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FINAL REPORT

# CONTINUUM ABSORPTION COEFFICIENT OF ATOMS AND IONS

NASA GRANT NSG 1581

BY

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UNIVERSITY OF MISSOURI - ROLLA

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ABSTRACT

The rate of heat transfer to the heat shield of a Jupiter probe has been estimated to be one order of magnitude higher than any previously experienced in an outer space exploration program. More than one-third of this heat load is due to an emission of continuum radiation from atoms and ions. The existing computer code for calculating the continuum contribution to the total load utilizes a modified version of Biberman's approximate method.

The continuum radiation absorption cross sections of a C - H - O - N ablation system have been examined in detail. The present computer code has been evaluated and updated by being compared with available exact and approximate calculations and correlations of experimental data. A detailed calculation procedure, which can be applied to other atomic species, is presented. The approximate correlations can be made to agree with the available exact and experimental data.

## NOMENCLATURE

| | |
|---|---|
| $A$ | = Atomic species |
| $A^+$ | = Ionized atomic species |
| $A^-$ | = Negative ion of an atomic species |
| $C_o$ | = $7.25 * 10^{-16}$ ($cm^2$ - $eV^2$) |
| $e^-$ | = Electron |
| $E_{b_\nu}$ | = Black body emissive power |
| $E_{b_\nu max}$ | = Maximum of black body emissive power |
| $g_i$ | = Statistical weight at state i |
| $h$ | = Planck constant ($6.6251 \times 10^{-27}$ erg-sec) |
| $I_k^-$ | = Ionization potential of negative ion of species k (eV) |
| $K$ | = Boltzmann's constant ($1.38044 \times 10^{-16}$ erg/$^o$K) |
| $n_i$ | = Principal quantum number of energy state i |
| $N_e$ | = Number density of electron in plasma (Part/$cm^3$) |
| $N_k$ | = Number density of species k, (Part/$cm^3$) |
| $N_k^-$ | = Number density of negative ion of species k (Part/$cm^3$) |
| $N_{i,k}$ | = Electronic number density of $i^{th}$ quantum level of species k (Part/$cm^3$) |
| $Q_k$ | = Partition function of species k |
| $Q_k^-$ | = Partition function of negative ion of species k |
| $T$ | = Temperature ($^o$K) |
| $u$ | = Frequency $h\nu$, (eV) |
| $u_i$ | = Ionization potential of state i, (eV), ($u_i = \epsilon_I - \epsilon_i$) |

$u_i'$ = Reduced ionization potential of state i, (eV), ($u_i' = u_i - 0.25$)

$Z_k$ = Effective nuclear charge

$\epsilon_I$ = Ionization energy (eV)

$\epsilon_T$ = Threshold energy (eV)

$\epsilon_i$ = Energy level of state i (eV)

$\lambda$ = Wave length

$\mu_k$ = Absorption coefficient, (1/cm)

$\mu_k^-$ = Absorption coefficient of negative ions (photodetachment) (1/cm)

$\nu$ = Frequency

$\sigma_i^b$ = Effective cross section due to bound-free transitions from state i to all accessible states ($cm^2$)

$\sigma_n^f$ = Effective cross section due to free-free transitions from state n to accessible states ($cm^2$)

$\sigma_k$ = Effective cross section due to bound-free and free-free transitions of species k ($cm^2$)

$\sigma_k^b$ = Effective cross section due to bound-free transition of species k ($cm^2$)

$\sigma_k^f$ = Effective cross section due to free-free transition of species k ($cm^2$)

$\sigma_{ij}^b$ = Absorption cross section due to bound-free transition $i \to j$ ($cm^2$)

$\sigma_{nm}^f$ = Absorption cross section due to free-free transition $n \to m$ ($cm^2$)

$\sigma_k^-$ = Absorption cross section of negative ions ($cm^2$)

$\theta$ = Temperature KT, (eV)

$\Gamma_k$ = Statistical weight factor

$\xi_k$ = Nonhydrogenic correction factor (Figure 3)

$\Phi_k$ = Function defined in Eq. (14)

## INTRODUCTION

In the past ten years, considerable effort has been expended in calculating the radiative flux that reaches the heat shield of a Jupiter probe[1,2,15-18]. These calculations indicate that the probe could be exposed to a severe aerothermodynamic environment. Unfortunately, the encountered environment cannot be duplicated in any existing ground experimental facilities; consequently, the design of the heat shield,which weighs half of the total weight of the probe,must rely extensively on analytical predictions. Specifically, the continuum radiation contributions, which have been estimated to be more than half of the total radiation load or about one-third of the total heat load, must be predicted accurately. An overly conservative approach would impose a severe weight penalty on the probe, and the reverse could result in the failure of the mission.

The theoretical methods for calculating the spectral continuum absorption coefficient of the atomic plasma of light elements are well developed[19,20]. These methods have been applied by Armstrong[21] to calculate the bound-free (photoionization) cross section for nitrogen and oxygen. A modified Kramer's semiclassical formula[20] has been used by Wilson and Nicolet[6] to calculate the free-free cross section for these two elements. They utilized Armstrong's results and their own calculations to tabulate an effective continuum cross section for them. Similarly detailed calculations of the continuum cross section of the heavier elements are not available in the literature.

The complexities and the calculation time associated with the detailed calculations of the continuum cross section make the procedure unsuitable for use in the overall investigation of heat transfer through the viscous shock layer of the probe. Approximate correlations for this property are normally used in the comprehensive calculations for heat transfer. Biberman and Norman[4] and Sibulkin[5] proposed approximate theories for calculating the continuum cross section of heavier elements, and the Biberman theory has been used by Wilson and Nicolet[6] to tabulate the continuum cross section for carbon. This theory is used with some modifications in the existing computer code[3]. The objective of the present study is to examine the accuracy of the computer code and to provide a systematic and detailed calculation procedure that can be applied to other atomic species.

## CONTINUUM ABSORPTION COEFFICIENT

The continuum radiation emitted or absorbed by a plasma is produced by the bound-free and free-free transitions of electrons from a specified energy level to a higher or lower level. The transitions can be described by

$$A + h\nu \rightleftarrows A^{+} + e^{-} \qquad \text{(bound-free)} \qquad (1)$$

and

$$A + e^{-} + h\nu \rightleftarrows A + e^{-}. \qquad \text{(free-free)} \qquad (2)$$

These and other transitions are shown schematically for the hydrogen atom in Figure 1.

The continuum abosorption coefficient for a given species k can be expressed by

$$\mu_k = \sum_{i,j} N_{i,k}\,\sigma^{b}_{i,j} + \sum_{n,m} N_{n,k}\,\sigma^{f}_{n,m} . \qquad (3)$$

It is expedient in many cases to define the effective absorption cross section, which accounts for all accessible transitions from state i, as

$$\mu_k = \sum_{i} N_{i,k}\,\sigma^{b}_{i} + \sum_{n} N_{n,k}\,\sigma^{f}_{n}. \qquad (4)$$

It is also very convenient for radiative heat transfer calculations to define the effective absorption cross section, which accounts for both bound-free and free-free transitions, as

$$\mu_k = N_k \sigma_k . \qquad (5)$$

The advantage of using the above expression is that it can be expressed directly in terms

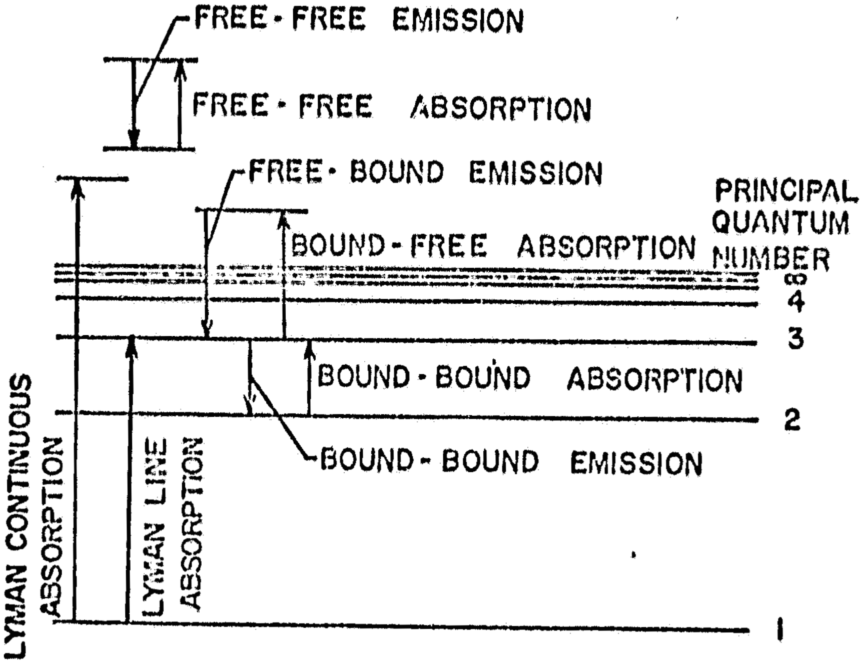


Fig. 1 Various radiation processes in the hydrogen atom

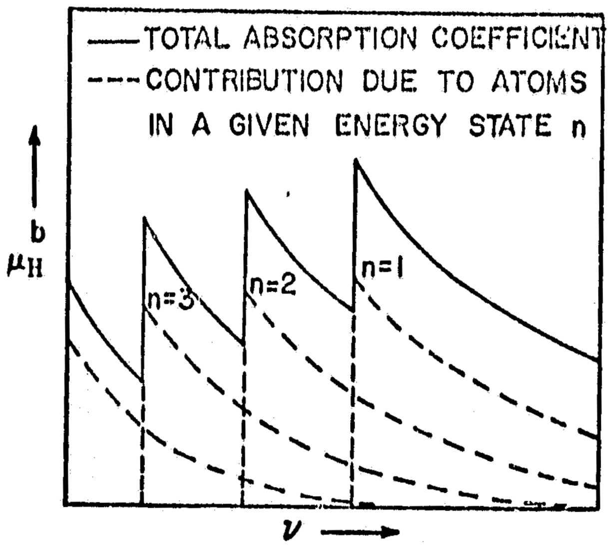


Fig. 2 Schematic of the bound-free absorption coefficient for hydrogen atom

of the number density of the species in the plasma. At equilibrium, the number density of species k in the $i^{th}$ quantum state is related to the number density of species k in the plasma by the Boltzmann relation:

$$N_{i,k}/N_k = g_i \, \mathrm{Exp}(-\epsilon_i/\theta)/Q_k. \qquad (6)$$

## Absorption Coefficient for Atomic Hydrogen

The hydrogen atom represents the simplest configuration of all atoms, because it has one electron, which can be treated as a free electron in a coloumb field. The simple analytical and classical solution for the bound-free and the free-free absorption coefficients, which is available in the literature[19,22], is given below:

$$\mu_H = N_H(\sigma_H^b + \sigma_H^f) \qquad (7)$$

in which

$$\sigma_H^b = (1.99 * 10^{-14}/u^3) \sum_{n=1} \mathrm{Exp}[-13.6(1-1/n^2)/\theta]/n^3, \qquad (8)$$

and

$$\sigma_H^f = 7.35 * 10^{-16}\theta \, \mathrm{Exp}(-13.6/\theta)/u^3. \qquad (9)$$

A schematic of the bound free absorption coefficient for hydrogen ($\mu_H^b = N_H\sigma_H^b$) is shown in Figure 2. This classical solution agrees well with a more detailed and exact solution for hydrogen.

## Absorption Coefficient for Heavier Atoms and Ions

The exact solution for the continuum absorption coefficient of atoms that are heavier than hydrogen is quite complex and, for most cases, is not available in the literature. Armstrong et al.[21] performed detailed nonhydrogenic quantum mechanical calculations of the continuum absorption coefficient for nitrogen and oxygen atoms, but because of the complexity of the calculations, their use in radiative transfer would require the results to be curve-fit to numerical data. Fortunately, Biberman and Norman[4] developed approximate analytical expressions for the continuum absorption coefficient resulting from free-free and bound-free transitions from excited states for a variety of atoms of low atomic number. Biberman's theory yields the following general expressions for the atomic continuum absorption coefficient:

when $0 < u < \epsilon_T$,

$$\mu_k = N_k \Gamma_k C_o Z_k^2 \theta \xi_k \, \mathrm{Exp}[(u-\epsilon_I)/\theta]/u^3; \tag{10}$$

and when $u > \epsilon_T$,

$$\mu_k = N_k \Gamma_k C_o Z_k^2 \theta \xi_k \, \mathrm{Exp}[(\epsilon_T-\epsilon_I)/\theta]/u^3 + N_k \Phi_k. \tag{11}$$

The quantity $\xi_k$ is Biberman's correction for a pure hydrogenic calculation, and it is presented for some atoms and ions in Figure 3. The parameter $\Gamma_k$ is defined by Biberman as

$$\Gamma_k = 2Q_k^+/Q_k \tag{12}$$

in which $Q_k$ and $Q_k^+$ are the partition functions for the parent atom and the residual ion respectively. Hoshizaki and Wilson[23, 24] obtained a better agreement between Biberman's approximate formula and Armstrong's detailed calculations[21] by setting

$$\Gamma_k = 2g_1^+/g_1. \tag{13}$$

In this equation, $g_1$ and $g_1^+$ are the statistical weights of the ground states of the parent atom and the residual ion respectively. The function $\Phi_k$ is the sum of the continuum cross section for bound-free absorption from low lying states, which are considered on an individual basis, such as

$$N_k \Phi_k = \sum_i N_{i,k} \sigma_i^b. \tag{14}$$

The continuum cross section for the bound-free absorption from low lying states of an atom or ion is approximated by treating the atom as a hydrogenlike atom. The hydro-

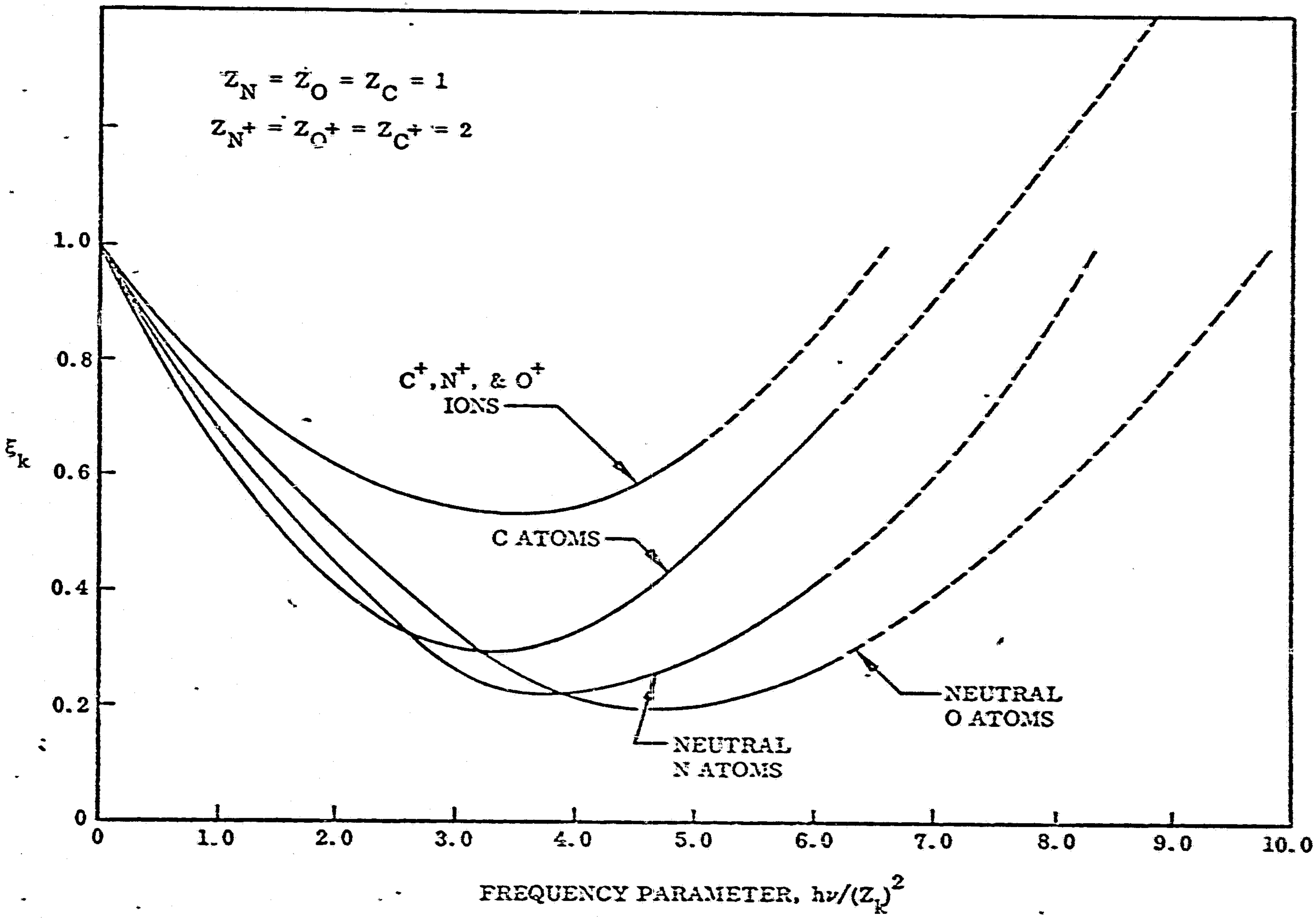


Fig. 3 Quantum - Mechanical Correction Factor

genlike approximation provides the following expression[19]:

$$\sigma_i^b = 1.99 * 10^{-14} Z_k^4/(n_i^5 u^3) \tag{15}$$

in which $n_i$ is the principal quantum number of energy state i, and $Z_k$ is the effective nuclear charge in the electron-neutral interaction of species k. For a hydrogenlike atom, these quantities are related by

$$Z_k^2 = n_i^2(\epsilon_I - \epsilon_i)/13.6. \tag{16}$$

By utilizing Eqs. (6, 14-16), the following relation can be obtained:

$$N_k\Phi_k = \sum_i 1.99 * 10^{-14} N_k g_i (u_i/13.6)^{2.5} \mathrm{Exp}(-\epsilon_i/\theta)/(u^3 Q_k Z_k). \tag{17}$$

In using the above relation with atoms and ions other than hydrogen, the spectral dependence, which appears in the denominator, $u^3$, can be eliminated by replacing it with $u_i^3$, which is the value at the photoionization edge of energy level i. This modification is justified[19], because complex atoms do not have the same spectral dependence as hydrogen atoms. Some atoms exhibit constant behavior as proposed herein, and others decrease with frequency but at a slower rate than the cubic behavior of hydrogen. In addition to the above modification, $u_i^3$ can be replaced by $u_i'^3$ to account for the reduction in the photoionization edges occasioned by the merging of line transitions near the series limit, which is considered to be a constant equivalent to 0.25 eV; $u_i' = u_i - 0.25$. Equation (17) becomes equivalent to

$$N_k\Phi_k = \sum_i 1.99 * 10^{-14} N_k g_i (u_i/13.6)^{2.5} \mathrm{Exp}(-\epsilon_i/\theta)/(Q_k Z_k u_i'^3) \tag{18}$$

The above expression is used with Eq. (11) to calculate the absorption coefficients of C, H, O, and N atoms and ions. Appropriate parameters and final correlations are summarized in the Appendix.

## Absorption Coefficient of Negative Ions

The bound-free photodetachment process, which also contributes to the continuum absorption of radiation, can be described by

$$A^- + h\nu = A + e^-. \tag{19}$$

The absorption coefficient for such a process can be expressed in terms of the absorption cross section and the number density by using

$$\mu_k^- = N_k^- \sigma_k^-. \tag{20}$$

It is convenient in some cases to express the absorption coefficient in terms of the number density of the neutral atom by utilizing the Saha equation[32]:

$$N_e N_k / N_k^- = 2(2\pi m_e \theta / h^2)^{1.5} Q_k \,\mathrm{Exp}(-I_k^- / \theta)\, Q_k^-. \tag{21}$$

Experimental and analytical photodetachment cross sections have been reported in the literature, and some, such as $H^-$, $O^-$, $N^-$, and $C^-$, are summarized in the following references: 25, 26 for $H^-$; 27, 10, 28 for $O^-$; 10, 29, 30 for $N^-$; and 31 for $C^-$. These results are discussed in more detail below and in the Appendix.

## RESULTS AND DISCUSSION

The formulation and calculation procedures, which are developed above, can be applied without any modification to calculate the continuum absorption cross section for both atoms and ions of H, O, C, and N. The detailed correlations and data for each species are included in the Appendix. In some of the cases, such as for positive ions and some atoms, the correlations can be modified to fit other reported data more accurately.

The normalized blackbody emissive power presented in Figure 4 can be used to demonstrate more clearly the frequency range that is of interest here. For example, at a temperature of 8000°K, the frequencies between 1 and 8 eV cover most of the emissive

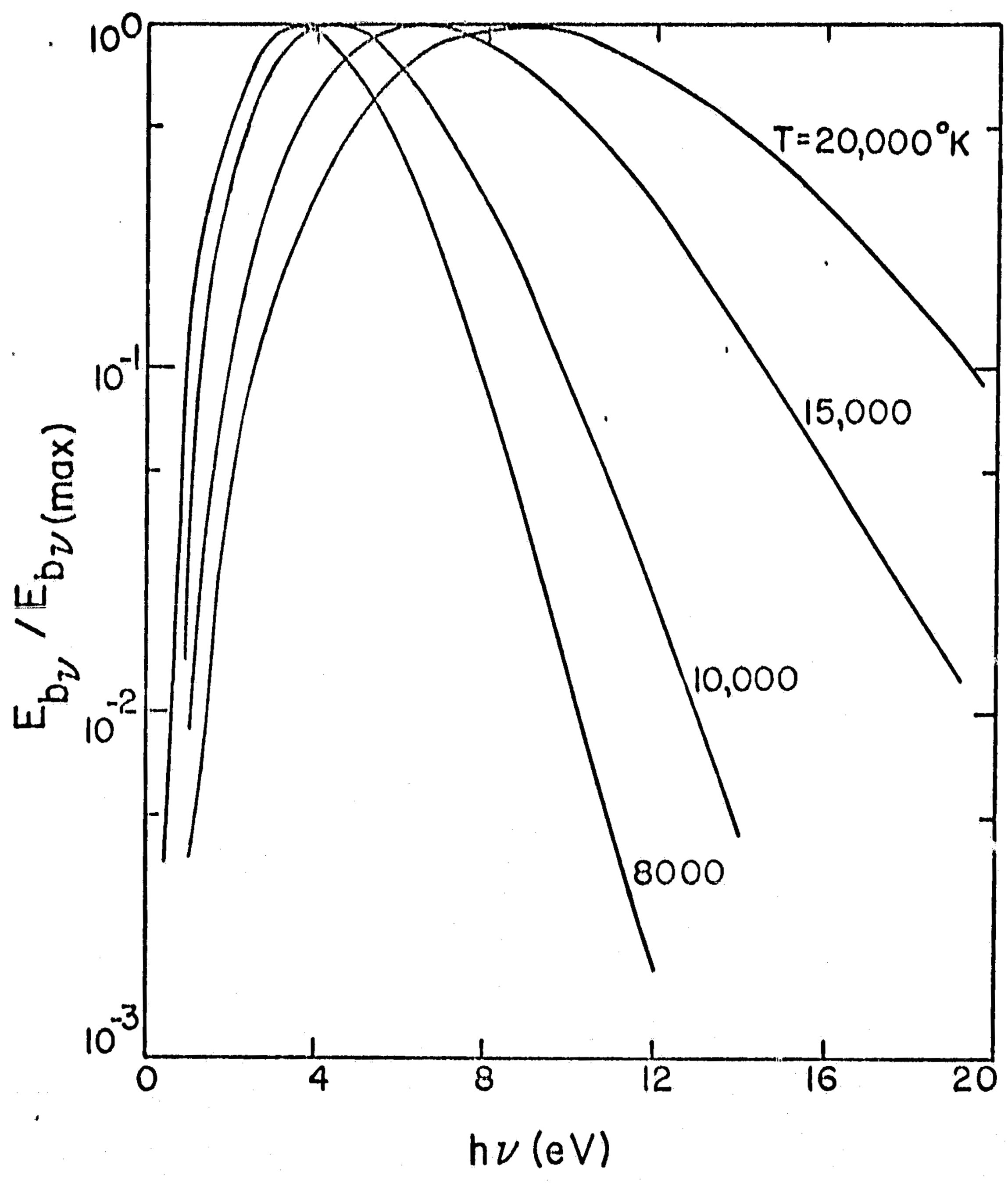


Fig. 4 Normalized blackbody emissive power distribution.

power range, whereas at 20,000°K, the important range extends from 2.5 to 20 eV. At these frequencies, the continuum absorption cross section can vary more than six orders of magnitude. The results for the hydrogen atom are presented in Figure 5 and are compared with existing experimental[9] and analytical data[3]. The difference between the present and reported results[3] is due to the reduction in the photoionization edge, $\Delta(h\nu) = 0.25$ eV, which is applied to all low lying energy states. The reported results also compare well with what could be predicted from the work of Zoby et al.[22].

The results for nitrogen, carbon, and oxygen atoms are presented in Figures 6 through 8. The results for nitrogen compare well with reported experimental[14,32,33] and analytical data[3,6]. On the other hand, significant differences exist between the present and the reported[3,6] calculations for carbon and oxygen. It appears that some low lying energy states were not included in the other calculations. This fact should be examined and confirmed by additional detailed calculations, because these differences could influence significantly the calculations of radiative transfer.

The results for the positive ions of nitrogen, carbon, and oxygen are presented in Figures 9 through 11. The results for the carbon ion compare very well with reported calculations[6], but this contribution has not been included in the program[3], and the contribution of the oxygen ion also has not been included. Present calculations for these two ions compare well with reported detailed calculations[6] up to frequencies of 12 eV. At higher frequencies, the direct application of the proposed method fails, but it could (as was done) empirically be modified to agree with reported data.

The contributions of the negative ions have not been modified by the present study, because the reported values[3] are the best available. The values for the hydrogen and oxygen

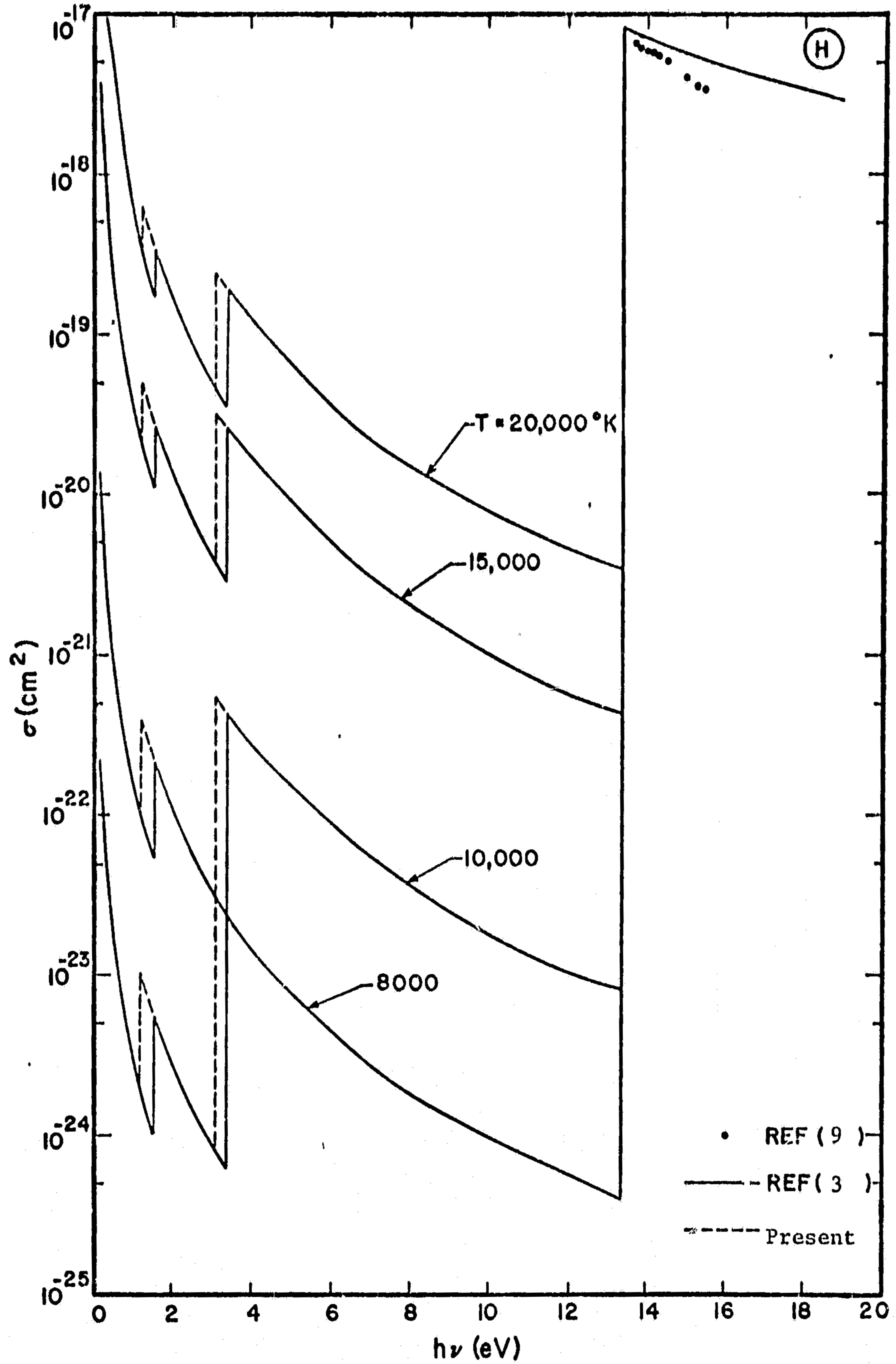


Fig. 5 Continuum absorption cross section of Hydrogen atom.

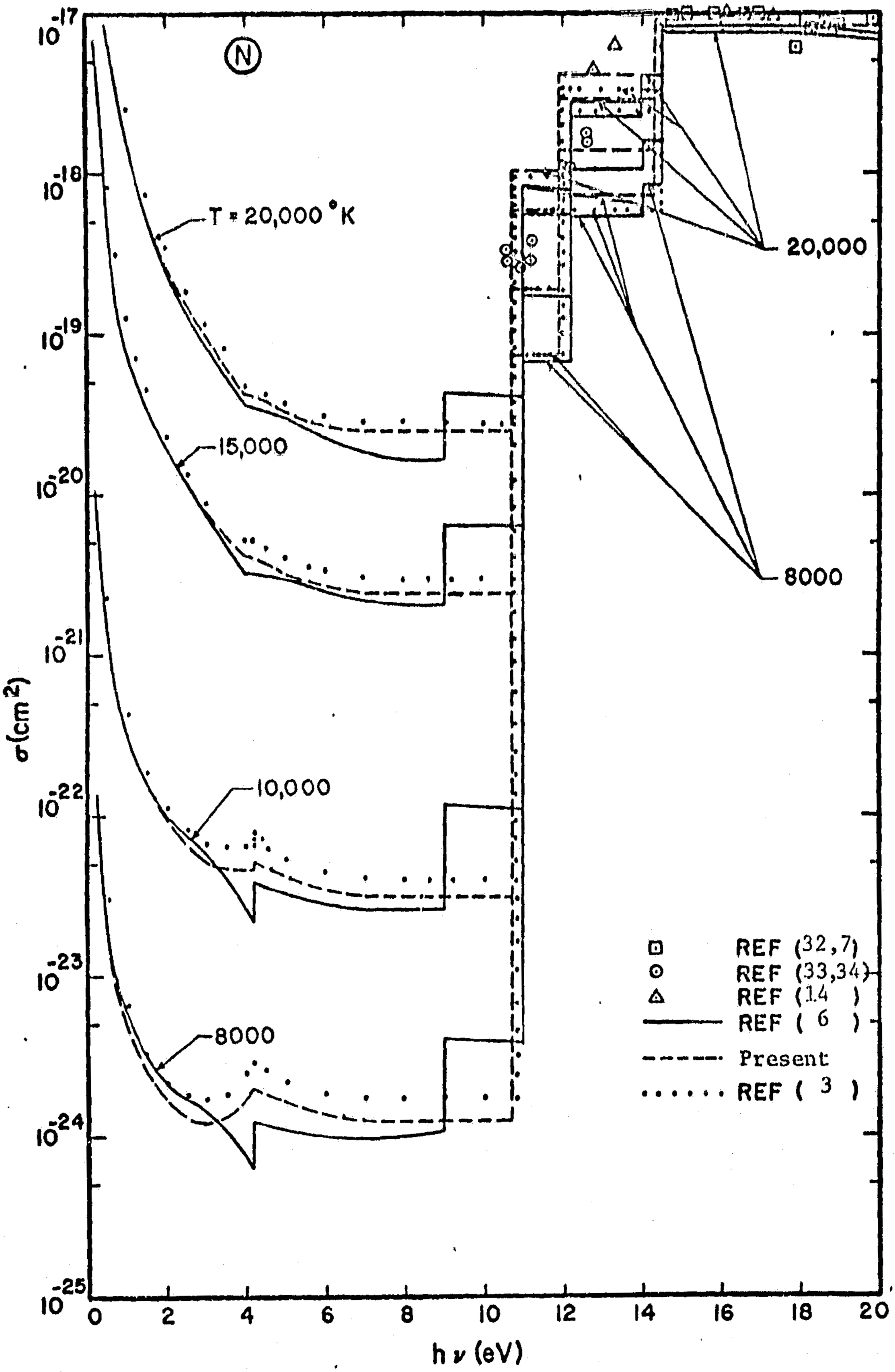


Fig. 6 Continuum absorption cross section for Nitrogen atom.

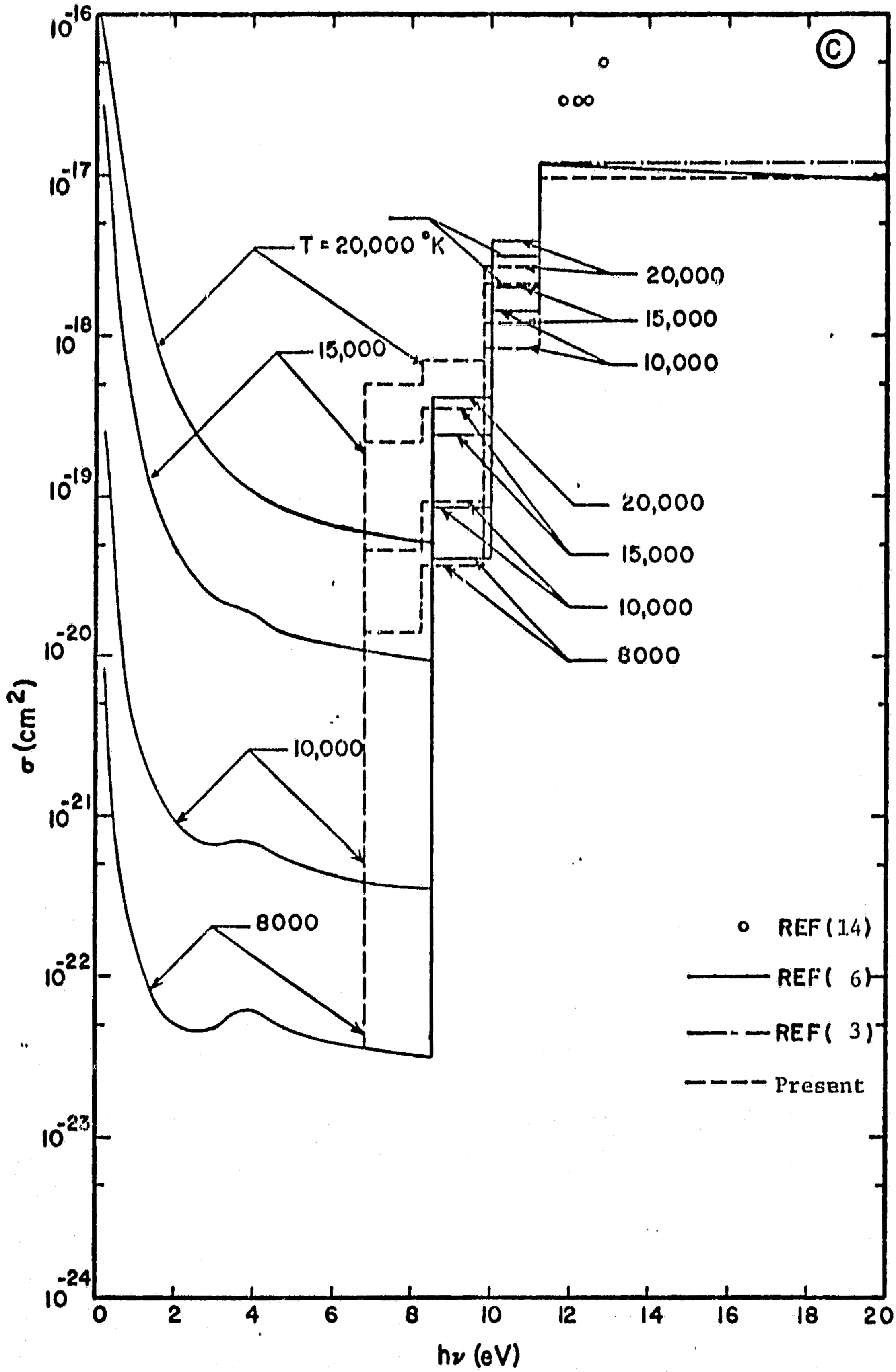


Fig. 7 Continuum absorption cross section for Carbon atom.

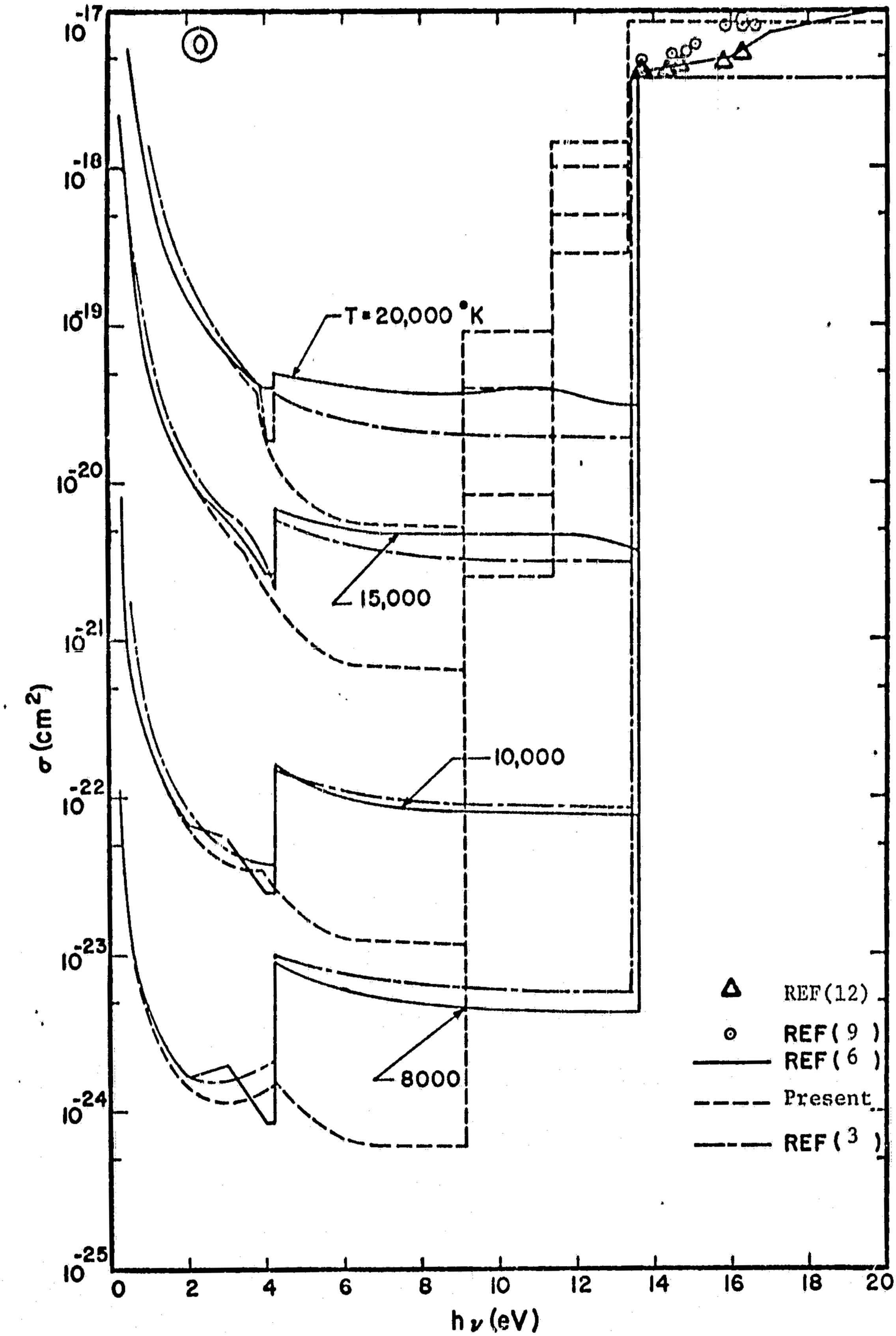


Fig. 8 Continuum absorption cross section for Oxygen atom.

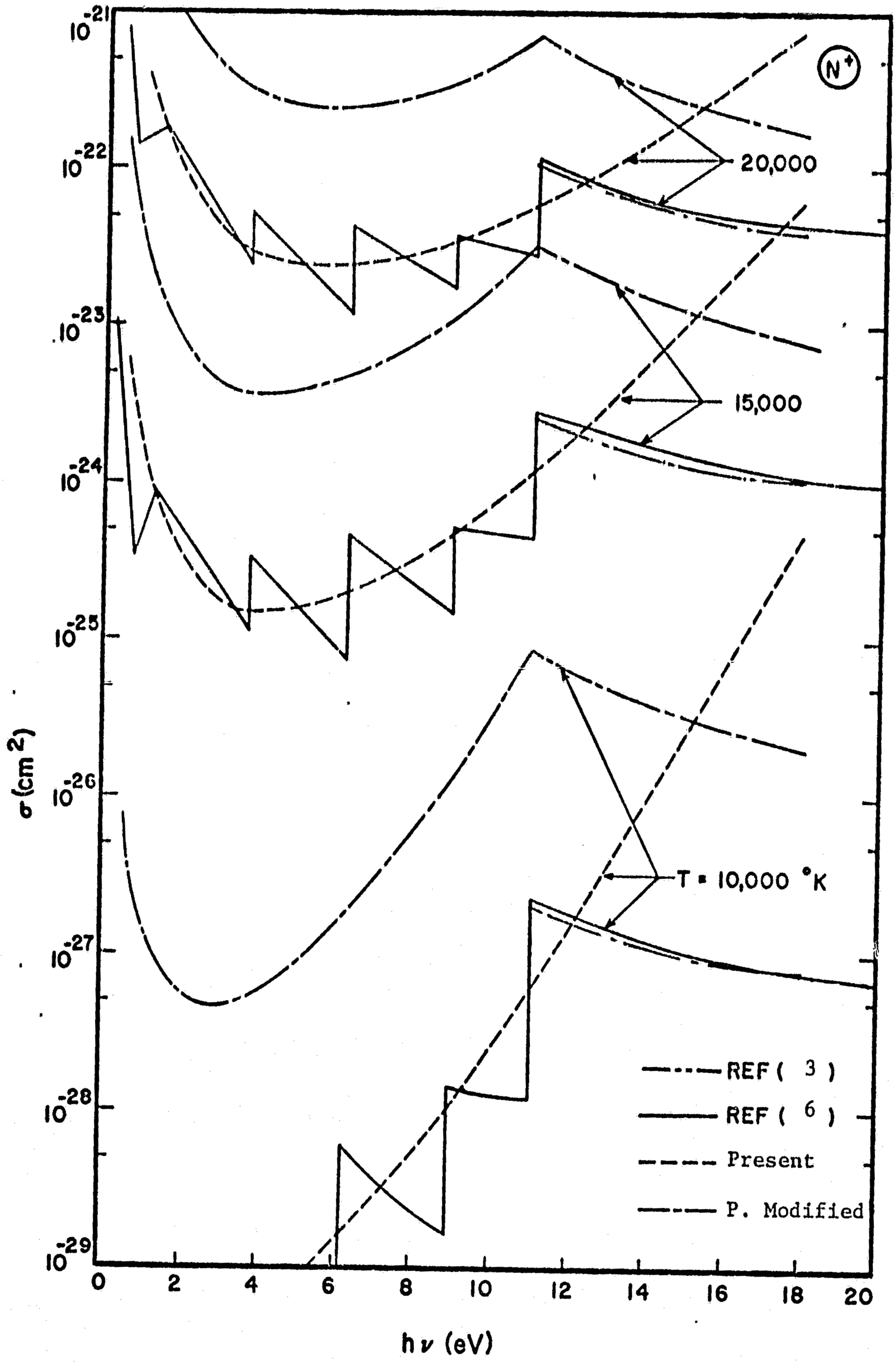


Fig. 9 Continuum absorption cross section for Nitrogen ion.

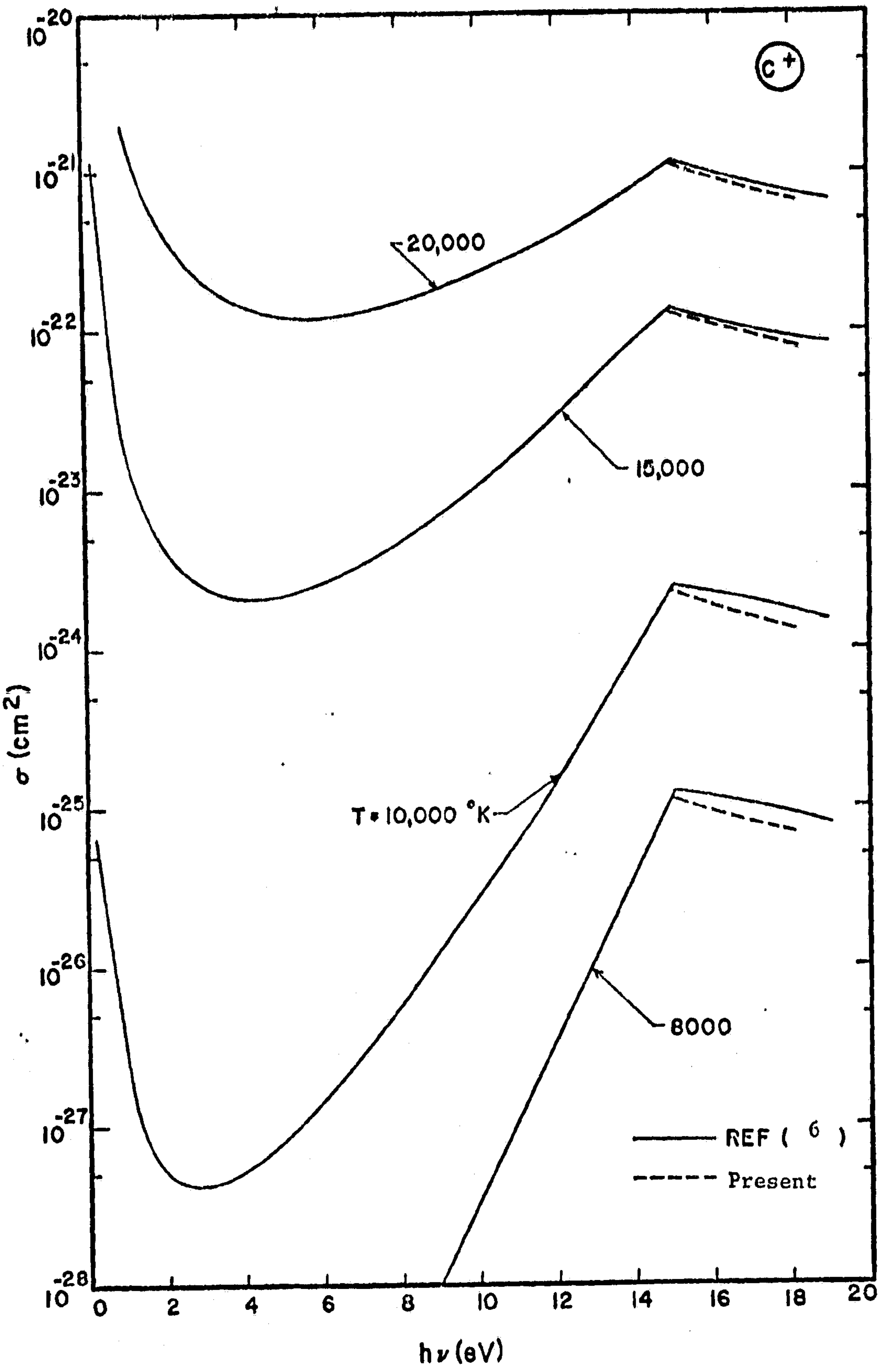


Fig. 10 Continuum absorption cross section for Carbon ion.

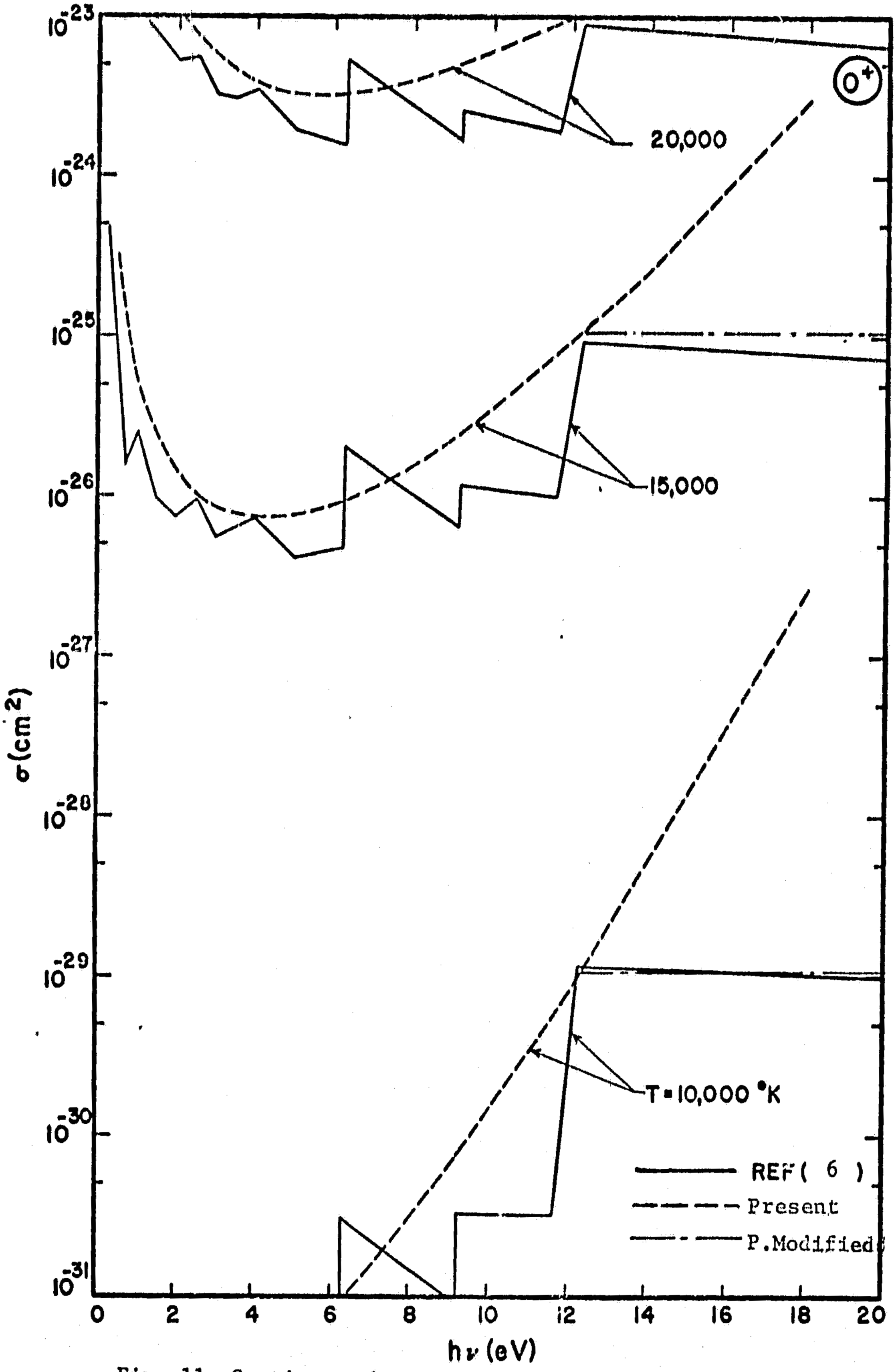


Fig. 11 Continuum absorption cross section for Oxygen ion.

ions are presented in Figures 12 and 13, and the values for carbon and nitrogen are considered to be constants at $1.4 \times 10^{-17}$ and $1.6 \times 10^{-16}$ $cm^2$ respectively. It should be noted that these cross sections are expressed in terms of the number density of the negative ions and are independent of temperature. Saha's equation can be used to express these results in terms of the number density of the parent atom, and such results are shown in Figure 14 for carbon.

The detailed numerical results of Thomas and Helliwell[7] cannot be used for comparison, because they cover a higher frequency range. The approximate calculation procedure of Sibulkin[5] is not as accurate as the one used in this study[4]. Henry's correlation[8] has been evaluated at the threshold frequency of the ground state of atoms (modified by Boltzmann relation) and compared with present results. For atoms, such a comparison can be made and the result is in good agreement, but for ions, the threshold frequencies of the ground states are outside the range of interest.

## CONCLUSIONS

A detailed procedure for calculating the continuum absorption cross section of atoms and ions is presented and compared with existing experimental and analytical results. The procedure uses Biberman's[4] approximate method to account for the combined contributions of free-free and the bound-free transitions. The accuracy of the available computer code[3] is evaluated by being compared with present and available data, and it appears that for most cases these results compare favorably except in some cases as indicated below.

1. The program[3] does not include contributions from the $C^+$ and $O^+$ ions.

   The contributions of these ions have been added in the present study.

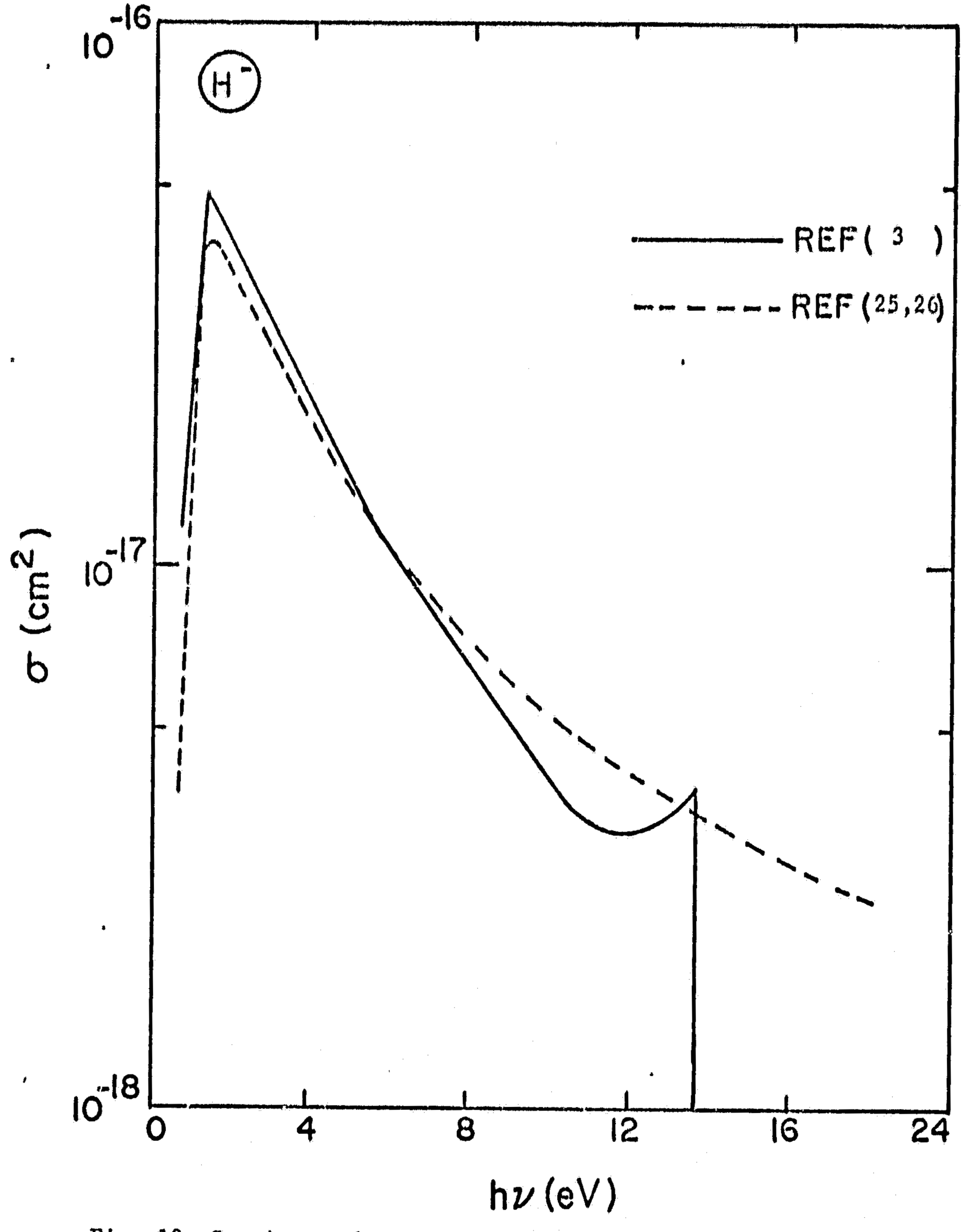


Fig. 12 Continuum absorption cross section of negative Hydrogen ion ($\mu_{H^-} = N_{H^-} \sigma_{H^-}$)

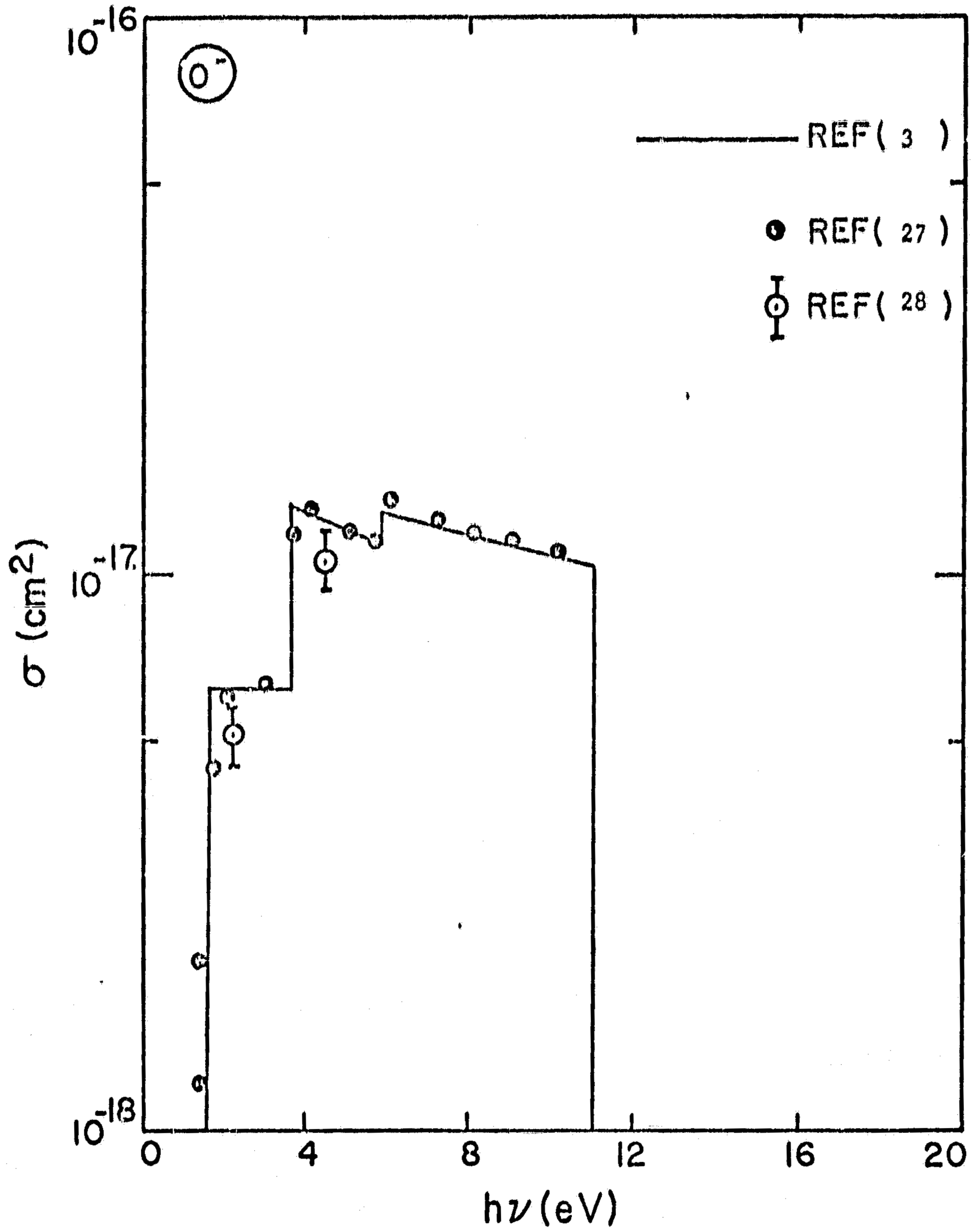


Fig. 13 Continuum absorption cross section for negative Oxygen ion ($\mu_{O^-}^- = N_{O^-} \sigma_{O^-}$)

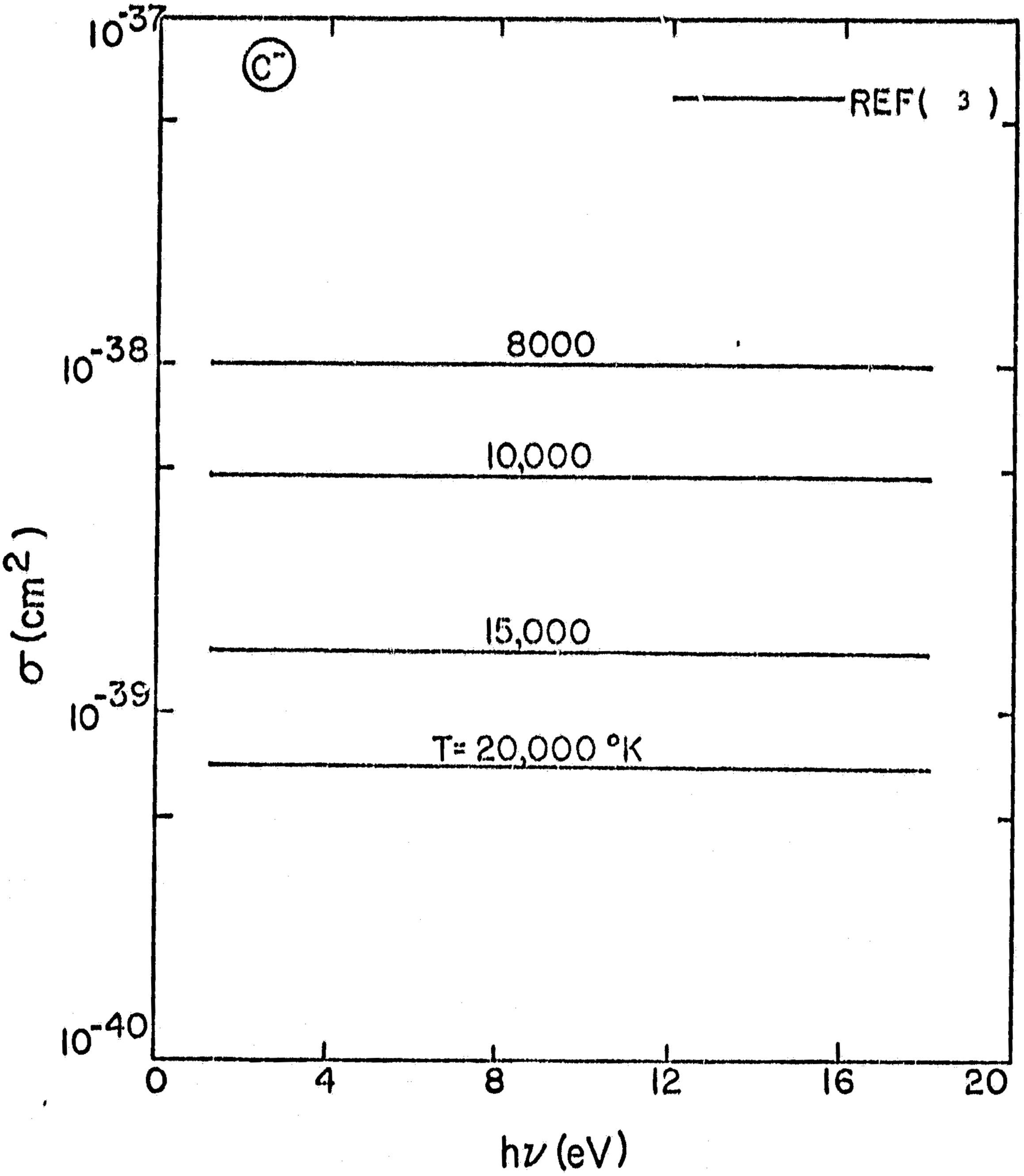


Fig. 14 Continuum absorption cross section for negative Carbon ion ($\mu^{-}_{C^-} = N_e N_C \sigma_{C^-}$)

2. The program[3] predicts a continuum absorption cross section for the $N^+$ ion, which is one or two orders of magnitude larger than reported data[6]. New correlations have been added in this study to improve this prediction.

3. The program[3] predicts results for O and N atoms that are significantly different than the results predicted in the present study. An additional detailed check should be performed to confirm this difference, because it could significantly affect the radiative transfer calculations.

4. The program[3] does not apply the reduction of the photoionization edges to all low lying energy states. This reduction has been applied in the present study.

In general, one can conclude that the direct application of Biberman's method[4] predicts with reasonable accuracy the contributions of the atoms, but the method needs to be empirically modified to predict accurately the contribution of the ions at high frequencies. Additional experimental data are needed for the lower frequencies, 6 through 10 eV, in order for one to evaluate the accuracy of available analytical results.

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## APPENDIX

### Correlations for the Continuum Absorption Coefficients of Atoms and Ions

The continuum absorption coefficients for neutral and ionized atoms were calculated by using Eqs. (7) and (8), which are repeated below:

$0 < u < \epsilon_T$

$$\mu_k = N_k \Gamma_k C_o Z_k^2 \theta \xi_k \, \mathrm{Exp}[(u-\epsilon_I)/\theta]/u^3$$

$u > \epsilon_T$

$$\mu_k = N_k \Gamma_k C_o Z_k^2 \theta \xi_k \, \mathrm{Exp}[(\epsilon_T - \epsilon_I)/\theta]/u^3$$

$$+ \sum_i 1.99 * 10^{-14} N_k g_i (u_i/13.6)^{2.5} \, \mathrm{Exp}(-\epsilon_i/\theta)/(Q_k Z_k u'^3)$$

The nomenclature for the above equations is given at the beginning of the attached report.

The above equations were applied to calculate the continuum absorption coefficients of neutral and ionized atoms of H, C, N, and O. The resulting expressions are presented below and are applicable for $u < 20$(eV).

The energy levels and the statistical weights of neutral and ionized atoms[8] that were used in the above equations to evaluate the contributions of low lying states are also tabulated. The reduction in the photoionization edges that is due primarily to the merging of line transitions near the series limit was considered constant and equivalent to 0.25 eV[7].

### EQUATIONS AND DATA FOR C

$Z_c = 1$ $\qquad \epsilon_T = 3.78$ $\qquad \Gamma_c = 1.33$ $\qquad \epsilon_I = 11.26$

| $\epsilon_i$ | $g_i$ | $u_i$ | $u'_i$ |
|---|---|---|---|
| 0 | 9 | 11.26 | 11.01 |
| 1.264 | 5 | 9.996 | 9.75 |
| 2.684 | 1 | 8.576 | 8.33 |
| 4.1825 | 5 | 7.077 | 6.83 |
| 7.9461 | 15 | 3.314 | 3.064 |

$0 < u < 3.78$

$$(\mu_c)_1 = 9.64 * 10^{-16} N_c \theta \xi_c \operatorname{Exp}[(u - 11.26)/\theta]/u^3$$

$3.78 < u < 6.83$

$$(\mu_c)_2 = 9.64 * 10^{-16} N_c \theta \xi_c \operatorname{Exp}(-7.48/\theta)/u^3$$

$6.83 < u < 8.33$

$$(\mu_c)_3 = (\mu_c)_2 + 6.10 * 10^{-17} N_c \operatorname{Exp}(-4.18/\theta)/Q_c$$

$8.33 < u < 9.75$

$$(\mu_c)_4 = (\mu_c)_3 + 1.09 * 10^{-17} N_c \operatorname{Exp}(-2.684/\theta)/Q_c$$

$9.75 < u < 11.01$

$$(\mu_c)_5 = (\mu_c)_4 + 4.97 * 10^{-17} N_c \operatorname{Exp}(-1.264/\theta)/Q_c$$

$u > 11.01$

$$(\mu_c)_6 = (\mu_c)_5 + 8.39 * 10^{-17} N_c/Q_c$$

EQUATIONS AND DATA FOR $C^+$

$Z_c{}^+ = 2$ $\qquad \epsilon_I = 24.4$ $\qquad \Gamma_c{}^+ = 0.333$ $\qquad \epsilon_T = 14.9$

| $\epsilon_i$ | $g_i$ | $u_i$ | $u'_i$ |
|---|---|---|---|
| 0 | 6 | 24.4 | 24.15 |
| 5.335 | 12 | 19.06 | 18.81 |
| 9.29 | 10 | 15.1 | 14.85 |
| 11.96 | 2 | 12.44 | 12.19 |

---

$0 < u < 14.9$

$$(\mu_c+)_1 = 9.657 * 10^{-16}(N_c+)\theta(\xi_c+)\ \mathrm{Exp}[u - 24.4)/\theta]/u^3$$

$14.9 < u < 18.81$

$$(\mu_c+)_2 = 9.657 * 10^{-16}\ (N_c+)\theta(\xi_c+)\ \mathrm{Exp}(-9.5/\theta)/u^3$$

$18.81 < u < 24.15$

$$(\mu_c+)_3 = (\mu_c+)_2 + 4.17 * 10^{-17}(N_c+)\mathrm{Exp}(-5.335/\theta)/Q_c+$$

EQUATIONS AND DATA FOR N

$Z_N = 1$ $\epsilon_I = 14.54$ $\Gamma_N = 4.5$ $\epsilon_T = 4.22$

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| $\epsilon_i$ | $g_i$ | $u_i$ | $u'_i$ |
|---|---|---|---|
| 0 | 4 | 14.54 | 14.29 |
| 2.384 | 10 | 12.16 | 11.91 |
| 3.576 | 6 | 10.96 | 10.71 |
| 10.93 | 12 | 3.61 | 3.36 |

---

$0 < u < 4.22$

$$(\mu_N)_1 = 32.625 * 10^{-16}N_N\theta\xi_N\ \mathrm{Exp}[(u - 14.54)/\theta]/u^3$$

$4.22 < u < 10.71$

$$(\mu_N)_2 = 32.625 * 10^{-16}N_N\theta\xi_N\ \mathrm{Exp}(-10.32/\theta)/u^3$$

$10.71 < u < 11.91$

$$(\mu_N)_3 = (\mu_N)_2 + 5.66 * 10^{-17} N_N \, Exp(-3.58/\theta)/Q_N$$

$11.91 < u < 14.29$

$$(\mu_N)_4 = (\mu_N)_3 + 8.9 * 10^{-17} N_N \, Exp(-2.384/\theta)/Q_N$$

$u > 14.29$

$$(\mu_N)_5 = (\mu_N)_4 + 3.22 * 10^{-17} N_N/Q_N$$

## EQUATIONS AND DATA FOR $N^+$

$Z_{N^+} = 2$ $\qquad \epsilon_I = 29.6$ $\qquad \Gamma_{N^+} = 1.33$ $\qquad \epsilon_T = 18.1$

| $\epsilon_i$ | $g_i$ | $u_i$ | $u'_i$ |
|---|---|---|---|
| 0 | 9 | 29.6 | 29.35 |
| 1.899 | 5 | 27.7 | 27.45 |
| 4.053 | 1 | 25.55 | 25.3 |
| 5.8 | 5 | 23.8 | 23.55 |
| 11.43 | 15 | 18.17 | 17.92 |
| 13.541 | 9 | 16.06 | 15.81 |

$0 < u < 18.1$

$$(\mu_{N^+})_1 = 38.57 * 10^{-16} (N_{N^+}) \theta (\xi_{N^+}) \, Exp[(u-29.6)/\theta]/u^3$$

$18.1 < u < 23.55$

$$(\mu_{N^+})_2 = 38.57 * 10^{-16} (N_{N^+}) \theta (\xi_{N^+}) \, Exp(-11.5/\theta)/u^3$$

The above correlations could be modified to fit more accurately reported calculations[6].

The modified correlations are presented below.

$0 < u < 11$

$$(\mu_{N^+})_1 = 3.857 * 10^{-15} (N_{N^+}) \theta (\xi_{N^+}) \, Exp[(u - 29.6)/\theta]/u^3$$

$u > 11$

$$(\mu_{N^+})_2 = 1.052 * 10^{-15} (N_{N^+})(\xi_{N^+}) \, \mathrm{Exp}(-18.6/\theta)/u^2$$

EQUATIONS AND DATA FOR 0

$Z_0 = 1$ $\epsilon_I = 13.61$ $\Gamma_0 = 0.888$ $\epsilon_T = 4.25$

| $\epsilon_i$ | $g_i$ | $u_i$ | $u_i'$ |
|---|---|---|---|
| 0 | 9 | 13.61 | 13.35 |
| 1.967 | 5 | 11.64 | 11.39 |
| 4.189 | 1 | 9.42 | 9.171 |
| 9.28 | 8 | 4.33 | 4.08 |

$0 < u < 4.25$

$$(\mu_0)_1 = 9.643 * 10^{-16} N_0 \theta \xi_0 \, \mathrm{Exp}[(u - 13.61)/\theta]/u^3$$

$4.25 < u < 9.171$

$$(\mu_0)_2 = 9.643 * 10^{-16} N_0 \theta \xi_0 \, \mathrm{Exp}(-9.36/\theta)/u^3$$

$9.171 < u < 11.39$

$$(\mu_0)_3 = (\mu_0)_2 + 1.03 * 10^{-17} N_0 \, \mathrm{Exp}(-4.189/\theta)/Q_0$$

$11.39 < u < 13.35$

$$(\mu_0)_4 = (\mu_0)_3 + 4.56 * 10^{-17} N_0 \, \mathrm{Exp}(-1.967/\theta)/Q_0$$

$u > 13.35$

$$(\mu_0)_5 = (\mu_0)_4 + 7.54 * 10^{-17} N_0/Q_0$$

The above correlations could be modified to fit more accurately reported calculations[6] and experimental results[9,12]. The modified correlations are presented below.

$0 < u < 4.22$

$$(\mu_0)_1 = 6.44 * 10^{-16} N_0 \theta \xi_0 \operatorname{Exp}[(u - 13.4)/\theta]/u^3$$

$4.22 < u < 13.4$

$$(\mu_0)_2 = (\mu_0)_1 * \operatorname{Exp}[(4.22 - u)/\theta]$$

$$+ 6.5 * 10^{-18} N_0 (8/Q_0) \operatorname{Exp}(-9.28/\theta) [1 + 0.09375(u - 4.22)$$

$$+ 0.00586 * (u - 4.22)^2]$$

$u > 13.4$

$$(\mu_0)_3 = (\mu_0)_2 + 3.6 * 10^{-17} N_0/Q_0 + (u - 13.3) * 10^{-18}$$

EQUATIONS AND DATA FOR $0^+$

$Z_0{+} = 2$ $\epsilon_I = 35.1$ $\Gamma_0{+} = 4.5$ $\epsilon_T = 20.4$

| $\epsilon_i$ | $g_i$ | $u_i$ | $u'_i$ |
|---|---|---|---|
| 0 | 4 | 35.1 | 34.85 |
| 3.325 | 10 | 31.775 | 31.525 |
| 5.02 | 6 | 30.08 | 29.78 |
| 14.87 | 12 | 20.23 | 19.98 |
| 20.57 | 10 | 14.53 | 14.28 |

$0 < u < 20.4$

$$(\mu_0{+})_1 = 130.5 * 10^{-16} (N_0{+}) \theta (\xi_0{+}) \operatorname{Exp}[(u - 35.1)/\theta]/u^3$$

The above correlations could be modified to fit more accurately reported calculations.

$0 < u < 12.3$

$$(\mu_0{+}) = 1.305 * 10^{-14} (N_0{+})(\xi_0{+}) \theta \operatorname{Exp}[(u - 35.1)/\theta] u^3$$

$u > 13.3$

$$(\mu_0^{+}) = 7.01 * 10^{-18}(N_0^{+})(\xi_0^{+})\theta \,\mathrm{Exp}(-22.8/\theta)$$

EQUATIONS AND DATA FOR H

$Z_H = 1$ $\epsilon_I = 13.6$ $\Gamma_H = 1$ $\epsilon_T = 0.8$

| $\epsilon_i$ | $g_i$ | $u_i$ | $u_i'$ |
|---|---|---|---|
| 0 | 2 | 13.6 | 13.35 |
| 10.2 | 8 | 3.4 | 3.15 |
| 12.08 | 18 | 1.51 | 1.26 |
| 12.75 | 32 | 0.85 | 0.6 |
| 13.056 | 50 | 0.544 | 0.294 |

$0 < u < 0.8$

$$(\mu_H)_1 = 7.25 * 10^{-16} N_H \theta \,\mathrm{Exp}[(u - 13.6)/\theta]/u^3$$

$0.8 < u < 1.26$

$$(\mu_H)_2 = 7.25 * 10^{-16} N_H \theta \,\mathrm{Exp}(-12.8/\theta)/u^3$$

$1.26 < u < 3.15$

$$(\mu_H)_3 = (\mu_H)_2 + 1.49 * 10^{-15} N_H \,\mathrm{Exp}(-12.08/\theta)/(Q_H u^3)$$

$3.15 < u < 13.35$

$$(\mu_H)_4 = (\mu_H)_3 + 4.975 * 10^{-15} N_H \,\mathrm{Exp}(-10.2/\theta)/(Q_H u^3)$$

$u > 13.35$

$$(\mu_H)_5 = (\mu_H)_4 + 3.98 * 10^{-14} N_H/(Q_H u^3)$$

Correlations for the continuum absorption cross section of negative ions are presented below. These correlations are identical with the ones used by Nicolet[3].

## EQUATIONS AND DATA FOR $H^-$

$0 < u < 0.75$

$$\mu^{-}_{H^-} = 0.0$$

$0.75 < u < 1.3$

$$\mu^{-}_{H^-} = (N_{H^-}) * 10^{-17} * (-4.51 + 7.15\, u)$$

$1.3 < u < 6$

$$\mu^{-}_{H^-} = (N_{H^-}) * 10^{-17} * (6.765 - 1.7u + 0.1258\, u^2)$$

$6 < u < 13.6$

$$\mu^{-}_{H^-} = (N_{H^-}) * 10^{-17} * (3.5 - 0.535\, u + 0.0225\, u^2)$$

$u > 13.6$

$$\mu^{-}_{H^-} = 0.0$$

## EQUATIONS AND DATA FOR $0^-$

$0 < u < 1.5$

$$\mu^{-}_{0^-} = 0.0$$

$1.5 < u < 3.5$

$$\mu^{-}_{0^-} = 6.2 * 10^{-18} (N_{0^-})$$

$3.5 < u < 5.7$

$$\mu^{-}_{0^-} = 10^{-18} (N_{0^-})(16.16 - 0.818\, u)$$

$5.7 < u < 11$

$$\mu^{-}_{0^-} = 10^{-18} (N_{0^-})(15.58 - 0.453\, u)$$

$u > 11$

$$\mu^{-}_{0^-} = 0.0$$

## EQUATIONS AND DATA FOR $C^-$

$0 < u < 1.25$

$$\bar{\mu}_{C^-} = 0.0$$

$u > 1.25$

$$\bar{\mu}_{C^-} = 1.4 * 10^{-17}(N_{C^-})$$

The above results could be expressed in terms of the number density of the atomic species by utilizing Saha's equation. The resulting correlations are presented below.

$0 < u < 1.25$

$$\bar{\mu}_{C^-} = 0.0$$

$u > 1.25$

$$\bar{\mu}_{C^-} = 1.159 * 10^{-32}(N_C N_e)/[T^{1.5}Q_C \text{ Exp}(-1.25/\theta)]$$

## EQUATIONS AND DATA FOR $N^-$

$0 < u < 1.22$

$$\bar{\mu}_{N^-} = 0.0$$

$u > 1.22$

$$\bar{\mu}_{N^-} = 1.6 * 10^{-16}(N_{N^-})$$

The above could also be expressed in terms of the number density of the atomic specie similar to the $C^-$ case. The resulting correlations are presented below.

$0 < u < 1.22$

$$\bar{\mu}_{N^-} = 0.0$$

$u > 1.22$

$$\bar{\mu}_{N^-} = 2.898 * 10^{-33} N_N N_e \, Q_{N^-}/[T^{1.5}Q_N \text{ Exp}(-1.22/\theta]$$