NASA Technical Paper 1485

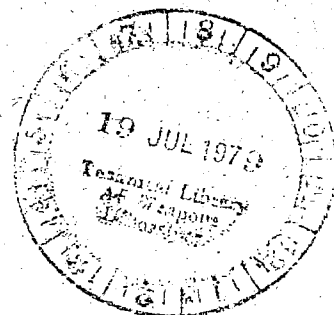# Design of High-Perveance Confined-Flow Guns for Periodic-Permanent-Magnet-Focused Tubes

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# Design of High-Perveance Confined-Flow Guns for Periodic-Permanent-Magnet-Focused Tubes

N. Stankiewicz
*Lewis Research Center*
*Cleveland, Ohio*

# SUMMARY

Immersing the cathode of an electron gun in a magnetic field produces a beam which is less sensitive to transverse forces. This technique, called confined flow, is used extensively to diminish beam scalloping due to the transverse velocities of thermal electrons and to improve beam transmission under radiofrequency operation with solenoidal fields. Confined flow can also be used to provide stiffer beams which resist the magnetic perturbations inherent in periodic-permanent-magnet (ppm) focusing.

This report gives an approach to the design of high-perveance, low-compression guns in which confinement is used to stabilize the beam for subsequent ppm focusing. The computed results for two cases are presented. A magnetic-boundary-value problem is solved for the scalar potential from which the axial magnetic field is computed. A solution is found by iterating between Poisson's equation and the electron trajectory calculations. Magnetic-field values are varied in magnitude until a laminar beam with minimum scalloping is produced.

# INTRODUCTION

The weight and power advantages of using periodic-permanent-magnet (ppm) stacks for focusing linear-beam microwave tubes have been known for some time (ref. 1). There is, however, an important class of tubes which continue to be equipped with solenoids. Characteristically these are high-power tubes with microperveances of order unity and greater.

A broad rule for obtaining stable laminar flow with ppm focusing requires that the magnetic period be less than the plasma wavelength (ref. 2). Because the plasma wavelength is proportional to the beam diameter divided by the square root of the perveance, it becomes increasingly difficult to fabricate magnetic circuits for perveances beyond certain values. It is usual in such cases to use solenoidal focusing.

If the cathode is immersed in a magnetic field, the beam is launched with an initial angular momentum. For an axisymmetric gun, the angular momentum is conserved, and this effect leads to Busch's theorem, which relates the beam rotation, radius, magnetic flux, and cathode flux (see, e.g., appendix A of ref. 3). The resultant centrifugal force due to the rotation adds to the space-charge force and requires a larger focusing field for balanced flow. This field, in turn, provides a larger restoring force

if the beam is perturbed from its equilibrium radius. Such beams are described as being stiffer or less sensitive to transverse forces (ref. 4).

The technique of cathode immersion in a magnetic field (or confined flow) is used to some extent in most linear-beam tubes to improve beam transmission through the tube in the presence of radiofrequency (rf) interactions. Confined flow is also used to diminish beam scalloping due to electrons being emitted from the cathode with other than zero and/or normal velocities. These transverse thermal effects are known to be amplified during beam compression (ref. 5). However, for the low-compression beams to which this study is addressed, thermal effects are of secondary importance.

It is the purpose of this report to investigate the use of confined flow for ppm-focused linear-beam tubes having high perveances and to establish a method for designing proper guns. A version of the Stanford Electron Trajectory Program (ref. 6) is used to simulate the gun and ppm stack. The program was adapted to run on the Lewis Research Center IBM 360 computer.

The assumptions and definitions used in this study are

(1) There is axial symmetry.

(2) The emissions from the cathode are space-charge limited.

(3) Thermal velocity effects can be ignored for the low compression ratios used in this study.

(4) Initially the magnetic and electric boundaries can be considered to be coincident. Later, because of the approximation used to compute off-axis magnetic fields, this assumption is modified.

(5) The cathode is assumed to be located at a magnetic equipotential surface. An auxiliary problem also considered in this report is the design of a magnetic circuit that will provide the proper field shaping.


METHOD OF SOLUTION

Electrostatic Problem

The assumed geometry of the gun and ppm stack is shown in figure 1. Its principal elements are (1) a cathode which is a spherical section having a radius $R_s$, (2) a focusing electrode which follows the Pierce design (ref. 7) ($67.5^o$ from the normal at the cathode edge), (3) an anode located a distance $Z_a$ from the cathode, and (4) a ppm stack of period $\lambda_m$ and of aperture $r_a$. This version of a gun and ppm stack is a simplified one which ignores such details as anode shaping and gap ferrules but does represent the fundamental elements.

From a design point of view, the anode voltage $V_o$, the beam current $I_o$, the beam radius $R_b$, the magnetic period, and the emitting area of the cathode (specified

by the cathode radius $r_c$) are set by external considerations. The gun model used in this study has, therefore, only two basic geometric parameters which are free to be varied: the spherical radius of the cathode $R_s$ and the anode spacing $Z_a$.

The gun design is begun by establishing the geometry. The program is run electrostatically, that is, with no impressed magnetic field. The parameters $R_s$ and $Z_a$ are varied until the desired beam current and beam radius are found. Because of the lack of a confinement magnetic field, the beam is focused to a minimum radius before space charge causes it to diverge. The beam minimum is taken to be the final beam radius.

For guns with little or no cathode flux, the location of the minimum radius defines the beginning of the confinement magnetic field. However, for confined flow with the perturbing influence of ppm focusing, the beam radius is one of the more uncertain quantities dealt with. This situation follows from the radial dependence of the magnetic field, and it is this dependence that is responsible for the iterative nature of the problem.

The electrostatic problem establishes the geometry and the beam current (hence the perveance). The assumptions are that space-charge-limited conditions exist at the cathode surface and that thermal electrons can be safely ignored. As mentioned in the INTRODUCTION, ignoring thermal electrons is acceptable for small compression ratios.

## Magnetostatic Problem

After the position of the anode and the spherical radius of the cathode are found, a second boundary-value problem is solved to determine the scalar magnetic potential. In this problem, the magnetic boundaries are assumed to be coincident with the electrostatic gun boundaries, and the Neumann boundary conditions remain identical. However, the Dirichlet boundary values are changed appropriately as required by a magnetic stack having alternating polarities (see fig. 1). Ideal magnetic conductors are assumed, and saturation effects are, therefore, not included in the computation.

The magnetic field is computed by differentiating a five-point Lagrange extrapolation formula. The Z-component of the field at the $i^{th}$ mesh point (for uniform mesh spacing h) is given by

$$\left(\frac{dU}{dz}\right)_{z=z_i} = \frac{1}{12h} \left(U_{i-2} - 8U_{i-1} + 8U_{i+1} - U_{i+2}\right) \tag{1}$$

where the $U_j$'s are the numerical values of the magnetic potentials at the stations $Z = Z_j$, which are found from the solution of the magnetic-boundary-value problem. Equation (1) is valid at a constant radius. Figure 2 shows the variation with radius of the Z-component of the magnetic field in one ppm gap as computed by using equation (1).

The Stanford Linear Accelerator Center (SLAC) electron optics program used in this study takes as input the magnetic field on the axis. The off-axis fields are computed by means of the axial expansion formula (see Gewartowski, ref. 8, p. 616, or Smythe, ref. 9, p. 60). Because this approach entails numerical differentiation (a procedure which can yield highly erratic results), care must be taken to smooth the axial field data properly. Even then the results are worthy only of second-order expansion (two differentiations). Figure 3 shows the ppm gap field as computed by the SLAC program. The axial field has been fitted to a parabola and agrees well with the on-axis field in figure 2. The off-axis components, however, are seriously underestimated.

Figure 4 is a plot of the peak field (center of the gap) as a function of radial mesh number. This plot gives a radial scale factor of approximately 1.23; that is, the radial dimensions of a real magnetic circuit must be larger by this factor in order to produce the peak fields shown in figure 3. The axial positions of the pole pieces remain fixed, however, so as to maintain the same periodicity.

The degree of confinement is the ratio of the cathode flux to the beam flux at the equilibrium radius. In ppm focusing, there is, of course, no constant field with which to compute the beam flux; hence some sort of average field must be used. In this study, a root-mean-square field $\langle B \rangle$ is used:

$$\langle B \rangle = \left\{ \frac{\int_0^b \int_0^L \left[ B_z(r_1 z) \right]^2 r \, dr \, dz}{\int_0^b \int_0^L r \, dr \, dz} \right\}^{1/2} \tag{2}$$

where $b$ is the equilibrium radius, and $L$ is the magnetic period.

These integrations are performed numerically. Let

$$I(r) = B_0^2 \int_0^L \left[ \frac{B_z(r_1 z)}{B_0} \right]^2 dz$$

$$I(r) = 2B_0^2 \int_0^{L/2} \left[ \frac{B_z(r_1 z)}{B_0} \right]^2 dz \tag{3}$$

The integrand is plotted in figure 3 for $r = 0$, 8, and 12. The numerical integration was carried out for these three values of $r$, and a Lagrange interpolation formula was used to fit these data points:

$$I(r) = B_0^2 \left[ I(r_0) \frac{(r - r_1)(r - r_2)}{(r_0 - r_1)(r_0 - r_2)} + I(r_1) \frac{(r - r_0)(r - r_2)}{(r_1 - r_0)(r_1 - r_2)} + I(r_2) \frac{(r - r_0)(r - r_1)}{(r_2 - r_0)(r_2 - r_1)} \right] \tag{4}$$

where $r_0$, $r_1$, and $r_2$ are replaced with 0, 8, and 12. The remaining integration over $r$ in equation (2) can now be carried out. The result is plotted in figure 5.

As mentioned previously, the cathode is assumed to be located at a magnetic equipotential surface. It is, of course, not possible to construct a cathode of ferromagnetic material to force it to be such a surface. Nor is it desirable to have a solid concentric pole piece immediately behind the cathode to produce the desired spherical shape for the potentials. However, a ring pole piece such as shown in figure 6 gives rise to a set of equipotential surfaces that could be used, with proper placement, to shape the cathode magnetic field.

Figure 7 shows the calculated radius of curvature (spherical radius) of the potentials over a range of radial distances (cylindrical radius). When these spherical surfaces are fitted, it is necessary to interpolate the potential between adjacent mesh points, and some uncertainty results. The shaded portions of the curves give the error spread. Nevertheless this sort of arrangement could probably provide the required equipotential surface which would be coincident with the spherical cathode surface.

Because of the way in which the SLAC program handles magnetic fields, it is difficult to assess the sensitivity of electron trajectories to variations in magnetic field across the cathode. In the present study, magnetic-field variations as high as 8.7 percent were present between the center and the edges of the cathode. Presumably this magnitude of error could be tolerated in an actual design.

## RESULTS AND DISCUSSION

The graphics output of the computer program for two cases, referred to as cases 1 and 2, is presented in figures 8 and 9. In both cases, the anode voltage is 7.25 kilo-

volts, the spherical radius of the cathode is 45 mesh units (0.393 m per mesh; 0.01 in. per mesh), the cathode diameter is 36 mesh units, and the magnetic periodicity is 24 mesh units.

In this report, the end of the gun section and the beginning of the tube are taken to be the second magnetic period of the stack. This starts at the middle of the third pole piece. In figure 8, this point is the station $Z = 53$ mesh units; in figure 9, it is the station $Z = 65$ mesh units. The reason for this distinction is the distortion in the magnetic field due to the finite length of the ppm structure. The interruption of the magnetic periodicity with the cathode structure results in an asymmetry of the magnetic field in the first two gaps of the stack. It has a pronounced effect in figure 8, and in figure 9 it has been compensated for by changing the magnetic-boundary value (the magnetic-field strength) in the first gap. The Laplace solution for the magnetic potentials shows that the field distortion is negligible in the second magnetic period.

In figure 8, case 1, the cathode-anode spacing is set at 29 mesh units. The cathode is immersed in a magnetic field of 0.018 tesla, and the peak confinement field (on-axis) is 0.028 tesla. The beam current is 1.343 amperes, and the perveance is $2.18 \times 10^{-6}$ A/V$^{3/2}$ (2.18 micropervs). The beam radius at the input to the tube section is 12 mesh units, and the flow is laminar. From figure 5, the root-mean-square field is $1.88 \times 0.028$ or 0.0526 tesla. The degree of confinement $\mathfrak{R}_c$ is, therefore, about 77 percent.

In figure 9, case 2, the cathode-anode spacing is 41 mesh units. The cathode field is 0.003 tesla, and the peak confinement field (on-axis) is 0.016 tesla. This increase in cathode-anode distance resulted in a beam current of 0.519 ampere with a perveance of $0.841 \times 10^{-6}$ A/V$^{3/2}$ (0.841 microperv). The beam radius at the tube entrance ($Z = 65$ mesh units) is 0.213 centimeter. The root-mean-square field from figure 5 is $1.28 \times 0.016$ or 0.0205 tesla. The degree of confinement $\mathfrak{R}_c$ for this case is 68 percent.

It is interesting to compare the root-mean-square fields used in these two cases with the theoretical constant fields required for the same confinement ratio $\mathfrak{R}_c$ and beam radius. The governing equation can be written

$$B_b = \frac{0.08303}{b} \left[ \frac{I_0}{V_0^{1/2} \left(1 - \mathfrak{R}_c^2\right)} \right]^{1/2} \tag{5}$$

where b is in centimeters, $I_0$ is in amperes, and $V_0$ is in volts.

For case 1, with $\mathfrak{R}_c = 0.77$ and $b = 0.3048$ centimeter, $B_b$ is 0.536 tesla. The root-mean-square field is 0.05264 tesla, and this agrees very well. However,

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for case 2, with $\mathfrak{R}_c = 0.68$ and $b = 0.2126$ centimeter, $B_b$ is 0.0415 tesla, which is twice as high as the root-mean-square field of 0.0205 used in figure 9.

It would be useful to extend this work by finding a general relation among confinement ratio, focusing field strength, perveance, and beam radius. The problem could be simplified by separating the geometry into two distinct regions, namely, the gun section and the ppm focusing stack. The study would then proceed by examining a complete set of assumed entrance conditions to the focusing system. Gun design would entail finding the compression ratio, cathode flux, and perveance to match these entrance conditions.

## CONCLUDING REMARKS

A computer study was conducted by using the SLAC program of reference 6 to establish a regimen for designing confined-flow, high-perveance guns for use with ppm-focused tubes.

In the point of view of this study, the variable parameters are the magnetic-field values at the cathode and in the periodic confinement structure. It is assumed that variables such as beam size, current, voltage, and perveance are fixed by other considerations.

The magnetic-field distributions are computed from a boundary-value problem in which the cathode is an equipotential surface. The magnitude of the field is varied until a stable laminar beam results.

The primary problem in the design is the magnetic circuit which produces a magnetic equipotential surface coincident with the cathode. There are, no doubt, numerous solutions to this problem. However, computer calculations show that the equipotential surfaces produced by a ring pole piece behind the cathode have an approximately spherical shape. The radii of curvature of these surfaces vary from very small values to infinity as a function of distance from the ring center. Hence the ring can be positioned so that coincidence with the cathode surface is found.

Lewis Research Center,
    National Aeronautics and Space Administration,
        Cleveland, Ohio, March 23, 1979,
            506-20.

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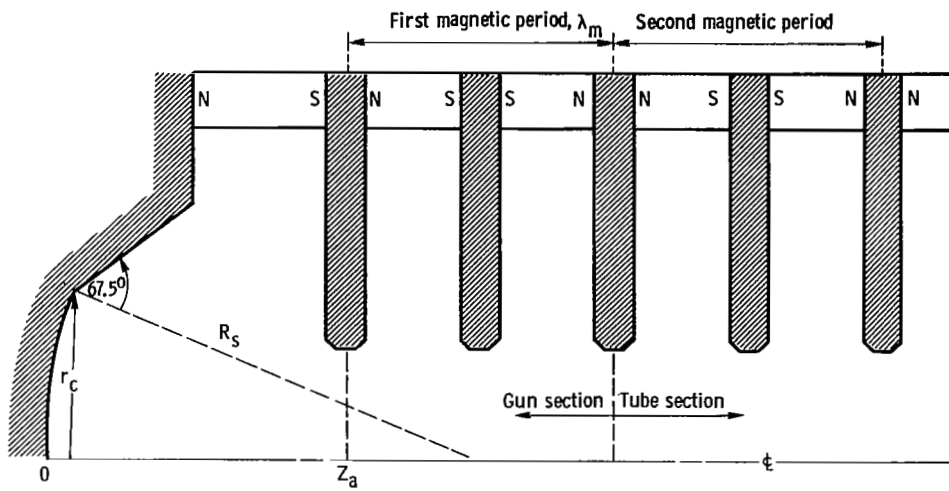
Figure 1. – Assumed geometry of confined-flow gun and ppm stack. (Shaded areas are magnetic equipotential surfaces.)
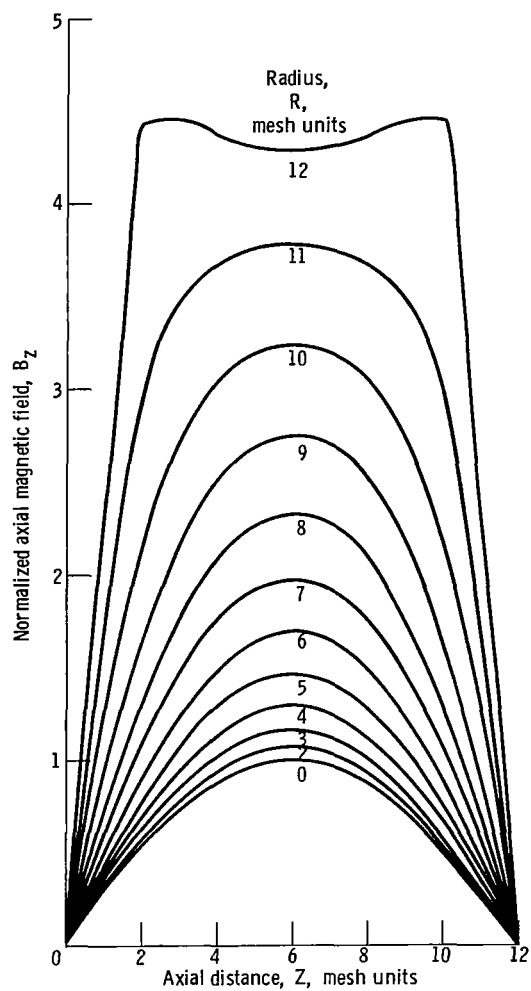
Figure 2. - Normalized axial magnetic field in gap region of assumed geometry as computed from Laplace solution.
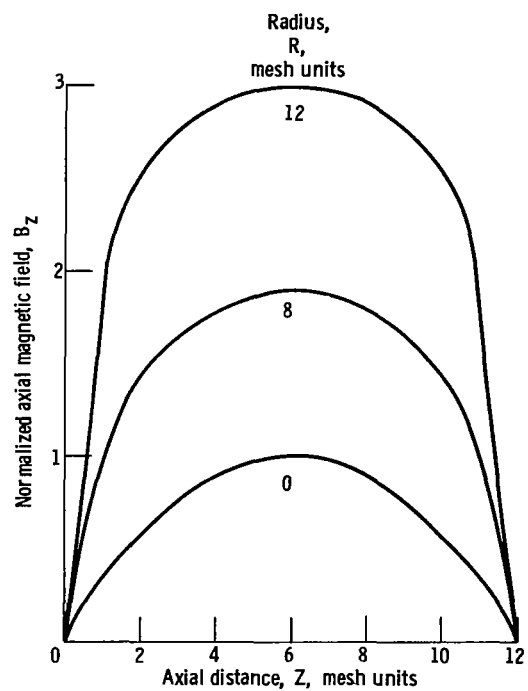


Figure 3. - Normalized axial magnetic field in gap region of assumed geometry as computed from axial expansion formula. (Internal to SLAC program.)
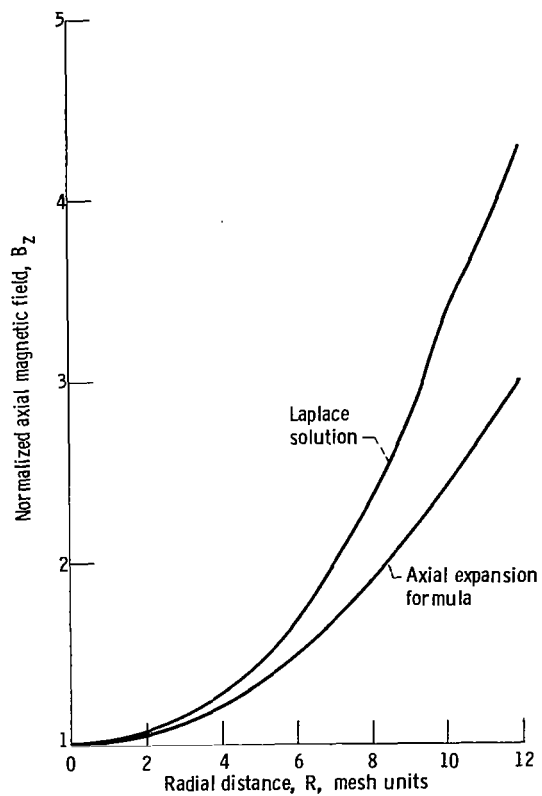
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Figure 4. - Comparison of peak axial magnetic fields from Laplace solution and axial expansion approximation for assumed gap geometry.
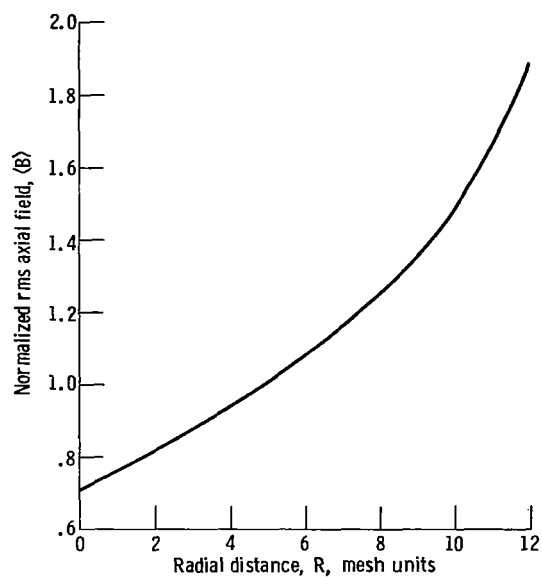


Figure 5. - Normalized root-mean-square axial magnetic field over one period of ppm structure (see eq. (2)).
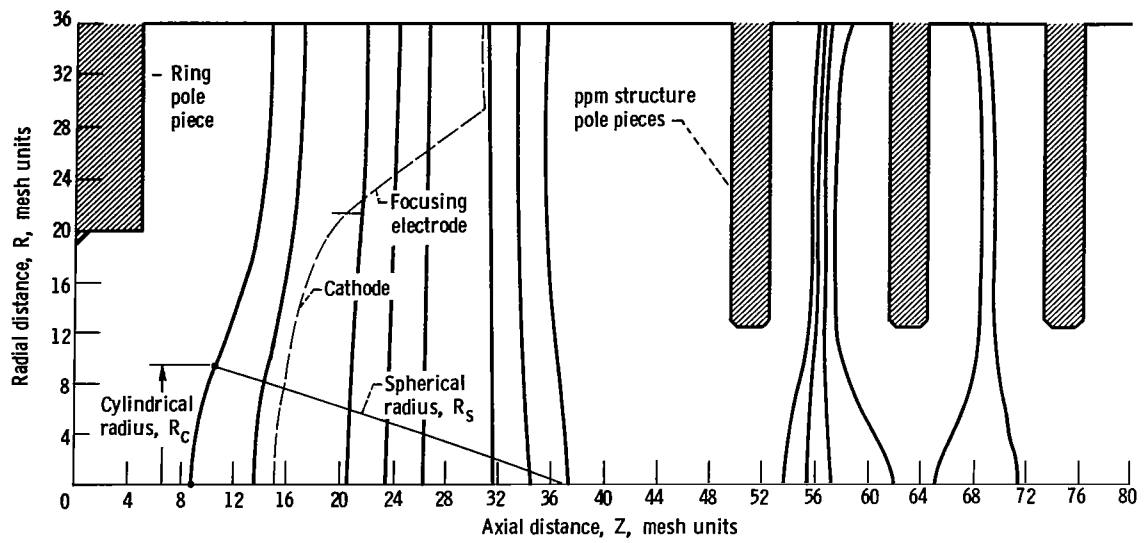
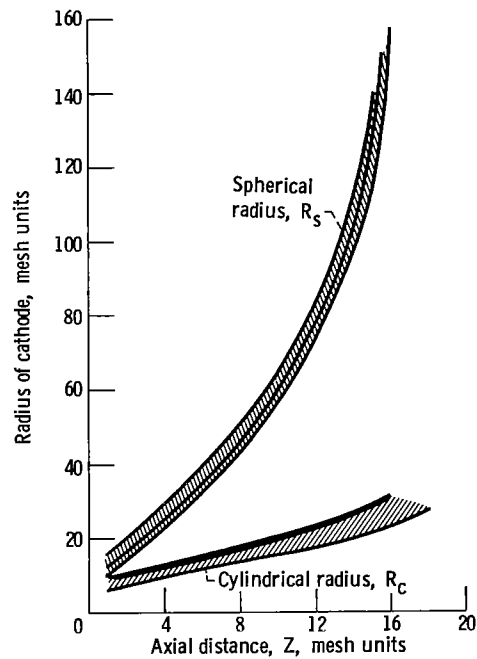Figure 6. – Laplace solution for magnetic scalar potential of ring pole piece and ppm structure.



Figure 7. – Spherical radius of curvature computed from curve-fitted scalar magnetic potentials of ring pole piece.
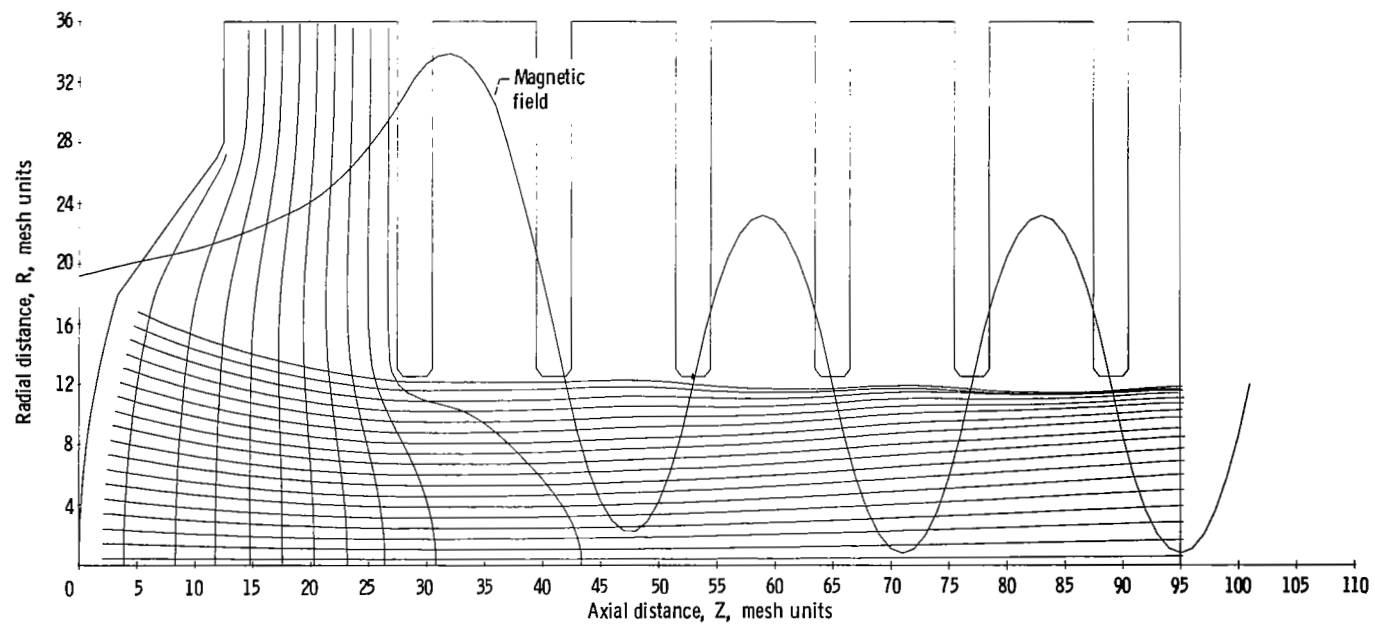
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Figure 8. - Electron trajectories from confined-flow gun with perveance of 2. 18x10$^{-6}$ A/V$^{2/3}$ (2. 18 micropervs) (case 1).
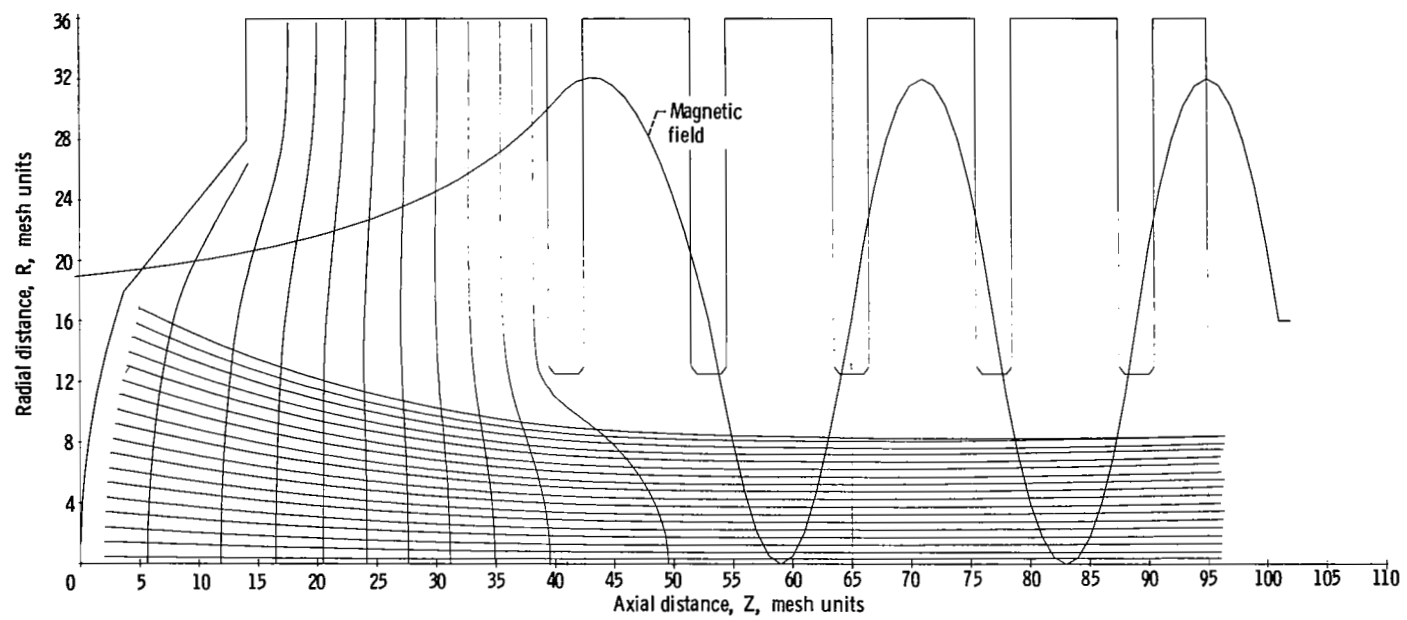
Figure 9. – Electron trajectories from confined-flow gun with perveance of $0.841\times10^{-6}$ A/V$^{3/2}$ (0.841 microperv) (case 2).

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15. Supplementary Notes

16. Abstract

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