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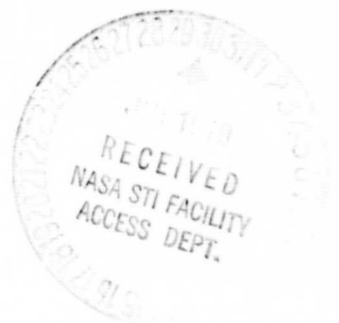
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THE MEAN FREE PATH OF LOW RIGIDITY COSMIC RAYS

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Abstract. A simple heuristic argument is presented which suggests that the large rigidity independent mean free paths observed for low rigidity solar and galactic cosmic rays can be understood in terms of weak turbulence diffusion theory if one assumes that the interplanetary magnetic turbulence consists of a combination of Alfvén waves propagating with constant field magnitude and a small ($\sim 5-10\%$) admixture of compressive fluctuations.

1. Introduction

In a recent series of papers Zwickel and Webber [1977, 1978] and Fisk [1979] have discussed what appear to be serious discrepancies between the theory of cosmic ray propagation and observations of low rigidity solar particles. In the first place, the mean free path, λ , for pitch angle scattering computed from weak turbulence theory, and the value inferred from solar particle events differ by more than an order of magnitude. In addition, at low rigidities λ is observed to be approximately constant with decreasing rigidity, P , below a few GV, while from theory one expects λ to decrease. The essence of the problem is apparent in Figure 1, taken from Fisk [1979]. The quasilinear resonant scattering theory [Jokipii, 1971] yields the solid curve denoted "theoretical upper limit." Nonlinear corrections to quasilinear theory [Völk, 1975; Goldstein, 1976; Jones et al., 1978] worsen the discrepancy because of the enhanced scattering at

pitch angles, α , near 90° (α is the particle's pitch angle defined with respect to the mean field, B_0).

The purpose of this letter is to suggest a simple physical argument which yields both the correct rigidity dependence and magnitude of λ . In the following section we briefly review those aspects of current theory leading to the prediction of very small mean free paths at low rigidities. We then argue that observations of interplanetary MHD waves indicate that the character of the magnetic turbulence has not been sufficiently well represented by current analyses. These observations suggest a simple quantitative resolution to the discrepancy.

2. Origin of the problem

The theoretical result that λ decreases with decreasing P , arises primarily from the very strong cyclotron resonance between charged particles and parallel propagating Alfvén waves (parallel being defined with respect to B_0). The inclusion of compressive MHD modes to the turbulence only enhances the wave-particle scattering because of the resultant strong Landau resonance [Goldstein et al., 1975; Lee and Völk, 1975]. Only in the case of turbulence composed of wavevectors, \mathbf{k} , propagating predominately at angles nearly 60° to the direction of B_0 will pitch angle scattering be inefficient in backscattering particles. This possibility has been developed in some detail by Morfill et al. [1976], but as far as is presently known the interplanetary turbulence consists primarily of Alfvén waves propagating significantly closer to the direction of B_0 than expected from WKB analysis [Denskat and Burlaga, 1977]. The observations are by no means exhaustive, and the suggestions of Morfill et al. [1976] may well prove important. In addition, the rigidity dependence

cannot easily be accounted for in that picture. For that reason, we feel that the basic explanation of the observed mean free paths lies elsewhere.

There is at least one area where the weak turbulence theory has proved successful. That is in the apparently excellent agreement between numerical simulations of the pitch angle scattering coefficient, D_μ ($\mu \equiv \cos \alpha$), conducted by Kaiser et al. [1978] and the predictions of the nonlinear theories [Goldstein, 1976; Jones et al., 1978]. In those comparisons the theories were carefully tailored to the simulations in that the power spectrum of magnetic turbulence was nearly identical in both treatments. In addition, the simulations were designed to minimize the effects of transients, and therefore modeled a purely steady state and diffusive situation. Similarly, because the adiabatic approximation is made in all the nonlinear theories, they also describe diffusive propagation. In a recent series of papers, Klimas et al. [1976a, b, 1977] have argued that the adiabatic approximation cannot be made, and that the nonadiabatic, or diabatic, behavior of the initial transients plays a crucial role in the long time relaxation of the particle distribution function toward isotropy. Thus a problem may exist in relating the simulations and nonlinear theories to interplanetary observations, where a steady state cannot be assumed and where the role of diabatic effects may be crucial. Unfortunately, the analysis of Klimas et al. was confined to the use of quasilinear, helical, particle orbits, which they showed to be inadequate. Because their theory has not been extended to include perturbed orbit effects the consequences of their arguments are presently unknown. With this reservation in mind, we will proceed with our discussion, taking the agreement between simulations and nonlinear theories as an indication that the theories do approximately describe charged particle propagation, and that the language and ideas of diffusion theory are appropriate for the

interplanetary problem.

When one attempts to use the correlation function employed in the simulations as an approximation to the interplanetary turbulence, the relative ease with which particles can propagate through $\alpha=90^\circ$ (which resulted in the close agreement between theory and numerical experiment) produces very small values of λ . The relationship between D_μ and the spatial diffusion coefficient, $\kappa_{||}$, from which one finds λ is given by [Hasselmann and Wibberenz, 1968; Earl, 1974]

$$\kappa_{||} = \frac{1}{2} \int_0^1 d\mu \frac{(1 - \mu^2)^2}{(2D_\mu)} \quad (1)$$

(The dimensionless notation follows Goldstein [1976 and 1977] where times are measured in gyroperiods and lengths in Larmor radii.) In the nonlinear theories, strong nonlinear perturbation of the particle trajectories produce nearly isotropic scattering, so that one could have just as well used the expression [e.g. Jokipii, 1971; Goldstein, 1977]

$$\kappa_{||} = \frac{2}{9} \left[\int_0^1 d\mu D_\mu \right]^{-1} \quad (2)$$

The physics of the scattering near $\mu=0$ has been analyzed by Völk [1975] and Jones et al. [1978], who showed it is due almost entirely to particles mirroring in the random fields. The mirroring arises even in the case of turbulence consisting of linearly or circularly polarized Alfvén waves (or the equivalent magnetostatic slab model) because of a second order change in field magnitude. This interpretation is also consistent with the scaling laws found for $D_\mu(\mu=0)$ when $\epsilon \equiv (\lambda_c/r_g) \gg 1$ (low rigidities); namely, $\epsilon D_\mu(\mu=0) \propto \eta^v$ with $v \approx 3$, approximately independent of ϵ . (λ_c is the correla-

tion length of the magnetic turbulence, and r_g is the particle's Larmor radius.) Goldstein et al. [1975] have pointed out that for $\epsilon \gg 1$ $\epsilon D_\mu(\mu=0)$ is independent of rigidity when mirroring predominates. One should note that for wave fields having the property that the total magnitude of B is a constant, the theories of Jones et al. [1978] and Völk [1975] would predict virtually no scattering through $\mu=0$. Such wave fields are, for example, characteristic of Alfvén waves observed in the solar wind [Burlaga and Turner, 1976].

Goldstein [1976] gave a different scaling at $\mu=0$ which resulted from the momentum dependent terms absent in either of the other nonlinear theories. In that case, the scaling became $\epsilon D_\mu(\mu=0) \propto \eta^\nu \epsilon$ for $\epsilon \gg 1$. Thus, there are additional physical effects tending to backscatter particles. However, at the low rigidities at which these momentum dependent terms become important, ϵ is so large that the validity of the weak turbulence theory itself is questionable [Goldstein, 1977].

3. The mean free path at low rigidities

Our proposed explanation for the observed behavior of low rigidity cosmic rays is based on our contention that the models of interplanetary turbulence thus far employed in simulations and theory are inaccurate in at least one very important respect.

Observations of interplanetary MHD waves by Burlaga and Turner [1976] have shown that the root mean square value of the fluctuation in $|B|$ is about 0.06 $|B|$ (or $\eta \approx 0.06$). In such a wave field, the amount of second order mirroring present and capable of scattering particles through $\mu=0$ is very limited. Thus we can conclude that both the extant numerical simulations and the theoretical analyses greatly overestimate the amount of

backscattering in the solar wind because all these models use correlation functions which do not preserve field magnitude in second order. This brings us to our basic hypothesis, whose implications we develop below: the interplanetary Alfvénic wave turbulence conserves $|B|$ and that residual fluctuations in $|B|$ are small, typically 5-10%. Obviously, in the vicinity of fast and slow stream interaction regions where $|B|$ changes, this requires modification. There the value of η for compressive modes probably exceeds 0.1 and particles encountering these interaction regions should be easily scattered [cf. Barouch and Burlaga, 1976] which seems consistent with recent observations of Gold and Roelof [1979].

If the Alfvénic turbulence conserves $|B|$, then in the absence of any other wave modes there should be greatly inhibited propagation through $\mu=0$. (Several years ago Sari, in an unpublished preprint discussed a similar suggestion.) In fact, $D_\mu(\mu=0)$ should be approximately given by the quasilinear formulas, and the strong, nearly isotropic scattering seen in the current simulations should be absent. If one now takes into account Denskat and Burlaga's [1977] observation that the angle, θ , between \mathbf{k} and \mathbf{B}_0 is in general small but nonzero, then from the well known properties of quasilinear theory we conclude that with $\theta \neq 0$ $\lambda \rightarrow \infty$ for interplanetary Alfvénic turbulence [see Völk, 1975 and Jones et al., 1978]. Recently, Klimas et al. [1976] have claimed that even for $\theta=0$ the quasilinear (adiabatic) theory does not allow particle propagation through $\mu=0$. Furthermore, when the adiabatic approximation is not made, Klimas et al. [1976a,b, 1977] showed that particle propagation through $\mu=0$ is greatly inhibited, with a logarithmic relaxation of the particle distribution function toward isotropy.

Thus we are led to the following ansatz: In the presence of magnetic turbulence consisting only of Alfvénic fluctuations conserving field

magnitude, particle propagation through $\mu=0$ is essentially zero. We now proceed to explore the consequences of this ansatz, leaving for future work the task of verifying it through more formal analysis. We go on to consider the residual effect of the 6% change in $|B|$ in determining both the magnitude and rigidity dependence of λ .

In the magnetostatic approximation, fluctuations in $|B|$ give rise to a Landau resonance of the form $D_\mu(\mu=0) \propto \delta(\mu)$ [Fisk et al., 1974; Goldstein et al., 1975; for a generalization to finite frequencies see Lee and Völk, 1975]. Völk [1975] has argued that the effect of nonlinear perturbations is to spread the Landau resonance over a finite region of μ -space while preserving its integrated weight. In that case, because D_μ (compressive modes) is maximum at $\mu=0$, and non-zero within the range $|\mu| \lesssim \pm 0.3$, equation (2) can be used to calculate $\kappa_{||}$. This has been done by Klimas and Sandri [1973]. Assuming $\eta=0.1$, we find that at low rigidities λ is independent of rigidity with a value of approximately 0.3 AU (cf. Fig. 3 of Klimas and Sandri [1973]). Note that Klimas and Sandri's calculation included the effects of some scattering by Alfvén waves in that they used a correlation function typical of isotropic turbulence. However, the Landau resonance is so efficient in scattering particles near $\mu=0$ that the cyclotron resonances can be completely ignored. This statement can be strengthened further by noting that because the observed Alfvén waves do not propagate precisely along B , the efficiency of the cyclotron resonance in even bringing particles close to $\mu=0$ is further reduced.

At high rigidities, $\epsilon \ll 1$, resonant scattering will again become important and the standard results should obtain using the full rms value of $\eta=0.3$ with $\lambda_c \approx 2 \times 10^{10}$ cm [Fisk and Sari, 1973]. The composite theoretical curve is shown in Figure 1.

4. Discussion and conclusions

The essential ideas of this letter can now be summarized. Alfvénic or slab turbulence conserving $|B|$ will not appreciably backscatter low rigidity particles. The residual 5-10% fluctuations in $|B|$ give rise to a value for λ which has both the correct magnitude and rigidity dependence. These ideas are amenable to test by two techniques. Numerical simulations can be constructed in which the turbulence conserves $|B|$ [Owens, private communication]. Furthermore, the nonlinear theories of Völk [1975], Goldstein [1976] and Jones et al. [1978] can be modified to use correlation functions subject to that same constraint.

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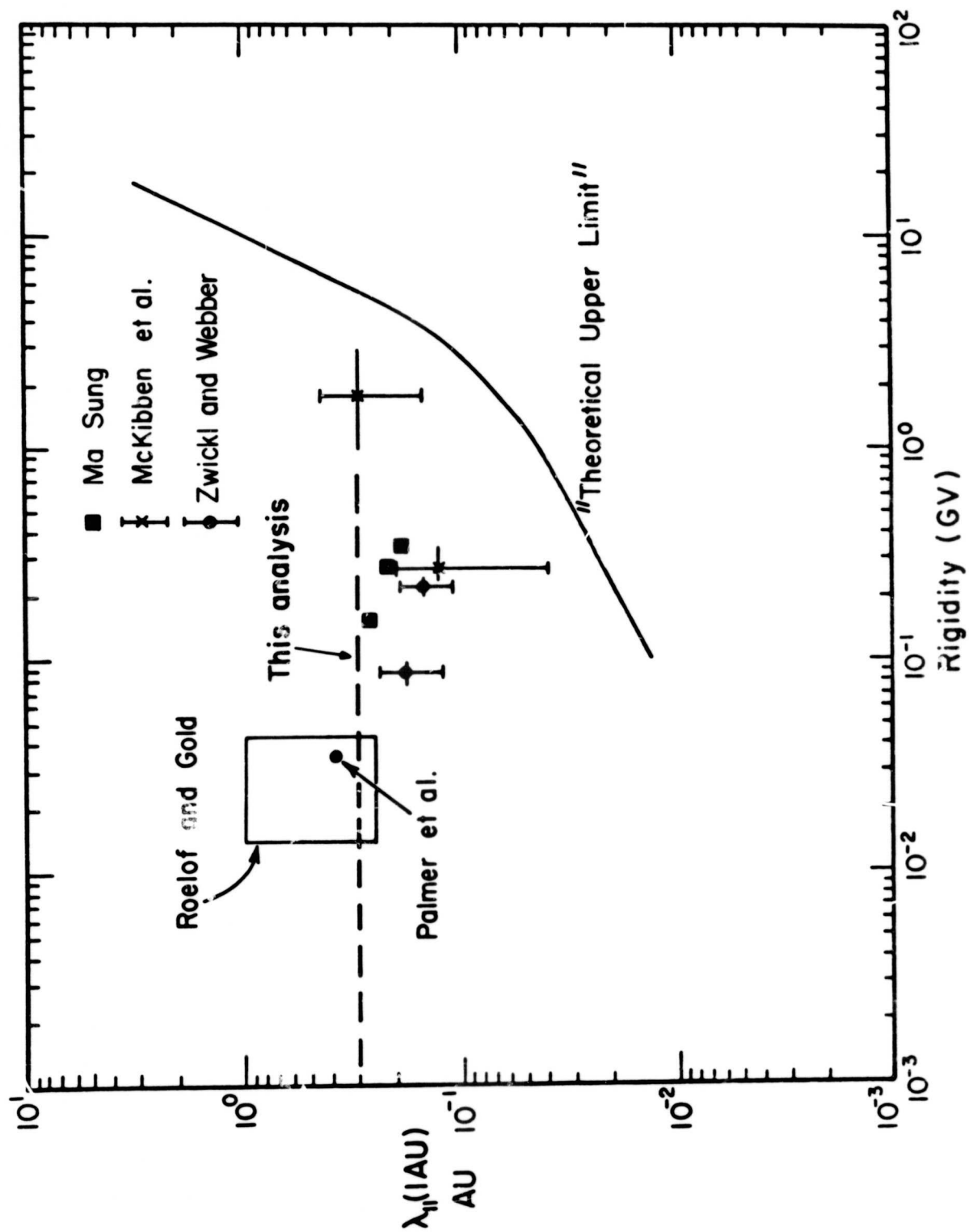
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Figure Caption

Representative values of $\lambda_{||}$ deduced from solar cosmic ray observations. The solid line labeled "Theoretical upper limit" is from weak turbulence calculations using equation (1) for $\kappa_{||}$ and correlation functions previously thought to model interplanetary magnetic turbulence. The dashed curve is our estimate of the residual scattering assuming that the Alfvénic turbulence conserves $|B|$ and all scattering results from the residual compressive fluctuations (from Zwickel and Webber [1977] and Fisk [1979]).



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