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COMPUTATION OF MINIMUM NOISE AIRCRAFT LANDING TRAJECTORIES

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Summary

This report is on our fourth year of research on minimization of aircraft noise to residents during landing. Results of our first three years of work are reviewed briefly. These consist of aircraft aerodynamic model, noise model, population model, performance index, and optimization procedure. Then the results of this past year's effort, the optimal trajectories from the three main near-terminal entry points, are presented via tables and graphs. The recommendation is that these minimal noise trajectories should now be tested as reference trajectories for the terminally configured aircraft to fly along.

I. Introduction

In 1973 a research project, under the sponsorship of NASA, Langley Research Center, on the subject of minimum noise aircraft landing trajectories was begun at the University of Virginia.

There were two main reasons for initiating such a project. First was the anticipated development of the microwave landing system (MLS). The MLS permits more accurate measurement of the aircraft's position than does the standard radar system. Coupled with this was the development of improved autopilots and navigational equipment as exemplified by the terminally configured vehicle (TCV). Had it not been for these developments, complex curved trajectories probably would not have been considered, as they would have greatly increased the pilot work load; however, with the new developments, it seemed quite natural to seek ways to take advantage of them. One way chosen was to precompute trajectories which yielded minimum noise to the population residing in the near-terminal area, that region within 20 miles of the terminal. Once obtained, these trajectories could be stored in the memory of the autopilot and used as reference trajectories for the plane to follow. Computing these optimal reference trajectories has been the subject of our research.

Section II of this report is a brief review of our previous work. It includes the noise model, population model, aircraft simulation model, and the optimization procedure. Section III describes how the entry points into the near-terminal area were calculated. These serve as the beginning points for the trajectories. Section IV presents and discusses the results. Tables and plots are utilized. Section V makes some recommendations for future work.

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

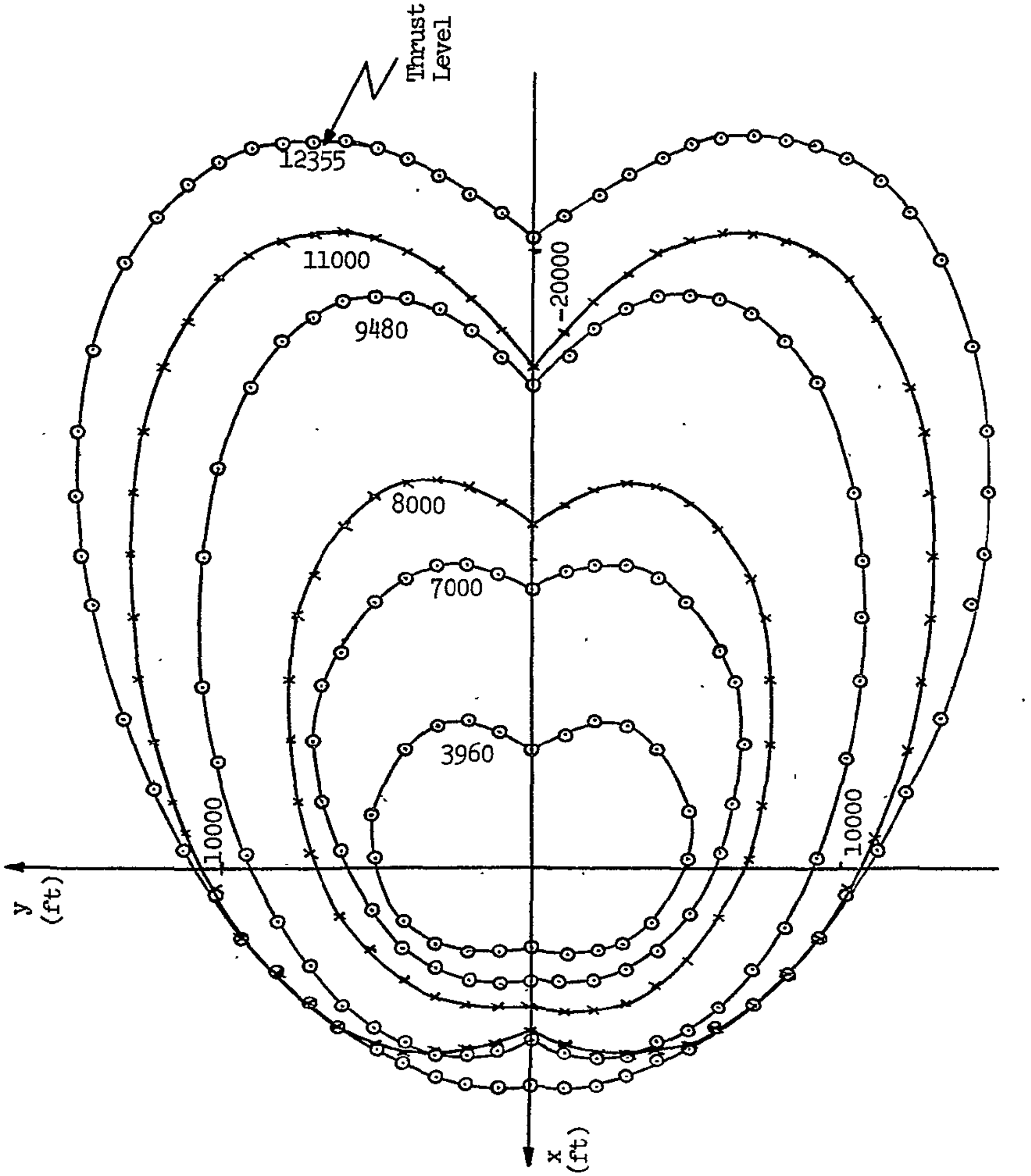

Figure 1

Intersection of the 70 db Surface with the Ground for Various Thrust Levels.
Aircraft is in Level Flight at 6000 ft. Altitude

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

II. Review of Previous Work

A. Noise Model

The model used to evaluate the noise effects of the aircraft under consideration, the Boeing 737, has been explained in considerable detail in our reports [1, 2]. We will present only a very brief summary here.

Earlier studies have indicated that almost no people object if the noise level is kept below 70 pn db. We thus decided to use, in our performance index, the number of persons receiving noise above 70 pn db and to integrate that number over the duration for which the objectionable noise exists.

Having made that decision, we turned to the problem of determining the noise footprint, that region on the ground receiving noise at, or above, 70 db pn. It was decided to model, with ellipsoids, the surface about the aircraft inside of which the noise is at, or above, 70 db. The coefficients of the ellipsoids depend on thrust, since the ellipsoids grow with thrust. Once this surface was modeled, it was fairly straightforward to calculate its intersection with the ground. Figure 1 shows some typical noise footprints. One can also determine the total area covered by the footprint as the aircraft flies along. This region is called the ground track.

B. Population Model

Reference [1] gives a detailed explanation of our population model. Briefly, our approach was to superimpose a uniform grid (one square mile per block was chosen for convenience) over a population map. For our study, we chose the Patrick Henry Airport at Newport News, Virginia (Figures 2 and 3). Within each square, the city blocks and other types of divisions were identified and their population determined. These numbers were then added together to yield the total number of persons residing in each square mile. The method

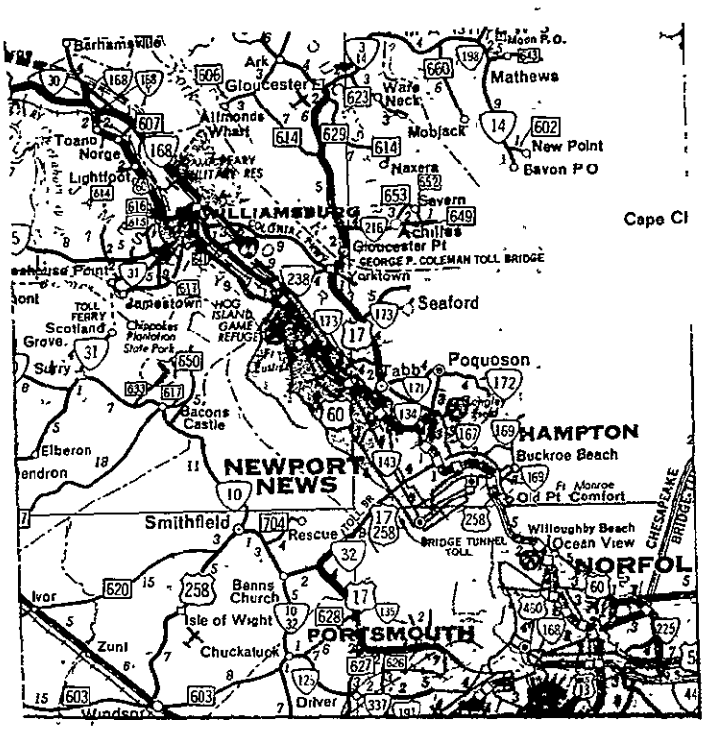

Figure 2.

Road Map of Newport News Area

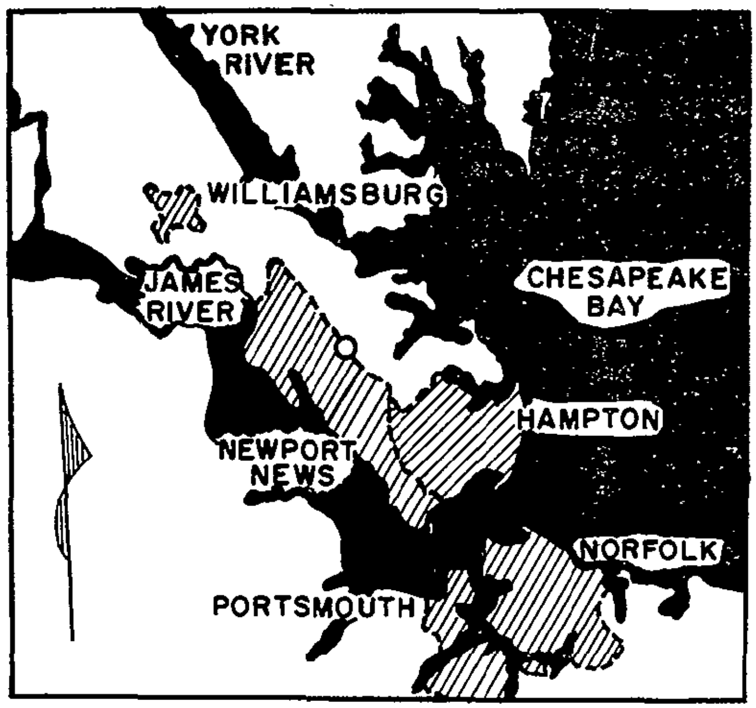

Figure 3

Land/Water Map of Newport News Area

is illustrated in Figure 4.

This uniformization of the data was necessary, since not all city blocks and other types of divisions used for census-taking were of the same size. Converting to land divisions, which were uniform, greatly simplified the problem of computer storage of the population data. Figure 5 shows a population contour map based on our population model.

To utilize the noise model in conjunction with the population model the footprint is first determined. Next, the area inside the footprint is calculated with a weighting equal to the population density. Since the footprint may cover several blocks of the population model, different portions of the footprint may have a different weighting. The result of this calculation is the instantaneous number of people receiving objectionable noise. This quantity is then integrated with respect to time to give a measure of duration of the objectionable noise. This procedure is repeated the full length of the ground track. The dimensions of this final measure of noise are people-seconds.

C. Aircraft Model

Our goal in modeling the aircraft for this study was to make the model as accurate as possible. References [3, 4] include most of the details. The model is 12th-order and contains six degrees of freedom (three translational and three rotational), is nonlinear, and uses wind-tunnel data in calculating the aerodynamic forces and moments. Wind has not been included, but provision has been made for that. The Milne-Reynolds method is used for numerical integration of the equations of motion, with fourth-order Runge-Kutta used for start-up. An integration step size of .1 seconds is used. This seemed to be the largest step size possible which would retain the proper behavior and the

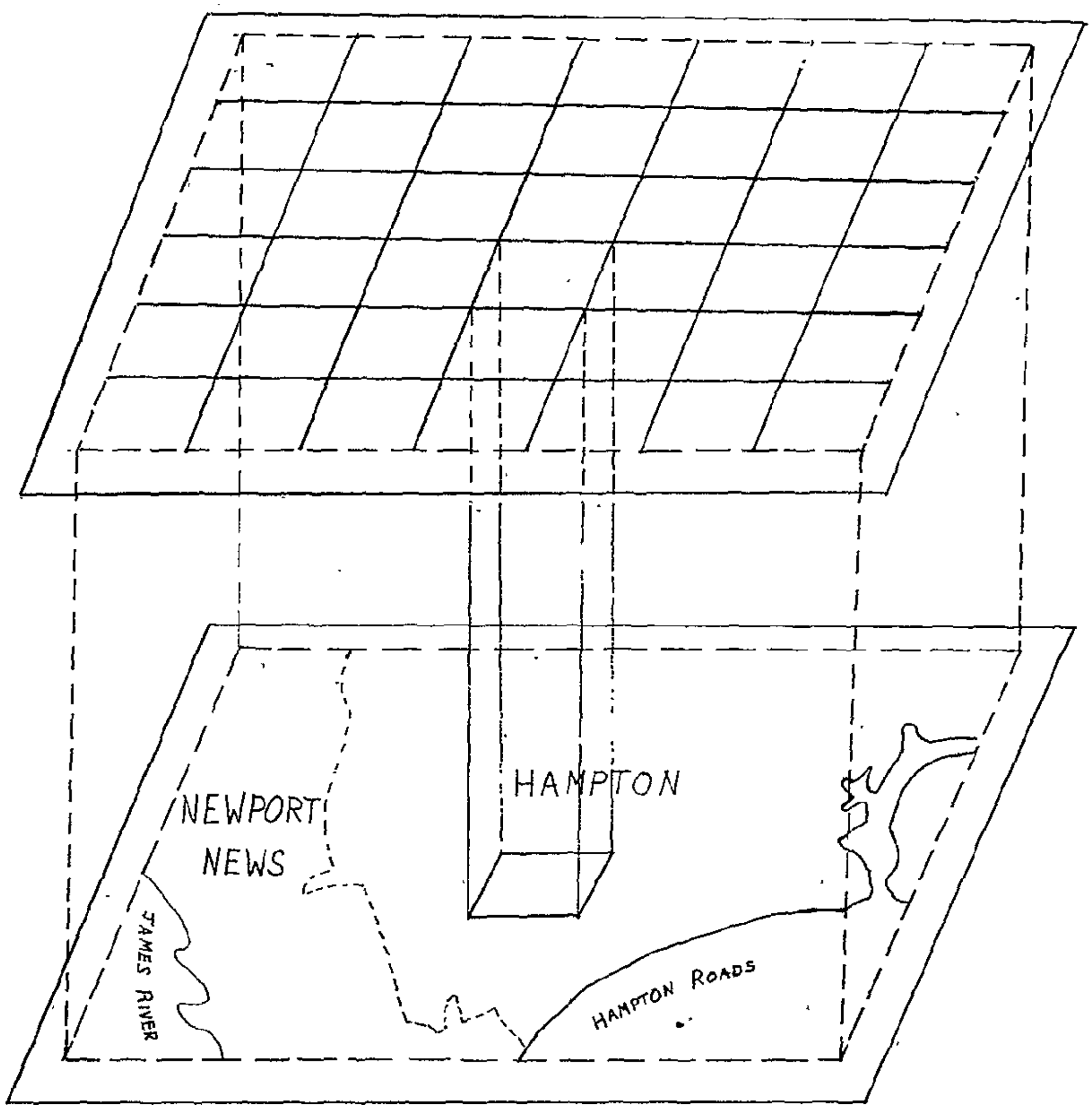

Figure 4a

Illustration of the Overlay Technique
for Determining Population within One Square Mile

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

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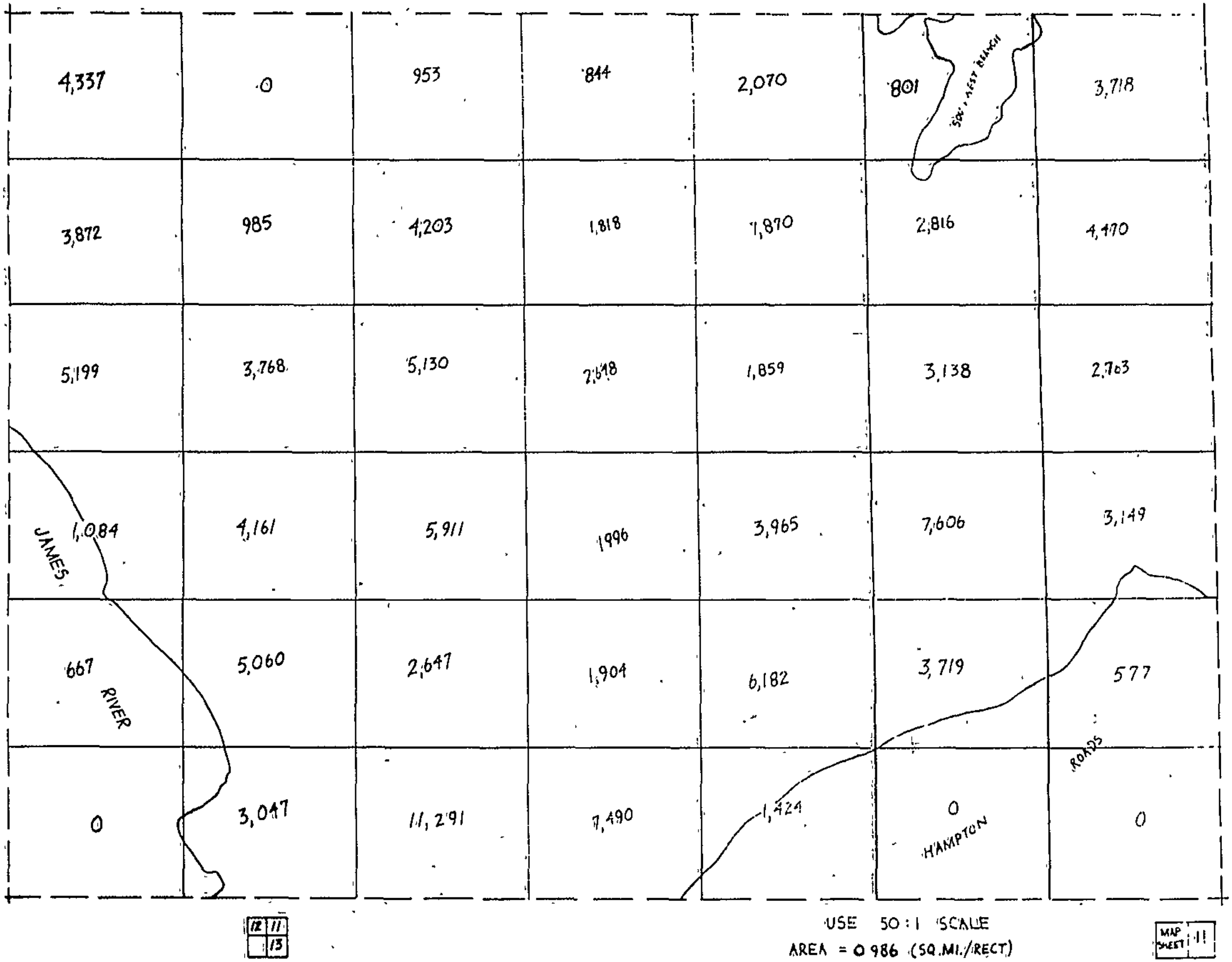

Figure 4b Compiled Population Data for Region II of the Newport News-Hampton Area

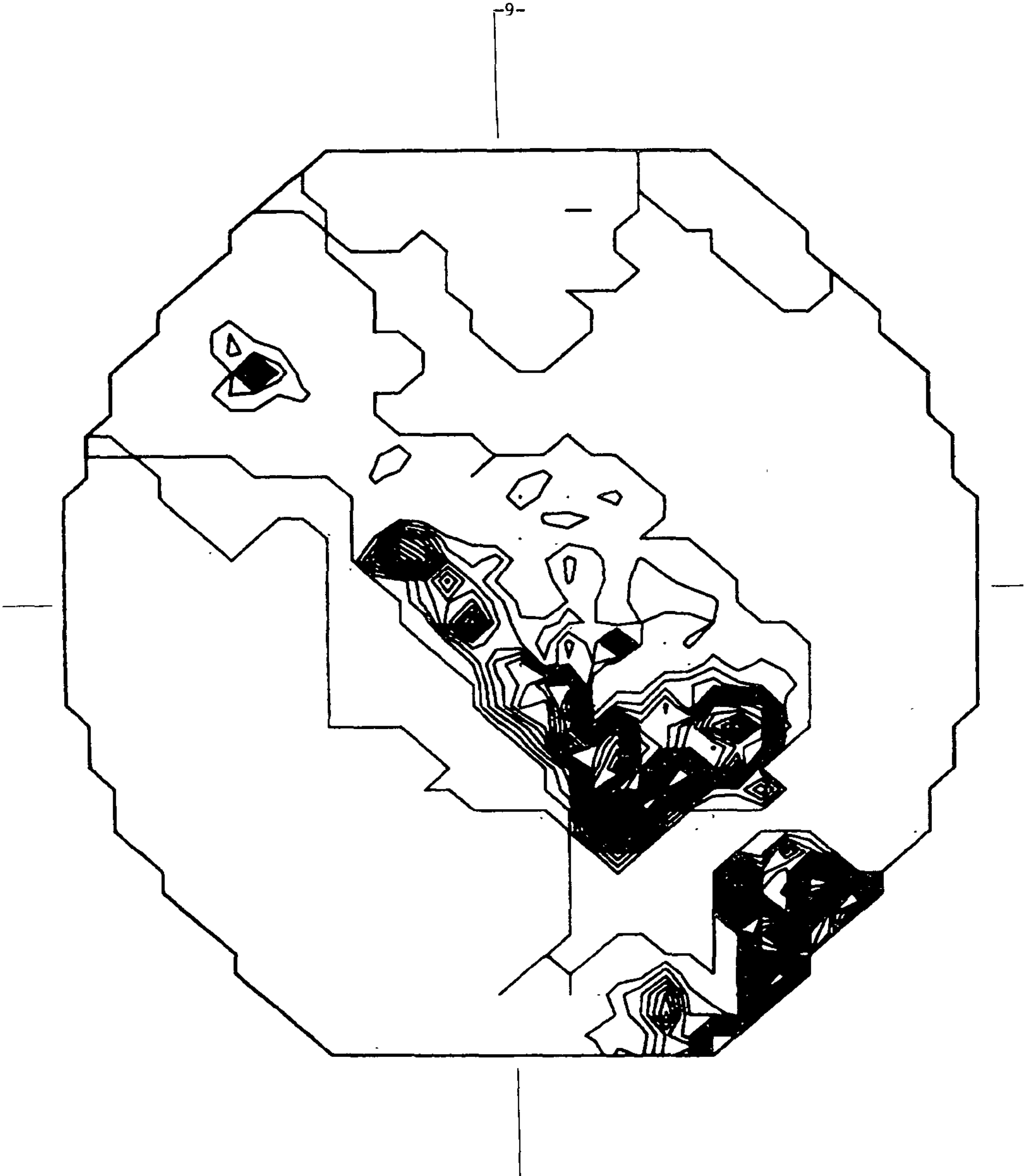

Figure 5a

Population Contour Map

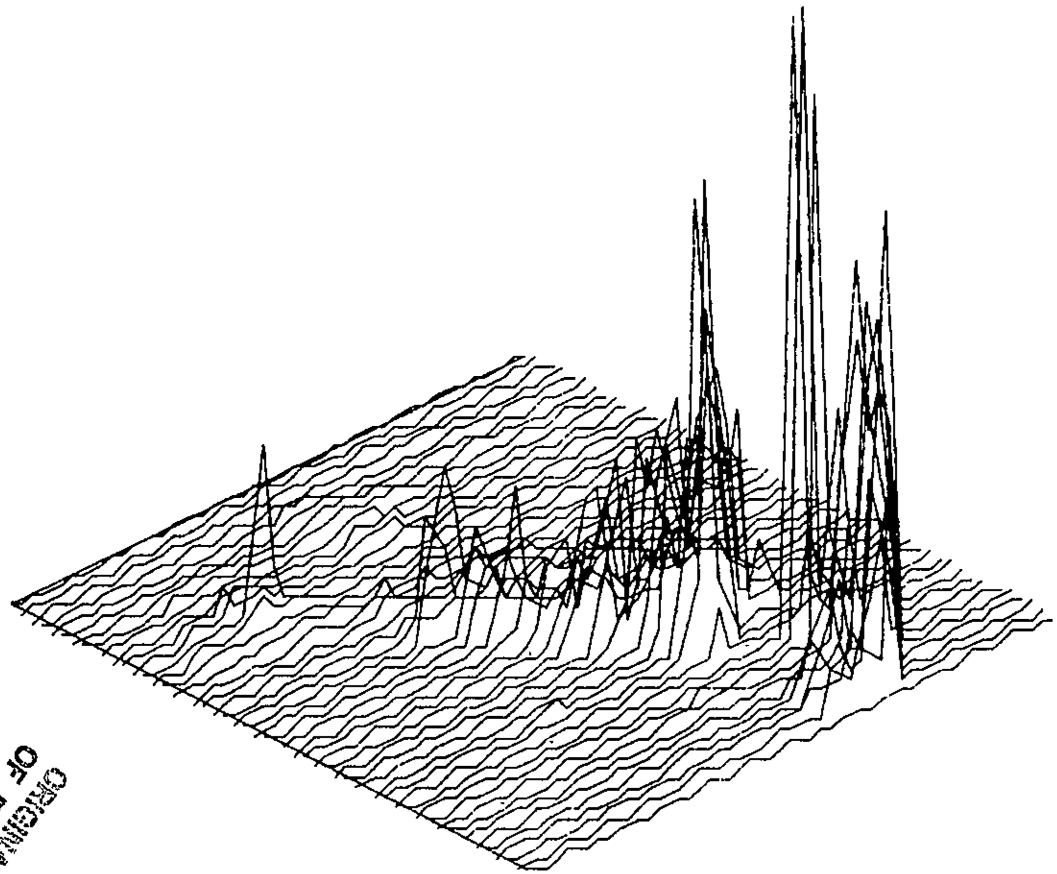

Figure 5b

Three-Dimensional Population Map

required accuracy. Our simulation was compared with one at NASA, Langley, and has very similar behavior in a three-degree glide slope, a six-degree glide slope, and a banked turn. We have no reason to think that the simulation is not accurate under all other conditions, also.

Perhaps the strongest criticism which could be made about our aircraft model is that it is too realistic. A simpler model could probably be almost as useful for our optimization studies and yet require somewhat less computation; however, we would rather err in this direction than to have a model so simple that it was markedly inaccurate.

D. Optimization Procedure

Having the aircraft simulation model and the means of determining the noise effects, we then addressed the optimization problem. The procedure selected was the method of steepest descent [5]. The reasons for using this particular method were its prior success when applied to aerospace problems and our previous experience with the method.

The user specifies an initial control history as the starting point. Next, the resulting trajectory and performance measured are computed, utilizing the aircraft simulation along with the noise and population models. A perturbation is then made from this trajectory, creating a neighboring trajectory which is nearer to the optimal. This procedure is repeated until very little improvement is achieved on successive iterations at which point the process is terminated. The information required in computing each perturbation is quite complex to obtain and requires considerable storage space; however, this is not uncommon for optimization procedures.

III. Starting Points for Near-Terminal Maneuver

As the aircraft which is about to land approaches the terminal, it follows a set pattern, depending on its origin and which runway it will utilize. It was decided to use these same constraints in determining the initial conditions for the optimal landing trajectory. It was assumed that the aircraft will proceed along the standard flight path until it enters the near-terminal area, the 20-mile radius. This entry point becomes the beginning point for the optimal trajectory. From this point on, of course, the trajectory may be quite different from what is presently being flown.

There are three VOR stations in the vicinity of the Patrick Henry Airport. These are Cape Charles to the northeast, Harcom to the northwest, and Cofield to the south. Incoming aircraft utilize a pattern based on these three stations and the four runways of Patrick Henry. The net result is that there are six entry points into the near-terminal area, and each one serves as the initial condition for one or more runways. For example, if the aircraft is arriving from south of Newport News, it comes via Cofield and then begins one pattern if it is to land on Runway 2 or a different pattern if it is to land on Runway 6, 20, or 24. Figure 6 illustrates the six entry points and their relationship to the runways.

The initial conditions were moved into a 17-mile radius. The noise footprint extended back about three miles, and we did not have population data to 23 miles. It was felt that using a 17-mile radius had little effect on the optimization, because the aircraft is still high and not yet creating a serious noise problem. Also, the shorter trajectory saved on the expensive computer costs. In some cases a hand-drawn curved aircraft trajectory was used between the 20- and 17-mile radius. This was done to help the convergence of the optimization procedure. These manual adjustments will be apparent in the Results Section that follows.

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

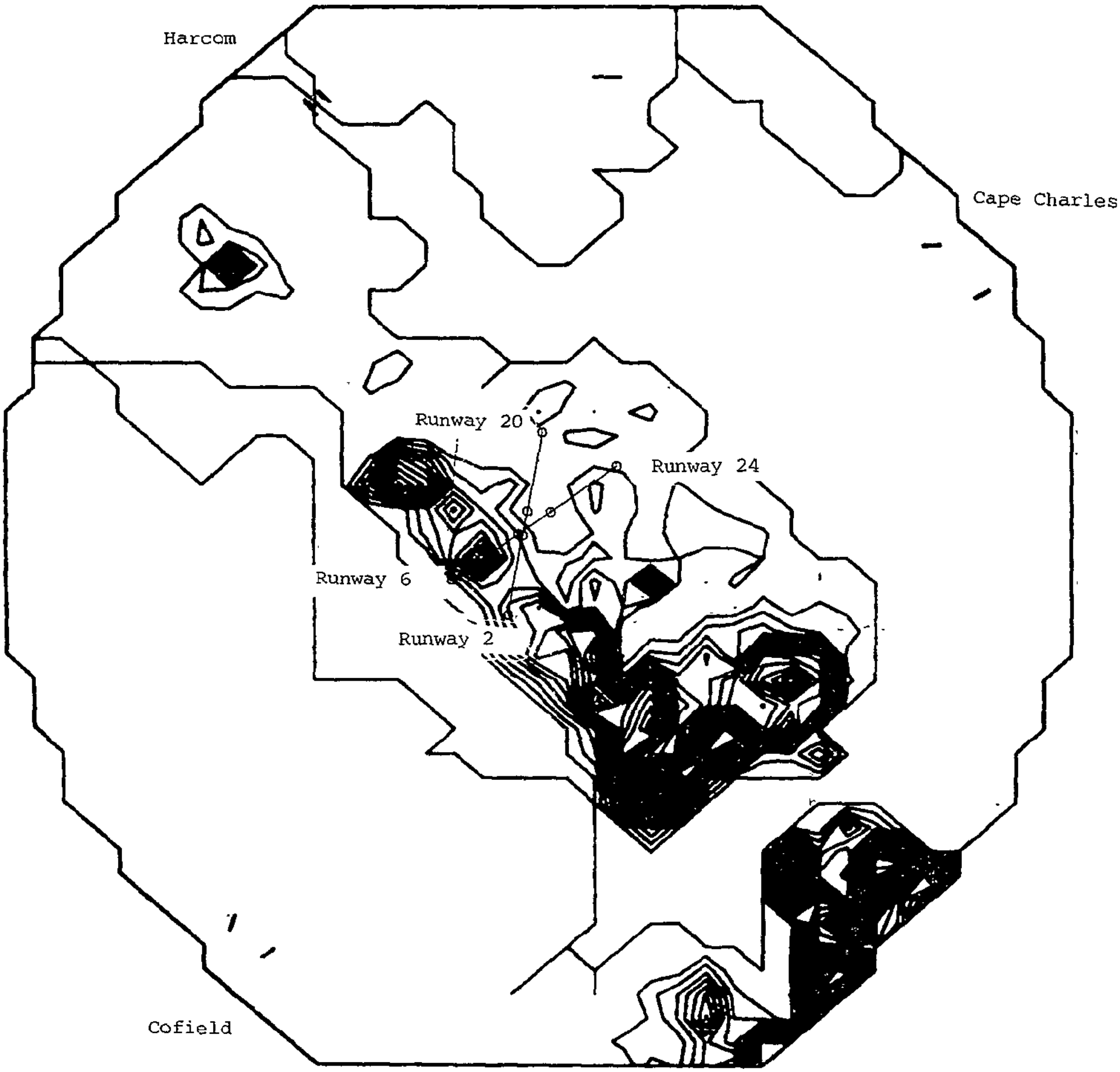

Figure 6

Entry Points for Near-Terminal Area

IV. Results

This section presents the results of optimizing the future flight trajectories into Patrick Henry Airport. In some cases the results could have been predicted by looking at maps. National Geological Survey Topographical maps locate water, marshes, and other areas of low population. Population maps generated from census data more precisely locate areas of low population. The population maps were projected in 3-D by computer graphics to dramatically show "valleys" of low population (Figure 5b).

It is not surprising that some results could be predicted. It is often the case in system studies that, once the problem is formulated and the required data obtained, the solution is fairly straightforward. For this reason, we feel the population model, in itself, is important and can be a useful tool in manual flight trajectory planning. On the other hand, there are other cases where the results would not have been as easy to predict, although, once obtained, do seem quite reasonable. Thus, something is gained in formulating the problem and using the optimizing procedure.

As the optimal trajectories are examined, it is important to keep in mind that they should serve as reference trajectories, but they do not have to be followed exactly. Our nonlinear aircraft simulation model prevents any of the trajectories from being anything the real airplane could not fly. Generally speaking, the trajectories are quite smooth. There are no sharp turns, no sharp bank angles, or other types of violent maneuvers. Nevertheless, for one reason or another, it may be desirable to make slight deviations from these trajectories during actual flights. This should not cause a great deal of sub-optimality. The optimization procedure had trouble meeting the final boundary conditions, so certainly some deviations will be required.

The computer program was very large and required more than 140 K (Base 8)

60-bit words. The computation time was very long, and the combination of large size and long time made the computer costs very expensive. For a 500-second flight, the forward integration of the nonlinear differential equations took approximately 250 seconds, one iteration (which includes the forward integration of the 12 states and the backward integration of the 24 adjoint variables) took approximately 100 seconds, and one iteration cost approximately $35.

To save money the optimization procedure was hand-helped. After some experience, we changed the initial conditions, the final time, and the shape of the nominal trajectory. Straight-line nominal trajectories worked well when one turn was required, but convergence was very slow when "s" or more complicated trajectories were needed. In those cases a heuristically chosen curved nominal trajectory was used.

Though it is difficult to be mathematically precise about this, it was felt that our choice of weights in the penalty function slowed down convergence. For instance, there is a tradeoff between minimizing noise and meeting the final boundary conditions. For various social, economic, and political reasons, the population close to an airport increases, as has occurred around Patrick Henry Airport. In order to minimize noise the optimization procedure "pushed" the aircraft trajectory away from the airport, and to meet the final boundary conditions the trajectory was "pushed" closer to the airport.

Three-degree glide slopes were used for all initial trajectories. Six-degree glide slopes cause less noise, because the 70 db "bubble" stays above ground for longer periods of time; however, the optimization procedure seemed to concentrate primarily on the x-y coordinates rather than modify the altitude profile, except for minor perturbations.

A. Flight #1: Cofield to Runway 6

Summary

The aircraft flies over the Cofield beacon and makes a gentle right turn into Runway 6. The flight of minimum noise is very nearly the flight of shortest distance.

The first half of the flight is over population density squares of 24 and has one square of 300 to miss. The middle of the flight is over water, and the very last part is over a high population density area.

Four different sets of iterations were computed. Operator intervention was used to change flight time and the initial yaw. The flight had difficulty converging on the runway. The final iteration from the fourth attempt is plotted in Figure 7, and the results are tabulated. The total number of iterations was 36, although only six were required after suitable initial conditions and flight time were established.

Initial and Final Boundary Conditions

The initial conditions for the fourth attempt (the final choice) were:

$$X_i = \text{RANGE} = -56520.0$$

$$Y_i = \text{SIDE} = 81530.0$$

$$Z_i = \text{ALT} = 6500.0$$

$$\text{Yaw}_i = \text{PSI} = -80.2$$

The final conditions were:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

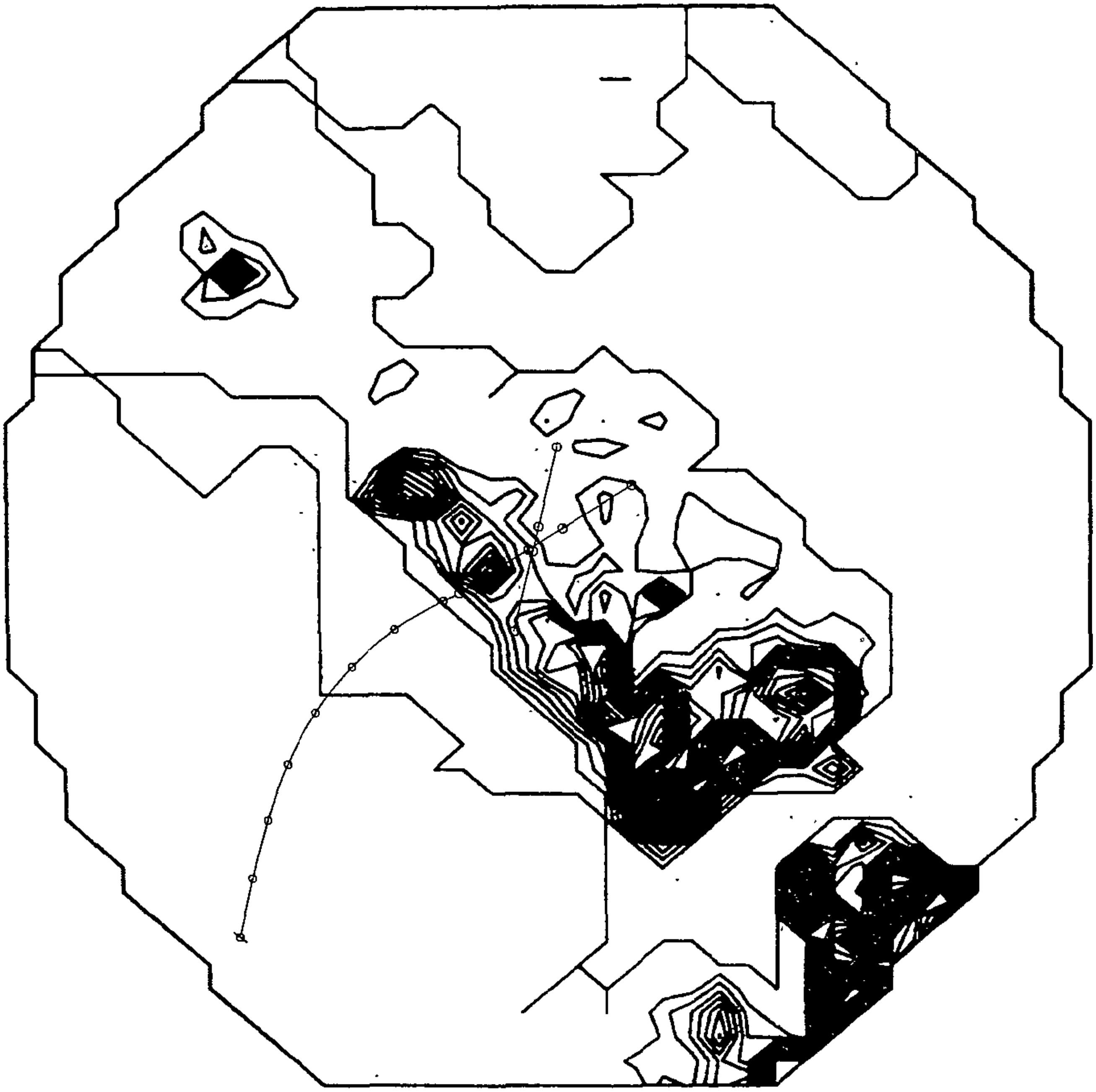

Figure 7

Optimal Trajectory - Flight 1

$$X_f = X_F = -14721.0$$

$$Y_f = Y_F = 10938.0$$

$$Z_f = H_F = 900.0$$

$$Yaw_f = PSI_F = -34.3$$

with runway coordinates

$$X_R = -1169.0$$

$$Y_R = 1693.0$$

$$T_F = 365.0$$

The A/C enters the 20 NM radius at:

$$x = -61630.0$$

$$y = 104730.0$$

$$yaw = -77.57$$

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.1

Summary of Flight #1 (fourth attempt)

Cofield to Runway 6

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	-2.59	2.24	900	-34.3				
#0	-2.59	2.27	1145	-24.8	2.55	25450	-.219E07	365
#1	-3.01	1.83	1125	-29.7	2.62	26139	-.204E07	365
#2	-2.61	3.32	1119	- 4.6	1.61	16091	-.964E07	365
#3	-3.85	2.45	1118	- 8.3	1.61	16098	-.962E07	365
#4	-4.97	1.07	1088	-28.3	1.66	16583	-.146E08	365
#5	-4.34	1.35	1088	-26.9	1.57	15718	-.864E07	365
#6	-3.83	1.57	1088	-26.3	1.63	16320	-.514E07	365

$d\psi$ - boundary condition error

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

B. Flight #2: Cofield to Runway 2

Summary

Starting at the Cofield beacon, the first half of the flight is over squares of population density equal to 24. The middle of the flight is over the James River and then over some high population areas close to the runway. The entire flight is a gentle left turn.

Three attempts were made, and the total number of iterations was 38. The first and second attempts started from the same initial condition as Flight #1. The flight path needed to be in the shape of an "s", but the computer was unable to perturb the flight in this manner. The third attempt used an initial condition that enabled the flight to make a gentle left turn. Convergence was good for the third attempt. The final trajectory is shown in Figure 8.

There were no high population density areas to steer around, and the optimal flight is very nearly the shortest flight. Operator intervention consisted of changing the flight time and the initial conditions.

Initial and Final Boundary Conditions

For the third attempt, final iteration (Iteration #13), the aircraft enters the 20 NM radius at the same point as Flight #1.

$$x = -61630.0 \text{ feet}$$

$$y = 104730.0 \text{ feet}$$

$$\text{yaw} = -77.57 \text{ degrees}$$

A right turn must be made in order to meet the initial conditions or the 16.33 NM radius.

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

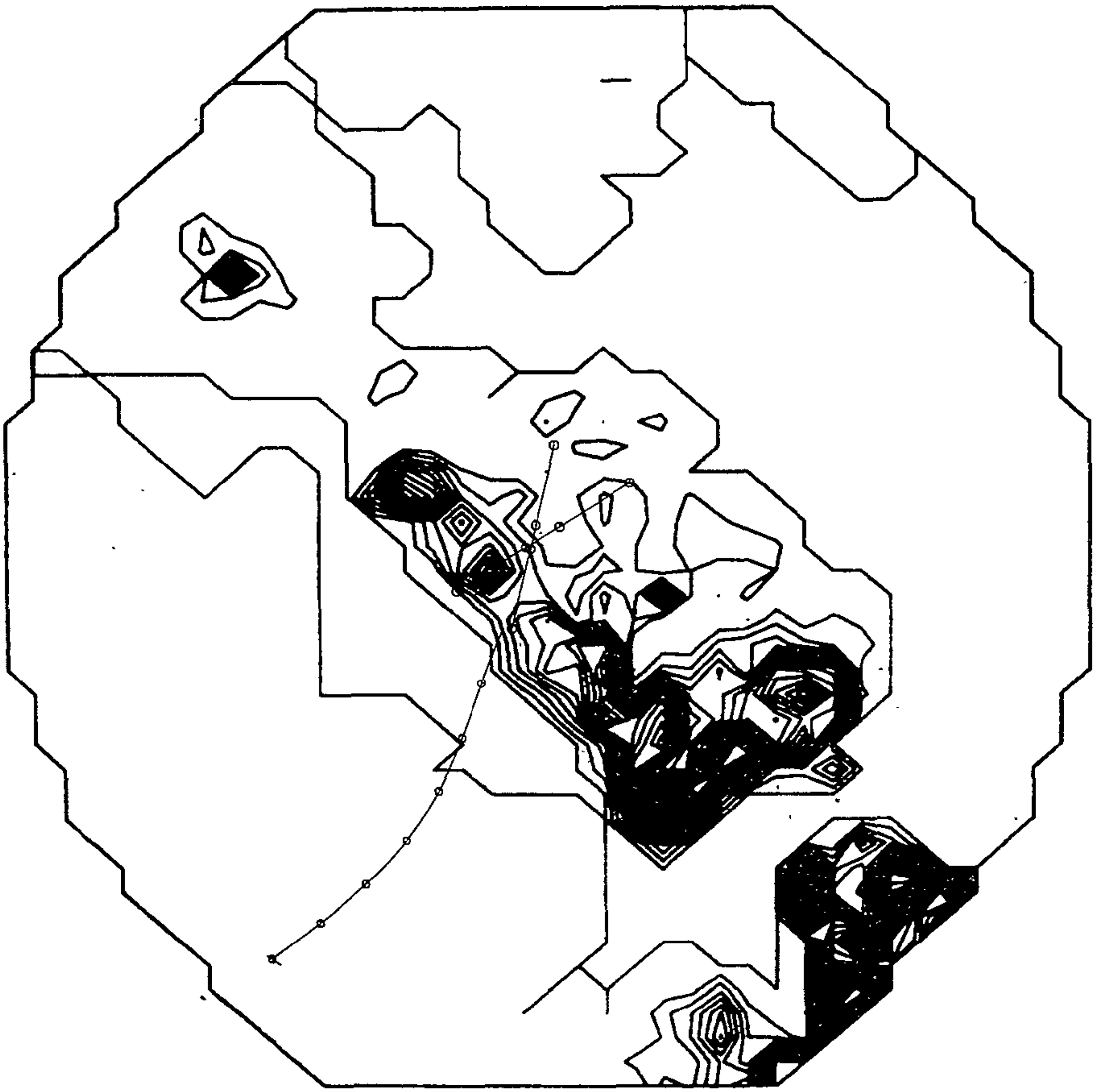

Figure 8

Optimal Trajectory - Flight 2

$$X_i = \text{RANGE} = -49158.0$$

$$Y_i = \text{SIDE} = 86169.0$$

$$Z_i = \text{ALT} = 7000.0$$

$$Yaw_i = \text{PSI} = -39.0$$

The final conditions are:

$$X_f = -3393.0$$

$$Y_f = 18099.0$$

$$Z_f = 900.0$$

$$Yaw_f = -79.2$$

$$X_R = -319.0$$

$$Y_R = 1985.0$$

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.2

Summary of Flight #2 (third attempt)

Cofield to Runway 2

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	-0.60	3.71	900	-79.2				
#0	3.20	3.85	1976	-45.0	59.11	591014	-.670E08	400
#1	2.57	3.31	990	-50.3	51.70	516919	-.270E08	400
#2	1.13	2.03	909	-44.0	31.97	319702	-.196E08	400
#3	-1.73	0.42	877	-45.8	20.32	203057	-.272E08	400
#4	-0.12	1.24	861	-42.3	15.66	156532	-.199E08	400
#5	-2.26	0.08	868	-66.2	18.21	182550	-.265E08	400
#6	-1.00	0.65	862	-58.4	17.76	177559		400
#7	-0.50	3.68	1659	-55.03	14.01	139946	-.189E08	350
#8	-5.12	2.09	1664	-84.1	1.49	14879	-.599E08	350
#9	-3.64	2.39	1676	-79.3	1.47	14721	-.364E08	350
#10	-2.59	2.78	1609	-75.5	1.92	19133	-.220E08	350
#11	-1.82	3.14	1522	-72.0	3.24	32320	-.136E08	350
#12	-1.25	3.41	1429	-68.9	7.72	77155	-.880E07	350
#13	-0.80	3.69	1342	-68.5	11.20	111861	-.583E07	350

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

C. Flight #3: Cofield to Runway 20

Summary

This is a long flight of 765 seconds and is very similar to Flight #4. The aircraft starts at the Cofield beacon and flies over an area of uniform population density of 24. The altitude is high enough so the 70 db noise envelope does not intersect the ground. The plane crosses the James River and takes a long and sharp right turn across to the York River and then down into Runway 20. The flight path is similar to Flight #4, except for the sharp turn into Runway 20.

One attempt with a total of 14 iterations was used to obtain the final result. Two forward integrations using manually adjusted sections were used to obtain the nominal. The final result was Iteration #10.

The final iteration is plotted in Figure 9, and the results are tabulated. The final performance index was 7.30, and most of that was obtained close to the runway.

Initial and Final Boundary Conditions

The aircraft crossed the 20 NM radius at the same x, y, and yaw as Flights #1 and #4.

X = -61630 feet

Y = 104730 feet

Yaw = -77.57 degrees

The 16.33 NM radius is crossed at:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

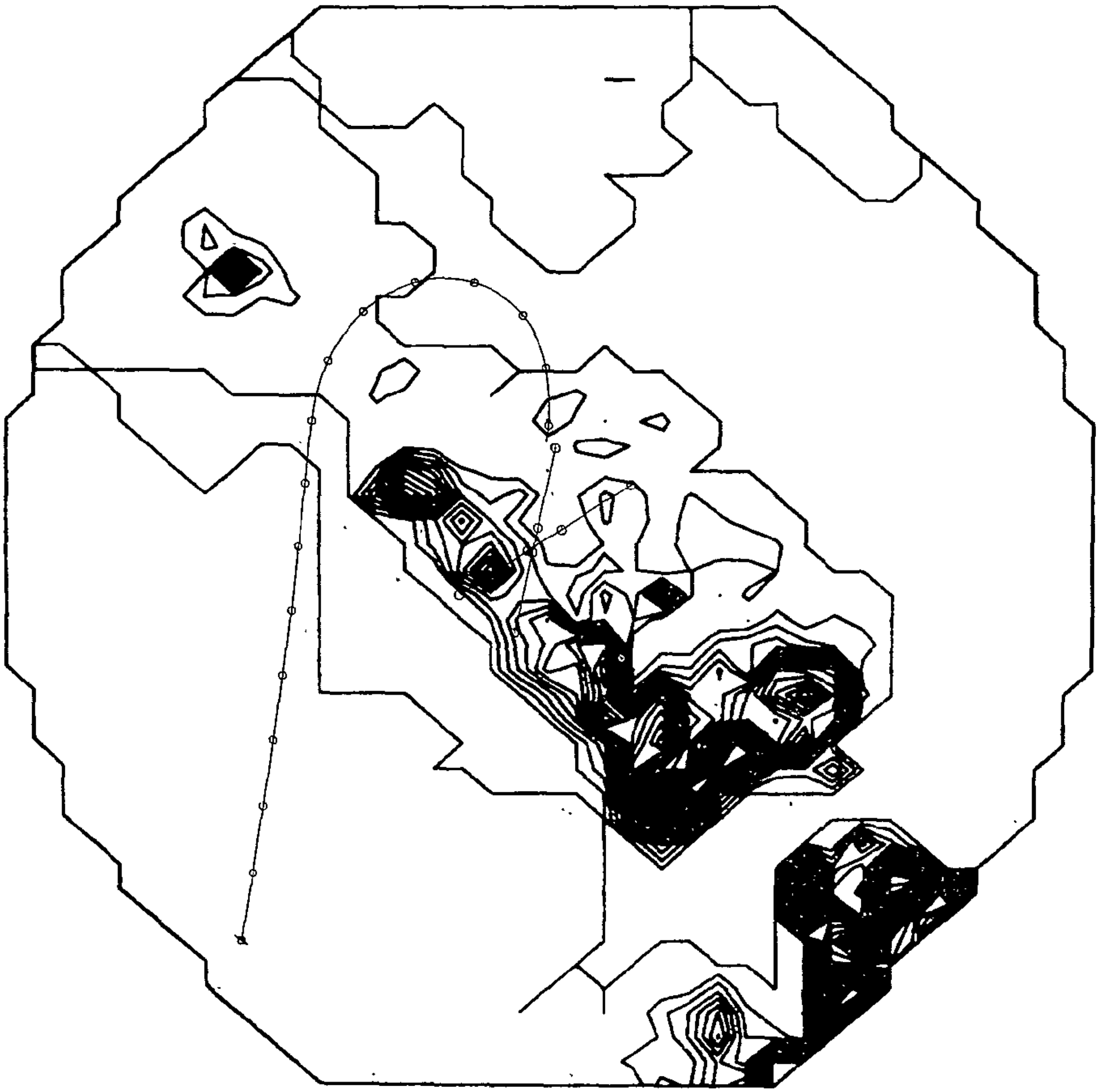

Figure 9

Optimal Trajectory - Flight 3

X_i = RANGE = -56520.0 feet

Y_i = SIDE = 81530.0 feet

Z_i = ALT = 13920.0 feet

Yaw_i = PSI = -82.3 degrees

The final conditions were:

$X_f = X_F$ = 3697 feet

$Y_f = Y_F$ = -19065 feet

$Z_f = H_F$ = 900 feet

Yaw_f = PSIF = 100.8 degrees

X_R = 623 feet

Y_R = -2951 feet.

T_F = 765 seconds

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.3

Summary of Flight #3 (first attempt)

Cofield to Runway 20

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	0.65	-3.91	900	100.8				
#0	0.96	-3.87	1899	100.14	6.48	64819	-.251E08	415
#1	0.92	-3.83	1892	100.66	6.64	66354	-.248E08	415
#2	0.59	-3.53	1851	105.02	7.70	76993	-.229E08	415
#3	1.00	-4.04	1819	99.19	6.04	60410	-.215E08	415
#4	0.45	-3.48	1785	106.60	7.93	79296	-.202E08	415
#5	1.02	-4.13	1761	98.68	5.82	58172	-.189E08	415
#6	0.42	-3.49	1728	106.88	7.92	79202	.178E08	415
#7	0.66	-3.02	2018	104.67	8.07	80745	-.325E08	765
#8	0.18	-4.28	1132	93.82	7.62	76126	-.236E07	765
#9	0.13	-4.21	986	94.68	7.62	76147	-.115E07	765
#10	0.34	-4.12	986	94.90	7.33	73228	-.716E06	765
#11	1.66	-5.07	994	84.83	3.83	38293	-.624E07	765
#12	2.67	-3.44	990	92.51	9.83	98245	-.926E07	765
#13	2.19	-4.36	991	87.93	4.95	49439	-.655E07	765
#14	1.66	-4.57	990	88.29	4.40	43959		765

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

D. Flight #4: Cofield to Runway 24

Summary

This was a long flight of 880 seconds. The aircraft started at the Cofield beacon and then flew over an area of uniform population density of 24. The height was such that the 70 db noise envelope did not touch the ground. The aircraft then crossed the James River to the left of the airport and made a long sweeping right turn to the York River, down the York River, and around to the runway. For such a long flight, the performance index was a low 5.14. High population areas were crossed between the James and York Rivers and close to the runway on the final approach.

Two attempts were made. The first had a nominal that needed a lot of correction -- particularly, in yaw. The flight was kicked off into the artificially high population areas encircling the map. Then, the procedure was restarted with rudder controls manually adjusted to obtain a nominal that was very close to what we felt was the optimal. On the second attempt a total of 14 iterations were run, and the final result was chosen from the tenth iteration.

One thing learned from this flight was that, when long flights are optimized, the nominal should be close to the optimal; or, else, convergence is extremely slow. Initially, there may even be large divergence.

The final trajectory is plotted in Figure 10, and the results are tabulated.

Initial and Final Boundary Conditions

The 20 NM radius is crossed at:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

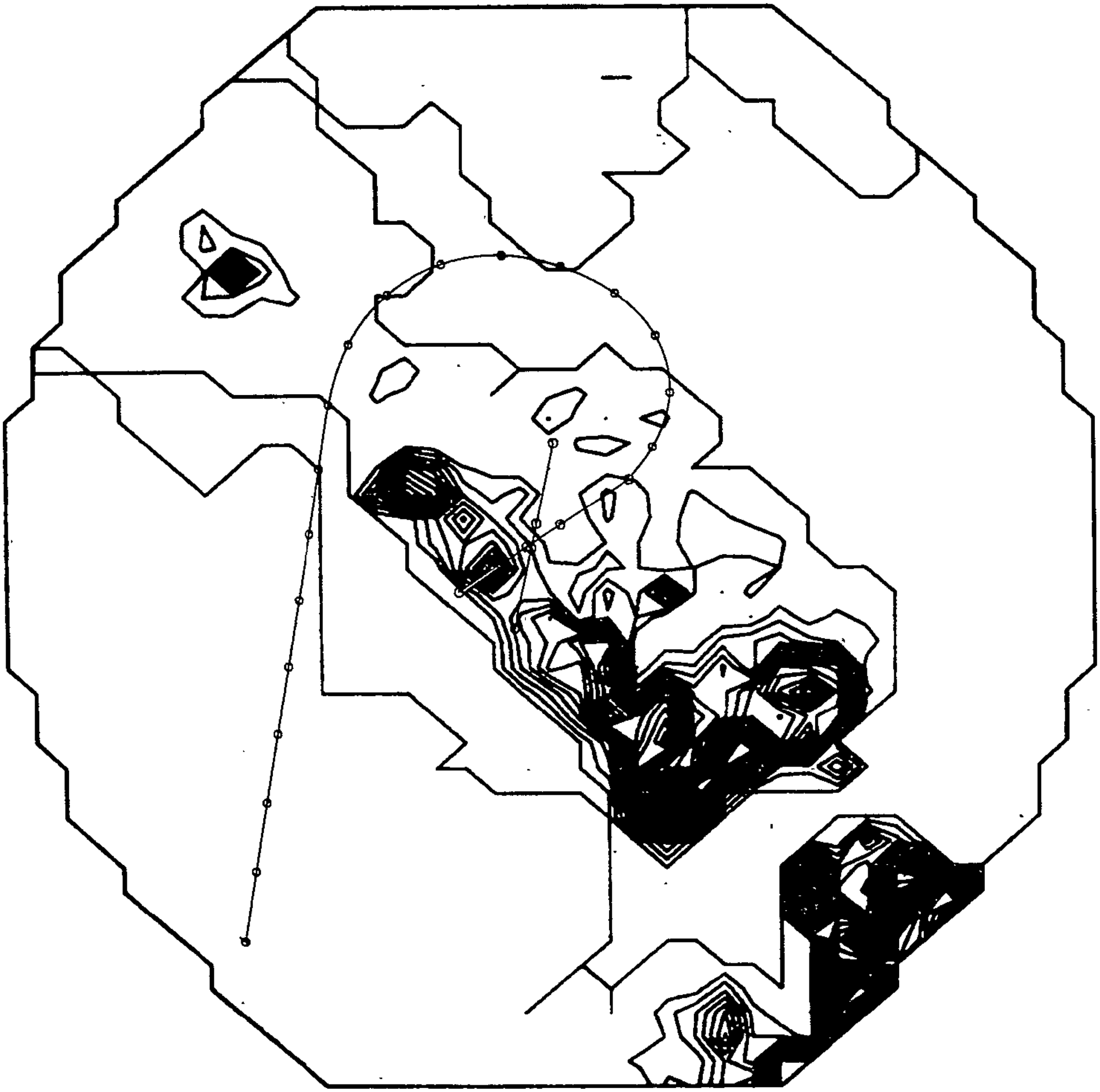

Figure 10

Optimal Trajectory - Flight 4

$X = -61630$ feet

$Y = 104730$ feet

Yaw = -77.57 degrees

The 16.33 NM radius is crossed at:

X_i = RANGE = -56520 feet

Y_i = SIDE = 81530 feet

Z_i = ALT = 16050 feet

Yaw_i = PSI = -82.3 degrees

The final conditions were:

$X_f = X_F = 18994$ feet

$Y_f = Y_F = 012062$ feet

$Z_f = H_F = 900$ feet

Yaw_f = PSIF = 145.7 degrees

$X_R = 5442$ feet

$Y_R = -2817$ feet

$T_F = 880$ seconds

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.4

Summary of Flight #4 (second attempt)

Cofield to Runway 24

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	3.33	-2.47	900	145.7				
#0	3.64	-2.03	1834	128.6	5.11	51139	-.245E08	530
#1	3.53	-1.96	1826	129.71	5.49	54922	-.239E08	530
#2	2.55	-1.53	1794	139.77	9.66	96543	-.288E08	530
#3	3.73	-2.79	1782	128.89	4.94	49343	-.221E08	530
#4	2.43	-1.70	1760	141.77	9.81	98090	-.212E08	530
#5	3.65	-2.92	1750	130.27	5.21	52147	-.203E08	530
#6	2.38	-1.80	1729	142.99	9.76	97574	-.197E08	530
#7	2.74	-1.95	1906	140.09	8.18	81752	-.267E08	880
#8	3.27	-3.12	1014	129.37	6.37	63659	-.299E07	880
#9	3.27	-3.10	947	129.16	6.28	62745	-.273E07	880
#10	3.45	-2.47	947	130.90	5.19	51843	-.180E07	880
#11	5.73	-0.35	949	133.49	3.38	33801	-.195E08	880
#12	5.90	-2.78	951	121.56	1.07	10689	-.179E08	880
#13	5.60	-0.51	948	133.62	3.14	31364	-.171E08	880
#14	5.77	-2.92	950	121.68	1.20	12001		880

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

E. Flight #5: Harcum to Runway 6

Summary

The aircraft comes from the Harcum beacon and enters the near-terminal area over the York River. From there, the aircraft makes a sweeping "s" turn, flies over the James River, and into the runway.

Twenty-four iterations were used to produce the final trajectory. Operator intervention consisted of lengthening the flight time and varying iteration step size.

The final iteration is plotted in Figure 11, and results are tabulated.

Initial and Final Boundary Conditions

Initial Conditions (used for Iterations 21 through 27)

The 20 NM radius is entered at:

X = -67240 feet

Y = -101220 feet

yaw = 83.54 degrees

The 16.33 NM radius is entered at:

X_i = RANGE = -44612 feet

Y_i = SIDE = -86376 feet

Z_i = ALT = 9200 feet

Yaw_i = PSI = 35 degrees

The boundary conditions were:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

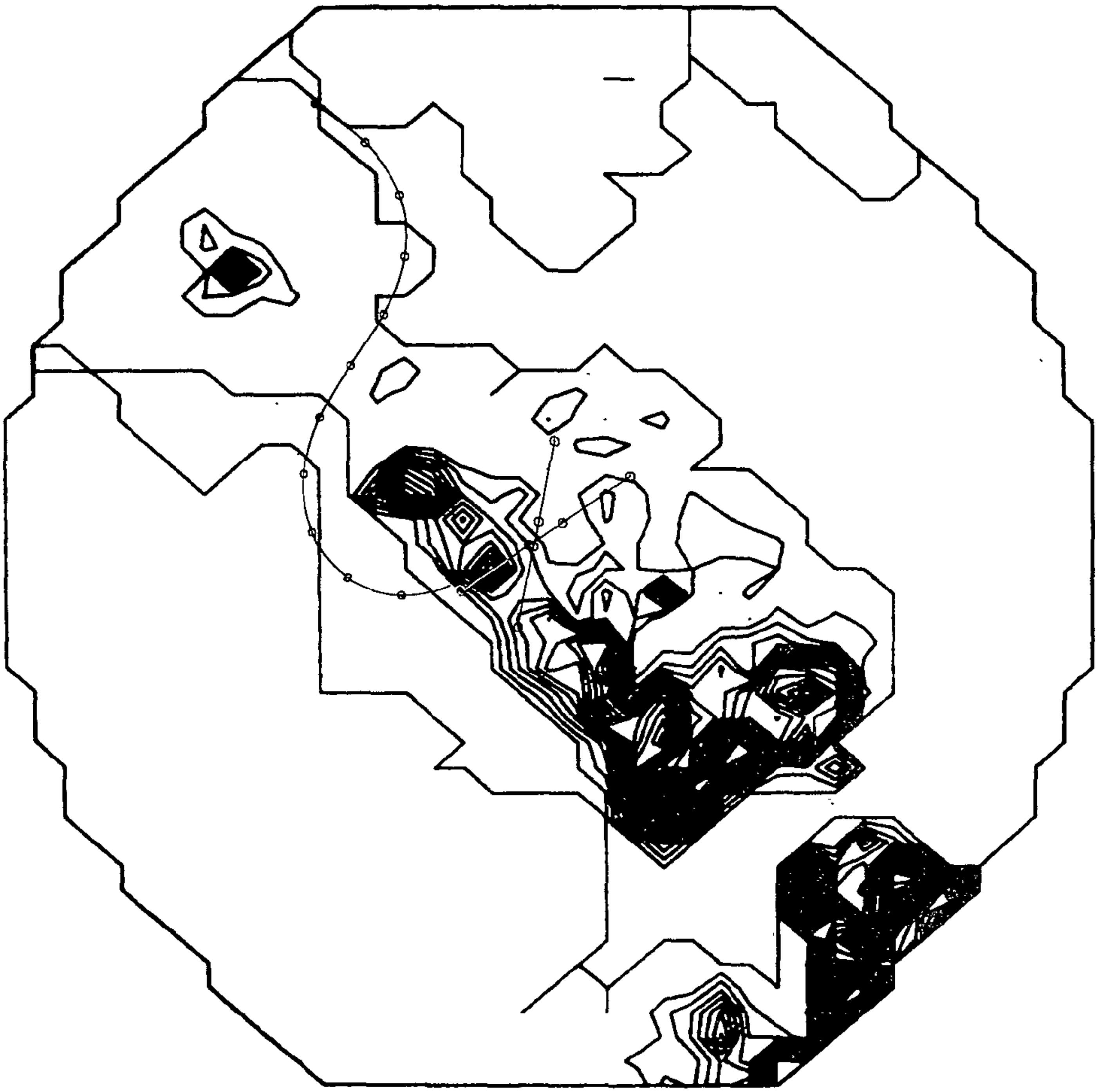

Figure 11

Optimal Trajectory - Flight 5

$$X_f = X_F = -14721.0 \text{ feet}$$

$$Y_f = Y_F = 10938.0 \text{ feet}$$

$$Z_f = H_F = 900.0 \text{ feet}$$

$$Yaw_f = PSIF = -34.3 \text{ degrees}$$

$$X_R = -1169.0 \text{ feet}$$

$$Y_R = 1693.0 \text{ feet}$$

$$T_F = 545.0 \text{ seconds}$$

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.5

Summary of Flight #5

Harcum to Runway 6

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	-2.59	2.24	900	-34.30				
#0	-10.91	3.04	-209	117.83	8.61	86094	-.348E09	515
#1	-9.65	3.73	-311	110.1	9.73	97273	-.299E09	515
#2	-7.21	4.55	-300	72.9	8.95	89508	-.174E09	515
#3	-5.85	3.86	-287	41.0	5.16	51631	-.104E09	515
#4	-5.25	2.31	-280	13.7	3.28	32837	-.667E08	515
#5	-1.76	2.83	-322	15.5	26.95	269514	-.582E08	515
#6	-3.07	1.61	-211	-2.8	4.85	48501	-.395E08	515
#7	-3.20	2.11	610	0.3	5.10	51015	-.120E08	515
#8	-3.00	2.03	603	-1.16	5.76	57611	-.110E08	515
#9	-2.51	1.44	612	-10.1	10.16	101509	-.751E07	515
#10	-1.18	0.56	626	-22.51	40.52	405104	-.112E08	515
#11	-1.96	0.89	620	-18.14	21.91	218917	-.749E07	515
#12	-2.81	1.29	624	-12.28	7.57	75659	-.705E07	515
#13	-1.99	0.89	629	-18.46	21.30	212876	-.718E07	515
#14	-4.71	2.78	859	-6.55	4.15	41469	-.159E08	545
#15	-4.25	2.65	786	-9.57	4.37	43714	-.108E08	545
#16	-4.07	2.03	782	-17.40	3.60	35989	-.699E07	545
#17	-3.82	1.38	779	-25.43	3.32	33159	-.511E07	545
#18	-3.54	1.93	800	-20.42	3.63	36249	-.367E07	545
#19	-3.36	1.43	808	-27.01	3.99	39901	-.279E07	545
#20	-3.33	2.17	824	-19.31	2.65	36473	-.296E07	545
#21	-3.32	2.17	821	-19.33	3.66	36609	-.294E07	545
#22	-3.13	2.09	821	-20.64	3.95	39519	-.219E07	545
#23	-3.11	1.70	829	-25.91	4.21	42032	-.163E07	545
#24	-2.90	2.20	842	-21.42	4.20	41962	-.154E07	545

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

F. Flight #6: Harcum to Runway 2

Summary

The first part of Flight #6 is almost exactly like Flight #5. The aircraft starts at the Harcum beacon, flies a short distance down the York River, and then makes a right turn across to the James River. The last part of Flight #6 continues down the James River and then makes a sharp left turn into Runway 2.

The manual adjustment method was used to obtain a nominal. Five single forward integrations were run. The total number of iterations was 18 (Flights #0 through #20). The numbering system is somewhat confusing, because the initial forward integration of each computer run is counted as an iteration, even though no optimization took place. The flights were not converging to the boundary condition -- probably, due to high population areas to the right of the runway. Iteration #16 was the closest and was used as the final result.

The final trajectory is plotted in Figure 12, and the results are tabulated. Operator intervention was extensive.

Initial and Final Boundary Conditions

These figures are for the final flight. The aircraft crosses the 20 NM radius at:

X	=	-67240	feet
Y	=	-101220	feet
Yaw	=	83.54	degrees

The 16.33 NM radius is crossed at:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

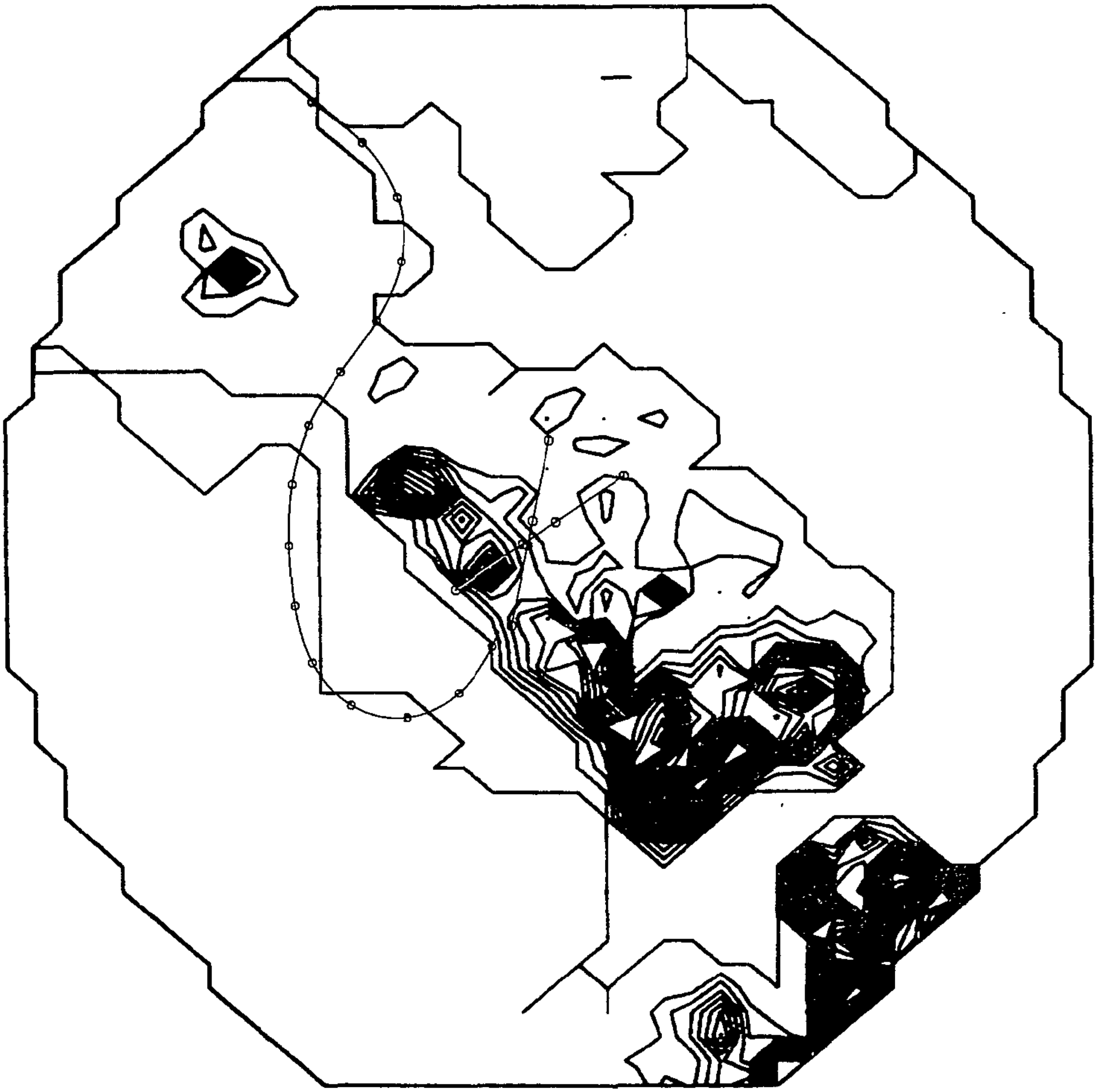

Figure 12

Optimal Trajectory - Flight 6

$$X_i = \text{RANGE} = -44612 \text{ feet}$$

$$Y_i = \text{SIDE} = -86376 \text{ feet}$$

$$Z_i = \text{ALT} = 11800 \text{ feet}$$

$$Yaw_i = \text{PSI} = 35 \text{ degrees}$$

The boundary condition is:

$$X_f = X_F = -3393 \text{ feet}$$

$$Y_f = Y_F = 18099 \text{ feet}$$

$$Z_f = H_F = 900 \text{ feet}$$

$$Yaw_f = \text{PSIF} = -79.2 \text{ degrees}$$

$$X_R = -319 \text{ feet}$$

$$Y_R = 1985 \text{ feet}$$

$$T_F = 715 \text{ seconds}$$

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.6

Summary of Flight #6 (first attempt)

Harcum to Runway 2

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	-0.60	3.71	900	-79.2				
#0	-0.92	2.71	921	-69.7	10.60	105953	-.238E07	330
#1	-0.91	2.92	828	-67.75	9.64	96254	-.225E07	330
#2	-0.89	3.69	839	-61.88	7.24	72285	-.255E07	330
#3	-0.95	3.26	838	-66.49	8.21	82026	-.187E07	330
#4	-1.03	3.03	842	-70.13	8.43	84168	-.179E07	330
#5	-0.98	3.44	847	-66.44	7.44	74297	-.173E07	330
#6	-1.06	3.06	848	-70.92	8.17	81648	-.164E07	330
#7	-1.13	3.06	1028	-72.03	8.49	84876	-.202E07	680
#8	-0.89	2.97	1028	-72.98	11.99	113837	-.170E07	680
#9	-3.21	4.98	1022	-53.25	9.95	19454	-.216E08	680
#10	-2.59	4.48	1033	-57.51	1.57	15139	-.129E08	680
#11	-1.79	4.57	1048	-57.21	1.61	10087	-.819E07	680
#12	-1.54	3.87	1041	-63.73	1.60	16022	-.547E07	680
#13	-1.77	3.16	1030	-71.12	2.59	25909	-.413E07	680
#14	-1.60	3.53	780	-71.06	3.20	36957	-.294E07	690
#15	-1.33	3.37	781	-72.68	5.70	56897	-.193E07	690
#16	-1.34	4.84	807	-61.15	2.78	27808	-.570E07	690
#17	-1.60	4.26	806	-66.62	2.63	26254	-.391E07	690
#18	-1.73	3.59	805	-73.00	2.56	24983	-.315E07	690
#19	-1.43	4.08	821	-69.67	3.81	38033.	-.243E07	690
#20	-1.58	3.45	823	-76.05	3.84	38402	-.226E07	690

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

G. Flight #7: Harcum to Runway 24

Summary

The aircraft enters the near-terminal area above the York River. The flight path follows the York River for about 250 seconds and then makes a right turn into Runway 24. The last part of the flight crossed high population density areas, and this caused some problems with convergence.

Two attempts, 22 iterations, and two forward integrations were used. The first attempt had errors in the input data, and the six iterations were of no value. The second attempt showed reasonable convergence. Iteration #8 had a better initial $\phi\psi$ than Iteration #13 and was used as the final result. Operator intervention consisted of changing the flight time.

The final iteration is plotted in Figure 13, and results of each iteration are tabulated.

Initial Conditions

The aircraft crosses the 20 NM radius at:

X = -57530 feet

Y = -107040 feet

yaw = 38.55 degrees

A slight left turn must be made to meet the ICs on the 16.33 NM radius:

X_i = RANGE = -44612 feet

Y_i = SIDE = -86376 feet

Z_i = ALT = 7000 feet

Yaw_i = PSI = 35 degrees

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

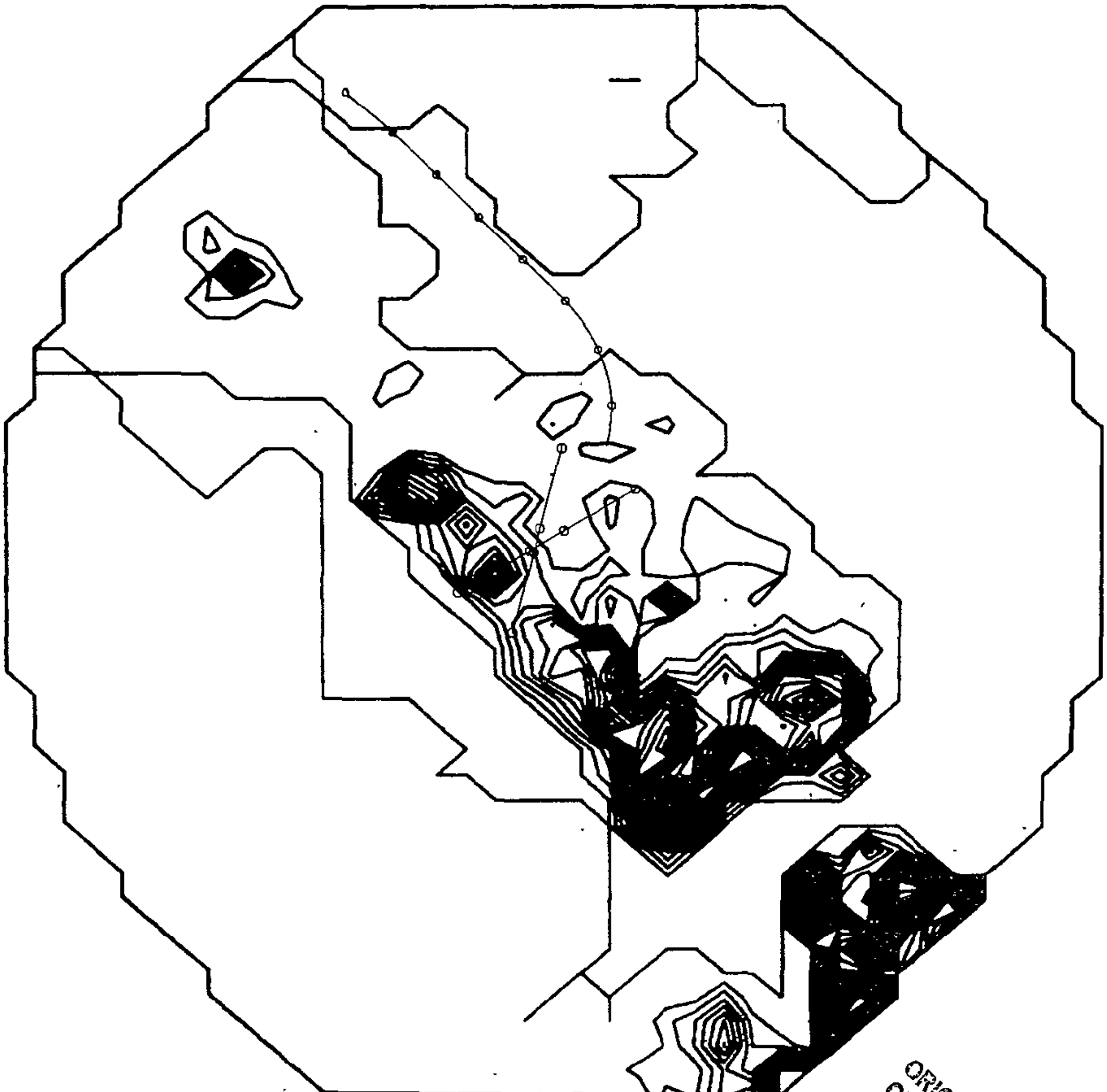

Figure 13

Optimal Trajectory - Flight 7

The final boundary conditions were:

$$X_f = X_F = 3697 \text{ feet}$$

$$Y_f = Y_F = -19065 \text{ feet}$$

$$Z_f = H_F = 900 \text{ feet}$$

$$Yaw_f = PSIF = 100.8 \text{ degrees}$$

$$X_R = 623 \text{ feet}$$

$$Y_R = -2951 \text{ feet}$$

$$T_F = 390 \text{ seconds}$$

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.7

Summary of Flight #7 (second attempt)

Harcum to Runway 20

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	0.65	-3.91	900	100.8				
#0	0.27	-3.46	1509	91.6	6.59	65871	-.105E08	370
#1	0.65	-3.66	1479	89.2	6.41	64047	-.950E07	370
#2	2.80	-5.53	1330	80.16	3.43	34262	-.209E08	370
#3	2.11	-4.64	1336	80.7	2.41	24097	-.129E08	370
#4	1.82	-4.56	1272	89.1	2.69	26865	-.188E07	370
#5	1.62	-4.54	1198	95.5	2.83	28289	-.493E07	370
#6	1.50	-4.40	1117	97.9	2.85	28526	-.306E07	370
#7	1.28	-3.39	1081	103.7	4.82	48196	-.208E07	390
#8	0.64	-3.14	891	108.3	5.73	57302	-.131E07	390
#9	1.16	-4.20	834	121.4	3.98	39804	-.398E07	390
#10	3.67	-5.74	843	95.5	4.59	45881	-.237E08	390
#11	3.09	-5.00	842	98.2	3.39	33840	-.139E08	390
#12	2.61	-4.41	840	99.8	2.57	25047	-.823E07	390
#13	2.20	-3.95	835	101.2	2.33	23280	-.496E07	390

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

H. Flight #8: Harcum to Runway 24

Summary

The aircraft starts at Harcum and enters the near-terminal area above the York River. The aircraft follows the York River, misses the Abingdon District 200 seconds into the flight, and then makes a sharp right turn into Runway 24.

Two attempts were made. The first started with a straight three-degree glide slope. Operator intervention was used to change the flight time and the iteration step size, and the boundary condition convergence was good; however, the aircraft flew over a high population district midway through the flight. The second attempt used a lot of operator intervention in order to steer the airplane around the high population district. The rudder controls were varied, and only the last part of the trajectory was optimized. The end result was remarkably good. A total of 27 iterations and seven single forward integrations were used. The final iteration is plotted in Figure 14, and the results are tabulated.

Initial and Final Boundary Conditions

These are for the final forward integration of Attempt #2. The aircraft enters the 20 NM radius at:

X = -57530 feet

Y = -107040 feet

yaw = 38.55 degrees

The 16.33 NM radius is entered at:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

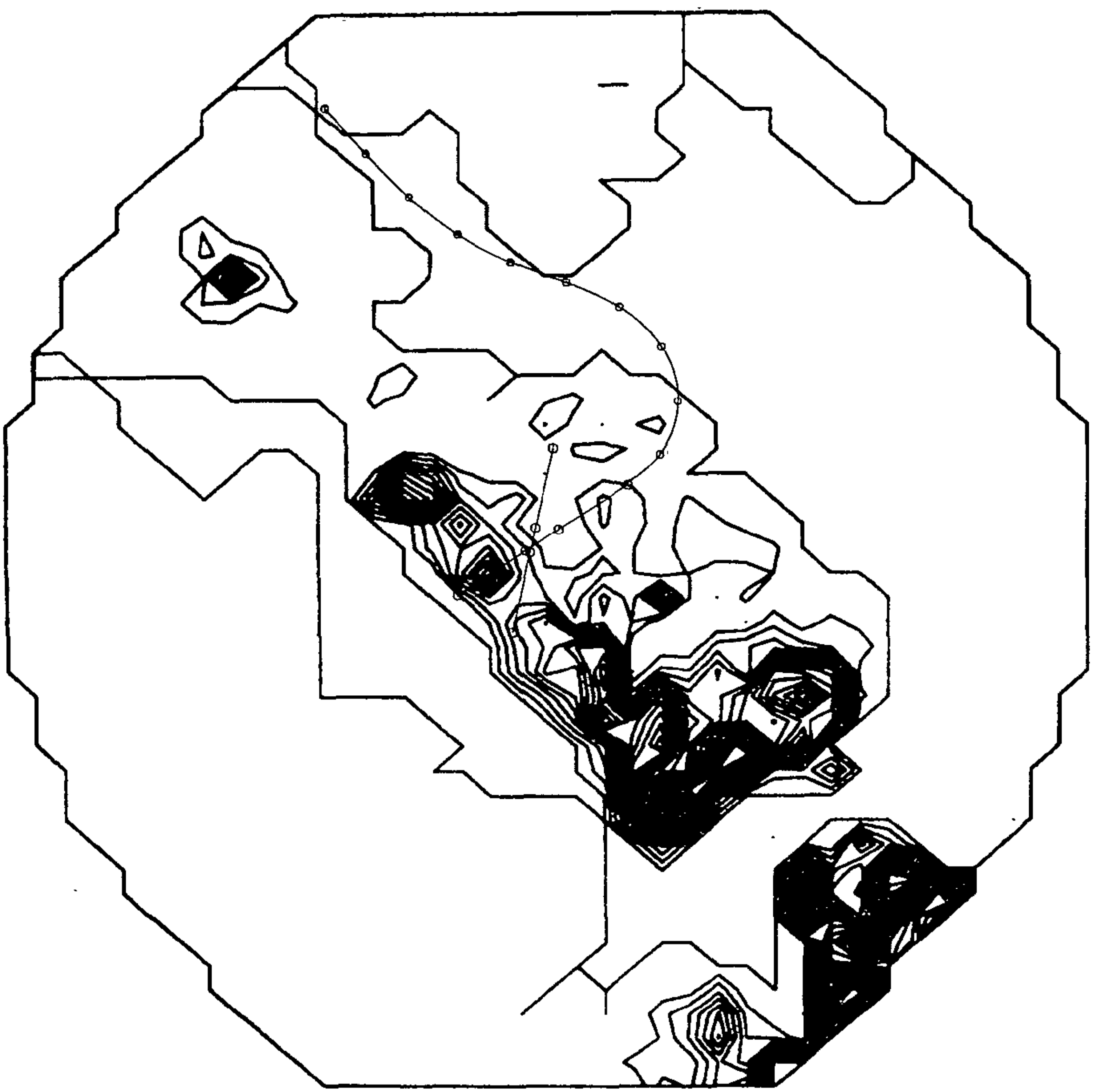

Figure 14

Optimal Trajectory - Flight 8

X_i = RANGE = -44612 feet

Y_i = SIDE = -86376 feet

Z_i = ALT = 8400 feet

Yaw_i = PSI = 46.3 degrees

The final condition was:

$X_f = X_F$ = 18944 feet

$Y_f = Y_F$ = -12062 feet

$Z_f = H_F$ = 900 feet

Yaw_f = PSIF = 145.7 degrees

X_R = 5442 feet

Y_R = -2817 feet

T_F = 495 seconds

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.8

Summary of Flight #8 (first attempt)

Harcum to Runway 24

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	3.33	-2.47	900	145.7				
#0	9.27	-7.63	1286	26.8	.661E03	66054	-.222E09	460
#1	8.67	-6.67	167	32.7	.630E03	63022	-.194E09	460
#2	6.43	-3.19	155	36.7	.373E03	37383	-.125E09	460
#3	5.68	-2.57	181	65.8	.443E03	44365	-.727E08	460
#4	5.23	-2.93	222	89.6	.966E03	46555	-.432E08	460
#5	4.15	-1.94	268	101.6	.497E03	49669	-.266E08	460
#6	4.79	-3.78	322	116.8	.424E03	42387	-.216E08	460
#7	3.67	-2.04	255	135.8	5.53	55336	-.116E08	500
#8	2.94	-1.79	200	141.6	7.32	73164	-.136E08	500
#9	6.79	-4.27	166	111.7	5.56	55619	-.513E08	500
#10	5.74	-2.88	164	117.1	3.65	36542	-.317E08	500
#11	4.54	-1.62	162	122.7	3.31	33130	-.216E08	500
#12	3.12	-1.04	178	135.5	5.90	59001	-.169E08	500
#13	4.72	-2.29	204	127.3	3.18	31845	-.186E08	500
#14	3.73	-1.93	667	136.3	5.66	56622	-.278E07	5-0
#15	3.66	-1.92	603	136.8	5.76	57540	-.346E07	500
#16	3.28	-1.80	606	140.0	6.74	67335	-.310E07	500
#17	3.51	-2.00	611	139.2	6.14	61817	-.281E07	500
#18	3.21	-1.93	615	142.0	6.98	69818	-.260E07	500
#19	3.54	-2.16	620	140.2	6.20	61988	-.242E07	500
#20	3.17	-2.03	624	143.3	7.12	71190	-.229E07	500

Second Attempt

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
#0	3.17	-2.19	236.9	146.6	3.89	38919	-.112E08	315
#1	3.22	-2.20	242	146.2	3.79	37849	-.110E08	315
#2	3.56	-2.24	292	142.6	3.01	30051	-.950E07	315
#3	3.03	-2.26	332	.49.0	4.34	43386	-.841E07	315
#4	3.75	-2.30	369	140.9	2.72	27227	-.760E07	315
#5	2.97	-2.29	393.3	150.1	4.54	45596	-.688E07	315
#6	3.77	-2.37	425.2	140.9	2.73	27288	-.632E07	315

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

I. Flight #9: Cape Charles to Runway 6

This is a long flight of 800 seconds. It starts at Cape Charles and flies across the Chesapeak Bay into the mouth of the York River. The aircraft flies up the York River and then crosses to the James River at approximately the same place as Flight #s 3, 4, 5, 6, and 10. The trajectory finishes with a left turn into Runway 6.

This flight proved to be troublesome, and this was not so much caused by difficulties in the flight path and population density as it was caused by programmer errors and bad luck. This first attempt was to optimize the entire trajectory. The nominal was not close, and the iterations diverged at first, and the succeeding iterations converged very slowly. The first attempt was abandoned after 13 iterations. The second attempt was to optimize only the last portion of the flight. Convergence was good, but the flight path swung over the east bank of the James River. The second attempt was abandoned after six iterations. The manual method was used to obtain a nominal for the third attempt. The third attempt was optimized over the entire trajectory, and convergence was very good.

The final iteration is plotted in Figure 15, and the results are tabulated.

Initial and Boundary Conditions

The aircraft crosses the 20 NM radius at:

X = 102420 feet

Y = -65400 feet

Yaw = 153.19 degrees

The 16.33 NM radius was crossed at:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

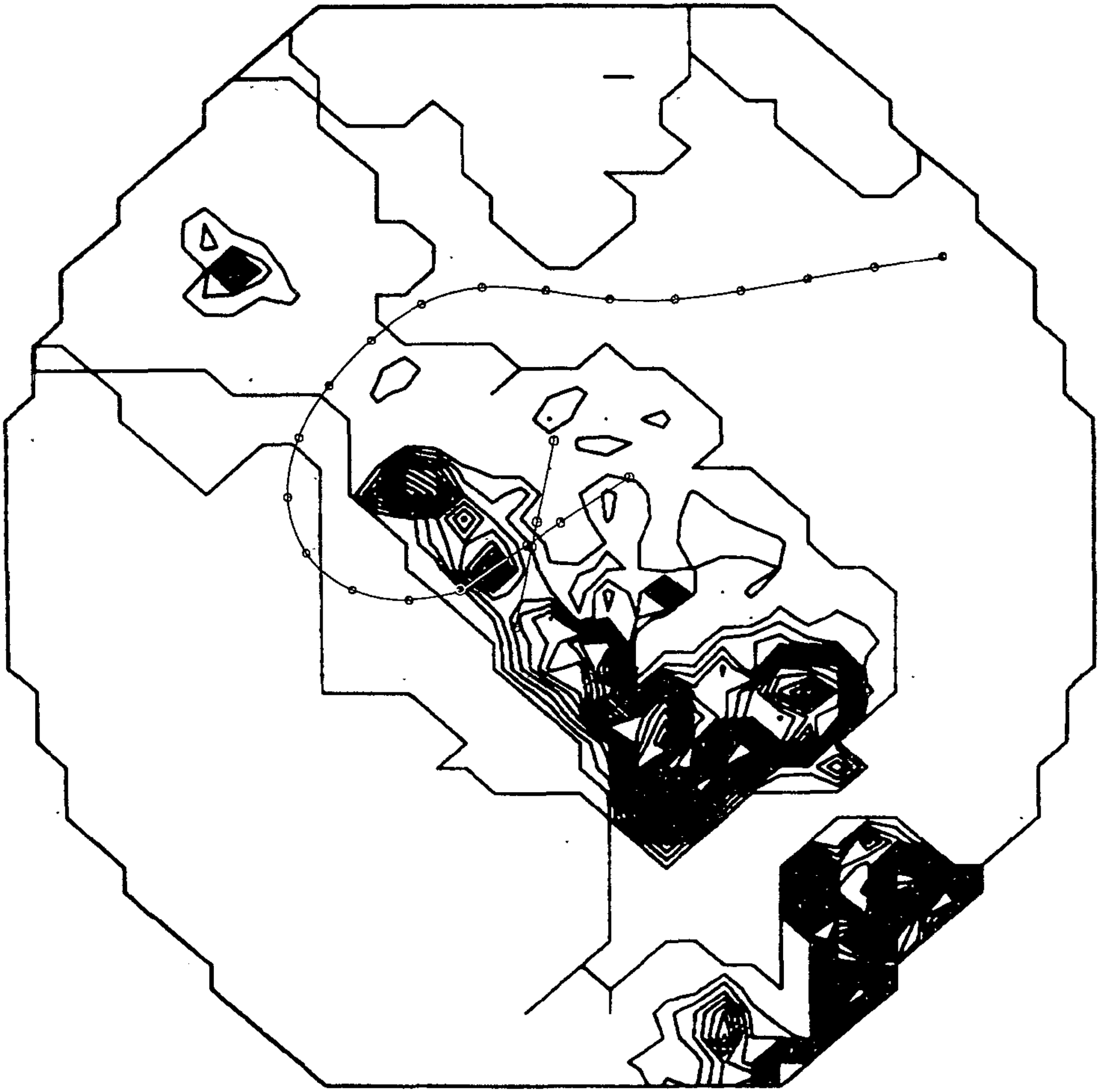

Figure 15

Optimal Trajectory - Flight 9

$$X_i = \text{RANGE} = 81267 \text{ feet}$$

$$Y_i = \text{SIDE} = -56925 \text{ feet}$$

$$Z_i = \text{ALT} = 14280 \text{ feet}$$

$$Yaw_i = \text{PSI} = 170 \text{ degrees}$$

The boundary condition is:

$$X_f = X_F = -14721 \text{ feet}$$

$$Y_f = Y_F = 10938 \text{ feet}$$

$$Z_f = H_F = 900 \text{ feet}$$

$$Yaw_f = \text{PSIF} = -34.3 \text{ degrees}$$

$$X_R = -1169 \text{ feet}$$

$$Y_R = 1693 \text{ feet}$$

$$T_F = 800 \text{ seconds}$$

TABLE 4.9

Summary of Flight #9 (first attempt)

Cape Charles to Runway 6

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $\partial\psi$	Time (Sec.)
Desired	-2.59	2.24	900	-34.30				
#0	-3.66	-1.68	1776	-70.05	14.02	140090	-.591E08	825
#1	-2.94	-.36	839	-67.20	13.97	139635	-.187E08	825
#2	-10.93	-10.14	847	19.01	27009.00	2.7E09	-.390E09	825
#3	-11.28	-4.40	849	17.31	20.85	208471	-.239E09	825
#4	-8.64	-5.31	838	-10.04	19.60	196018	-.163E09	825
#5	-6.60	-4.35	848	-26.78	34.99	349947	-.102E09	825
#6	-7.46	-1.56	904	-14.82	16.11	161065	-.722E08	825
#7	-7.46	-1.57	906	-14.88	16.12	161250	-.724E08	825
#8	-6.65	-.09	907	-20.62	7.87	78711	-.428E08	825
#9	-8.15	6.77	938	2.77	7.10	70981	-.103E08	825
#10	-8.68	1.53	933	-4.58	8.03	80328	-.875E08	825
#11	-7.13	7.30	946	-3.16	13.24	132420	-.909E08	825
#12	-6.41	6.04	942	-14.80	4.59	45926	-.537E08	825
#13	-6.50	3.79	938	-21.33	3.47	34683	-.357E08	825

Second Attempt

#0	-2.25	1.08	1709	-25.6	12.44	124250	-.192E08	455
#1	-2.55	1.65	1700	-26.78	5.71	57044	-.179E08	455
#2	-5.44	4.51	1531	6.21	1.81	18105	-.465E08	455
#3	-3.83	4.35	1549	-.29	1.51	15085	-.290E08	455
#4	-2.15	3.70	1580	-6.31	1.42	14182	-.211E08	455
#5	-2.73	2.94	1529	-15.27	1.56	15570	-.134E08	455
#6	-3.05	2.36	1446	-23.37	1.70	17030	-.881E07	455

Third Attempt

#0	-1.76	1.59	1459	-20.4	12.49	128760	-.113E08	800
#1	-2.17	1.84	1084	-15.8	6.94	69316	-.404E07	800
#2	-2.11	3.77	1075	-14.07	5.02	50187	-.780E07	800
#3	-3.93	2.86	1077	-3.83	2.59	25890	-.120E08	800
#4	-3.31	2.73	1077	-8.25	2.68	25987	-.735E07	800
#5	-2.83	2.47	1074	-12.33	2.92	29225	-.463E07	800
#6	-2.40	2.21	1070	-16.90	4.19	41891	-.310E07	800

J. Flight #10: Cape Charles to Runway 2

Summary

This was the longest flight, and it was 940 seconds. Most of the flight was the same as Flight #9. The aircraft starts at Cape Charles, flies across the Chesapeak Bay, flies up the York River, and then turns left and flies down the James River. At this point, Flight #9 turns into Runway 6, but Flight #10 continues down the James River and makes a sharp left turn into Runway 2.

This was one of the most trouble-free flights, and the major reason for this was a good nominal. The manual method was used to obtain the nominal, and only one forward integration was needed. More would have been needed if Flight #9 had not previously been done. The entire trajectory was optimized for six iterations, and the program worked extremely well. The allowable iteration step size was very low, and only one of the six iterations had any control energy left for optimization with respect to noise (all the energy being used to satisfy the boundary conditions). The sixth iteration was used as the final result.

The final iteration is plotted in Figure 16, and the results are tabulated.

Initial Conditions and Boundary Conditions

The aircraft intersects the 20 NM radius at:

$$X = 102420$$

$$Y = -65400$$

$$\text{Yaw} = 153.19$$

A turn must be made to meet the initial conditions on the 16.33 NM radius.

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

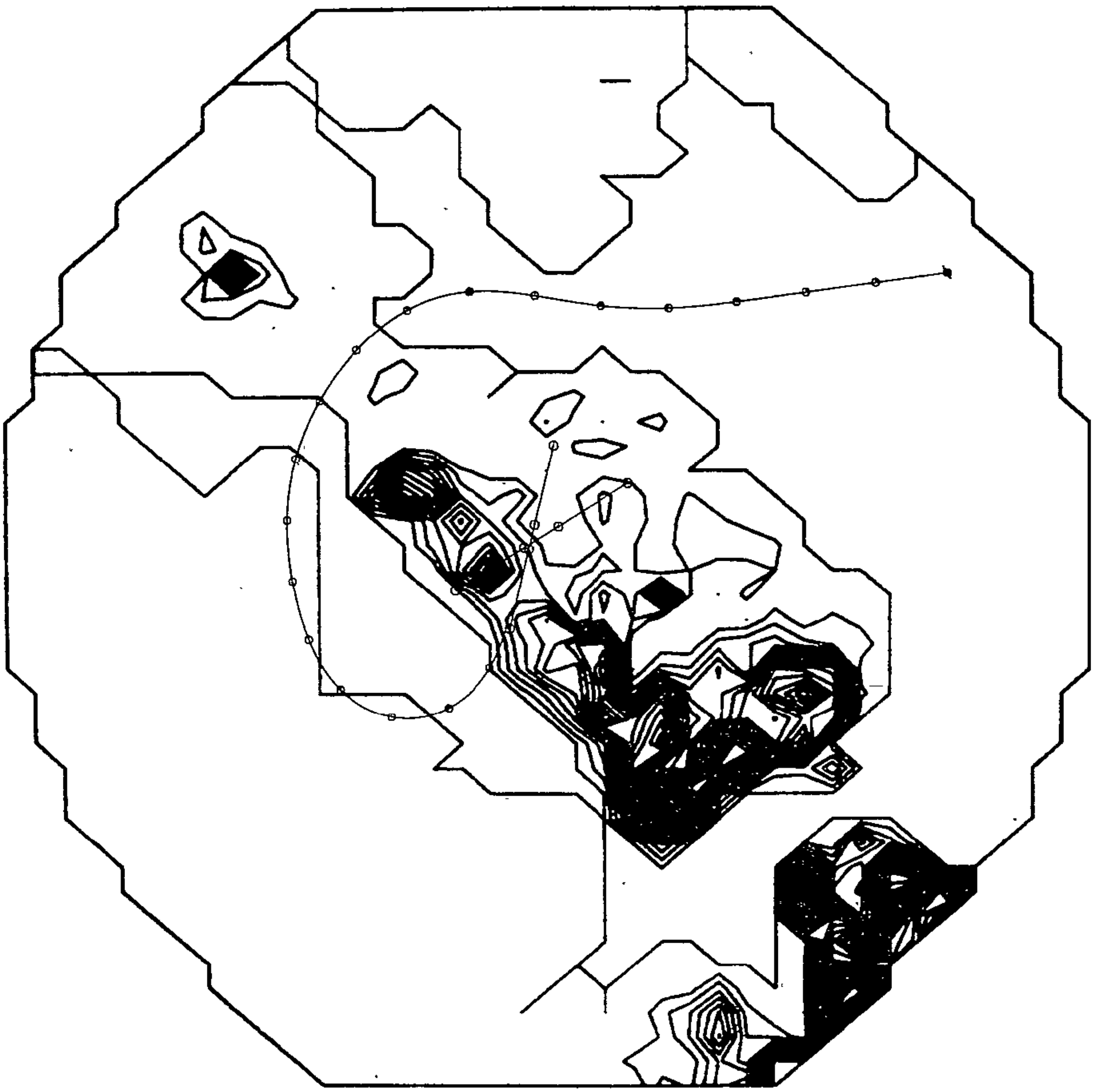

Figure 16

Optimal Trajectory - Flight 10

X_i = RANGE = 81267 feet

Y_i = SIDE = -56925 feet

Z_i = ALT = 16800 feet

Yaw_i = PSI = 170 degrees

The boundary conditions are:

$X_f = X_F$ = -3393 feet

$Y_f = Y_F$ = 18099 feet

$Z_f = H_F$ = 900 feet

Yaw_f = PSIF = -79.2 degrees

X_R = -319 feet

Y_R = 1985 feet

T_F = 940 seconds

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.10

Summary of Flight #10

Cape Charles to Runway 2

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	-0.60	3.71	900	-79.2				
#0	-1.61	4.08	612	-65.8	2.33	23311	-.572E07	940
#1	-1.41	4.38	239	-64.8	2.75	27423	-.145E08	940
#2	-.76	3.34	252	-73.85	9.39	93857	-.109E08	940
#3	.76	3.98	272	-80.09	18.22	182165	-.137E08	940
#4	-.21	3.85	276	-75.88	11.72	117153	-.101E08	940
#5	-1.06	3.57	285	-72.28	6.33	63191	-.103E08	940
#6	-.58	3.64	287	-74.95	9.80	97921	-.954E07	940

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

K. Flight #11: Cape Charles to Runway 20

Summary

The aircraft starts over the Cape Charles beacon, and the first two-thirds of the flight is across the Chesapeak Bay. The trajectory then goes into the mouth of the York River and makes a left turn into Runway 20.

One attempt with a total of 13 iterations was used to optimize this flight. The nominal trajectory was a straight three-degree glide slope. The program made a turn in the first six iterations but seemed unable to cross the population areas between the York river and the runway in order to meet the boundary conditions. To help the trajectory meet the boundary conditions we pivoted the entire trajectory about its intersection with the 20 NM radius and increased the flight length from 380 to 390 seconds. In the Iterations 7 through 13 the flight converged to a final result.

The final iteration is plotted in Figure 17 and tabulated.

Initial Conditions and Boundary Conditions

These are for the final result. The aircraft crosses the 20 NM radius at:

$$X = 104270 \text{ feet}$$

$$Y = -59400 \text{ feet}$$

$$\text{yaw} = 146 \text{ degrees}$$

The plane must make a turn to meet the initial conditions on the 16.33 NM radius.

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

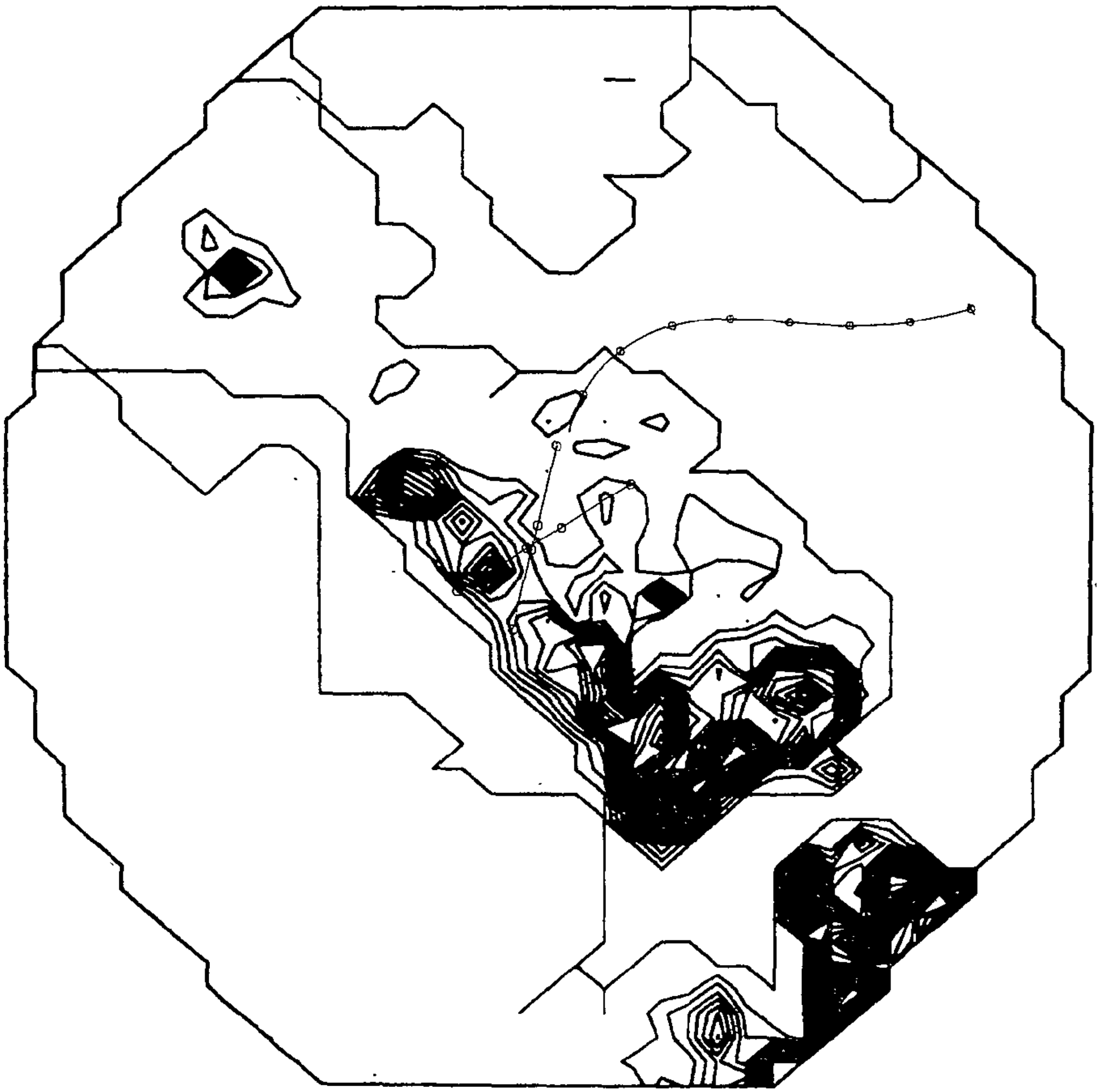

Figure 17

Optimal Trajectory - Flight 11

X_i = RANGE = 85565 feet

Y_i = SIDE = -50233 feet

Z_i = ALT = 7000 feet

Yaw_i = PSI = 163 degrees

The boundary conditions are:

$X_f = X_F$ = 3697 feet

$Y_f = Y_F$ = -19065 feet

$Z_f = H_F$ = 900 feet

Yaw_f = PSIF = 100.8 degrees

X_R = 623 feet

Y_R = -2951 feet

T_F = 390 seconds.

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.11

Summary of Flight #11

Cape Charles to Runway 20

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	0.65	-3.91	900	100.8				
#0	-0.87	-7.31	2208	169.2	0.068	684	-.100E09	380
#1	-0.64	-6.52	1268	164.0	1.20	12004	-.473E08	380
#2	-0.44	-6.48	1232	143.2	0.42	4182	-.287E08	380
#3	-0.03	-6.74	1180	123.9	0.11	1058	-.189E08	380
#4	0.61	-6.94	1096	104.4	0.04	389	-.147E08	380
#5	0.93	-6.48	902	96.4	0.30	2970	-.101E08	380
#6	0.12	-6.33	885	111.3	0.37	3651	-.114E08	380
#7	0.65	-3.64	852	109.2	3.84	39420	-.704E06	390
#8	0.83	-3.49	776	105.9	3.69	36881	-.905E06	390
#9	1.07	-3.77	746	100.3	3.25	32502	-.981E06	390
#10	0.91	-4.51	727	101.3	3.01	30084	-.141E07	390
#11	1.30	-3.94	726	95.3	2.80	27945	-.183E07	390
#12	1.08	-4.31	728	98.5	2.82	28136	-.138E07	390
#13	1.03	-4.29	737	99.6	2.92	29191	-.118E07	390

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

L. Flight #12: Cape Charles to Runway 24

Summary

The aircraft flies over the Cape Charles beacon which is on the Delmarva Pennisula. Most of the flight is over the Chesapeak Bay, and only the last portion of the flight is over populated areas.

Only one attempt of 13 iterations was needed to produce a final result. We call these final results "near optimums," because that sounds better than "suboptimum." There were no population spikes to steer around. The flight path is almost straight. The final result is a stretched-out "S"; but, if the final time is decreased, it will probably straighten out.

The final iteration is plotted in Figure 18, and the results are tabulated. Operator intervention consisted of decreasing the flight time.

Initial and Boundary Conditions

The aircraft enters the 20 NM radius at:

$$X = 104270 \text{ feet}$$

$$Y = -59400 \text{ feet}$$

$$\text{yaw} = 146 \text{ degrees}$$

The 16.33 NM radius is crossed at:

$$X_i = \text{RANGE} = 86968 \text{ feet}$$

$$Y_i = \text{SIDE} = -47730 \text{ feet}$$

$$Z_i = \text{ALT} = 6150 \text{ feet}$$

$$\text{Yaw}_i = \text{PSI} = 146 \text{ degrees}$$

The boundary conditions are:

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

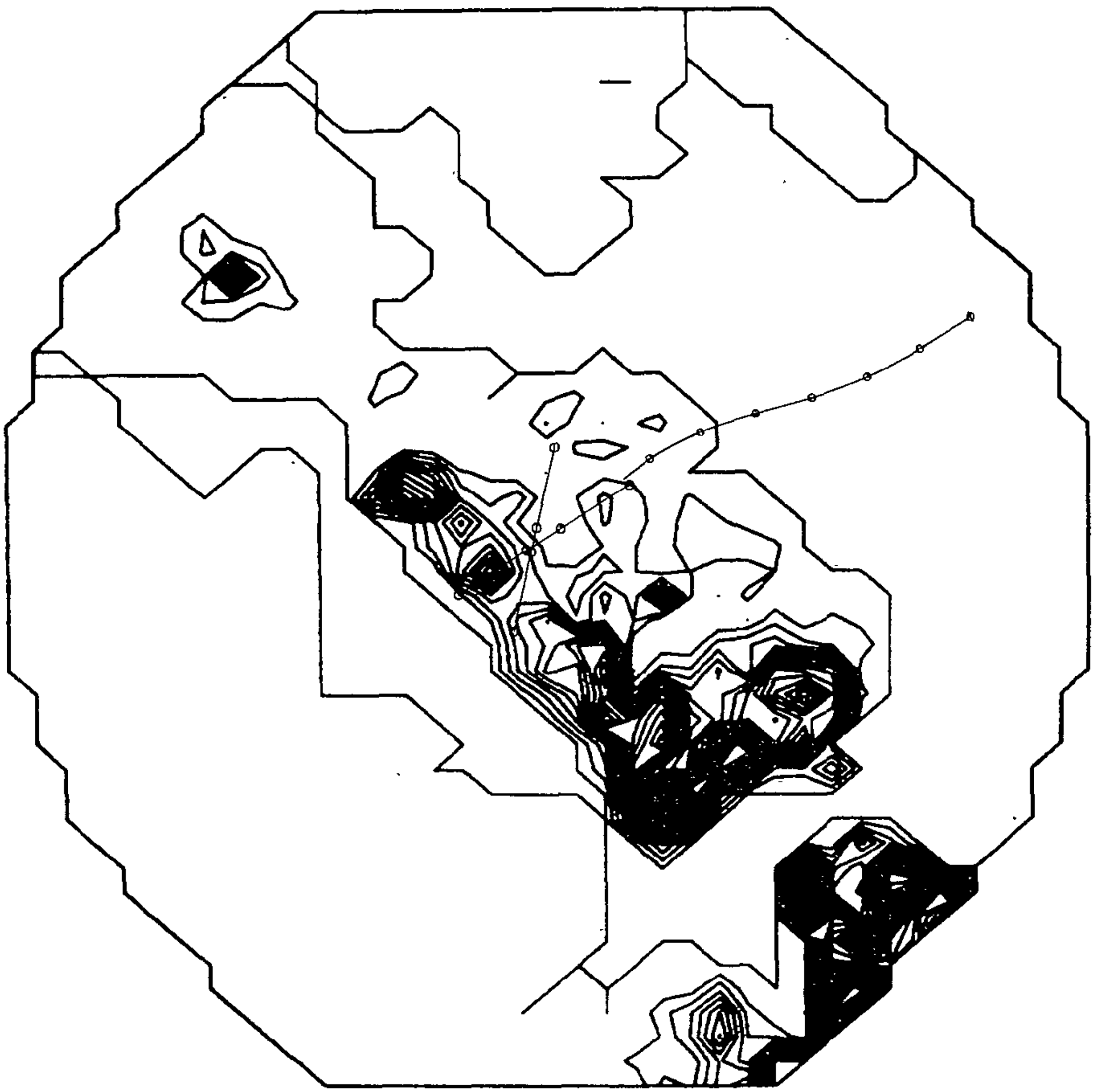

Figure 18

Optimal Trajectory - Flight 12

$$X_f = X_F = 18944 \text{ feet}$$

$$Y_f = Y_F = -12062 \text{ feet}$$

$$Z_f = H_F = 900 \text{ feet}$$

$$Yaw_f = PSIF = 145.7 \text{ degrees}$$

$$X_R = 5442 \text{ feet}$$

$$Y_R = -2817 \text{ feet}$$

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

TABLE 4.12

Summary of Flight #12 (first attempt)

Cape Charles to Runway 24

Traj.	X_f Blocks	Y_f Blocks	Z_f Feet	Yaw_f Degs.	Perf. Index J	People-Time-Sum (People-Sec.)	Initial $d\psi$	Time (Sec.)
Desired	3.33	-2.47	900	145.7				
#0	2.51	0.27	1887	146.0	10.1	101143	-.369E08	370
#1	2.22	-0.42	973	150.7	8.07	80629	-.907E07	370
#2	1.82	-1.39	908	140.0	8.60	85949	-.663E07	370
#3	0.73	-5.10	804	174.4	5.32	53207	-.305E08	370
#4	0.85	-4.19	797	159.5	6.61	66037	-.185E08	370
#5	1.08	-3.45	799	145.1	7.65	76459	-.119E08	370
#6	1.50	-2.64	800	130.2	7.84	78347		370
#7	2.65	-3.86	1433	140.6	4.95	49479	-.111E08	330
#8	3.50	-2.28	1446	117.6	3.54	35388	-.136E08	330
#9	3.23	-2.57	1385	127.5	3.77	37674	-.843E07	330
#10	3.05	-2.77	1310	135.3	3.79	37882	-.531E07	330
#11	3.63	-1.46	1233	128.8	2.78	27785	-.665E07	330
#12	3.32	-2.15	1225	130.9	3.12	31136	-.447E07	330
#13	3.14	-2.45	1196	136.6	3.31	33047	-.289E07	330

Note: 1 mile = 1.6093 kilometers; 1 foot = .3048 meters; 1 square mile = 2.589998 square kilometers; and 1 nautical mile (NM) = 1.852 kilometers.

In addition to seeing the final trajectories, it is also of interest to observe the pattern of convergence of the optimization procedure. Figure 19 shows a series of six iterations for Flight #9. Here, the convergence was very poor, and a new initial trajectory had to be selected. Figure 20 shows a series of five iterations for Flight #5. The convergence here was excellent, and only a few more iterations were required to determine an acceptable trajectory. What these results are telling us is that: (a) the method is problem-dependent; and (b) operator intervention is required. This is not news, and anyone experienced with numerical optimization techniques realizes this. What is important is that the method does give reasonable results which do accomplish the end objective which is low noise reference trajectories.

V. Future Work

The methodology for determining minimum noise flight trajectories has been developed and demonstrated. There are many possible areas for extension of this work. Obviously, the technique can be used for other types of aircraft and other airports by modifying the airplane model and the population model. Also, one could optimize trajectories for takeoff as well as for landing.

To test the trajectories for flying ease it is important that actual aircraft attempt to land using the optimal trajectories as references. The coordinates of the optimal trajectories as functions of time would be stored on magnetic tape, and the autopilot for the TCV would read off these coordinates and steer the aircraft along the path using a suitable control strategy. It is believed that the aircraft would have very little difficulty executing the required maneuvers.

Additional insight into our results can be obtained by examining Figure 21 which shows all 12 optimal trajectories superimposed. An interesting

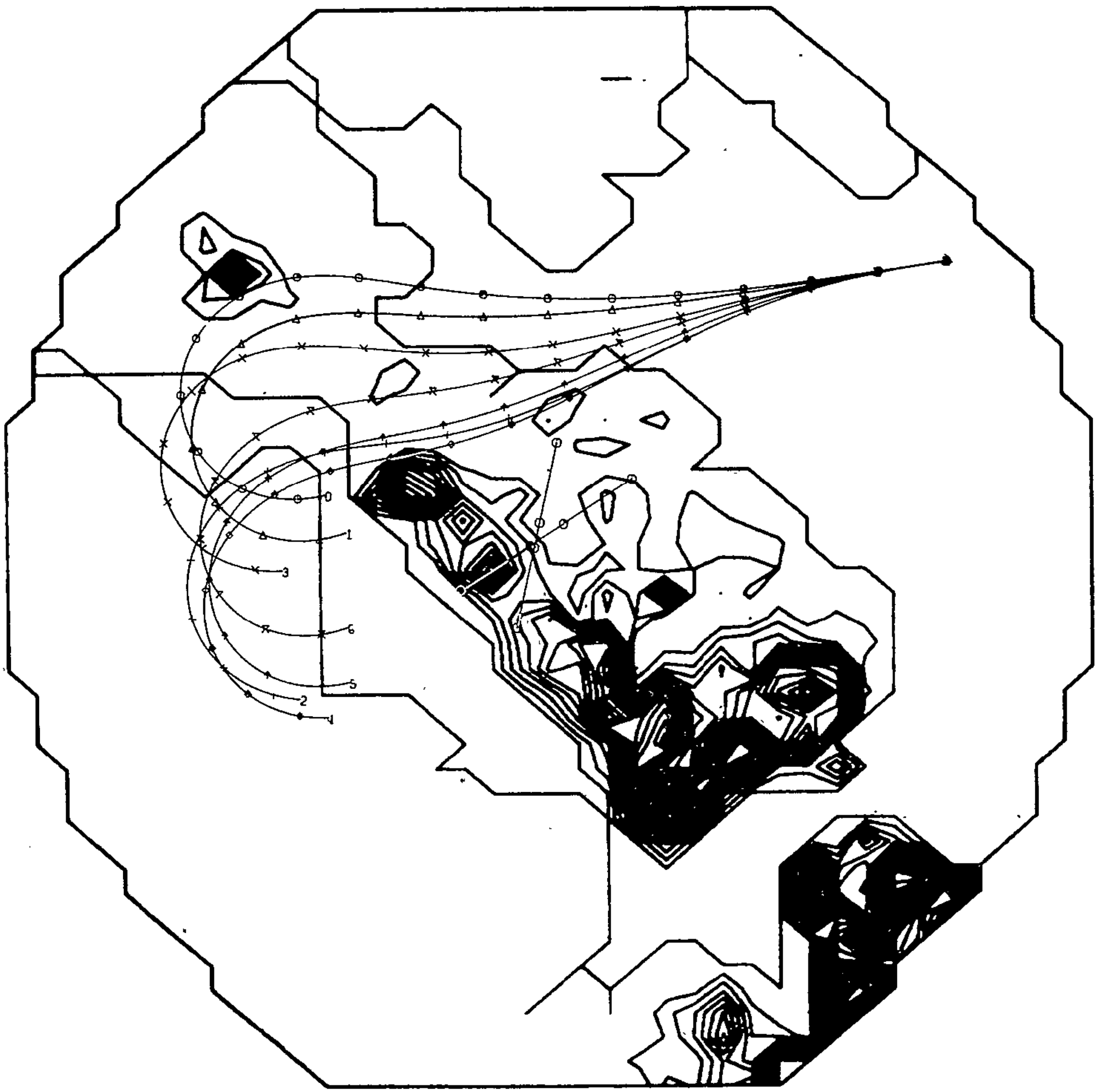

Figure 19

Several Iterations · Flight 9

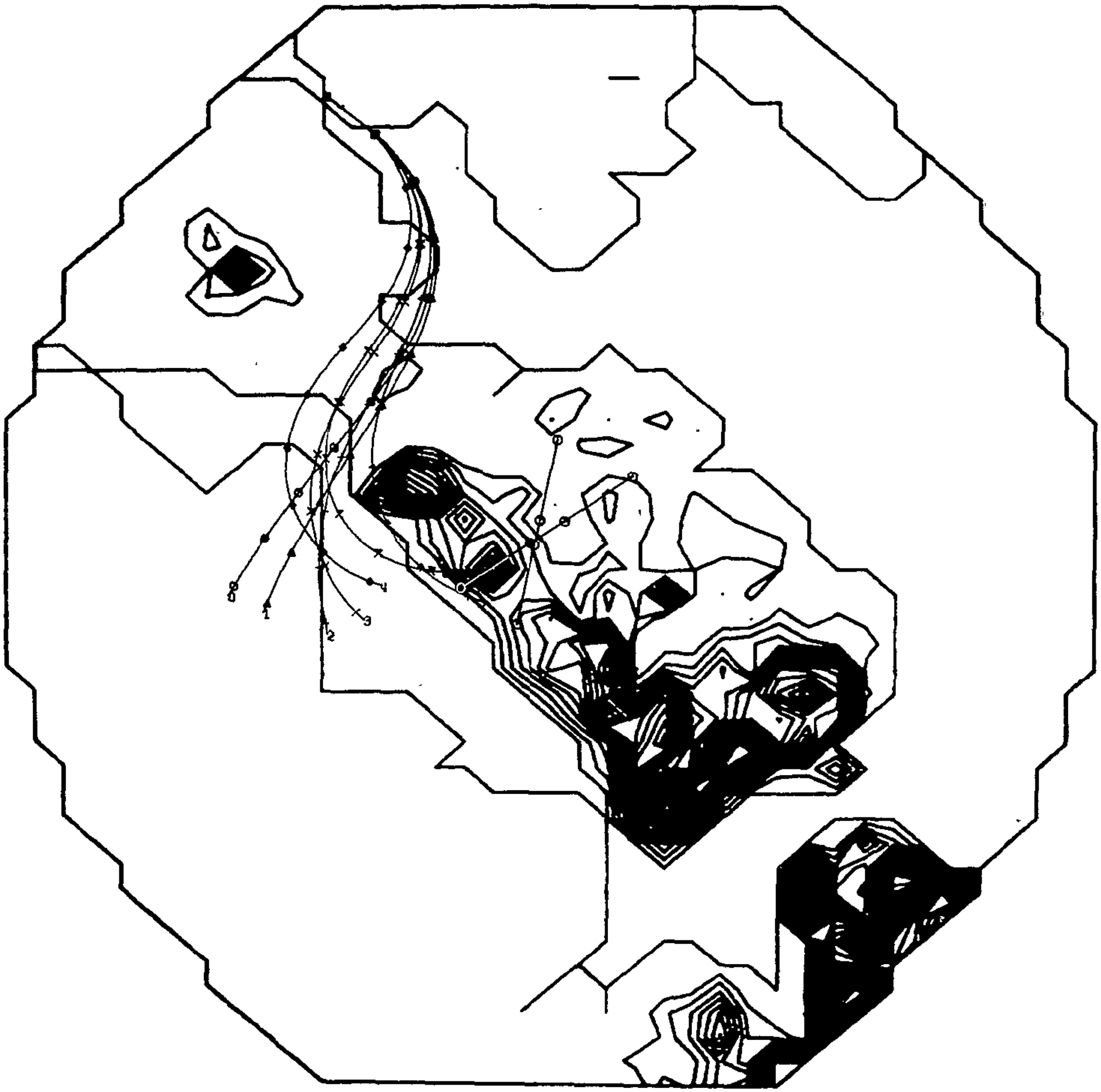

Figure 20

Several Iterations - Flight 5

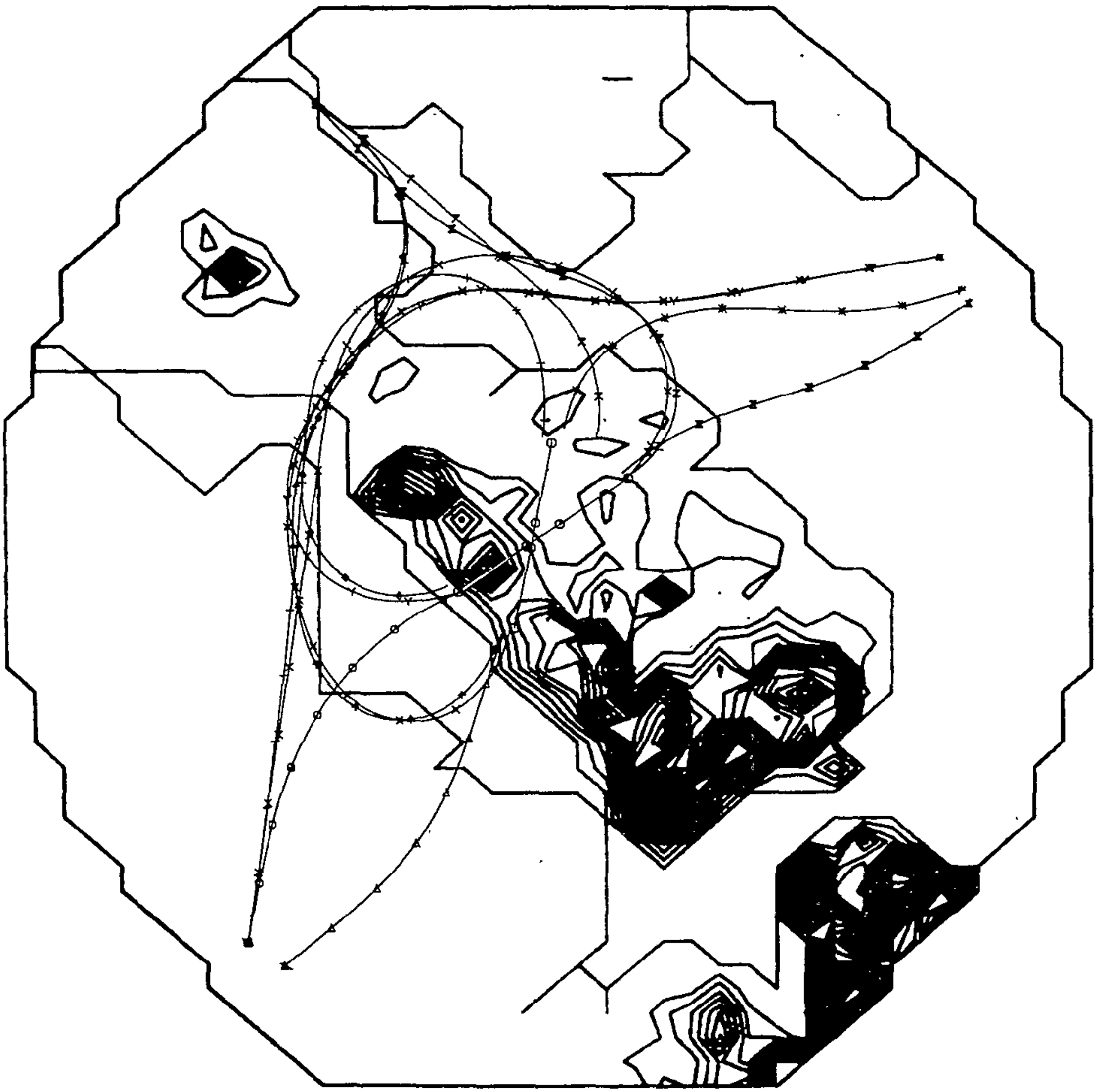

Figure 21

Composite of All 12 Optimal Flight Paths feature is that many of these 12 trajectories cross over the same low population density areas. While this makes sense, when one considers only one trajectory at a time, the cummulative effect of repeated flyovers may become significant even if the population density is low. Thus, while the trajectories which are optimal on an individual basis are important, one should probably consider the traffic schedule for a full day and simultaneously optimize the set of landings to minimize the total annoyance effect. It is expected that work in this area will be performed under the sponsorship of a different group within NASA.

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2. Barkana, A., and Cook, G., "An Aircraft Noise Pollution Model for Trajectory Optimization," IEEE Trans. on Aerospace Electron., Vol. AES-12, No. 2, Mar. 1976.

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4. Witt, R. M., "Software Model Development for a Boeing 737 Optimal Landing Trajectory Program," Master of Science Thesis, Dept. of Elec. Eng., Univ. of Virginia, May 1976.

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