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A SEMI-SPECTRAL NUMERICAL MODEL FOR THE LARGE SCALE STRATOSPHERIC CIRCULATION

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CONTENTS

1. INTRODUCTION

A numerical model for simulation of the global circulation of the stratosphere and mesosphere is currently under development at the University of Washington. The complete model is a semi-spectral model in which the longitudinal dependence is represented by expansion in zonal harmonics while the latitude and height dependencies are represented by a finite difference grid. Since many of the dynamical processes which occur in the stratosphere and mesosphere are the result of interactions between the zonal mean flow and planetary waves, it is useful to formulate a model in which the zonal mean and wave portions are explicitly separated, as is done here.

The model is based on the primitive equations in the log pressure coordinate system as given by Holton (1975). In order to avoid the problems inherent in simulating tropospheric meteorological processes, the lower boundary of the model domain is set at the 100 mb level (i.e., near the tropopause) and the effects of tropospheric forcing are included in the lower boundary condition. The upper boundary is at approximately 96 km, and the latitudinal extent is either global or hemispheric.

In this report we first outline the basic differential equations and boundary conditions. We next describe the finite difference equations. We then discuss the initial conditions and present a sample calculation. Finally, the Fortran code is given in the appendix.

2. BASIC EQUATIONS

In setting down the basic equations we will make use of the following symbols:

λ	longitude
θ	latitude
z	a measure of "height" [$\equiv -H \ln (p/p_s)$]
H	scale height [$\equiv RT_s/g$]
R	gas constant for dry air
T_s	a constant stratospheric mean temperature
g	gravitational acceleration
p	pressure
p_s	a constant reference pressure
u	eastward velocity
v	northward velocity
w	a measure of "vertical velocity" [$\equiv dz/dt$]
T_o	a basic state temperature [$\equiv T_o(z)$]
Φ_o	a basic state geopotential [$\equiv \Phi_o(z)$]
T	departure of local temperature from $T_o(z)$
Φ	departure of local geopotential from $\Phi_o(z)$
Ω	angular velocity of earth
a	radius of earth
J	diabatic heating rate per unit mass
c_p	specific heat at constant pressure
κ	ratio of gas constant to specific heat at constant pressure [$\equiv R/c_p$]
dx	eastward distance increment [$\equiv a \cos \theta \, d\lambda$]
dy	northward distance increment [$\equiv a d\theta$].

The horizontal momentum equations can then be written in flux form as

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{1}{\cos^2 \theta} \frac{\partial}{\partial y} (uv \cos^2 \theta)$$

$$+ \frac{1}{\rho_o} \frac{\partial}{\partial z} (\rho_o wu) - 2\Omega v \sin \theta = - \frac{\partial \Phi}{\partial x} + D_1(u) \tag{2.1}$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x} (uv) + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (v^2 \cos \theta)$$

$$+ \frac{1}{\rho_o} \frac{\partial}{\partial z} (\rho_o vw) + \frac{u^2 \tan \theta}{a} + 2\Omega u \sin \theta = - \frac{\partial \Phi}{\partial y} + D_2(v) \tag{2.2}$$

Here, $\rho_o \equiv \rho_s \exp(-z/H)$, where ρ_s is the mean density at $z = 0$. $D_1(u)$ and $D_2(v)$ represent subgrid scale momentum diffusion. Explicit forms for these terms will be given in Section 4.

Using the above notation the hydrostatic approximation and continuity equation become

$$\frac{d\Phi_o}{dz} = \frac{RT_o}{H} , \quad \frac{\partial \Phi}{\partial z} = \frac{RT}{H} , \tag{2.3}$$

and

$$\frac{\partial u}{\partial x} + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (v \cos \theta) + \frac{1}{\rho_o} \frac{\partial}{\partial z} (\rho_o w) = 0 \tag{2.4}$$

The variables T_o and Φ_o define a hydrostatically balanced basic state which is specified to be the U.S. standard atmosphere. Using (2.3) we can write the thermodynamic energy equation for the departure from the basic state as follows[1]

$$\frac{\partial \Phi_z}{\partial t} + \frac{\partial}{\partial x} (u\Phi_z) + \frac{1}{\cos\theta}\frac{\partial}{\partial y} (v\Phi_z \cos\theta)$$

$$+ \frac{1}{\rho_o}\frac{\partial}{\partial z} (\rho_o \Phi_z w) + wN^2 = \kappa J/H + D_2(\Phi_z) \qquad (2.5)$$

where

$$N^2 \equiv \frac{R}{H}\left(\frac{dT_o}{dz} + \frac{\kappa T_o}{H}\right)$$

is the buoyancy frequency squared, and we have let $D_2(\Phi_z)$ denote the subgrid scale diffusion.

The basic state temperature profile is assumed to be in radiative equilibrium (see Section 9), so that the horizontal average of the diabatic heating will vanish provided that the horizontally averaged temperature equals the basic state temperature $T_o(z)$. Because of the nonlinearity of (2.5) the horizontally averaged temperature need not remain equal to $T_o(z)$ as the flow evolves in time. However, in practice we find that departures of the horizontally averaged total temperature from $T_o(z)$ are at most a few degrees so that for practical purposes the horizontally averaged diabatic heating remains very small, and J can be regarded as the differential heating.

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[1]Following Holton (1975) we here neglect the small term $w\kappa T/H$ compared to $w\kappa T_o/H$. This approximation is necessary if we wish to define available potential energy in terms of the temperature variance.

3. ZONAL HARMONIC EXPANSION

The basic equations of the model were given as (2.1), (2.2), (2.4), and (2.5). In order to develop the semi-spectral model we expand the basic equations in zonal harmonic series by letting

$$f(\lambda,y,z,t) = e^{z/2H} \sum_{n=-\infty}^{n=+\infty} F_n(y,z,t)e^{in\lambda} \tag{3.1}$$

where $f(\lambda,y,z,t)$ stands for any field variable and F_n is the Fourier transform of f defined by

$$F_n = \frac{e^{-z/2H}}{2\pi} \int_{-\pi}^{+\pi} f(\lambda,y,z,t)e^{-in\lambda}\, d\lambda \tag{3.2}$$

so that $F_{-n} \equiv F_n^*$, where the asterisk denotes the complex conjugate.

To transform the nonlinear terms in the basic equations we need the convolution theorem:

$$\frac{1}{2\pi}\int_{-\pi}^{+\pi} [f(\lambda)g(\lambda)]e^{-in\lambda}\, d\lambda = e^{z/H} \sum_{m=-\infty}^{+\infty} G_m F_{n-m}$$

from which we find

$$\frac{1}{2\pi}\int_{-\pi}^{+\pi} \left[\frac{\partial f}{\partial \lambda} g\right] e^{-in\lambda}\, d\lambda = e^{z/H} \sum_{m=-\infty}^{+\infty} im\, G_{n-m} F_m$$

To Fourier transform (2.1), (2.2), (2.4), and (2.5) we define the following transform pairs:

$f(\lambda)$:	u	v	w	Φ	$\kappa J/H$
F_n :	U_n	V_n	W_n	Ψ_n	Q_n

The transformed equations are as follows:

$$\frac{\partial U_n}{\partial t} - fV_n = \frac{-\,in}{a\cos\theta}\Psi_n + D_\ell(U_n)$$

$$- e^{z/2H}\sum_{m=-\infty}^{+\infty}\left[\frac{2im}{a\cos\theta}U_m U_{n-m}\right.$$

$$\left. + \frac{1}{\cos^2\theta}\frac{\partial}{\partial y}(U_m V_{n-m}\cos^2\theta) + \frac{\partial}{\partial z}(U_m W_{n-m})\right] \tag{3.3}$$

where $\ell = 1$ for $n = 0$, $\ell = 2$ for $n \neq 0$.

$$\frac{\partial V_n}{\partial t} + fU_n = -\frac{\partial \Psi_n}{\partial y} + D_2(V_n)$$

$$- e^{-z/2H}\sum_{m=-\infty}^{+\infty}\left[\frac{im}{a\cos\theta}(U_m V_{n-m} + V_m U_{n-m})\right.$$

$$+ \frac{1}{\cos\theta}\frac{\partial}{\partial y}(V_m V_{n-m}\cos\theta)$$

$$\left. + \frac{\partial}{\partial z}(V_m W_{n-m}) + \frac{\tan\theta}{a}(U_m U_{n-m})\right] \tag{3.4}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{\partial}{\partial z}+\frac{1}{2H}\right)\Psi_n\right]+N^2W_n=$$

$$Q_n+D_2\left[\left(\frac{\partial}{\partial z}+\frac{1}{2H}\right)\Psi_n\right]-e^{z/2H}\sum_{m=-\infty}^{+\infty}\left\{\frac{im}{a\cos\theta}\left[U_m\left(\frac{\partial}{\partial z}+\frac{1}{2H}\right)\Psi_{n-m}\right.\right.$$

$$\left.+\left(\frac{\partial}{\partial z}+\frac{1}{2H}\right)\Psi_m U_{n-m}\right]+\frac{1}{\cos\theta}\frac{\partial}{\partial y}\left[\cos\theta\, V_{n-m}\left(\frac{\partial}{\partial z}+\frac{1}{2H}\right)\Psi_m\right]$$

$$\left.+\frac{\partial}{\partial z}\left[W_{n-m}\left(\frac{\partial}{\partial z}+\frac{1}{2H}\right)\Psi_m\right]\right\} \tag{3.5}$$

$$\frac{inU_n}{a\cos\theta}+\frac{1}{\cos\theta}\frac{\partial}{\partial y}(\cos\theta\, V_n)+\left(\frac{\partial}{\partial z}-\frac{1}{2H}\right)W_n=0 \tag{3.6}$$

We now severely truncate the wave spectrum by assuming that the flow consists of a single wave of wavenumber $n = s$, and the zonal mean $n = 0$. To exclude all other wave modes we must replace the summations in (3.3) - (3.5) by a summation over the two values $m = 0$ and $m = s$.

3.1 The zonal mean equations

If we set $n = 0$ in (3.3) - (3.6) and replace $(\ )_o$ by $\overline{(\ )}$ for all field variables we obtain the zonal mean equations:

$$\frac{\partial\overline{U}}{\partial t}-f\overline{V}=-e^{z/2H}\left[\frac{1}{\cos^2\theta}\frac{\partial}{\partial y}(\overline{U}\,\overline{V}\cos^2\theta)+\frac{\partial}{\partial z}(\overline{U}\,\overline{W})\right]$$

$$+F_M+D_1(\overline{U}) \tag{3.7}$$

$$\frac{\partial \overline{V}}{\partial t} + f\overline{U} = - \frac{\partial \overline{\Psi}}{\partial y} - e^{z/2H}\, \overline{U}^2\, \frac{\tan\theta}{a} + D_2(\overline{V}) \tag{3.8}$$

$$\frac{1}{\cos\theta}\frac{\partial}{\partial y}(\overline{V}\cos\theta) + \left(\frac{\partial}{\partial z} - \frac{1}{2H}\right)\overline{W} = 0 \tag{3.9}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{H}\right) + N^2\overline{W} = + F_T - e^{z/2H}\left\{\frac{1}{\cos\theta}\frac{\partial}{\partial y}\left[\overline{V}\cos\theta\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\overline{\Psi}\right]\right.$$

$$\left. + \frac{\partial}{\partial z}\left[\overline{W}\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\overline{\Psi}\right]\right\} + D_2\left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H}\right) + \overline{Q} \tag{3.10}$$

where $f \equiv 2\Omega \sin\theta$ is the Coriolis parameter. Here F_M denotes the convergence of the momentum flux due to zonally asymmetric motions (e.g., planetary waves) while F_T denotes the convergence of the eddy heat flux.

We have neglected the advection by the mean meridional circulation and the eddy momentum flux terms in (3.8) since the mean zonal wind is nearly in gradient wind balance. The terms $\partial\overline{V}/\partial t$ and $D_2(\overline{V})$ are also very small but must be retained for our method of numerical solution.

With the aid of (3.9) we can define a mean meridional streamfunction, $\overline{X}$, by letting

$$\overline{W}\cos\theta = \frac{\partial\overline{X}}{\partial y} \quad , \quad \overline{V}\cos\theta = -\left(\frac{\partial}{\partial z} - \frac{1}{2H}\right)\overline{X} \tag{3.11}$$

The $\overline{X}$ field proves useful in specifying boundary conditions and solving the zonal mean component equations.

The eddy flux convergence terms in (3.7) and (3.10) have the following forms:

$$F_M = - e^{z/2H}\left\{\frac{1}{\cos^2\theta}\frac{\partial}{\partial y}\left[(U_s V_s^* + U_s^* V_s)\cos^2\theta\right]\right.$$

$$\left. + \frac{\partial}{\partial z}(U_s W_s^* + U_s^* W_s)\right\} \tag{3.12a}$$

and

$$F_T = - e^{z/2H} \left\{ \frac{1}{\cos\theta} \frac{\partial}{\partial y} \left[\cos\theta \left(V_s \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi_s^* + V_s^* \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi_s \right) \right] \right.$$

$$\left. + \frac{\partial}{\partial z} \left[W_s^* \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi_s + W_s \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi_s^* \right] \right\} \tag{3.12b}$$

3.2 The eddy equations

When we set n = s in (3.3) - (3.6) and again designate zonal means by an overbar rather than the m = 0 subscript, we obtain the eddy equations,

$$\frac{\partial U_s}{\partial t} - fV_s = \frac{-is}{a\cos\theta} \Psi_s - e^{z/2H} \left\{ \frac{isU_s\overline{U}}{a\cos\theta} + \frac{V_s}{\cos\theta} \frac{\partial}{\partial y} (\overline{U}\cos\theta) \right.$$

$$\left. + W_s \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \overline{U} \right\} + D_2(U_s) \tag{3.13}$$

$$\frac{\partial V_s}{\partial t} + fU_s = - \frac{\partial \Psi_s}{\partial y} - e^{z/2H} \left\{ \frac{2\overline{U}\, U_s \tan\theta}{a} + \frac{isV_s\overline{U}}{a\cos\theta} \right\} + D_2(V_s) \tag{3.14}$$

$$\frac{\partial}{\partial t} \left[\left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi_s \right] + N^2 W_s = Q_s - e^{z/2H} \left\{ \frac{is\overline{U}}{a\cos\theta} \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi_s \right.$$

$$\left. + W_s \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right)^2 \overline{\Psi} + V_s \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \overline{\Psi} \right\}$$

$$+ D_2 \left[\left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi_s \right] \tag{3.15}$$

$$\frac{isU_s}{a\cos\theta} + \frac{1}{\cos\theta} \frac{\partial}{\partial y} (\cos\theta\, V_s) + \left(\frac{\partial}{\partial z} - \frac{1}{2H} \right) W_s = 0 \tag{3.16}$$

Here all terms involving advection by the mean meridional circulation, $\overline{V}$, $\overline{W}$ have been neglected.

4. BOUNDARY CONDITIONS

Conditions for the zonal mean:

The model can be integrated on either a hemispheric or global domain. For global integrations the horizontal boundary conditions are as follows:

$$\overline{X} = \overline{U} = \overline{V} = \partial\overline{\Psi}/\partial y = 0 \text{ at } \theta = \pm\ \pi/2 \tag{4.1}$$

For hemispheric integrations the same boundary conditions are used at $\theta = 0,\ \pi/2$ except that a value of $\overline{U}$ not equal to zero may be specified at $\theta = 0$.

Vertical boundary conditions are specified as follows:

$$\overline{U} \equiv \overline{U}_B(y,t) \text{ at } z = 0 \tag{4.2a}$$

where $z = 0$ designates the lower boundary (i.e., the tropopause level) and $\overline{U}_B$ is an externally specified mean zonal wind. The boundary mean zonal flow is assumed to be in gradient wind balance. Thus from (2.10) we see that at $z = 0$

$$\overline{V} = 0 \tag{4.2b}$$

and

$$-\frac{\partial\overline{\Psi}}{\partial y} = f\overline{U} + \overline{U}^2\,\frac{\tan\theta}{a}\,e^{z/2H} \tag{4.2c}$$

Using the conditions (4.2a) we can integrate (4.2c) to obtain $\overline{\Psi}(y,t)$ at $z = 0$, provided that we let the horizontal average of $\overline{\Psi}(y,0,t)$ vanish.

At the upper boundary ($z = z_T$) we assume that the vertical shear of the mean zonal wind, the mean meridional wind, and the mean geopotential all vanish. Thus,

$$\frac{\partial}{\partial z}\left(\overline{U}e^{z/2H}\right) = \left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\overline{U} = 0 \tag{4.3a}$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\overline{V} = 0 \tag{4.3b}$$

and

$$\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\overline{\Psi} = 0 \tag{4.3c}$$

Condition (4.3c) of course implies that the zonal mean temperature must equal the basic state $T_o(z_T)$ at $z = z_T$.

In addition to these conditions it is clear from (3.7) and (3.10) that boundary conditions are also required for the vertical momentum and heat fluxes associated with the mean meridional circulation. We wish to avoid specifying $\overline{W}$ or the fluxes themselves at $z = 0$. Instead we assume that the flux divergences vanish at the lower boundary:

$$\frac{\partial}{\partial z}(\overline{U}\,\overline{W}) = \frac{\partial}{\partial z}\left[\overline{W}\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\overline{\Psi}\right] = 0 \tag{4.4}$$

However, for simplicity we assume that the fluxes themselves vanish at the upper boundary. If in addition we let $\overline{Q} = F_T = 0$ at the upper boundary, then from (3.10) we have

$$\overline{W} = 0 \text{ at } z = z_T \tag{4.5}$$

Boundary conditions for the eddy equations:

The boundary conditions for the eddy motions are analogous to the conditions for the zonal mean. However, the case $s = 1$ must be treated separately because V_s does not vanish at the poles for $s = 1$. Thus, for integrations on the global domain we have at $\theta = \pm \pi/2$:

$$\Psi_s = 0$$

$$U_s = V_s = 0 \text{ for } s > 1 \tag{4.6}$$

$$\partial U_s/\partial\theta = \partial V_s/\partial\theta = 0 \text{ for } s = 1$$

For a hemispheric domain the conditions at $\theta = 0$ depend on the symmetry conditions assumed. If geopotential is symmetric we have

$$\partial\Psi_s/\partial\theta = \partial U_s/\partial\theta = V_s = 0 \text{ at } \theta = 0 \tag{4.7}$$

If geopotential is antisymmetric we have

$$\Psi_s = U_s = \partial V_s/\partial\theta = 0 \text{ at } \theta = 0 \tag{4.8}$$

Conditions at the horizontal boundaries are specified as follows: At the lower boundary a geopotential height perturbation is specified so that

$$\Psi_s(y,t) = gh_s(y,t) \text{ at } z = 0 \tag{4.9}$$

while at the upper boundary the wave perturbations are assumed to vanish

$$\Psi_s = 0 \text{ at } z = z_T \tag{4.10}$$

The latter condition requires that we impose strong damping in the layers near z_T to prevent spurious reflection of wave energy from the upper boundary. Finally, in order to compute F_M and F_T at the upper and lower boundaries we assume that the vertical momentum and heat flux divergences vanish at the boundaries.

5. ENERGETICS

It can be shown that the eddy equations (3.13) - (3.16) are energetically consistent with the mean flow equations (3.7) - (3.10). In fact the system is governed by a Lorenz type energy cycle which (neglecting the diffusion terms) may be written as follows:

$$\frac{d\overline{K}}{dt} = \langle K_s \cdot \overline{K} \rangle + \langle \overline{A} \cdot \overline{K} \rangle + B(\overline{K}) \tag{5.1}$$

$$\frac{d\overline{A}}{dt} = - \langle \overline{A} \cdot \overline{K} \rangle - \langle \overline{A} \cdot \overline{A}_s \rangle + \overline{G} + B(\overline{A}) \tag{5.2}$$

$$\frac{dK_s}{dt} = - \langle K_s \cdot \overline{K} \rangle + \langle A_s \cdot K_s \rangle + B(K_s) \tag{5.3}$$

$$\frac{dA_s}{dt} = \langle \overline{A} \cdot A_s \rangle - \langle A_s \cdot K_s \rangle + G_s \tag{5.4}$$

where

$$\overline{K} \equiv \int_0^\infty \int_0^{\pi/2} \left(\frac{\overline{U}^2 + \overline{V}^2}{2} \right) \cos\theta \, d\theta \, dz$$

$$\overline{A} \equiv \int_0^\infty \int_0^{\pi/2} \frac{1}{2N^2} \left(\frac{\partial \overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H} \right)^2 \cos\theta \, d\theta \, dz$$

$$\langle K_s \cdot \overline{K} \rangle \equiv - \int_0^\infty \int_0^{\pi/2} \overline{U} e^{z/2H} \left\{ \frac{1}{\cos\theta} \frac{\partial}{\partial y} \left[(U_s V_s^* + U_s^* V_s) \cos^2\theta \right] \right.$$

$$\left. + \cos\theta \frac{\partial}{\partial z} (U_s W_s^* + U_s^* W_s) \right\} d\theta \, dz$$

$$\left\langle \overline{A} \cdot \overline{K} \right\rangle \equiv \int_0^\infty \int_0^{\pi/2} \overline{W}\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right) \overline{\Psi} \cos\theta \, d\theta \, dz$$

$$B(K) \equiv \int_0^{\pi/2} \{[\overline{W}(\overline{\Psi} + \overline{U}^2/2) + \overline{U}(U_s W_s^* + U_s^* W_s)] \cos\theta \, d\theta\}_{z=0}$$

$$\left\langle \overline{A} \cdot A_s \right\rangle \equiv \int_0^\infty \int_0^{\pi/2} \frac{e^{z/2H}}{N^2} \left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H}\right) \left\{\frac{\partial}{\partial y} \left\{\cos\theta \left[V_s\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s^*\right.\right.\right.$$

$$\left.\left. + V_s^*\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s\right]\right\} + \cos\theta \frac{\partial}{\partial z} \left[W_s^*\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s\right.$$

$$\left.\left. + W_s\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s^*\right]\right\} d\theta \, dz$$

$$\overline{G} \equiv + \int_0^\infty \int_0^{\pi/2} \frac{\overline{Q}}{N^2} \left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H}\right) \cos\theta \, d\theta \, dz$$

$$B(\overline{A}) \equiv \int_0^{\pi/2} \left\{\left[\frac{1}{2N^2} \left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H}\right)^2 \overline{W} + \frac{1}{N^2} \left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H}\right)\left(W_s^* \left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s\right.\right.\right.$$

$$\left.\left.\left. + W_s\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s^*\right)\right] \cos\theta \, d\theta\right\}_{z=0}$$

$$K_s \equiv \int_0^\infty \int_0^{\pi/2} (|U_s|^2 + |V_s|^2) \cos\theta \, d\theta \, dz$$

$$A_s \equiv \int_0^\infty \int_0^{\pi/2} \frac{1}{N^2} \left[\frac{\partial \Psi_s}{\partial z} + \frac{\Psi_s}{2H}\right]^2 \cos\theta \, d\theta \, dz$$

$$\langle A_s \cdot K_s \rangle \equiv \int_0^\infty \int_0^{\pi/2} \left[W_s\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s^* + W_s^*\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\Psi_s\right] \cos\theta \, d\theta \, dz$$

$$B(K_s) \equiv \int_0^{\pi/2} [(\Psi_s W_s^* + \Psi_s^* W_s) \cos\theta \, d\theta]_{z=0}$$

$$G_s = \int_0^\infty \int_0^{\pi/2} \frac{1}{N^2} \left[Q_s\left(\frac{\partial \Psi_s^*}{\partial z} + \frac{\Psi_s^*}{2H}\right) + Q_s^*\left(\frac{\partial \Psi_s}{\partial z} + \frac{\Psi_s}{2H}\right)\right] \cos\theta \, d\theta \, dz$$

Thus, in the above equations the terms enclosed by angle brackets represent transfers of energy among the components $\overline{K}$, $\overline{A}$, K_s, and A_s while the terms $\overline{G}$ and G_s represent diabatic heat sources and the terms $B(\overline{K})$, $B(\overline{A})$, and $B(K_s)$ represent energy fluxes across the lower boundary. Summing (5.1) - (5.4) we see that the total energy $\overline{K} + \overline{A} + K_s + A_s$ is conserved in the absence of diabatic heating and boundary fluxes.

6. FINITE DIFFERENCE EQUATIONS

6.1 The grid mesh

All field variables are represented on a staggered grid in the meridional plane with grid points identified by the indices (j,k). Here, j increases southwards and k increases upwards. To minimize truncation errors the grid points are staggered as shown in Fig. 1. The grid staggering is arranged in the horizontal so that $\overline{U}$, $\overline{V}$, F_M, $\overline{X}$, Ψ' and W' are defined at the meridional points given by $y = (\pi a/2) - (j - 1)\Delta y$, $j = 1,2,\ldots J_m$ where

$$\Delta y \equiv \begin{cases} \pi a/(J_m - 1), \text{ global domain} \\ \\ \dfrac{\pi a}{2(J_m - 1)}, \text{ hemispheric domain} \end{cases}$$

The variables $\overline{\Psi}$, U', V', $\overline{W}$ and F_T are defined at the meridional points

$$y = \frac{\pi a}{2} - (j - 1/2)\Delta y, \quad j = 1,2,\ldots J_m - 1 \quad .$$

Thus, $\overline{U}$, $\overline{V}$, $\overline{X}$, Ψ' and W' are defined at the horizontal boundary points while $\overline{\Psi}$, U', V', and $\overline{W}$ are defined a distance $\Delta y/2$ inside the boundaries. This form of staggering is natural for use with the horizontal boundary conditions (4.1).

The vertical staggering is arranged so that $\overline{U}$, $\overline{V}$, $\overline{\Psi}$, U', V', Ψ' and F_M are defined at the levels

$$z = (k - 1)\Delta z, \quad k = 1,2,\ldots K_N$$

where Δz is the vertical grid increment; while the variables $\overline{W}$, $\overline{X}$, W' and F_T are defined at the levels

$$z = (k - 1/2)\Delta z, \quad k = 1,2,\ldots (K_N - 1) \quad .$$

6.2 The difference equations for the zonal mean

For time differencing we choose a semi-implicit method in which the inertia-gravity terms (i.e., the Coriolis, pressure gradient, and adiabatic heating terms) are treated implicitly while the nonlinear advection terms and forcing terms are represented by centered differences. However, in order to prevent a weak time splitting of the solutions, which occurs due to the "leapfrog" scheme for the advection terms, a forward time step is used once every 48 steps.

The time differencing scheme can be expressed efficiently if we define a time average as follows

$$\hat{F} \equiv \beta_1 F^{n+1} + \beta_2 F^n + \beta_3 F^{n-1} \tag{6.1}$$

Here F stands for any dependent variable, n is the time index given by

$$t = n\Delta t, \quad n = 0,1,2....$$

where Δt is the time step, and β_1, β_2, β_3 are defined as follows:

(a) leapfrog step, $\beta_1 = 1/4, \quad \beta_2 = 1/2, \quad \beta_3 = 1/4$

(b) forward step, $\beta_1 = 1/2, \quad \beta_2 = 1/2 \quad \beta_3 = 0$

For leapfrog steps the time difference can then be written as

$$\left(\frac{\partial F}{\partial t}\right)^n \simeq \frac{F^{n+1} - F^{n-1}}{2\Delta t} = \frac{\hat{F} - 1/2(F^n + F^{n-1})}{(\Delta t/2)} \tag{6.2a}$$

While for forward steps we have

$$\left(\frac{\partial F}{\partial t}\right)^n \simeq \frac{F^{n+1} - F^n}{\Delta t} = \frac{\hat{F} - F^n}{(\Delta t/2)} \tag{6.2b}$$

In writing out the space differences it is convenient to use the following differencing and averaging operators:

$$\delta_{j+1/2}(\ ) = [(\ )_j - (\ )_{j+1}]/\Delta y \tag{6.3a}$$

$$\langle\ \rangle_{j+1/2} = [(\ )_j + (\ )_{j+1}]/2 \tag{6.3b}$$

Furthermore, to write the required vertical differences we let

$$\left(\frac{\partial F}{\partial z} + \frac{F}{2H}\right) = e^{-z/2H}\frac{\partial}{\partial z}(Fe^{z/2H}) \simeq (F_{k+1}e^{+} - F_k e^{-})/\Delta z \tag{6.4a}$$

where $e^{+} \equiv e^{\Delta z/4H}$ and $e^{-} \equiv e^{-\Delta z/4H}$. Similarly, we have

$$\left(\frac{\partial F}{\partial z} - \frac{F}{2H}\right) \simeq (F_{k+1}e^{-} - F_k e^{+})\Delta z \tag{6.4b}$$

where in each case the difference is centered at the k + 1/2 level.

Using the operators defined in (6.1) - (6.4) we can write finite difference approximations to (3.7) - (3.10) as follows:

$$\hat{\hat{U}} - (f\Delta t/2)\hat{\hat{V}} = \overline{A} \tag{6.5}$$

$$\hat{\hat{V}} + (f\Delta t/2)\hat{\hat{U}} + (\Delta t/2)\delta_{j-1/2}(\hat{\hat{\Psi}}) = \overline{B} \tag{6.6}$$

$$(\hat{\hat{W}}_k e^{-} - \hat{\hat{W}}_{k-1}e^{+}) + \frac{\Delta z}{\cos\theta}\,\delta_{j+1/2}(\hat{\hat{V}}_k \cos\theta^*) = 0 \tag{6.7}$$

$$(\hat{\hat{\Psi}}_{k+1}e^{+} - \hat{\hat{\Psi}}_k e^{-}) + \frac{N^2\Delta t\Delta z}{2}\hat{\hat{W}}_k = R \tag{6.8}$$

Here the terms involving the unknown variables have been collected on the left hand sides, and the source terms involving known quantities at time levels n and n - 1 appear on the right hand sides. In writing out these equations the subscripts j,k have been omitted wherever no ambiguity would result. In the continuity equation $\cos\theta$ is required at both the $\overline{V}$ and $\overline{W}$ grid points. Thus, we define

$$\theta_j = \pi/2 - (j - 1/2)\Delta y/a \tag{6.9a}$$

$$\theta^*_j = \pi/2 - (j - 1)\Delta y/a \tag{6.9b}$$

The source terms $\overline{A}$ and $\overline{B}$ are defined as follows:

$$\begin{aligned}\overline{A} = {} & \mu_1\overline{U}^n + \mu_2\overline{U}^{n-1} - \frac{\Delta t}{2}\, e^{z/2H}\left\{\frac{1}{\cos^2\theta^*}\,\delta_j\left(\langle\overline{U}^n\cos\theta^*\rangle_j\langle\overline{V}^n\cos\theta^*\rangle_j\right)\right. \\ & + \frac{1}{2\Delta z\cos\theta^*}\left[(\overline{U}^n_{j,k}e^- + \overline{U}^n_{j,k+1}e^+)\langle\overline{W}^n\cos\theta\rangle_{j-1/2,k}\right. \\ & \left.\left. - (\overline{U}^n_{j,k}e^+ + \overline{U}^n_{j,k-1}e^-)\langle\overline{W}^n\cos\theta\rangle_{j-1/2,k-1}\right]\right\} \\ & + \frac{\Delta t}{2}\left[F_M + D_1(\overline{U}^{n-1})\right]\end{aligned} \tag{6.10a}$$

$$\begin{aligned}\overline{B} = {} & \mu_1\overline{V}^n + \mu_2\overline{V}^{n-1} + \frac{\Delta t}{2}\, e^{z/2H}\left(\frac{\overline{U}^n_j}{4\Delta y}\right)\left[\overline{U}^n_{j-1}\left(\frac{\cos\theta^*_{j-1}}{\cos\theta^*_j} - \frac{\cos\theta^*_j}{\cos\theta^*_{j-1}}\right)\right. \\ & \left. + \overline{U}^n_{j+1}\left(\frac{\cos\theta^*_j}{\cos\theta^*_{j+1}} - \frac{\cos\theta^*_{j+1}}{\cos\theta^*_j}\right)\right] + \frac{\Delta t}{2}\, D_2(\overline{V}^{n-1})\end{aligned} \tag{6.10b}$$

Here the coefficients μ_1 and μ_2 are defined as follows:

(a) leapfrog step, $\mu_1 = \mu_2 = 1/2$

(b) forward step, $\mu_1 = 1$, $\mu_2 = 0$

In formulating $\overline{B}$ we have used a special form for the so-called "metric" term, $\overline{U}^2 \tan\theta/a$, which is required to keep the difference equations energy conserving in adiabatic, frictionless flow. To derive this form we note that

$$-\frac{\overline{U}^2 \tan\theta}{a} = \frac{\overline{U}^2}{2\cos^2\theta}\frac{d}{dy}(\cos^2\theta)$$

This may be approximated as

$$\simeq \frac{\overline{U}_j}{4\Delta y}\left[\overline{U}_{j-1}\left(\frac{\cos^2\theta^*_{j-1} - \cos^2\theta^*_j}{\cos\theta^*_j \cos\theta^*_{j-1}}\right) + \overline{U}_{j+1}\left(\frac{\cos^2\theta^*_j - \cos^2\theta^*_{j+1}}{\cos\theta^*_j \cos\theta^*_{j+1}}\right)\right]$$

which easily reduces to the form given in (6.10b).

In order to write the thermodynamic source term, $\overline{R}$, in a compact form we define a density weighted "thickness" by letting

$$\overline{S}^n_{j,k} = \left(\overline{\Psi}^n_{j,k+1}\, e^+ - \overline{\Psi}^n_{j,k}\, e^-\right) \tag{6.11}$$

We then have:

$$\begin{aligned}\overline{R} = {} & \mu_1 \overline{S}^n + \mu_2 \overline{S}^{n-1} + \frac{\Delta t}{2}\left[\Delta z(F_T + Q) + D_2(\overline{S}^{n-1})\right] \\ & - \frac{\Delta t}{2} e^{z/2H}\left\{\frac{1}{2\cos\theta}\,\delta_{j+1/2}\left[\cos\theta^* \left(\overline{V}^n_{j,k+1} + \overline{V}^n_{j,k}\right)\langle \overline{S}^n\rangle_{j-1/2,k}\right]\right. \\ & + \frac{1}{4\Delta z}\left[\left(\overline{W}^n_{j,k+1} + \overline{W}^n_{j,k}\right)\left(\overline{S}^n_{j,k+1}\, e^+ + \overline{S}^n_{j,k}\, e^-\right)\right. \\ & \left.\left. - \left(\overline{W}^n_{j,k} + \overline{W}^n_{j,k-1}\right)\left(\overline{S}^n_{j,k}\, e^+ + \overline{S}^n_{j,k-1}\, e^-\right)\right]\right\}\end{aligned} \tag{6.12}$$

6.3 The difference equations for the eddies

Using the above notation the equations may be written as follows:

$$\hat{U}_s - (f\Delta t/2)\hat{V}_s = \frac{-im_s\Delta t}{2}\langle\Psi_s\rangle_{j+1/2} + A_s \tag{6.13}$$

$$\hat{V}_s + (f\Delta t/2)\hat{U}_s = \frac{\Delta t}{2}\,\delta_{j+1/2}\,(\hat{\Psi}_s) + B_s \tag{6.14}$$

$$(\hat{\Psi}_{s,k+1}\,e^+ - \hat{\Psi}_{s,k}\,e^-) + \frac{N^2\Delta t\Delta z}{2}\hat{W}_s = R_s \tag{6.15}$$

$$(\hat{W}_{s,k}e^- - \hat{W}_{s,k-1}e^+) + \frac{\Delta z}{\cos\theta^*}\,[i\langle m_s\hat{U}_s\cos\theta\rangle_{j-1/2} + \delta_{j-1/2}(\hat{V}_s\cos\theta)] = 0 \tag{6.16}$$

Where here $m_s \equiv s/(a\cos\theta)$

$$A_s \equiv \mu_1 U_s^n + \mu_2 U_s^{n-1} - \frac{\Delta t}{2}e^{z/2H}\{i\langle m_s^*\overline{U}^n\rangle_{j+1/2}\,U_s^n + \frac{V_s^n}{\cos\theta}\,\delta_{j+1/2}(\overline{U}\cos\theta^*)$$

$$+ \frac{1}{2\Delta z}\,[\langle W_{s_{j,k}}(\overline{U}_{j,k+1}\,e^+ - \overline{U}_{j,k}\,e^-)\rangle_{j+1/2}$$

$$+ \langle W_{s_{j,k-1}}(\overline{U}_{j,k}\,e^+ - \overline{U}_{j,k-1}e^-)\rangle_{j+1/2}]\} + \frac{\Delta t}{2}D_2(U_s^{n-1}) \tag{6.17}$$

$$B_s \equiv \mu_1 V_s^n + \mu_2 V_s^{n-1} - \frac{\Delta t}{2}e^{z/2H}\left\{i\langle m_s^*\overline{U}^n\rangle_{j+1/2}\,V_s^n\right.$$

$$- \frac{U_s}{\Delta y}\left[\overline{U}_j\left(\frac{\cos\theta_j^*}{\cos\theta_j} - \frac{\cos\theta_j}{\cos\theta_j^*}\right)\right.$$

$$\left.\left. + \overline{U}_{j+1}\left(\frac{\cos\theta_j}{\cos\theta_{j+1}^*} - \frac{\cos\theta_{j+1}^*}{\cos\theta_j}\right)\right]\right\} + \frac{\Delta t}{2}D_2(V_s^{n-1}) \tag{6.18}$$

(Note that if $\overline{U}$ is symmetric about the equator then A_s and B_s have the same symmetry as U_s and V_s, respectively.)

In formulating B_s we have used a special form for the metric term, $-2\overline{U}U_s \tan\theta/a$, which is required to keep the difference equations energy conserving in adiabatic frictionless flow. To derive this form note that

$$- 2\cdot\overline{U}\, U_s \tan\theta/a = \frac{\overline{U}\, U_s}{\cos^2\theta} \frac{d}{dy} (\cos^2\theta)$$

This may be approximated as

$$\simeq \frac{U_s}{\Delta y} \left[\overline{U}_j \left(\frac{\cos^2\theta^*_j - \cos^2\theta_j}{\cos\theta^*_j \cos\theta_j} \right) + \overline{U}_{j+1} \left(\frac{\cos^2\theta_j - \cos^2\theta^*_{j+1}}{\cos\theta_j \cos\theta^*_{j+1}} \right) \right]$$

which easily reduces to the form given in (6.18).

The term R_s in (6.15) can be written in a fairly compact form if we first define a density weighted "thickness"

$$\mathcal{S}_{s,j,k} \equiv (\Psi_{s,j,k+1}\, e^{+} - \Psi_{s,j,k}\, e^{-})$$

We then can write

$$R_s \equiv \mu_1 \mathcal{S}^{n}_{s,j,k} + \mu_2 \mathcal{S}^{n-1}_{s,j,k} + \frac{\Delta t}{2} Q_s + \frac{\Delta t}{2} D_2 (\mathcal{S}^{n-1}_s)$$

$$- \frac{\Delta t}{2} e^{z/2H} \left[\frac{-im^*_s}{2} (\overline{U}^n_{j,k+1} + \overline{U}^n_{j,k})\, \mathcal{S}^n_{s,j,k} \right.$$

$$+ \frac{W^n_{s,j,k}}{2\Delta z} \left(\langle \overline{\mathcal{S}}^n \rangle_{j-\frac{1}{2},k+1} e^{\Delta z/2H} - \langle \overline{\mathcal{S}}^n \rangle_{j-\frac{1}{2},k-1} e^{-\Delta z/2H} \right)$$

$$\left. + \left(\frac{\langle V^n_s \rangle_{j-\frac{1}{2},k+1} + \langle V^n_s \rangle_{j-\frac{1}{2},k}}{2} \right) \delta_{j-\frac{1}{2}}\, \overline{\mathcal{S}}^n_k \right] \qquad (6.19)$$

7. SOLUTION METHOD

7.1 The zonal mean equations

The system (6.5) - (6.8) is a set of simultaneous equations for the unknowns $\hat{\hat{U}}$, $\hat{\hat{V}}$, $\hat{\hat{\Psi}}$, and $\hat{\hat{W}}$. To solve this set we first eliminate $\hat{\hat{U}}$ between (6.5) and (6.6) to obtain

$$\hat{\hat{V}} = \gamma_j \left[- \frac{\Delta t}{2} \delta_{j-1/2} \hat{\hat{\Psi}} + \overline{B} - \frac{f\Delta t}{2} \overline{A}\right] \tag{7.1}$$

where $\gamma_j \equiv (1 + f^2\Delta t^2/4)^{-1}$.

We next substitute from (7.1) and (6.8) into (6.7) to eliminate $\hat{\hat{W}}$ and $\hat{\hat{V}}$. The result is an elliptic difference equation in $\hat{\hat{\Psi}}$:

$$\Gamma_k \hat{\hat{\Psi}}_{j,k+1} - (\Gamma_{k-1} e^{\Delta z/2H} + \Gamma_k e^{-\Delta z/2H}) \hat{\hat{\Psi}}_{j,k}$$

$$+ \Gamma_{k-1} \hat{\hat{\Psi}}_{j,k-1} + A_j \hat{\hat{\Psi}}_{j-1,k} + B_j \hat{\hat{\Psi}}_{j,k} + C_j \hat{\hat{\Psi}}_{j+1,k} = D_{j,k} \tag{7.2}$$

Here we have let $N^2(z) = N_o^2/\Gamma(z)$ where N_o^2 = constant, and then expressed $\Gamma(z)$ (the vertical variation of static stability) at $z = (k - 1/2)\Delta z$ as Γ_k. The coefficients, A_j, B_j, C_j, D_j are then defined by

$$A_j \equiv \frac{N_o^2\Delta z^2\Delta t^2}{4\Delta y^2 \cos\theta_j} (\gamma_j \cos\theta_j^*)$$

$$C_j \equiv \frac{N_o^2\Delta z^2\Delta t^2}{4\Delta y^2 \cos\theta_j} (\gamma_{j+1} \cos\theta_{j+1}^*)$$

$$B_j \equiv - A_j - C_j$$

$$D_{jk} \equiv (\Gamma_K \overline{R}_{j,k}\, e^- - \Gamma_{k-1}\, \overline{R}_{j,k-1}\, e^+$$

$$+ \frac{N_o^2 \Delta t \Delta z^2}{2 \cos \theta_j}\, \delta_{j+1/2}\, [\gamma \cos \theta^* \, (\overline{B} - f\Delta t/2\overline{A})] \quad .$$

The elliptic system (7.2) may be solved for $\overline{\Psi}_{j,k}$ using the NCAR subroutine BLKTRI (Swarztrauber and Sweet, 1975). In this case the solution is carried out for the grid points in the range

$$1 \leq j \leq J_m - 1 \quad , \quad 2 \leq k \leq K_N \leq 1$$

The lateral boundary condition (4.1) is incorporated by setting

$$A_1 = 0 \text{ and } C_{J_m - 1} = 0$$

The lower boundary condition is incorporated by setting $\hat{\overline{\Psi}}_{j,1}$ equal to the value obtained from integrating (4.2c). The upper boundary condition (2.16c) in finite difference form requires

$$\hat{\overline{\Psi}}_{j,K_N}\, e^+ = \hat{\overline{\Psi}}_{j,K_N-1}\, e^- \tag{7.3}$$

Once $\hat{\overline{\Psi}}$ has been obtained by inversion of (7.2) it is a simple matter to compute the remaining fields. If, however, one attempts to compute $\hat{\overline{V}}$ from (7.1) the results are rather unsatisfactory due to large truncation errors. This problem arises due to the fact that the first and third terms on the right side are generally two orders of magnitude greater than $\hat{\overline{V}}$ so that $\hat{\overline{V}}$ is obtained as a small residual of two large but opposite terms. To avoid this problem it is useful to utilize the meridional streamfunction defined by (3.11). In finite difference form we have

$$\hat{\overline{W}}_{j,k} = \frac{1}{\cos\,\theta}\,\delta_{j+1/2}\,\hat{\overline{X}} \tag{7.4a}$$

$$\hat{\overline{V}}_{m,k} = -\frac{1}{\Delta z\ \cos\,\theta^*}\,(\hat{\overline{X}}_{j,k}\ e^- - \hat{\overline{X}}_{j,k-1}\ e^+) \tag{7.4b}$$

which identically satisfies the finite difference form of the continuity equation (6.7).

Substituting from (7.4a) into (6.8) and noting that $\hat{\overline{X}}_{1,k} = 0$ we can solve for $\hat{\overline{X}}_{j,k}$:

$$\hat{\overline{X}}_{j+1,k} = \hat{\overline{X}}_{j,k}\ \frac{-2\Delta y\ \cos\,\theta}{N^2\Delta z\Delta t}\ [\overline{R} - (\hat{\overline{\Psi}}_{k+1}\ e^+ - \hat{\overline{\Psi}}_k\ e^-)] \tag{7.5}$$

We next use (7.4b) to solve for $\hat{\overline{V}}$ and finally use (6.5) to obtain $\hat{\overline{U}}$. The final step of the solution is then to use the definition (6.1) to obtain all fields at time n + 1. For example,

$$\overline{U}^{n+1} = (\hat{\overline{U}} - \beta_2\overline{U}^{n} - \beta_3\overline{U}^{n-1})/\beta_1 \tag{7.6}$$

and similarly for $\overline{V}^{n+1}$, $\overline{\Psi}^{n+1}$, and $\overline{W}^{n+1}$.

7.2 The eddy equations

The system (6.13) - (6.16) is a set of simultaneous equations for the unknowns $\hat{U}_s$, $\hat{V}$, $\hat{\Psi}_s$, $\hat{W}_s$ which is exactly analogous to the mean flow set discussed above. To solve this set we first solve for $\hat{U}_s$ and $\hat{V}_s$ in terms of $\hat{\Psi}_s$

using (6.13) and (6.14):

$$\hat{U}_s = \gamma_j \left[\frac{-im_s\Delta t}{2} \langle \hat{\Psi}_s \rangle_{j+1/2} - \frac{f^2\Delta t}{4} \delta_{j+1/2} \hat{\Psi}_s + A_s + \frac{f\Delta t}{2} B_s\right] \tag{7.7}$$

$$\hat{V}_s = \gamma_j \left[im_s f \frac{\Delta t^2}{4} \langle \hat{\Psi}_s \rangle_{j+1/2} - \frac{\Delta t}{2} \delta_{j+1/2} \hat{\Psi}_s + B_s - \frac{f\Delta t}{2} A_s\right] \tag{7.8}$$

where $\gamma_j \equiv (1 + f^2\Delta t^2/4)^{-1}$.

Combining (6.15), (6.16), (7.7), and (7.8) we get a single equation for $\hat{\Psi}_s$:

$$\Gamma_k \hat{\Psi}_{s_{j,k+1}} = (\Gamma_{k-1} e^{\Delta z/2H} + \Gamma_k e^{-\Delta z/2H}) \hat{\Psi}_{s,j,k} + \Gamma_{k-1} \hat{\Psi}_{s,j,k-1}$$

$$+ D_{s,j} \hat{\Psi}_{s,j-1,k} + E_{s,j} \hat{\Psi}_{s,j,k} + F_{s_j} \hat{\Psi}_{s,j+1,k} = T_{s,j,k} \tag{7.9}$$

where Γ_k is as defined below (7.2), and the coefficients D_s, E_s, F_s are

$$D_{s,j} \equiv \frac{N_o^2\Delta z^2\Delta t^2}{4 \cos \theta_j^*} \left[\frac{\gamma_{j-1} \cos \theta_{j-1}}{\Delta y^2} - \frac{m^2_{s_{j-1}} \gamma_{j-1} \cos \theta_{j-1}}{4}\right]$$

$$F_{s,j} \equiv \frac{N_o^2\Delta z^2\Delta t^2}{4 \cos \theta_j^*} \left[\frac{\gamma_j \cos \theta_j}{\Delta y^2} - \frac{m_s^2 \gamma_j \cos \theta_j}{4}\right]$$

$$E_{s,j} \equiv - \frac{N_o^2 \Delta z^2 \Delta t^2}{4 \cos \theta_j^*} \left[\frac{\gamma_{j-1} \cos \theta_{j-1} \gamma_j + {}_j \cos \theta_j}{\Delta y^2}\right.$$

$$+ \frac{m^2_{s_{j-1}} \cos \theta_{j-1} \gamma_{j-1} + m^2_{s_j} \cos \theta_j \gamma_j}{4}$$

$$\left. + \frac{i\Delta t}{\Delta y} (\gamma_{j-1} m_{s_{j-1}} \cos \theta_{j-1} f_{j-1} - \gamma_j m_{s_j} \cos \theta_j f_j)\right]$$

and the source term is

$$T_{s,j,k} \quad (\Gamma_k R_{s,j,k}\, e^- - \Gamma_{k-1} R_{s,j,k}\, e^+)$$

$$+ \frac{N_o^2\, \Delta t\, \Delta z^2}{2 \cos \theta_j^*} \left[(\gamma_{j-1} \cos \theta_{j-1}\, q_{s,j-1} - \gamma_j \cos \theta_j\, q_{s,j})/\Delta y \right.$$

$$\left. + \frac{i}{2} (m_{s,j-1}\, \gamma_{j-1} \cos \theta_{j-1}\, p_{s,j-1} + m_{s,j}\, \gamma_j \cos \theta_j\, p_{s,j}) \right]$$

where

$$p_{s,j} = A_s + \frac{f\Delta t}{2} B_s \ , \quad q_{s,j} = B_s - \frac{f\Delta t}{2} A_{s,j}$$

The elliptic system (7.9) may be solved for $\hat{\Psi}_{s,j,k}$ using the NCAR subroutine CBLKTRI.

For global or hemispheric antisymmetric modes the solution is carried out for grid points

$$2 \le j \le J_m - 1 \ ; \quad 2 \le k \le K_N - 1$$

However, for <u>symmetric</u> hemispheric calculations the solution includes the point $j = J_m$.

In the global or antisymmetric hemispheric case we thus require

$$\hat{\Psi}_{s,1,k} = \hat{\Psi}_{s,J_m,k} = 0 \tag{7.10}$$

while for the symmetric hemispheric case we must have $\hat{\Psi}_{s,1,k} = 0$ and

$$\hat{\Psi}_{s,J_{m-1},k} = \hat{\Psi}_{s,J_{m+1},k} \tag{7.11}$$

Condition (7.10) is incorporated by letting $D_{s,2} = 0$ and $F_{s,J_{m-1}} = 0$ while the condition (7.11) requires replacing D_{s,J_m} as defined above by $D_{s,J_m} + F_{s,J_m}$.

In all cases the upper boundary condition is $\hat{\Psi}_{s,j,K_N} = 0$ and the lower boundary condition is a specified forcing

$$\hat{\Psi}_{s,j.1} = gh(y,t) \tag{7.12}$$

Once $\hat{\Psi}_{s,j,k}$ is obtained from (7.9) we compute $\hat{W}_{s,j,k}$ from (6.15)

$$\hat{W}_{s,j,k} = \frac{2\Gamma_k}{\Delta t\ N_o^2\ \Delta z}\,[R_{s,j,k} - (\hat{\Psi}_{s_{j,k+1}}\ e^+ - \hat{\Psi}_{s,j,k}\ e^-)] \tag{7.13}$$

We then use (7.7) and (7.8) to solve for $\hat{U}_s$ and $\hat{V}_s$. Finally, these results are used to obtain U_s^{n+1}, V_s^{n+1}, Ψ_s^{n+1} by a formula analogous to (7.6). A similar treatment of $\hat{W}_s$ proved unstable. Therefore in computing fluxes and vertical advection terms $\hat{W}_s$ is used in place of W_s^{n+1}.

7.3 The eddy flux terms

The eddy momentum flux convergence (3.11) and the eddy heat flux convergence (3.12) must be written in finite differences so that the energy integrals of the flow remain satisfied. It turns out that energetically consistent forms are:

$$F_m = - e^{z/2H} \left\{ \frac{1}{\cos^2 \theta_j^*} \delta_{j-1/2} \; [(U_s V_s^* + U_s^* V_s) \cos^2 \theta] \right.$$

$$+ \frac{1}{2\Delta z} [\langle U_{s,j,k} \, e^- + U_{s,j,k+1} \, e^+ \rangle_{j-1/2} \, W^*_{s,j,k}$$

$$- \langle U_{s,j,k} \, e^+ + U_{s,j,k-1} \, e^- \rangle_{j-1/2} \, W^*_{s,j,k-1}$$

$$+ \langle U^*_{s,j,k} \, e^- + U^*_{s,j,k+1} \, e^+ \rangle_{j-1/2} \, W_{s,j,k}$$

$$\left. - \langle U^*_{s,j,k} \, e^+ + U^*_{s,j,k-1} \, e^- \rangle_{j-1/2} \, W_{s,j,k-1}] \right\} \tag{7.14}$$

and,

$$\Delta z F_T = - \frac{e^{(z+\Delta z/2)/H}}{2 \cos \theta_j} \delta_{j+1/2} \left\{ \cos \theta_j^* \; [(\langle V_s \rangle_{j-1/2,k+1} + \langle V_s \rangle_{j-1/2,k}) \; S^*_{s,j,k} \right.$$

$$\left. + (\langle V_s^* \rangle_{j-1/2,k+1} + \langle V_s^* \rangle_{j-1/2,k}) \; S_{s,j,k}] \right\}$$

$$- \frac{e^{z/2H}}{2\Delta z} [\langle W_s^* S_s \rangle_{j+1/2,k+1} - \langle W_s^* S_s \rangle_{j+1/2,k-1}$$

$$+ \langle W_s S_s^* \rangle_{j+1/2,k+1} - \langle W_s S_s^* \rangle_{j+1/2,k-1}] \tag{7.15}$$

8. INTEGRAL CONSTRAINTS AND SUBGRID SCALE DIFFUSION

8.1 Integral constraints for the zonal mean equations

The basic equations of the model (3.7) - (3.10) satisfy certain integral constraints which also must be satisfied by the finite difference equations if satisfactory long term integrations are to be obtained. It is easily verified that when the forcing terms F_M, F_T, and $\overline{Q}$ are omitted, and subgrid scale diffusion is neglected, the rate of change of zonal mean kinetic plus available potential energy is equal to the energy flux through the lower boundary:

$$\frac{d}{dt}\,(\overline{P} + \overline{K}) = \int_A \left\{\left[\frac{\overline{U}^2 + (\partial\overline{\Psi}/\partial z + \overline{\Psi}/2H)^2/N^2}{2} + \overline{\Psi}\right]\overline{W}\right\}_{z=0} dA \qquad (8.1)$$

where

$$\overline{P} = \int_\tau \frac{1}{2}\left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H}\right)/N^2 d\tau \qquad (8.2a)$$

$$\overline{K} = \int_\tau \frac{1}{2}\,(\overline{U}^2 + \overline{V}^2)d\tau \qquad (8.2b)$$

and

$$dA = a^2 \cos\theta\, d\theta\, d\lambda$$

$$d\tau = a^2 \cos\theta\, d\theta\, d\lambda\, dz$$

Another important constraint is the conservation of relative angular momentum. If we multiply (3.7) by $\exp(-z/2H)\cos\theta$ and integrate the result over the entire domain we find that relative angular momentum is conserved except for the flux of angular momentum through the lower boundary:

$$\frac{d}{dt}\int_{\tau} e^{-z/2H}\,\overline{U}\cos\theta\, d\tau = \int_{A} [\overline{U}\,\overline{W}\cos\theta]_{z=0}\, dA + \int_{A} f\overline{X}(z=0)\,dA \tag{8.3}$$

In deriving (8.3) we have neglected eddy momentum fluxes through the lower boundary. It is important to note in connection with (8.3) that horizontal diffusion can not change relative angular momentum so that we require

$$\int_{A} D_1(\overline{U})\cos\theta\, dA = 0 \tag{8.4}$$

which constrains the possible forms for the operatore $D_1(\ )$.

Also, horizontal diffusion can not change the horizontally averaged temperature (thickness) on a horizontal surface. Thus from (3.10):

$$\int_{A} D_2\left(\frac{\partial\overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H}\right) dA = 0 \tag{8.5}$$

The constraints (8.1), (8.3), and (8.5) must also be satisfied by our system of finite difference equations[2] if satisfactory results are to be obtained. In finite difference form the integrals are replaced by sums over the grid points:

$$\int (\ )d\tau \approx \sum_{j,k} (\ )2\pi a\cos\theta\,\Delta y\,\Delta z \tag{8.6a}$$

$$\int_{A} (\ )dA \approx \sum_{j} (\ )2\pi a\cos\theta\,\Delta y \tag{8.6b}$$

[2]Except for the effects of time truncation errors.

The value of θ_j used in (8.6a) or (8.6b) is given by either (6.9a) or (6.9b) depending on the location of the dependent variable; e.g., in (8.5) we use (6.9a) and in (8.3) we used (6.9b).

Next, multiplying (6.5) by $e^{-z/2H} \cos^2 \theta^*(2\pi a\Delta y\Delta z)$ we find after summing over all grid points ($2 \le j \le J_M' - 1$, $2 \le k \le K_N - 1$), and using 6.2a):

$$\frac{d}{dt} \sum_{j,k} e^{-z/2H}\, \overline{U} \cos^2 \theta^* = \frac{1}{\Delta z} \sum_j [f\, \overline{X}_{j,1} \cos \theta^*]$$

$$+ \frac{1}{\Delta z} \sum_j [(\overline{U}_{j,2}\, e^+ + \overline{U}_{j,1}\, e^-) \langle \overline{W} \cos \theta \rangle_{j-\frac{1}{2},1} \cos \theta^*] \tag{8.7}$$

which is consistent with the differential form for angular momentum conservation, (8.3). (We have here assumed that (8.4) holds for the finite difference form of subgrid scale diffusion.)

The finite difference analogy to the energy integral (8.1) may be obtained by multiplying (6.5) by $\overline{U} \cos \theta^*$, (6.6) by $\overline{V} \cos \theta^*$, and (6.8) by $(\cos \theta)/N^2\Delta z^2)(\overline{\Psi}_{k+1}\, e^+ - \overline{\Psi}_k\, e^-)$ then adding the three resulting equations together and summing over all gridpoints. Using (6.2) to express the time derivatives in differential form we can then write

$$\frac{d}{dt}\left[\sum_{\substack{j=2,J_m-1\\ k=2,K_N-1}} \frac{(\overline{U}^2_{jk} + \overline{V}^2_{jk})\cos \theta^*_j}{2} + \sum_{\substack{j=1,J_m\\ k=1,N}} \frac{(\overline{\Psi}_{k+1}\, e^+ - \overline{\Psi}_k\, e^-)^2 \cos \theta_j}{2\Delta z^2 N^2}\right]$$

$$= \frac{1}{\Delta z} \sum_{j=1,J_m} \left[\overline{W}_{j,1}\, \overline{\Psi}_{j,1}\, e^- \cos \theta_j + \frac{\overline{U}_{j,2}\, \overline{U}_{j,1}}{2} \langle \overline{W} \cos \theta \rangle_{j-\frac{1}{2},1}\, e^+ \right.$$

$$\left. + \frac{(\overline{W}_{j,2} + \overline{W}_{j,1})(\overline{\Psi}_{j,3}\, e^+ - \overline{\Psi}_{j,2}\, e^-)(\overline{\Psi}_{j,2}\, e^+ - \overline{\Psi}_{j,1}\, e^-)\, e^+}{4N^2\, \Delta z^2} \right] \tag{8.8}$$

where $F_M = F_T = Q = 0$ and friction terms have all been neglected. Clearly, (8.9) is a reasonable analogue to the differential relationship (8.1).

8.2 Subgrid scale diffusion for the mean flow equations

In order to suppress nonlinear instability it is necessary to smooth all fields in the meridional direction. In order to prevent this smoothing from damping the large scale motions we have chosen to use a fourth order linear diffusion operator. In applying the diffusion in the zonal momentum and thermodynamic energy equations we must recall that both relative angular momentum and horizontal average temperature must be conserved [see (8.4) and (8.5)]. In addition the diffusion terms should make negative definite contributions to the energy equation.

In order to satisfy both these requirements it turns out that in the zonal momentum equation relative angular velocity should be diffused. Thus,

$$D_1(\overline{U}) = - \frac{K}{\cos^2 \theta} \frac{\partial^4}{\partial y^2} \left(\frac{\overline{U}}{\cos \theta}\right) \tag{8.9}$$

This automatically satisfies (8.4) provided that $(\partial^3/\partial y^3)(\overline{U}/\cos \theta) = 0$ at the meridional boundaries. If we multiply (8.9) by $\overline{U} \cos \theta$ and integrate the result in y we obtain

$$\int_A \overline{U} \cos \theta \, D_1(\overline{U}) dA = - K \int \left[\frac{\partial^2}{\partial y^2} \left(\frac{\overline{U}}{\cos \theta}\right)\right]^2 dy \, d\lambda$$

which is negative definite. Thus, the diffusion term (8.9) acts as an energy sink. In finite difference form we write

$$D_1(\overline{U}) = \frac{-K}{\cos^2\theta^*\,\Delta y^4}\left[\left(\frac{\overline{U}}{\cos\theta^*}\right)_{j-2} - 4\left(\frac{\overline{U}}{\cos\theta^*}\right)_{j-1}\right.$$

$$\left. + 6\left(\frac{\overline{U}}{\cos\theta^*}\right)_{j} - 4\left(\frac{\overline{U}}{\cos\theta^*}\right)_{j+1} + \left(\frac{\overline{U}}{\cos\theta^*}\right)_{j+2}\right] \tag{8.10}$$

In order that this finite difference form satisfy the difference analogue of (8.4) i.e., $\sum_j \cos^2\theta^*\, D_1(\overline{U}) = 0$ the formula must be modified at the points adjacent to the boundaries. Thus

$$[D_1(\overline{U})]_{j=2} = \frac{-K}{\cos^2\theta^*\,\Delta y^4}\left[\left(\frac{2\overline{U}}{\cos\theta^*}\right)_{j=2} - \left(\frac{3\overline{U}}{\cos\theta^*}\right)_{j=3} + \left(\frac{\overline{U}}{\cos\theta^*}\right)_{j=4}\right] \tag{8.11a}$$

$$[D_1(\overline{U})]_{j=3} = \frac{-K}{\cos^2\theta^*\,\Delta y^4}\left[-3\left(\frac{\overline{U}}{\cos\theta}\right)_{j=2} + 6\left(\frac{\overline{U}}{\cos\theta^*}\right)_{j=3}\right.$$

$$\left. - 4\left(\frac{\overline{U}}{\cos\theta^*}\right)_{j=4} + \left(\frac{\overline{U}}{\cos\theta^*}\right)_{j=5}\right] \tag{8.11b}$$

with analogous expressions for $j = J_m - 1$ and $j = J_m - 2$. Again using the notation of (6.11) we can write the finite difference diffusion term in the thermodynamic energy equation as follows:

$$D_2(\overline{S}) = \frac{-K}{\cos\theta\,\Delta y^4}\,[\overline{S}_{j-2} - 4\overline{S}_{j-1} + 6\overline{S}_j - 4\overline{S}_{j-1} + \overline{S}_{j+2}] \tag{8.12}$$

Again the points adjacent to the boundaries require special treatment:

$$[D_2(\overline{S})]_{j=1} = \frac{-K}{\cos\theta\,\Delta y^4}\,[2\overline{S}_{j=1} - 3\overline{S}_{j=2} + \overline{S}_{j=3}] \tag{8.13}$$

$$[D_2(\overline{S})]_{j=2} = \frac{-K}{\cos\theta\,\Delta y^4}[-3\overline{S}_{j=1} + 6\overline{S}_{j=2} - 4\overline{S}_{j=3} + \overline{S}_{j=4}] \tag{8.14}$$

$$[D_2(\overline{S})]_{j=J_m-1} = \frac{-K}{\cos\theta\,\Delta y^4}[\overline{S}_{j=J_m-3} - 3\overline{S}_{j=J_m-2} + 2\overline{S}_{j=J_m-1}] \tag{8.15}$$

Finally we write

$$D_2(\overline{V}) = \frac{-K}{\cos\theta^*\,\Delta y^4}[\overline{V}_{j-2} - 4\overline{V}_{j-1} + 6\overline{V}_j - 4\overline{V}_{j+1} + \overline{V}_{j+2}] \tag{8.16}$$

with the special cases

$$[D_2(\overline{V})_{j=2} = \frac{-K}{\cos\theta^*\,\Delta y^4}[3\overline{V}_{j=2} - 3\overline{V}_{j=3} + \overline{V}_{j=4}] \tag{8.17}$$

and an analogous form for $j = j_m - 1$.

8.3 Subgrid scale diffusion for the eddy equations

To filter out small scale noise so as to suppress nonlinear instability the eddy equations include fourth order linear diffusion terms similar to those discussed in Section 8.2 for the zonal mean flow. $D_2(U_s)$ and $D_2(V_s)$ have the same form as $D_2(\overline{S})$ given in (8.12), while $D_2(\mathcal{S}_s)$ has the form of $D_2(\overline{V})$ given in (8.16). These forms must, however, be modified next to the boundaries to insure that diffusion does not change the meridional average of any field. For a global domain the modification to $D_2(\mathcal{S}_s)$ is identical to that given in (8.17) for $D_2(\overline{V})$.

For the U_s and V_s field the situation is more complicated since the boundary conditions are different in the $s = 1$ and $s > 1$ cases.

For the case $s = 1$, $D_2(U_s)$ and $D_2(V_s)$ are computed using formulas analogous to (8.12), (8.13), and (8.14). For global integrations formulas similar to (8.13) and (8.14) are applied at $j = J_m-1$ and $j = J_m - 2$.

For the case $s > 1$ the polar boundary condition requires that

$$D_2(U_s)_{j=1} = \frac{K}{\cos\theta\ \Delta y^4}\left[4U_{s,j=1} - 3U_{s,j=2} + U_{s,j=3}\right] \tag{8.18}$$

$$D_2(U_s)_{j=2} = -\frac{K}{\cos\theta\ \Delta y^4}\left[-5U_{s,j=1} + 6U_{s,j=2} - 4U_{s,j=3} + U_{s,j=4}\right] \tag{8.19}$$

with similar expressions for V_s.

In the case of hemispheric integrations the diffusion operators at $j = J_m - 1$ and $j = J_m - 2$ are modified as follows:

For antisymmetry conditions on $\hat{\Psi}$ the form of $D_2(S_s)$ is the same as in the global case; however, since $U_{s,J_m} = -U_{s,J_m-1}$, and $V_{s,J_m} = +V_{s,J_m-1}$ we use a diffusion form analogous to (8.18) and (8.19) for $D_2(U_s)_{J_m-1}$ and $D_2(U_s)_{J_m-2}$ while for $D_2(V_s)_{J_m-1}$ and $D_2(V_s)_{J_m-2}$ we use forms analogous to (8.14) and (8.15).

For symmetric conditions on $\hat{\Psi}$ the form of $D_2(S_s)$ must be modified as follows:

$$D_2(S_s)_{J_m-1} = \frac{-K}{\cos\theta\ \Delta y^4}\left[-4S_{s,J_m} + 7S_{s,J_m-1} - 4S_{J_m-2} + S_{J_m-3}\right] \tag{8.20}$$

$$D_2(S_s)_{J_m} = \frac{-K}{\cos\theta\ \Delta y^4}\left[3S_{J_m} - 4S_{J_m-1} + S_{J_m-2}\right] \tag{8.21}$$

and since $U_{s,J_m} = U_{s,J_m-1}$ and $V_{s,J_m} = -V_{s,J_m-1}$ we use forms similar to (8.14) and (8.15) for $D_2(U_s)_{J_m-1}$ and $D_2(U_s)_{J_m-2}$ and forms analogous to (8.18) and (8.19) for $D_2(V_s)_{J_m-1}$ and $D_2(V_s)_{J_m-2}$.

9. DIABATIC HEATING COMPUTATION

9.1 Infrared heating

This study has utilized Dickinson's (1973) parameterization of infrared cooling consisting of the sum of the cooling for a reference temperature T_o and a Newtonian cooling approximation for the departures from that profile. Thus the net heating terms take the following forms: For the eddies,

$$Q_s = - \alpha T_s$$

while for the mean flow

$$\overline{Q} = \overline{Q}_e - (\overline{Q}_r + \alpha \overline{T})$$

where $\overline{Q}_e$ is the diabatic heating due to the absorption of solar radiation by ozone. Q_r is the net infrared cooling at each level for the reference temperature profile, and α is the Newtonian cooling coefficient. $\overline{Q}_e$ and $\overline{T}$ are functions of altitude and latitude while $\overline{Q}_r$, α, and T_o depend on altitude alone.

The values of the Newtonian cooling coefficients have been calculated for levels between 30 and 80 km by Dickinson (1973). Below 30 km Trenberth's (1973) values are adopted. Although the accuracy of the Newtonian cooling representation breaks down above about 70 km, it shall be retained at this time for lack of a better representation. Following Schoeberl and Strobel (1978), the value of α between 80 and 96 km was taken to be the CO_2 cooling rate in the fundamental band at 15µ (see Fig. 2).

Dickinson's (1973) careful computations of α and $\overline{Q}_r$ were made for atmospheric temperature profiles that differ little from the reference temperature profile. Because the actual temperatures may vary considerably from this reference profile, especially in the winter polar region, alternative values of Q_r are here computed in the following manner.

At a given level the globally averaged diabatic heating $\tilde{\bar{Q}}$ is given by

$$\tilde{\bar{Q}} = \tilde{\bar{Q}}_e - (\overline{Q}_r - \alpha\tilde{\bar{T}})$$

where ($\tilde{\ }$) designates a horizontal average on the sphere. Since the observed globally averaged temperature profile T_o is fairly well known, we choose $\overline{Q}_r$ so that global radiative equilibrium ($\tilde{\bar{Q}} = 0$) is achieved when the globally averaged temperature profile $\tilde{\bar{T}}$ is equal to T_o. Therefore

$$\tilde{\bar{Q}}_r = \tilde{\bar{Q}}_e \ .$$

9.2 Solar Heating

Below 96 km ozone is the only significant absorber of solar radiation. The parameterizations of Lacis and Hansen (1974) are used to compute the solar heating term Q_s. The diurnally averaged solar heating is calculated by fixing the sun angle at its average value between sunrise and sunset (approximation 1 of Cogley and Borucki, 1976). The sun angle may remain fixed for the duration of a given run, or may be varied according to the objectives of the particular run.

10. A TEST APPLICATION OF THE MODEL

In order to demonstrate the capabilities of the model we have computed the zonal mean annual cycle for the stratosphere and mesosphere for conditions of zonal mean forcing only. In this experiment the eddy forcing was set to zero at the lower boundary. The mean zonal winds at the lower boundary (16 km) were specified to vary over the annual cycle according to the observations of Labitzke (1972) for the northern hemisphere and Taljaard et al. (1969) for the southern hemisphere. The diabatic heating was also specified to vary on the annual cycle by including seasonal variations in the solar zenith angle and sun-earth distance.

10.1 Rayleigh friction parameterization

In order to produce a realistic mean wind profile it proved necessary to specify strong damping in the mean momentum equations above 70 km. In the atmosphere the mechanical damping of the mean wind near the mesopause is probably due to the breaking of gravity waves and tides. For the present model this effect is parameterized in the simplest possible form by using a height dependent Rayleigh friction coefficient

$$\kappa_R = \kappa_o + \kappa_1 \left[1. + \tanh\left(\frac{z - 71}{10.}\right)\right]$$

where κ_o = 1/80 days, κ_1 = 1/4 days and z is in kilometers. This profile is shown in Fig. 2.

The biharmonic horizontal diffusion coefficient is given the value $K/\Delta y^4 = 10^{-8}\ s^{-1}$ which is the minimum necessary to suppress nonlinear computational instability when Δt = 1 hr.

10.2 The zonal mean annual cycle

Figs. 3-8 show the zonal mean wind, mean meridional wind, and vertical velocity profiles for southern hemisphere winter solstice and spring equinox conditions computed using the above described parameters and a grid resolution of 10° latitude and 5 km height. During the solstice season there is a thermally direct mean meridional circulation with rising motion in the summer hemisphere and sinking in the winter hemisphere. At the equinox, on the other hand there is a two cell meridional circulation with rising in the equatorial zone and sinking near both poles. Zonal mean winds computed in both seasons are quite realistic. This example shows that a zonal mean model is capable of simulating many important features of the general circulation in the middle atmosphere. Further details of this annual cycle simulation are reported in Holton and Wehrbein (1979).

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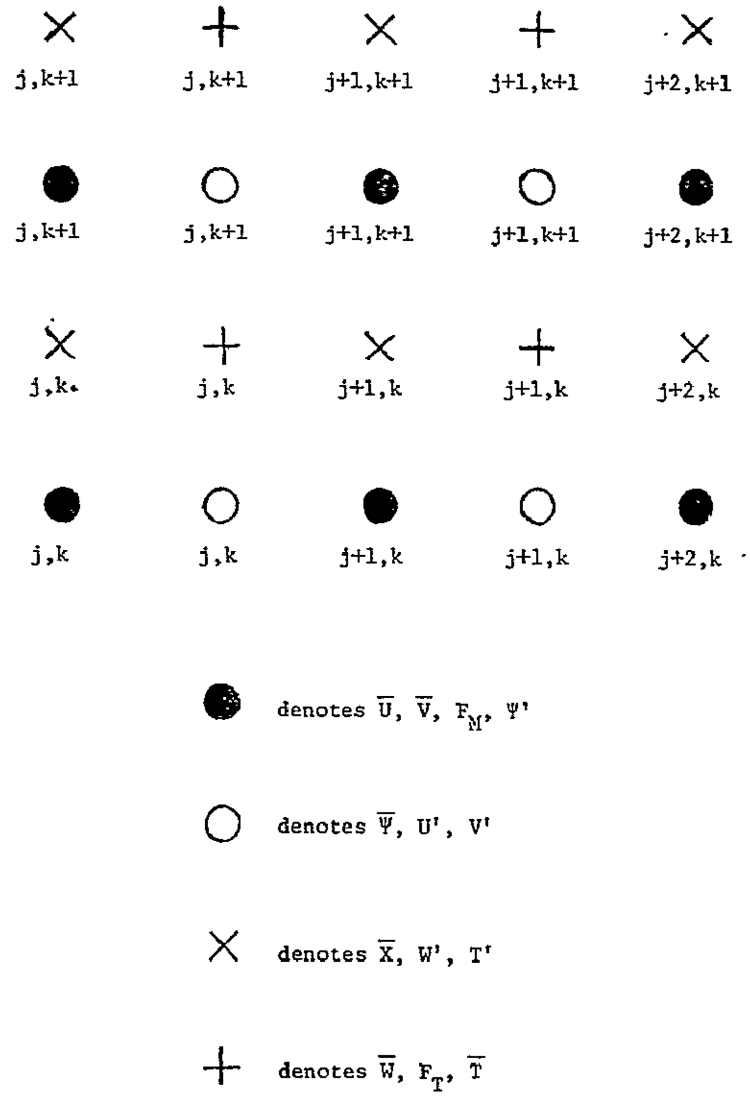

Figure 1: A portion of the grid mesh in the meridional plane showing the arrangement of variables on the staggered grid.

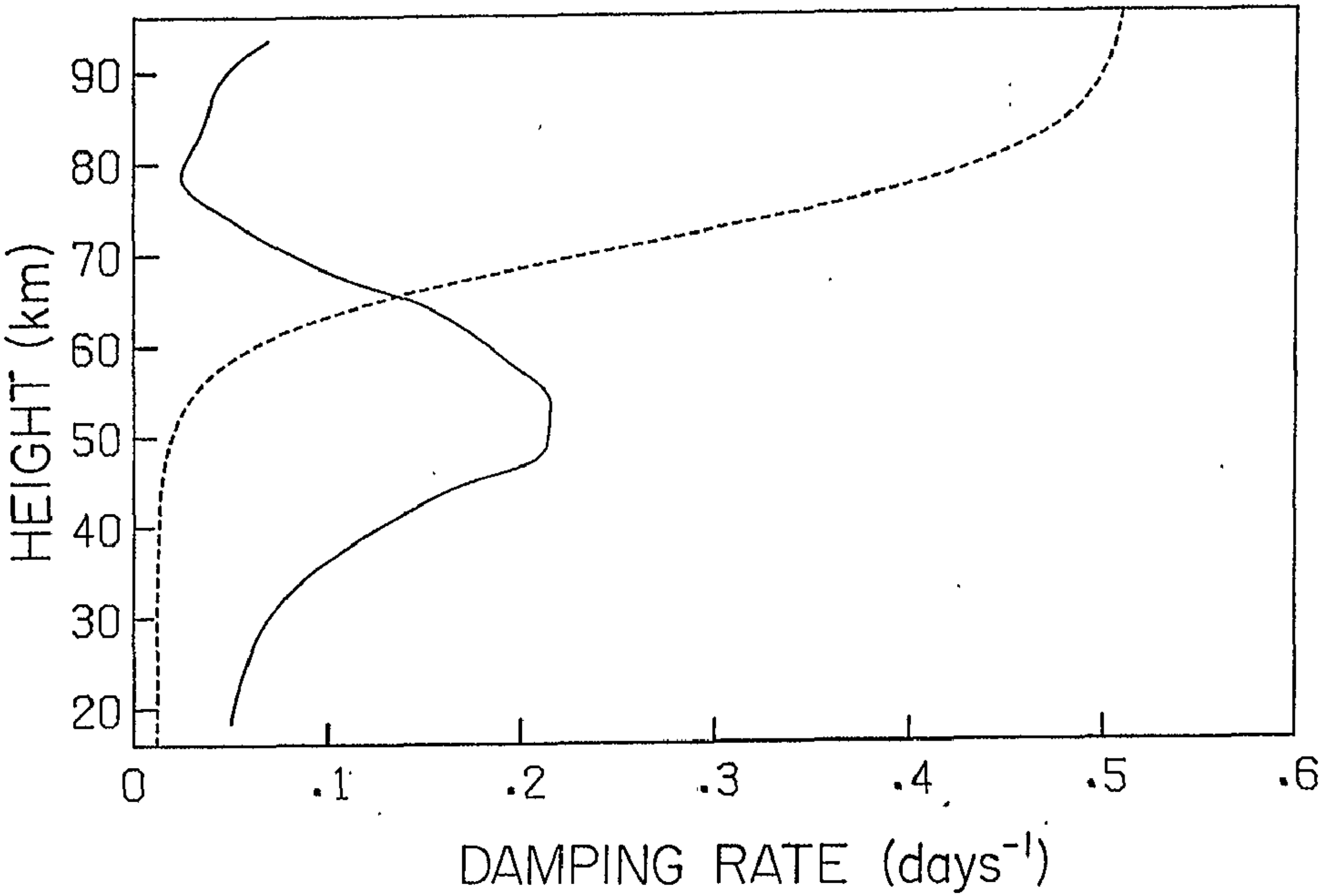

Figure 2: Vertical profiles of the Newtonian cooling coefficient (solid line) and Rayleigh friction coefficient (dashed line) in units of d^{-1}.

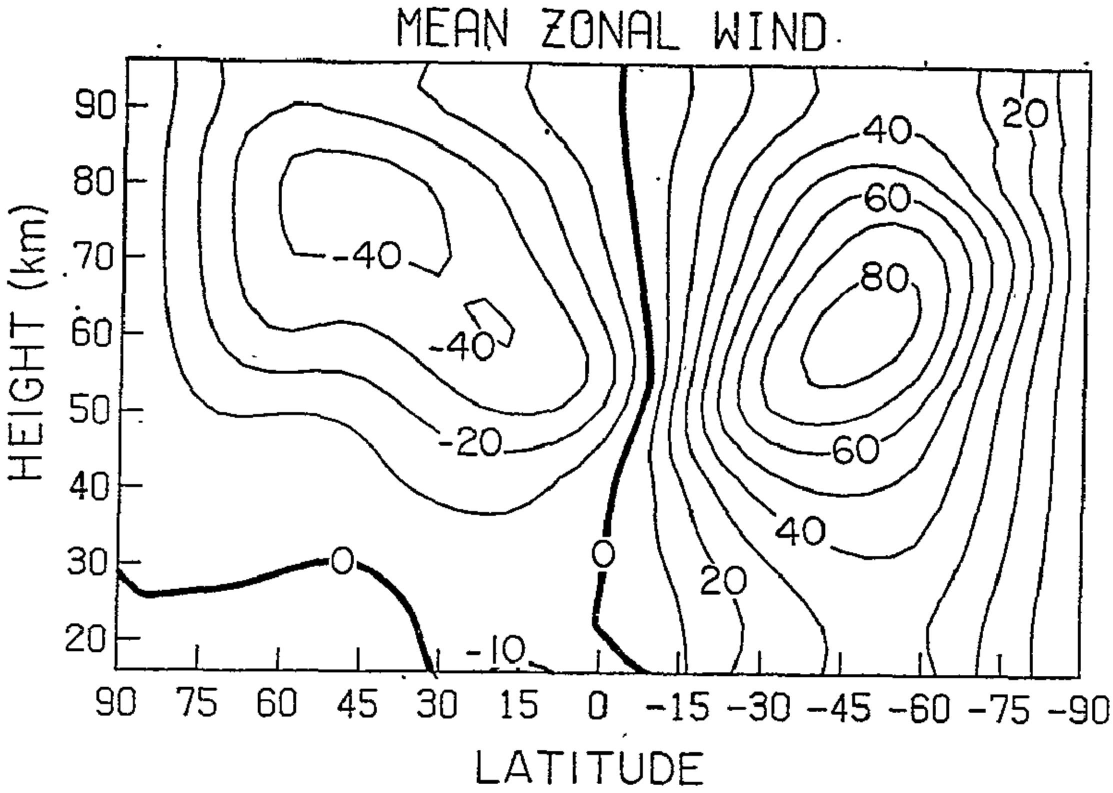

Figure 3: Computed mean zonal winds ($m\ s^{-1}$) for the Southern Hemisphere winter solstice.

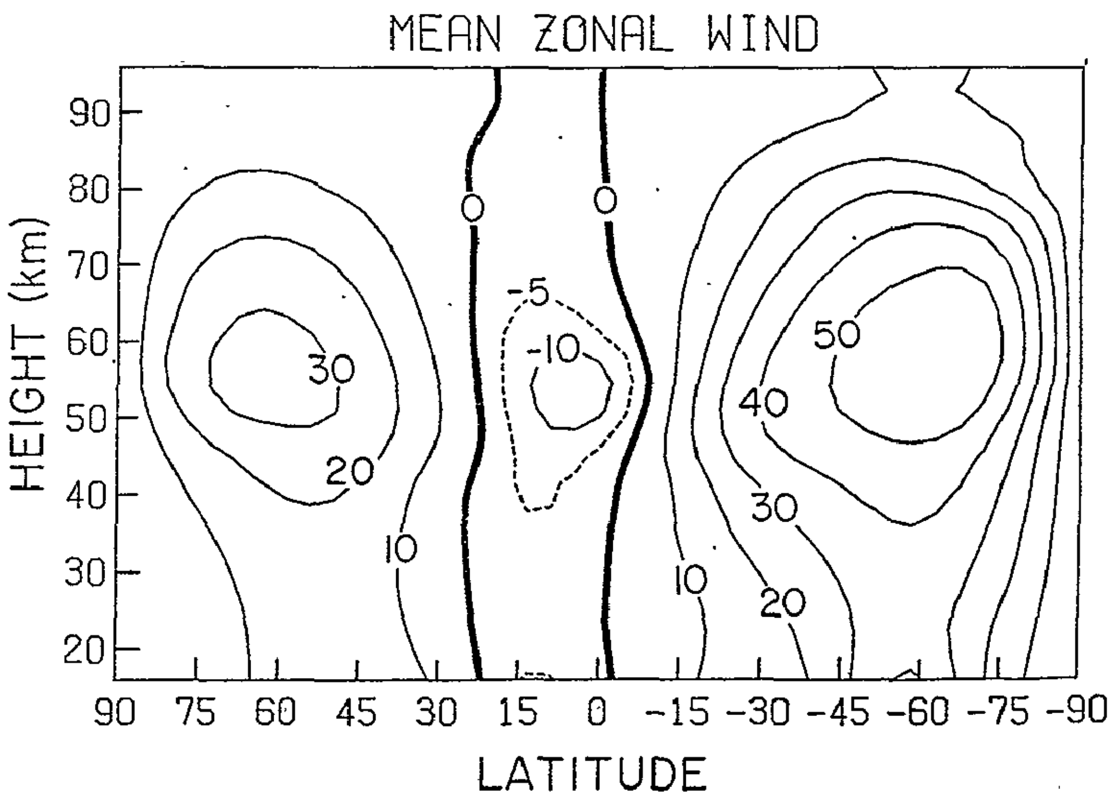

Figure 4: Computed mean zonal winds ($m\ s^{-1}$) for the Southern Hemisphere vernal equinox.

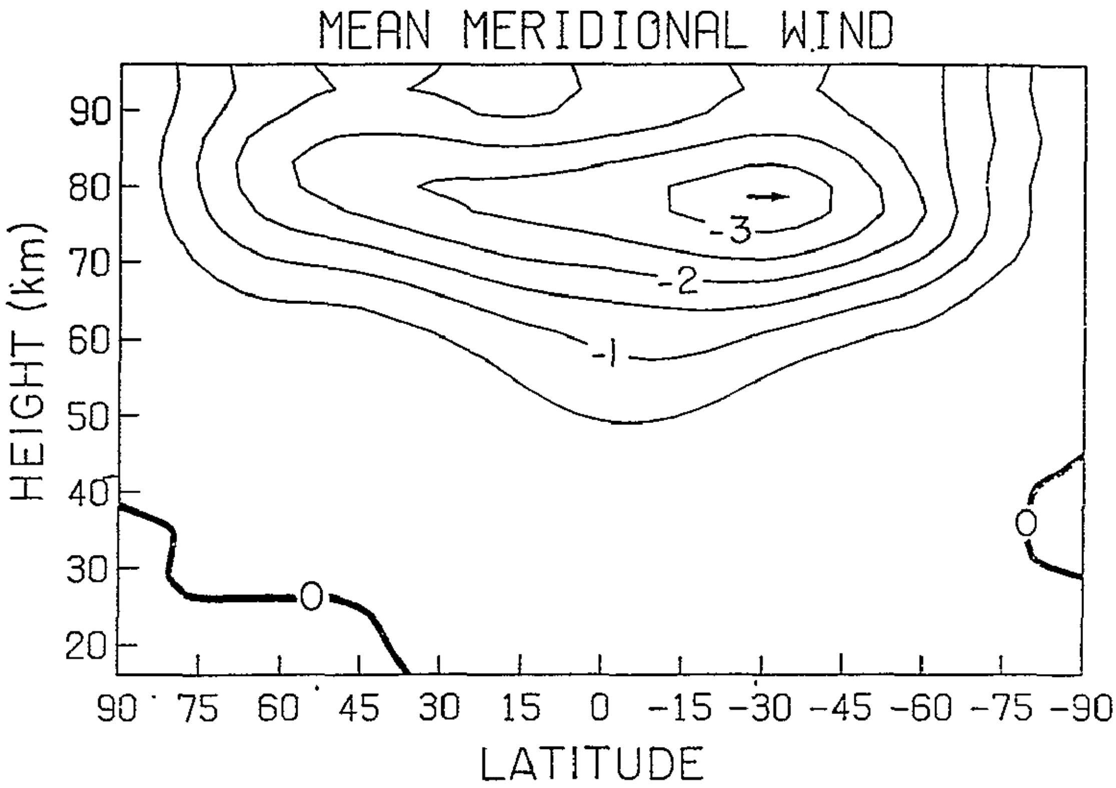

Figure 5: Computed mean meridional wind (m s^{-1}) for the Southern Hemisphere winter solstice.

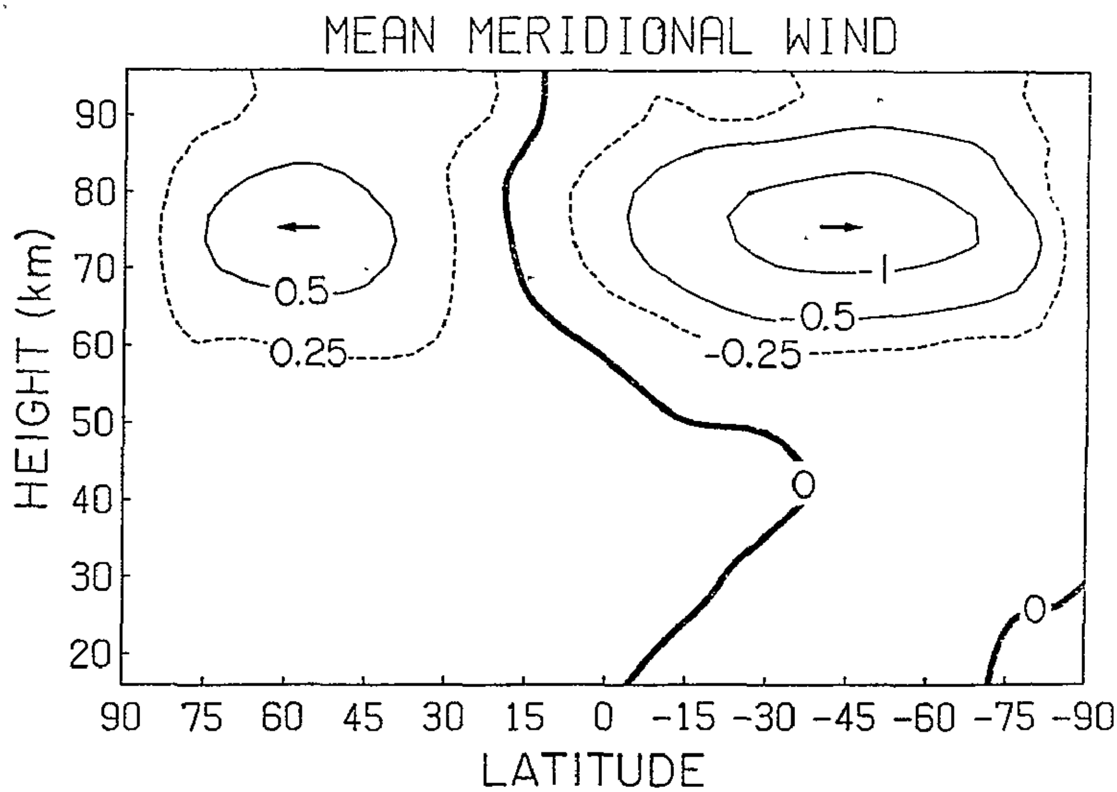

Figure 6: Computed mean meridional wind (m s^{-1}) for the Southern Hemisphere vernal equinox.

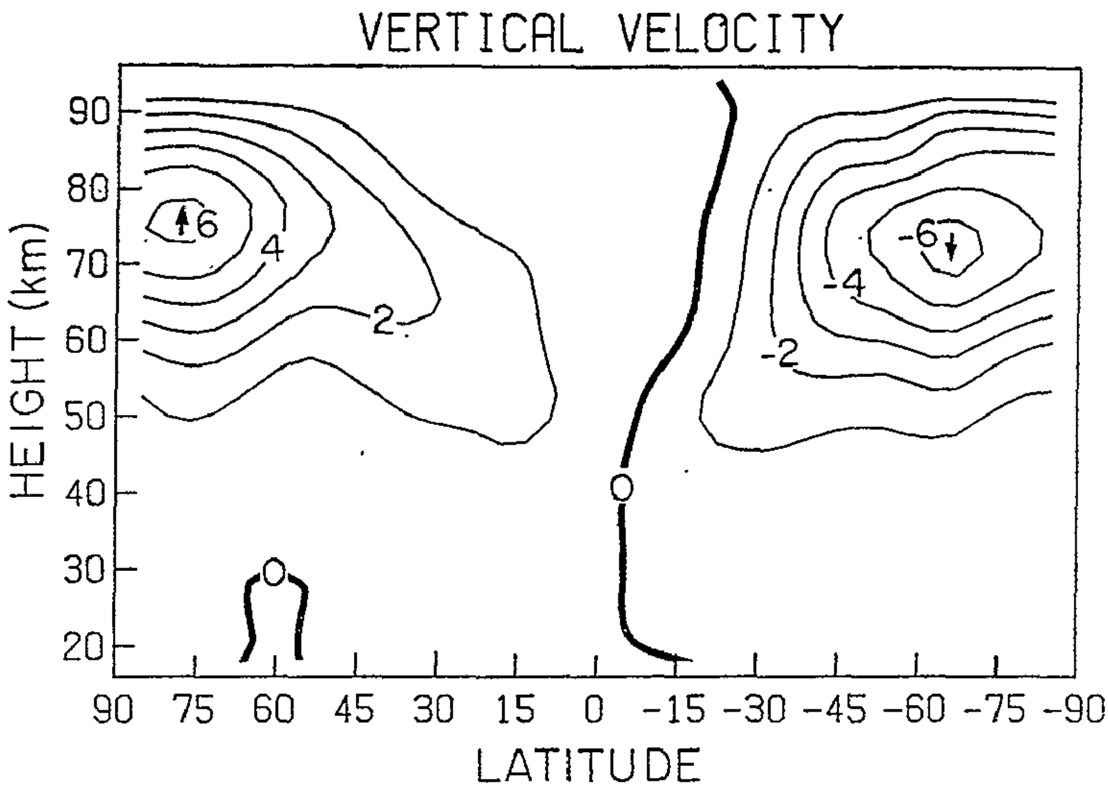

Figure 7: Computed mean vertical velocity (mm s^{-1}) for the Southern Hemisphere winter solstice.

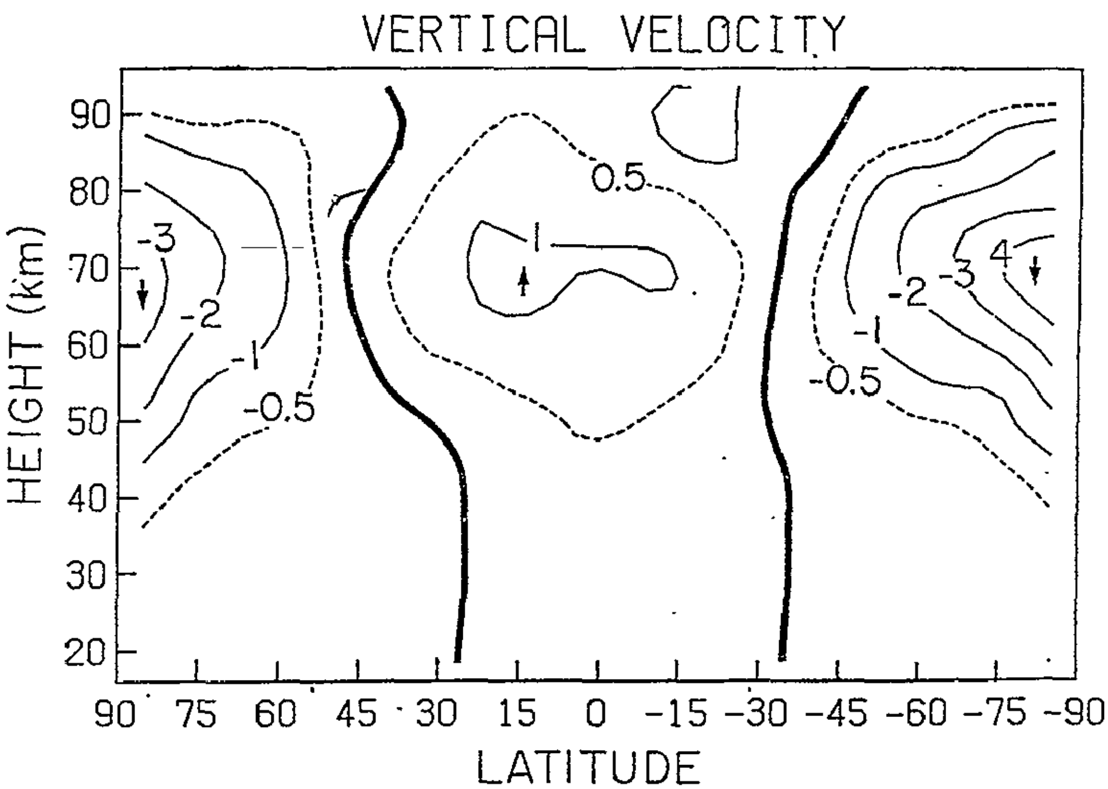

Figure 8: Computed mean vertical velocity (mm s^{-1}) for the Southern Hemisphere vernal equinox.

APPENDIX

FORTRAN CODE FOR THE SEMI-SPECTRAL MODEL

The present version of the model computes the interaction of a single wave mode with the zonal mean flow. The program consists of the main program, PROGRAM WAVE2, in which the fields are initialized and the calling sequence for the various subroutines is established. The main dynamical computations are carried out in SUBROUTINE ASTREAM (mean flow equations) and SUBROUTINE EDDY (wave equations). The radiative heating calculations are carried out in SUBROUTINE HEAT, SUBROUTINE RADEQU, FUNCTION DELT, and FUNCTION OZUV. The output fields are created in SUBROUTINE AOUT. All of the above routines are given in the following program listing. In addition the program requires the NCAR library routines SUBROUTINE BLKTRI and SUBROUTINE CBLKTRI.

In order to facilitate reading of the code a dictionary of the principal FORTRAN sysmbols is provided below. In addition to defining the symbols, we have indicated the location in the program where each symbol first appears.

Dictionary of FORTRAN Symbols

FORTRAN Symbol	Definition	Location
A	α, Newtonian cooling coefficient	C 24
AIRDEN	air density	C 36
ALA(J)	$\Delta t/\Delta y\ (2\Omega \sin\theta)$	A 71
ALB(J)	$\Delta t(2\Omega \sin\theta)\gamma_{j+1/2}$	A 72
AM(J)	A_j coefficient in (7.2)	A 103
AN(K-1)	Coefficient of $\hat{\hat{\Psi}}_{k-1}$ in (7.2)	A 175
ANG(J)	$s\ \overline{U}/(a \cos\theta^*)$	G 74
AR(K)	Coefficient of 1st term in (7.9)	A 180
AS	$\overline{A}$ (6.10a)	F 106
AZEN	Magnification factor	C 38
BLKTRI	Subroutine to solve (7.2)	F 139
BM(J)	B_j Coefficient in (7.2)	A 106
BN(K-1)	Coefficient of 2nd term in (7.2)	A 177
BR(K)	Coefficient of 2nd term in (7.9)	A 182
BS	$\overline{B}$ (6.10b)	F 113
BVF	N^2 (buoyancy frequency)	A 29
B1(J)	$E_{s,j}$ coefficient in (7.9)	A 188
CCO2	Infrared heating	C 25
CDUM(J,K)	Dummy Array	G 42
CG(J,K)	(unused)	A 42
CHI(J,K)	$\overline{R}_{j,k}$ (6.12)	F 61
CI	$(-1)^{1/2}$	A 52
CLOUD	fractional cloudiness	C 41

FORTRAN Symbol	Definition	Location
CM(J)	C_j coefficient in (7.2)	A 105
CN(K-1)	Coefficient of $\hat{\bar{\Psi}}_{k+1}$ in (7.2)	A 176
CNA(J)	$N^2\Delta z^2\Delta t/(2\Delta y \cos\theta)$	A 77
CNB(J)	$\Delta t/(2\Delta y \cos\theta)$	A 68
COR(J)	$2\Omega \sin\theta$	A 65
CORIOL(J)	$2\Omega \sin\theta^*$	A 66
COZONE	(unused)	C 8
CR(K)	Coefficient of 3rd term in (7.9)	A 180
CS(J)	$\cos\theta$	A 63
CSA(J)	$\cos\theta^*$	A 64
CTS(J,K)	(unused)	A 42
CUB(J)	$\gamma \cos\theta^*$	A 78
CURTIS(J,K)	(unused)	A 37
C1(J)	$F_{s,j}$ coefficient in (7.9)	A 187
DCA(K)	$\alpha_k\Delta t/2$	A 227
DCOS(J)	$1./(a \cos\theta^*)$	A 87
DELAY	forcing switch-on time delay	A 229
DEL(J)	$N^2\Delta t\Delta z^2/\cos\theta^*$	A 85
DELT	Δt	A 30
DELTA	solar declination	B 24
DENS(K)	$\exp(z/2H)$	A 94
DKY	$K/\Delta y^4$	A 228
DR(J)	$\overline{B} - \overline{A}f\Delta t/2$	F 130
DT	$\Delta t/2$	A 31

FORTRAN Symbol	Definition	Location
DTODY	$\Delta t/(4\Delta y)$	A 57
DUM(J)	dummy	I 18
DY	Δy	A 51
DY2	$(\Delta y)^2$	A 54
DZ	Δz	A 29
DZ2	Δz^2	A 53
DZN	$N^2\Delta z^2\Delta t^2/(4\Delta y^2 \cos\theta)$	A 103
EC	Orbital eccentricity of the earth	B 21
EMI	$\exp(-\Delta z/4H)$	A 59
EPL	$\exp(+ z/4H)$	A 58
FDAY	fraction of day that sun shines	B 49
FM(J,K)	F_M (7.14)	H 38
FREQ	Ω	A 55
FT(J,K)	F_T (7.15)	H 22
GAM(J)	$\gamma_j + \frac{1}{2}$	A 67
GAMB(J)	γ	A 76
GBV(K)	Γ_k	A 95
GMEP(J)	vertical advection of $\overline{T}$	F 76
GM1(J)	$.5 i s \Delta t/(a \cos\theta)$	A 89
GTIME	growth time for forcing	A 231
ICLOUD	altitude layer of clouds	C 43
ICT	Index for forward difference	A 33
IEND	Total time steps for run	A 45
IFD	Frequency of forward steps	A 32
IFLG	IFLG = 0 initializes BLKTRI	A 29

FORTRAN Symbol	Definition	Location
IMAT	Index for output	A 111
INIT	Input Flag $\neq 0$ for TAPE 1 input on continuation runs	A 45
IPHAS(J)	Dummy	I 89
IPRINT	Frequency of output	A 45
IRAD	Index for calls to HEAT	A 99
IRCT	Frequency of calls to HEAT	A 100
ITIME	Index for time step n	A 110
JM	J_m	A 29
JML	$J_m - 1$	A 47
KAP(K)	α_k, Newtonian cooling	A 97
KN	K_N	A 29
KNL	$K_N - 1$	A 48
KZ	Diffusion Coefficient	A 34
M	$J_m - 2$	A 49
N	$K_N - 2$	A 50
NGC(J)	$\gamma_{j+\frac{1}{2}} \cos \theta$	A 81
NU1(J)	$.5i\ s(\gamma \cos \theta)/(a \cos \theta)$	A 83
OZUV	Solar energy absorbed by ozone	D 1
P(J)	$p_{s,j}$	G 140
PB(J,K)	$\overline{\Psi}^{n+1}$	A 159
PBA(J,K)	Dummy array	A 138
PBO(J,K)	$\overline{\Psi}^{n}$	A 160
PERH	Date of herhelion after Vernal Equinox	B 18
PHB1(J)	Amplitude of boundary forcing for wave	A 69

FORTRAN Symbol	Definition	Location
PHI	Latitude	B 30
PI	π	A 46
PL1(J)	$\hat{\Psi}_s(k=1)$	A 234
PRS(J,K)	Contains Fourier Coefficients of $\overline{U}_B(y,t)$ on input	A 39
P1(J,K)	Ψ_s^{n+1}	A 243
P1A(J,K)	Dummy array	A 239
P10(J,K)	Ψ_s^{n}	A 243
Q(J)	$q_{s,j}$	G 141
QA(K)	Globally averaged net radiative heating	A 259
QB(J,K)	$\overline{Q}$, diabatic heating	B 93
QDOT( )	Net radiative heating function	B 89
QO(J,K), QOG(J)	Ozone density/Lochschmidt's number	A 40
QOZS(J,K), QOSZG(J)	Ozone column abundance	A 41
QR	Radiative cooling of reference atmosphere	C 23
QRS(K)	Globally averaged solar heating	A 115
QS(J,K)	Q_s, diabatic heating in wave	A 195
R(J,K)	$T_{s,j,k}$ (on input)	G 143
RAB	Effective albedo of lower atmosphere	C 49
RAD	a (radius of earth)	A 29
RAYF(K)	κ_R, Rayleigh friction	A 225
RB1	Albedo of reflecting region	C 51
RDB	Spherical albedo of reflecting region	C 50
RED(6,K)	Newtonian cooling parameters	A 43
RG	Ground reflectivity	C 42

FORTRAN Symbol	Definition	Location
RHO	Distance to sun in A.U.	B 22
RHZ	H/287.1	A 56
RR(J,K)	$D_{j,k}$ (on input)	F 135
S	Planetary Wavenumber	A 26
SA	$\theta*$	A 62
SB	θ	A 61
SH	H	A 29
SO3	Ozone UV heating	C 63
STAB(J)	$N^2\Delta t^2\Delta z^2/\cos\theta*$	A 86
STRDAY	Starting day (vernal equinox = 80)	A 36
SU(J)	$2\Omega \sin\theta\ \Delta t^2\gamma/\Delta y$	A 80
SV(J)	$\gamma\Delta t/\Delta y$	A 79
SZT(J,K)	Ozone UV heating field	B 67
T(J,K), TG(J)	Standard Atmosphere temperatures in each zone	A 38
TAU	Optical depth of clouds in visible	C 40
TB(J,K)	$\overline{T}$, deviation of zonally averaged temperature from global mean	B 76
TEMP	Dummy	A 147
TIME	$t = n\Delta t$	A 113
THETA	Polar angle	B 59
TN(J)	$(4\Delta y)^{-1}(\cos\theta^*_{j-1}/\cos\theta^*_j - \cos\theta^*_j/\cos\theta^*_{j-1})$	A 91
TNA(J)	$(\cos\theta^*_j/\cos\theta_j - \cos\theta_j/\cos\theta^*_j)/\Delta y$	A 82
TNB(J)	$(\cos\theta_j/\cos\theta^*_{j+1} - \cos\theta^*_{j+1}/\cos\theta_j)/\Delta y$	A 90
TR	Reference temperature profile	C 22
TSTAR	Local time of sunset	B 40

FORTRAN Symbol	Definition	Location
TU1(J)	$i\,\gamma\,s\,\Delta t/(2a\cos\theta)$	A 74
TV1(J)	$i\,\gamma\,s\,\Delta t^2(2\Omega\sin\theta)/2a\cos\theta)$	A 75
TZO(K)	Global radiative equilibrium temperature	B 61
T1	μ_1	F 25
T2	μ_2	F 26
T3	β_2/β_1	F 27
T4	β_3/β_1	F 28
T5	$1/\beta_1$	F 29
UB(J,K)	$\overline{U}^{n+1}$	A 127
UBO(J,K)	$\overline{U}^{n}$	A 128
UT	ozone path	C 57
U1(J,K)	U_s^{n+1}	A 243
U1O(J,K)	U_s^{n}	A 244
VB(J,K)	$\overline{V}^{n+1}$	A 250
VBO(J,K)	$\overline{V}^{n}$	A 250
V1(J,K)	V_s^{n+1}	A 244
V1O(J,K)	V_s^{n}	A 244
WB(J,K)	$\overline{W}^{n+1}$	A 250
WBO(J,K)	$\overline{W}^{n}$	A 250
WRA(I)	Work Array	A 249
WRØ(I)	Work Array in BLKTRI	A 244
W1(J,K)	W_s^{n+1}	A 244
XBA(J,K)	$\overline{X}$	A 250
XMA1(J)	$s/(a\cos\theta^*)$	A 88

FORTRAN Symbol	Definition	Location
XM1(J)	$s/(a \cos \theta)$	A 73
Z(K)	z	A 93
ZEN	Average value of cos(solar zenith angle)	B 37
ZT	$z + \Delta z/2$	I 31

```
C     $$$$$$$$$$$$$ WAVE ZONAL FLOW INTERACTION $$$$$$$$$$$$$$
      PROGRAM WAVE2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE2,
     $TAPE3)
      COMPLEX U1(19,17),U10(19,17),V1(19,17),V10(19,17),P1(19,17),P10(19
     $,17),W1(19,16),W10(19,16),P1A(19,17),B1(17),R(17,15),P(19),Q(19),
     $NU1(19),CI,GM1(19),TU1(19),TV1(19),CDUM(19,16),PL1(19),QS(19,16)
      COMPLEX Y2,PHBK(19)
      REAL UB(19,17),UBO(19,17),VB(19,17),VBO(19,17),PB(19,17),PBO(19,17
     $),WB(19,17),PBA(19,17),WRO(400),AN(15),BN(15),CN(15),CS(19),KAP(17
     $),Z(17),CSA(19),DCA(17),DR(19),CORIOL(19),GAMB(19),FM(19,17),FT(19
     $,17),QB(19,17),DENS(17),CNA(19),CNB(19),CUB(19),TN(19),DCOS(19),
     $RAYF(17),CHI(19,17),RR(18,15),AM(18),BM(18),CM(18),GMEP(19),XBA(19
     $,17),WBO(19,17),GBV(17),DUM(19)
      REAL PHB1(19),XM1(19),DEL(19),STAB(19),ALA(19),ALB(19),TNA(19),SU
     $(19),SV(19),GAM(19),COR(19),XMA1(19),NGC(19),A1(17),C1(17),ANG(19)
     $,TNB(19),WRA(400),AR(15),BR(15),CR(15),TB(19,17)
      REAL KZ,TZO(17)
      INTEGER IPHAS(19),S,SYM
      COMMON UB,PB,U1,U10,V1,V10,P1,P10,W1,UBO,VBO,PBO,WB,QB
      COMMON /BUMLEG/ CURTIS(13,19),TG(18),T(18,19),PRSG(18),PRS(18,19),
     $QOG(18),QO(18,19),QOZSG(18),QOZS(18,19),CTS(18,19),CG(18,19),ZN(18
     $),AZN(18),FDY(18),TQ(18,16),CZT(19,17),SZT(19,17),CDT(19,17),RED(6
     $,16),QRS(16)
      DIMENSION QA(17)
      EQUIVALENCE (XBA,PBA), (TB,PBA)
      S = 1
C     SYM =0 FOR GLOBAL DOMAIN, SYM =1 FOR ANTISYMMETRIC HEMISPHERE
      SYM = 0.
      DATA JM,KN,IFLG/19,17,0/,RAD,DZ,BVF,SH/6.37E6,5000.,4.E-4,7.0E3/
      DELT = 1800.*2.
      DT = DELT/2.
      IFD = 48
      ICT = IFD-1
      KZ = 0.
C     SETUP CONSTANTS AND INITIALIZE
      STRDAY = 80.
      READ (2,190) CURTIS
      READ (2,200) TG,T
      READ (2,180) ((PRS(I,J),I=1,4),J=1,19)
      READ (2,195) QOG,QO
      READ (2,195) QOZSG,QOZS
      READ (2,195) CTS,CG
      READ (2,205) RED
      WRITE (6,185) ((RED(I,J),I=1,6),J=1,16)
      DATA INIT,IEND,IPRINT/1,1,1/
      PI = 2.*ASIN(1.)
      JML = JM-1
      KNL = KN-1
      M = JM-2
      N = KN-2
      DY = PI*RAD/JML
      CI = (0.,1.)
      DZ2 = DZ**2
      DY2 = DY**2
      FREQ = 7.292E-5
      RHZ = SH/287.1
      DTODY = DT/(4.*DY)
      EPL = EXP(DZ/(4.*SH))
      EMI = EXP(-DZ/(4.*SH))
      DO 10 J=1,JM
        SB = PI*(JM-2.*J)/(2.*JML)
        SA = PI*(JM+1-2.*J)/(2.*JML)
        CS(J) = COS(SB)
        CSA(J) = COS(SA)
        COR(J) = 2.*FREQ*SIN(SB)
        CORIOL(J) = 2.*FREQ*SIN(SA)
        GAM(J) = 1./(1.+COR(J)**2*DT**2)
        CNB(J) = 1./(DY*CS(J))
        PHB1(J) = 150.*(SIN(3.*(SA-PI/6)))**2*9.8
        IF (SA.GT.-PI/6.) PHB1(J) = 0.
        ALA(J) = DT/DY*COR(J)
        ALB(J) = COR(J)*DT*GAM(J)
        XM1(J) = S/(RAD*CS(J))
        TU1(J) = GAM(J)*CI*XM1(J)*DT/2.
        TV1(J) = TU1(J)*COR(J)*DT
        GAMB(J) = 1./(1.+CORIOL(J)**2*DT**2)
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      CNA(J) = BVF*DZ2*DT/(DY*CS(J))
      CUB(J) = GAMB(J)*CSA(J)
      SV(J) = GAM(J)*DT/DY
      SU(J) = COR(J)*DT*SV(J)
      NGC(J) = GAM(J)*CS(J)
      TNA(J) = (CSA(J)/CS(J)-CS(J)/CSA(J))/DY
 10   NU1(J) = CI*.5*XM1(J)*GAM(J)*CS(J)
      DO 15 J=2,JM
        DEL(J) = BVF*DT*DZ2/CSA(J)
        STAB(J) = DEL(J)*DT
        DCOS(J) = 1./(RAD*CSA(J))
        XMA1(J) = S/(RAD*CSA(J))
        GM1(J) = .5*CI*XMA1(J)*DT
        TNB(J-1) = (CS(J-1)/CSA(J)-CSA(J)/CS(J-1))/DY
 15   TN(J) = 1./(4.*DY)*(CSA(J-1)/CSA(J)-CSA(J)/CSA(J-1))
      DO 20 K=1,KN
        Z(K) = (K-1)*DZ
        DENS(K) = EXP(Z(K)/(2.*SH))
        GBV(K) = 1.
        IF (K.GT.KNL) GO TO 20
        KAP(K) = (RED(5,K)/86400.)*DT
 20   CONTINUE
      IRAD = 0
      IRCT = 1
      WRITE (6,195) (KAP(K),K=1,KN)
      DO 25 J=1,JML
        DZN = BVF*DZ2/(CS(J)*DY2)*DT**2
        AM(J) = DZN*GAMB(J)*CSA(J)
        CM(J) = DZN*GAMB(J+1)*CSA(J+1)
 25   BM(J) = -AM(J)-CM(J)
      BM(1) = BM(1)+AM(1)
      BM(JML) = BM(JML)+CM(JML)
      AM(1) = CM(JML) = (0.,0.)
      ITIME = 0
      IMAT = 0
      MT = 1.
      TIME = 0.
      IF (INIT.NE.0) GO TO 90
      QRS(1) = -101.
C     READ INITIAL ZONAL FLOW
      DO 30 J=1,JM
        Y = 2.*PI*STRDAY/360.
        RZ = PRS(1,J)
        R1 = PRS(2,J)
        S1 = PRS(3,J)
        R2 = PRS(4,J)
        UBO(J,1) = RZ/2.+R1*COS(Y)+S1*SIN(Y)+R2*COS(2.*Y)
 30   CONTINUE
      DO 40 J=1,JM
        DO 35 K=1,KN
          UBO(J,K) = UBO(J,1)
          UB(J,K) = UBO(J,K)
 35     CONTINUE
 40   CONTINUE
C     COMPUTE INITIAL GEOPOTENTIAL
      DO 55 K=1,KN
        DO 45 J=1,JM
          UBO(J,K) = UBO(J,K)/DENS(K)
 45     CONTINUE
        PBA(1,K) = 0.
        DO 50 J=2,JML
          PBA(J,K) = PBA(J-1,K)+DY*(CORIOL(J)*UB(J,K)-UB(J,K)*(UB(J-1,K)
     $    *TN(J)+UB(J+1,K)*TN(J+1)))/DENS(K)
 50     CONTINUE
 55   CONTINUE
      DO 60 K=1,KN
        PBA(JM,K) = PBA(JML,K)
 60   CONTINUE
      DO 75 K=1,KN
        SUM1 = 0.
        TEMP = 0.
        DO 65 J=1,JML
          TEMP = TEMP+CS(J)
          SUM1 = SUM1+PBA(J,K)*CS(J)
 65     CONTINUE
        DO 70 J=1,JML
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          PBA(J,K) = PBA(J,K)-SUM1/TEMP
   70   CONTINUE
        PBA(JM,K) = PBA(JML,K)
   75 CONTINUE
      DO 85 J=1,JM
        DO 80 K=1,KN
          PB(J,K) = PBA(J,K)
          PBO(J,K) = PBA(J,K)
   80   CONTINUE
   85 CONTINUE
      GO TO 95
C
   90 CONTINUE
C     FOR CONTINUATION RUNS READ INPUT DATA
      READ (1) TIME,PI,PB,PIO,U1,UB,U1O,V1,V1O,W1,PBO,UBO,VB,VBO,WB,WBO,
     $GBV,TZO,ZN,AZN,FDY,TQ,QRS,SZT
   95 CONTINUE
C     GLOBAL MEAN STABILITY PROFILE AND INITIAL HEATING
      QA(1) = 101.
      CALL HEAT (TIME,GBV,PB,TB,QB,RHZ,DENS,DZ,JM,KN,TZO,BVF,SH,PI,QA,
     $STRDAY)
      DO 100 K=1,N
        AN(K) = GBV(K)
        CN(K) = GBV(K+1)
        BN(K) = -GBV(K)*EPL**2-GBV(K+1)*EMI**2
  100 CONTINUE
      DO 105 K=1,N
        AR(K) = AN(K)
        CR(K) = CN(K)
  105 BR(K) = BN(K)
      BR(N) = BR(N)+EMI/EPL*GBV(KNL)
      AR(1) = CR(N) = 0.
      DO 110 J=2,JML
        A1(J-1) = STAB(J)*(NGC(J-1)/DY2-XM1(J-1)**2*NGC(J-1)/4.)
        C1(J-1) = STAB(J)*(NGC(J)/DY2-XM1(J)**2*NGC(J)/4.)
        B1(J-1) = -STAB(J)*((XM1(J-1)**2*NGC(J-1)+XM1(J)**2*NGC(J))/4.+
     $  (NGC(J-1)+NGC(J))/DY2+CI*(NGC(J-1)*XM1(J-1)*ALA(J-1)-NGC(J)*XM1
     $  (J)*ALA(J)))
  110 CONTINUE
C     COMPUTE EDDY NONNEWTONIAN HEATING
  115 DO 120 J=1,JM
        DO 120 K=1,KNL
  120 QS(J,K) = (0.,0.)
      DAY = TIME/(24.*60.*60.)
      DO 125 J=1,JM
        Y = 2.*PI*(STRDAY+DAY+DELT/86400.)/360.
        RZ = PRS(1,J)
        R1 = PRS(2,J)
        S1 = PRS(3,J)
        R2 = PRS(4,J)
        PRS(5,J) = RZ/2.+R1*COS(Y)+S1*SIN(Y)+R2*COS(2.*Y)
  125 CONTINUE
      PRS(6,1) = 0.
      DO 130 J=2,JML
        PRS(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PRS(5,J)-PRS(5,J)*(PRS(5,J-1)
     $  *TN(J)+PRS(5,J+1)*TN(J+1))
  130 CONTINUE
      PRS(6,JM) = PRS(6,JML)
      SUM = 0.
      TEMP = 0.
      DO 135 J=1,JML
        TEMP = TEMP+CS(J)
        SUM = SUM+PRS(6,J)*CS(J)
  135 CONTINUE
      DO 140 J=1,JML
        PRS(6,J) = PRS(6,J)-SUM/TEMP
  140 CONTINUE
      PRS(6,JM) = PRS(6,JML)
      DO 145 J=1,JM
        PRS(7,J) = (PRS(6,J)+2.*PB(J,1)+PBO(J,1))/4.
  145 CONTINUE
      DO 150 K=1,KN
        RAYF(K) = DT*(0.5/(960.*3600.)+(1.+TANH((Z(K)-55000.)/10000.))/
     $  (96.*3600.))*(1.-EXP(-.4E-05*TIME))
  150 DCA(K) = KAP(K)*(1.-EXP(-(TIME+DELT)*.4E-5))
      DKY = 1.E-6*(1.-EXP(-.4E-5*TIME))
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```
      DELAY = 0.
      DELTIM = TIME-DELAY
      GTIME = 4.32E+05
      IF (DELTIM.LT.0.) GO TO 160
      DO 155 J=2,JM
        PL1(J) = (PHB1(J)*(1.-EXP(-(DELTIM+DELT)/GTIME))+2.*P1(J,1)+P10
     $  (J,1))/4.
  155 CONTINUE
  160 CONTINUE
      DO 165 K=1,KNL
        P1A(1,K) = (0.,0.)
  165 P1A(JM,K) = (0.,0.)
      CALL EDDY (A1,B1,C1,ALB,BVF,ANG,CI,CS,CSA,COR,DCA,DEL,DENS,DKY,
     $DTODY,DT,DZ,DY,EMI,EPL,GAM,GM1,ICT,IFLG,IFD,JM,KNL,M,S,NGC,NU1,
     $RAYF,PL1,P,Q,SJ,SV,TU1,TV1,TNA,TNB,XM1,XMA1,PB,P1A,P1,P10,R,UB,U1,
     $U10,V1,V10,W1,WRO,CDUM,AN,BN,CN,GBV,SYM,QS,KZ)
      CALL FLUX (JM,KN,DY,DZ,CSA,CS,DENS,EPL,EMI,U1,V1,W1,P1,P1A,FM,FT,
     $KZ,UB0,CDUM,CNB)
      IFLG = IFLG-2
      ICT = ICT-1
      CALL ASTREAM (AM,BM,CM,AR,BR,CR,IFLG,IFD,ICT,DT,DZ,DY,CHI,QB,WRA,
     $RR,XBA,UB,UBO,VB,VBO,WB,WBO,PB,PBO,PBA,CS,CSA,FM,FT,EPL,EMI,RAYF,
     $DCA,DKY,TN,DR,DENS,CNB,GMEP,CORIOL,CNA,CUB,BVF,JM,JML,KN,N,GBV)
      IMAT = IMAT+1
      ITIME = ITIME+1
      TIME = TIME+DELT
      IF (ICT.EQ.IFD) ICT = 0
      IRAD = IRAD+1
      IF (IRAD.LT.IRCT) GO TO 170
      QA(1) = 0.
      CALL HEAT (TIME,GBV,PB,TB,QB,RHZ,DENS,DZ,JM,KN,TZO,BVF,SH,PI,QA,
     $STRDAY)
      IRAD = 0
  170 CONTINUE
      IF (IMAT.EQ.IPRINT) GO TO 175
      GO TO 115
C
  175 CONTINUE
      CALL AOUT (TIME,DT,KN,JM,DUM,DENS,IPHAS,Z,JML,DZ,RHZ,COR,DY,CI,XM1
     $,IEND,ITIME,MT,IMAT,P1,PB,U1,UB,V1,VB,W1,WB,FM,FT,S,QS,CS,QB,CZT,
     $SZT,COT,TQ)
      REWIND 1
      WRITE (1) TIME,P1,PB,P10,U1,UB,U10,V1,V10,W1,PBO,UBO,VB,VBO,WB,WBO
     $,GBV,TZO,ZN,AZN,FDY,TQ,QRS,SZT
      END FILE 1
      END FILE 3
      IF (ITIME.LT.IEND) GO TO 115
      STOP
C
C
  180 FORMAT (4F15.4)
  185 FORMAT (5X,6F14.7)
  190 FORMAT (6F10.4,/,5F10.4)
  195 FORMAT (9E10.3)
  200 FORMAT (9F10.3)
  205 FORMAT (6F10.0)
      END
```

```
      SUBROUTINE HEAT(TIME,GBV,PB,TB,QB,RHZ,DENS,DZ,JM,KN,TZO,BVF,SH,PI,
     $QA,STRDAY)
      COMMON /BUMLEG/ R(13,19),TG(18),T(18,19),PRSG(18),PRS(18,19),QOG
     $(18),QO(18,19),QOZSG(18),QOZS(18,19),CS(18,19),CG(18,19),ZN(18),
     $AZN(18),FDY(18),TQ(18,16),CZT(19,17),SZT(19,17),CDT(19,17),RED(6,
     $16),QRS(16)
      DIMENSION GBV(KN), PB(JM,KN), TB(JM,KN), QB(JM,KN), DENS(KN), TZO
     $(KN), QA(KN)
      JML = JM-1
      KNL = KN-1
      LBOT = 4
      LTOP = 19
C     COMPUTE SOLAR GEOMETRY FACTORS
      DAY = TIME/(24.*60.*60.)
      IDAY = DAY
      RES = DAY-IDAY
      IF (RES.GT.0.05) GO TO 60
      PERH = (101.21972+180.)*360./360.+80.
      VD = STRDAY+DAY-PERH
      V = VD*2.*PI/360.
      EC = 0.016722
      RHO = (1.-EC*EC)/(1.+EC*COS(V))
C     DELTA=SOLAR DECLINATION
      DELTA = 0.4093198*SIN(2.*PI*(STRDAY+DAY-80.)/360.)
      DO 30 L=1,18
        LL = 19-L
        RL = LL-9
        PHD = RL*10.-5.
C     PHI=TERRESTRIAL LATITUDE
        PHI = PHD*PI/180.
C     TSTAR=TIME OF SUNSET (OR NEGATIVE TIME OF SUNRISE)
C     ZEN=AVERAGE VALUE OF COS(SZA) BETWEEN -TSTAR AND TSTAR
        SUN = TAN(DELTA)*TAN(PHI)
        ISUN = SUN
        IF (ISUN) 10,15,20
 10     TSTAR = 0.
        ZEN = COS(DELTA-PHI)
        GO TO 25
C
 15     TSTAR = (12./PI)*ACOS(-SUN)
        ZEN = SIN(DELTA)*SIN(PHI)+(12./(PI*TSTAR))*COS(DELTA)*COS(PHI)*
     $  SIN(PI*TSTAR/12.)
        GO TO 25
C
 20     TSTAR = 12.
        ZEN = SIN(DELTA)*SIN(PHI)
 25     CONTINUE
        AZEN = 35./SQRT(1224.*ZEN*ZEN+1.)
        FDAY = TSTAR/12.
        ZN(L) = ZEN
        AZN(L) = AZEN
        FDY(L) = FDAY
 30   CONTINUE
C     COMPUTE GLOBAL RADIATIVE EQUILIBRIUM TEMPERATURE
      IF (TIME.GT.0.) GO TO 45
      DO 40 K=1,KNL
        TZO(K) = 0.
        DO 35 J=1,JML
          THETA = 175-(J-1)*10
          THETA = THETA*PI/180.
          TZO(K) = TZO(K)+T(J,K+3)*SIN(THETA)*PI/36.
 35     CONTINUE
 40   CONTINUE
 45   CONTINUE
      DO 55 J=1,JM
        DO 50 K=1,KNL
          SZT(J,K) = SOL(TZO,J,K+3,RHO)
 50     CONTINUE
        SZT(J,KN) = 0.
 55   CONTINUE
      CALL RADEQU (TZO,TB,QB,RHO)
 60   CONTINUE
C     COMPUTE ZONAL MEAN TEMPERATURE (DEVIATION FROM GLOBAL AVERAGE)
      DO 65 J=1,JML
        DO 65 K=1,KNL
 65   TB(J,K) = RHZ*(PB(J,K+1)*DENS(K+1)-PB(J,K)*DENS(K))/DZ
```

```
      IF (TIME.GT.0.) GO TO 75
C     COMPUTE STATIC STABILITY FROM TZO
      N = KNL-1
      DO 70 K=2,N
   70 GBV(K) = BVF*RHZ/(2./7.*TZO(K)/SH+(TZO(K+1)-TZO(K-1))/(2.*DZ))
      GBV(1) = GBV(2)
      GBV(KNL) = GBV(N)
   75 CONTINUE
      IF (QA(1).LT.100.) GO TO 80
      WRITE (6,105) (GBV(I),I=1,KNL)
   80 CONTINUE
C     COMPUTE HEATING RATE
      CALL QDOT (TZO,TB,QB,RHO)
      GTIME = 1.73E+06
      DO 90 K=1,KNL
        DO 85 L=1,JML
          QB(L,K) = QB(L,K)/(86400.*RHZ*DENS(K))*(1.-EXP(-TIME/GTIME))*
     $    DZ
   85   CONTINUE
   90 CONTINUE
      QA(1) = 0.
      QA(15) = 0.
      QA(16) = 0.
      DO 100 M=LBOT,LTOP
        I = M-3
        QA(I) = 0.
        DO 95 L=1,JML
          THETA = 175-(L-1)*10
          THETA = THETA*PI/180.
          QA(I) = QA(I)+QB(L,I)*SIN(THETA)*PI/36.
   95   CONTINUE
  100 CONTINUE
      RETURN
C
C
  105 FORMAT (5X,8E15.3)
      END
```

```
      FUNCTION DELT(TP,L,IM,RHO)
      COMMON /BUMLEG/ R(13,19),TG(18),T(18,19),PRSG(18),PRS(18,19),QOG
     $(18),QO(18,19),QOZSG(18),QOZS(18,19),CS(18,19),CG(18,19),ZN(18),
     $AZN(18),FDY(18),TQ(18,16),CZT(19,17),SZT(19,17),CDT(19,17),RED(6,
     $16),QRS(16)
      DIMENSION TP(17), TD(19)
      I = IM-3
      COZONE = 0.
      CCO2 = 0.
      SO3 = 0.
      IF (I.EQ.16) GO TO 20
      KNL = 16
      DO 10 J=1,KNL
        TD(J+3) = TP(J)
   10 CONTINUE
C     FOR Z .LT 18.5 KM U. S. STANDARD ATMOSPHERE ASSUMED
      DO 15 J=1,3
        TD(J) = T(L,J)
   15 CONTINUE
      SO3 = SZT(L,I)
C     DICKINSON INFRARED COOLING
      TR = RED(3,I)
      QR = QRS(I)
      A = RED(5,I)
      CCO2 = -QR-A*(TD(IM)-TR)
   20 CONTINUE
      CDT(L,I) = CCO2
      CZT(L,I) = COZONE
      DELT = SO3+CCO2+COZONE
      RETURN
      ENTRY SOL
      I = IM-3
      DZ = 5000.
      SH = 7000.
      Z = (I-1)*DZ
      AIRDEN = 1.5391E-04*EXP(-Z/SH)
      ZEN = ZN(L)
      AZEN = AZN(L)
      FDAY = FDY(L)
      TAU = 10.
      CLOUD = 0.446
      RG = 0.5
      ICLOUD = 2
C     HEATING BY THE ABSORPTION OF SOLAR RADIATION BY OZONE
C     FROM LACIS AND HANSEN, J ATMOS SCI 31 118-133
      SO3 = 0.
      IF (FDAY.LT.0.0001) GO TO 25
C     CLEAR SKY
      RAB = 0.219/(1.+0.816*ZEN)
      RDB = 0.144
      RB1 = RAB+(1.-RAB)*(1.-RDB)*RG/(1.-RDB*RG)
      RB = RB1
C     CLOUDY SKY
      RAB = 0.13*TAU/(1.+0.13*TAU)
      RDB = RAB
      RB1 = RAB+(1.-RAB)*(1.-RDB)*RG/(1.-RDB*RG)
      UT = AZEN*QOZS(L,ICLOUD)+1.9*(QOZS(L,ICLOUD)-QOZS(L,IM))
      A1 = OZUV(UT)
      UT = AZEN*QOZSG(L)+1.9*(QOZSG(L)-QOZS(L,IM))
      A2 = OZUV(UT)
      U = AZEN*QOZS(L,IM)
      A3 = OZUV(U)
      SO3 = 76.374*QO(L,IM)*1.E+05*(1./AIRDEN)*ZEN*FDAY*(AZEN*A3+(1.-
     $CLOUD)*RB*1.9*A2+CLOUD*RB1*1.9*A1)
      SO3 = SO3/(220.*2.67*RHO*RHO)
   25 CONTINUE
      DELT = SO3
      RETURN
      END
```

```
      FUNCTION OZUV(U)
C     SUBROUTINE TO CALCULATE SOLAR ENERGY ABSORBED BY OZONE
      F1 = 1.0+(0.042*U)+(3.23E-4*(U**2.0))
      F2 = (0.0212/F1)*(1.0-((U/F1)*(0.042+(6.46E-4*U))))
      F1 = 1.0+(136.6*U)
      F2 = F2+(1.082/(F1**0.805))*(1.0-((138.6*0.805*U)/F1))
      F1 = 1.0+((103.6*U)**3.0)
      OZUV = F2+((0.0658/F1)*(1.0-(3.0*(F1-1.0)/F1)))
      RETURN
      END
```

```
      SUBROUTINE RADEQU(TZO,TB,QB,RHO)
      DIMENSION TZO(17), TP(17), TB(19,17), QB(19,17), A3(2)
      COMMON /BUMLEG/ R(18,19),TG(18),T(18,19),PRSG(18),PRS(18,19),QOG
     $(18),QO(18,19),QOZSG(18),QOZS(18,19),CTS(18,19),CG(18,19),ZN(18),
     $AZN(18),FDY(18),TQ(18,16),CZT(19,17),SZT(19,17),CDT(19,17),RED(6,
     $16),QRS(16)
      DATA LBOT,LTOP,ITOT,DT,EPS/4,19,20,0.1,0.1/
      DATA JML,KNL/18,16/
      PI = ACOS(-1.)
      IF (QRS(1).GT.-100.) GO TO 20
      DO 15 K=LBOT,LTOP
        I = K-3
        QRS(I) = 0.
        DO 10 J=1,JML
          THETA = 175-(J-1)*10
          THETA = THETA*PI/180.
          W = SIN(THETA)*PI/36.
          QRS(I) = QRS(I)+SOL(TZO,J,K,1.)*W
10      CONTINUE
15    CONTINUE
20    CONTINUE
      DO 40 IT=1,ITOT
        SUM = 0.
        LMP = LTOP-1
        DO 35 M=LBOT,LMP
          I = M-3
          TZO(I) = TZO(I)+2.*DT
          DO 30 J=1,2
            TZO(I) = TZO(I)-DT
            A3(J) = 0.
            DO 25 L=1,JML
              THETA = 175-(L-1)*10
              THETA = THETA*PI/180.
              W = SIN(THETA)*PI/36.
              A3(J) = A3(J)+DELT(TZO,L,M,RHO)*W
25          CONTINUE
30        CONTINUE
          DQ = (A3(1)-A3(2))/DT
          DIFF = A3(2)/DQ
          TN = TZO(I)-DIFF
          DTP = DIFF
          IF (DTP.GT.20.) DTP = 20.
          IF (DTP.LT.-20.) DTP = -20.
          TN = TZO(I)-DTP
          DIFF = ABS(DIFF)
          IF (DIFF.GT.SUM) SUM = DIFF
          TZO(I) = TN
35      CONTINUE
        IF (SUM.LT.EPS) GO TO 45
40    CONTINUE
      PRINT 65, ITOT
      STOP 1
45    PRINT 70, IT
      WRITE (6,75) TZO
      RETURN
      ENTRY QDOT
C     COMPUTE RADIATIVE HEATING IN KELVIN PER DAY
      DO 60 L=1,JML
        DO 50 J=1,KNL
          TP(J) = TB(L,J)+TZO(J)
50      CONTINUE
        QB(L,1) = 0.
        DO 55 M=LBOT,LTOP
          I = M-3
          QB(L,I) = DELT(TP,L,M,RHO)
55      CONTINUE
60    CONTINUE
      RETURN
C
C
65    FORMAT (5X,*TEMPERATURE PROFILE FAILED TO CONVERGE AFTER*,I3,
     $* ITERATIONS*)
70    FORMAT (5X,*TEMPERATURE CONVERGED AFTER *,I2,* ITERATIONS*)
75    FORMAT (1X,18F7.2)
      END
```

```
      SUBROUTINE ASTREAM(AM,BM,CM,AN,BN,CN,IFLG,IFD,ICT,DT,DZ,DY,CHI,QB,
     $WRO,RR,XBA,UB,UBO,VB,VBO,WB,WBO,PB,PBO,PBA,CS,CSA,FM,FT,EPL,EMI,
     $RAYF,DCA,DKY,TN,DR,DENS,CNB,GMEP,CORIOL,CNA,CUB,BVF,JM,JML,KN,N,
     $GBV)
C
C
      DIMENSION AM(JML), BM(JML), CM(JML), PBA(JM,KN), CORIOL(JM), QB(JM
     $,KN), DCA(KN), UB(JM,KN), UBO(JM,KN), VB(JM,KN), VBO(JM,KN), PB(JM
     $,KN), PBO(JM,KN), TN(JM), DR(JM), WB(JM,KN), CHI(JM,KN), WRO(400),
     $ RR(JML,15), GMEP(JM), XBA(JM,KN), DENS(KN), CS(JM), CSA(JM), FM
     $(JM,KN), FT(JM,KN), CNA(JM), CNB(JM), CUB(JM), RAYF(KN), WBO(JM,KN
     $), AN(15), BN(15), CN(15), GBV(KN)
      COMMON /BUMLEG/ CURTIS(13,19),TG(18),T(18,19),PRSG(18),PRS(18,19)
      M = JML-1
      KNL = N+1
      KN = N+2
C
C     CHOOSE LEAPFROG OR FORWARD DIFFERENCE
C
      DO 10 J=1,JM
        DO 10 K=2,KN
 10   UB(J,K) = UB(J,K)/DENS(K)
      ICT = ICT+1
      IF (ICT.LT.IFD) GO TO 15
      T1 = 1.
      T2 = 0.
      T3 = 1.
      T4 = 0.
      T5 = 2.
      GO TO 20
C
 15   T1 = T2 = .5
      T3 = 2.
      T4 = 1.
      T5 = 4.
 20   CONTINUE
C
C     UB  SMOOTHING
C
      DO 30 K=2,KNL
        DO 25 J=3,M
 25     FM(J,K) = FM(J,K)-DKY*(UBO(J-2,K)/CSA(J-2)-4.*UBO(J-1,K)/CSA(J-1
     $  )+6.*UBO(J,K)/CSA(J)-4.*UBO(J+1,K)/CSA(J+1)+UBO(J+2,K)/CSA(J+2))
     $  /CSA(J)**2
        FM(3,K) = FM(3,K)-DKY*(UBO(2,K)/CSA(2))/CSA(3)**2
        FM(M,K) = FM(M,K)-DKY*(UBO(JML,K)/CSA(JML))/CSA(M)**2
        FM(2,K) = FM(2,K)-DKY*(2.*UBO(2,K)/CSA(2)-3.*UBO(3,K)/CSA(3)+UBO
     $  (4,K)/CSA(4))/CSA(2)**2
        FM(JML,K) = FM(JML,K)-DKY*(UBO(JML-2,K)/CSA(JML-2)-3.*UBO(JML-1,
     $  K)/CSA(JML-1)+2.*UBO(JML,K)/CSA(JML))/CSA(JML)**2
 30   CONTINUE
C
C     THICKNESS TENDENCY
C
      DO 40 J=1,JM
        DO 35 K=1,KNL
 35     PBA(J,K) = PB(J,K+1)*EPL-PB(J,K)*EMI
 40   PBA(J,KN) = 0.
      DO 65 K=1,KNL
        DO 50 J=1,JML
          CHI(J,K) = T1*PBA(J,K)+T2*(PBO(J,K+1)*EPL-PBO(J,K)*EMI)+DT*(QB
     $    (J,K)+FT(J,K))
          IF (J.EQ.1) GO TO 45
          CHI(J,K) = CHI(J,K)-DT*DENS(K)*EPL/(4.*CS(J)*DY)*((VB(J,K+1)+
     $    VB(J,K))*CSA(J)*(PBA(J-1,K)+PBA(J,K))-(VB(J+1,K+1)+VB(J+1,K))*
     $    CSA(J+1)*(PBA(J,K)+PBA(J+1,K)))
          GO TO 50
C
 45       CHI(J,K) = CHI(J,K)-DT*DENS(K)*EPL/(4.*CS(J)*DY)*((VB(J,K+1)+
     $    VB(J,K))*CSA(J)*(2.*PBA(J,K))-(VB(J+1,K+1)+VB(J+1,K))*CSA(J+1)
     $    *(PBA(J,K)+PBA(J+1,K)))
 50     CONTINUE
        IF (K.EQ.1) GO TO 65
        IF (K.EQ.KNL) GO TO 65
        DO 55 J=1,JML
          GMEP(J) = -DT/(4.*DZ)*EPL*DENS(K)*((WB(J,K+1)+WB(J,K))*(PBA(J,
```

```
     $     K+1)*EPL+PBA(J,K)*EMI)-(WB(J,K)+WB(J,K-1))*(PBA(J,K)*EPL+PBA(J
     $      ,K-1)*EMI))
   55      CONTINUE
           DO 60 J=1,JML
   60      CHI(J,K) = CHI(J,K)+GMEP(J)
   65    CONTINUE
C        THICKNESS SMOOTHING
         DO 70 J=1,JM
           CHI(J,KNL) = 0.
           DO 70 K=1,KNL
   70    PBA(J,K) = PBO(J,K+1)*EPL-PBO(J,K)*EMI
         DO 80 K=1,KNL
           DO 75 J=3,M
   75      CHI(J,K) = CHI(J,K)-DT*DKY*(PBA(J-2,K)-4.*PBA(J-1,K)+6.*PBA(J,K)
     $    -4.*PBA(J+1,K)+PBA(J+2,K))/CS(J)
           CHI(2,K) = CHI(2,K)-DT*DKY*(-3.*PBA(1,K)+6.*PBA(2,K)-4.*PBA(3,K)
     $    +PBA(4,K))/CS(2)
           CHI(1,K) = CHI(1,K)-DT*DKY*(2.*PBA(1,K)-3.*PBA(2,K)+PBA(3,K))/CS
     $    (1)
           CHI(JML,K) = CHI(JML,K)-DT*DKY*(PBA(JML-2,K)-3.*PBA(JML-1,K)+2.*
     $    PBA(JML,K))/CS(JML)
   80    CONTINUE
C
C        UB AND VB TENDENCIES
C
         DR(JM) = 0.
         DR(1) = 0.
         DO 100 K=2,KNL
           DO 95 J=2,JML
             AS = T1*UB(J,K)+T2*UBO(J,K)
             AS = AS+DT*FM(J,K)+DENS(K)*(-DT/(CSA(J)**2*DY*4.)*((UB(J-1,K)*
     $       CSA(J-1)+UB(J,K)*CSA(J))*(VB(J-1,K)*CSA(J-1)+VB(J,K)*CSA(J))-
     $       (UB(J,K)*CSA(J)+UB(J+1,K)*CSA(J+1))*(VB(J,K)*CSA(J)+VB(J+1,K)*
     $       CSA(J+1)))-DT/(4.*DZ*CSA(J))*((UB(J,K)*EMI+UB(J,K+1)*EPL)*(WB
     $       (J-1,K)*CS(J-1)+WB(J,K)*CS(J))-(UB(J,K)*EPL+UB(J,K-1)*EMI)*(WB
     $       (J-1,K-1)*CS(J-1)+WB(J,K-1)*CS(J))))-RAYF(K)*UBO(J,K)
             BS = T1*VB(J,K)+T2*VBO(J,K)
             BS = BS+DT*DENS(K)*UB(J,K)*(UB(J-1,K)*TN(J)+UB(J+1,K)*TN(J+1))
     $     -RAYF(K)*VBO(J,K)
C
C        VB SMOOTHING
C
             IF (J.EQ.2) GO TO 85
             IF (J.EQ.JML) GO TO 90
             BS = BS-DT*DKY*(VBO(J-2,K)-4.*VBO(J-1,K)+6.*VBO(J,K)-4.*VBO(J+
     $       1,K)+VBO(J+2,K))/CSA(J)
             GO TO 95
C
   85        BS = BS-DT*DKY*(+3.*VBO(2,K)-3.*VBO(3,K)+VBO(4,K))/CSA(2)
             GO TO 95
C
   90        BS = BS-DT*DKY*(VBO(JML-2,K)-3.*VBO(JML-1,K)+3.*VBO(JML,K))/
     $       CSA(JML)
   95      DR(J) = BS-CORIOL(J)*DT*AS
C
C        SOLVE ELLIPTIC SYSTEM FOR PB
C
           DO 100 J=1,JML
             RR(J,K-1) = (CHI(J,K)*EMI*GBV(K)-CHI(J,K-1)*EPL*GBV(K-1))+CNA
     $       (J)*(CUB(J)*DR(J)-CUB(J+1)*DR(J+1))
             IF (K.EQ.2) RR(J,K-1) = RR(J,K-1)-PRS(7,J)*GBV(1)
  100    CONTINUE
  105    CALL BLKTRI (IFLG,1,N,AN,BN,CN,1,JML,AM,BM,CM,JML,RR,IER,WRO)
         IFLG = IFLG+1
         IF (IFLG-1) 105,105,110
C
C        COMPUTE MERIDIONAL STREAMFUNCTION
C
  110    DO 115 K=1,KN
           XBA(1,K) = 0.
           XBA(JM,K) = 0.
  115    CONTINUE
         DO 125 J=1,M
           DO 120 K=2,N
  120      XBA(J+1,K) = XBA(J,K)-DY*CS(J)*(CHI(J,K)-(RR(J,K)*EPL-RR(J,K-1)*
     $     EMI))/(BVF*DT*DZ)*GBV(K)
```

```
      XBA(J+1,KNL) = XBA(J,KNL)-DY*CS(J)*(CHI(J,KNL))/(BVF*DT*DZ)*GBV
     $ (KNL)
      XBA(J+1,1) = XBA(J,1)-DY*CS(J)*(CHI(J,1)-(RR(J,1)*EPL-PRS(7,J)*
     $ EMI))/(BVF*DT*DZ)*GBV(1)
      XBA(J+1,KN) = 0.
  125 CONTINUE
C
C     COMPUTE NEW UB, VB, WB, PB
C
      DO 130 K=2,KNL
        DO 130 J=2,JML
          AS = T1*UB(J,K)+T2*UBO(J,K)
          AS = AS+DT*FM(J,K)+DENS(K)*(-DT/(CSA(J)**2*DY*4.)*((UB(J-1,K)*
     $    CSA(J-1)+UB(J,K)*CSA(J))*(VB(J-1,K)*CSA(J-1)+VB(J,K)*CSA(J))-
     $    (UB(J,K)*CSA(J)+UB(J+1,K)*CSA(J+1))*(VB(J,K)*CSA(J)+VB(J+1,K)*
     $    CSA(J+1)))-DT/(4.*DZ*CSA(J))*((UB(J,K)*EMI+UB(J,K+1)*EPL)*(WB
     $    (J-1,K)*CS(J-1)+WB(J,K)*CS(J))-(UB(J,K)*EPL+UB(J,K-1)*EMI)*(WB
     $    (J-1,K-1)*CS(J-1)+WB(J,K-1)*CS(J))))-RAYF(K)*UBO(J,K)
          UBO(J,K) = -UBO(J,K)*T4-UB(J,K)*T3+(AS+CORIOL(J)*DT*(XBA(J,K-1
     $    )*EPL-XBA(J,K)*EMI)/(DZ*CSA(J)))*T5
  130 VBO(J,K) = -VBO(J,K)*T4-T3*VB(J,K)+T5*(XBA(J,K-1)*EPL-XBA(J,K)*EMI
     $)/(DZ*CSA(J))
      DO 135 J=1,JML
        DO 135 K=1,KNL
  135 WBO(J,K) = -WBO(J,K)*T4-T3*WB(J,K)+T5*(XBA(J,K)-XBA(J+1,K))/(DY*CS
     $(J))
      DO 140 J=1,JML
        PBA(J,1) = PRS(7,J)
        UBO(J,1) = PRS(5,J)
        DO 140 K=2,KNL
  140 PBA(J,K) = RR(J,K-1)
      DO 145 K=1,KN
  145 PBA(JM,K) = PBA(JML,K)
      DO 150 J=1,JM
        PBA(J,KN) = PBA(J,KNL)*EMI/EPL
        VBO(J,KN) = VBO(J,KNL)*EMI/EPL
  150 UBO(J,KN) = UBO(J,KNL)*EMI/EPL
      DO 155 J=1,JM
        DO 155 K=1,KN
          CHI(J,K) = T5*PBA(J,K)-T4*PBO(J,K)-T3*PB(J,K)
          PBO(J,K) = PB(J,K)
          PB(J,K) = CHI(J,K)
          CHI(J,K) = WBO(J,K)
          WBO(J,K) = WB(J,K)
          WB(J,K) = CHI(J,K)
          IF (J.EQ.1) GO TO 155
          CHI(J,K) = UBO(J,K)
          UBO(J,K) = UB(J,K)
          UB(J,K) = CHI(J,K)
          CHI(J,K) = VBO(J,K)
          VBO(J,K) = VB(J,K)
          VB(J,K) = CHI(J,K)
  155 UB(J,K) = UB(J,K)*DENS(K)
      RETURN
      END
```

```
      SUBROUTINE EDDY(AA,BB,CC,ALB,BVF,ANG,CI,CS,CSA,COR,DCA,DEL,DENS,
     $DKY,DTODY,DT,DZ,DY,EMI,EPL,GAM,GM,ICT,IFLG,IFD,JM,KNL,M,S,NGC,NU,
     $RAYF,PL1,P,Q,SU,SV,TU,TV,TNA,TNB,XM,XMA,PB,PHIA,PHI,PHIO,R,UB,U,UO
     $,V,VO,W,WRO,CDUM,AN,BN,CN,GBV,SYM,QS,KZ)
      DIMENSION AA(1), BB(1), CC(1), R(M,1), WRO(1), PL1(1), PHIA(JM,1),
     $ PHIO(JM,1), PHI(JM,1), U(JM,1), UO(JM,1), V(JM,1), VO(JM,1), ANG
     $(1), XMA(1), UB(JM,1), CS(1), CSA(1), TNA(1), P(1), Q(1), COR(1),
     $DEL(1), NGC(1), NU(1), DCA(1), GM(1), PB(JM,1), TU(1), SU(1), TV(1
     $), SV(1), ALB(1), DENS(1), GAM(1), RAYF(1), TNB(1), W(JM,1), XM(1)
     $, CDUM(JM,1), AN(1), BN(1), CN(1), GBV(1), QS(JM,1)
      REAL NGC,KZ
      INTEGER S,SYM
      COMPLEX BB,R,PHIA,PHIO,PHI,U,UO,V,VO,CI,P,Q,NU,GM,TU,TV,AV,BV,PL1,
     $W,CDUM,QS
      KN = KNL+1
      N = KNL-1
      JML = M+1
      DO 10 J=1,JM
        DO 10 K=2,KN
   10 UB(J,K) = UB(J,K)/DENS(K)
C     CHOOSE LEAPFROG OR FORWARD DIFFERENCE
      ICT = ICT+1
      IF (ICT.LT.IFD) GO TO 15
      T1 = T3 = 1.
      T2 = T4 = 0.
      T5 = 2.
      GO TO 20
C
   15 T1 = T2 = .5
      T3 = 2.
      T4 = 1.
      T5 = 4.
   20 CONTINUE
      ISW = 1
C     THICKNESS TENDENCY
      DO 30 J=1,JM
        DO 25 K=1,KNL
   25   PHIA(J,K) = PHIO(J,K+1)*EPL-PHIO(J,K)*EMI
   30 PHIA(J,KN) = (0.,0.)
      DO 35 K=1,KNL
        DO 35 J=2,JML
          CDUM(J,K) = (T2-DCA(K))*PHIA(J,K)+(T1-DENS(K)*EPL*GM(J)*(UB(J,
     $    K+1)+UB(J,K)))*(PHI(J,K+1)*EPL-PHI(J,K)*EMI)-DTODY*DENS(K)*EPL
     $    *(V(J-1,K+1)+V(J,K+1)+V(J-1,K)+V(J,K))*((PB(J-1,K+1)-PB(J,K+1)
     $    )*EPL-(PB(J-1,K)-PB(J,K))*EMI)+DT*QS(J,K)
          IF (K.EQ.1) GO TO 35
          CDUM(J,K) = CDUM(J,K)-DT*DENS(K)*EPL/(4.*DZ)*W(J,K)*(((PB(J-1,
     $    K+2)+PB(J,K+2))*EPL-(PB(J-1,K+1)+PB(J,K+1))*EMI)*EPL**2-((PB(J
     $    -1,K)+PB(J,K))*EPL-(PB(J-1,K-1)+PB(J,K-1))*EMI)*EMI**2)
   35 CONTINUE
C     THICKNESS SMOOTHING
      DO 45 J=2,JM
        DO 40 K=3,N
   40   CDUM(J,K) = CDUM(J,K)-DT*KZ*(PHIA(J,K-2)-4.*PHIA(J,K-1)+6.*PHIA
     $  (J,K)-4.*PHIA(J,K+1)+PHIA(J,K+2))
        CDUM(J,2) = CDUM(J,2)-DT*KZ*(-PHIA(J,1)+3.*PHIA(J,2)-3.*PHIA(J,3
     $  )+PHIA(J,4))
        CDUM(J,KNL) = CDUM(J,KNL)-DT*KZ*(PHIA(J,N-1)-3.*PHIA(J,N)+PHIA(J
     $  ,KNL)*3.)
   45 CONTINUE
      DO 55 K=1,KNL
        DO 50 J=3,M
   50   CDUM(J,K) = CDUM(J,K)-DT*DKY*(PHIA(J-2,K)-4.*PHIA(J-1,K)+6.*PHIA
     $  (J,K)-4.*PHIA(J+1,K)+PHIA(J+2,K))/CSA(J)
        CDUM(2,K) = CDUM(2,K)-DT*DKY*(3.*PHIA(2,K)-3.*PHIA(3,K)+PHIA(4,K
     $  ))/CSA(2)
        CDUM(JML,K) = CDUM(JML,K)-DT*DKY*(3.*PHIA(JML,K)-3.*PHIA(JML-1,K
     $  )+PHIA(JML-2,K))/CSA(JML)
   55 CONTINUE
C     MOMENTUM TENDENCY
   60 DO 115 K=1,KNL
        IF ((K.EQ.1).AND.(ISW.EQ.1)) GO TO 115
        DO 110 J=1,JML
          ANG(J) = (XMA(J)*UB(J,K)+XMA(J+1)*UB(J+1,K))/2.
          AV = T1*U(J,K)+(T2-RAYF(K))*UO(J,K)-DT*DENS(K)*(CI*ANG(J)*U(J,
     $    K)+V(J,K)/(CS(J)*DY)*(UB(J,K)*CSA(J)-UB(J+1,K)*CSA(J+1))+(W(J+
```

```
     $      1,K)*(UB(J+1,K+1)*EPL-UB(J+1,K)*EMI)+W(J,K)*(UB(J,K+1)*EPL-UB
     $      (J,K)*EMI))/(4.*DZ))
            IF (K.GT.1) AV = AV-DT*DENS(K)*(W(J+1,K-1)*(UB(J+1,K)*EPL-UB(J
     $      +1,K-1)*EMI)+W(J,K-1)*(UB(J,K)*EPL-UB(J,K-1)*EMI))/(4.*DZ)
            BV = T1*V(J,K)+(T2-RAYF(K))*VO(J,K)-DT*DENS(K)*(CI*ANG(J)*V(J,
     $      K)-U(J,K)*(UB(J,K)*TNA(J)+UB(J+1,K)*TNB(J)))
C     MOMENTUM SMOOTHING
            IF (K.EQ.1) GO TO 75
            IF (K.EQ.2) GO TO 65
            IF (K.EQ.KNL) GO TO 70
            AV = AV-DT*KZ*(UO(J,K-2)-4.*UO(J,K-1)+6.*UO(J,K)-4.*UO(J,K+1)+
     $      UO(J,K+2))
            BV = BV-DT*KZ*(VO(J,K-2)-4.*VO(J,K-1)+6.*VO(J,K)-4.*VO(J,K+1)+
     $      VO(J,K+2))
            GO TO 75
C
 65         AV = AV-DT*KZ*(-UO(J,1)+3.*UO(J,2)-3.*UO(J,3)+UO(J,4))
            BV = BV-DT*KZ*(-VO(J,1)+3.*VO(J,2)-3.*VO(J,3)+VO(J,4))
            GO TO 75
C
 70         AV = AV-DT*KZ*(UO(J,N-1)-3.*UO(J,N)+3.*UO(J,KNL))
            BV = BV-DT*KZ*(VO(J,N-1)-3.*VO(J,N)+3.*VO(J,KNL))
 75         IF (J.EQ.1) GO TO 80
            IF (J.EQ.2) GO TO 85
            IF (J.EQ.JML-1) GO TO 90
            IF (J.EQ.JML) GO TO 95
            AV = AV-DT*DKY*(UO(J-2,K)-4.*UO(J-1,K)+6.*UO(J,K)-4.*UO(J+1,K)
     $      +UO(J+2,K))/CS(J)
            BV = BV-DT*DKY*(VO(J-2,K)-4.*VO(J-1,K)+6.*VO(J,K)-4.*VO(J+1,K)
     $      +VO(J+2,K))/CS(J)
            GO TO 100
C
 80         IF (S.EQ.1) CF = 2.$ IF (S.NE.1) CF = 4.
            AV = AV-DT*DKY*(CF*UO(1,K)-3.*UO(2,K)+UO(3,K))/CS(1)
            BV = BV-DT*DKY*(CF*VO(1,K)-3.*VO(2,K)+VO(3,K))/CS(1)
            GO TO 100
C
 85         IF (S.EQ.1) CF = 3.$ IF (S.NE.1) CF = 5.
            AV = AV-DT*DKY*(-CF*UO(1,K)+6.*UO(2,K)-4.*UO(3,K)+UO(4,K))/CS
     $      (2)
            BV = BV-DT*DKY*(-CF*VO(1,K)+6.*VO(2,K)-4.*VO(3,K)+VO(4,K))/CS
     $      (2)
            GO TO 100
C
 90         IF (S.EQ.1) CF = 3.$ IF (S.NE.1) CF = 5.
            IF (SYM.EQ.1) CF = 5.
            AV = AV-DT*DKY*(-CF*UO(JML,K)+6.*UO(JML-1,K)-4.*UO(JML-2,K)+UO
     $      (JML-3,K))/CS(JML-1)
            IF (SYM.EQ.1) CF = 3.
            BV = BV-DT*DKY*(-CF*VO(JML,K)+6.*VO(JML-1,K)-4.*VO(JML-2,K)+VO
     $      (JML-3,K))/CS(JML-1)
            GO TO 100
C
 95         IF (S.EQ.1) CF = 2.$ IF (S.NE.1) CF = 4.
            IF (SYM.EQ.1) CF = 4.
            AV = AV-DT*DKY*(CF*UO(JML,K)-3.*UO(JML-1,K)+UO(JML-2,K))/CS
     $      (JML)
            IF (SYM.EQ.1) CF = 2.
            BV = BV-DT*DKY*(CF*VO(JML,K)-3.*VO(JML-1,K)+VO(JML-2,K))/CS
     $      (JML)
 100        CONTINUE
C     COMPUTE FORCING TERM IN ELLIPTIC EQUATION
            IF (ISW.EQ.2) GO TO 105
            P(J) = AV+COR(J)*DT*BV
            Q(J) = BV-COR(J)*DT*AV
            IF (J.EQ.1) GO TO 110
            R(J-1,K-1) = CDUM(J,K)*EMI*GBV(K)-CDUM(J,K-1)*EPL*GBV(K-1)+DEL
     $      (J)*((NGC(J-1)*Q(J-1)-NGC(J)*Q(J))/DY+NU(J-1)*P(J-1)+NU(J)*P(J
     $      ))
            GO TO 110
C     NEW VALUES FOR U AND V
 105        UO(J,K) = -UO(J,K)*T4+T5*(-TU(J)*(PHIA(J+1,K)+PHIA(J,K))-SU(J)
     $      *(PHIA(J,K)-PHIA(J+1,K))+AV*GAM(J)+ALB(J)*BV)-T3*U(J,K)
            VO(J,K) = -VO(J,K)*T4+T5*(TV(J)*(PHIA(J+1,K)+PHIA(J,K))-SV(J)*
     $      (PHIA(J,K)-PHIA(J+1,K))+BV*GAM(J)-ALB(J)*AV)-T3*V(J,K)
 110     CONTINUE
```

```
  115 CONTINUE
      IF (ISW.EQ.2) GO TO 145
      DO 120 J=1,M
        PHIA(J+1,KNL+1) = (0.,0.)
  120 R(J,1) = R(J,1)-PL1(J+1)*GBV(1)
C     INVERT ELLIPTIC EQUATION FOR PHI
  125 CALL CBLKTRI (IFLG,1,N,AN,BN,CN,1,M,AA,BB,CC,M,R,IER,WRO)
      IFLG = IFLG+1
      IF (IFLG-1) 125,125,130
  130 CONTINUE
      DO 135 J=2,JML
        PHIA(J,1) = PL1(J)
        DO 135 K=2,KNL
  135 PHIA(J,K) = R(J-1,K-1)
      DO 140 K=1,KNL
        PHIA(1,K) = (0.,0.)
  140 PHIA(JM,K) = (0.,0.)
      ISW = 2
      GO TO 60
C
  145 CONTINUE
      DO 150 J=2,JML
        DO 150 K=1,KNL
  150 W(J,K) = (CDUH(J,K)-(PHIA(J,K+1)*EPL-PHIA(J,K)*EMI))/(BVF*DT*DZ)*
     $GBV(K)
C     NEW VALUES FOR DEPENDENT VARIABLES
      DO 155 J=1,JML
        DO 155 K=1,KN
          PHIA(J,K) = T5*PHIA(J,K)-T4*PHIO(J,K)-T3*PHI(J,K)
          PHIO(J,K) = PHI(J,K)
          PHI(J,K) = PHIA(J,K)
          PHIA(J,K) = UO(J,K)
          UO(J,K) = U(J,K)
          U(J,K) = PHIA(J,K)
          PHIA(J,K) = VO(J,K)
          VO(J,K) = V(J,K)
          V(J,K) = PHIA(J,K)
  155 CONTINUE
      DO 160 K=1,KN
        U(JM,K) = -U(JML,K)
        V(JM,K) = V(JML,K)
        DO 160 J=2,JM
          UB(J,K) = UB(J,K)*DENS(K)
  160 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FLUX(JM,KN,DY,DZ,CSA,CS,DENS,EPL,EMI,U,V,W,P,T,FM,FT,KZ
     $,UBO,VT,CNB)
      DIMENSION CSA(1), CS(1), DENS(1), U(JM,1), V(JM,1), W(JM,1), P(JM,
     $1), T(JM,1), FM(JM,1), FT(JM,1), UBO(JM,1)
      REAL KZ
      DIMENSION VT(JM,1), CNB(1)
      COMPLEX U,V,W,P,T
      KNL = KN-1
      JML = JM-1
      N = KNL-1
      DO 10 J=1,JM
        DO 10 K=1,KNL
   10 T(J,K) = P(J,K+1)*EPL-P(J,K)*EMI
      DO 15 K=1,KNL
        VT(1,K) = VT(JM,K) = 0.
        DO 15 J=2,JM
   15 VT(J,K) = DENS(K)*EPL*CSA(J)*(REAL(V(J-1,K+1)+V(J,K+1)+V(J-1,K)+V
     $(J,K))*REAL(T(J,K))+AIMAG(V(J-1,K+1)+V(J,K+1)+V(J-1,K)+V(J,K))*
     $AIMAG(T(J,K)))/2.
      DO 20 J=1,JML
        DO 20 K=1,N
   20 FT(J,K) = -CNB(J)*(VT(J,K)-VT(J+1,K))
      DO 25 J=1,JML
        VT(J,KN) = 0.
        VT(J,KNL) = 0.
        DO 25 K=1,N
   25 VT(J,K) = REAL(W(J+1,K))*REAL(T(J+1,K))+REAL(W(J,K))*REAL(T(J,K))+
     $AIMAG(W(J+1,K))*AIMAG(T(J+1,K))+AIMAG(W(J,K))*AIMAG(T(J,K))
      DO 30 K=2,N
        DO 30 J=1,JML
   30 FT(J,K) = FT(J,K)-DENS(K)*EPL*(VT(J,K+1)-VT(J,K-1))/(2.*DZ)
      DO 45 J=2,JM
        DO 45 K=1,KNL
          FM(J,K) = -DENS(K)*((REAL(U(J-1,K))*REAL(V(J-1,K))+AIMAG(U(J-1
     $    ,K))*AIMAG(V(J-1,K)))*CS(J-1)**2-(REAL(U(J,K))*REAL(V(J,K))+
     $    AIMAG(U(J,K))*AIMAG(V(J,K)))*CS(J)**2)/(CSA(J)**2*DY)*2.
          IF (K.EQ.1) GO TO 45
          FM(J,K) = FM(J,K)-DENS(K)*((REAL(U(J-1,K)+U(J,K))*EMI+REAL(U(J
     $    -1,K+1)+U(J,K+1))*EPL)*REAL(W(J,K))+(AIMAG(U(J-1,K)+U(J,K))*
     $    EMI+AIMAG(U(J-1,K+1)+U(J,K+1))*EPL)*AIMAG(W(J,K))-(REAL(U(J-1,
     $    K)+U(J,K))*EPL+REAL(U(J-1,K-1)+U(J,K-1))*EMI)*REAL(W(J,K-1))-
     $    (AIMAG(U(J-1,K)+U(J,K))*EPL+AIMAG(U(J-1,K-1)+U(J,K-1))*EMI)*
     $    AIMAG(W(J,K-1)))/(2.*DZ)
          IF (K.EQ.2) GO TO 35
          IF (K.EQ.KNL) GO TO 40
          FM(J,K) = FM(J,K)-KZ*(UBO(J,K-2)-4.*UBO(J,K-1)+6.*UBO(J,K)-4.*
     $    UBO(J,K+1)+UBO(J,K+2))
          GO TO 45
C
   35     FM(J,2) = FM(J,2)-KZ*(-UBO(J,1)+3.*UBO(J,2)-3.*UBO(J,3)+UBO(J,
     $    4))
          GO TO 45
C
   40     FM(J,KNL) = FM(J,KNL)-KZ*(UBO(J,KNL-2)-3.*UBO(J,KNL-1)+3.*UBO
     $    (J,KNL)-UBO(J,KN))
   45 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AOUT(TIME,DT,KN,JM,DUM,DENS,IPHAS,Z,JML,DZ,RHZ,COR,DY,
     $CI,XM1,IEND,ITIME,MT,IMAT,P1,PB,U1,UB,V1,VB,W1,WB,FM,FT,S,QS,CS,QB
     $,CZT,SZT,CDT,TQ)
      DIMENSION DUM(JM), DENS(KN), Z(9), PB(JM,KN), UB(JM,KN), VB(JM,KN)
     $, WB(JM,KN), CS(JM), QB(JM,KN), CZT(JM,KN), SZT(JM,KN), CDT(JM,KN)
     $, TQ(JML,16), TDQ(18,16), LAT(19)
      DIMENSION IPHAS(JM), COR(JM), XM1(JM), FM(JM,KN), FT(JM,KN)
      COMPLEX CI,P1(JM,KN),U1(JM,KN),V1(JM,KN),W1(JM,16),QS(JM,16)
      INTEGER S
      PI = 2.*ASIN(1.)
      DAY = (TIME)/(3600.*24.)
      KNL = KN-1
      WRITE (6,105)
      WRITE (6,130) DAY
      DO 15 KK=1,KN
        K = KN-KK+1
        DO 10 J=1,JM
          DUM(J) = 0.0
          DUM(J) = VB(J,K)*DENS(K)
10      CONTINUE
        ZZ = Z(K)+16000.
        WRITE (6,190)
        WRITE (6,110) ZZ,(DUM(J),J=1,JM)
15    CONTINUE
      WRITE (6,105)
      WRITE (6,135)
      DO 30 KK=1,KNL
        K = KN-KK
        DO 20 J=1,JM
20      DUM(J) = RHZ*(PB(J,K+1)*DENS(K+1)-PB(J,K)*DENS(K))/DZ
        ZT = Z(K)+DZ/2.+16000.
        SUN = 0.
        DO 25 J=1,JML
          TDQ(J,K) = DUM(J)-TQ(J,K)
25      SUN = SUN+DUM(J)*CS(J)
        SUN = SUN*PI/36.
        WRITE (6,190)
        WRITE (6,140) ZT,(DUM(J),J=1,JML),SUN
30    CONTINUE
      WRITE (6,105)
      WRITE (6,145)
      DO 35 KK=1,KN
        K = KN-KK+1
        ZZ = Z(K)+16000.
        WRITE (6,190)
        WRITE (6,125) ZZ,(UB(J,K),J=1,JM)
35    CONTINUE
      DO 40 L=1,19
        LL = (10-L)*10
        LAT(L) = LL
40    CONTINUE
      WRITE (6,150) LAT
      WRITE (6,105)
      WRITE (6,155)
      DO 50 KK=1,KN
        K = KN-KK+1
        DO 45 J=1,JM
45      DUM(J) = WB(J,K)*DENS(K)*1.E3
        ZZ = Z(K)+16000.+DZ/2.
        WRITE (6,190)
        WRITE (6,120) ZZ,(DUM(J),J=1,JML)
50    CONTINUE
      WRITE (6,105)
      WRITE (6,160)
      DO 60 KK=1,KN
        K = KN-KK+1
        DO 55 J=1,JM
55      DUM(J) = QB(J,K)*DENS(K)*86400.*RHZ/DZ
        ZD = Z(K)+DZ/2.+16000.
        WRITE (6,190)
        WRITE (6,115) ZD,(DUM(J),J=1,JML)
60    CONTINUE
      WRITE (6,105)
      WRITE (6,100)
      DO 65 KK=1,KN
        K = KN-KK+1
```

```
        ZD = Z(K)+DZ/2.+16000.
        WRITE (6,190)
        WRITE (6,115) ZD,(SZT(J,K),J=1,JML)
 65   CONTINUE
      WRITE (6,105)
      WRITE (6,165) DAY,S
      DO 75 KK=1,KN
        K = KN-KK+1
        DO 70 J=1,JM
          DUM(J) = CABS(P1(J,K))
          IF (DUM(J).EQ.0.) GO TO 70
          DUM(J) = DUM(J)*DENS(K)/9.8*2.
          IPHAS(J) = ATAN2(AIMAG(P1(J,K)),REAL(P1(J,K)))*180./PI
 70     CONTINUE
        ZD = Z(K)+16000.
        WRITE (6,180) ZD,(DUM(J),J=1,JML)
        WRITE (6,185) (IPHAS(J),J=1,JML)
        WRITE (6,190)
 75   CONTINUE
      WRITE (6,105)
      WRITE (6,170)
      DO 85 KK=1,KNL
        K = KN-KK
        DO 80 J=1,JM
 80     DUM(J) = FM(J,K)*DENS(K)*1.E6
        DUM(19) = 0.
        ZD = Z(K)+16000.
        WRITE (6,190)
 85   WRITE (6,125) ZD,(DUM(J),J=1,JM)
      WRITE (6,105)
      WRITE (6,175)
      DO 95 KK=1,KNL
        K = KN-KK
        DO 90 J=1,JM
 90     DUM(J) = FT(J,K)*DENS(K)*RHZ/DZ*1.E6
        ZD = Z(K)+16000.
        WRITE (6,190)
 95   WRITE (6,125) ZD,(DUM(J),J=1,JM)
      MT = 1
      IMAT = 0
      RETURN
C
C
 100  FORMAT (*  SOLAR HEATING BY OZONE *)
 105  FORMAT (1H1)
 110  FORMAT (-3PF6.1,19(0PF6.2))
 115  FORMAT (-3PF6.1,18(0PF6.2))
 120  FORMAT (-3PF6.1,18(0PF6.1))
 125  FORMAT (-3PF6.1,19(0PF6.1))
 130  FORMAT (29H  MEAN MERIDIONAL WIND DAY=  ,F6.2)
 135  FORMAT (20H  MEAN TEMPERATURE  )
 140  FORMAT (-3PF6.1,18(0PF6.1),6X,F10.4)
 145  FORMAT (18H  MEAN ZONAL WIND )
 150  FORMAT (/,6X,19I6)
 155  FORMAT (26H VERTICAL VELOCITY, MM/S  )
 160  FORMAT (30H    RADIATIVE HEATING K/D       )
 165  FORMAT (23H  GEOPOTENTIAL, DAY =  ,F6.2,12H WAVENUMBER ,I4)
 170  FORMAT (30H EDDY MOMENTUM FLUX DIVERGENCE)
 175  FORMAT (26H EDDY HEAT FLUX DIVERGENCE)
 180  FORMAT (-3PF6.1,19(0PF6.1))
 185  FORMAT (6X,19I6)
 190  FORMAT (1H )
      END
```