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Approximation Methods for Combined Thermal/Structural Design

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SUMMARY

Two approximation concepts for combined thermal/structural design are evaluated. The first concept is an approximate thermal analysis based on the first derivatives of structural temperatures with respect to design variables. Two commonly used first-order Taylor series expansions are examined. The direct and reciprocal expansions are shown to be special members of a general family of approximations, and it is shown that for some conditions other members of that family of approximations are more accurate. Several examples are used to compare the accuracy of the different expansions.

The second approximation concept is the use of critical time points for combined thermal and stress analyses of structures with transient loading conditions. It is shown that significant time savings may be realized by identifying critical time points and performing the stress analysis for those points only. This approach is used to design an insulated panel which is exposed to transient heating conditions.

INTRODUCTION

One of the major obstacles to the widespread use of automated structural design (member sizing) procedures is the need for large computational resources to perform repeated analyses of a structure throughout the design process. To alleviate this problem, it is now common practice during major portions of the design process to use approximations for response functions such as displacements, stresses, buckling loads, or flutter speeds. (See refs. 1 to 6.) Such approximations are often based on the first derivatives of the response functions with respect to the design variables. One obvious technique in the use of these approximations is to employ a first-order Taylor series (i.e., a linear approximation), herein called the direct expansion. An alternate approach is to approximate the desired function by a first-order Taylor series in the reciprocals of the design variables. Such a technique, herein called the reciprocal expansion, is exact for stresses and displacements of statically determinate trusses and has been shown to be more accurate than the direct expansion for other structures (ref. 5).

Another useful approximation concept is that of constraint deletion (e.g., ref. 1). Constraints which are far from being critical are ignored during parts of this design process. Periodic updates are made of the number of constraints that are retained. For constraints which are functions of a parameter such as time (parametric constraints), the concept of constraint deletion can be extended (ref. 7). When the dependence on the parameter is sufficiently smooth, critical values of that parameter (critical points) may be identified. The constraints for other values of the parameter may be ignored; however, the values of the critical points are updated periodically as the design changes.

This paper is concerned with application of the two concepts, response approximation and critical points, to combined thermal/structural design. Both direct and reciprocal expansions of the response functions are investigated for accuracy in predicting temperature fields for steady-state and transient thermal problems. The time savings associated with the use of these expansions are also evaluated with the aid of two examples; one of a steady-state response and the other of transient thermal response. In the second example, use is also made of the second approximation concept (i.e., critical time points), and an investigation is made of the resulting accuracy and time savings.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

a_1, a_2, a_3 bar element areas, mm^2 (in^2)

E Young's modulus, Pa (psi)

f(V) response function

$f_D(V)$ direct approximation to function f(V)

$f_R(V)$ reciprocal approximation to function f(V)

g constraint function

g_c critical constraint function

k thermal conductivity, W/m-°C (Btu-in/sec-ft^2-°F)

m(V) mass of structure, kg (lb)

n_c number of constraints

n_v number of design variables

n_τ number of time points

N_x, N_y, N_{xy} applied loads, kN/m (lbf/in.)

$\dot{Q}$ heat rate, W/m^2 (Btu/in^2-sec)

t thickness, mm (in.)

t_1, t_2, t_3, t_4 plate element thicknesses, mm (in.)

t_0, t_{45}, t_{90} lamina thicknesses, mm (in.)

T temperature, °C (°F)

T_a allowable temperature, °C (°F)

T_{eq} transient, surface equilibrium temperature, °C (°F)

v_i ith design variable

v_{mi} offset value of ith design variable

V vector of design variables

α coefficient of thermal expansion, per °C (per °F)

ρ mass density, kg/m^3 (lb/in^3)

τ time, sec

τ_c critical time, sec

Subscripts:

max maximum

o initial value

APPROXIMATION CONCEPTS

First-Order Taylor Series Expansions of Response Functions

The design problem considered herein is to find a vector of design variables V (with components v_j; $j = 1, 2, \ldots, n_v$) that minimizes the mass $m(V)$ of a structure and/or its thermal protection system subject to a set of n_c constraints

$$g_i(V,\tau) > 0 \qquad (i = 1, 2, \ldots, n_c; \quad 0 < \tau < \tau_{max}) \tag{1}$$

where τ is time. The constraint functions g_i represent design limits on response functions such as displacements, stresses, or temperatures. Many algorithms for resizing the structure require evaluation of the derivatives of the response functions with respect to the design variables. These derivatives may be used to obtain approximations for the response functions which can replace the full analysis during parts of the design process. One approximation for a response function $f(V)$, which is based on the value of the function and its derivatives at a design point V_o, is a first-order Taylor series expansion

$$f_D(V) = f(V_o) + \sum_{j=1}^{n_v} \frac{\partial f(V_o)}{\partial v_j}(v_j - v_{oj}) \tag{2}$$

where v_{oj} are the components of V_o. This approximation is herein called the direct expansion. An alternate expression (e.g., refs. 1 to 3, 5, and 6), herein called the reciprocal expansion, is a first-order Taylor series in the reciprocals of the design variables. It can be written as

$$f_R(V) = f(V_o) + \sum_{j=1}^{n} \frac{\partial f(V_o)}{\partial v_j} \frac{v_{oj}}{v_j}(v_j - v_{oj}) \tag{3}$$

The reciprocal expansion was originally proposed for approximating stresses and displacements in structural analyses. For statically determinate trusses, stresses and displacements are indeed linear functions of the reciprocals of the cross-sectional areas of the members. For statically indeterminate structures, the reciprocal expansion is not exact, but studies (ref. 5) have shown it to be superior to the direct expansion for a number of problems. The reciprocal expansion also has the added attraction that it is usually more conservative than the direct expansion in estimating response functions such as stresses, displacements, or temperatures (ref. 2). If $f(V)$ is the temperature at a point and V is a vector of insulation thicknesses, it may be expected that $\partial f(V)/\partial v_j < 0$. Since all design variables are positive, it is easy to check from equations (2) and (3) that $f_R(V) > f_D(V)$.

The reciprocal expansion predicts an infinite value for $f(V)$ when any one of the variables v_j goes to zero. It is, therefore, especially suitable for approximating functions, such as stresses and displacements in a statically determinate truss, which go to infinity when the member areas go to zero. Approximation of stresses in redundant structures (or temperatures in situations where multiple heat paths exist) is slightly different in that several variables must vanish simultaneously for $f(V)$ to become infinite.

Some understanding of the relative performance of the direct and reciprocal expansions for a redundant structure may be gained by an examination of the following simple example:

$$f(V) = \frac{11}{10v_1 + v_2} \tag{4}$$

This example illustrates a redundant situation where one of the design variables has a much larger effect than the other. Assume that $f(V)$ is approximated by using its value and derivatives at $v_{o1} = v_{o2} = 1.0$. The direct and reciprocal expansions for several other values of v_1 and v_2 are contained in table 1. Two conclusions may be drawn from table 1. First, the reciprocal expansion is better for approximating $f(V)$ for changes in the dominant design variable v_1 but not as good for approximations with changes in the other design variable v_2. Second, the reciprocal expansion is much better with changes in the design variables which reinforce each other ($v_1 = v_2 = 2$) than with changes that cancel each other ($v_1 = 0.5$, $v_2 = 6$).

Generalized Approximation

The fact that the function $f(V)$ becomes very large only when several variables go to zero simultaneously can be accounted for by the following more general form of the approximation:

$$f(V) \cong f(V_o) + \sum_{i=1}^{n_V} (v_i - v_{oi}) \frac{v_{oi} - v_{mi}}{v_i - v_{mi}} \frac{\partial f}{\partial v_i}(V_o) \qquad (5)$$

where v_{mi} is the ith component of a vector of constants chosen to account for the fact that as one variable goes to zero the function remains finite because other variables may have nonzero values. The direct and reciprocal expansions are special conditions of equation (5) with the direct expansion corresponding to $v_{mi} \to -\infty$ and the reciprocal expansion corresponding to $v_{mi} = 0$. Because the approximation becomes unbounded when $v_i = v_{mi}$, v_{mi} may be chosen as the value of v_i which drives the exact function $f(V)$ to infinity if all other variables v_i remain constant (i.e., $v_j = v_{oj}$ where $j \neq i$).

For the function of equation (4) with $v_{o1} = v_{o2} = 1$, such a consideration leads to the choice of $v_{m1} = -0.1$ and $v_{m2} = -10$. The last column in table 1 shows that the general approximation for this choice is generally superior to the other two, in the range considered. In general, it may not be possible to pick optimum values of v_{mi}, but small negative values of v_{mi} may be used to avoid large errors in the reciprocal expansion when one variable goes to zero.

Critical Time Points

The use of response approximations applies to designs under either steady-state or transient thermal constraints; however, a second approximation concept may also be useful in transient problems. For transient thermal loads, the thermal/structural design problem is characterized by constraint functions (see eq. (1)) which must be enforced for an entire time interval. Numerically, this means that a set of closely spaced time points τ_j, where $j = 1, 2, \ldots, n_\tau$, must be chosen such that serious constraint violations at intermediate time points are unlikely. Equation (1) may then be replaced by

$$g_{ij}(V) = g_i(V,\tau_j) \geq 0 \qquad \begin{pmatrix} i = 1, 2, \ldots, n_c \\ j = 1, 2, \ldots, n_\tau \end{pmatrix} \tag{6}$$

In most practical applications, the product $n_c n_\tau$ will be a very large number and carrying out the design process with this full complement of constraints may become prohibitively expensive.

Instead of monitoring a constraint at a set of n_τ time points, it may be monitored only at its most critical points (ref. 7). This concept is explained by figure 1 which schematically shows the variation of a constraint function with time. The times of the local minima of the constraint function (A, B, and C in fig. 1) are herein called critical time points. Instead of monitoring the constraint function at all times, it is monitored only at those times. However, the critical time points drift as the design changes, and the constraint must be calculated at many time points to accurately locate the critical time points. The usefulness of the critical-time-points concept derives from the fact that, for portions of the design process, the drift of the critical time points may be neglected. For these portions of the design process, the constraint function has to be evaluated for only a relatively small number of time points.

To prove that the drift in the location of the critical points may be neglected to a first-order approximation, it is assumed at first that a constraint function $g(V,\tau)$ has a single local minimum in the time region of interest. Equation (1) for the constraint $g_i(V,\tau)$ is replaced by

$$g_{ci}(V) = g_i(V,\tau_c) = \min\{g_i(V,\tau)\} \geq 0 \tag{7}$$

where τ_c is the time for which g_i is most critical (the critical time point). The partial derivative of g_{ci} with respect to a design variable v_j is

$$\frac{\partial g_{ci}(V)}{\partial v_j} = \frac{\partial g_i(V,\tau_c)}{\partial v_j} + \frac{\partial g_i(V,\tau_c)}{\partial \tau}\frac{\partial \tau_c}{\partial v_j} \tag{8}$$

For interior critical points (A and B in fig. 1), $\partial g_i/\partial \tau = 0$, and for boundary minimum points (point C), $\partial \tau_c/\partial v_j = 0$ if $\partial g_i/\partial \tau \neq 0$. Therefore,

$$\frac{\partial g_{ci}(V)}{\partial v_j} = \frac{\partial g_i(V,\tau_c)}{\partial v_j} \tag{9}$$

Equation (9) shows that the drift in the critical time point may be neglected to a first-order approximation. Drift may be neglected not because it is small but because it does not affect the first derivatives of the constraint g_{ci}. In practice, the critical time points are calculated periodically and are frozen between updatings. However, the concept remains useful, even if the critical points require frequent updating, because of the savings in the derivative calculation.

When the constraint function has more than one local minimum as in figure 1, each critical point must be assigned a separate constraint g_{ci}; otherwise, equation (7) may yield a discontinuous derivative of g_{ci} when the global minimum is switched from one local minimum to another.

EVALUATION OF APPROXIMATION TECHNIQUES

To judge the usefulness of an approximation concept, the question of what magnitude of change in design parameters results in unacceptable errors must be answered. Although it is desirable to obtain a general answer to this question, this did not appear possible in the present studies. Instead, the present work uses examples of both steady-state and transient thermal/structural design results to assess the usefulness of the two approximation concepts discussed in the preceding section. The steady-state example is a titanium panel with integral aluminum bars, and the transient one is an insulated panel.

Approximate Thermal Analysis of a Titanium Panel

The accuracy of both the direct and reciprocal expansions for steady-state thermal analysis was investigated in reference 8 for a configuration which consisted of a titanium panel with aluminum bars as shown in figure 2. This configuration is representative of a class of structures in which one material satisfies strength requirements and the other acts as an efficient conductor to transfer incident heat to a heat sink. The sink is represented by the panel edge maintained at $T = -18^{\circ}$ C (0° F). The incident heat load, material properties, and remaining boundary conditions are also shown in figure 2. The accuracy of the direct and reciprocal expansions for predicting the maximum temperature is shown in figure 3 for changes in the thickness of one-quarter of the plate and in the area of one aluminum bar (both variables are changed proportionately). Initially the eight triangular plate elements were 12.7 mm (0.5 in.) thick and the four bar elements had areas of 645.2 mm^2 (1.0 in^2).

The results shown in figure 3 indicate that the reciprocal expansion is more accurate than the direct expansion. The portions of the plate and the bars that were resized constitute the major path of heat conduction to the heat sink. When their thickness and area are reduced to zero, the temperature becomes unbounded so that the reciprocal expansion may be expected to be better than the direct one. The bias toward the reciprocal expansion can be carried to an extreme by uniformly increasing or decreasing the thickness and areas of all elements. When this is done, the error in the reciprocal expansion becomes minute. A different situation occurs when some of the design variables are increased and others are decreased (a common occurrence in optimization problems). Performance of the expansions for such mixed changes in design variables is shown in figure 4. The percentage change in maximum plate temperature is shown as a function of the percentage of the total change in design variables from the original design to the final design. This is indicated in the figure inset. For these conditions, the direct expansion is better. The reciprocal expansion does not perform as well because of the redundancy of the heat paths. This redundancy creates a situation similar to that of the simple example of equation (4) previously noted.

Approximate Thermal Analysis of an Insulated Panel

The quality of the approximate thermal analysis was investigated for transient heating of the insulated panel shown in figure 5. A transient surface equilibrium temperature T_{eq}, typical of a reentry heating trajectory, is applied to the outer surface of the insulation layer. The insulation protects a balanced, symmetric, graphite/epoxy composite panel with 0°, $\pm45^{\circ}$, and 90° plies. The material properties of the composite panel and insulation are given in table 2. The thermal and stress analyses of the panel were carried out using a modified version of the computer program employed to obtain the results of reference 8.

The dependence of the laminate temperature on insulation thickness is shown in figure 6 for one time point. Also shown are the direct and reciprocal expansions for that temperature based on derivatives calculated for a 1.52-mm (0.060-in.) laminate thickness and 102-mm (4.0-in.) insulation thickness. The reciprocal expansion is reasonably good over the entire range of insulation thicknesses, which is equal to the base value ±50 percent. The direct expansion is considerably less accurate. The better performance of the reciprocal expansion is understandable because the temperature becomes very high when the insulation thickness goes to zero.

The dependence of the laminate temperature on laminate thickness is shown in figure 7. The temperature is much less sensitive to the laminate thickness than to the insulation thickness so that the errors for both approximations are smaller. For this set of calculations however, the direct expansion is better than the reciprocal one. The reason is that the temperature is not expected to become very high when the laminate thickness goes to zero because of the presence of the insulation (the same phenomenon was noted in the example of eq. (4) for changes in v_2). It is also shown in figure 7 that an even better approximation can be obtained with $v_m = -4.06$ mm (-0.16 in.).

For the calculations illustrated in figure 7, the design variable was assigned to the laminate thickness. Often, design variables are assigned to the thickness of each ply. When one ply thickness is changed while others are kept constant, the reciprocal expansion will be even less accurate. This inaccuracy was demonstrated by repeating the calculations presented in figure 7 with individual design variables assigned to the 0°, $\pm45^{\circ}$, and 90° plies. The initial laminate thickness, 1.52 mm (0.060 in.), consisted of 0.51 mm (0.020 in.) for each ply direction. Then, the 90°- and $\pm45^{\circ}$-ply thicknesses were kept constant and only the 0°-ply thickness was increased. The reciprocal expansion for these conditions is shown in figure 7 by the circular symbols. The direct expansion is not affected by assigning design variables in this manner.

The curve with the circular symbols in figure 7 shows the danger of choosing $v_{mi} = 0$ (reciprocal expansion) for a highly redundant system such as a composite laminate. The choice of v_{mi} is based on estimations of the combined effect of all other design variables when v_i goes to zero. Based on the insulated panel example, the following recommendations can be made:

1. If the response function is not expected to become very large when v_i goes to zero, the direct expansion is appropriate and v_{mi} should be chosen to be a large negative number.

2. If the response function may be expected to become large but finite when $v_i = 0$, choose v_{mi} to be a small negative fraction of a typical value of v_i. This is a safety measure to guard against excessive errors in the approximation when v_i changes by an order of magnitude.

Use of Critical Time Points

The approximation concept of following only critical time points was also checked for the insulated panel. Shown in figure 8 are typical changes in temperature histories as the design changes. Indicated in the figure is an appreciable drift in the critical time for the maximum temperature constraint. The same phenomenon is illustrated in figure 9 by showing the difference between following the maximum temperature, that takes into account the drift of the critical time point, and following the temperature at the time point which was maximum for the base design (insulation thickness = 101.6 mm (4.0 in.), laminate thickness = 1.52 mm (0.06 in.)). The two curves in figure 9 have the same derivatives at the base design point as predicted by equation (9). However, the difference between the two curves becomes substantial due to the drift of the time point of maximum temperature. There are indications in figures 8 and 9 that the critical time points require updating more often than the approximate thermal analyses. (Compare figs. 6, 8, and 9.)

APPLICATION TO OPTIMIZATION

Optimization of a Titanium Plate

The titanium plate with aluminum bars shown in figure 2 was designed for minimum mass under a constraint on the maximum temperature. A resizing algorithm based on optimality criteria (ref. 9) was used for optimization. Derivatives of the temperatures with respect to design variables were calculated analytically.

Mass iteration histories for this design problem are shown in figure 10. During the first few iterations, all design variables were reduced until constraint violations occurred; thus, a reciprocal expansion may be expected to be more accurate than a direct expansion. Succeeding design iterations tend to redistribute material among the structural elements to satisfy the design constraints; therefore, a direct expansion may be expected to be more accurate for that phase of the optimization process. Accordingly, three separate calculations were made using approximate thermal analysis: in the first, a reciprocal expansion was used in both phases of the optimization process; in the second, a direct expansion was used in both phases; and in the third, a reciprocal expansion was used in the first phase and a direct expansion was used in the second phase. For the initial design, the plate thickness was set at 76 mm (3.0 in.) and the aluminum bars had areas of 9290 mm^2 (14.4 in^2) each. Derivatives were calculated for the initial design, and the temperature was approximated up to iteration 10. At that point, a new full analysis was used to generate a new approximation which was used for the rest of the design process. The decision to generate a new approximation at iteration 10 was based on full analysis results where changes in the design variables became mixed rather than uniform. Although the mass changed very little after about the 40th iteration, the optimality criteria required an additional 40 iterations to accurately satisfy the temperature constraints.

The final designs and maximum temperature errors (which were obtained from a full analysis of that final design) are given in table 3. It can be seen that the direct expansion lacks sufficient accuracy to guide the optimization process completely to a converged solution. The reciprocal expansion also begins to diverge after about 30 iterations. This results in a design substantially different from the full analysis with 20 percent greater mass and a maximum temperature error of 49.9° C (89.8° F). However, the use of the reciprocal expansion in the first phase of the optimization and the direct expansion in the second phase yields a final design very close to the full analysis design with only a 1.1 percent mass difference and a maximum temperature error of 14.9° C (26.8° F). Additionally, the approximate thermal analysis reduces the total optimization time by about 40 percent. The optimization algorithm was modified to automatically select the expansion concept based on whether the changes in design variables all have the same sign or different signs. The results obtained by such a modified algorithm were essentially the same as before.

Optimization of an Insulated Panel

The insulated panel shown in figure 5 was designed for minimum mass under temperature and stress constraints. The stress constraint was based on the Tsai-Wu failure criterion (ref. 10). The applied loads and the material strength allowables are given in table 2.

The optimizer for the programs for analysis and resizing of structures (PARS), described in reference 11, was used for optimization. PARS is a collection of programs developed for the minimum mass design of structures subject to stress, displacement, and flutter constraints. The PARS optimizer employs the sequential unconstrained minimization technique (SUMT), described in reference 12, with a quadratic extended interior penalty function, described in reference 13. Newton's method with approximate second derivatives (ref. 14) is used to generate directions for one-dimensional searches for each unconstrained minimization. The program approximates the constraint functions during one-dimensional searches using values of v_{mi} specified by the user.

In resizing the panel, new temperatures were approximated based on the values and derivatives of the initial design temperatures without any updating. That is, a full thermal analysis was performed only twice; once for the initial design and once for the final design. Exact calculations of the stresses from the approximate temperatures were performed once for each one-dimensional search. During each search, the stresses were approximated using the same values of v_{mi} used for approximating the temperatures.

Starting with two initial designs, studies were carried out to test the accuracy and time savings associated with the two approximation concepts. The first initial design had a 101.6-mm (4.0-in.) insulation thickness and ply thickness of $t_0 = t_{90} = 0.76$ mm (0.03 in.) and $t_{45} = 1.52$ mm (0.06 in., combined thickness of +45° and -45° plies). This is fairly close to the optimum insulation thickness, but far from the optimum laminate thickness. The second initial design had a 203-mm (8.0-in.) insulation thickness and ply thickness of $t_0 = t_{45} = t_{90} = 0.508$ mm (0.02 in.), which is close to the optimum laminate thickness but far from the optimum insulation thickness.

Performance using response approximations.- The results of the optimization starting with the first initial design are summarized in the first three rows of table 4. For this design, both approximations compare favorably with the full analysis. This may be expected, because the final design differs from the initial design primarily in the laminate thickness which has a weak effect on the temperature. (See fig. 7.) The final design thicknesses determined using the reciprocal expansion are substantially different from those obtained from the full analysis. However, the mass is only 3.6 percent greater than that from the full analysis design. The maximum temperature error for the reciprocal expansion final design is only 3.67° C (6.6° F), which is smaller than the error for the direct expansion. This does not indicate that the reciprocal expansion is superior to the direct expansion; rather, it indicates that the difference between the initial and final design thicknesses from the reciprocal expansion is less than that from the direct expansion.

Considerable time savings are achieved by using the approximate thermal analysis. The approximate thermal analysis requires less than 5 percent of the time required for a full analysis. However, the total optimization time can only be reduced to 30 percent of the time required for full analyses because of the time needed for the initial full analysis and for the operation of the PARS optimizer.

The first three rows of table 5 summarize the results obtained with the second initial design. This time there was a large change in insulation thickness, and as a result, both approximations performed rather poorly. The direct expansion yielded an infeasible design with a maximum temperature error of 292° C (525° F). The reciprocal expansion produced a much better design which is very close to the true optimum with a maximum temperature difference of 66.7° C (120° F). Based on the temperature error, the final design is, in fact, much closer to the true optimum than may be expected. This is because the error peaks for noncritical time points. For the critical time points, the temperature error is an order of magnitude smaller.

The results in tables 4 and 5 for the reciprocal expansion were obtained with $v_{mi} = 0$ for the ply thickness design variables. When the same calculations were repeated for $v_{mi} = -1.02$ mm (-0.04 in.), there was little change. However, the changes in ply thickness from the initial design to the final design were moderate. When ply thicknesses are changed drastically during the design process, it becomes necessary to compensate for the thickness of other plies via v_{mi}. An example is an initial design with insulation thickness of 122 mm (4.8 in.) and ply thickness of $t_0 = t_{90} = 0.127$ mm (0.005 in.) and $t_{45} = 1.27$ mm (0.05 in.). With $v_{mi} = -1.02$ mm (-0.04 in.), the final mass per unit area was 20.67 kg/m^2 (4.233 lb/ft^2), or 3.6 percent heavier than the true optimum, with a maximum temperature error of 7.4° C (13.3° F). With $v_{mi} = 0$ mm, the final mass per unit area was 21.35 kg/m^2 (4.372 lb/ft^2), or 7.1 percent heavier than the true optimum, with a maximum temperature error of 16.4° C (29.6° F).

Based on these studies, it appears that it may be feasible to employ an approximate thermal analysis using the reciprocal expansion without updating the approximation. If the temperature error for the final design is large enough to put the optimality of that design into doubt, the optimization may be restarted from that final design. This was done for the final design given in table 5 for the reciprocal expansion. The restarted optimization required 14.0 seconds of central processing unit (CPU) time to converge to the exact optimum in a few iterations.

Performance using critical times.- The performance of the critical-time approach was evaluated next. Because of the drift in the critical time point discussed in "Use of Critical Time Points" and shown in figures 8 and 9, critical times were updated at the beginning of each one-dimensional search. For each one-dimensional search, stresses were calculated only at two time points: the time of maximum stress ratio and that of maximum temperature. The results of the optimization using this concept are summarized in the last two rows of tables 4 and 5. It can be seen that additional savings of about 50 percent in computation time are realized without any substantial change in the details of the final design.

CONCLUDING REMARKS

Two approximation concepts were evaluated for use in the design of thermal/structural systems for minimum mass. The first concept was the use of response approximations, and the second concept was the use of critical time points. The evaluations were carried out by the use of steady-state and transient example problems. The following conclusions are therefore valid only to the extent that they apply to those examples.

The first concept which was evaluated made use of approximate thermal analysis. Two commonly used approximations, the direct and reciprocal expansions, were shown to be special members of a more general family of approximations characterized by a set of parameters v_{mi} which controls the point where the approximations become unbounded. The following conclusions were reached regarding the performance of the approximations.

1. The reciprocal expansion is more accurate than the direct expansion when the temperature becomes very high as a single design variable goes to zero.

2. If the temperature does not become very high when a single design variable goes to zero but as several design variables go to zero simultaneously (redundant system), the reciprocal expansion provides a better approximation when these design variables either increase or decrease together. The direct expansion is better when some design variables are increased as others decrease.

3. The reciprocal expansion is accurate enough for approximating temperatures without needing updating by a full analysis for changes of up to 50 percent in the values of design variables during the design process.

4. For a redundant system such as a composite laminate, the generalized approximation with a value of the parameter which compensates for the redundancy is found to be better than either the direct or reciprocal expansions.

The second concept that was evaluated, the use of critical time points, was found to work well if the critical time points were updated periodically. For transient example problems, the combined use of the two approximation concepts resulted in an order of magnitude reduction in computation time.

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TABLE 1.- APPROXIMATIONS FOR $f(V) = 11/(10v_1 + V_2)$ BASED ON VALUES OF FUNCTION AND DERIVATIVES AT $v_{o1} = v_{o2} = 1$

v_1	v_2	f(V)	Direct approximation (eq. (2))	Reciprocal approximation (eq. (3))	Generalized approximation $v_{m1} = -0.1$, $v_{m2} = -10$ (eq. (5))
1.0	1.0	1.000	1.000	1.000	1.000
.5	1.0	1.833	1.455	1.909	1.833
2.0	1.0	.524	.091	.545	.524
1.0	.2	1.078	1.073	1.364	1.078
1.0	5.0	.733	.636	.927	.733
2.0	2.0	.500	.000	.500	.441
.5	6.0	1.00	1.000	1.833	1.521

TABLE 2.- MATERIAL PROPERTIES AND LOADING FOR INSULATED PANEL

(a) Mechanical properties for graphite epoxy

Property	Symbol	Units	Value at -	
			Room temperature	177° C (350° F)
Material properties				
Young's modulus	E_1	GPa psi	155 22.5×10^6	155 22.5×10^6
	E_2	GPa psi	8.83 1.28×10^6	5.58 0.81×10^6
Poisson's ratio, major	ν_{12}	----------------	0.30	0.23
Shear modulus	G_{12}	GPa psi	5.10 0.74×10^6	0.551 0.08×10^6
Coefficients of thermal expansion	α_1	per °C per °F	-0.234×10^{-6} -0.13×10^{-6}	-0.126×10^{-6} -0.07×10^{-6}
	α_2	per °C per °F	34.0×10^{-6} 18.9×10^{-6}	78.7×10^{-6} 43.7×10^{-6}

TABLE 2.- Continued

(a) Concluded

Property	Symbol	Units	Value at -	
			Room temperature	177° C (350° F)
Material allowables				
Strength component in fiber direction, tensile and compressive	x_t	GPa psi	1.11 0.161×10^6	1.03 0.150×10^6
	x_c	MPa psi	-972 -0.141×10^6	-848 -0.123×10^6
Strength component in direction normal to fibers, tensile and compressive	y_t	MPa psi	35.8 5.19×10^3	21.0 3.04×10^3
	y_c	MPa psi	-170 -24.7×10^3	-119 -17.3×10^3
Shear strength	s	MPa psi	57.9 8.40×10^3	25.9 3.76×10^3
Loads				
Applied loads	N_x	kN/m lbf/in.	-700 -4000	
	N_y	kN/m lbf/in.	228 1300	
	N_{xy}	kN/m lbf/in.	22.4 128	

TABLE 2.- Concluded

(b) Thermal properties

Property	Symbol	Units	Material	
			Graphite/Epoxy	Insulation
Mass density	ρ	kg/m^3 lb/in^3	1550 56×10^{-3}	144 5.2×10^{-3}
Thermal conductivity	k	$W/m\text{-}^{\circ}C$ $Btu\text{-}in/sec\text{-}ft^2\text{-}^{\circ}F$	1.64 3.16×10^{-3}	0.18 0.35×10^{-3}
Heat capacity	c	$J/kg\text{-}^{\circ}C$ $Btu/lb\text{-}^{\circ}F$	962 0.23	1200 0.291

TABLE 3.- FINAL DESIGNS FOR TITANIUM PLATE WITH ALUMINUM BARS

Procedure	Final mass		Design variables			Maximum temperature error		CPU[1] per iteration, sec	CPU[1] for total optimization, sec
	kg	lb	Symbol	SI Units	U.S. Customary Units	°C	°F		
Full analysis	5.35	11.79	t_1	21.23 mm	0.836 in.	0	0	1.99	167
			t_2	6.91	.272				
			t_3	4.70	.185				
			t_4	2.62	.103				
			a_1	901.9 mm^2	1.398 in^2				
			a_2	2491.0	3.861				
			a_3	314.8	.488				
Reciprocal expansion	6.41	14.14	t_1	28.85 mm	1.136 in.	49.9	89.8	1.12	100
			t_2	8.70	.342				
			t_3	4.66	.183				
			t_4	2.76	.109				
			a_1	1036.8 mm^2	1.607 in^2				
			a_2	2507.7	3.887				
			a_3	342.6	.531				
Direct expansion	Algorithm diverges; final design not obtained								
Combined reciprocal and direct expansions	5.29	11.66	t_1	20.84 mm	0.821 in.	14.9	26.8	1.12	100
			t_2	6.91	.272				
			t_3	4.63	.182				
			t_4	2.55	.101				
			a_1	917.4 mm^2	1.422 in^2				
			a_2	2485.8	3.853				
			a_3	302.6	.469				

[1]Central processing unit time on CDC Cyber 173; FTN compiler; OPT = 2.

TABLE 4.- FINAL DESIGNS OF INSULATED COMPOSITE PANEL

[Initial design: t_1 = 101.6 mm, t_0 = 0.76 mm, t_{45} = 1.52 mm, and t_{90} = 0.76 mm]

Procedure	Final mass, kg/m^2 (lb/ft^2)	Design variables, mm (in.)				Maximum temperature error, °C (°F)	CPU[1] per analysis, sec	Total CPU,[1] sec
		t_1	t_0	t_{45}	t_{90}			
Full analysis	19.94 (4.084)	121.4 (4.781)	0.848 (0.0334)	0.175 (0.0069)	0.546 (0.0215)	0 (0)	11.00	230.0
Direct expansion	19.98 (4.093)	121.7 (4.792)	0.859 (0.0338)	0.173 (0.0068)	0.541 (0.0213)	9.17 (16.5)	0.49	59.1
Reciprocal expansion	20.70 (4.239)	113.5 (4.468)	0.724 (0.0285)	1.384 (0.0545)	0.688 (0.0271)	3.67 (6.6)	0.49	63.0
Direct expansion plus critical time points	19.97 (4.090)	121.7 (4.790)	0.853 (0.0336)	0.163 (0.0064)	0.528 (0.0208)	9.50 (17.1)	0.23	28.9
Reciprocal expansion plus critical time points	20.68 (4.235)	113.6 (4.472)	0.673 (0.0265)	1.422 (0.056)	0.681 (0.0268)	3.72 (6.7)	0.23	28.7

[1]Central processing unit time on CDC Cyber 173; FTN compiler, OPT = 2.

TABLE 5.- FINAL DESIGNS OF INSULATED COMPOSITE PANEL

[Initial design: t_1 = 203.2 mm and $t_0 = t_{45} = t_{90}$ = 0.51 mm]

Procedure	Final mass, kg/m^2 (lb/ft^2)	Design variables, mm (in.)				Maximum temperature error, °C (°F)	CPU[1] per analysis, sec	Total CPU,[1] sec
		t_1	t_0	t_{45}	t_{90}			
Full analysis	19.93 (4.081)	121.5 (4.783)	0.848 (0.0334)	0.163 (0.0064)	0.549 (0.0216)	0 (0)	11.00	208.0
Direct expansion	13.91 (2.849)	80.6 (3.175)	0.864 (0.0340)	0.102 (0.0040)	0.508 (0.0200)	292 [2](525)	0.49	53.6
Reciprocal expansion	20.73 (4.246)	127.6 (5.023)	0.833 (0.0328)	0.124 (0.0049)	0.551 (0.0217)	66.7 (120)	0.49	58.6
Direct expansion plus critical points	14.49 (2.968)	79.9 (3.145)	0.777 (0.0306)	0.480 (0.0189)	0.663 (0.0261)	275 [2](495)	0.23	21.5
Reciprocal expansion plus critical points	20.67 (4.234)	127.4 (5.014)	0.836 (0.0329)	0.114 (0.0045)	0.538 (0.0212)	68.3 (123)	0.23	21.4

[1]Central processing unit time on CDC Cyber 173; FTN compiler, OPT = 2.
[2]Infeasible design.

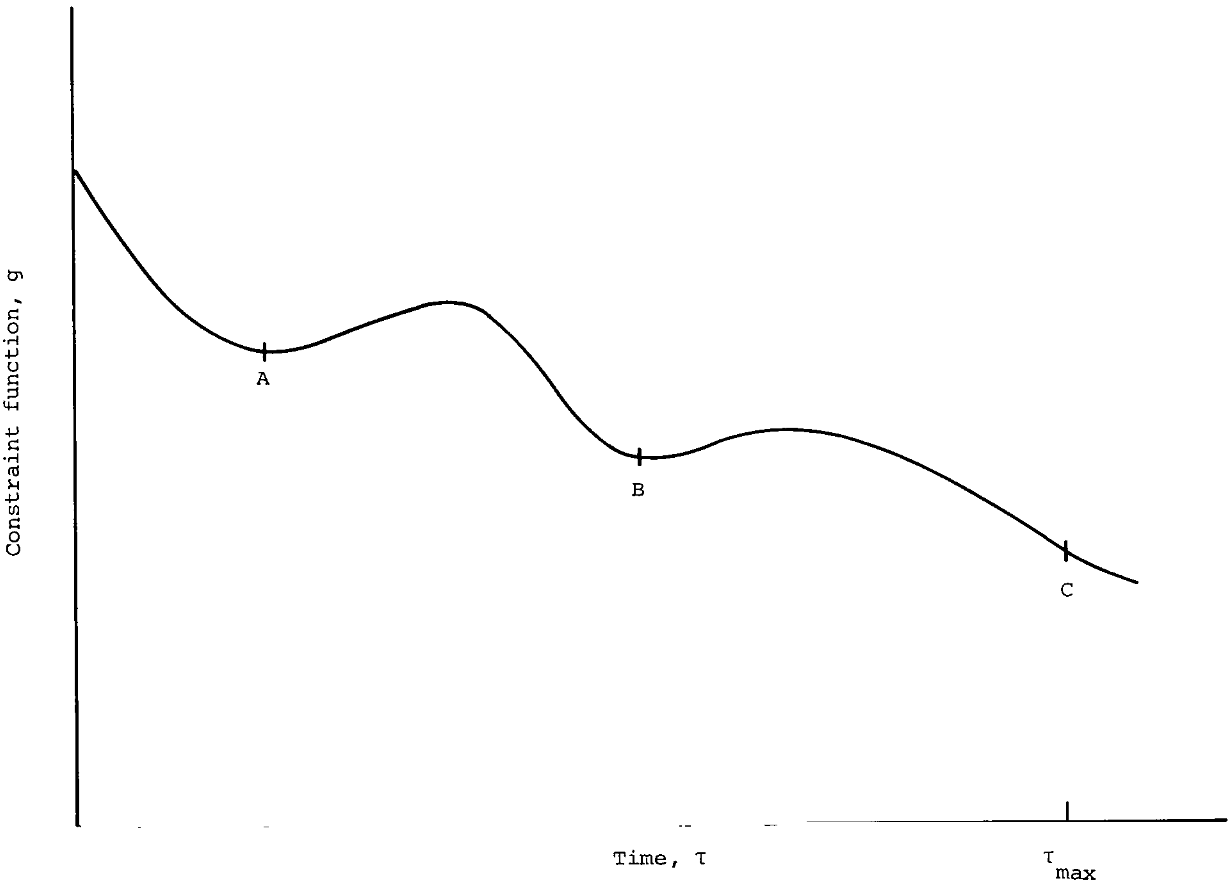
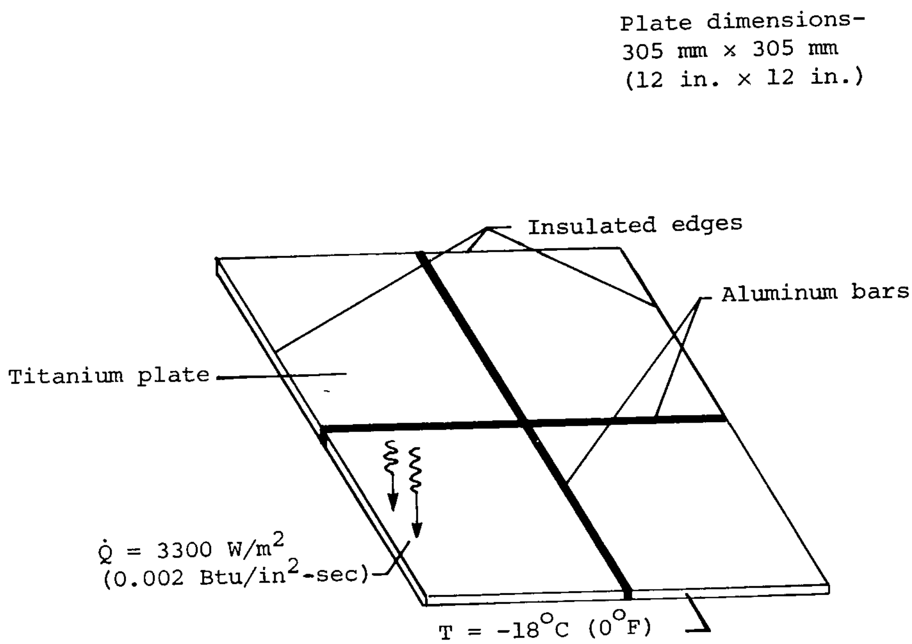


Figure 1.- Typical variation of constraint function with time.



(a) Configuration and loads.

Property		Material	
		Aluminum	Titanium
E	GPa	69.6	103
	psi	10.1×10^6	15×10^6
α	Per °C	23.8×10^{-6}	10.4×10^{-6}
	Per °F	13.2×10^{-6}	5.8×10^{-6}
ρ	kg/m^3	2770	4437
	lb/in^3	0.100	0.160
k	$\frac{W}{m\text{-}^{\circ}C}$	234	9.49
	$\frac{Btu\text{-}in}{sec\text{-}ft^2\text{-}^{\circ}F}$	0.451	0.018
T_a	°C	177	260
	°F	350	500

(b) Material properties.

Figure 2.- Titanium plate with aluminum bars.

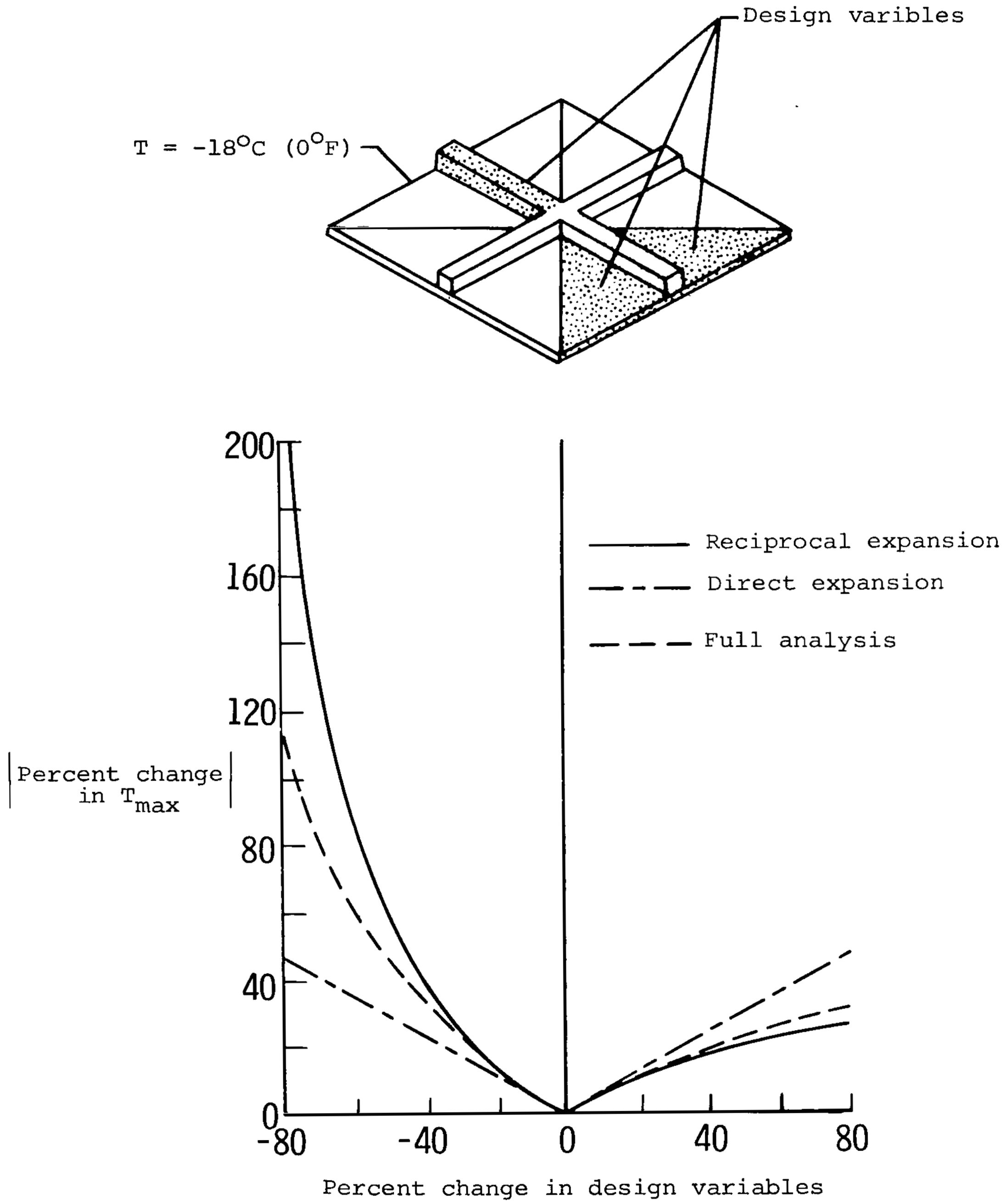
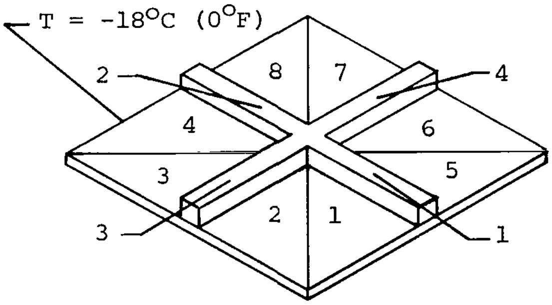


Figure 3.- Approximations of maximum temperature in titanium plate with aluminum bars (from ref. 8).



Design variable	Original design		Final design		Percent difference
	S.I. unit	U.S. Cust.	S.I. unit	U.S. Cust.	
t_1	13.11 mm	0.516 in.	21.03 mm	0.828 in.	+60.5
t_2	11.68	.460	6.92	.272	-40.8
t_3	4.18	.164	4.69	.185	+12.2
t_4	3.26	.128	2.62	.103	-19.5
a_1	650.3 mm^2	1.008 in^2	901.3 mm^2	1.397 in^2	+38.6
a_2	2350.3	3.643	2489.7	3.859	+ 5.9
a_3	418.1	.648	316.1	.490	-24.4

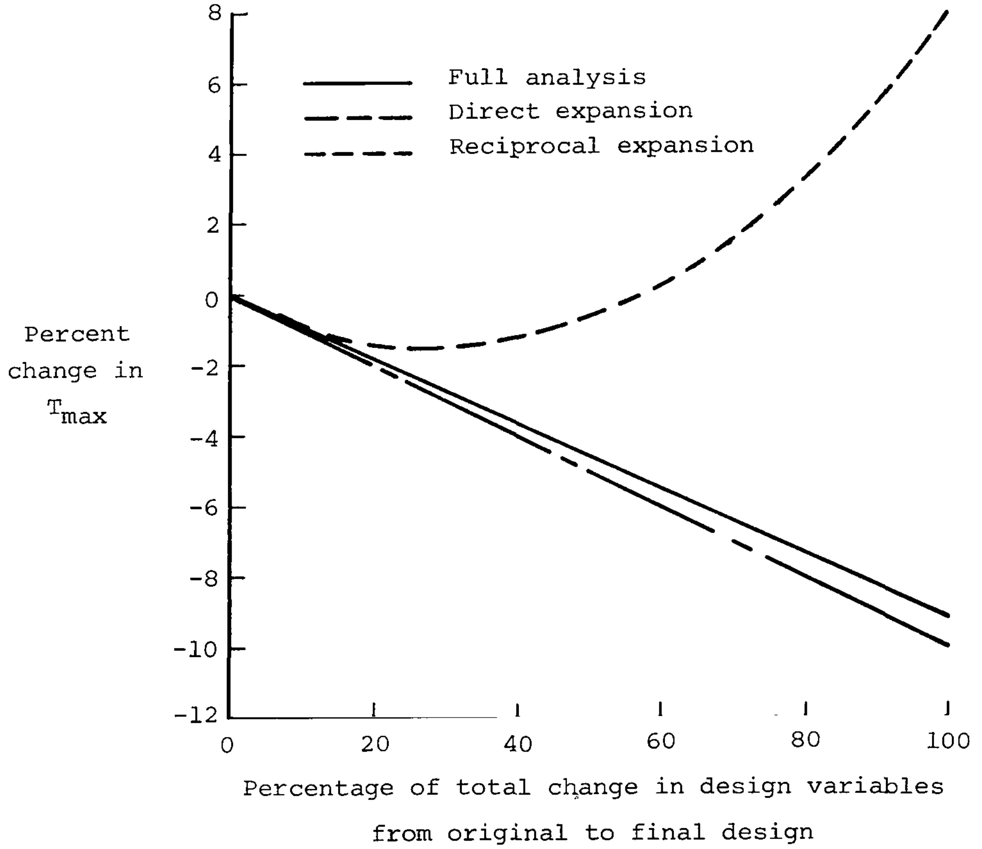


Figure 4.- Approximation performance for mixed changes in design variables for titanium plate with aluminum bars.

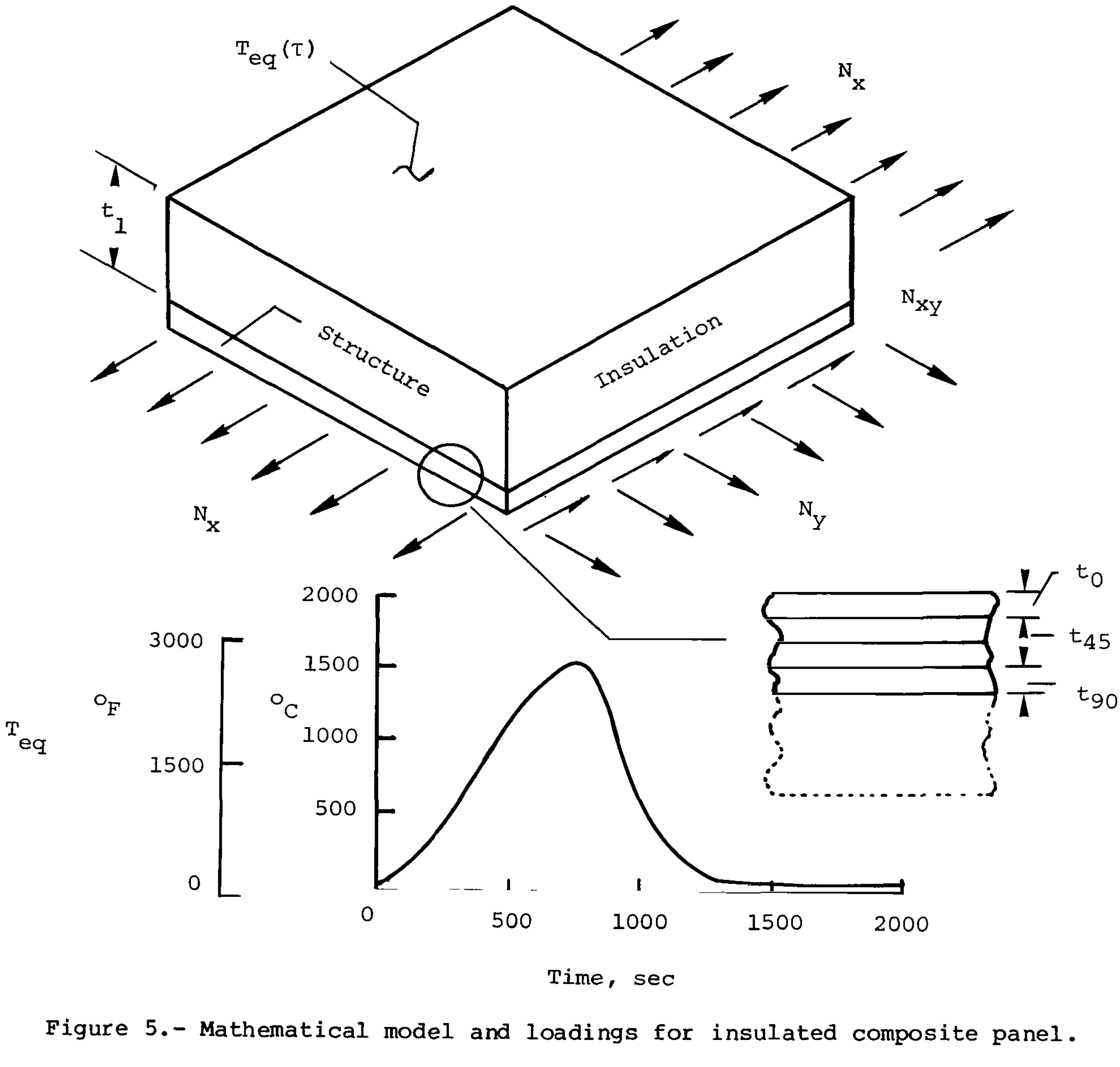


Figure 5.- Mathematical model and loadings for insulated composite panel.

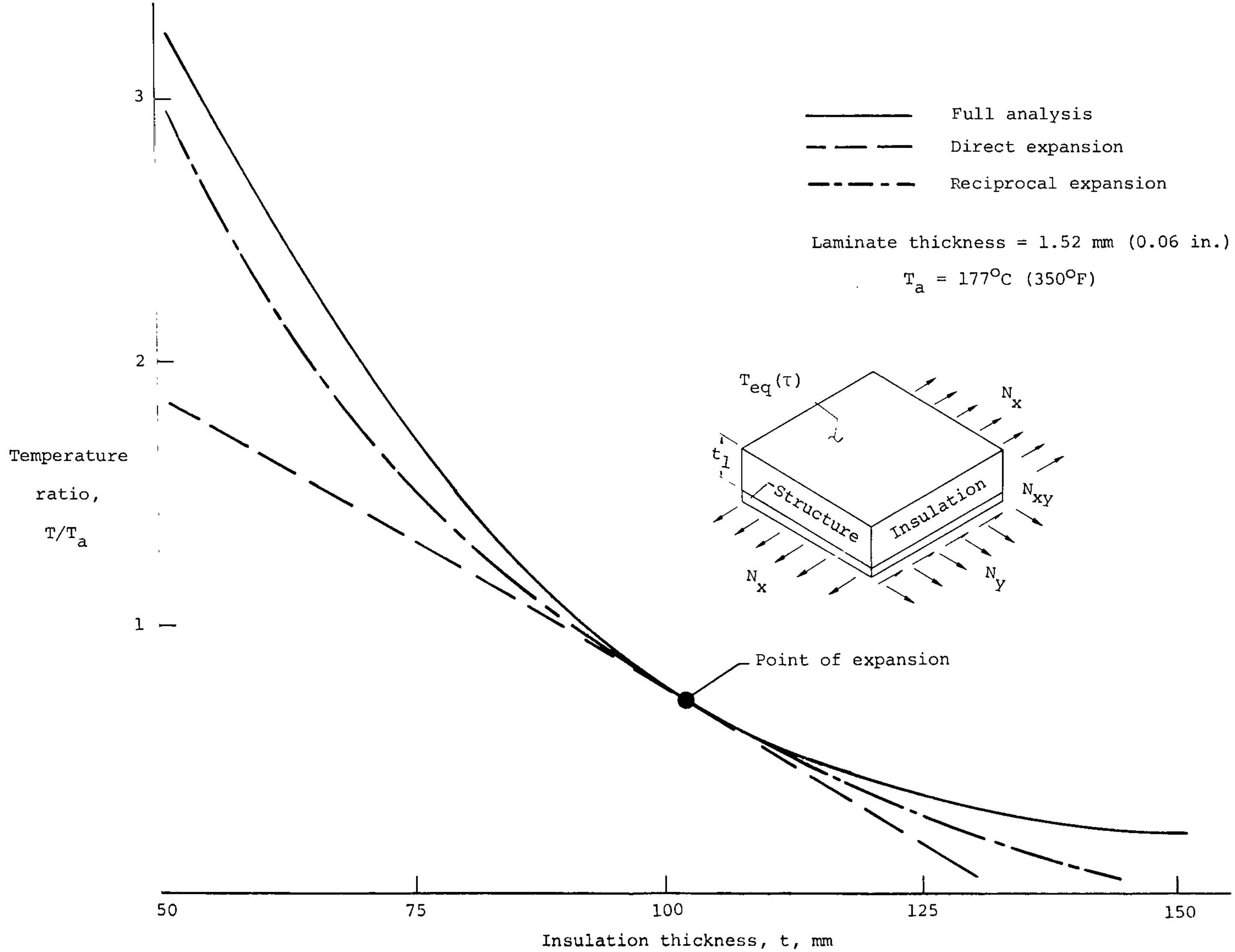


Figure 6.- Quality of approximations in estimating temperature dependence on insulation thickness; τ = 2000 sec.

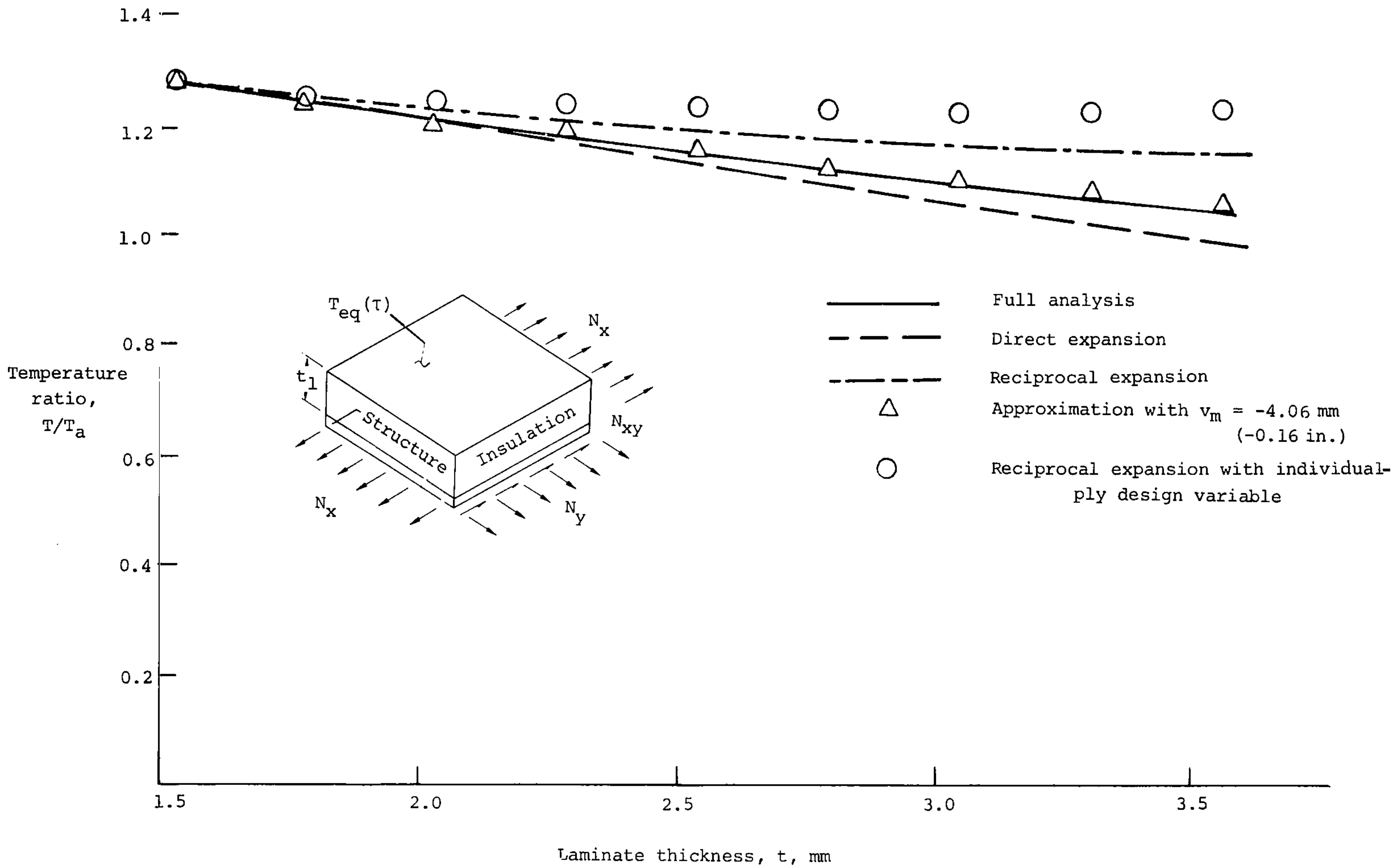


Figure 7.- Quality of approximation in estimating temperature dependence on laminate thickness; τ = 3200 sec, t_1 = 101.6 mm (4.0 in.).

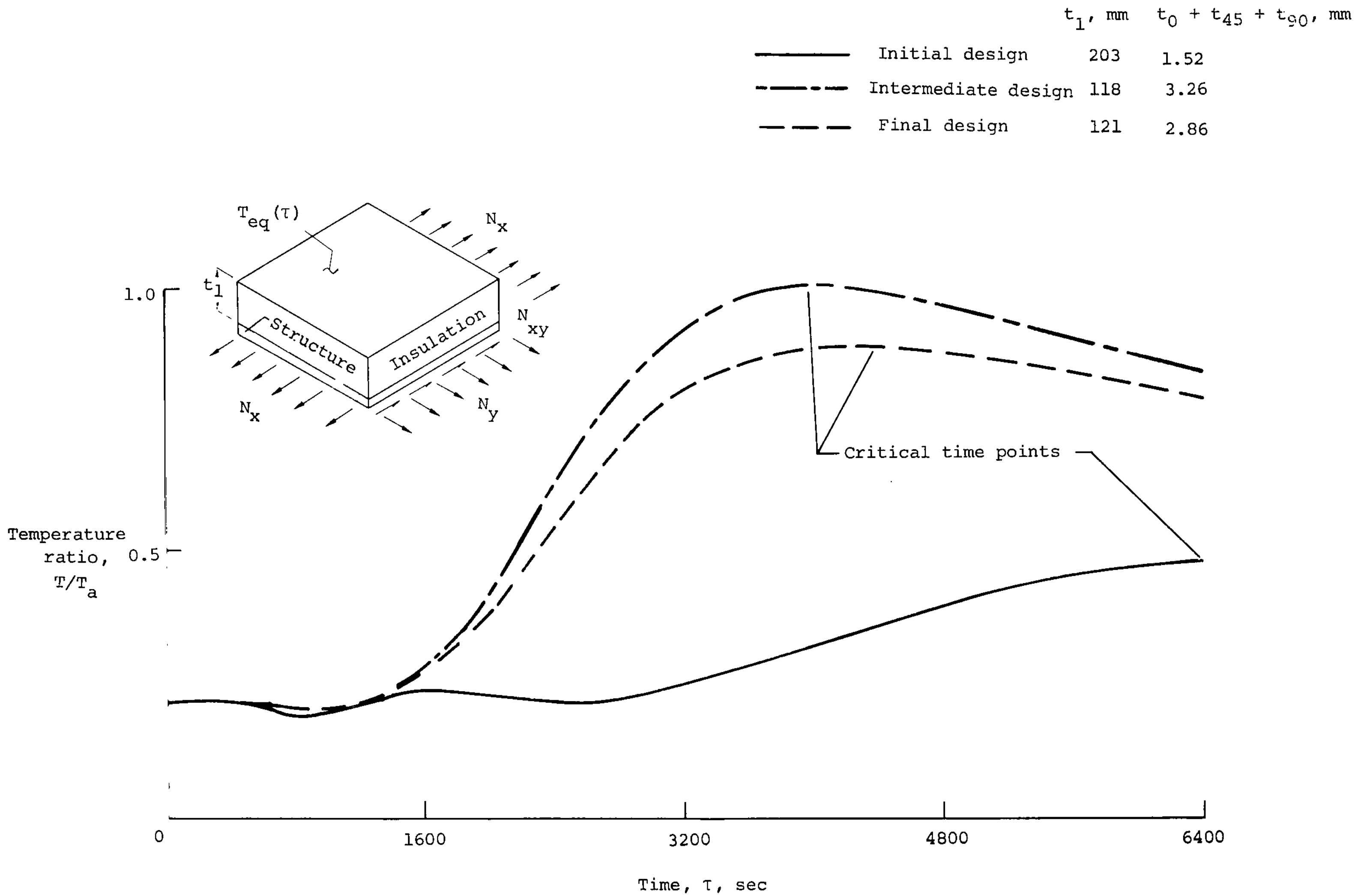


Figure 8.- Laminate temperature history for different insulated composite-panel design.

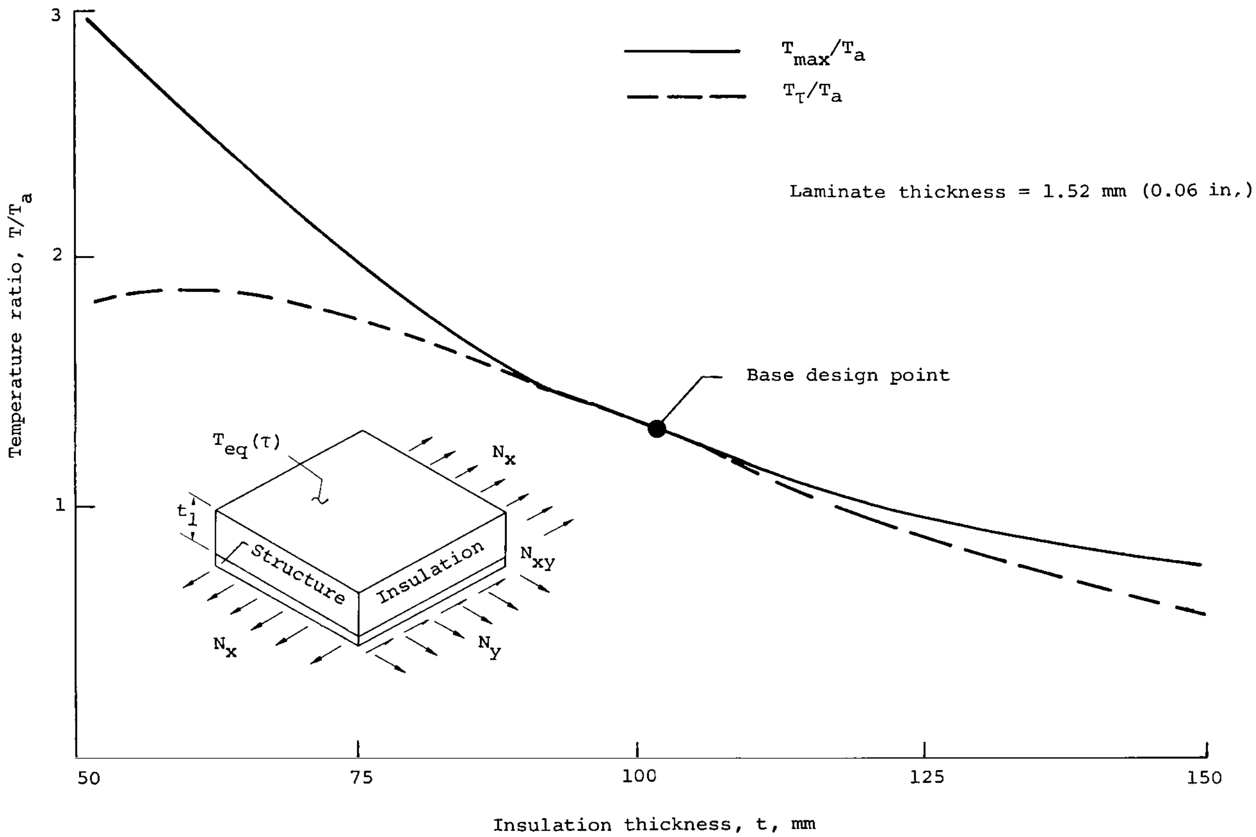


Figure 9.- Variation with insulation thickness of maximum laminate temperature and laminate temperature at τ = 3200 sec for insulated composite panel.

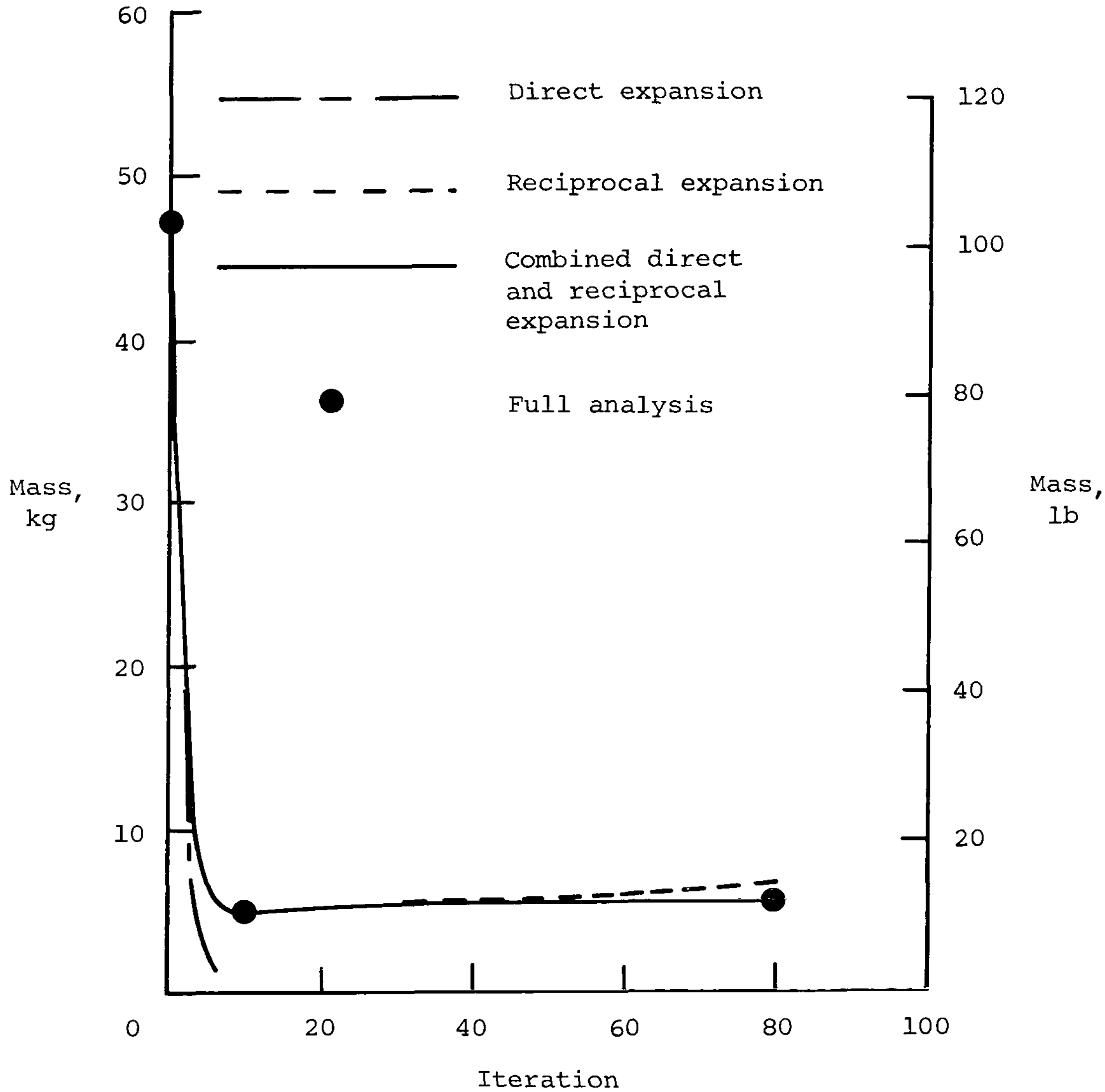


Figure 10.- Performance of Taylor series approximations in optimization of titanium plate with aluminum bars.

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15. Supplementary Notes

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16. Abstract

Two approximation concepts for combined thermal/structural design are evaluated. The first concept is an approximate thermal analysis based on the first derivatives of structural temperatures with respect to design variables. Two commonly used first-order Taylor series expansions are examined. The direct and reciprocal expansions are shown to be special members of a general family of approximations, and it is shown that for some conditions other members of that family of approximations are more accurate. Several examples are used to compare the accuracy of the different expansions.

The second approximation concept is the use of critical time points for combined thermal and stress analyses of structures with transient loading conditions. It is shown that significant time savings may be realized by identifying critical time points and performing the stress analysis for those points only. This approach is used to design an insulated panel which is exposed to transient heating conditions.

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