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THE DYNAMIC MODELLING OF A SLOTTED TEST SECTION

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1. Introduction.

In designing the control system for the National Transonic Facility, it was necessary to develop a mathematical model of the wind tunnel dynamics. Due to the limitations of present day modelling, it was necessary to restrict the modelling techniques to the use of one dimensional unsteady flow. With this limitation it was still imperative that the dynamic characteristics of a slotted test section be incorporated into the model. This report describes the model developed to accomplish these objectives.

2. Objectives.

The following objectives were set for the test section-plenum model to be developed.

a. The model should serve as a means for determining the dynamic performance of the NTF and the subsequent design of optimal control configurations.

b. The model should be of the one dimensional flow type so as to facilitate computer simluation.

c. The model should allow for the simulation of the effects of test section blockage, boundary layer losses, slot flow required for supersonic flow generation, reentry flap position, and auxiliary plenum suction.

d. The model should be capable of validation by comparison to previous tests made on the eight foot tunnel at Langley.

e. The model should be capable of being updated as operating data from the NTF becomes available.

f. The model should lead heuristically to simpler models so that the principles of operation of this part of the tunnel are more clearly understood.

The model presented in this report meets most of the objectives. The shortcomings of the current representation are:

a. There is no representation of reentry flap position.

b. The boundary layer coefficients Cxm are not known to a sufficient degree of precision.

c. The test section divergence is not represented.

d. There is no simplified representation available so as to better understand the test section dynamics.

e. The actual value of Ar* is not known. Ar is an equivalent area and may not be equal to the geometric area. Ar may even vary with mach number.

f. The assumption of a constant ρV^2 loss coefficient for the diffuser may not be a good one.

3. Basic Configuration.

The basic configuration of the slotted test section and plenum is illustrated in figure 1. The following equations are used:

3.1 The flow entering the test section is given by:

$$\text{Qthrt} = \begin{cases} \dfrac{144\cdot\text{Atest}\cdot\text{Pstag}\cdot\text{Mach}}{(1 + .2\cdot\text{Mach}^2)^{3}} \sqrt{\dfrac{1\cdot4\cdot g}{R\cdot\text{Tstag}}} & \text{if Mach} \leq 1 \\[2ex] \dfrac{144\cdot\text{Atest}\cdot\text{Pstag}}{(1.2)^3} \sqrt{\dfrac{1\cdot4\cdot g}{R\cdot\text{Tstag}}} & \text{if Mach} > 1 \end{cases} \tag{1}$$

* see Appendix B

where:

Atest	is the test section area, Atest = 66.77 ft^2
R	is the universal gas constant, R = 55.1 ft/°R
g	is the gravity constant, g = 32.2 ft/sec^2
Pstag	is the stagnation pressure (lb/in^2)
Tstag	is the stagnation temperature (°R)
Mach	is the test section mach number

Pstag and Tstag are determined by the tunnel simulation.

3.2 The mach number is given by:

$$\text{Mach} = \sqrt{5\left[\frac{\text{Pstag}}{\text{Pstest}}\right]^{2/7} - 5} \tag{2}$$

where:

Pstest is the static test section pressure (lb/in^2)

3.3 The flow (lbs/sec) entering the plenum from the test section via the slots is given by:

$$\text{Qslot} = \text{Qthrt}\cdot\text{Cxm} + (\text{Qthrt}-\text{Qtest}) + \text{Qdm} \tag{3}$$

where:

Qb = Qthrt•Cxm is the component due to boundary layer.

Cxm is a function of mach number (see section 3).

Qt = (Qthrt-Qtest) is the component due to supersonic flow generation.

Qdm = Qthrt•Qdmx is the component due to model blockage.

Qdmx is an input variable.

3.4 The flow exiting the test section is given by:

$$Qexit = Qthrt - Qslot$$

3.5 Qtest is the test section flow and is given by:

$$Qtest = \frac{144 \cdot Atest \cdot Pstag \cdot Mach}{(1 + .2 \cdot Mach^2)^3} \sqrt{\frac{1.4 \cdot g}{R \cdot Tstag}} \qquad (4)$$

3.6 The flow leaving the plenum and flowing into the diffuser via the reentry flaps is given by:

$$Qflap = Qr - Qexit \qquad (5)$$

where:

Qr is the total flow into the diffuser.

3.7 The total weight of gas in the plenum is given by:

$$Wpl = \int (Qslot - Qflap)\, dt \qquad (6)$$

3.8 The static pressure in the plenum is given by:

$$Ppl = \frac{Wpl \cdot R \cdot Tpl}{144 \cdot Vpl} \qquad (7)$$

where:

Tpl is the plenum static temperature. It is assumed that the total and static temperature in the plenum are equal to each other as well as to the test section total temperature.

Vpl is the plenum volume, $Vpl = 36{,}000\ ft^3$

3.9 The test section static pressure is assumed equal to the plenum static pressure.

$$Pstest = Ppl \tag{8}$$

3.10 The static temperature in the test section is given by:

$$Tstest = \frac{Ttest}{(1 + .2\ Mach^2)} \tag{9}$$

Ttest is equal to Tstag less any thermal losses.

3.11 The total temperature in the plenum is given by:

$$Tpl = Epl/(Wpl \cdot Cv) \tag{10}$$

$$\dot{E}pl = Cp(Qslot \cdot Ttest - Qflap \cdot Tpl) \tag{11}$$

3.12 The velocity in the test section is given by:

$$Utest = Mach \sqrt{1.4 \cdot g \cdot R \cdot Tstest} \tag{12}$$

3.13 The ejector action of the diffuser is represented by:

$$144 \cdot Ar \cdot Pstest + Qexit \cdot Utest/g = 144 \cdot Ar \cdot Pr\ (1 + 1.4 \cdot Mr^2) \tag{13}$$

where:

The balance is taken across an equivalent area Ar. The exact value of this area is not known. A nominal value of $Ar = 1.16\ Atest = 77.45\ ft^2$ was chosen. The value of Ar was varied during the simulation so as to obtain a measure of the sensitivity of the system to the value of Ar. This is the familiar momentum equation applied across a line. The equation balances the sum of the potential and and kinetic forces on either side of a line. On the left face of Ar, the potential force is 144•Ar•Pstest. The kinetic force is given by

Qexit•Utest/g, since we may assume that the kinetic energy associated with the flow Qflap is negligible. The energy on the right face of Ar is obtained by using the identity.

$$144 \cdot Ar \cdot Pr \cdot g \; 1.4 \; Mr^2 = Qr \cdot Ur \tag{14}$$

3.14 The remainder of the tunnel, up to the fan, is represented by a sequence of momentum, energy, and continuity equations. The first equation encountered is:

$$\dot{Q}_9 = \left[144 \cdot A_9 \cdot g \; (Pr - P_{10} - Xq_9 Q_9 U_9) + QrUr - Q_{10} U_{10} \right] / XL_9 \tag{15}$$

where:

The equation is written across a section of the diffuser of length XL_9, XL_9 = 30ft, and an equivalent area of A_9, $A_9 = 90ft^2$. The state of the gas entering the section is (Pr,Qr,Ur). The state of the gas leaving is (P_{10}, Q_{10}, U_{10}). Qr and Q_9 are assumed equal. This assumption is valid since it is equivalent to saying that the flow entering a volume of short length is equal to the average flow in that volume.

Xq_9 is the ρV^2 loss coefficient in this section.

$Xq_9 = .1111 \; 10^{-6} \; sec^2/(in^2 ft)$.

4. Solution of Equations

The equations of 3.1 to 3.14 are solved simultaneously in an iterative manner. Care must be taken in the calculations so as to remain on the proper branch of the solution. This will assure that choking and the shock equation are properly represented. The procedure used is outlined in Appendix A. In general, the procedure proceeds as follows:

Let

$$Fm = 144 \cdot Pstest \cdot Ar + Qexit \cdot Utest/g \tag{16}$$

and

$$d = Qr^2 R\ Ttr\ /(1.4 \cdot Fm^2 \cdot g) \tag{17}$$

where Ttr is the total temperature on the right face of Ar. If there is no loss of heat in the plenum Ttr = Ttest. Mr is given by the quadratic expression.

$$(5\text{-}49\ d) \cdot Mr^4 + (25 - 70d) \cdot Mr^2 - 25\ d = 0. \tag{18}$$

The remaining variables are given by:

$$Pr = Fm/(144 \cdot Ar(1 + 1.4Mr^2)) \tag{19}$$

$$Tr = Ttr/(1 + .2Mr^2) \tag{20}$$

$$Ur = 49.84 \cdot Mr \sqrt{Tr} \tag{21}$$

If Mr > 1, choking occurs and we set Mr equal to 1.

Qr is then given by:

$$Qr = Fm \sqrt{5(1.4)g/(24 \cdot R \cdot Ttr)} \tag{22}$$

Q_9 is set equal to Qr, and $\dot{Q}_9$ is constrained so that $\dot{Q}_9 \leq 0$.

5. The Distributions of Losses Around the Tunnel.

For simulation purposes, the tunnel has been divided into 15 stations. Where:

Station 1 is at the fan outlet

8 is at the stagnation chamber

10 is at the entrance to the high speed diffuser

15 is at the fan inlet.

From the report by D. M. Rao (11), it may be inferred that when operating at 14.7 psia, 500°R the tunnel drops will be distributed as follows:

station 1 to 4	13.91%
station 4 to 6	3.21%
station 6 to 8	11.37%
station 8 to 10	60.86%
station 10 to 12	5.46%
station 12 to 14	5.19%

From the report by B. Gloss (2) the fan ratio as a function of mach number is given as:

Mach No.	Fan Ratio
.2	1.0120
.3	1.0180
.4	1.0245
.5	1.0330
.6	1.0430
.7	1.0550
.8	1.0700
.9	1.0880
1.0	1.1120
1.1	1.1420
1.2	1.1850

5.1 Using the above data the losses around the tunnel may be established if we further assume:

a. The losses are ρV^2 losses, with constant loss coefficients.

b. That the slot flow due to boundary layer at Mach 1 is 3% of the flow at the throat.

c. That the slot flow is a function of mach number.

5.2 The coefficients Xq_i, $i = 3,5,7,9,11,13$ are defined by:

$$DP_i = Xq_i \cdot Q_i \cdot U_i \tag{23}$$

where:

DP_i is the pressure loss in the ith section.

Xq_i is the loss coefficient in the ith section.

Q_i is the flow in the ith section.

U_i is the velocity in the ith section.

The coefficients Cxm_i are defined by:

$$Qb = Cxm_i \cdot Qthrt \tag{24}$$

where:

The subscript i denotes the mach number at which the computation is made.

Qb is the slot flow due to boundary layer.

Cxm_i is the slot flow coefficient.

5.3 The coefficients Xq and Cxm are determined as follows:

a. Establish the tunnel simulation at 14.7 psi, 500°R, Mach 1.0. Fix Cxm at .03 and vary all of the Xq_i in an iterative fashion until the fan ratio is 1.112 and the losses are distributed as indicated in 5.0.

b. Establish the mach number at .1 to 1.2 in steps of .1 and for each mach number vary Cxm_i until the proper fan ratio is obtained. If this is done, the following values are obtained:

$$Xq_3 = .8055\ 10^{-7}$$

$$Xq_5 = .3491\ 10^{-7}$$

$$Xq_7 = .1934\ 10^{-6}$$

$$Xq_9 = .1111\ 10^{-6}$$

$$Xq_{11} = .1791\ 10^{-7}$$

$$Xq_{13} = .2013\ 10^{-7}$$

Mach No.	Cxm
.1	.4501
.2	.1612
.3	.0807
.4	.0445
.5	.0300
.6	.0232
.7	.0206
.8	.0216
.9	.0243
1.0	.0300
1.1	.0299
1.2	.0216

5.4 The data given in 5.3 is suspect due to the large values of Cxm at the low mach numbers. This is more than likely due to the inaccuracies of the assumed fan ratios. In order to correct for this error we introduce the energy ratio.

$$CE = \frac{\text{Qthrt Utest}^2}{2(550)g\ \text{HP}} \tag{25}$$

where HP is the horsepower needed to drive the tunnel.

The coefficients Cxm are then redetermined. Below mach numbers of .6 the fan ratios as given in 3.0 are ignored. Instead the values of Cxm are chosen so that the value of CE is held constant and equal to that at Mach = .6. This results in coefficients which appear to be more realistic. It is these values that are to be used to represent the test section. The results of this calculation is tabulated on the following page.

Mach No.	Cxm	CE	Cr	Crl
.1	.0137	4.70	1.0012	1.0080
.2	.0144	4.70	1.0048	1.0120
.3	.0155	4.70	1.0107	1.0180
.4	.0169	4.70	1.0189	1.0245
.5	.0210	4.70	1.0299	1.0330
.6	.0232	4.70	1.0430	1.0430
.7	.0206	4.73	1.0550	1.0550
.8	.0216	4.96	1.0700	1.0700
.9	.0243	4.89	1.0880	1.0880
1.0	.0300	4.65	1.1120	1.1120
1.1	.0299	4.39	1.1420	1.1420
1.2	.0215	4.01	1.1850	1.1850

Where Cr is the resulting fan ratio and Crl is the fan ratio given in 5.0.

6.0 The time response of the test section.

6.1 It was suspected from the system equations that the dynamics of the test section can be represented by a simple first order system. To verify this, the time response of mach number due to a 2 degree step change in guide vane angle was determined by the NTF dynamic model. The quantity

$$\ln \left\{ \frac{\text{Mach} - \text{Mf}}{\text{Mi} - \text{Mf}} \right\} \tag{26}$$

where Mach, Mf, and Mi are the current, final, and initial values of mach number, was plotted as a function of time on figure 5. The quantification of the points is due to the fact that Mach, Mf, and Mi were only observed to 3 significant digits.

Since a straight line fits through these points it is verified that the system is of the first order.

The x intercept of .35 corresponds to a dead time of .35 seconds. This agrees with the acoustic propagation time of

$$343 \text{ ft}/(863 \text{ ft/sec}) = .397 \text{ sec} \tag{27}$$

It may be shown that the slope of the line is the reciprocal of the time constant. This yields a time constant of 1/.411 = 2.43 sec. If the time constant is determined in the usual fashion, by defining the time constant as the time required to reach .632 of the final change, a value of 2.8 seconds is obtained. Note that 2.43 + .35 = 2.78.

From this we conclude that one can model the test section response by:

$$\Delta M = \frac{k \cdot e^{-Td \cdot S}}{(Tc \cdot S + 1)} \quad \Delta G \tag{28}$$

where:

ΔG is the change in guide vane angle.

ΔM is the corresponding change in Mach number.

Tc is the time constant.

Td is the dead time.

k is the gain.

S is the Laplace transform operator.

In designing the mach number control system, the above parameters prove useful. In the next section we examine how these parameters vary with the operating point. It is seen that they are a function of pressure, mach number, and Ar. In addition, a nonlinear affect is observed in that the time constant is dependent on the step size. Further, for small values of Ar choking occurs at supersonic operation. This choking occurs at the boundary of Ar and causes velocity limiting which in turn slows down the response.

6.2 The variation of gain and time constant.

On figures 2a and 2b the gain and time constant of the test section is plotted as a function of stagnation pressure and mach number. This data was obtained by runs made on the NTF simulation. With the pressure and temperature control active a 2 degree step change in guide vane angle was made. The time constant was determined as the time required for the mach number to make .632 of the total change. Since we have already shown that the system is indeed a first order, this calculation is valid. However, the time constants shown are the true time constant plus the system dead time. Since the distance between the test section and the fan is approximately 343 ft, the dead time varies between .307 and .487 seconds for the temperature range of 500 to 200°R. To obtain this data, runs were made at pressures of 30, 60, 90, and 120 psi; temperatures of 200, 300, 400, and 500°R; and mach numbers of .2, .3, .4, .5, .6, .7, .8, .9, 1.0, and 1.1. There was no change in either gain or time constant with temperature. Since it is known that the dead time does vary with temperature, it must be true that the actual time constant also varies with temperature in such a manner as to maintain the apparent time constant (as plotted in fig. 2a) fixed. The gain plotted on fig. 2b was obtained by taking the ratio of mach number change and guide angle change and multiplying by 100.

6.3 The time constant variation due to Ar.

It was observed that the value of Ar had a strong effect on the time constant. The curves of figure 4 show the variation of time constant with Ar. It is noted that for low values of mach number that the value of Ar has little if any effect on the time constant. The time constant for transonic operation is strongly dependent on Ar. These curves clearly show the need for obtaining better information as to the value of Ar.

6.4 The time constant due to blockage.

On figure 3, the time constant due to a 2% blockage is plotted. Comparing this with figure 2a, it is seen that the time constant for blockage and guide vane deflection is almost the same. That this is so is clearly seen by examining figures 5 and 6. From these we conclude that the response due to blockage differs from that due to guide vane deflection only in that blockage does not have any dead time associated with it. Note that figure 5 has an x intercept of .35 seconds, while the line of figure 6 goes through the origin.

6.5 Comparison with the eight foot tests.

In order to assist in the validation of the NTF simluation transient tests were made on the eight foot tunnel at Langley. These tests were made in order to determine the response of the plenum-test section blockage. The findings of these tests are documented in ref. (10). The table below which appears on page 11 of the above report summarizes the pertinent results.

Test Run	Nominal Mach No.	Average Time Constant, sec SEL (#44)	FM(101T)	Eqs. (11) to (13)
19	1.05	7.7	9.2	13.5
20	1.20	3.7	5.2	3.5
21	.95	7.1	6.7	4.5
22	.80	5.3	5.5	4.6

In order to compare the present NTF simulation with these tests, the NTF simulation was altered so that it would exhibit the properties of the eight foot tunnel. This was accomplished by changing volumes and lengths in the tunnel proper. This change is described in ref. (8). The plenum volume was increased from 36,000 to 72,000 ft^3. The value of Ar was left at 77.45 ft^2, and a linear fan representation was utilized. The inbleed and outbleed valves were locked and a 2% step change in blockage was made in the test section. Four runs were made: run 2219, Tstag = 500, Pstag = 14.71, Mach = 1.05; run 2220, Tstag = 500, Pstag = 14.71, Mach = 1.2; run 2221, Tstag = 500, Pstag = 14.71, Mach = .95; and run 2222, Tstag = 500, Pstag = 14.71, Mach = .8. These correspond to runs 19, 20, 21, and 22 respectively. A typical response is shown on figure 8.

An analysis of the output of figure 8, is made by the use of figure 7. From this figure we conclude:

a. The response of mach number is of the first order. For run 2219, with an Ar of 77.45 ft^2, the time constant is 3.95 sec, there is no system dead time.

b. The response of the test section pressure is of the first order. For run 2219, with an Ar of 77.45, the time constant is 3.98, there is no system dead time.

c. The response of stagnation pressure, is of the third order. For run 2219, with an Ar of 77.45 ft^2, there is a time constant of 3.69 seconds, with an additional underdamped second order of approximately 3 hertz. There is a system dead time of .35 seconds.

The table below compares the eight foot tests with the simulation results.

Run	8 ft tests	simulation Ar = 70	Ar = 72	Ar = 74	Ar = 77.45
19	7.7, 9.2, 13.5	C	6.3	5.3	4.0
20	3.7, 5.2, 3.5	C	C	19.0	2.9
21	7.1, 6.7, 4.5	6.2	5.0	5.6	3.9
22	5.3, 5.5, 4.6	4.9	4.6	4.4	4.7

The listed values are measured time constants in seconds. A C indicates that the simulation failed to run due to choking at Ar.

Examining the above, it may be concluded that the simulation concurs with the test. In order to obtain better correlation or detect differences between the simulation and the tests, it will be necessary to obtain more consistent test data and establish more precise values of Ar.

7.0 Mach Number Control

From the low band-pass of the mach number response, it is apparent that the guide vane actuator will be the limiting factor on the response of the mach number control system.

The process time constant is of the order of 3 seconds, while the actuator is a second order system of 6 radians with a damping factor of .8. Together these two yield a phase angle lag of 130° at 3 radians, which limits the response to 3 radians per second. This is in agreement with the analysis presented in (6).

The settling time, to within 1%, for a second order system with a natural frequency of 3 radians per second and a damping factor of .8, is obtained by solving

$$\frac{e^{-.8(3)t}}{\sqrt{1 - .8^2}} \doteq .01$$

This yields a settling time of 2 seconds.

This is not consistent with the system performance as exhibited by the NTF simulation. A typical simulation output is contained on figures 9a, 9b, and 9c. This run corresponds to a step change in mach number set point of .1, with the tunnel operating at 100 psi, 300°R, and mach = .8. In this run a settling time of 20 seconds was required. This is far different from that expected for a properly tuned system. The discrepancy is due to cross coupling, the presence of reset in the control, improper tuning and perhaps some nonlinear affect unaccounted for. This discrepancy is to be investigated in a later study, meanwhile the target values of 2 to 3 seconds for settling times of the mach number control system is to be maintained.

8.0 Conclusions and Recommendations.

From the results presented in this report, it may be inferred that the proposed test section math model is realistic. It is therefore reasonable to proceed with the dynamic investigation of the NTF utilizing the model established in this report.

The following action items have arisen as a result of this study:

1. Obtain better data as to the distribution of losses in the tunnel.

2. Establish more precise values of fan ratios as a function of mach number.

3. Obtain better data so as to establish the boundary layer losses.

4. Obtain data as to the variable losses in the high speed diffuser.

5. Determine more precise values for the parameter Ar. Establish any variation in Ar with mach number.

6. Design an experiment on an operating tunnel to facilitate further validation of the simulation.

7. Proceed with the design of a suboptimal mach number control system.

9.0 Acknowledgments.

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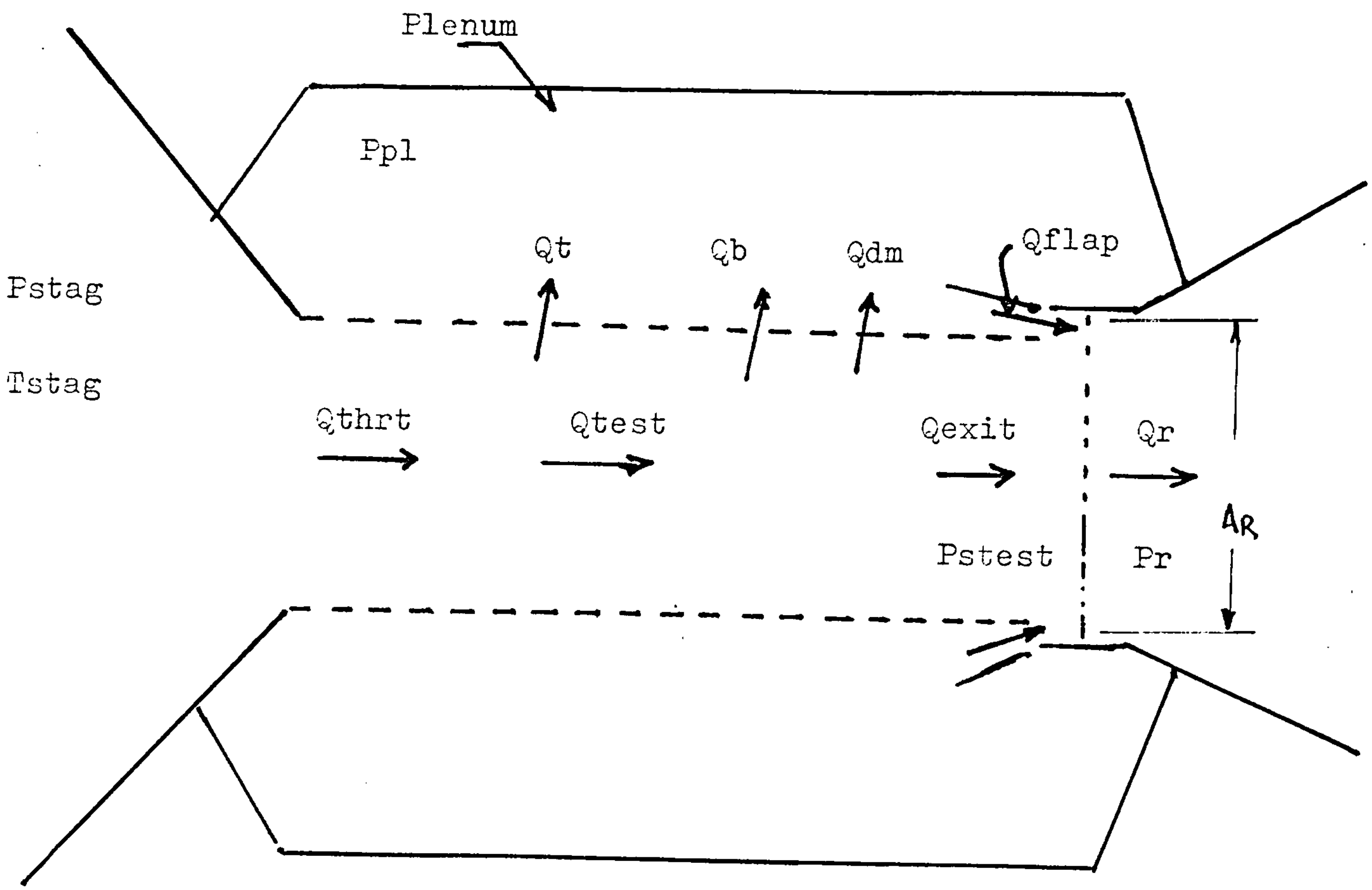


Figure 1 Schematic of Test Section and Plenum Configuration.

Fig. 2a Test section time constant

2 degree step in guide vane

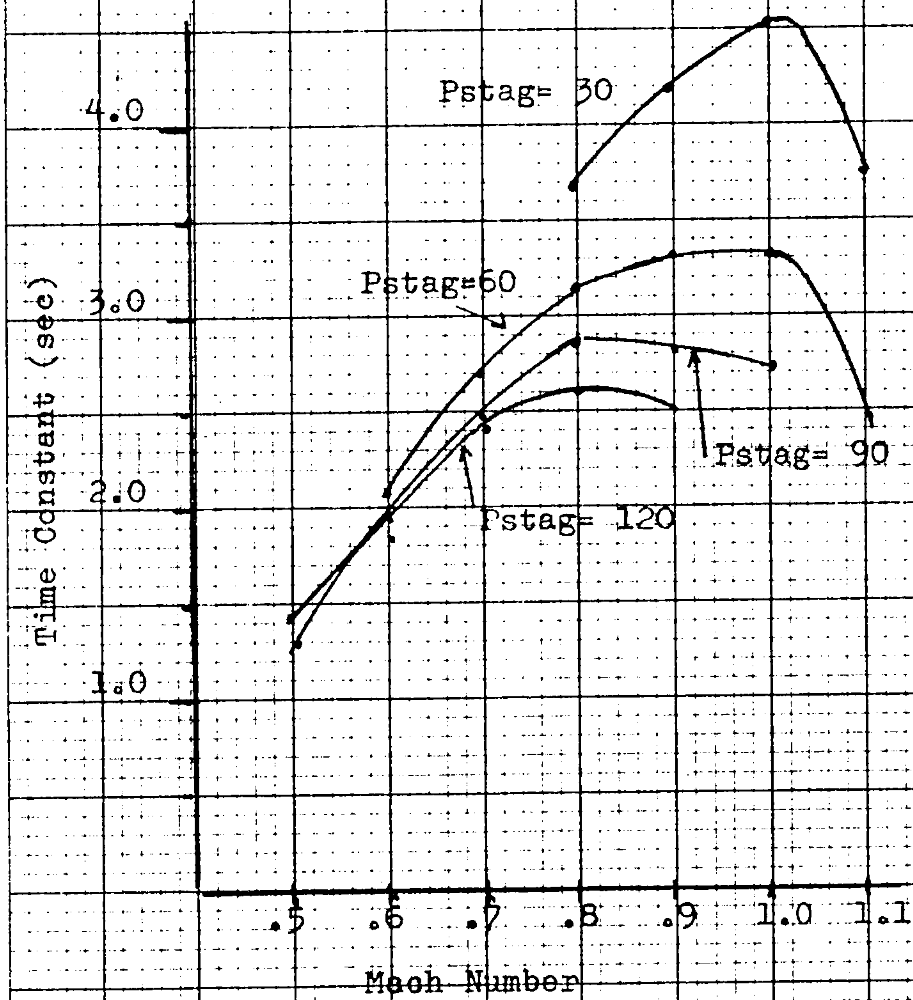


Fig. 2b Test section gain

2 degree step in guide vane

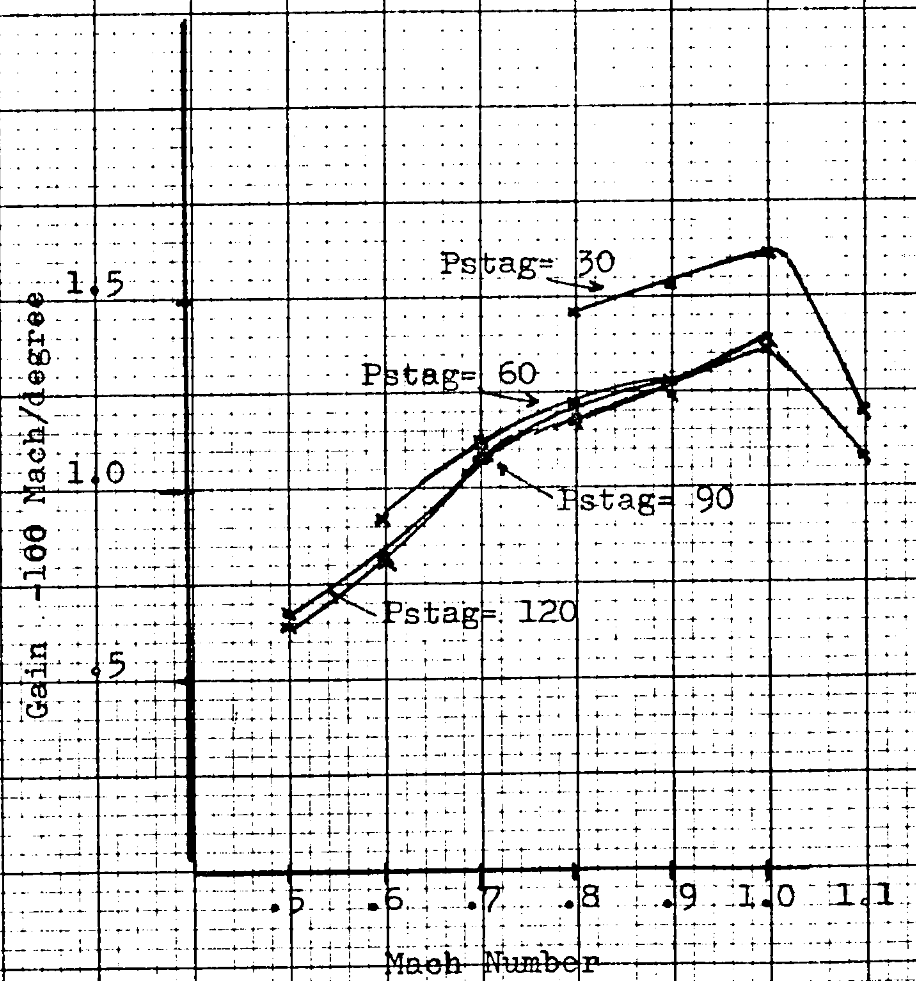


Fig. 3 Test Section Time Constant

2% step in blockage

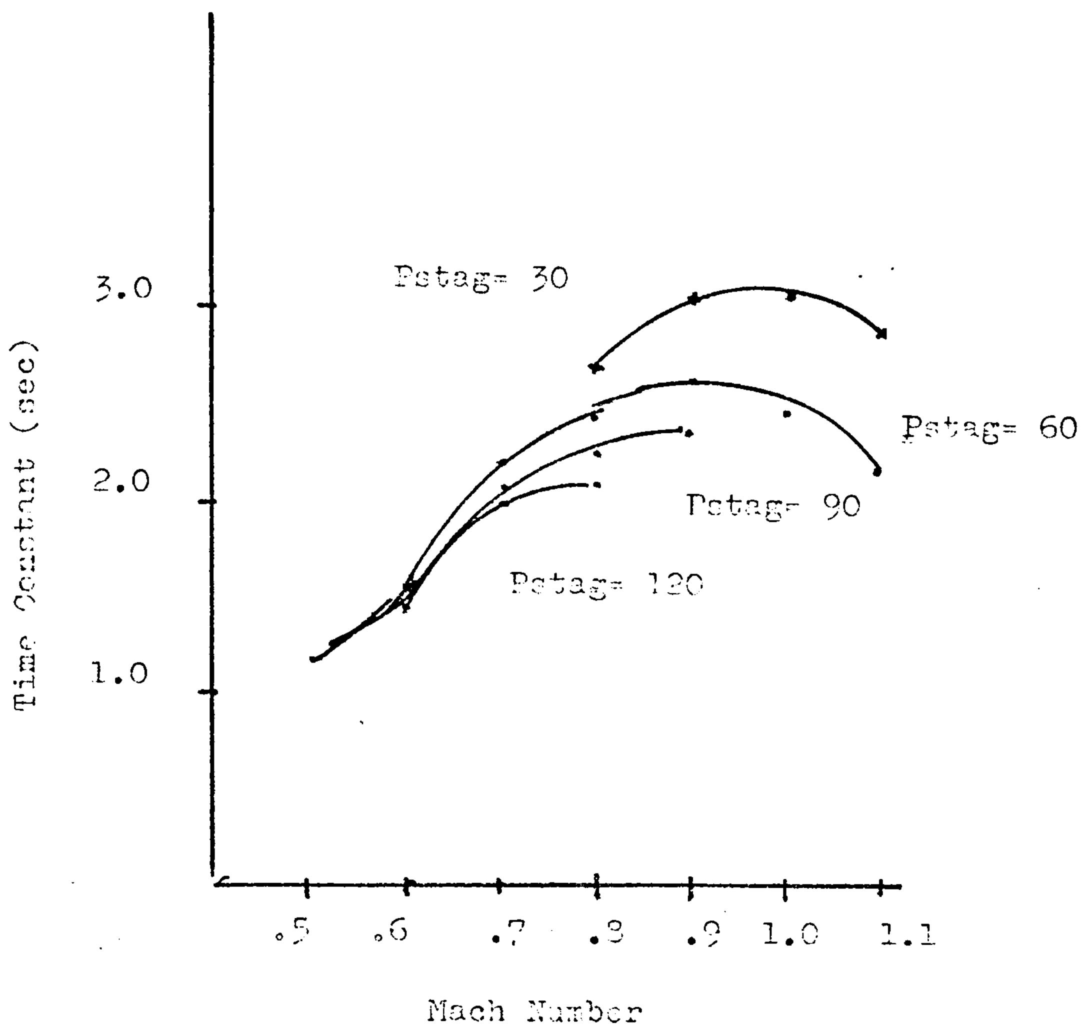


Fig. 4 Test section Time Constant

Pstag= 100, Tstag=300

10 degree step in guide vane

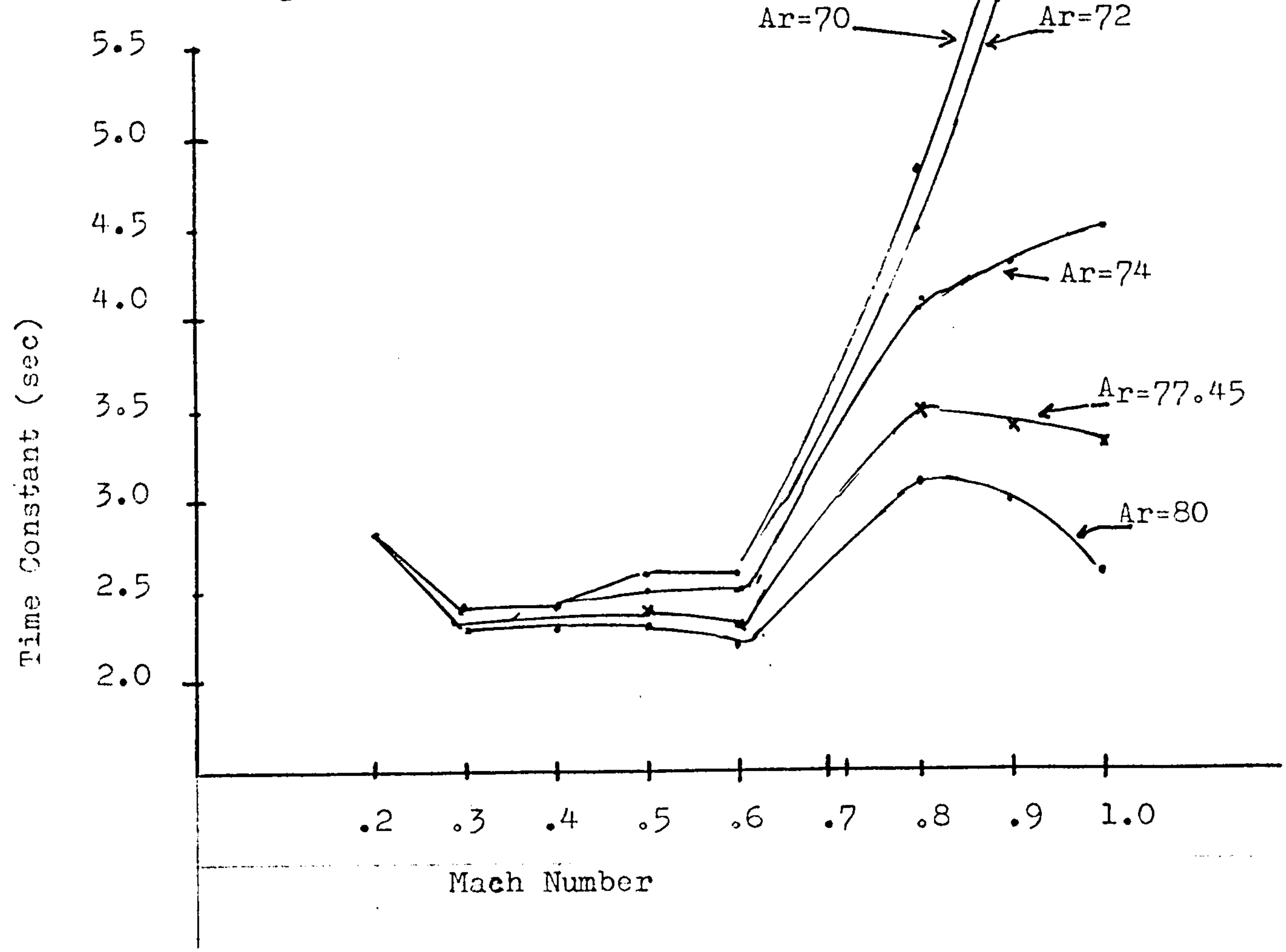


Fig. 5 Response of test section

Pstag= 100., Tstag= 300., Mach= .8 ,

Fan at synchronous speed, 2 degree step in

guide vane angle.

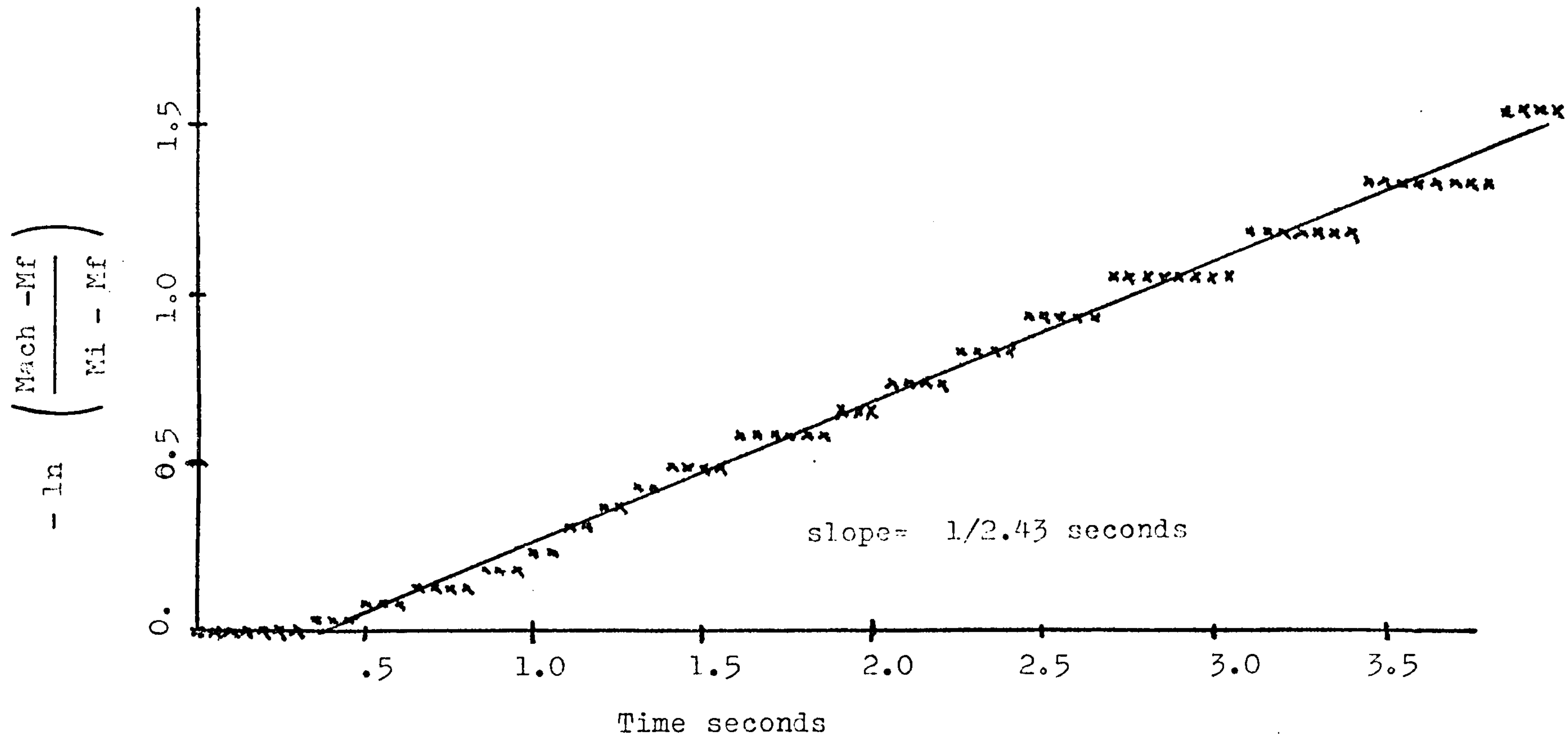


Fig. 6 Response of test section

Pstag= 100., Tstag= 300., Mach= .8,

Fan at synchronous speed

2% blockage

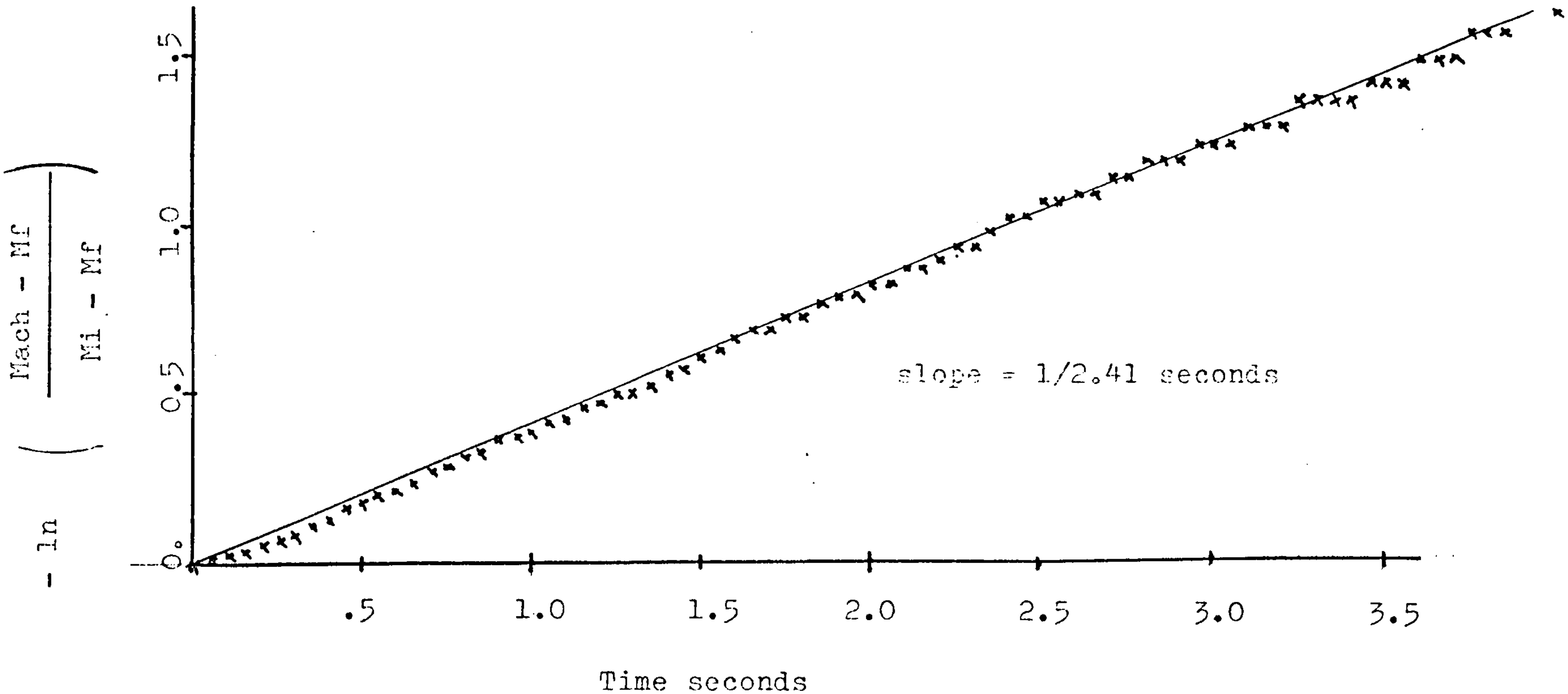
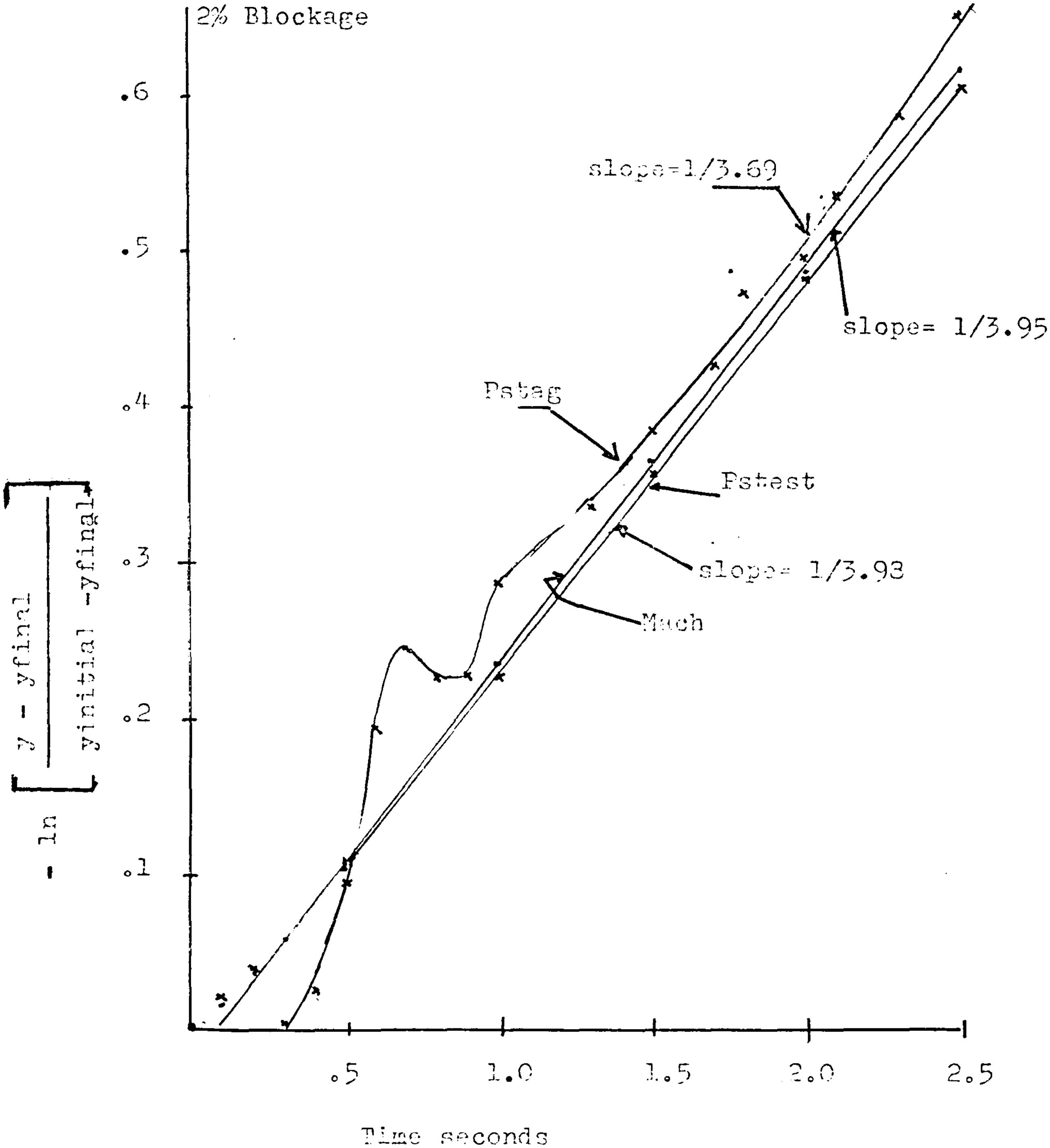


Fig. 7 Response of test section

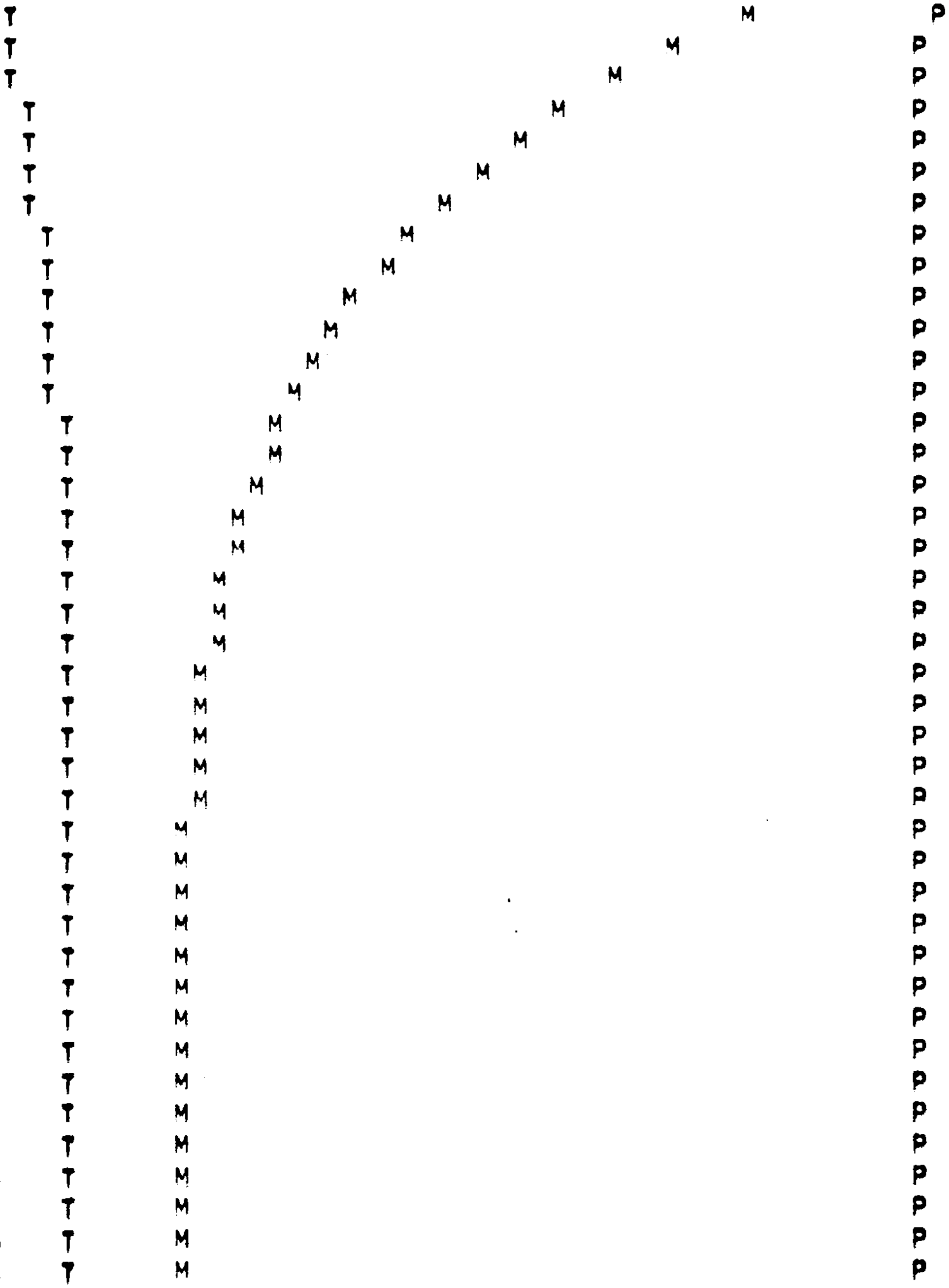
Eight foot tunnel simulation

Pstag= 14.71, Tstag= 500., Mach= 1.05



IRUN= 2219.

TIME	PSTEST	PSTAG	MACH
0.00	7.3237	14.7100	1.0500
.50	7.4024	14.6973	1.0404
1.00	7.4788	14.6740	1.0304
1.50	7.5550	14.6644	1.0214
2.00	7.6166	14.6543	1.0139
2.50	7.6715	14.6415	1.0070
3.00	7.7227	14.6340	1.0009
3.50	7.7657	14.6260	.9957
4.00	7.8046	14.6188	.9909
4.50	7.8385	14.6123	.9868
5.00	7.8677	14.6060	.9832
5.50	7.8943	14.6007	.9800
6.00	7.9181	14.5972	.9772
6.50	7.9388	14.5939	.9747
7.00	7.9572	14.5909	.9725
7.50	7.9736	14.5881	.9705
8.00	7.9885	14.5843	.9687
8.50	8.0015	14.5816	.9671
9.00	8.0122	14.5800	.9658
9.50	8.0212	14.5787	.9647
10.00	8.0296	14.5771	.9637
10.50	8.0377	14.5757	.9627
11.00	8.0449	14.5743	.9619
11.50	8.0512	14.5733	.9611
12.00	8.0565	14.5725	.9605
12.50	8.0607	14.5717	.9600
13.00	8.0644	14.5706	.9595
13.50	8.0680	14.5700	.9591
14.00	8.0711	14.5697	.9587
14.50	8.0738	14.5695	.9584
15.00	8.0762	14.5691	.9581
15.50	8.0786	14.5686	.9578
16.00	8.0809	14.5678	.9575
16.50	8.0825	14.5676	.9574
17.00	8.0836	14.5676	.9572
17.50	8.0844	14.5675	.9571
18.00	8.0856	14.5673	.9570
18.50	8.0869	14.5670	.9568
19.00	8.0881	14.5669	.9567
19.50	8.0891	14.5669	.9566
20.00	8.0897	14.5669	.9565



TIME CONSTANT = 4.00

Figure 8 2% step in blockage.

```
 TRUN= 2234.        DT=  .010       TCIR= 5.179
TEST SECTION
MACH= .8000  PSTAG= 100.00  TSTAG=  300.0      QTEST= 27990.
FAN
QFAN= 28374.8; CR=1.0697; PIN=   94.36; POUT=  100.93; HP=   .66505E+05
GVA=  7.43       RPM=   360.0  ETA= .83  KFAN=  2  GEAR=SYNC
MACH NUMBER
MSET=  .9000      ;GAIN=   20.000      CR= 1.0697   RESET=  .500
 AR= 77.45 A9= 20.00
OUTBLEED
QLOUT=  385.3      ;GAIN=417.399      PSET=100.0   RESET=  .200
INBLEED=
QLIN=  385.3      GAIN=  9.512      TSET=300.0   RESET=  .050
DISTANCE
   0.0    52.5   152.5   236.5   343.5   396.8   443.5   496.0
       7.5   102.5   194.5   270.5   373.5   420.2   488.5
AREA
  123.    240.    446.    662.     90.    146.    174.    123.
      165.    344.    491.    619.    104.    186.    154.
FLOW VOLUME
    15026.  34262.  41162.  57872.   9840.  17776.  18637.
TOTAL PRESSURE
     100.9   100.4   100.3   100.0    95.3    94.5    94.4
WIN=  53243.  WOUT= 146210.  WPL=  20574.
PRESSURE=
      97.1    99.5    99.9    99.7    84.6    91.4    89.8
TOTAL TEMPERATURE
      300.    300.    300.    300.    300.    293.    293.
TEMPERATURE
      297.    299.    300.    300.    290.    290.    289.
FLOW
28375.  28375.  28375.  27990.  27990.  27990.  28375.  28375.
    28375.  28375.  28182.  27990.  27990.  28182.  28375.
MACH NUMBER
      .234    .110    .076    .060    .417    .217    .268
DENSITY
 .8556   .8624   .8702   .8703   .7338   .7926   .8174   .8119
     .8556   .8593   .8711   .8696   .7624   .8228   .8119
 $DATA1 TRUN=34.,PSTAG=100.,TSTAG=300.,MACH=.8,MSET=.9,TFINAL=40.,NPRINT=4,
 GAINM=20.,$END
A STEP CHANGE IN MACH NUMBER OF    .100
```

Figure 9a. Step Response in Mach Number.

IRUN= 2234.

TIME	TEMP	PRESS	MACH
0.00	300.0	100.00	.800
.25	300.0	100.00	.800
.50	300.1	100.21	.802
.75	300.3	100.73	.808
1.00	300.2	100.99	.816
1.25	300.2	101.14	.827
1.50	300.1	101.17	.839
1.75	300.0	101.16	.851
2.00	299.9	100.97	.859
2.25	300.0	100.92	.866
2.50	300.1	101.01	.874
2.75	300.2	101.06	.881
3.00	300.1	100.98	.886
3.25	300.0	100.86	.890
3.50	300.0	100.81	.893
3.75	300.0	100.75	.896
4.00	300.0	100.68	.898
4.25	300.2	100.65	.900
4.50	300.7	100.66	.903
4.75	301.4	100.64	.904
5.00	301.8	100.57	.905
5.25	301.8	100.44	.905
5.50	301.5	100.34	.904
5.75	301.2	100.24	.900
6.00	300.9	100.16	.898
6.25	300.4	100.12	.897
6.50	299.9	100.15	.900
6.75	299.6	100.16	.903
7.00	299.6	100.09	.906
7.25	299.6	99.98	.909
7.50	299.6	99.95	.912
7.75	299.6	99.90	.914
8.00	299.6	99.76	.913
8.25	299.6	99.65	.911
8.50	299.7	99.66	.910
8.75	299.8	99.69	.909
9.00	299.9	99.67	.908
9.25	299.9	99.61	.906
9.50	299.8	99.56	.905
9.75	299.8	99.54	.904
10.00	299.8	99.53	.903
10.25	299.8	99.51	.902
10.50	299.9	99.52	.902
10.75	300.1	99.56	.902
11.00	300.3	99.56	.902
11.25	300.3	99.51	.902
11.50	300.1	99.48	.901

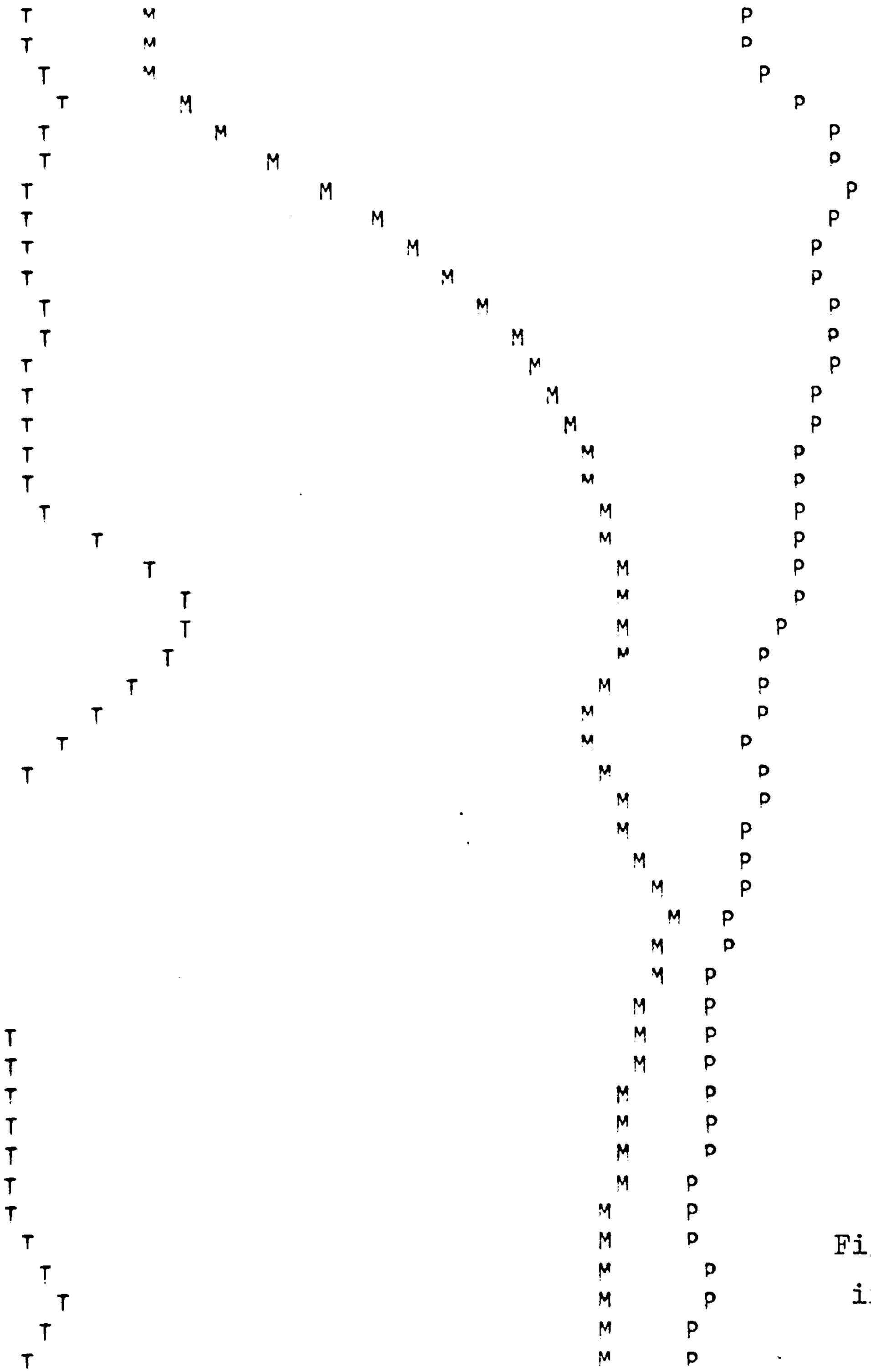


Figure 9b Step Response in Mach Number.

IRUN= 2234.

TIME	TSTAG	PSTAG	QLIN	QLOUT	QTEST	MACH	GVA	CP	TFIN	PIN	PT(10)	PSTEST
0.0	300.000	100.00	395.3	395.3	27989.5	.800	7.435	1.070	293.1	94.356	75.253	65.602
.3	300.000	100.00	399.6	400.8	27989.4	.800	6.389	1.072	293.1	94.268	75.257	65.602
.5	300.025	100.04	398.6	420.7	28003.3	.800	5.399	1.074	293.1	94.135	75.280	65.605
.7	300.059	100.14	405.1	462.1	28042.5	.802	4.953	1.075	293.1	94.052	75.015	65.588
1.0	300.043	100.18	408.4	496.2	28072.9	.803	4.717	1.075	292.9	94.000	74.617	65.515
1.2	300.032	100.21	414.9	519.6	28106.9	.805	4.519	1.076	292.8	93.998	74.274	65.390
1.5	300.020	100.23	426.1	538.7	28140.9	.808	4.329	1.077	292.7	93.957	73.982	65.228
1.7	300.014	100.24	431.1	553.0	28173.7	.811	4.148	1.077	292.7	93.931	73.956	65.061
2.0	299.999	100.23	431.4	559.5	28193.1	.813	3.972	1.078	292.5	93.939	74.032	64.914
2.2	300.022	100.25	432.5	566.1	28220.3	.815	3.797	1.078	292.4	93.929	74.008	64.778
2.5	300.042	100.28	436.9	585.6	28253.4	.818	3.627	1.078	292.4	93.896	73.873	64.643
2.7	300.044	100.31	439.1	606.4	28285.9	.820	3.465	1.079	292.4	93.877	73.741	64.497
3.0	300.025	100.31	440.3	620.5	28309.7	.822	3.310	1.079	292.4	93.863	73.685	64.350
3.2	300.015	100.31	441.4	627.1	28330.0	.824	3.158	1.079	292.3	93.846	73.620	64.209
3.5	300.020	100.32	443.5	637.9	28352.9	.827	3.010	1.080	292.3	93.826	73.565	64.078
3.7	300.022	100.33	445.6	649.7	28375.2	.829	2.865	1.080	292.3	93.797	73.469	63.944
4.0	300.017	100.33	447.5	660.3	28396.1	.831	2.726	1.080	292.2	93.773	73.436	63.809
4.2	300.055	100.33	449.6	669.5	28415.1	.833	2.592	1.081	292.2	93.751	73.352	63.674
4.5	300.155	100.35	452.0	682.6	28433.3	.835	2.463	1.081	292.2	93.724	73.243	63.542
4.7	300.275	100.35	453.5	695.4	28448.8	.837	2.341	1.081	292.2	93.691	73.164	63.409
5.0	300.350	100.35	454.9	706.3	28462.7	.839	2.222	1.082	292.1	93.682	73.133	63.280
5.2	300.364	100.34	457.5	712.3	28474.6	.841	2.107	1.082	292.1	93.663	73.076	63.156
5.5	300.340	100.33	459.6	716.5	28486.7	.843	1.992	1.082	292.1	93.631	73.038	63.045
5.7	300.308	100.32	460.4	719.6	28495.5	.844	1.878	1.082	292.0	93.598	73.019	62.958
6.0	300.276	100.31	462.8	722.8	28505.1	.845	1.763	1.083	292.0	93.585	73.005	62.865
6.2	300.207	100.30	469.7	724.9	28516.9	.847	1.649	1.083	292.0	93.562	72.904	62.763
6.5	300.132	100.30	479.1	729.7	28535.7	.848	1.543	1.083	292.0	93.517	72.752	62.640
6.7	300.099	100.29	483.2	736.8	28554.1	.850	1.447	1.083	292.0	93.471	72.637	62.503
7.0	300.107	100.28	483.8	741.5	28566.5	.852	1.361	1.083	292.0	93.452	72.583	62.367
7.2	300.106	100.27	484.0	742.5	28576.7	.854	1.279	1.084	291.9	93.441	72.513	62.235
7.5	300.095	100.26	484.9	744.6	28591.7	.856	1.202	1.084	291.9	93.407	72.431	62.091
7.7	300.085	100.25	483.9	747.3	28603.8	.858	1.131	1.084	291.9	93.366	72.409	61.960
8.0	300.077	100.23	479.8	746.1	28609.8	.860	1.062	1.084	291.8	93.344	72.415	61.849
8.2	300.072	100.21	476.5	742.6	28614.0	.861	.991	1.084	291.8	93.325	72.391	61.751
8.5	300.075	100.20	476.4	741.3	28621.7	.863	.920	1.085	291.8	93.295	72.329	61.652
8.7	300.083	100.19	478.2	743.3	28630.5	.864	.852	1.085	291.8	93.262	72.268	61.552
9.0	300.092	100.18	479.3	744.7	28637.6	.866	.790	1.085	291.8	93.237	72.203	61.453
9.2	300.088	100.16	479.8	743.1	28642.0	.867	.730	1.085	291.8	93.214	72.158	61.355
9.5	300.073	100.14	480.1	738.5	28645.4	.868	.673	1.085	291.8	93.187	72.124	61.261
9.7	300.061	100.12	480.5	734.1	28649.6	.869	.616	1.085	291.7	93.160	72.100	61.173
10.0	300.051	100.11	480.7	730.8	28654.1	.871	.562	1.086	291.7	93.133	72.068	61.089
10.2	300.039	100.09	481.3	727.3	28657.8	.872	.510	1.086	291.7	93.109	72.030	61.007
10.5	300.041	100.08	482.5	723.3	28661.7	.873	.460	1.086	291.7	93.082	71.975	60.923
10.7	300.069	100.07	483.6	720.9	28665.9	.874	.414	1.086	291.7	93.055	71.922	60.839
11.0	300.101	100.06	484.3	718.6	28668.9	.875	.371	1.086	291.7	93.032	71.876	60.756

Figure 9c

Appendix A. The solution of the diffuser-ejector equation

1. The momentum balance across the area Ar is given by:

$$144 \text{ Ar Pstest} + \text{Qexit Utest}/g = 144 \text{ Ar Pr } (1 + 1.4 \text{ Mr}^2) \tag{29}$$

2. The momentum equation between Pr and P_{10} is given by:

$$\dot{Q}_9 = [144 \cdot A_9 \cdot g \; (Pr - P_{10} - Xq_9 \cdot Q_9 \cdot U_9) + Qr \cdot Ur - Q_{10} \cdot U_{10}]/XL_9 \tag{30}$$

3. To solve these, in terms of Pstest, Mach, and P_{10}, we write:

$$Mr = \frac{Ur}{\sqrt{1\ 4 \cdot R \cdot g \cdot Tr}} = \frac{Qr}{Ar \cdot \rho_r \sqrt{1.4 \cdot R \cdot g \cdot Tr}} \tag{31}$$

$$= \frac{Qr \cdot (5 + 7\ Mr^2)}{Fm \cdot 5\ (5 + Mr^2)} \sqrt{\frac{Ttr \cdot R}{1.4 \cdot g}} \tag{32}$$

where:

$$Fm \cdot 144 \cdot pstest \cdot Ar + Qexit \cdot Utest/g \tag{33}$$

Letting

$$d = \frac{Qr^2 \cdot R \cdot Ttr}{1.4 \cdot Fm^2 \cdot g} \tag{34}$$

the above equation may be written as:

$$\frac{5 \cdot Mr^2\ (5 + Mr^2)}{(5 + 7 \cdot Mr^2)^2} = d \tag{35}$$

or

$$(5 - 49\ d)\ Mr^4 + (25 - 70\ d)\ Mr^2 - 25\ d = 0 \tag{36}$$

5. The above system is solved by letting:

$$b = -\frac{25 - 70\, d}{2\,(5 - 49\, d)}\ ; \qquad c = -\frac{25\, d}{(5 - 49\, d)} \tag{37}$$

which yields the quadratic

$$Mr^4 - 2b\, Mr^2 + c = 0 \tag{38}$$

We consider three cases

<u>case 1</u>. $d \leq 5/49$

Then $b < 0$ and $c < 0$ and since $Mr^2 \geq 0$, the unique solution is

$$Mr^2 = b + \sqrt{b^2 - c} \tag{39}$$

<u>case 2</u>. $5/49 < d < 5/24$

There are two real solutions for Mr^2. Moreover if x is a solution for Mr^2, then y is the dual solution. With x and y satisfying:

$$x = \frac{5 + y}{7y - 1}; \qquad \text{and} \qquad y = \frac{5 + x}{7x - 1} \tag{40}$$

further if $x \geq 1$, then $y \leq 1$, and visa versa. *

In this case we chose the solution for $Mr < 1$ or

$$Mr^2 = b - \sqrt{b^2 - c} \tag{41}$$

For $d \geq 5/24$, the solution is degenerate. In this case we set Mr equal to 1. The value of Qr is obtained from:

* We note that X=1/7 is degenerate. From the shock tables (NACA Report 1135) it is noted that the mach number upstream of a normal shock wave is assymptotic to the square root of 1/7 as the downstream mach number goes to infinity.

$$Qr = Fm \sqrt{\frac{5 \cdot 1 \cdot 4 \cdot g}{24 \cdot R \cdot Ttr}} \tag{42}$$

4.0 The ejector equation is seen to be consistent with the normal shock equation, if Qr = Qexit. For this case we get due to the symmetry between Mach and Mr.

$$\frac{5\ Mr^2\ (5 + Mr^2)}{5 + 7\ Mr^2)^2} = d = \frac{5\ Mach^2\ (5 + Mach^2)}{(5 + 7\ Mach^2)^2} \tag{43}$$

If Mach < 1, then $d \leq 5/24$ and either case one or case two holds. For case one Mr = Mach, since there is only one solution. For case two, again Mr = Mach, since the other solution

$$Mr^2 = \frac{5 + Mach^2}{7\ Mach^2 - 1} \tag{44}$$

is not physically realizable (Mr can not be greater than one).

If Mach $\geq$ 1, then $5/49 < d \leq 5/24$, and case two holds. There are now two possible solutions

Mr = Mach, and $Mr = \sqrt{(5 + Mach^2)/(7\ Mach^2 - 1)}$

which is the well-known shock equation.

APPENDIX B - The Reentry Area A_R

Immediately downstream of the test section there is a step increase in tunnel area. The increased area is approximately 1.16 times the test section area. In this region test section flow and plenum flow merge to form diffuser inlet flow. The ejector action of the diffuser maintains the required plenum flow through the flaps. During supersonic flow a shock wave is established in this region.

For simulation purposes the above transition is assumed to occur at a line. The effective cross sectional area at this line being the numerical value of A_R used in the simulation. Equation (13) equates the momentum balance across this line. As in shown in this report equation 13 as well as the value of A_R are the main significant factors in the characterization of the test section flow phenomena.

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