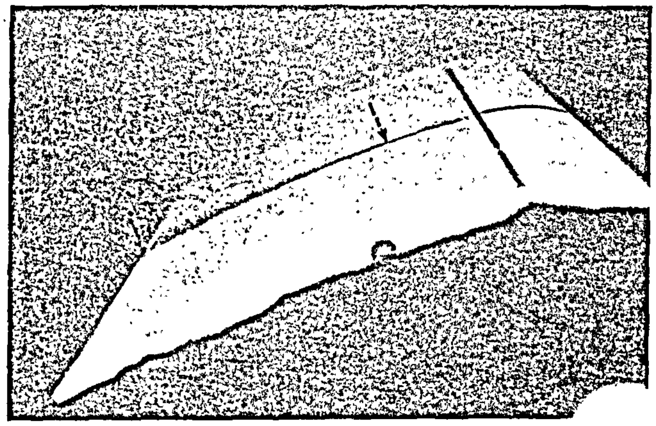
$\alpha_{\infty} = 16^{\circ}$; $c_{\mu} = 0{,}0036$.

Figure 14.



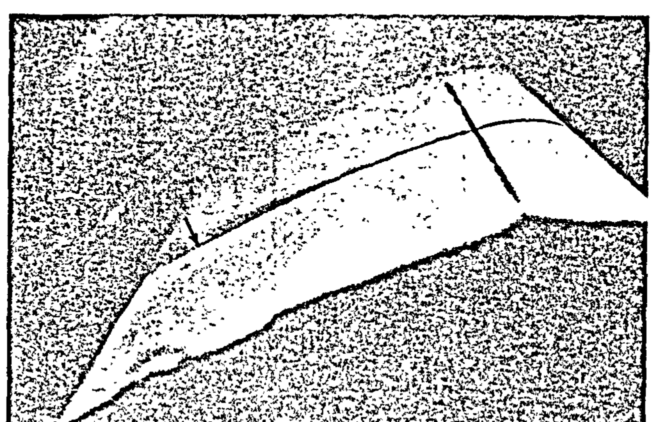
$\alpha_{\infty} = 19^{\circ}$; $c_{\mu} = 0{,}0045$.

Figure 15.



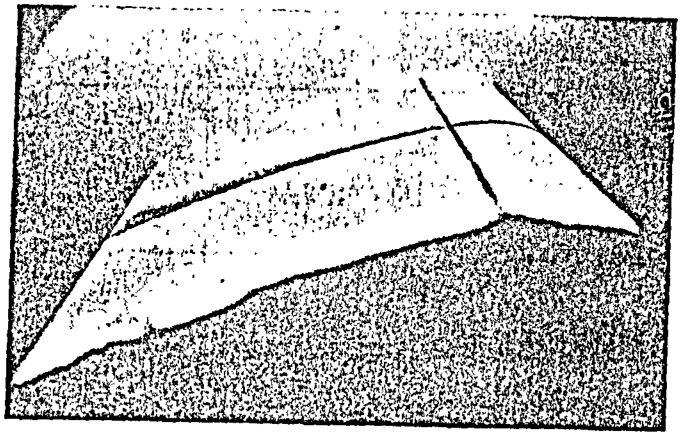
$a_{oo} = 19^{o}$; $c_Q = 0,0107$.

Figure 16.



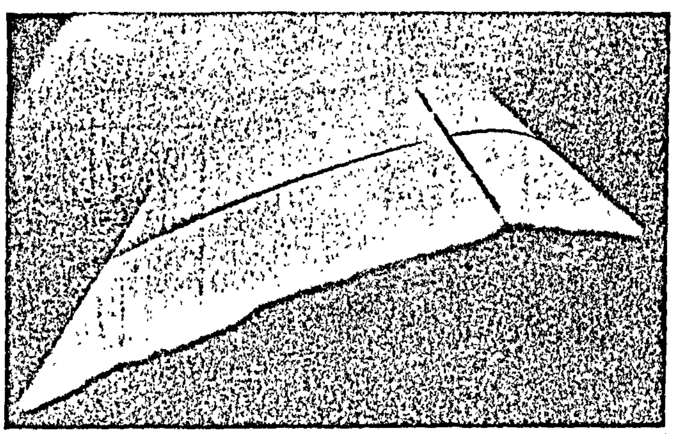
$a_{oo} = 21^{o}$; $c_Q = 0,0050$.

Figure 17.



$\alpha_\infty = 16°$; $c_Q = 0{,}0046$

Figure 18. Profile with wing flap.



$\alpha_\infty = 16°$; $c_Q = 0{,}0110$

Figure 19. Profile with wing flap.

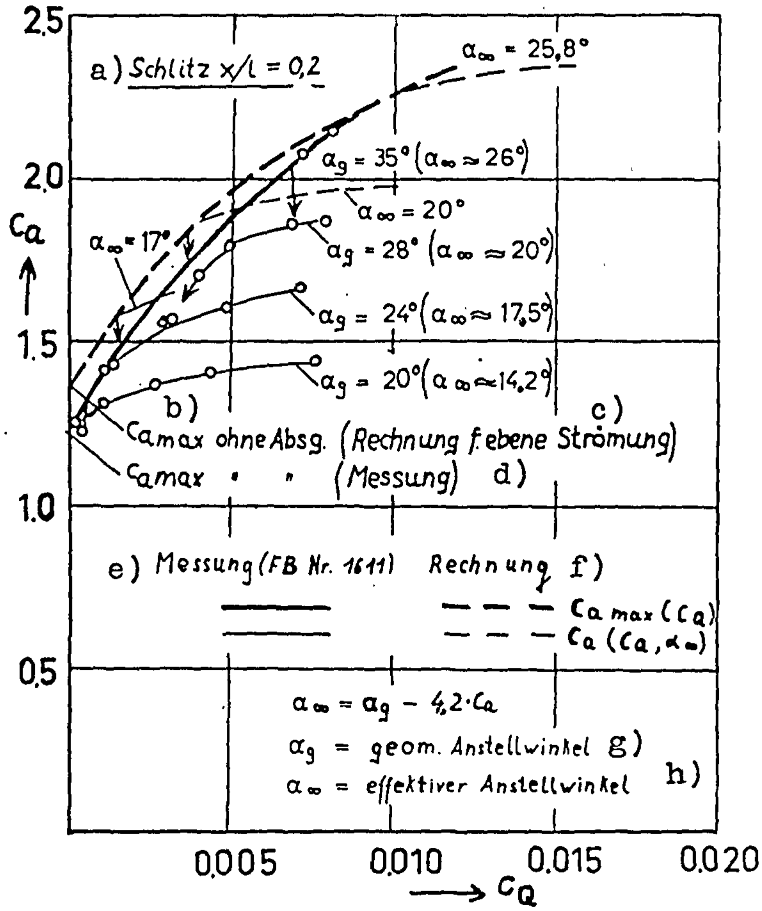
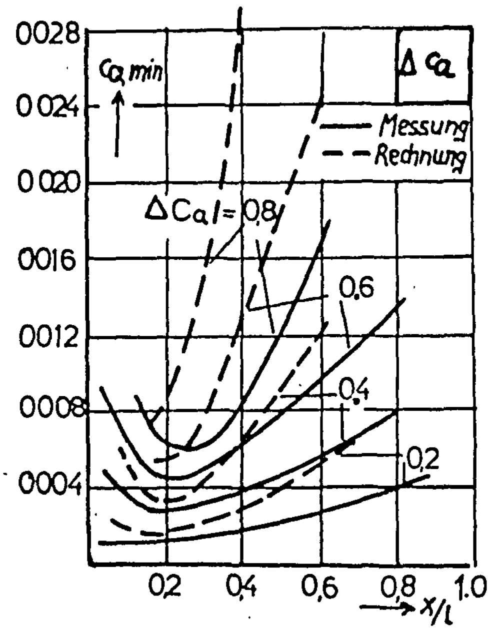
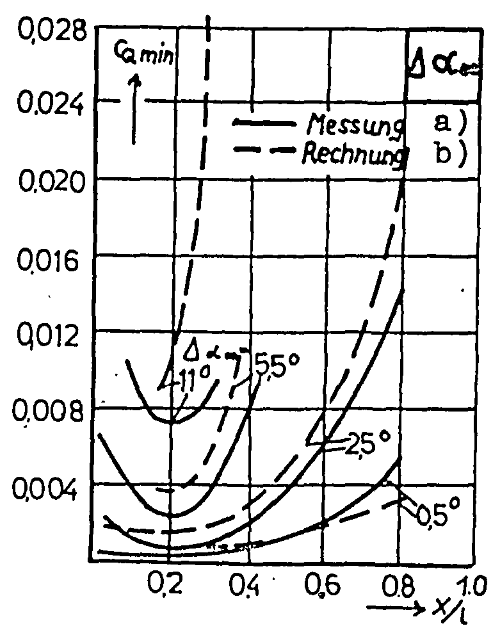


Figure 20. Key: a) slit; b) $C_{a\ max}$ without suction; c) (calculation for even flow); d) (measurement); e) measurement (FB Nr. 1611); f) calculation; g) geometrical angle of attack; h) effective angle of attack.



a)
b)

Figure 21. Key: a) measurement; b) calculation.



a)
b)

Figure 22. Key: a) measurement; b) calculation.

REFERENCES

1. Prandtl, L., "On fluid movement with very small friction", Address to the Heidelberg Mathematicians' Congress, 1904.

2. Doetsch, H., "Investigations on several profiles with small resistance in the region of small c_a values", Jahrb. d. deutsch. Luftfahrtforschung, I, p. 54, (1940[a]).

3. Abbott, J.H., A.E. von Doenhoff, and L.S. Stivers, "Summary of airfoil data", NACA Report 824, (1954).

4. Walz, A., "Theoretical resistance calculations on a laminar profile with different shapes of tail portion", ZWB, UM Nr. 3160, (1944)*.

5. Walz, A., "Measurements on the laminar profile 2315 BIS with suction in the neighborhood of the profile nose, with and without nose wing flaps", ZWB, UM Nr. 3195, (1944)*.

6. Eppler, R., "Results of a mutual application of boundary layer theory and profile theory", ZFW 8, (1960), 247/260.

7. Wortmann, F. X., "Experimental investigations on new laminar profiles for gliders and helicopters", ZFW 5, (1957), No. 8, p. 228.

8. Görtler, H., "Several new results for hydrodynamic stability theory", ZFW 8, (1960), p. 2/8.

9. Blasius, H., "Boundary layers in fluids with small friction", Z. f. Math. u. Phys., 56/1, (1908).

10. Schlichting, H., Grenzschichttheorie, [Boundary Layer Theory], 3rd edition, G. Braun Verlag, Karlsruhe, 1958. (For boundary layer suction, see pp. 249-267).

11. Prandtl, L., Führer durch die Strömungslehre, [A Guide to Flow Theory], Fr. Vieweg u. Sohn, Braunschweig, 1942, p. 128.

12. Ehlers, F. and A. Walz, "On the improvement of profile characteristics by boundary layer influence", Publication in honor of the 60th birthday of A. Betz, 25 Dec. 1945, AVA Göttingen.*

*These reports are obtainable at the Zentralstelle für Luftfahrtdokumentation und -information, München, Flughafen.

13. Walz, A., "Experiments with suction of the frictional layer on a NACA 23015 airfoil profile with variable disposition of the suction slit along the wing depth", ZWB Report, FB Nr. 1611, (1942)*.

14. Holstein, H., "Measurements for the maintenance of boundary layer laminarity on a wing", Lilienthal Report, S 10, (1940), p. 17-27.

15. Pfenninger, W., "Investigations on the reduction of friction on airfoils, especially with the help of boundary layer suction", Mitteilung des Inst. f. Aerodynamik der ETH Zürich, Nr. 13, (1946).

16. Pfenninger, W., "Recent developments in the field of resistance reduction by boundary layer suction", Address to an AGARD conference in Stuttgart, (1960).

17. Head, H. R., "A brief review of laminar boundary layer research in Great Britain and the USA", Rep. of Cambridge Univ., (June, 1957).

18. Wuest, W., "The development of a laminar boundary layer behind a suction site", Ing. Archiv, 17, (1949), pp. 199-206.

19. Wuest, W., "Approximate calculation and stability behavior of laminar boundary layers with suction by single slits", Ing. Archiv, 21, (1953), pp. 90-103.

20. Rheinboldt, W., "Toward the calculation of stationary boundary layers in the case of continuous suction with an unstably fluctuating suction velocity", J. Rational Mechanics and Analysis, 5/3, (1956), pp. 539-604, (Indiana U.).

21. Walz, A., "Theoretical speculations for the suction of the frictional layer", ZWB Report Nr. 1775, (AVA Göttingen Report Nr. 43 (W/15), 1943*).

22. Gruschwitz, E., "The turbulent frictional layer in regular flow with pressure drop and pressure rise", Ing. Archiv, 2, (1931), p. 321.

23. Truckenbrodt, E., "A quadrature procedure for calculation of the laminar and the turbulent boundary layer during regular and rotationally symmetric flow", Ing. Archiv, 20, (1952), pp. 211-228.

24. Walz, A., "A contribution to the approximation theory of compressible, turbulent boundary layers", DVL Reports 84 (1959) and 136 (1960).

25. Ehlers, F., "A potential theory calculation of the effect of a suction sink on the flow field about an airfoil profile", (unpublished work, AVA Göttingen, 1944).