ON GLOBAL OPTIMAL SAILPLANE FLIGHT STRATEGY

G. Sander and F. X. Litt
University of Liège, Belgium

SUMMARY

The present paper concentrates on the derivation and intepretation of the necessary conditions that a sailplane cross-country flight has to satisfy to achieve the maximum global flight speed. Simple rules are obtained for two specific meteorological models. The first one uses concentrated lifts of various strengths and unequal distance. The second one takes into account finite, non-uniform space amplitudes for the lifts and allows, therefore, for dolphin-style flight. In both models, altitude constraints consisting of upper and lower limits are shown to be essential to model realistic problems. Numerical examples illustrate the difference with existing techniques based on local optimality conditions.

INTRODUCTION

The problems associated with the optimization of sailplane flight paths to achieve maximum cross-country speeds have recently received special attention in the literature. This has been stimulated by the modern competitive soaring which consists almost exclusively in racing and by the development of high performance sailplanes allowing for new, highly efficient flight techniques. Starting with the now classical MacCready [1] results, most of the investigations have been concerned essentially with local optimization problems, that is, finding the optimum flight strategy for various specific situations encountered in a short section of a flight [1 to 10].

In recent papers [2, 4, 5, 8] the optimum speeds to fly in a variety of atmospheric vertical velocity distributions have been determined from the basic assumption that the corresponding flight segments had to be crossed with zero net altitude loss. Conditions under which a transition from the circling mode of climb to the dolphin or essing modes has to be decided have been examined [4]. Although such results yield extremely valuable guidelines for selecting a flight strategy, they only optimize the speed over a limited portion of the total flight.

It is well known, however, in optimization theory that a succession of locally optimum solutions does not, in general, lead to a globally optimum result [11]. It is worth pointing out that a globally optimum flight strategy can only be determined if the assumption is made that the distribution of atmospheric velocities over the whole flight path is known in advance and is independent of time. Although this is never achieved in practice, it is felt that the derivation of global optimality conditions allows for a new insight into the sailplane flight technique by giving a posteriori the decisions that the pilot should have taken and the influence of factors that have been up to now neglected in the analysis, such as the effect of the unequal distribution and strength of the lifts and the effect of minimum and maximum altitude limitations. Such altitude constraints reveal an essential ingredient in the formulation. Their necessity appears as follows. If they are absent and if the lifts are of unequal strength, the globally optimum solution turns out to be trivial and consists of a glide until the maximum lift existing along the path is reached and a climb to an altitude that allows completion of the task, the speed on both segments corresponding to the MacCready setting for that strongest lift [12].

The present paper provides simple rules for global optimality for two simple atmospheric models. These appear to be in agreement with the techniques intuitively used by good competition pilots.

PROBLEM FORMULATION

In both atmospheric models used in the following, the horizontal (wind) velocity of the air mass is assumed to be either zero or to be taken into account by an appropriate modification of the polar equation. The vertical velocity (lift) of the air mass c_i is defined by the so-called netto value. It is constant in the vertical direction between the altitude limits. The flight path is supposed to be constituted by a succession of segments of variable lengths in which the air mass exhibits vertical velocities c_i which are constant along a given segment but vary from one to the other. The altitude constraints consist in constant upper and lower limits denoted $\bar{h}$ and $\underline{h}$. Note that variable altitude limits could be easily incorporated. For simplicity the lower altitude limit is taken as zero ($\underline{h} = 0$).

Concentrated Lift Model

In a first model, the lengths of the segments where a positive vertical velocity is encountered are supposed to be negligible, that is, the lifts are considered as concentrated. The air mass between the lifts is supposed to be stable. Climbing is therefore achieved only by circling at fixed locations corresponding to the lifts. If a lift is not used, its crossing is supposed not to affect the glide and the dynamical aspects of the transition from gliding to circling are not considered. The model is illustrated in figure 1 which is drawn in the vertical plane. The signs of the velocities are taken according to the positive sign of the axes.

The flight consists of a succession of climbs in the selected lifts followed by a glide at constant speed which possibly crosses discarded lifts. The pilot controls the selection of the lifts where he decides to gain altitude, the amount of altitude gained, and the speed to fly between the selected lifts. During the climbs the vertical speed of the sailplane is simply taken as the algebraic sum of the air mass vertical velocity and the mirimum sinking speed of the glider. The increase of sinking speed due to variations in bank angle is not considered explicitely and should be incorporated in the polar definition.

The variations in altitudes are given by

$$h_{2i+1} - h_{2i} = \Delta h_i \qquad\qquad i = 0,1,\dots,n-1 \qquad (1)$$

$$h_{2i+2} - h_{2i+1} = \frac{w(v_i)}{v_i}\,\ell_i$$

where the sinking speed $w(v_i)$ is given by the polar equation. The classical quadratic approximation has been used for numerical examples

$$w = A\,v^2 + B\,v + C \qquad (2)$$

The time spent at each step consists of the sum of the time used in climbing and the transition time between lifts

$$t_{2i} = \frac{\Delta h_i}{c_i + w_m} \qquad\qquad i = 0,1,\dots,n-1 \qquad (3)$$

$$t_{2i+1} = \frac{\ell_i}{v_i}$$

The achieved rate of climb is the sum of the air mass velocity c_i and the glider minimum sinking speed w_m. The constraints on the altitude and altitude gains are expressed by

A control constraint $\quad \Delta h_i \geq 0 \qquad (4)$

Initial and terminal constraints, say $\quad h_o = 0 \quad h_{2n} = 0 \qquad (5)$

Altitude constraints at each step $\quad h_{2i+1} \leq \bar{h}; \quad h_{2i} \geq 0 \qquad (6)$

$i = 0,1,\dots,n-1$

The mathematical problem can be treated by the classical discrete optimal control theory [11] and consists of finding the sequence(s) Δh_o, v_o, Δh_1, v_1, ..., v_{n-1} which satisfy the relations (1), (4), (5), and (6) and minimizes the total flight time

$$T = \sum_{i=0}^{n-1} (t_{2i} + t_{2i+1}) \qquad (7)$$

Distributed Lift Model

In this second model, the length of the segments is always finite and non-vanishing. Between lifts the air mass may present negative vertical velocities so that any desired air mass vertical balance can be achieved. The model is illustrated in figure 2. An important difference with the preceding model is brought by the possibility of crossing a lifting segment at a horizontal speed v_i less than the speed corresponding to the minimum sinking speed w_m. This is achieved by using the equivalent polar illustrated in figure 3 and already used in other similar works [4]. For horizontal speeds less than that corresponding to the minimum sinking speed, the sinking speed remains constant. This appears to be sufficiently accurate to simulate the transition from pure dolphin flight to essing or circling or a combination of equivalent manoeuvers achieved to cross a lifting area in the time required to gain a certain amount of altitude. Note that the same approximation is used as the basis for speed control in some modern instrumentation. In the numerical applications, the quadratic approximation (2) remains applicable at speeds higher than $v(w_m)$.

The variations in altitude are governed by

$$h_{i+1} - h_i = \frac{w(v_i) + c_i}{v_i} \ell_i \qquad i = 0,1,\ldots,n-1 \qquad (8)$$

If $v_i \leqq v(w_m)$, then $w = w_m = \text{Constant}$ and thus the altitude gain is entirely controlled by the equivalent horizontal speed v_i. The altitude gain no longer appears as an explicit control variable. The time spent in each segment is given by

$$t_i = \frac{\ell_i}{v_i} = \frac{h_{i+1} - h_i}{w(v_i) + c_i} \qquad (9)$$

while the altitude constraints read

At initial and terminal points $\qquad h_o = 0 \quad h_n = 0 \qquad (10)$

At each step $\qquad 0 \leqq h_i \leqq \bar{h} \qquad i = 1,2,\ldots,n-1 \qquad (11)$

The mathematical problem consists of finding the sequence(s) $v_o, v_1, \ldots, v_{n-1}$ satisfying the relations (8), (10), and (11) and minimizing the total flight time

$$T = \sum_{i=0}^{n-1} t_i \qquad (12)$$

NECESSARY OPTIMALITY CONDITIONS

The first-order necessary conditions for optimality can be deduced by the classical methods of discrete optimal control [11]. Such methods have been used in previous work [4, 6]. A detailed treatment can be found in [12, 13] for the two atmospheric models presented here, as well as for certain more complex situations. The conclusions are summarized as follows.

Concentrated Lift Model

The Hamiltonian turns out to be

$$H(h_{2i}, \Delta h_i, p_{2i+1}, p^o, 2i) = p^o \frac{\Delta h_i}{c_i + w_m} + p_{2i+1} \Delta h_i$$
$$H(h_{2i+1}, v_i, p_{2i+2}, p^o, 2i+1) = p^o \frac{\ell_i}{v_i} + p_{2i+2} \frac{w(v_i)}{v_i} \ell_i \tag{13}$$

It has to be maximized with respect to Δh_i or v_i for each i. The so-called adjoint variables p_i have to satisfy the relations

$$\begin{aligned}
&p^o - p_1 = 0 && p^o \leqq 0 \\
&p_{2i+1} - p_{2i+2} = \lambda_{2i+1} && i = 0,1,\ldots,n-1 \\
&\lambda_{2i+1}(h_{2i+1} - \bar{h}) = 0 && \lambda_{2i+2} \leqq 0 \\
&p_{2i} - p_{2i+1} = -\lambda_{2i} && i = 1,2,\ldots,n-1 \\
&\lambda_{2i}\, h_{2i} = 0 && \lambda_{2i} \leqq 0
\end{aligned} \tag{14}$$

where the λ_i are Lagrange multipliers. From those conditions, it can be shown [12] that a reduced set of adjoint variables $\eta_o, \eta_1, \ldots, \eta_{n-1}, \psi_o, \psi_1, \ldots, \psi_{n-1}$ and of Lagrange multipliers $\mu_1, \mu_2, \ldots, \mu_{n-1}$ can be derived which, in an optimal solution, have to satisfy the following conditions

$$\mu_{2i+1}(h_{2i+1} - \bar{h}) = 0 \qquad \mu_{2i+1} \leqq 0 \qquad i = 0,1,\ldots,n-1 \tag{15}$$

$$\mu_{2i}\, h_{2i} = 0 \qquad \mu_{2i} \leqq 0 \qquad i = 1,2,\ldots,n-1 \tag{16}$$

$$\Delta h_i \geqq 0 \qquad \eta_i \geqq 0 \qquad \eta_i\, \Delta h_i = 0 \qquad i = 0,1,\ldots,n-1 \tag{17}$$

$$\eta_{i+1} = \eta_i + c_i - c_{i+1} + \mu_{2i+1} - \mu_{2i+2} \qquad i = 0,1,\ldots,n-2 \tag{18}$$

$$v_i \frac{dw(v_i)}{dv_i} - w(v_i) + \psi_i = 0 \qquad \psi_i > 0 \qquad i = 0,1,\ldots,n-1 \tag{19}$$

$$\psi_{i+1} = \psi_i - \mu_{2i+2} + \mu_{2i+3} \qquad i = 0,1,\ldots,n-2 \qquad (20)$$

$$\psi_i = \eta_i + c_i + w_m + \mu_{2i+1} \qquad i = 0,1,\ldots,n-1 \qquad (21)$$

Distributed Lift Model

The optimal solution (h_i, v_i) must be such that the Hamiltonian

$$H(h_i, v_i, p_{i+1}, p^o, i) = p^o \frac{\ell_i}{v_i} + p_{i+1} \frac{w(v_i) + c_i}{v_i} \ell_i \qquad (22)$$

$$i = 0,1,\ldots,n-1$$

is maximized with respect to v_i for each i. The adjoint variables p_i have to satisfy the relations

$$\left.\begin{array}{ll} p^o - p_1 = 0 & p^o \leqq 0 \\ p_i - p_{i+1} = -\lambda_i^{(1)} + \lambda_i^{(2)} & i = 1,2,\ldots,n-1 \\ \lambda_i^{(1)} h_i = 0 & \lambda_i^{(1)} \leqq 0 \\ \lambda_i^{(2)} (h_i - \bar{h}) = 0 & \lambda_i^{2} \leqq 0 \end{array}\right\} \qquad (23)$$

where $\lambda_i^{(1)}$ and $\lambda_i^{(2)}$ are Lagrange multipliers. Again a set of reduced variables and Lagrange multipliers

$$\left(\psi_o,\ldots,\psi_{n-1},\mu_1^{(1)},\ldots,\mu_{n-1}^{(1)},\mu_1^{(2)},\ldots,\mu_{n-1}^{(2)}\right)$$

allows to present the optimality conditions in a more suitable form which turns out to be [13]

$$\left.\begin{array}{ll} \mu_i^{(1)} h_i = 0 & \mu_i^{(1)} \leq 0 \\ \mu_i^{(2)} (h_i - \bar{h}) = 0 & \mu_i^{(2)} \leq 0 \end{array}\right\} \qquad i = 1,2,\ldots,n-1 \qquad (24)$$

$$v_i \frac{dw(v_i)}{dv_i} - w(v_i) - c_i + \psi_i = 0 \qquad i = 0,1,\ldots,n-1 \qquad (25)$$

$$\psi_i = \psi_{i-1} - \mu_i^{(1)} + \mu_i^{(2)} \qquad i = 1,2,\ldots,n-1 \qquad (26)$$

$$\psi_i > 0 \qquad \psi_i \geq w_m + c_i \qquad i = 0,1,\ldots,n-1 \qquad (27)$$

PHYSICAL INTERPRETATION

The interpretation of the two sets of optimality conditions follows similar lines. A first conclusion is drawn from equation (19) or (25) which governs the speed to fly in a segment. Note that equation (25) reduces to equation (19) if the air mass vertical velocity c_i is zero, as assumed in the first model. From figure 3, the reduced adjoint variable ψ_i appears to correspond to a classical MacCready setting and indeed equation (25) appears in most other works on optimization [4, 5, 8]. In the following, the notation $MCS(c_i)$ denotes the setting corresponding to an air mass velocity c_i as defined by equation (25). The next interpretation concerns the Lagrange multipliers μ_i in equation (15), (16), or (24). These multipliers are zero in entering $\left(\mu_{2i} \text{ or } \mu_i^{(1)}\right)$ or leaving $\left(\mu_{2i+1} \text{ or } \mu_i^{(2)}\right)$ a segment if the LAL (lower altitude limit) or UAL (upper altitude limit) is not reached.

From equation (20) or (26) the important conclusion is drawn that the MCS cannot change from a segment to the next one unless either the UAL or the LAL has been reached, that is, if one of the Lagrange multipliers μ becomes negative. If, and only if, the UAL is reached, then the MCS may be reduced. Conversely, the MCS may be increased only if the LAL is touched. To proceed further with the interpretation requires distinguishing between the two models.

Concentrated Lift Model

The reduced adjoint variables η_i appears from equation (17) as indicators of whether the lifts c_i may be used ($\eta_i = 0$) to climb or not ($\eta_i > 0$). With these results in mind, it becomes easy to deduce from equations (18), (20), and (21) the logic for deriving iteratively the optimum solution.

Consider the beginning of the flight at a location where a lift A exists. Denote by B the first lift stronger than A along the flight path. The following iterative reasoning can be made. If B can be reached from the UAL in A with a MCS(A), that is, a MCS corresponding to the lift A, then one has to climb in A just enough to reach B at LAL with a MCS(A). If B cannot be reached with MCS(A) even when climbing at UAL in A, then climb to UAL in A and look for the next best lift, denoted C, between A and B. Evidently $C < B$ and $C \leq A$. Try to reach B at LAL with a MCS inferior to MCS(A) but superior to MCS(C). If this is impossible because C cannot be reached, then restart the reasoning with A unchanged and B replaced by C; if this is impossible because B cannot be reached, then take a MCS(C) to reach C and restart the reasoning with B unchanged and A replaced by C.

Consider now the case where a lift A is stronger than all remaining ones on the flight path. Denote by B the strongest of the remaining lifts. Unless in the special case where the task could be ended from A with a MCS(A) without climbing up to the UAL, one necessarily has to climb up to the UAL and take a MCS superior to the MCS(B) but equal or inferior to the MCS(A). If the task cannot be ended, reach B with a MCS(B) and climb in it up to the UAL. Repeat the reasoning in B for the next strongest lift. If B cannot be reached from A with MCS(B), look for the best lift between A and B, denoted C, and try if B can be reached at LAL with a MCS between MCS(B) and MCS(C). If necessary climb in C if the MCS is equal MCS(C). If it is impossible to reach C at LAL with a MCS(C) look for the best lift, say D, between A and C and repeat the reasoning. If there is no lift, try MCS(O). If it still does not work, the flight is evidently impossible.

A combination of the two reasonings proposed above for increasing or decreasing lifts allows the construction of the optimal solution. Note that it is not necessarily unique.

It is worth pointing out that the optimal solution leads always when going from a lift A to a lift B to use a MCS corresponding to the weaker of the two lifts. If A > B the UAL has to be taken in A while B has to be reached at LAL if A < B. Similar conclusions have been obtained independently by a variational approach in [14].

Distributed Lift Model

In this model, the decision to gain altitude in a lift is dictated by equation (27). If $\psi_i > w_m + c_i$ the speed v_i is larger than $v_i(w_m)$ and therefore is uniquely determined by equation (25). The lift has then to be crossed at the corresponding speed. This appears to be a pure dolphin mode. If, and only if, $\psi_i = w_m + c_i$ the decision of climbing may be taken as the speed v_i becomes equal to or smaller than the speed $v(w_m)$. Indeed, due to the form used for the equivalent polar, the speed v_i cannot be computed by equation (25). Its value is dictated by the need to gain a certain amount of altitude in that lift, given by

$$\Delta h_i = \psi_i \ t_i = (w_m + c_i) \frac{\ell_i}{v_i} \tag{28}$$

If the speed v_i is inferior to $v_i(w_m)$, Δh_i is larger than the value obtained in pure dolphin mode which implies that some sort of manoeuver like essing and/or circling is achieved while flying through the segment.

Except for the process of gaining altitude in a non-dolphin mode, the conclusions reached for the preceding model are still valid. From equation (26) the MCS may not be changed unless the altitude limits are reached. It may increase only if the LAL is touched and may decrease only when reaching the UAL. The process for constructing numerically an optimum solution is as follows.

Start by trying to use the MCS of the best existing lift, say A, for all the segments. If the LAL is not reached, increase the setting until either the task can be ended or the LAL is reached. At that point, the MCS may be increased.

If the MCS corresponding to the best lift allows reaching the LAL before the segment where it occurs, look for the strongest lift between the present point and A. Denote it by B. Then try to reach A at the LAL with a MCS(B). If this is not possible, climbing in B is allowed. If B cannot be reached with a MCS(B), look for the best lift between the present point and B and repeat the reasoning as necessary, keeping in mind the rules that allow climbing in a lift and those that allow changing the MCS.

NUMERICAL EXAMPLES

Concentrated Lift Model

As a simple example, a 300 km flight is schematized in figure 4. The lifts are equidistant (10 km) for simplicity although it is by no means implied in the preceding rules for optimality. The lift strengths are indicated in m/sec along the y-axis. They increase progressively during the flight, then decrease, but are in general unequal. The altitude limits are 0 and 1000 m. The sailplane polar is approximated by

$$w = -1.65\ 10^{-3}\ v^2 + 61.6\ 10^{-3}\ v - 1.026 \qquad \text{(m/sec)}$$

and corresponds to a dry open class ship. The optimal strategy for that lift distribution and altitude constraints is illustrated in figure 4. The MCS for each glide is indicated. It follows as a simple and systematic application of the rules for optimality established above. Note that the flight strategy consists in hitting systematically the altitude limits, except at 110 km and 170 km where the altitude just necessary for reaching the next best lift at LAL is gained. Note also that the MCS is not always equal to the strength of one of the two lifts defining the glide. Finally, note that this example justifies the practical rule of flying "low" when the lifts are improving and keeping "high" when they are deteriorating. The cruising speed for that flight is 81.01 km/h.

As a test for the sensitivity of the speed with respect to the MCS, the same problem has been solved with the additional constraints of keeping the same MCS which implies the same speed between any two selected lifts. The optimal MCS for that situation has been obtained in [13] in the form

$$\psi = \frac{L}{\sum \frac{\ell_i}{c_i + w_m}} \qquad (29)$$

where L is the total length of the flight (300 km) and $c_i + w_m$ is the achieved rate of climb in each selected lift. Note that the selection of the lifts is highly dependent upon the altitude limits. In the present example $\psi = 1.23$ m/sec and the corresponding constant speed between the thermals is 133 km/h. The new constrained optimal speed becomes 79.51 km/h which differs by only 1.85% from the exact optimum. Although some restrictions have to be mentioned (see [13]) concerning the applicability of equation (29) in a general case, it indicates clearly that in an atmosphere corresponding to the present model, the MCS is much less important than the correct selection of the lifts. This is again well known for many competition pilots [15].

Distributed Lift Model

The flight polar has been approximated by

$$w(v) = -1.896\ 10^{-3}\ v^2 + 77.8\ 10^{-3}\ v - 1.27 \qquad \text{(m/sec)}$$

which corresponds also to a dry open class glider and to the model used in [4] and [5]. Three distributions of lifts have been selected and are presented in figures 5, 6, and 7 and denoted flights I, II, and III. These flights are all 200 km long and correspond to different weather conditions. In flight I the lifts are relatively concentrated except at two places and their strengths are rather different from each other. The length of the lifting zones represents 36% of the total which is rather critical for the transition from thermaling to dolphining [4]. The air mass balance is positive, that is, the average over the distance of the air mass (netto) vertical velocities yields 0.39 m/sec. In flight II the lifting zones represent 31% of the distance and their strengths are much more similar to each other. The lifting and sinking areas exactly balance each other; that is, not only the average vertical velocity is zero, but the air mass is organized in a succession of cells which are 20 to 40 km long where the exact air mass balance is also achieved. This allows for using the classical rules for local optimality [4] in crossing these cells and compares with the globally optimal solution. In flight III the lifts are weaker and their strengths still closer to each other. The lifting zone represents 49% of the total. The air mass balance yields 0.236 m/sec and the lifting zones are again organized in cells in which approximately the same air mass balance is maintained. For each of these atmospheric models three upper altitude limits have been considered $\bar{h}$ = 1000 m, 1500 m, and 2000 m. The LAL has been kept at $\underline{h} = 0$ which is evidently not necessarily the ground level.

The numerically obtained optimal solutions are illustrated in tables I, II, and III in digital form and in figures 8, 9, and 10 in graphical form. The satisfaction of the optimality conditions described above are easily verified. The lifts in which gaining altitude in circling or essing are indicated as well as the corresponding equivalent horizontal speed which is then smaller than the speed of minimum sink $v(w_m)$ = 20.52 m/sec. In the other segments, crossed in dolphin mode, the optimum MCS is given. Note that w_m = 0.47 m/sec. The influence of the altitude limits is illustrated by the following table:

	Δh = 1000 m	Δh = 1500 m	Δh = 2000 m	Δh = Unlimited
Flight I	v = 94.5	97.94	100.19	100.57
Flight II	v = 73.76	81.2	83.10	84.20
Flight III	v = 85.87	87.98	88.16	88.16

where Δh is the allowed altitude range and v is the optimum average speed in km/h. The application of various non-globally optimal flight strategies based on the use of existing rules for optimizing the speed in each individual meteorological cell [4, 5] resulted in average speed from 5 to 15% inferior depending on the allowed altitude limits and on the various conditions selected in applying these rules.

CONCLUSIONS

Simple rules for obtaining numerically the optimum flight strategy in two meteorological models have been obtained. Their applications reveal that the altitude limits imposed for the flight may have, as known from experience, a much more significant influence on the achieved speed than the selection of MCS. Additional investigation is required to determine the relation beween the various possible weather profiles and the optimum flight strategies as well as altitude limits influence. The in flight recording of such atmospheric profiles over rather long distances would allow for studying systematically the optimum solution corresponding to a number of classical situations.

SYMBOLS

A,B,C	flight polar coefficients
c_i	air mass netto vertical velocity
h_i	altitude
$\bar{h}$	upper altitude limit
H	Hamiltonian
ℓ_i	flight segment length
p_i	adjoint variable
t_i	elapsed time
T	total flight time
v_i	horizontal speed
w	sailplane sinking speed
w_m	minimum sinking speed
λ_i, η_i, μ_i	Lagrange multipliers
ψ_i	reduced adjoint variable = MCS
$\bar{\ }_i$	index of a flight segment
UAL, LAL	upper (lower) altitude limits
MCS	MacCready setting

REFERENCES

1. MacCready, P.: Optimum Speed Selector. Soaring, Apr. 1954.

2. Reichmann, H.: Zum Problem der Fahrtoptimierung im Streckensegelflug. Diss. Univ. Karlsruhe, Int. Bericht no. 76/2.

3. Reichmann, H.: Strecken-Segelflug. Motor Buch Verlag, Stuttgart, 1976.

4. Metzger, D.; and Hedrick, J.: Optimum Flight Paths for Soaring Flights. J. of Aircraft, vol. 12, no. 11, Nov. 1975.

5. Arho, R.: Some Notes on Soaring Flight Optimization Theory. Tech. Soaring, vol. IV, no. 2, 1977.

6. Irving, F.: Cloud-Street Flying. Tech. Soaring, vol. III, no. 1, 1976.

7. Gedeon, J.: Dynamic Analysis of Dolphin Style Thermal Cross Country Flight. Tech. Soaring, vol. III, no. 1, 1976.

8. Arho, R.: Optimal Dolphin Soaring as a Variational Problem. Tech. Soaring, vol. III, no. 1, 1976.

9. Bohli, H.: Optimale Dolphinfluggeschwindigkeit auf Streckensegelflug. Aero Revue, no. 8, Aug. 1971.

10. Pirker, H.: Some Computer Calculations on Optimum Water-Bollest of Sailplanes. 15th OSTIV Congress, Räyskälä, Finland, 1976.

11. Cannon, M.; Cullum, C.; and Polak, E.: Theory of Optimal Control and Mathematical Programming. McGraw Hill, 1970.

12. Litt, F. X.; and Sander, G.: Optimal Strategy in a Given Space Distribution of Lifts With Minimum and Maximum Altitude Constraints. Report SART 78/03, Service de Régulation et Automatique, Univ. Liège, Belgium, June 1978. Also presented at XVI OSTIV Congress in Châteauroux, France.

13. Litt, F. X.; and Sander, G.: Global Optimization of Sailplane Flight Techniques. Report SART 79/02, Service de Régulation et Automatique, Univ. Liège, Belgium, Feb. 1979.

14. de Jong, J. L.: De optimale MacCready-ring instellingen by inachtname van verticale begrenzingen van de bruikbare thermiek hoogte. COSOR R-78-05, Tech. Hogeschool Eindhoven, Holland, April 1978.

15. Schuemann, W.: The Price You Pay for MacCready Speeds. Proc. Symp. on Competitive Soaring, 1972, U.S.A.

TABLE I.- OPTIMAL SOLUTION FOR FLIGHT I

	H_{max} = 1000 m				H_{max} = 2000 m			
Segment no.	MCS	h_{out}	v_i	Mode	MCS	h_{out}	v_i	Mode
1	0.53	425	0.6	C	0.53	425	0.6	C
2	0.53	0	30.8	D	0.53	0	30.8	D
3	1.03	481	1.0	C	1.03	481	1.0	C
4	1.03	0	34.8	D	1.03	0	34.8	D
5	2.03	732	2.7	C	2.03	732	2.7	C
6	2.03	288	41.7	D	2.03	288	41.7	D
7	2.03	213	38.4	D	2.03	213	38.4	D
8	2.03	233	34.8	D	2.03	233	34.8	D
9	2.03	0	44.7	D	2.03	0	44.7	D
10	3.03	164	34.8	D	3.03	164	34.8	D
11	3.03	1000	18.1	C	3.03	1781	9.3	C
12	1.57	646	47.8	D	3.03	1396	55.3	D
13	1.57	356	45.0	D	3.03	1069	52.8	D
14	1.57	249	35.0	D	3.03	826	44.7	D
15	1.57	350	31.0	D	3.03	749	41.7	D
16	1.57	712	21.0	D	3.03	913	34.8	D
17	1.57	53	42.0	D	3.03	121	50.3	D
18	1.57	0	35.1	D	3.03	0	44.7	D
19	4.61	268	42.2	D	4.03	370	38.4	D
20	4.61	1000	27.0	D	4.03	2000	12.3	C
21	1.38	428	43.8	D	2.69	1365	51.1	D
22	1.38	0	40.7	D	2.69	858	48.5	D
23	1.53	858	8.9	C	2.69	1056	32.1	D
24	1.53	437	38.4	D	2.69	506	45.7	D
25	1.53	0	41.7	D	2.69	0	48.5	D
Speed	94.54 km/h				100.19 km/h			

h_{out} = altitude at the end of the segment (meters)

Mode = D for dolphin

C for climbing with $v_i < v_i(w_m)$

MCS, v_i = m/sec

TABLE II.- OPTIMAL SOLUTION FOR FLIGHT II

	H_{max} = 1000 m				H_{max} = 2000 m			
Segment no.	MCS	h_{out}	v_i	Mode	MCS	h_{out}	v_i	Mode
1	0.53	447	2.9	C	0.53	447	2.9	C
2	0.53	284	30.8	D	0.53	284	30.8	D
3	0.53	541	20.5	D	0.53	541	20.5	D
4	0.53	0	38.4	D	0.53	0	38.4	D
5	1.03	617	8.3	C	1.03	617	8.3	C
6	1.03	0	38.4	D	1.03	0	38.4	D
7	2.03	1000	10.1	C	2.03	2000	5.0	C
8	1.03	443	41.7	D	2.03	1398	47.6	D
9	1.03	237	38.4	D	2.03	1165	44.7	D
10	1.03	181	30.8	D	2.03	1014	38.4	D
11	1.03	895	7.2	C	2.03	1149	30.6	D
12	1.03	278	38.4	D	2.03	450	44.7	D
13	1.03	0	41.7	D	2.03	149	47.6	D
14	2.03	1000	10.1	C	2.03	2000	5.5	C
15	0.53	729	38.4	D	1.53	1711	44.7	D
16	0.53	144	34.8	D	1.53	1055	41.7	D
17	0.53	890	10.6	C	1.53	1215	30.8	D
18	0.53	781	30.8	D	1.53	1074	38.4	D
19	0.53	0	34.8	D	1.53	200	41.7	D
20	1.53	1000	7.6	C	1.53	1092	8.6	C
21	0.78	0	36.7	D	1.53	0	41.7	D
Speed	73.76 km/h				83.10 km/h			

h_{out} = altitude at the end of the segment (meters)

Mode = D for dolphin

C for climbing with $v_i < v_i(w_m)$

MCS, v_i = m/sec

TABLE III.- OPTIMAL SOLUTION FOR FLIGHT III

	H_{max} = 1000 m				H_{max} = 2000 m			
Segment no.	MCS	h_{out}	v_i	Mode	MCS	h_{out}	v_i	Mode
1	0.53	364	3.6	C	0.53	364	3.6	C
2	0.53	200	30.8	D	0.53	200	30.8	D
3	0.53	586	20.6	D	0.53	586	20.6	D
4	0.53	0	34.8	D	0.53	0	34.8	D
5	1.03	999	20.6	D	1.03	999	20.6	D
6	1.03	594	47.6	D	1.03	594	47.6	D
7	1.03	37	41.7	D	1.03	37	41.7	D
8	1.03	968	16.6	C	1.03	968	16.6	C
9	1.03	411	41.7	D	1.03	411	41.7	D
10	1.03	0	38.4	D	1.03	0	38.4	D
11	1.53	1000	15.3	C	1.53	1737	8.8	C
12	0.56	608	35.0	D	1.53	1299	41.7	D
13	0.56	208	44.9	D	1.53	887	50.3	D
14	0.56	587	20.8	D	1.53	1046	30.8	D
15	0.56	0	35.0	D	1.53	390	41.7	D
16	1.03	673	15.3	C	1.53	759	26.2	D
17	1.03	617	30.8	D	1.53	656	34.8	D
18	1.03	0	38.4	D	1.53	0	41.7	D
Speed	85.87 km/h				88.16 km/h			

h_{out} = altitude at the end of the segment (meters)

Mode = D for dolphin

C for climbing with $v_i < v_i(w_m)$

MCS, v_i = m/sec

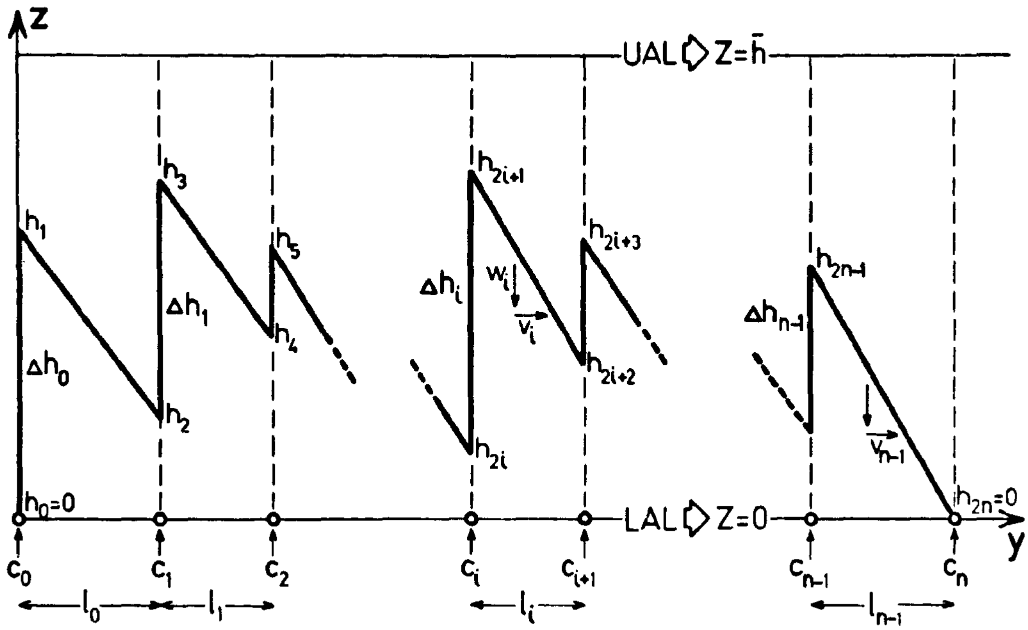


Figure 1.- Concentrated lift model.

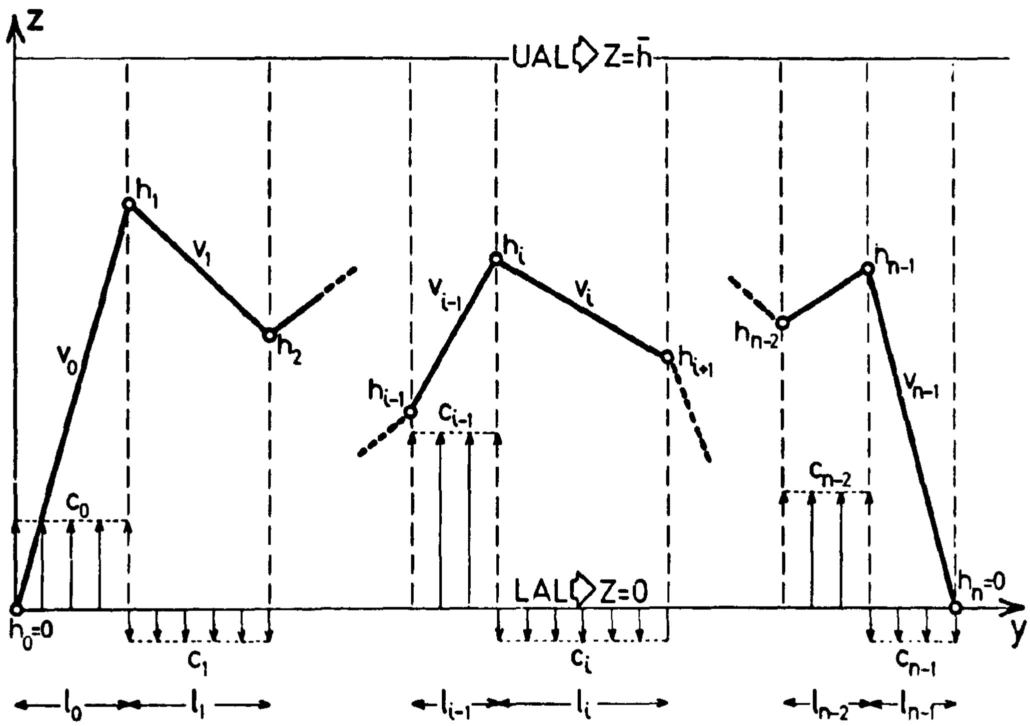


Figure 2.- Distributed lift model.

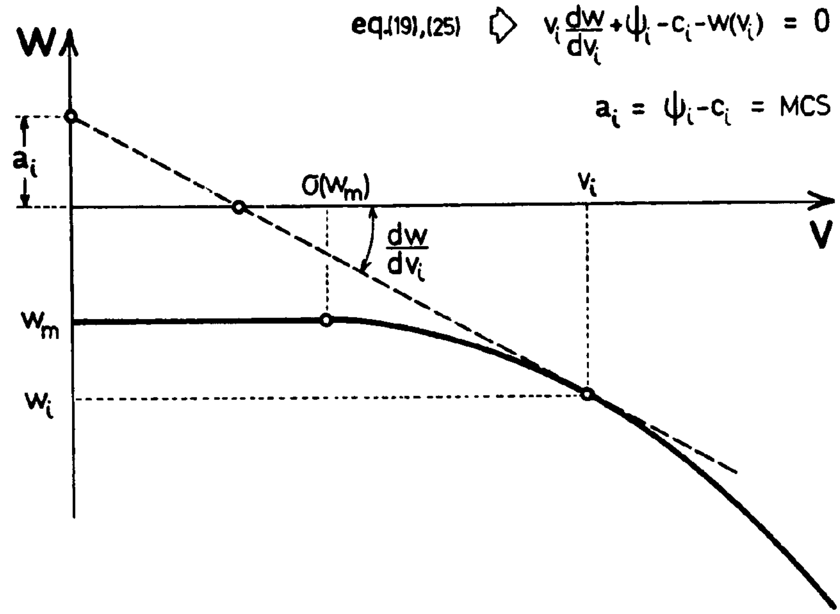


Figure 3.- The equivalent polar.

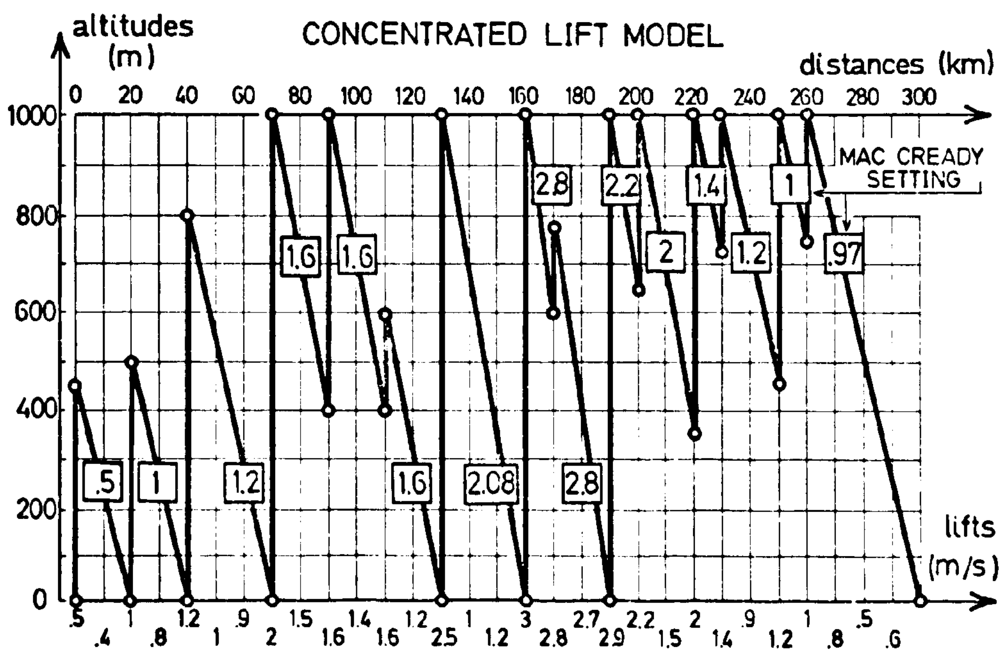
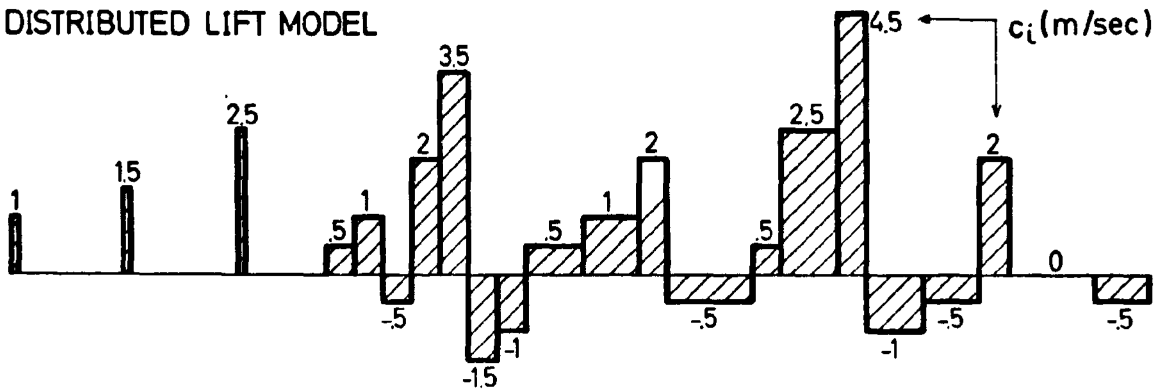


Figure 4.- Optimum flight strategy.



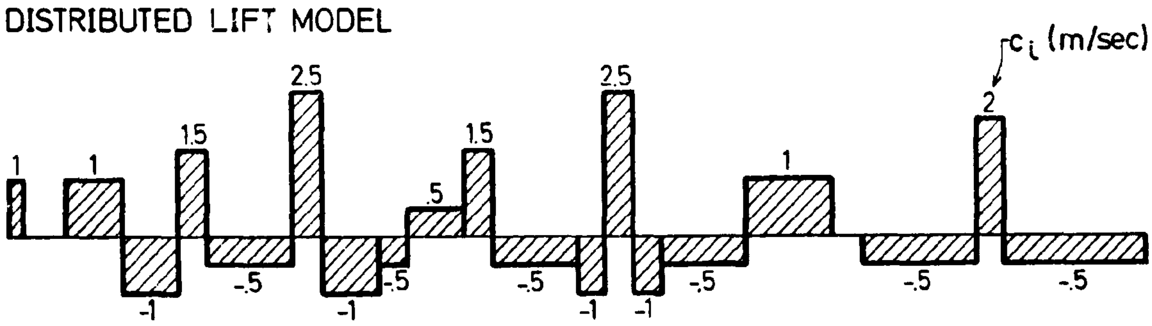
flight segment no.					1	2	3	4	5	6	7	8	9	10
c_i (m/sec)					1	0	1.5	0	2.5	0	.5	1	-.5	2
l_i (km)					.5	19.5	.5	19.5	1	14	5	5	5	5
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
3.5	-1.5	-1	.5	1	2	-.5	.5	2.5	4.5	-1	-.5	2	0	-.5
5	5	5	10	10	5	15	5	10	5	10	10	5	15	10

$L = \sum l_i = 200$ km

$\bar{c} = \frac{\sum c_i l_i}{L} = .39 \frac{m}{sec}$

LIFTING ZONES = 36 %

Figure 5.- Lift distribution for flight I.



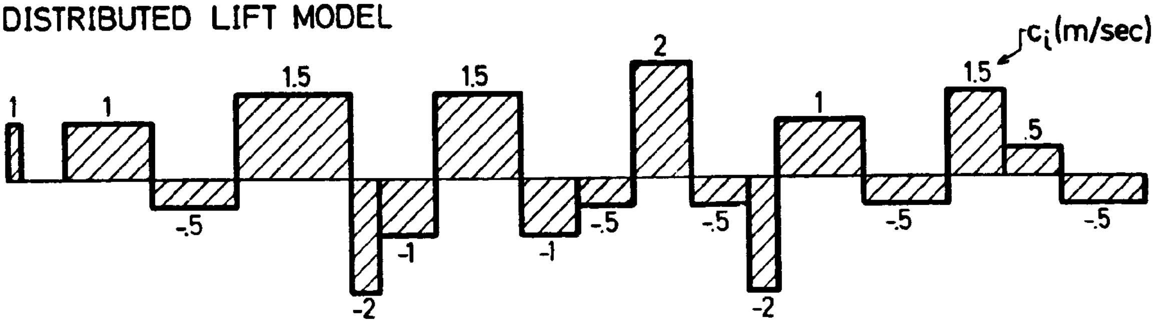
flight segment no.		1	2	3	4	5	6	7	8	9	10
c_i (m/sec)		1	0	1	-1	1.5	-.5	2.5	1	-.5	.5
l_i (km)		2.5	7.5	10	10	5	15	5	10	5	10
	11	12	13	14	15	16	17	18	19	20	21
	1.5	-.5	-1	2.5	-1	-.5	1	0	-.5	2	-.5
	5	15	5	5	5	15	15	5	20	5	25

$L = \sum l_i = 200$ km

$\bar{c} = \frac{\sum c_i l_i}{L} = 0$ m/sec

LIFTING ZONES = 31 %

Figure 6.- Lift distribution for flight II.



flight segment no.	1	2	3	4	5	6	7	8	9	10
c_i (m/sec)	1	0	1	-.5	1.5	-2	-1	1.5	-1	-.5
l_i (km)	2.5	7.5	15	15	20	5	10	15	10	10

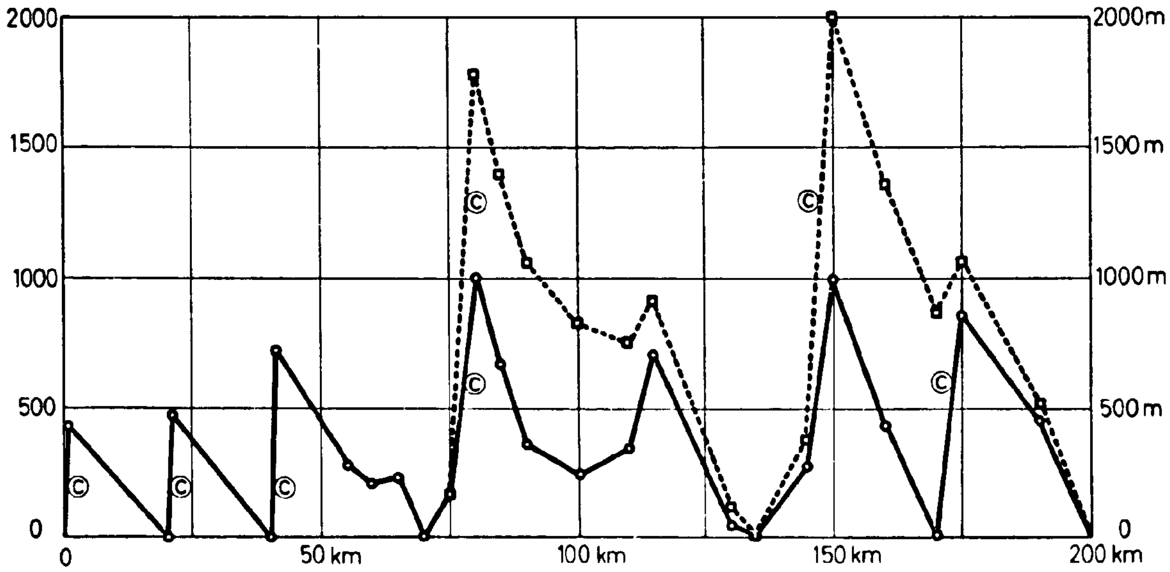
flight segment no.	11	12	13	14	15	16	17	18
c_i (m/sec)	2	-.5	-2	1	-.5	1.5	.5	-.5
l_i (km)	10	10	5	15	15	10	10	15

$L = \sum l_i = 200$ km

$\bar{c} = \frac{\sum c_i l_i}{L} = .236 \frac{m}{sec}$

LIFTING ZONES = 49%

Figure 7.- Lift distribution for flight III.



© climbing with $v_i < v_i(w_m)$

Figure 8.- Optimal solution for flight I.

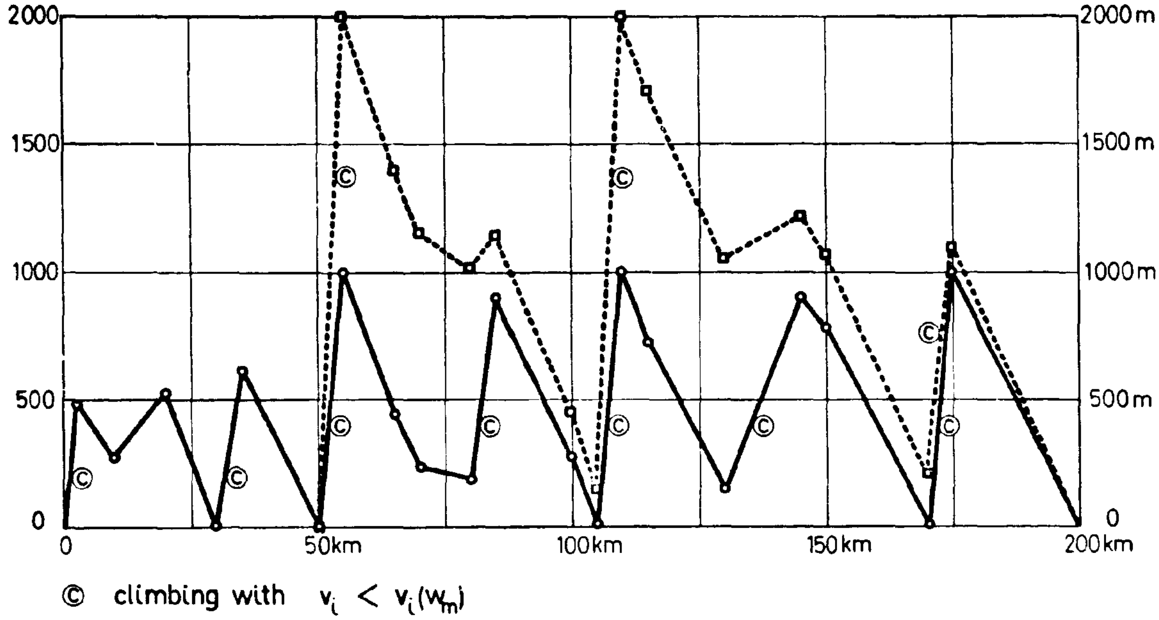


Figure 9.- Optimal solution for flight II.

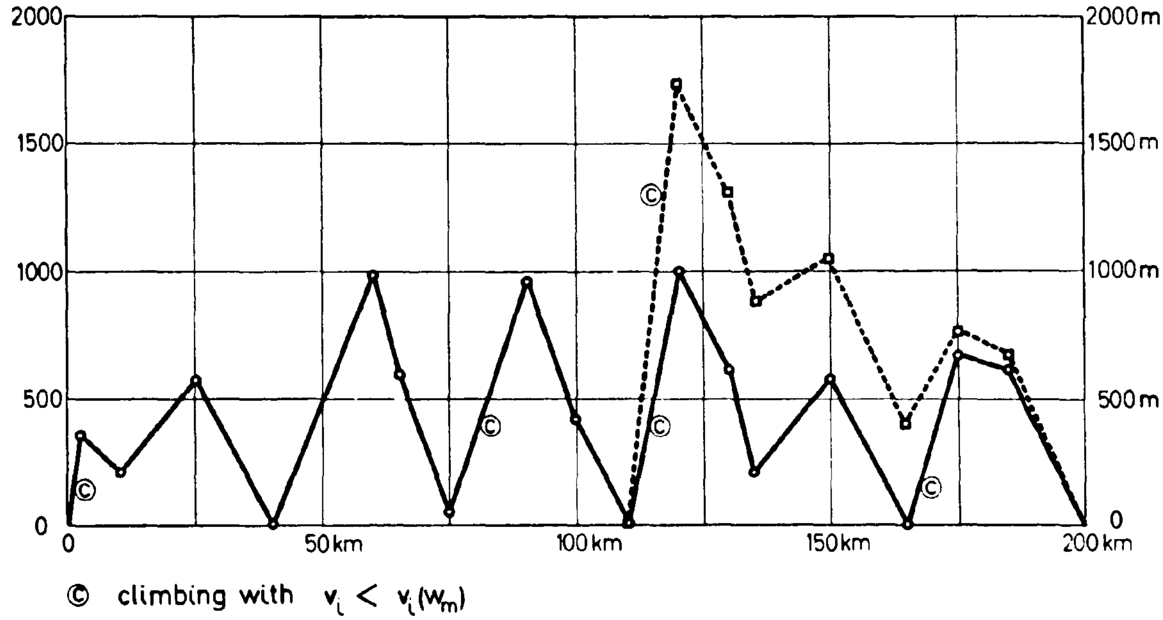


Figure 10.- Optimal solution for flight III.