RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

UNIVERSITY OF VIRGINIA

Charlottesville, Virginia 22901

Annual Report
on NASA Grant NSG-1509

EVALUATING AND MINIMIZING NOISE IMPACT DUE TO AIRCRAFT FLYOVER

Submitted to:

NASA Scientific and Technical Information Facility
P.O. Box 8757
Baltimore/Washington International Airport
Baltimore, MD 21240

Submitted by:

Ira D. Jacobson
Associate Professor

Gerald Cook
Professor

Report No. UVA/528166/MAE79/101

May 1979

RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

Members of the faculty who teach at the undergraduate and graduate levels and a number of professional engineers and scientists whose primary activity is research generate and conduct the investigations that make up the school's research program The School of Engineering and Applied Science of the University of Virginia believes that research goes hand in hand with teaching Early in the development of its graduate training program, the School recognized that men and women engaged in research should be as free as possible of the administrative duties involved in sponsored research In 1959, therefore, the Research Laboratories for the Engineering Sciences (RLES) was established and assigned the administrative responsibility for such research within the School

The director of RLES—himself a faculty member and researcher—maintains familiarity with the support requirements of the research under way He is aided by an Academic Advisory Committee made up of a faculty representative from each academic department of the School This Committee serves to inform RLES of the needs and perspectives of the research program

In addition to administrative support, RLES is charged with providing certain technical assistance Because it is not practical for each department to become self-sufficient in all phases of the supporting technology essential to present-day research, RLES makes services available through the following support groups Machine Shop, Instrumentation, Facilities Services, Publications (including photographic facilities), and Computer Terminal Maintenance

Annual Report
on NASA Grant NSG-1509

EVALUATING AND MINIMIZING NOISE IMPACT DUE TO AIRCRAFT FLYOVER

Submitted to:

NASA Scientific and Technical Information Facility
P.O. Box 8757
Baltimore/Washington International Airport
Baltimore, MD 21240

Submitted by:

Ira D. Jacobson
Associate Professor

Gerald Cook
Professor

Report No. UVA/528166/MAE79/101

May 1979

N79-24769#

I. INTRODUCTION

This report presents the results of a study on the evaluation and reduction of noise impact to a community due to aircraft operation. Existing techniques have been used to assess the noise impact and to optimize the flight paths of an approaching aircraft with respect to the annoyance produced. Major achievements have been: (1) the development of a population model suitable for determining the noise impact, (2) generation of a numerical computer code which uses this population model along with the steepest descent algorithm to optimize approach/ landing trajectories, (3) implementation of this optimization code in several fictitious cases as well as for the community surrounding Patrick Henry International Airport.

Previous work has centered on developing noise annoyance criteria for flyover (i.e. NEF, NNI, CNR, etc.) and ground noise signatures for aircraft. Some of these criteria are discussed in References 1-5 with a review of many of the noise effect measures being summarized in Ref. 6. Typical of the noise footprint work is Ref. 7. The annoyance criterion used in the study is the noise impact index (NII).

The details of the models used, their advantages and disadvantages and the results obtained are outlined in the following sections.

II. PROBLEM FORMULATION

A. Overview

Analysis of the problem consists of six parts: (1) aircraft noise signatures, (2) population models, (3) cost (annoyance) function, (4) aircraft flight-path model, (5) aircraft constraints, and (6) approach/landing path optimization. A modular concept has been employed so that modification of any of these segments may be effected with relative ease. The sections below describe each of these parts in detail.

B. A/C Noise Signature

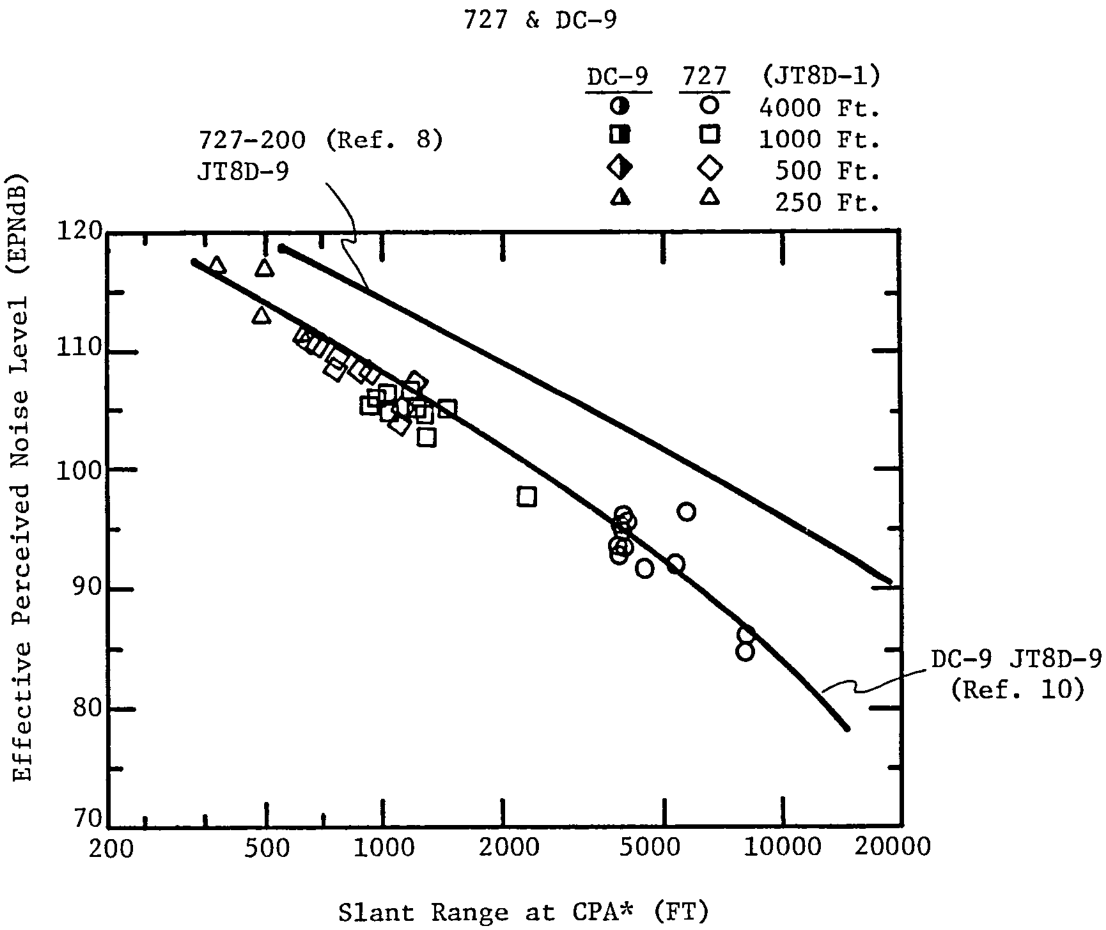
The aircraft noise signature is obtained using data from Ref. 3. Here the effective perceived noise level (EPNdB) is given as a function of slant range to the closest point of approach for a variety of aircraft. A typical plot of the slant range variation is shown in Figure 1. These data were fit using standard least squares techniques to yield an expression for EPNdB given by

$$\text{EPNdB} = 115-22.5 \log_{10} x \text{ (Slant Range)}. \qquad (1)$$

This equation is used for calculation of the maximum noise level at each location for a flyover. A typical footprint for a straight in approach along a 3-degree glide slope is shown in Figure 2.

C. Population Model

To model the population, a map of the community is overlaid with a grid and the population in each section of the grid determined. The population distribution within each section is assumed to be uniform. Several grid geometries were examined (see Figure 3). These geometries



FLYBY NOISE LEVEL
(1.93 - 1.95 EPR 727 Aircraft) FIG. A-1**
(1.94 EPR DC-9 Aircraft) FIG. D-1**

*Closest Point of Approach
**FAA-RD-71-83 (Ref. 6)

Figure 1. EPNL vs. Slant Range

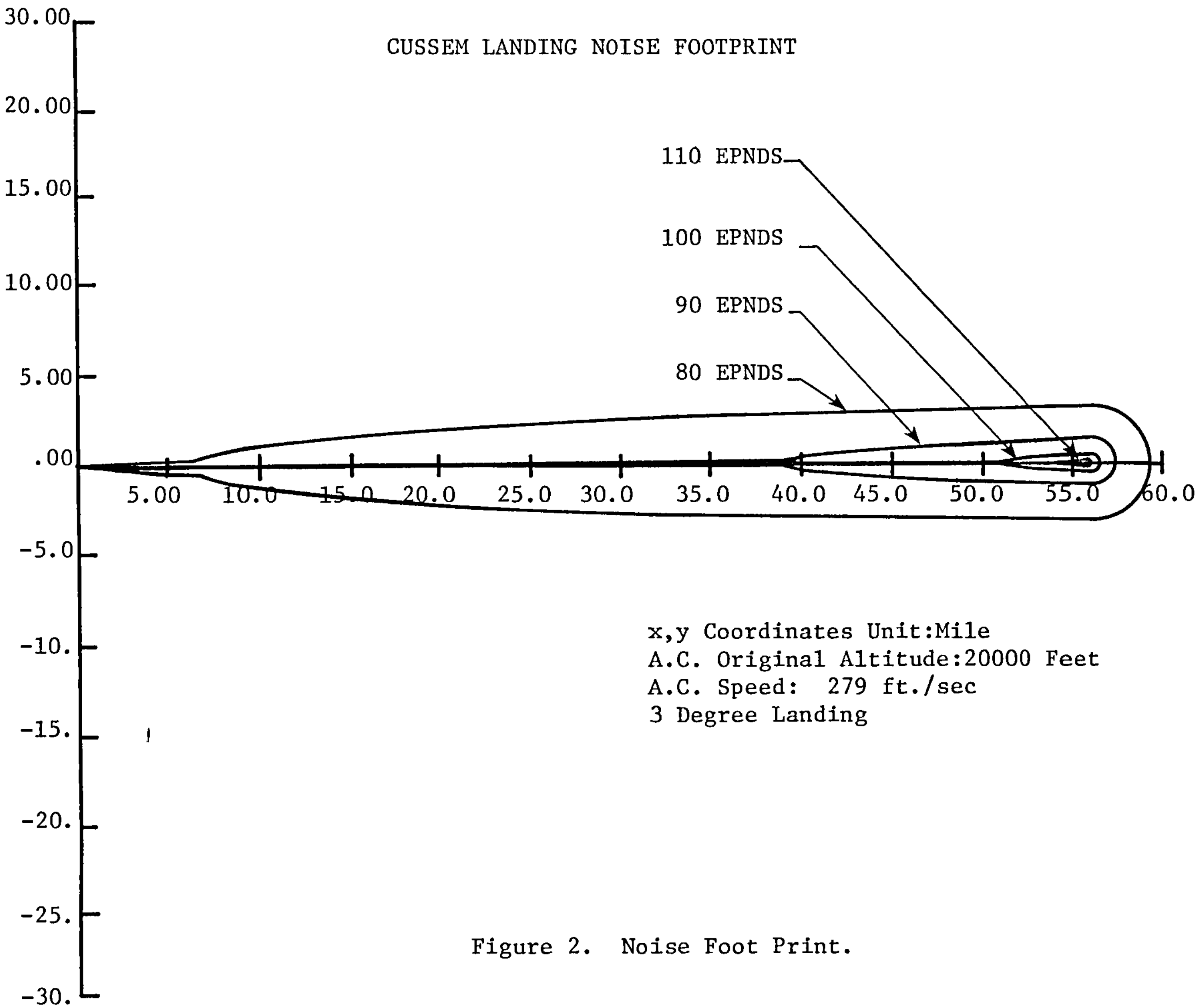
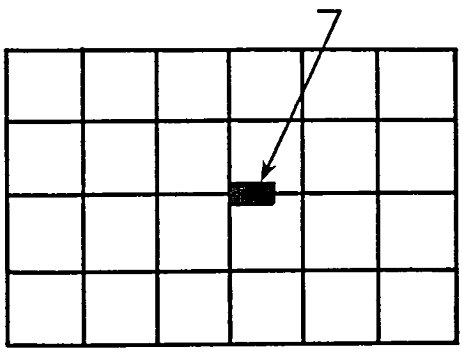
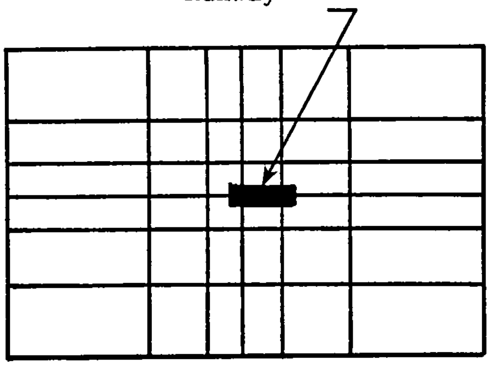


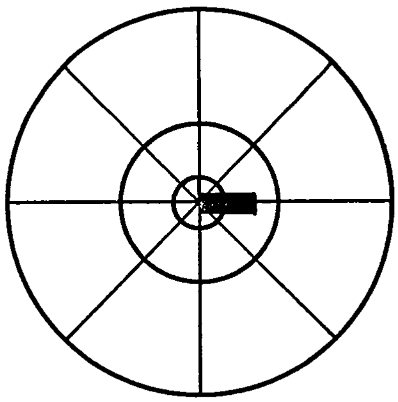
Figure 2. Noise Foot Print.



1. Equal Size Blocks



2. Variable Size Blocks



3. Concentric Circles

Figure 3. Grid Geometries.

included: (1) rectangular sections of equal size, (2) rectangular sections whose dimensions increased with distance from the airport runway, and (3) concentric circles divided by several radial lines. The second scheme was chosen since it requires fewer rectangular sections than the first and is easier to implement than the third. Computer time required for determining the optimum trajectory varies directly with the number of grid sections. Furthermore, in light of the dependence of noise levels on distance and the fact that the aircraft has higher altitude when further from the runway, the need for high resolution of the population density diminishes with distance from the airport. While the third population scheme could offer this same advantage, it is somewhat more difficult to determine the population and noise impact for each grid section with such a geometry.

Within a grid section, the population is determined by use of the SITE II system, (Ref. 8), available on the CDC 7600 computer at the NASA-Langley facility. This system requires as input the latitude and longitude of a reference point and the coordinates of the corners of each rectangular section. Although SITE II allows for simple retrieval of 1970 census data, there is some question about its resolution capabilities for small grid sections. In addition, in rapidly growing areas the population data may lag actual population. The SITE II program is capable of producing detailed census information as shown in Figure 4. However for the present analysis only population information is used.

```
                              DEMOGRAPHIC PROFILE REPORT           PAGE  1
SEVEN CORNERS
SALES       TERRITORY
SITE TOTAL                      * * * * * * * * * * * * * * * * * * * * * *
                                *                                1970-1975 *
          DEG  MIN  SEC         *                          1975   CHANGE   *
LATITUDE    38   52   10        * POPULATION             369003   -18006   *
LONGITUDE   77    9   20        * HOUSEHOLDS             138552     1076   *
                                * PER CAPITA INCOME      $ 7464   $ 2384   *
                                * *
                                *    ANNUAL COMPOUND GROWTH  -0.9%         *
                                * * * * * * * * * * * * * * * * * * * * * *

                              1970 CENSUS DATA

POPULATION                AGE AND SEX
TOTAL    387009  100.0%                 MALE              FEMALE          TOTAL
WHITE    367224   94.9%   0-5      19328  10.4%     18646   9.2%          9.8%
NEGRO     15414    4.0%   6-13     26757  14.5%     25269  12.5%         13.4%
OTHER      4371    1.1%   14-17    13645   7.4%     13194   6.5%          6.9%
                          18-20     7536   4.1%     10413   5.2%          4.6%
SPAN      13839    3.6%   21-29    35499  19.2%     39587  19.6%         19.4%
                          30-39    23840  12.9%     22964  11.4%         12.1%
                          40-49    23476  12.7%     27719  13.7%         13.2%
FAMILY INCOME (000)       50-64    27112  14.7%     30045  14.9%         14.8%
$0-5       7945    7.8%   65 +      7859   4.2%     14113   7.0%          5.7%
$5-7       6942    6.8%   TOTAL   185052           201950
$7-10     14752   14.4%   MEDIAN(AGE)      27.4               28.6        28.0
$10-15    25949   25.4%
$15-25    32623   31.9% HOME VALUE (000)          OCCUPATION
$25-50    12867   12.6% $0-10       339    0.7%   MGR/PROF    68537   41.8%
$50 +      1109    1.1% $10-15     1084    2.1%   SALES       12291    7.5%
TOTAL    102187         $15-20     4450    8.6%   CLERICAL    48735   29.8%
                        $20-25     8491   16.3%   CRAFT       12810    7.8%
AVERAGE  $15763         $25-35    17183   33.1%   OPERTIVS     6010    3.7%
MEDIAN   $14134         $35-50    14380   27.7%   LABORER      2144    1.3%
                        $50 +      6012   11.6%   FARM          114    0.1%
                          TOTAL   51939           SERVICE     11469    7.0%
RENT                                              PRIVATE      1663    1.0%
$0-100     8737  10.5%  AVERAGE $34161
$100-150  35292  42.5%  MEDIAN  $31754
$150-200  28662  34.5%  % OWNER   38.5            EDUCATION   ADULTS > 25
$200-250   6645   8.0%                            0-8         20729    9.6%
$250 +     3792   4.6%                            9-11        24297   11.3%
TOTAL     83128         AUTOMOBILES               12          69170   32.0%
                        NONE      13451    9.8%   13-15       37764   17.5%
AVERAGE   $ 150         ONE       71744   52.2%   16 +        64003   29.6%
MEDIAN    $ 147         TWO       44475   32.3%
% RENTER   61.5         THREE+     7872    5.7%
                                                  HOUSEHOLD PARAMETERS
                                                  FAM POP    335153   86.6%
UNITS IN STRUCTURE      HOUSEHOLDS WITH:          INDIVIDS    45881   11.9%
1        66945  48.7%  TV        126239  91.8%    GRP QTRS     5975    1.5%
2         1304   0.9%  WASHER     71594  52.1%    TOT POP    387009
3-4       5510   4.0%  DRYER      54258  39.5%
5-9      11809   8.6%  DISHWSH    56277  40.9%   NO OF HH'S    137476
10-49    31569  23.0%  AIRCOND    79438  57.8%   NO OF FAM'S   101961
50 +     20288  14.7%  FREEZER    28600  20.8%   AVG HH SIZE      2.8
MOBILE     125   0.1%  2 HOMES     2856   2.1%   AVG FAM SIZE     3.3
                                                                CACI,INC
```

Figure 4. Demographic Profile Report

D. Flight Path Model

There are two ways in which the trajectory of the aircraft can be determined. In one, a discrete time integration of the equations of motion with control deflections yields point by point spatial coordinates and orientation. Although this allows the flexibility of building in control constraints as well as dynamical constraints (e.g. max roll angle) it requires a considerable number of states to be stored in the optimization routine. In a multi-aircraft, multi-runway problem, as is anticipated, these storage requirements become prohibitive.

Thus, another method was adopted which utilizes only the functional form of the trajectory to describe the flight path. First a starting path was assumed which went from the initial point to the desired runway and ended up with the proper heading, i.e., velocity vector aligned with the runway. The following equation was used to generate this starting trajectory. (See Figure 5).

$$y_s(x) = \left[\left(\frac{y_f - y_p}{x_f - x_p}\right) (x-x_p) + (y_p-y_o)\right] \text{EXP} \left[-C(x-x_f)/(x_o-x_f)\right] + y_o \tag{2}$$

For the vertical motion a simple three degree descent path was assumed.

Next the first five Fourier sine harmonics were used to introduce deviation from this starting path. One advantage in using this type representation is the fact that each of the forms contributes zero at the end points. Therefore if the starting path satisfies the boundary conditions the curve with the deviations will also. An exponential decay at the final point was used to eliminate heading deviations.

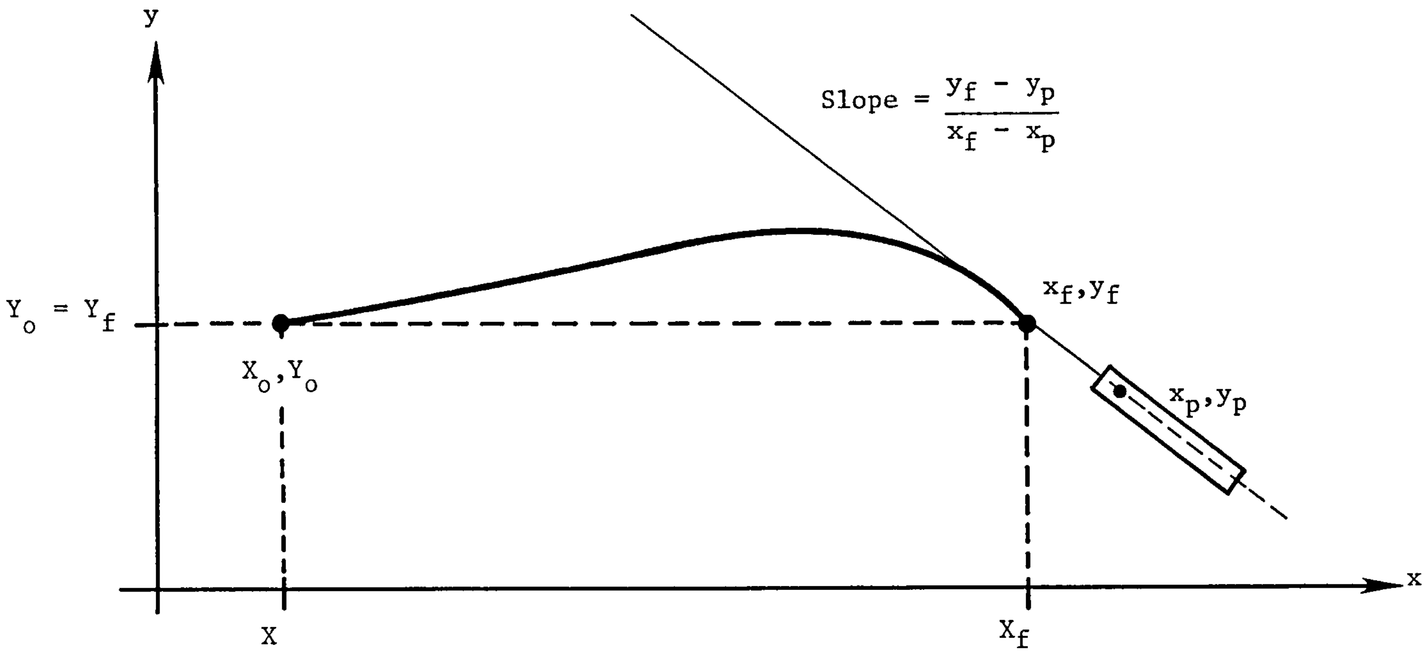


Figure 5. Rotated Coordinate System for Establishing Nominal Flight Trajectory from Initial Point to Runway Approach.

The equations with the deviations thus become

$$y(x) = \left\{ \sum_{i=1}^{5} \alpha_i \sin[\pi i(x-x_o)/(x_f-x_o)] \right\} \left\{ 1-\mathrm{EXP}[(x-x_f)/C_i] \right\} + y_s(x) \tag{3a}$$

$$Z(x) = \left\{ \sum_{i=1}^{5} \beta_i \sin[\pi i(x-x_o)/(x_f-x_o)] \right\} \left\{ 1-\mathrm{EXP}[(x-x_f)/C_i] \right\} + Z_s(x) \tag{3b}$$

A second advantage to using a Fourier Series representation for the curve is that it provides a means of representing a function with a finite number of parameters. This reduces the optimization problem from a variational one to an ordinary one.

E. Constraints

The use of a functional form of the flight path for the trajectory requires the reformulation of constraints into parameters which can be used in the optimization. This is accomplished by translating the steady state solutions of the lateral and longitudinal perturbation equations into geometric constraints. An exact derivation is given in Appendix A. The constraints are incorporated by determining maximum curvature and slope parameters as a function of aerodynamics and physical constraints. For example the constraint of a maximum roll angle, ϕ_{max}, yields

$$\frac{\frac{d^2y}{dx^2}}{1+\left(\frac{dy}{dx}\right)^2} \leq \frac{C_2+C_1C_3}{C_4+C_1C_5} \; \frac{\phi_{max}}{V_{avg}} \tag{4}$$

where C_1 through C_5 depend upon aircraft stability and control derivations (see Appendix A for details) and V_{avg} is the average velocity. Similar expressions are given in Appendix A for constraints on aileron rudder and elevator deflection, flight path angle and pitch rate limits.

F. Cost Function

A large number of criteria have been proposed by evaluating noise annoyance (e.g., EPNdB, NII, sleep interference index, speech interference index, etc.). The recent trend in noise assessment work is toward a universal measure--the noise impact index (NII). This measure is a weighted day-night model which accounts for population density. It is described in detail in Ref. 9. Briefly, the total population exposed to each incremental average day-night model sound level is multiplied by the weighting function for the level. The weighting function used in shown in Figure 6. This weighting factor $W(L_{dn})$ multiplied by the population exposed to that L_{dn} is summed and normalized by the total population giving the Noise Impact Index for the area.

$$NII = \frac{\sum_{L_{dn}} P(L_{dn})W(L_{dn})}{\sum_{L_{dn}} P(L_{dn})} \tag{5}$$

The cost function or payoff for the optimization procedure is taken to be the NII plus penalties for violating constraints. Basically the optimization procedure is set up to "drive" the aircraft trajectory to the path which will minimize the NII and at the same time not violate any constraints. As an example of the constraint of flight path angle not exceeding a maximum descent angle, γ_d, nor a maximum climb angle, γ_c, is written as

$$\tan\gamma_c < \frac{dZ}{dx} < \tan\gamma_d \tag{6}$$

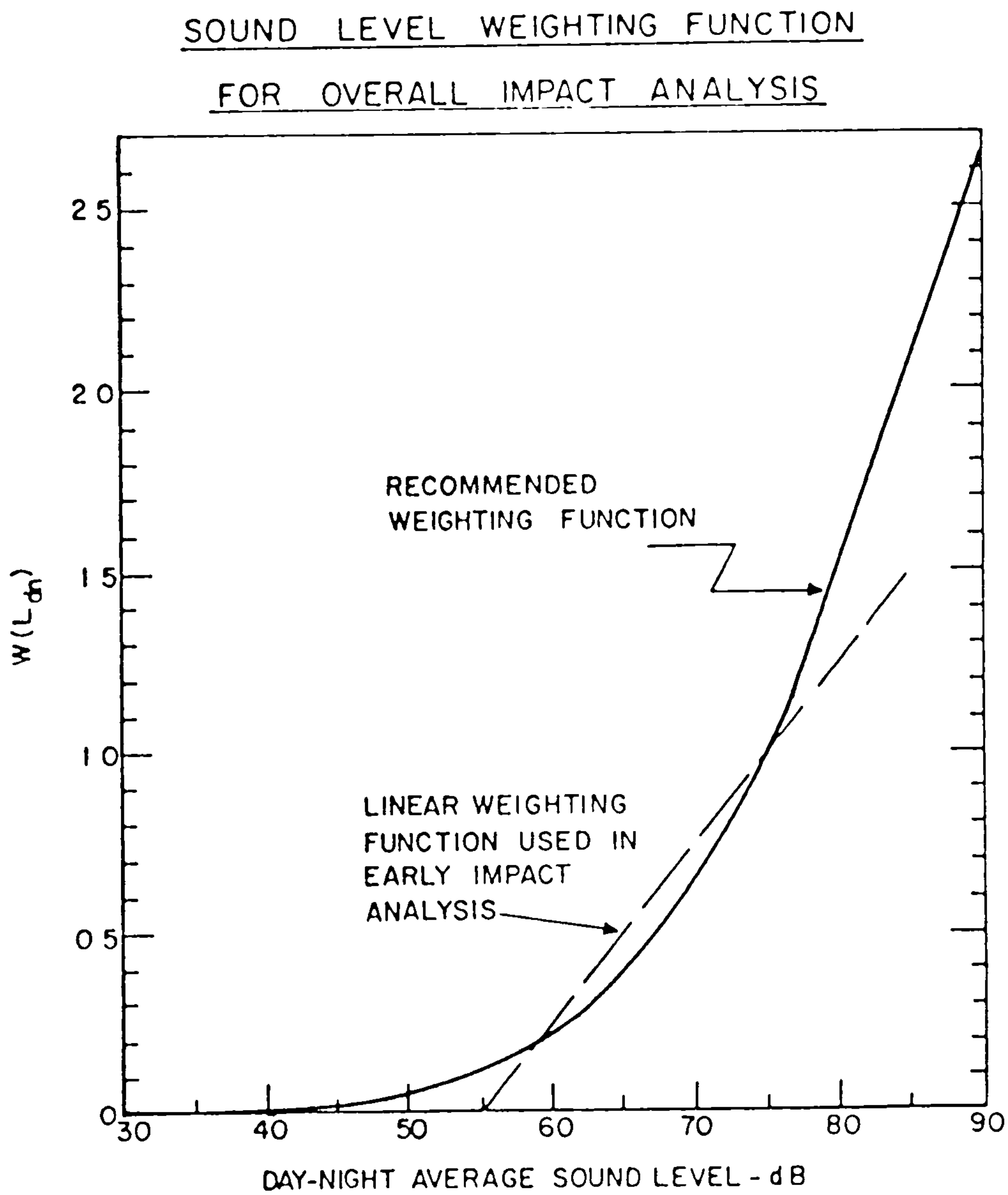


Figure 6. Sound Level Weighting Function for Overall Impact Analysis.

Each is converted to a penalty which is added to the NII in the form

$$\text{Cost} = \text{NII}+(\frac{dZ}{dx}/\tan\gamma_d)^{20} + (\tan\gamma_c/\frac{dZ}{dx})^{20} \tag{7}$$

As is seen for values of the flight path angle within the allowable range the penalty is negligible; however for values outside this range the penalty and thus the increase in cost is great. Other terms are added in a like manner.

III. OPTIMIZATION

The optimum trajectory is determined by calculating values of the α_i's and β_i's (Eq. 2) which minimize the total cost (NII plus penalties). A steepest descent algorithm is employed here. Basically, this method computes the gradient of the cost function, C, with respect to the α_i's and β_i's, then searches along the negative gradient direction for values of α_i's and β_i's which reduce the cost. The change in cost is given by

$$\Delta C = \sum_{i=1}^{5} \left(\frac{\partial C}{\partial \alpha_i} \Delta\alpha_i + \frac{\partial C}{\partial \beta_i}\Delta\beta_i\right) \tag{8}$$

The process continues iteratively until the cost converges to within a specified tolerance. While implementation of the algorithm is fairly straightforward, convergence near the optimal set of α_i's and β_i's is inherently slow. Most of the cost reduction, however, occurs in the first few iterations.

A. The Optimization Algorithm

A computer code has been developed which implements the functions described above. Figure 7 shows a flow chart for this code. Initial data (population map, aircraft constraints, initial and final aircraft positions, etc.) are required for each airport/airplane configuration to be evaluated. To facilitate calculation of the Fourier coefficients, the coordinate axes are rotated such that a line joining the initial and final aircraft positions is made to be parallel to the x axis. A nominal trajectory is generated which constrains the heading of the aircraft to asymptotically approach the runway. The steepest descent search then begins and continues until the stopping criterion is met. This criterion

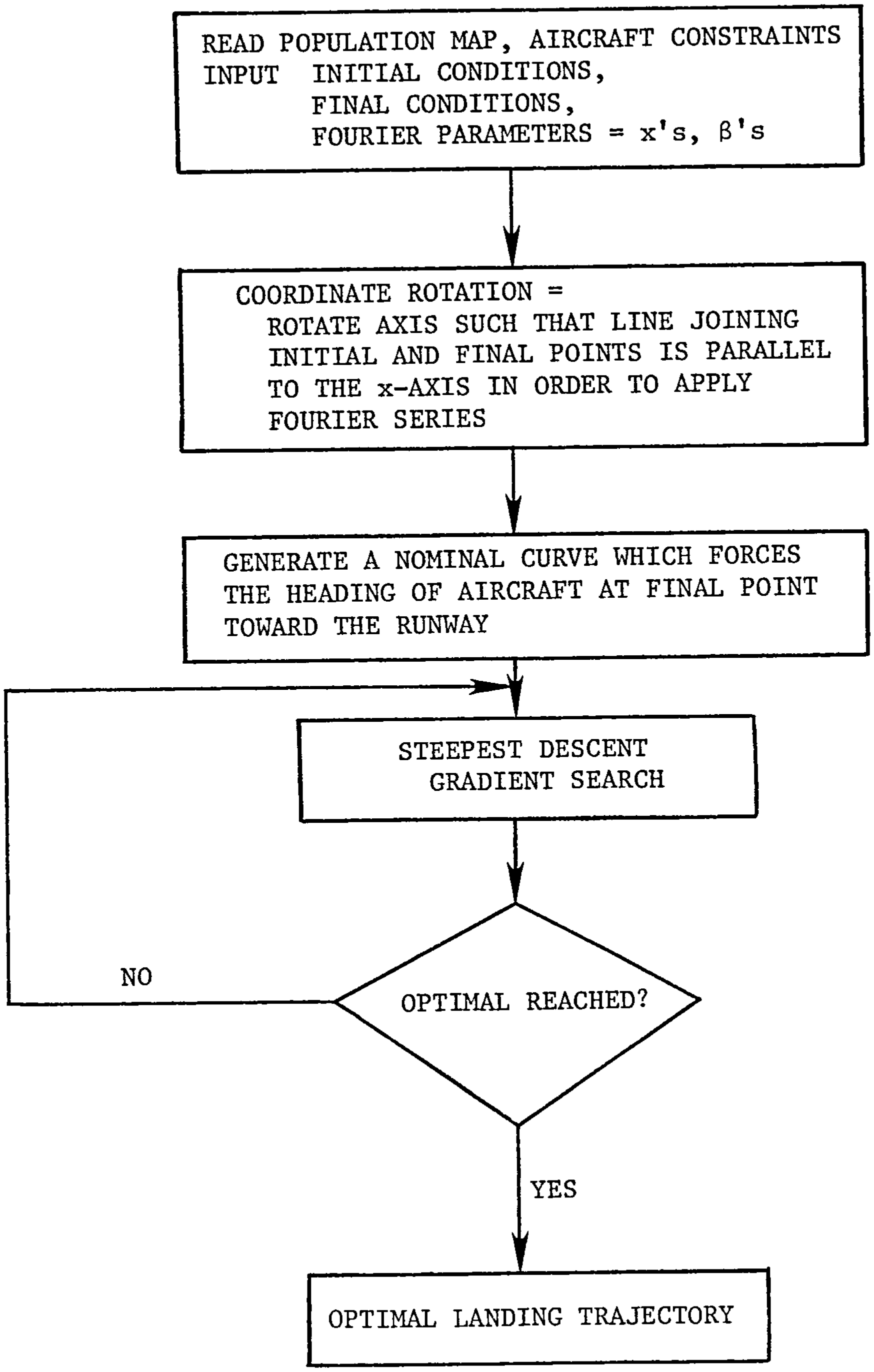


Figure 7. Flow Chart.

is met if successive improvements become negligible.

In order to provide a more accurate noise impact in each population section the impact is integrated using quadratures. This procedure can be found in Ref. 9.

The various functions such as the population model, the cost function, and the aircraft signature are incorporated as subroutines. This will allow ease of upgrading or modification if different models are desired.

Appendix B contains the Fortran code as written for a CDC Cyber 172 machine.

B. Results

Several cases have been run to test the benefits that can be obtained by this approach. First, a fictitious set of data incorporating a population valley is used. As can be seen in Figure 8 the optimization algorithm moves the aircraft (a Convair 880) towards the valley (i.e. fewer people impacted) with a corresponding improvement in the NII of 32%.

The second case models the Patrick Henry Airport in Hampton, Virginia. Here the SITE II program was used to generate the census data for each block as shown in Figure 9. Two initial trajectories were flown. One entering the area from the northwest over the Swing VOR station and the other from the southwest over the Franklin VOR station. Both of these paths are specified IFR trajectories (Figure 10). The aircraft enters the area approximately 30,000 meters from the runway. In addition several straight-in paths were evaluated. Figure 9 shows each of the trajectories. The associated

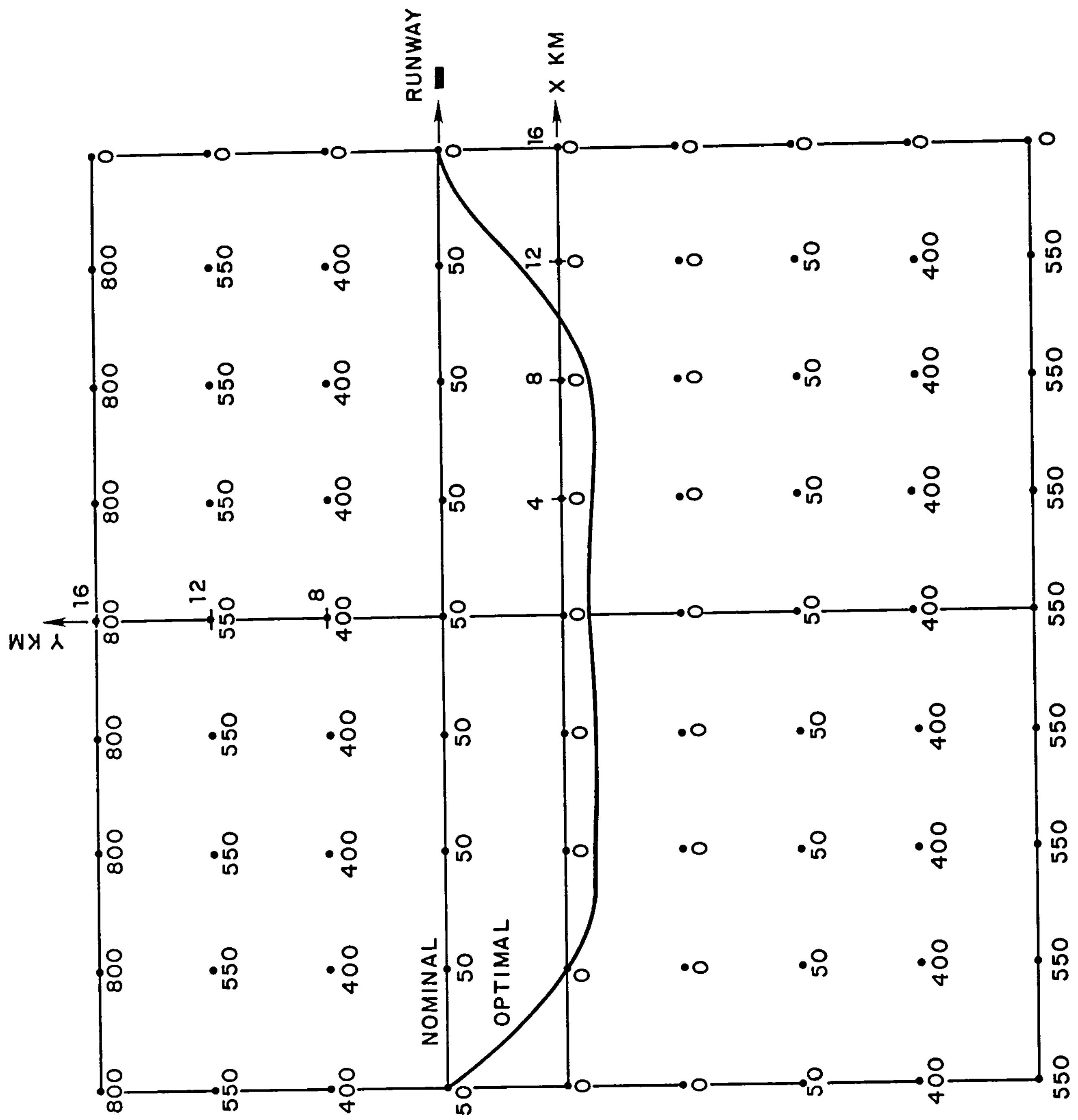


Figure 8. Optimization Results Using Fictitious Population Data.

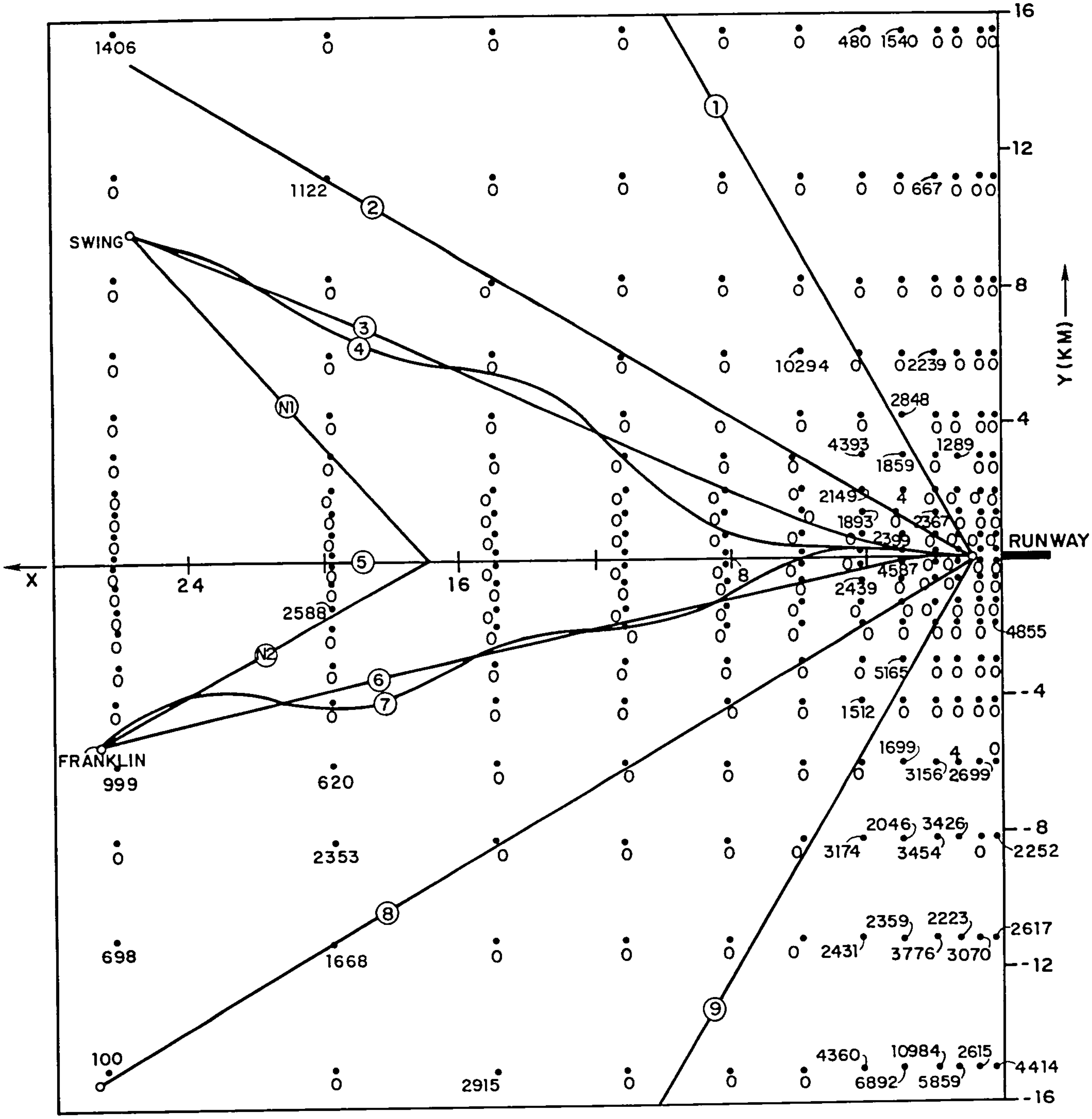


Figure 9. Population Model and Optimization Results for Patrick Henry Airport.

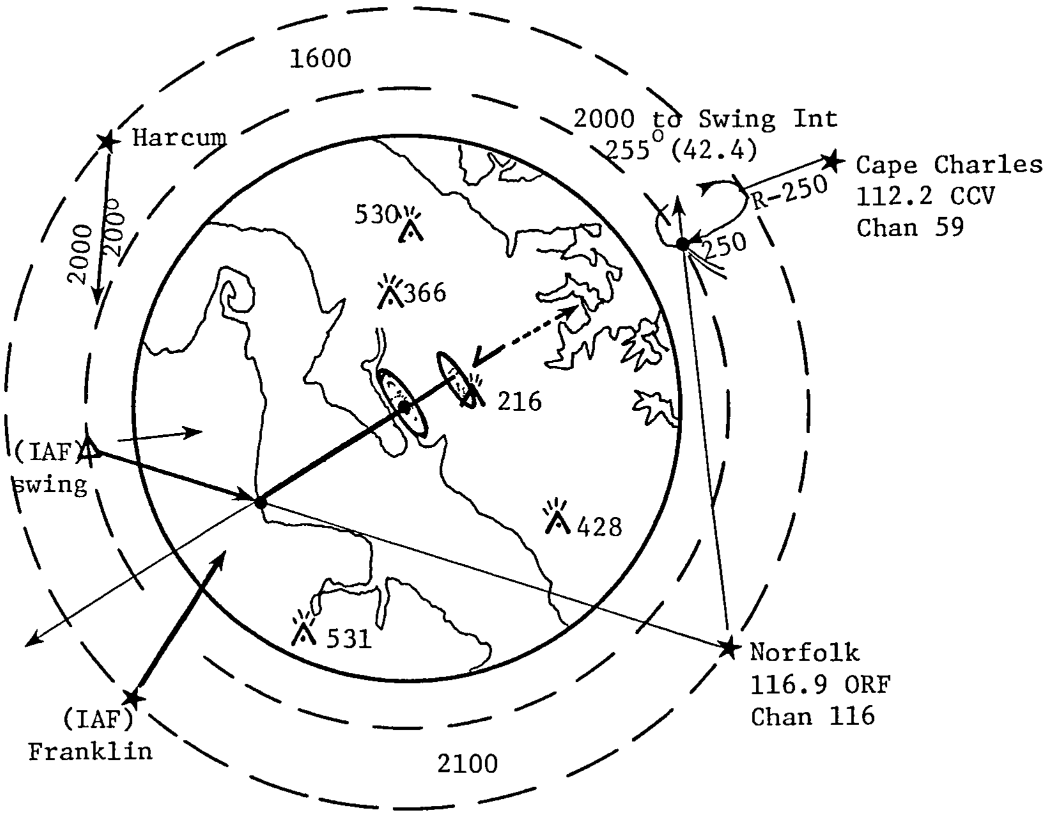


Figure 10. Conventional Approach Pattern

Table I

Northwest Approach

Entry Point: Swing

Traj. No.	Description	Cost (NII x 10^{-2})	% Change from Present
1	60 deg wrt runway	2.373	+3.2%
2	30 deg wrt runway	2.438	+6.0%
3	Initial iteration	2.27	-1.3%
4	Optimal	2.213	-3.8%
5	Straight in	2.316	+.1%
N1	Presently used	2.300	0%

Table II

Southwest Approach

Entry Point: Franklin

Traj. No.	Description	Cost (NII x 10^{-2})	% Change From Present
5	Straight in	2.316	-1.3%
6	Initial iteration	2.408	+2.6%
7	Optimal	2.241	-4.5%
8	30 deg wrt runway	2.598	+10.7%
9	60 deg wrt runway	2.687	+14.5%
N2	Presently used	2.346	0%

NII's are summarized in Tables I and II.

As is seen that even for this case, where the population is sparse away from the runway and congested near the end of it, an improvement of 3 to 5% is achieved using the optimization algorithm. It should also be noted that this technique allows not only the optimization of the path but also can be used to evaluate existing or proposed paths, such as the nominal and straight-in paths indicated.

Conclusion

A method has been formulated to optimize the path of an aircraft during approach or take-off from any airport. Models have been developed using available data where possible for population, aircraft signature, noise impact, constraints and flight path. An algorithm using steepest descent has been implemented and tested. This approach allows

1) The evaluation of the noise impact of existing flight paths,
2) The evaluation of the noise impact of proposed flight paths, and
3) The optimization of the flight path to minimize the noise impact under constraints.

This method has been applied to the Patrick Henry International Airport. Both nominal and other straight paths were evaluated. Also an optimal path was determined for each of two terminal area entry points. Performance ranged from a 15% degradation of the NII to 4.5% improvement compared to the presently used approaches. It is significant that as much as 3 to 5% improvement could be achieved in light of the fact that most of the population is concentrated at the end of the runway.

REFERENCES

1. Kirji, F.K.I., and Waters, D.M., "Preliminary Design Charts for the Assessment of Airport Noise Nuisance," Loughborough Univ. of Tech. Dept. of Transport Tech., England, Report No. 11 7412 Nov. 1974, N76-16107.

2. Bishop, Dwight E., "Community Noise Exposure Resulting From Aircraft Operations: Application Guide for Predictive Procedure," AMRL-TR-73-105 Nov. 1974.

3. Kolk, Franklin W., "A Method for Assessing the Relative Effectiveness of Various Noise Abatement Strategies," American Airlines Report June 1, 1976.

4. U. S. Environmental Protection Agency, "Information on Levels of Environmental Noise Requisite to Protect Public Health and Welfare with an Adequate Margin of Safety," March 1974, Report No. 55019-74-004.

5. Edge, P.M., Jr., Cawthorn, J.M., "Selected Methods for Quantification of Community Exposure to Aircraft Noise," Feb. 1976, NASA TN D7977.

6. Jacobson, I.D., "Environmental Criteria for Human Comfort - A Study of the Related Literature," RLES Report No. BE-4088-101-74, eb. 1974.

7. Gleck, J.M., Shevell, R.S., and Bowies, J.V., "Evaluation of Methods of Reducing Community Noise Impact Around San Jose Municipal Airport," Nov. 1975, NASA TMX 62503.

8. Annonymous, "SITE II User's Manual", CACI Inc., Arlington, Va. 1976.

9. Abramowitz, M., and Stegun, I., Handbook of Mathematical Functions, NBS Applied Math Series 55, June 1964.

APPENDIX A

Derivation of Parameterized Trajectory Constraints

Lateral perturbation equations

Y eq'n: $-\frac{b}{2V_r} C_{y_p}\dot{\phi} - \frac{mg}{q_\infty S}\cos\phi_o\phi + (\frac{mV_T}{q_\infty S} - \frac{b}{2V_T} C_{y_r})\dot{\Psi} - \frac{mg}{q_\infty S}\sin\theta_o\Psi$

$+ \frac{mV_T}{q_\infty S}\dot{\beta} - C_{y_\beta}\beta = C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r$

L eq'n: $\frac{I_{xx}}{q_\infty Sb}\ddot{\phi} - \frac{b}{2V_T} C_{\ell_p}\phi - \frac{I_{xz}}{q_\infty Sb}\ddot{\Psi} - \frac{b}{2V_T} C_\ell \dot{\Psi} - C_{\ell_\beta}\beta = C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r$

N eq'n: $-\frac{I_{xz}}{q_\infty Sb}\ddot{\phi} - \frac{b}{2V_T} C_{n_p}\dot{\phi} + \frac{I_{zz}}{q\ Sb}\ddot{\Psi} - \frac{b}{2V_T} C_{n_r}\Psi - C_{n_\beta}\beta = C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r$ (1)

If we assume all turns to be coordinated (no sideslip)

Then letting $-\frac{b}{2V_T} C_{y_p} = \overline{C}_{y_p}$, etc.

$\frac{mg}{q_\infty S}\cos\theta_o = \overline{g}_1 \quad \frac{mg}{q_\infty S}\sin\theta_o = \overline{g}_2$

$\frac{I_{xx}}{q_\infty Sb} = i_x$, etc. $\frac{mV_T}{q_\infty S} = \overline{m}$

L eq'n: $i_x\ddot{\phi} - C_{\ell_p}\dot{\phi} - i_x Z\ddot{\Psi} - \overline{C}_{\ell r}\dot{\Psi} = C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r$

N eq'n: $-ixZ\ddot{\phi} - \overline{C}_{np}\dot{\phi} + iZ\ddot{\Psi} - \overline{C}_{n_r}\dot{\Psi} = C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r$

Y eq'n: $-\overline{C}_{y_p}\dot{\phi} - \overline{g}_1 + (\overline{m} - \overline{C}_{yr})\dot{\Psi} - \overline{g}_2\Psi = C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r$ (2)

Taking the Laplace transform (I.C.'s = 0)

L eq'n: $(i_x s^2 - \overline{C}_{\ell_p} s)\ \phi\ (s) + (-i_x Z s^2 - \overline{C}_{\ell_r} s)\ \Psi\ (s) = C_{\ell_{\delta_a}}\delta_a(s) + C_{\ell_{\delta_r}}\delta_r(s)$

$$\text{N eq'n:} \quad (-i_{xZ}s^2-\overline{C}_{n_p})\ \phi(s) + (i_Z s^2 - C_{n_r}s)\ \Psi(s) = C_{n_{\delta a}}\delta_a(s) + C_{n_{\delta r}}\delta_r(s)$$

$$\text{Y eq'n:} \quad (-\overline{C}_{y_p}s-\overline{g}_1)\ \phi(s) + [(\overline{m}-\overline{C}_{y_p})s-\overline{g}_2]\Psi(s) = C_{y_{\delta_a}}\delta_a(s) + C_{y_{\delta_r}}\delta_r(s) \tag{3}$$

To determine the required δ_a for a given δ_r we consider δ_a an unknown along with $\phi(s)$ and $\Psi(s)$ [i.e. move δ_a to the left hand side of the equations] and solve for δ_a/δ_r using Cramer's rule

$$\frac{\delta_a}{\delta_r} = \frac{\begin{vmatrix} i_x s^2-\overline{C}_{\ell_p}s & -i_{xZ}s^2-\overline{C}_{\ell_r}s & +C_{\ell_{\delta_r}} \\ -i_x Z s^2-\overline{C}_{n_p}s & i_Z s^2-C_{n_r}s & +C_{n_{\delta_r}} \\ -\overline{C}_{y_p}s-\overline{g}_1 & (\overline{m}-\overline{C}_{y_p})s-\overline{g}_2 & +C_{y_{\delta_r}} \end{vmatrix}}{\begin{vmatrix} i_x s^2-\overline{C}_{\ell_p}s & -i_x Z s^2-\overline{C}_{\ell_r}s & -C_{\ell_{\delta_a}} \\ -i_x Z s^2-\overline{C}_{n_p}s & +i_Z s^2-\overline{C}_{n_r}s & -C_{n_{\delta_a}} \\ -\overline{C}_{y_p}s-\overline{g}_1 & (\overline{m}-\overline{C}_{y_p})s-\overline{g}_2 & -C_{y_{\delta_a}} \end{vmatrix}} = \frac{N(s)}{\Delta(s)} \tag{4}$$

The denominator (characteristic eqn.) is given by:

$$\begin{aligned}\Delta(s) = {} & s^4\{-C_{y_{\delta a}}(i_x i_Z - i_x^2 Z)\} + s^3\{C_{y_{\delta a}}[i_Z\overline{C}_{\ell_p} + i_x\overline{C}_{n_r} + i_{xZ}(\overline{C}_{\ell_r} + \overline{C}_{n_p})] \\ & + C_{n_{\delta a}}[-i_x Z\overline{C}_{y_p} + (\overline{m}-\overline{C}_{y_r})i_x] + \overline{C}_{\ell_{\delta a}}[i_x\overline{C}_{y_p} - (\overline{m}-\overline{C}_{y_r})i_x Z]\} \\ & + s^2\{C_{y_{\delta a}}(\overline{C}_{n_p}\overline{C}_{\ell_r} - \overline{C}_{\ell_p}\overline{C}_{n_r}) + C_{n_{\delta a}}[-i_x Z\overline{g}_1 - i_x\overline{g}_2 - \overline{C}_{y_p}\overline{C}_{\ell_r} - (\overline{m}-\overline{C}_{y_r})\overline{C}_{\ell_p}] \\ & + C_{\ell_{\delta a}}[\overline{g}_1 i_Z - \overline{g}_2 i_x Z - \overline{C}_{y_p}\overline{C}_{n_r} + (\overline{m}-\overline{C}_{y_r})\overline{C}_{n_r}]\} \\ & + s\{C_{n_{\delta a}}(\overline{g}_2\overline{C}_{\ell p} - \overline{g}_1\overline{C}_{\ell r}) + C_{\ell_{\delta a}}(\overline{g}_2\overline{C}_{np} - \overline{g}_1\overline{C}_{n_r})\}\end{aligned} \tag{5}$$

The numerator is:

$$N(s) = s^4\{C_{y_{\delta_r}}(i_x i_Z - i_x^2 Z\} + s^3\{-C_{y_{\delta_r}}[i_Z\overline{C}_{\ell_p} + i_x\overline{C}_{n_r} + i_{x_Z}(\overline{C}_{\ell_r} + \overline{C}_{n_p})]$$

$$-C_{n_{\delta_r}}[-i_x Z\overline{C}_{y_p} + (\overline{m} - \overline{C}_{y_r})i_x] - C_{\ell_{\delta_r}}\ i_x\overline{C}_{y_p} - (\overline{m} - \overline{C}_{y_r})i_x Z]\}$$

$$+s^2\{-C_{y_{\delta_r}}(\overline{C}_{n_p}\overline{C}_{\ell_r} - \overline{C}_{\ell_p}\overline{C}_{n_r}) - C_{n_{\delta_a}}[-i_x\ \overline{g}_1 - i_x\overline{g}_2 - \overline{C}_{y_p}\overline{C}_{\ell_r} - (\overline{m} - \overline{C}_{y_r})\overline{C}_{\ell_p}]$$

$$-C_{\ell_{\delta_r}}[\overline{g}_1 i_Z - \overline{g}_2 i_x Z - \overline{C}_{y_p}\overline{C}_{n_r} + (\overline{m} - \overline{C}_{y_r})\overline{C}_{n_r}]\}$$

$$+s\{-C_{n_{\delta_r}}(\overline{g}_2\overline{C}_{\ell_p} - \overline{g}_1\overline{C}_{\ell_r}) - C_{\ell_{\delta_r}}(\overline{g}_2\overline{C}_{n_p} - \overline{g}_1\overline{C}_{n_r})\} \tag{6}$$

Now assuming that only the steady state (st. st.) condition is of interest,

$$\lim_{s\to 0}\frac{N(s)}{\Delta(s)} = \left(\frac{\delta_a}{\delta_r}\right) \text{ st. st.}$$

we get

$$\left(\frac{\delta_a}{\delta_r}\right) \text{ st. st.} = \frac{-C_{n_{\delta_r}}(\overline{g}_2\overline{C}_{\ell_p} - \overline{g}_1\overline{C}_{\ell_r}) - C_{\ell_{\delta_r}}(\overline{g}_2\overline{C}_{n_p} - \overline{g}_1\overline{C}_{n_r})}{C_{n_{\delta_a}}(\overline{g}_2\overline{C}_{\ell_p} - \overline{g}_1\overline{C}_{\ell_r}) + C_{\ell_{\delta_a}}(\overline{g}_2\overline{C}_{n_p} - \overline{g}_1\overline{C}_{n_r})} \tag{7}$$

$$\left(\frac{\delta_a}{\delta_r}\right) \text{ st. st.} = \frac{\cos\theta_o(C_{n_{\delta_r}}C_{\ell_r} + C_{\ell_{\delta_r}}C_{n_r}) - \sin\theta_o(C_{n_{\delta_r}}C_{\ell_p} + C_{\ell_{\delta_r}}C_{n_p})}{-\cos\theta_o(C_{n_{\delta_a}}C_{\ell_r} + C_{\ell_{\delta_a}}C_{n_r}) + \sin\theta_o(C_{n_{\delta_a}}C_{\ell_p} + C_{\ell_{\delta_a}}C_{n_p})} \tag{8}$$

For small initial flight path angle (i.e. $\theta_o \simeq 0$)

$$\left(\frac{\delta_a}{\delta_r}\right) \text{ st. st.} = -\frac{C_{n_{\delta_r}}C_{\ell_r} + C_{\ell_{\delta_r}}C_{n_r}}{C_{n_{\delta_r}}C_{\ell_r} + C_{\ell_{\delta_r}}C_{n_r}} = C_1 \tag{9}$$

Assuming $\theta_o = 0$ to simplify we can write the transfer functions for ϕ and $\dot{\Psi}$ as (in the st. st.)

$$\frac{\dot{\Psi}}{\delta_r} = \frac{C_{\ell_{\delta_r}} C_{n_\beta} - C_{n_{\delta_r}} C_{\ell_\beta}}{C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r}} = C_2 \tag{10}$$

$$\frac{\dot{\Psi}}{\delta_a} = \frac{C_{\ell_{\delta_a}} C_{n_\beta} - C_{n_{\delta_a}} C_{\ell_\beta}}{C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r}} = C_3 \tag{11}$$

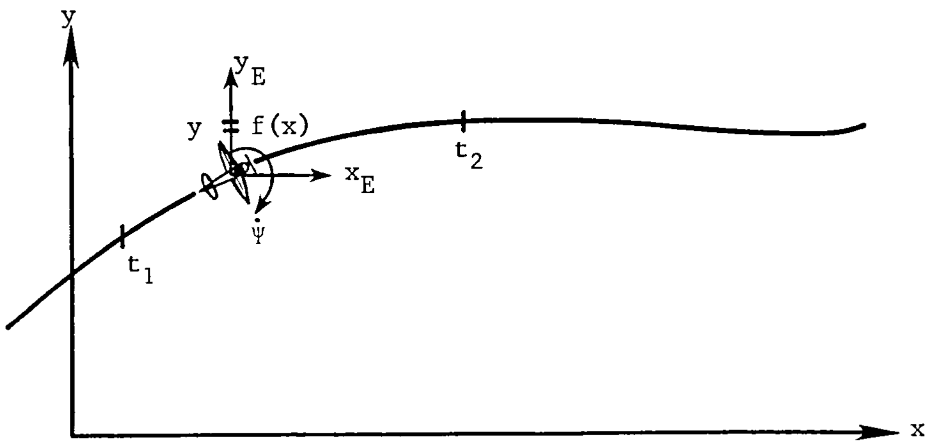
$$\frac{\phi}{\delta_r} = \frac{C_{y_{\delta_r}} (\overline{C}_{\ell_r} C_{n_\beta} - C_{\ell_\beta} \overline{C}_{n_r}) + C_{\ell_{\delta_r}} (C_{y_p} \overline{C}_{n_r} + C_{n_\beta} (\overline{m} - \overline{C}_{y_r})) + C_{n_{\delta_r}} (C_{\ell_\beta} (\overline{m} - \overline{C}_{y_r}) + C_{y_\beta} \overline{C}_{\ell_r})}{\frac{mg}{q_\infty S} (C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r})}$$

$$= C_4 \tag{12}$$

$$\frac{\phi}{\delta_r} = \frac{C_{y_{\delta_a}} (\overline{C}_{\ell_r} C_{n_\beta} - C_{\ell_\beta} \overline{C}_{n_r}) + C_{\ell_{\delta_a}} (C_{y_\beta} \overline{C}_{n_r} + C_{n_\beta} (\overline{m} - \overline{C}_{y_r})) + C_{n_{\delta_a}} (C_{\ell_\beta} (\overline{m} - \overline{C}_{y_r}) + C_{y_\beta} \overline{C}_{\ell_r})}{\frac{mg}{q_\infty S} (C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r})}$$

$$= C_5 \tag{13}$$

Consider the aircraft trajectory shown



The slope at any point is $\frac{dy}{dx}$ and the angle the slope makes with the x axis is $\tan^{-1}(\frac{dy}{dx})$.

The angular rate $\dot{\Psi}$ is then $\frac{d}{dt} \tan^{-1}(\frac{dy}{dx})$

or $\frac{\partial}{\partial x} \{\tan^{-1} \frac{dy}{dx}\} \frac{dx}{dt} = V_{avg} \frac{\partial}{\partial x} \{\tan^{-1}(\frac{dy}{dx})\}$

$$\text{Then } \Psi = V_{avg} \frac{\frac{d^2y}{dx^2}}{1+(\frac{dy}{dx})^2} = V_{avg} \{\frac{f''(x)}{1+ f''[(x)]^2}\} \qquad (14)$$

If we know δ_r we can determine δ_a from $\delta_a = C_1\delta_r$

$$\text{Also } \dot{\Psi} = C_2\delta_r+C_3\delta_a = (C_2+C_1+C_3)\delta_r \quad . \qquad (15)$$

We can also write

$$\phi = C_4\delta_r+C_5\delta_a = (C_4+C_1C_5)\delta_r \quad , \qquad (16)$$

Constraining δ_a to be $\leq \delta_{a_{max}}$, (17)

δ_r to be $\leq \delta_{r_{max}}$, (18)

and ϕ to be $\leq \phi_{max}$ ($\simeq$ max bank angle) (19)

we get the following expressions

$$\delta_{r_1} \leq \frac{\phi max}{C_4+C_1C_5} \qquad (20)$$

$$\delta_{r_2} \leq \delta_{r_{max}} \qquad (21)$$

$$\delta_{r_3} \leq \frac{\delta_{a_{max}}}{C_1} \qquad (22)$$

The constraining value is given by

$$\delta_{r_{max}} = \min(\delta_{r_1}, \delta_{r_2}, \delta_{r_3}) \tag{23}$$

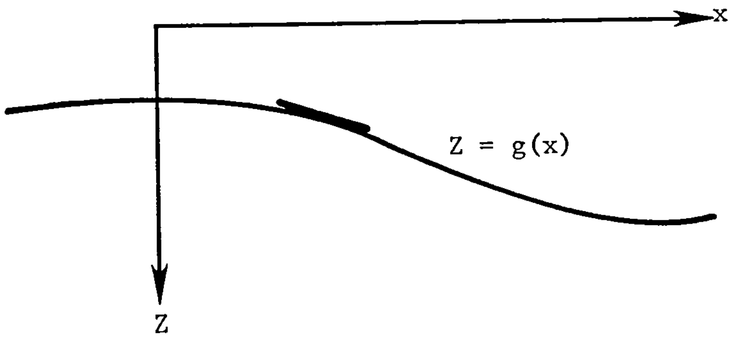
which yields

$$\dot{\Psi}_{max} = (C_2 + C_1 C_3) \min(\delta_{r_1}, \delta_{r_2}, \delta_{r_3}) \tag{24}$$

This condition incorporates all three constraints ((17)-(19)) as

$$\frac{\frac{d^2y}{dx^2}}{1+(\frac{dy}{dx})^2} = \frac{f''(x)}{1+ f'(x)^2} \leq \frac{(C_2+C_1C_3)}{V_{avg}} \min\ (\delta_{r_1}, \delta_{r_2}, \delta_{r_3})$$

Longitudinally we wish to constrain the behavior of the trajectory so that we restrict γ (the flight path angle) and θ (the pitching rate). The trajectory is given by



Then, assuming the aircraft center of mass follows this trajectory γ is given by

$$\gamma = \tan^{-1} \frac{dz}{dx}$$

or

$$\frac{dz}{dx} = \tan \gamma$$

We wish to constrain γ to a maximum descent angle, $\gamma_{d_{max}}$ and a maximum angle, $\gamma_{c_{max}}$.

Thus

$$\tan\gamma_{c_{max}} \leq \frac{dz}{dx} \leq \tan \gamma_{d_{max}}$$

APPENDIX B

```
      PROGRAM NOISE (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE97,TAPE98
     1,TAPE99)
      COMMON ALFA(5),BETA(5),POSIT(53,3),ARRAY(578,9),NMAP
      COMMON /CURVE/ YCURVE(51),ADY(51),ADDY(51)
      COMMON /LABEL/ LINFO(4),LLOC(3)
      COMMON /AIRPORT/ XPORT,YPORT,ZPORT
      COMMON /SCALE/ XMIN,XINC,YMIN,YINC
      INTEGER COUNT,HALF
      DIMENSION ALFAOD(5), BETAOD(5), GY(5), GZ(5), DALFA(5), DBETA(5)
      DIMENSION AGY(5), BGY(5), AGZ(5), BGZ(5)
C
C ...................................................................
C .                                                                 .
C .   READ MAP FROM DISC INTO MEMORY                                .
C .                                                                 .
C ...................................................................
C
      READ (5,*) A11,A12
      READ (5,*) NMAP,XPORTO,YPORTO
      READ (5,*) ((ARRAY(I,J),J=1,9),I=1,NMAP)
C
C ...................................................................
C .                                                                 .
C .   INPUT INITIAL CONDITIONS                                      .
C .                                                                 .
C ...................................................................
C
      READ (5,*) MAXIT,YALLOW,ZALLOW,(ALFA(I),I=1,5),(BETA(I),I=1,5),XO,
     1YO,ZO,XF,YF,ZF
      READ (5,9100) (LLOC(I),I=1,3),(LINFO(I),I=1,4)
      WRITE (6,9110) (LLOC(I),I=1,3),(LINFO(I),I=1,4)
      WRITE (6,9120) MAXIT
      WRITE (6,9010) XO,YO,ZO,XF,YF,ZF,XPORTO,YPORTO
      WRITE (6,9130) YALLOW,ZALLOW
      WRITE (6,9020) A11,A12
      WRITE (6,9140) (I,ALFA(I),BETA(I),I=1,5)
C
C ...................................................................
C .                                                                 .
C .   COORDINATE ROTATION                                           .
C .                                                                 .
C ...................................................................
C
      THETA = ATAN2((YF-YO),(XF-XO))
      A = (XF*COS(THETA)+YF*SIN(THETA))-(XO*COS(THETA)+YO*SIN(THETA))
      PHI = ATAN2((ZF-ZO),A)
      XOCAP = XO*COS(THETA)*COS(PHI)+YO*COS(PHI)*SIN(THETA)+ZO*SIN(PHI)
      XFCAP = XF*COS(THETA)*COS(PHI)+YF*SIN(THETA)*COS(PHI)+ZF*SIN(PHI)
      XPORT = XPORTO*COS(THETA)*COS(PHI)+YPORTO*SIN(THETA)*COS(PHI)
      YOCAP = -XO*SIN(THETA)+YO*COS(THETA)
      YFCAP = -XF*SIN(THETA)+YF*COS(THETA)
      YPORT = -XPORTO*SIN(THETA)+YPORTO*COS(THETA)
      ZOCAP = -XO*COS(THETA)*SIN(PHI)-YO*SIN(THETA)*SIN(PHI)+ZO*COS(PHI)
      ZFCAP = -XF*COS(THETA)*SIN(PHI)-YF*SIN(THETA)*SIN(PHI)+ZF*COS(PHI)
      ZPORT = -XPORTO*COS(THETA)*SIN(PHI)-YPORTO*SIN(THETA)*SIN(PHI)
C
C ...................................................................
```

```
      C .                                                                  .
      C .   START OPTIMIZATION                                             .
      C .                                                                  .
      C ....................................................................
      C
            INDEX = 0
            DLXCAP = (XFCAP-XOCAP)/50.
      C
      C ....................................................................
      C .                                                                  .
      C .   FIRST FIND A CURVE WHICH FORCES THE HEADING OF THE             .
      C .   AIRCRAFT TOWARD THE RUNWAY AT THE FINAL POINT                  .
      C .                                                                  .
      C ....................................................................
      C
            SLOPE = (YFCAP-YPORT)/(XFCAP-XPORT)
            YCURVE(1) = YOCAP
            ADY(1) = 0.
            ADDY(1) = 0.
            XCAP = XOCAP
            DO 10 I = 1,50
              XCAP = XCAP+DLXCAP
              EXPO = -5.*(XCAP-XFCAP)/(XOCAP-XFCAP)
              YCURVE(I+1) = (SLOPE*(XCAP-XPORT)+(YPORT-YOCAP))*EXP(EXPO)+YOCAP
              ADY(I+1) = -5./(XOCAP-XFCAP)*(YCURVE(I+1)-YOCAP)+(SLOPE)*EXP(EXP
           1  O)
              ADDY(I+1) = ((-5./(XOCAP-XFCAP))**2)*(YCURVE(I+1)-YOCAP)+(-5./(X
           1  OCAP-XFCAP))*SLOPE*EXP(EXPO)*2.
         10   CONTINUE
            COUNT = 0.
      C
      C ....................................................................
      C .                                                                  .
      C .   INITIAL COST                                                   .
      C .                                                                  .
      C ....................................................................
      C
            XMIN = -40000
            XINC = 2500
            YMIN = -40000
            YINC = 2500
            H = 0.07
            CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
           1TY)
            COST1 = TOTAL
            A = COST1-PNALTY
            WRITE (6,9150) COUNT,COST1,A,PNALTY
            WRITE (6,9220)
            WRITE (6,9230) ((POSIT(I,J),J=1,3),I=1,51)
            WRITE (6,9030)
            DO 20 I = 1,5
              WRITE (6,9040) I,ALFA(I),BETA(I)
         20   CONTINUE
            WRITE (6,9050)
         30 DO 40 I = 1,5
              DALFA(I) = A11
         40   DBETA(I) = A12
```

```
      C
      C ...................................................................
      C .                                                                 .
      C .   CALCULATE GRADIENT                                            .
      C .                                                                 .
      C ...................................................................
      C
   50 DO 60 I = 1,5
         ALFA(I) = ALFA(I)+DALFA(I)
         CALL COST (1,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PN
     1   ALTY)
         COST2 = TOTAL
         GY(I) = (COST2-COST1)/ABS(DALFA(I))
         IF (INDEX.EQ.0) AGY(I) = GY(I)
         IF (INDEX.EQ.1) BGY(I) = GY(I)
         WRITE (6,9160) I,GY(I)
   60    ALFA(I) = ALFA(I)-DALFA(I)
      DO 70 I = 1,5
         BETA(I) = BETA(I)+DBETA(I)
         CALL COST (1,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PN
     1   ALTY)
         COST2 = TOTAL
         GZ(I) = (COST2-COST1)/ABS(DBETA(I))
         GZ(I) = 0.
         IF (INDEX.EQ.0) AGZ(I) = GZ(I)
         IF (INDEX.EQ.1) BGZ(I) = GZ(I)
         WRITE (6,9170) I,GZ(I)
   70    BETA(I) = BETA(I)-DBETA(I)
      IF (INDEX.EQ.1) GO TO 190
      GYMAX = ABS(GY(1))
      GZMAX = ABS(GZ(1))
      DO 80 I = 2,5
         IF (GYMAX.LT.ABS(GY(I))) GYMAX = ABS(GY(I))
   80    IF (GZMAX.LT.ABS(GZ(I))) GZMAX = ABS(GZ(I))
      C
      C ...................................................................
      C .                                                                 .
      C .   DETERMINE SIZE OF STEP CHANGE                                 .
      C .                                                                 .
      C ...................................................................
      C
      YALLOW = (YALLOW-A11)*0.95+A11
      ZALLOW = (ZALLOW-A12)*0.95+A12
      IF (GYMAX.EQ.0.) YRATIO = 0.
      IF (GYMAX.NE.0.) YRATIO = YALLOW/GYMAX
      IF (GZMAX.EQ.0.) ZRATIO = 0.
      IF (GZMAX.NE.0.) ZRATIO = ZALLOW/GZMAX
      DO 90 I = 1,5
         ALFAOD(I) = ALFA(I)
         ALFA(I) = ALFA(I)-YRATIO*GY(I)
         BETAOD(I) = BETA(I)
   90    BETA(I) = BETA(I)-ZRATIO*GZ(I)
      CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
     1TY)
      COST2 = TOTAL
      IF (COST2.GE.COST1) GO TO 150
  100 PRCENT = ABS(COST2-COST1)/COST1
```

```
C
C ....................................................................
C .                                                                  .
C .   STOP CRITERION -- PERCENTAGE CHANGE IN COST INSIGNIFICANT      .
C .                                                                  .
C ....................................................................
C
      IF (PRCENT.GE.1.E-5) GO TO 110
      COUNT = COUNT+1
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
     1TY)
      WRITE (6,9180) COUNT
      CALL MONIT (COUNT,COST2,PNALTY)
      STOP
  110 CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
     1TY)
      COST1 = TOTAL
      COUNT = COUNT+1
      A = COST1-PNALTY
      WRITE (6,9150) COUNT,COST1,A,PNALTY
      WRITE (6,9220)
      WRITE (6,9230) ((POSIT(I,J),J=1,3),I=1,51)
      DO 120 I = 1,5
C
C ....................................................................
C .                                                                  .
C .   STOP CRITERION -- ALL GRADIENT COMPONENTS EQUAL TO ZERO        .
C .                                                                  .
C ....................................................................
C
        IF (GY(I).NE.0.) GO TO 130
        IF (GZ(I).NE.0.) GO TO 130
  120   CONTINUE
      WRITE (6,9060) COUNT
      CALL MONIT (COUNT,COST1,PNALTY)
      STOP
  130 WRITE (6,9070)
      DO 140 I = 1,5
        WRITE (6,9190) I,ALFA(I),BETA(I)
  140   CONTINUE
      COST2 = TOTAL
C
C ....................................................................
C .                                                                  .
C .   STOP CRITERION -- MAXIMUM NUMBER OF ITERATIONS REACHED         .
C .                                                                  .
C ....................................................................
C
      IF (COUNT.LT.MAXIT) GO TO 30
      WRITE (6,9200)
      CALL MONIT (COUNT,COST1,PNALTY)
      STOP
  150 HALF = 1
C
C ....................................................................
C .                                                                  .
C .   REDUCE SIZE OF STEP CHANGE BY HALF                             .
```

```
      C .   IF COST HAS NOT DECREASED                                   .  A 2290
      C .                                                               .  A 2300
      C .................................................................  A 2310
      C                                                                    A 2320
            DO 170 J = 1,3                                                 A 2330
              DO 160 I = 1,5                                               A 2340
                ALFA(I) = (ALFA(I)+ALFAOD(I))/2.                           A 2350
        160     BETA(I) = (BETA(I)+BETAOD(I))/2.                           A 2360
              HALF = J                                                     A 2370
              WRITE (6,9210) HALF                                          A 2380
              CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PN  A 2390
           1  ALTY)                                                        A 2400
              COST2 = TOTAL                                                A 2410
              IF (COST2.LT.COST1) GO TO 100                                A 2420
        170   CONTINUE                                                     A 2430
            HALF = 4                                                       A 2440
            INDEX = 1                                                      A 2450
            DO 180 I = 1,5                                                 A 2460
              DALFA(I) = -DALFA(I)                                         A 2470
        180   DBETA(I) = -DBETA(I)                                         A 2480
      C                                                                    A 2490
      C .................................................................  A 2500
      C .                                                               .  A 2510
      C .   PERTURB CURVE IN THE OPPOSITE DIRECTION                     .  A 2520
      C .                                                               .  A 2530
      C .................................................................  A 2540
      C                                                                    A 2550
            GO TO 50                                                       A 2560
        190 DO 200 I = 1,5                                                 A 2570
              IF (AGY(I).LT.0.) GO TO 220                                  A 2580
              IF (BGY(I).LT.0.) GO TO 220                                  A 2590
              IF (AGZ(I).LT.0.) GO TO 220                                  A 2600
              IF (BGZ(I).LT.0.) GO TO 220                                  A 2610
        200   CONTINUE                                                     A 2620
            WRITE (6,9080)                                                 A 2630
            DO 210 I = 1,5                                                 A 2640
              ALFA(I) = ALFAOD(I)                                          A 2650
        210   BETA(I) = BETAOD(I)                                          A 2660
            CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 2670
           1TY)                                                            A 2680
            CALL MONIT (COUNT,COST1,PNALTY)                                A 2690
            STOP                                                           A 2700
        220 BGYMAX = ABS(BGY(1))                                           A 2710
            BGZMAX = ABS(BGZ(1))                                           A 2720
            DO 230 I = 2,5                                                 A 2730
              IF (BGYMAX.LT.ABS(BGY(I))) BGYMAX = ABS(BGY(I))              A 2740
        230   IF (BGZMAX.LT.ABS(BGZ(I))) BGZMAX = ABS(BGZ(I))              A 2750
        240 WRITE (6,9210) HALF                                            A 2760
      C                                                                    A 2770
      C .................................................................  A 2780
      C .                                                               .  A 2790
      C .   CHECK EACH GRADIENT COMPONENT TO DETERMINE SIZE              .  A 2800
      C .   OF STEP CHANGE                                              .  A 2810
      C .                                                               .  A 2820
      C .................................................................  A 2830
      C                                                                    A 2840
            DO 320 I = 1,5                                                 A 2850
```

```
      IF (HALF.EQ.7) GO TO 250
      IF (GYMAX.NE.0.) AY = YALLOW/GYMAX/FLOAT(HALF-3)*AGY(I)
      IF (BGYMAX.NE.0.) BY = YALLOW/BGYMAX/FLOAT(HALF-3)*BGY(I)
      IF (GZMAX.EQ.0.) AZ = 0.
      IF (BGZMAX.EQ.0.) BZ = 0.
      IF (GYMAX.EQ.0.) AY = 0.
      IF (BGYMAX.EQ.0.) BY = 0.
      IF (GZMAX.NE.0.) AZ = ZALLOW/GZMAX/FLOAT(HALF-3)*AGZ(I)
      IF (BGZMAX.NE.0.) BZ = ZALLOW/BGZMAX/FLOAT(HALF-3)*BGZ(I)
      GO TO 260
250   AY = -DALFA(I)
      BY = DALFA(I)
      AZ = -DBETA(I)
      BZ = DBETA(I)
260   IF (AGY(I).LE.0.) GO TO 270
      IF (BGY(I).GE.0.) ALFA(I) = ALFAOD(I)
      IF (BGY(I).LT.0.) ALFA(I) = ALFAOD(I)+BY
      GO TO 290
270   IF (AGY(I).LT.0.) GO TO 280
      IF (BGY(I).GE.0.) ALFA(I) = ALFAOD(I)
      IF (BGY(I).LT.0.) ALFA(I) = ALFAOD(I)+BY
      GO TO 290
280   IF (AGY(I).LT.BGY(I)) ALFA(I) = ALFAOD(I)-AY
      IF (AGY(I).GE.BGY(I)) ALFA(I) = ALFAOD(I)+BY
290   IF (AGZ(I).LE.0.) GO TO 300
      IF (BGZ(I).GE.0.) BETA(I) = BETAOD(I)
      IF (BGZ(I).LT.0.) BETA(I) = BETAOD(I)+BZ
      GO TO 320
300   IF (AGZ(I).LT.0.) GO TO 310
      IF (BGZ(I).GE.0.) BETA(I) = BETAOD(I)
      IF (BGZ(I).LT.0.) BETA(I) = BETAOD(I)+BZ
      GO TO 320
310   BETA(I) = BETA(I)-AZ
320   CONTINUE
    CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
   1TY)
    COST2 = TOTAL
    IF (COST2.LT.COST1) GO TO 420
    HALF = HALF+1
    IF (HALF.LT.7) GO TO 240
    WRITE (6,9210) HALF
    GYMIN = AGY(1)
    J = 1
    GZMIN = AGZ(1)
    K = 1
    DO 340 I = 2,5
      IF (GYMIN.LE.AGY(I)) GO TO 330
      GYMIN = AGY(I)
      J = I
330   IF (GZMIN.LE.AGZ(I)) GO TO 340
      GZMIN = AGZ(I)
      K = I
340   CONTINUE
    DO 360 I = 1,5
      IF (GYMIN.LE.BGY(I)) GO TO 350
      GYMIN = BGY(I)
      J = I+5
```

```
  350 IF (GZMIN.LE.BGZ(I)) GO TO 360
      GZMIN = BGZ(I)
      K = I+5
  360 CONTINUE
      IF ((GYMIN.LT.0.0).OR.(GZMIN.LT.0.0)) GO TO 370
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
     1TY)
      COUNT = COUNT+1
      WRITE (6,9090) COUNT
      CALL MONIT (COUNT,COST1,PNALTY)
      STOP
  370 DO 380 I = 1,5
      ALFA(I) = ALFAOD(I)
  380 BETA(I) = BETAOD(I)
      IF ((GYMIN.LT.0.0).AND.(GZMIN.GE.0.0)) GO TO 390
      IF ((GYMIN.LT.0.).AND.(GZMIN.LT.0.)) GO TO 400
      IF (K.LE.5) BETA(K) = BETA(K)-DBETA(K)
      IF (K.GT.5) BETA(K-5) = BETA(K-5)+DBETA(K-5)
      GO TO 420
  390 IF (J.LE.5) ALFA(J) = ALFA(J)-DALFA(J)
      IF (J.GT.5) ALFA(J-5) = ALFA(J-5)+DALFA(J-5)
      GO TO 420
  400 IF (J.LE.5) ALFA(J) = ALFA(J)-DALFA(J)
      IF (J.GT.5) ALFA(J-5) = ALFA(J-5)+DALFA(J-5)
      IF (K.LE.5) BETA(K) = BETA(K)-DBETA(K)
      IF (K.GT.5) BETA(K-5) = BETA(K-5)+DBETA(K-5)
      CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
     1TY)
      COST2 = TOTAL
      IF (COST2.LT.COST1) GO TO 420
      DO 410 I = 1,5
  410 BETA(I) = BETAOD(I)
  420 INDEX = 0
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
     1TY)
      GO TO 100
C
C
 9010 FORMAT (5X,14HINITAL X,Y,Z: ,3(F12.2,3X),7H METERS,/,5X,13HFINAL X
     1,Y,Z: ,3(F12.2,3X),7H METERS,/,5X,23HAIRPORT LOCATION, X,Y: ,2(F12
     2.2,3X),7H METERS)
 9020 FORMAT (5X,43HPERTURB TRAJECTORY IN Y AND Z DIRECTONS BY ,F6.2,5H
     1AND ,F6.2,42H METERS, RESPECTIVELY FOR CALCULATING GRAD,5HIENTS)
 9030 FORMAT (13X,4HALFA,16X,4HBETA)
 9040 FORMAT (10X,I1,1PE16.9,4X,1PE16.9)
 9050 FORMAT (////)
 9060 FORMAT (////,1X,13HAT ITERATION ,I2,49H ALL GRADIENTS EQUAL TO
     1   ZERO, PROGRAM STOPS)
 9070 FORMAT (10X,2HNO,1X,4HALFA,16X,4HBETA)
 9080 FORMAT (5X,43HALL GRADIENTS PERTURBED BOTH DIRECTIONS > 0)
 9090 FORMAT (1X,13HAT ITERATION ,I2,16H OPTIMUM REACHED)
 9100 FORMAT (3A10,/,4A10)
 9110 FORMAT (1H1,20X,3A10,//,4A10,////)
 9120 FORMAT (1X,19HINFORMATION INPUT: ,//,5X,21HMAXIMUM ITERATION SET,1
     1H:,I3)
 9130 FORMAT (5X,47HMAXIMUM ALLOWED CHANGES PER ITERATON IN Y AND Z,27H
     1DIRECTIONS, RESPECTIVELY: ,1PE10.3,5H AND ,1PE10.3,7H METERS)
```

```
      9140 FORMAT (5X,22HINITIAL ALFA AND BETA:,/,13X,4HALFA,16X,4HBETA,5(/,1
          10X,I1,1X,1PE16.9,4X,1PE16.9))
      9150 FORMAT (////,1X,10HITERATION ,I3,//,5X,14HTOTAL COST IS ,1PE16.9,/
          1,5X,22HTRUE ANNOYACE(NII) IS ,1PE16.9,/,5X,42HPENALTY DUE TO AI
          2RCRAFT CONSTRAINTS IS ,1PE16.9,//)
      9160 FORMAT (10X,I2,17HTH Y-GRADIENT IS ,1PE16.9)
      9170 FORMAT (10X,I2,17HTH Z-GRADIENT IS ,1PE16.9)
      9180 FORMAT (////,1X,13HAT ITERATION ,I2,24H PERCENTAGE CHANGE IN CO,33
          1HST LESS THAN .001%, PROGRAM STOPS)
      9190 FORMAT (10X,I1,2X,1PE16.9,4X,1PE16.9)
      9200 FORMAT (10X,41HREACH MAXIMUM ITERATION SET, PROGRAM STOP)
      9210 FORMAT (10X,7HHALF = ,I2)
      9220 FORMAT (10X,10HTRAJECTORY,/,10X,12HX COORDINATE,8X,12HY COORDINATE
          1,8X,12HZ COORDINATE,/,12X,7H(METER),13X,7H(METER),13X,7H(METER))
      9230 FORMAT (10X,3(1PE16.9,4X))
           END
```

45000B CM STORAGE USED 7.828 SECONDS

```
      SUBROUTINE COST (IGRAD,IWRITE,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA
     1,PHI,TOTAL,PNALTY)
      COMMON ALFA(5),BETA(5),POSIT(53,3),ARRAY(578,9),NMAP
      COMMON /CURVE/ YCURVE(51),ADY(51),ADDY(51)
      COMMON /AIRPORT/ XPORT,YPORT,ZPORT
      COMMON /AC/ X,Y,Z
      EXTERNAL FCN
      PNALTY = 0.
      XCAP = XOCAP
      PI = ATAN(1.)*4.
      C2 = PI/ABS(XFCAP-XOCAP)
      C3 = ABS(XFCAP-XOCAP)/4.
      DO 10 I = 1,NMAP
        ARRAY(I,4) = 0.
   10   ARRAY(I,5) = 0.
C
C ..................................................................
C .                                                                .
C .    MULTIPLY BY EXPONENTIAL TERM SUCH THAT THE FINAL            .
C .    HEADING OF AIRCRAFT IS TOWARD THE RUNWAY                    .
C .                                                                .
C ..................................................................
C
       DO 50 I = 1,51
         Y2 = 1.-EXP(-(XFCAP-XCAP)/C3)
         Y5 = (Y2-1.)/C3
         Y9 = 0.0
         Y8 = Y9
         Y7 = Y8
         Y6 = Y7
         Y3 = Y6
C
C ..................................................................
C .                                                                .
C .    GENERATE SINE HARMONICS                                     .
C .                                                                .
C ..................................................................
C
         DO 20 J = 1,5
           TRIGOX = FLOAT(J)*(XCAP-XOCAP)*C2
           Y3 = Y3+ALFA(J)*SIN(TRIGOX)
           Y8 = Y8+BETA(J)*SIN(TRIGOX)
           Y6 = Y6+FLOAT(J)*C2*ALFA(J)*COS(TRIGOX)
           Y7 = Y7-FLOAT(J**2)*(C2**2)*ALFA(J)*SIN(TRIGOX)
   20      Y9 = Y9+FLOAT(J)*C2*BETA(J)*COS(TRIGOX)
         DLYCAP = Y2*Y3
         DLZCAP = Y2*Y8
         ZCAP = ZOCAP+DLZCAP
         YCAP = DLYCAP+YCURVE(I)
C
C ..................................................................
C .                                                                .
C .    AIRCRAFT CONSTRAINTS                                        .
C .                                                                .
C ..................................................................
C
         DY = Y2*Y6+Y3*Y5
```

```
            DY = DY+ADY(I)
            DDY = Y2*Y7+2.*Y5*Y6+Y3*Y5/C3
            DDY = DDY+ADDY(I)
            DDY = DDY/(1+DY**2)
            DZ = Y2*Y9+Y5*Y8
            DZ = DZ+TAN(PHI)
            DZ = 0.
            PNALTY = PNALTY+(DDY/.001)**(20)+(DZ/.14)**(20)
            X = XCAP*COS(THETA)*COS(PHI)-YCAP*SIN(THETA)-ZCAP*COS(THETA)*SIN
         1  (PHI)
            Y = XCAP*SIN(THETA)*COS(PHI)+YCAP*COS(THETA)-ZCAP*SIN(THETA)*SIN
         1  (PHI)
            Z = XCAP*SIN(PHI)+ZCAP*COS(PHI)
            DO 40 K = 1,NMAP
              RANGE = ((X-ARRAY(K,1))**2+(Y-ARRAY(K,2))**2+Z**2)**.5
              DB = 115.-22.5*ALOG10(3.281*RANGE/500.)
              IF (DB.LE.ARRAY(K,4)) GO TO 40
              ARRAY(K,4) =  A
              IF (ARRAY(K,4).LT.55.) GO TO 40
              IF (ARRAY(K,3).EQ.0.) GO TO 30
C
C ..............................................................
C .                                                            .
C .  ANNOYANCE INTEGRATION OVER A SINGE BLOCK                  .
C .                                                            .
C ..............................................................
C
              SMALLP = ARRAY(K,3)/(ARRAY(K,7)-ARRAY(K,6))/(ARRAY(K,9)-ARRAY(
         1    K,8))
              CALL GAUSS (ARRAY(K,6),ARRAY(K,7),ARRAY(K,8),ARRAY(K,9),FCN,TE
         1    MP)
              ARRAY(K,5) = TEMP*SMALLP
              GO TO 40
       30     ARRAY(K,5) = 0.
       40     CONTINUE
            IF (IWRITE.EQ.0) GO TO 50
            II = I
            POSIT(II,1) = X
            POSIT(II,2) = Y
            POSIT(II,3) = Z
       50   XCAP = XCAP+DLXCAP
C
C ..............................................................
C .                                                            .
C .  TOTAL POPULATON EXPOSED TO NOISE ABOVE 55 EPNDB            .
C .                                                            .
C ..............................................................
C
          PEOPLE = 0.
          DO 60 K = 1,NMAP
            IF (ARRAY(K,5).EQ.0.0) GO TO 60
            PEOPLE = ARRAY(K,3)+PEOPLE
       60   CONTINUE
          FX = 0.
          DO 70 K = 1,NMAP
            ARRAY(K,5) = ARRAY(K,5)/PEOPLE
            FX = FX+ARRAY(K,5)
```

```
          70   CONTINUE                                                  B 1150
             TOTAL = FX+PNALTY                                           B 1160
             RETURN                                                      B 1170
             END                                                         B 1180
```

41000B CM STORAGE USED   .874 SECONDS

```
      SUBROUTINE MONIT (IA,AA,BB)
      COMMON ALFA(5),BETA(5),POSIT(53,3),ARRAY(578,9),NMAP
      COMMON /LABEL/ LINFO(4),LLOC(3)
      COMMON /SCALE/ XMIN,XINC,YMIN,YINC
      DIMENSION PCRIT(10)
      DIMENSION XM(1026), YM(1026)
      DIMENSION XP(53), YP(53), ZP(53), NA(5), NB(3)
      EQUIVALENCE (XM(1),ARRAY(1,1)), (YM(1),ARRAY(1,2))
      EQUIVALENCE (XP(1),POSIT(1,1)), (YP(1),POSIT(1,2)), (ZP(1),POSIT(1
     1,3))
      DATA NB/10HTOTAL POPU,10HLATION ANN,9HOYANCE = /
C
C ...................................................................
C .                                                                 .
C .   DOCUMENTATION                                                 .
C .                                                                 .
C ...................................................................
C
      CC = AA-BB
      WRITE (6,9010)
      WRITE (6,9020) IA,AA,CC,BB
      DO 10 I = 1,51
        WRITE (6,9030) (POSIT(I,J),J=1,3)
   10   CONTINUE
      WRITE (6,9040)
      DO 20 I = 1,NMAP
        WRITE (6,9050) (ARRAY(I,J),J=1,5)
   20   CONTINUE
      WRITE (97,9060) ((POSIT(I,J),J=1,3),I=1,51)
      WRITE (97,9070) ((ARRAY(I,J),J=1,5),I=1,NMAP)
      RETURN
C
 9010 FORMAT (10X,55HOPTIMUM TRAJECTORY FOR LANDING AT PATRICK HENRY AIR
     1PORT,//,10X,59HNOISE BELOW 55 EPNDB IS CONSIDERED NOT NOISY, ANNOY
     2ANCE = 0,/,10X,23HUNIT FOR NOISE IS EPNDB,/,10X,50HUNIT FOR COORDI
     3NATES, AIRCRAFT TRAJECTORY IS METER,//)
 9020 FORMAT (10X,13HAT ITERATION ,I2,/,15X,14HTOTAL COST IS ,1PE16.9,/,
     115X,23HTRUE ANNOYANCE(NII) IS ,1PE16.9,/,15X,11HPENALTY IS ,1PE16.
     29,//,10X,18HOPTIMUM TRAJECTORY,/,10X,12HX COORDINATE,8X,12HY COORD
     3INATE,8X,12HZ COORDINATE,/,12X,7H(METER),13X,7H(METER),13X,7H(METE
     4R))
 9030 FORMAT (10X,3(1PE16.9,4X))
 9040 FORMAT (//,10X,32HPOPULATION-NOISE-ANNOYANCE CHART,/,10X,10HX-POSI
     1TION,5X,10HY-POSITION,5X,10HPOP. INDEX,5X,11HNOISE LEVEL,4X,9HANNO
     2YANCE)
 9050 FORMAT (10X,3(F10.3,5X),2(1PE10.3,5X))
 9060 FORMAT (3E12.6)
 9070 FORMAT (5E12.6)
      END
```

41000B CM STORAGE USED .284 SECONDS

```
      SUBROUTINE GAUSS (XN,XX,YN,YX,FCN,FINT)
      COMMON /AC/ XA,YA,ZA
      DIMENSION X(5), Y(5), F(5), XI(5), W(5)
      DATA XI,W,N/-0.577350269,0.577350269,0,0,0,1.,1.,0,0,0,2/
C
C ....................................................................
C .                                                                  .
C .   GAUSSIAN QUADRATURE INTEGRATION WITH FOUR POINTS                .
C .                                                                  .
C ....................................................................
C
      DO 10 I = 1,N
        Y(I) = (YX-YN)/2.*XI(I)+(YX+YN)/2.
   10   X(I) = (XX-XN)/2.*XI(I)+(XX+XN)/2.
      FINT = 0.
      DO 30 J = 1,N
        F(J) = 0.
        DO 20 I = 1,N
   20     F(J) = F(J)+W(I)*FCN(X(I),Y(J))
        F(J) = F(J)*(XX-XN)/2.
   30   FINT = FINT+W(J)*F(J)
      FINT = FINT*(YX-YN)/2
      RETURN
      END
```

41000B CM STORAGE USED .186 SECONDS

```
FUNCTION FCN        73/172  TS                FTN 4.6+452        04/27/79  11.45.47        PAGE    1

                 FUNCTION FCN (X,Y)
                 COMMON /AC/ XA,YA,ZA
                 RANGE = SQRT((X-XA)**2+(Y-YA)**2+ZA**2)
                 ARG = 129.12-22.5*ALOG10(RANGE)
                 FCN = (3.36E-6*10.**(.103*ARG))/(.2*10.**(.03*ARG)+1.43E-4*10.**(.
                108*ARG))
                 RETURN
                 END

41000B CM STORAGE USED          .100 SECONDS
```

```
                    >>> COST REPORT FOR LISTOAF <<<

              04/27/79                                11.45.59

      RESOURCE               BILLING RATE                UNITS USED          COST

CENTRAL PROCESSOR    -     $105.00  /HOUR                 9.314 CP SECONDS    $    .27
PERIPHERAL PROCESSOR         20.00  /HOUR                 9.737 PP SECONDS         .05
I/O                          80.00  /HOUR                 2.926 IO SECONDS         .07
FIELD LENGTH                  3.00  /KILO-WRD-HOUR      205.576 KILO-WRD-SECS.     .17
                                                                              --------
(BASIC COST EXCLUDES LINES PRINTED, CARDS PUNCHED                  BASIC COST      .56
  AND PLOTTER TIME CHARGES)

JOB PRIORITY  3         PRIORITY COST FACTOR  1.00        APPROXIMATE ADJUSTED COST     .56

      AS OF LAST ACCOUNT UPDATE, ACCOUNT EXPIRES 04/30/79, FUNDS LEFT $  6037.31


  04/27/79    UVA NOS/BE 1.2   LEVEL 454-03/11/78
11.45.47.LISTOAF  FROM *GD/AB
11.45.47.LIST,M3117A,T100.
11.45.47.ATTACH,Q,NEWTIDY.
11.45.47.PF CYCLE NO. = 002
11.45.47.FTN(I=Q)
11.45.59.  45000B   CM STORAGE USED
11.45.59.     9.292 CP SECONDS COMPILATION TIME
11.45.59.     STOP
11.46.00.EJ  END OF JOB, AB

PRINT COST $000.88 LISTOAF //// END OF LIST ////   0000803 LINES
```

DISTRIBUTION LIST

Copy No.	
1 - 2	NASA Scientific and Technical Information Facility P.O. Box 8757 Baltimore/Washington International Airport Baltimore, MD 21240
3 - 5	Mr. Richard DeLoach National Aeronautics and Space Administration Langley Research Center Hampton, Va 23665
6 - 7	I. D. Jacobson
8 - 9	G. E. Cook
10	L. S. Fletcher
11	E. A. Parrish, Jr.
12	I. A. Fischer Office of Sponsored Programs
13 - 15	E. H. Pancake Science/Technology Information Center Clark Hall
16	RLES Files

9967:hfc

UNIVERSITY OF VIRGINIA

School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,000 students with a graduate enrollment of 350 There are approximately 120 faculty members, a majority of whom conduct research in addition to teaching

Research is an integral part of the educational program and interests parallel academic specialties These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering, and Applied Mathematics and Computer Science In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics All departments offer the doctorate, the Biomedical and Materials Science Departments grant only graduate degrees

The School of Engineering and Applied Science is an integral part of the University (approximately 1,400 full-time faculty with a total enrollment of about 14,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, and Business Administration In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service

End of Document