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TECHNICAL MEMORANDUM

FURTHER EVALUATION OF PROCEDURE 1 SECONDARY ERROR ANALYSIS

By

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1. INTRODUCTION

At the close of the Phase III crop year of the Large Area Crop Inventory Experiment (LACIE), several investigations were outlined in support of the Classification and Mensuration Subsystem. The goal of the secondary error analysis plan was to evaluate as many of the error sources as possible in the Procedure 1 (P1) small grains' estimate for 5- by 6-nautical-mile segments in the U.S. Great Plains. An evaluation of analyst labeling errors on type 1 and 2 dots was completed on a total of 25 test segments.[1] However, because of the scope of the test, more analyses were required to provide an understanding of the results. The further evaluation of P1 error analysis was defined to include the following studies:

1. A comparison of the classification results obtained in the P1 secondary error analysis study to the P1 study by the International Business Machines Company (IBM).[2]

2. A re-evaluation of the classification results based on three criteria: (a) winter- vs. spring-grain segments, (b) computation of the probability of correct classification (PCC) for small grains only, and (c) the use of a signed difference between the proportion estimates and ground-truth (GT) estimates.

3. The computation of the variance of the estimate and the corresponding reduction coefficient.

These three evaluations are considered in this document, and the results for each evaluation are presented.

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[1] The test segments are the following: 1005, 1032, 1033, 1853, 1861 (Kansas); 1512, 1520 (Minnesota); 1544, 1739 (Montana); 1582 (Nebraska); 1604, 1606, 1648, 1661, 1902 (North Dakota); 1231, 1242, 1367 (Oklahoma); 1677, 1690, 1803, 1805 (South Dakota); and 1056, 1059, 1060 (Texas).

[2] IBM memorandum from S. G. Wheeler to R. P. Heydorn, dated June 20, 1977, re "Procedure 1 Evaluation Experiment with Ground-Truth Labeling."

2. COMPARISON OF RESULTS TO IBM P1 STUDY

The IBM study, performed before the delivery of LACIE software (version 6) on the Earth Resources Interactive Processing System, was aimed at determining the best set of parameters for use in P1. Several parameter sets and cluster labeling procedures were tested, including the nearest-neighbor cluster parameter set and the cluster labeling technique now used in P1. Because of the wide scope of the IBM study, only one table of results was directly comparable to that of the secondary error analysis. Table 1 presents the IBM results of average differences between the estimated and the GT wheat proportions. The IBM study used GT-labeled picture elements (pixels) from field centers and nearest-neighbor cluster parameters to classify seven test segments. The seven test segments used were 1033, 1961, 1988, 1865, 1178, 1046, and 1978. These seven segments were reclassified using varying numbers of channels: 8, 12, and 16. The average differences found in table 1 indicate the amount of bias introduced by the P1 classifier trained using GT-labeled samples. The pixels were allocated by two methods; random and stratified, both from a dot-grid laid over the image. This was compared to average differences between the estimated and the GT proportions for the two treatments (table 2), which used GT labeling in the secondary error analysis study; i.e., the random dot-grid and the uniform dot-grid treatments.

To test for method differences between the IBM study and the secondary error analysis study, an analysis of variance was performed on the data presented in tables 1 and 2 using a split plot design of the following form:

$$y_{\text{ave difference}} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + (\beta\gamma)_{jk} + \varepsilon_{ijk}$$

where

α_i, i = 1, 2, 3, represents channels 8, 12, and 16.

β_j, j = 1, 2, represents methods IBM and secondary error analysis.

γ_k, k = 1, 2, represents random (R) and uniform (U) treatments.

μ = overall mean.

TABLE 1.— IBM RESULTS OF AVERAGE DIFFERENCES BETWEEN ESTIMATED AND GT WHEAT PROPORTIONS

No. of channels	No. of segments	Treatments	
		Random dot allocation from a grid	Stratified dot allocation from a grid
8	7	-0.187	-0.012
12	7	2.032	1.535
16	7	-0.497	-1.064
Average difference .		0.449	0.153

TABLE 2.— SECONDARY ERROR ANALYSIS — AVERAGE DIFFERENCES BETWEEN ESTIMATED AND GT SMALL-GRAIN PROPORTIONS

No. of channels	No. of segments	Treatments	
		Random	Uniform
8	9	-1.489	-2.044
12	6	-1.050	1.450
16	12	-3.508	1.658
Average difference . .		-2.016	0.355

Results presented in table 3 showed no significant differences for any of the effects. However, due to the limited amount of data, the power of this analysis of variance is low.

TABLE 3.— ANALYSIS OF VARIANCE — AVERAGE DIFFERENCES

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F-value†
Channels	2	9.48438	4.74219	7.16
Methods	1	3.84201	3.84201	5.24
Error (a)*	2	1.46523	.73261	
Treatment	1	3.22611	3.22611	1.54
Treatment by methods	1	5.33333	5.33333	2.55
Error (b)**	4	8.36291	2.09073	
Total	11	31.71397	2.88309	

†No significant differences between treatments at the 5-percent level were noted.
*Coefficient of variation for error (a) = 323 percent.
**Coefficient of variation for error (b) = 546 percent.

3. RE-EVALUATION OF SECONDARY ERROR ANALYSIS

The secondary error analysis experiment (reported on in ref. 1) presented the PCC calculation for type 1 and 2 dots, denoted PCC1 and PCC2, respectively, for three different treatments: GT labeling of a random dot grid (R), GT labeling of a uniform dot grid (U), and analyst-interpreter (AI) labeling of a random dot grid. The proportion estimate for each classification was compared to the GT estimate by computation of the absolute value of the difference, Δ_i, where i varied over treatments. The data set consisted of 25 Phase III blind sites in the U.S. Great Plains. (A complete description of the data set and the experiment is given in ref. 1.) The re-evaluation of this experiment required that the PCC be computed for small grains for type 1 and 2 dots; i.e., PCG1 and PCG2, respectively. After the segments were divided into winter- and spring-grain segments, the proportion differences were calculated using the signed difference between the GT and treatment estimates (Δ_i') to show the amount and direction of bias.[3] Each of these response variables, PCG1, PCG2, and Δ_i', underwent an analysis-of-variance test. A Newman-Keuls multiple compairson test (ref. 3) was planned in the event that any of these analysis-of-variance tests indicated significant treatment differences.

Table 4 presents the number of dots labeled for each treatment. Table 5 presents the PCG1 values for each of the three treatments. Three missing PCG1 values for the AI treatment were estimated from the block means by using the method of least squares. Since these data are presented in terms of percentages ranging in value from 0 to 100, these data were transformed using the arc sine of the square root of the raw data (arc sin $\sqrt{\text{PCG1}}$). The transformed PCG1 data are presented in table 6. The results of the analysis

[3]Since the Phase III GT did not identify barley, rye, and oats as winter or spring grains, the GT proportions were established by econometric models of the categories (ref. 2).

TABLE 4.— NUMBER OF DOTS LABELED FOR EACH TREATMENT

Segment	State	AI- and GT-labeled random grids		GT-labeled uniform grids	
		No. of type 1 dots	No. of type 2 dots	No. of type 1 dots	No. of type 2 dots
1005	Colo.	39	60	43	60
1032	Kans.	39	60	49	60
1033	Kans.	50	60	50	60
1053	Kans.	31	60	44	60
1061	Kans.	45	60	47	60
1512	Minn.	46	60	47	59
1520	Minn.	31	60	41	60
1544	Mont.	35	60	45	60
1739	Mont.	43	60	43	60
1582	Neb.	45	60	47	59
1604	N. Dak.	41	60	42	60
1606	N. Dak.	47	47	30	40
1640	N. Dak.	43	60	42	58
1661	N. Dak.	32	54	36	50
1902	N. Dak.	50	60	45	59
1231	Okla.	36	60	42	59
1242	Okla.	32	50	32	53
1367	Okla.	40	60	48	59
1677	S. Dak.	42	53	45	52
1690	S. Dak.	40	60	48	60
1803	S. Dak.	49	60	47	58
1805	S. Dak.	50	99	32	40
1056	Tex.	40	60	48	60
1059	Tex.	49	60	50	60
1060	Tex.	46	60	48	60

TABLE 5.– PCG1 DATA

Segment	State	Treatment		
		Random	Uniform	AI
1005	Colo.	66.7	81.3	60.0
1032	Kans.	82.4	93.3	72.7
1033	Kans.	20.0	33.3	0.0
1853	Kans.	83.3	80.0	77.8
1861	Kans.	83.3	94.4	[a]65.5
1512	Minn.	68.8	57.1	62.5
1520	Minn.	100.0	61.5	[a]70.0
1544	Mont.	80.0	71.4	66.7
1739	Mont.	77.8	71.4	[a]73.5
1582	Neb.	88.9	100.0	88.9
1504	N. Dak.	94.1	76.0	33.3
1506	N. Dak.	91.3	100.0	58.3
1648	N. Dak.	75.0	70.0	66.7
1661	N. Dak.	66.7	90.9	60.0
1902	N. Dak.	33.3	100.0	0.0
1231	Okla.	100.0	100.0	95.8
1242	Okla.	100.0	100.0	90.0
1367	Okla.	91.7	72.0	64.7
1677	S. Dak.	72.7	66.7	72.7
1690	S. Dak.	100.0	66.7	50.0
1803	S. Dak.	100.0	100.0	50.0
1805	S. Dak.	72.7	62.5	30.0
1056	Tex.	100.0	83.3	57.1
1059	Tex.	78.3	85.0	70.8
1060	Tex.	87.5	88.9	53.8

[a]Estimate for missing value.

TABLE 6.— TRANSFORMED PCG1 DATA

Segment	State	Treatment		
		Random	Uniform	AI
1005	Colo.	54.76	64.38	50.77
1032	Kans.	65.20	75.00	58.50
1033	Kans.	26.56	35.24	0.0
1853	Kans.	65.88	63.44	61.89
1861	Kans.	65.88	76.31	53.91
1512	Minn.	56.04	49.08	52.24
1520	Minn.	90.00	51.65	56.79
1544	Mont.	63.44	57.67	54.71
1739	Mont.	61.89	57.67	59.02
1582	Neb.	70.54	90.00	70.54
1604	N. Dak.	75.94	60.67	35.24
1606	N. Dak.	72.34	90.00	49.78
1648	N. Dak.	60.00	56.79	54.76
1661	N. Dak.	54.76	72.44	50.77
1902	N. Dak.	35.24	90.00	0.0
1231	Okla.	90.00	90.00	78.17
1242	Okla.	90.00	90.00	71.56
1367	Okla.	73.26	58.05	53.55
1677	S. Dak.	58.50	54.76	58.50
1690	S. Dak.	90.00	54.76	45.00
1803	S. Dak.	90.00	90.00	45.00
1805	S. Dak.	58.50	52.24	33.21
1056	Tex.	90.00	65.88	49.08
1059	Tex.	62.24	67.21	57.29
1060	Tex.	69.30	70.54	47.18

of variance performed on the PCG1 transformed data are given in table 7. The state-by-treatment means of the PCG1 values were computed using the untransformed data and excluding the segment treatments for which a missing value was computed. These means are presented in table 8. The Newman-Keuls multiple comparison test of the table 7 analysis of variance appears in table 9.

In the analysis of variance, an α-level of 0.05 was used to perform the F-tests. A significant difference was found in treatment effects. The Newman-Keuls test showed the AI-labeled treatment to be significantly different from either of the GT-labeled treatments. However, the GT-labeled treatments were not significantly different from each other.

Table 10 presents the PCG2 values for each of the three treatments. The three missing PCG2 values for the AI treatment were estimated in the same manner as for the PCG1 values. Again, the data were transformed using the arc sine of the square root of the raw data; the data are given in table 11. The results of the analysis of variance performed on the transformed data appear in table 12. The state-by-treatment means computed in the same manner as the PCG1 values appear in table 13, and the treatment means of the Newman-Keuls are ranked in table 14.

An α-level of 0.05 was again used to perform the F-tests with significant differences being found for the treatment effect and the segment within-state effect. The Newman-Keuls test ranked the AI-labeled treatment with the GT-labeled uniform-dot treatment. These two treatments were significantly different from the GT-labeled random-dot treatment.

One can conclude from these analyses that the GT labeling should improve the probability of correctly classifying small grains. However, from the varying results of the Newman-Keuls tests, it is unclear which dot grid is preferable for the labeling. The random dot grid was consistently the highest ranked according to treatment means; but for type 1 dots, it did not test as significantly different from the uniform dot grid.

TABLE 7.— ANALYSIS OF VARIANCE FOR PCG1[a]

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-value
Total	[b]71	25 469.91	358.73	
States	8	4 089.49	511.19	[c]2.91
Segment within state	16	8 363.35	522.71	[c]2.98
Treatment	2	5 159.30	2 579.65	[d]14.69
Treatment by state	16	2 765.37	172.84	[c]<1
Error[e]	[b]29	5 092.40	175.60	

[a]The analysis was based on transformed data using arc sin $\sqrt{PCG1}$.

[b]Three missing values were estimated and used in the analysis.

[c]No significant differences between means at the 5-percent level were noted.

[d]Differences between means are significant at the 1-percent level.

[e]Coefficient of variation for this error = 21.5 percent.

TABLE 8.— STATE-BY-TREATMENT MEANS

State	No. of segments	Treatment			State average
		Random	Uniform	AI[a]	
Colo.	1	66.7	81.3	60.0	69.3
Kans.	4	67.2	75.2	50.2[b]	65.5
Minn.	2	84.4	59.3	62.5[c]	70.0
Mont.	2	78.9	71.4	66.7[d]	73.5
Neb.	1	88.9	100.0	88.9	92.6
N. Dak.	5	72.1	87.4	43.7	67.7
Okla.	3	97.2	90.7	83.5	90.5
S. Dak.	4	86.4	74.0	50.7	70.3
Tex.	3	88.6	85.7	60.6	78.3
Treatment average		80.6	80.2	60.0	73.6

[a]The AI treatment averages for each state did not include the segments for which a missing value was calculated.

[b]Three segments were used for Kansas.

[c]One segment was used for Minnesota.

[d]One segment was used for Montana.

TABLE 9.— NEWMAN-KEULS TEST OF PCG1[a]

Treatment	Mean
AI	60.0
Uniform	80.2
Random	80.6

[a]The W constants are derived as follows:

$W_2 = (2.89)(2.65) = 7.66$

$W_3 = (3.49)(2.65) = 9.25$

TABLE 10.– PCG2 DATA

Segment	State	Treatment		
		Random	Uniform	AI
1005	Colo.	71.4	62.1	30.4
1032	Kans.	70.8	62.5	73.9
1033	Kans.	50.0	0.0	0.0
1853	Kans.	100.0	65.2	78.6
1861	Kans.	12.5	66.7	[a]52.7
1512	Minn.	18.2	29.4	57.1
1520	Minn.	54.2	94.1	[a]50.6
1544	Mont.	91.7	45.8	61.7
1739	Mont.	69.2	39.1	[a]61.5
1582	Neb.	85.7	83.3	75.0
1604	N. Dak.	82.1	53.1	41.9
1606	N. Dak.	67.9	77.3	56.0
1648	N. Dak.	50.0	56.3	33.3
1661	N. Dak.	82.4	37.5	59.1
1902	N. Dak.	0.0	0.0	0.0
1231	Okla.	90.2	100.0	97.9
1242	Okla.	95.8	75.9	76.7
1367	Okla.	67.6	61.8	82.8
1677	S. Dak.	57.1	36.8	46.7
1690	S. Dak.	72.7	70.6	45.5
1803	S. Dak.	100.0	50.0	50.0
1805	S. Dak.	100.0	75.0	21.4
1056	Tex.	30.8	77.8	50.0
1059	Tex.	78.1	78.6	80.0
1060	Tex.	100.0	61.5	47.1

[a]Estimate for missing value.

TABLE 11.— TRANSFORMED PCG2 DATA

Segment	State	Treatment		
		Random	Uniform	AI
1005	Colo.	57.67	52.00	33.46
1032	Kans.	57.29	52.24	59.28
1033	Kans.	45.00	0.0	0.0
1853	Kans.	90.00	53.85	62.44
1861	Kans.	20.70	54.76	46.55
1512	Minn.	25.25	32.83	49.08
1520	Minn.	47.41	75.94	45.34
1544	Mont.	73.26	42.59	51.77
1739	Mont.	56.29	38.70	51.65
1582	Neb.	67.78	65.88	60.0
1604	N. Dak.	64.97	46.78	40.34
1606	N. Dak.	55.49	61.55	48.45
1648	N. Dak.	45.00	48.62	35.18
1661	N. Dak.	65.20	37.76	50.24
1902	N. Dak.	0.0	0.0	0.0
1231	Okla.	71.76	90.0	81.67
1242	Okla.	78.70	60.60	61.14
1367	Okla.	55.30	51.83	65.50
1677	S. Dak.	49.08	37.35	43.11
1690	S. Dak.	58.50	57.17	42.42
1803	S. Dak.	90.0	45.0	45.0
1805	S. Dak.	90.0	60.0	27.56
1056	Tex.	33.71	61.89	45.0
1059	Tex.	62.10	62.44	63.44
1060	Tex.	90.0	51.65	43.34

TABLE 12.— ANALYSIS OF VARIANCE FOR PCG2[a]

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-value
Total	[b]71	30 930.86	435.65	
States	8	6 151.91	768.99	[c]3.30
Segment within state	16	13 623.90	851.50	[d]3.65
Treatment	2	1 877.34	938.67	[d]4.02
Treatment by state	16	2 511.62	156.98	[c]<1
Error[e]	[b]29	6 766.09	233.31	

[a]The analysis was based on transformed data using arc sin $\sqrt{PCG2}$.

[b]Three missing values were estimated and used in the analysis.

[c]No significant differences between means at the 5-percent level were noted.

[d]Differences between means are significant at the 5-percent level.

[e]Coefficient of variation for this error = 29.8 percent.

TABLE 13.— STATE-BY-TREATMENT MEANS FOR PCG2

State	No. of segments	Treatment			State average
		Random	Uniform	AI[a]	
Colo.	1	71.4	62.1	30.4	54.6
Kans.	4	58.3	48.6	50.8[b]	52.7
Minn.	2	36.2	61.8	57.1[c]	50.6
Mont.	2	80.4	42.4	61.7[d]	61.5
Neb.	1	85.7	83.3	75.0	81.3
N. Dak.	5	56.5	44.8	38.1	46.5
Okla.	3	84.5	79.2	85.8	83.2
S. Dak.	4	82.4	58.1	40.9	60.5
Tex.	3	69.6	72.6	59.0	67.1
Treatment average		67.9	58.4	53.0	57.7

[a]The AI treatment averages for each state did not include the segments for which a missing value was calculated.

[b]Three segments were used for Kansas.

[c]One segment was used for Minnesota.

[d]One segment was used for Montana.

TABLE 14.– NEWMAN-KEULS
Test of PCG2[a]

Treatment	Means
AI	53.0
Uniform	58.4
Random	67.9

[a]The W constants are derived as follows:

$W_2 = (3.05)(2.89) = 8.8$
$W_3 = (3.05)(3.49) = 10.6$

TABLE 15.— SMALL-GRAIN PROPORTION ESTIMATES

Segment	State	GT label		AI label	GT
		Random	Uniform		
1005	Colo.	38	48	20	34.7
1032	Kans.	37	40	23	38.6
1033	Kans.	9	13	2	9.5
1853	Kans.	35	35	26	30.3
1861	Kans.	6	25	31	[a]35.3
1512	Minn.	16	28	31	33.7
1520	Minn.	22	22	21	30.0
1544	Mont.	60	40	43	38.3
[b]1739	Mont.	21 12	21 16	10 10	25.4 3.0
1582	Neb.	16	14	18	19.4
1604	N. Dak.	53	54	35	52.4
1606	N. Dak.	25	33	19	32.9
1648	N. Dak.	33	26	36	37.9
1661	N. Dak.	37	35	33	41.0
1902	N. Dak.	11	8	7	8.6
1231	Okla.	72	74	76	74.1
1242	Okla.	50	50	51	47.2
1367	Okla.	62	58	36	54.0
1677	S. Dak.	28	40	24	34.1
1690	S. Dak.	18	26	9	21.3
1803	S. Dak.	2	3	2	1.1
[b]1805	S. Dak.	1 16	0 19	0 12	1.2 14.6
1056	Tex.	17	26	32	22.6
1059	Tex.	38	43	38	44.5
1060	Tex.	20	22	17	23.1

[a]Based on a 400-dot estimate.

[b]The first estimates are for winter wheat; the second for spring wheat.

The small-grain proportion estimates and the GT estimates are presented in table 15. For one segment, a 400-dot count estimate was used in lieu of the GT estimate because of incomplete GT coverage. These proportions were transformed using the arc sine of the square root of each proportion estimate. The transformed data for winter grain segments are listed in table 16 and for spring grain segments in table 17. To analyze these data, the AI subtracted the transformed GT estimates from the transformed proportion estimates with the differences denoted as $\Delta'R$, $\Delta'U$, and $\Delta'AI$. In each case, if the Δ'-value is positive, it indicates an overestimate of the GT for that particular procedure. Tables 18 and 19 present the differences for the winter grain estimates and the spring grain estimates, respectively. The results of the analyses of variance performed on the difference tables appear in tables 20 and 21.

Using an α-level of 0.05, no significant differences were found for any of the effects for the winter grain proportion differences. Because the winter grain areas consist of relatively large field sizes, the estimates were expected to be fairly close to the GT values. Thus, no statistical significances were expected from this analysis of variance. However, for the spring grain proportion differences, significant results were found for all effects tested: state, segment within state, treatment, and state by treatment.

These significant results can be attributed to several problem areas that have been associated with spring grain estimation in previous phases of LACIE, such as strip fields, confusion crops, and adverse weather conditions. Tables of state-by-treatment means are presented for winter and spring grain segments in tables 22 and 23, respectively. Because the state-by-treatment interaction is statistically significant for spring grains, the comparisons are based on the state-by-treatment means.

The least significant difference (LSD) values for comparing any two treatment means of the same state were computed (presented in table 23). Results indicate that for Minnesota and Montana, the uniform and AI treatments were significantly better than the random treatment but were not significantly different from each other. For North Dakota and South Dakota, the random and

TABLE 16.— TRANSFORMED WINTER GRAIN PROPORTION ESTIMATES

Segment	State	Labeling procedure			GT
		Random	Uniform	AI	
1005	Colo.	38.06	43.85	26.56	36.09
1032	Kans.	37.47	39.23	28.66	38.41
1033	Kans.	17.46	21.13	8.13	17.95
1853	Kans.	36.27	36.27	30.66	33.40
1861	Kans.	14.18	30.00	33.83	36.45
1739	Mont.	27.28	27.28	18.44	30.26
1582	Neb.	23.58	21.97	25.10	26.13
1231	Okla.	58.05	59.34	60.67	59.41
1242	Okla.	45.00	45.00	45.57	43.39
1367	Okla.	51.94	49.60	36.87	47.29
1803	S. Dak.	8.13	9.98	8.13	6.02
1805	S. Dak.	5.74	0.00	0.00	6.29
1056	Tex.	24.35	30.66	34.45	28.38
1059	Tex.	38.06	40.98	38.06	41.84
1060	Tex.	26.56	27.97	24.35	28.73

TABLE 17.— TRANSFORMED SPRING GRAIN PROPORTION ESTIMATES

Segment	State	Labeling procedure			GT
		Random	Uniform	AI	
1512	Minn.	23.58	31.95	33.83	35.49
1520	Minn.	27.97	27.97	27.28	33.21
1544	Mont.	50.77	39.23	40.98	38.23
1739	Mont.	20.27	23.58	18.44	9.98
1604	N. Dak.	46.72	47.29	36.27	47.52
1606	N. Dak.	30.00	35.06	25.84	35.00
1648	N. Dak.	35.06	30.66	36.87	38.00
1661	N. Dak.	37.47	36.27	35.06	40.34
1902	N. Dak.	19.37	16.43	15.34	17.05
1677	S. Dak.	31.95	39.23	29.33	35.73
1690	S. Dak.	25.10	30.66	17.46	27.49
1805	S. Dak.	23.58	25.84	20.27	22.46

TABLE 18.— DIFFERENCES BETWEEN WINTER GRAIN PROPORTION ESTIMATES AND GT

Segment	State	Labeling procedure		
		Δ′R	Δ′U	Δ′AI
1005	Colo.	1.97	7.76	-9.53
1032	Kans.	-0.94	0.82	-9.75
1033	Kans.	-0.49	3.18	-9.82
1853	Kans.	2.87	2.87	-2.74
1861	Kans.	-22.27	-6.45	-2.62
1739	Mont.	-2.98	-2.98	-11.82
1582	Neb.	-2.55	-4.16	-1.03
1231	Okla.	-1.36	-0.07	1.26
1242	Okla.	1.61	1.61	2.18
1367	Okla.	4.65	2.31	-10.42
1803	S. Dak.	2.11	3.96	2.11
1805	S. Dak.	-0.55	-6.29	-6.29
1056	Tex.	-4.03	2.28	6.07
1059	Tex.	-3.78	-0.86	-3.78
1060	Tex.	-2.17	-0.76	-4.38

Δ′ = estimate - GT.

TABLE 19.— DIFFERENCES BETWEEN TRANSFORMED SPRING GRAIN PROPORTION ESTIMATES AND GT

Segment	State	Labeling procedure		
		Δ´R	Δ´U	Δ´Ai
1512	Minn.	-11.91	-3.54	-1.66
1520	Minn.	-5.24	-5.24	-5.93
1544	Mont.	12.54	1.00	2.75
1739	Mont.	10.29	13.60	8.46
1604	N. Dak.	-0.80	-0.23	-11.25
1606	N. Dak.	-5.00	0.06	-9.16
1648	N. Dak.	-2.94	-7.34	-1.13
1661	N. Dak.	-2.87	-4.07	-5.28
1902	N. Dak.	2.32	-.62	-1.71
1677	S. Dak.	-3.78	3.50	-6.40
1690	S. Dak.	-2.39	3.17	-10.03
1805	S. Dak.	1.12	3.38	-2.19

Δ´ = estima'e - GT.

TABLE 20.— ANALYSIS OF VARIANCE FOR WINTER GRAIN PROPORTION DIFFERENCES

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-value†
State	6	154.07	25.68	0.86
Segment within state	8	330.76	41.35	1.39
Treatment	2	135.62	67.81	2.28
State by treatment	12	216.64	18.05	0.61
Error*	16	475.48	29.72	
Total	44	1[illegible]2.57		

†The proportion differences are not significantly different at the 5-percent level.

*Coefficient of variation for error = 2.88 percent.

TABLE 21.— ANALYSIS OF VARIANCE FOR SPRING GRAIN PROPORTION DIFFERENCES

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-value†
State	3	303.93	101.31	10.02
Segment within state	8	510.20	63.78	6.31
Treatment	2	99.89	49.95	4.94
State by treatment	6	196.88	32.81	3.25
Error*	16	161.83	10.11	
Total	35	1272.73		

†A significant difference between proportion differences at the 5-percent level were noted.

*Coefficient of variation for error = 2.72 percent.

TABLE 22.— STATE-BY-TREATMENT MEANS
FOR WINTER GRAIN SEGMENTS

State	No. of segments	Treatment			State average
		Random	Uniform	AI	
Colo.	1	3.0	13.0	-15.0	0.3
Kans.	4	-9.5	-3.0	-4.8	-5.8
Mont.	1	-4.0	-4.0	-15.0	-7.6
Neb.	1	-3.0	-5.0	-1.0	-3.0
Okla.	3	3.0	2.3	-4.0	0.4
S. Dak.	2	0.5	0.5	0.0	0.3
Tex.	3	-5.3	0.0	-1.3	-2.2
Treatment average		-2.2	0.5	-5.9	-2.5

TABLE 23.— STATE-BY-TREATMENT MEANS
FOR SPRING GRAIN SEGMENTS

State	No. of segments	Treatment[a]			State average	LSD values	
		Random	Uniform	AI		At 5%	At 1%
Minn.	2	*-13.0	†-7.0	†-6.0	-8.6	5.6	9.3
Mont.	2	*15.5	†7.5	†6.0	9.7	5.6	9.3
N. Dak.	5	*-2.8	*-3.4	†-8.6	-4.9	3.5	5.9
S. Dak.	3	*-2.7	*5.0	†-8.3	-2.0	4.5	7.6
Treatment average		-0.8	-0.5	-4.2	-1.5		

[a]Any two treatment means superscribed by the same symbol (asterisk or dagger) are not significantly different from each other at the 5% level.

uniform treatments were not significantly different from each other but were both significantly better than the AI treatment. Consideration of all spring grain states investigated indicates that the uniform treatment was not significantly different from the best of the treatments.

4. COMPUTATION OF THE VARIANCE OF THE ESTIMATE AND THE REDUCTION COEFFICIENT

In this section, the variance of the estimate and the reduction coefficient (R) are derived.

Let $x_1, x_2, \ldots, x_n$ denote the spectral samples of type 2 dots and θ_{ij} be a function of x_n, where

$$\theta_{ij} = \begin{cases} 1 \text{ if pixel } j \text{ of class } i \text{ is wheat} \\ 0 \text{ if pixel } j \text{ of class } i \text{ is nonwheat} \end{cases}$$

Let

N = total number of type 2 dots.

N_1 = number of type 2 dots in wheat strata.

$N - N_1$ = number of type 2 dots in nonwheat strata.

λ = machine estimate of wheat.

The proportion estimate can be expressed as

$$\hat{P}_N = \frac{\lambda}{N_1} \sum_{i=1}^{N_1} \theta_{1i} + \frac{1 - \lambda}{N - N_1} \sum_{i=1}^{N-N_1} \theta_{0i} \qquad (1)$$

$$= \lambda P_{11} + (1 - \lambda) P_{10}$$

where

$$P_{11} = \Pr[\text{labeled wheat} \mid \text{classified wheat}] = \frac{1}{N_1} \sum_{i=1}^{N_1} \theta_{1i}$$

$$P_{10} = \Pr[\text{labeled wheat} \mid \text{classified nonwheat}] = \frac{1}{N - N_1} \sum_{i=1}^{N-N_1} \theta_{0i}$$

The variance of the estimator is expressed as

$$\mathrm{Var}(\hat{P}_N) = \lambda^2 \frac{P_{11}(1 - P_{11})}{N_1} + (1 - \lambda)^2 \frac{P_{10}(1 - P_{10})}{N - N_1} \qquad (2)$$

Assume π is the probability that an analyst labels a pixel wheat and

$$\begin{aligned}
\lambda P_{11} &= \Pr(\text{classified W}) \cdot \Pr(\text{labeled W} \mid \text{classified W}) \\
&= \Pr(\text{labeled W, classified W}) \\
&= \Pr(\text{labeled W}) \cdot \Pr(\text{classified W} \mid \text{labeled W}) \\
&= \pi\pi_{11}
\end{aligned}$$

where

$\pi_{11} = \Pr(\text{classified W} \mid \text{labeled W})$,

$\pi_{01} = \Pr(\text{classified N} \mid \text{labeled W})$,

and

$N_1 = \lambda N \quad , \quad N - N_1 = (1 - \lambda)N.$

Then equation (2) can be expressed as

$$\begin{aligned}
\mathrm{Var}(\hat{P}_N) &= \frac{\lambda P_{11} - \lambda(1 - P_{11})}{N_1} + \frac{(1 - \lambda)P_{10} \cdot (1 - \lambda)(1 - P_{10})}{N - N_1} \\
&= \frac{\Pr(\text{labeled W, classified W})\Pr(\text{labeled N, classified W})}{N_1} \\
&\quad + \frac{\Pr(\text{labeled W, classified N}) \cdot \Pr(\text{labeled N, classified N})}{N - N_1} \\
&= \frac{\pi\pi_{11}(1 - \pi)\pi_{10}}{N_1} + \frac{\pi\pi_{01}(1 - \pi)\pi_{00}}{N - N_1}
\end{aligned}$$

$$\text{Var}(\hat{P}_N) = \pi(1 - \pi)\left[\frac{\pi_{11}\pi_{10}}{N\lambda} + \frac{\pi_{01}\pi_{00}}{N(1 - \lambda)}\right]$$

$$= \frac{\pi(1 - \pi)}{N}\left[\frac{\pi_{11}\pi_{10}}{\lambda} + \frac{\pi_{01}\pi_{00}}{(1 - \lambda)}\right] \qquad (3)$$

Using $\pi_{11} = 1 - \pi_{01}$, $\pi_{00} = 1 - \pi_{10}$,

then λ = Pr(classified W)

= Pr(classified W, labeled W) + Pr(classified W, labeled N)

$= \pi\pi_{11} + (1 - \pi)\pi_{10}$

$= \pi(1 - \pi_{01}) + (1 - \pi)\pi_{10}$

and

$1 - \lambda$ = Pr(classified N)

= Pr(classified N, labeled W) + Pr(classified N, labeled N)

$= \pi\pi_{01} + (1 - \pi)\pi_{00}$

$= \pi\pi_{01} + (1 - \pi)(1 - \pi_{10})$

Thus, substituting into equation (3)

$$\text{Var}(\hat{P}_N) = \left[\frac{(1 - \pi_{01})\pi_{10}}{\pi(1 - \pi_{01}) + (1 - \pi)\pi_{10}} + \frac{\pi_{01}(1 - \pi_{10})}{\pi\pi_{01} + (1 - \pi)(1 - \pi_{10})}\right]\frac{\pi(1 - \pi)}{N}$$

$$= R\left(\frac{\pi(1 - \pi)}{N}\right) \qquad (4)$$

where R is known as the reduction coefficient and the expression $\left(\frac{\pi(1 - \pi)}{N}\right)$ is generally known as the sampling error. The expression for R is easily computed from the omission and commission errors for the type 2 dots and can be viewed as an indication of how much the machine classification improves the proportion estimation.

The R-values were computed for GT-labeled machine classifications which were performed for the secondary error analysis study using the random grid

system and for AI-labeled machine classifications which were performed for Phase III LACIE processing. The machine classifications in both cases were compared to GT labels. For three segments (1520, 1739, and 1861), the Phase III processing results were unavailable for analysis. Tables 24 and 25 present the raw data and computed R-values for the GT-labeled machine classifications and the AI-labeled machine classifications, respectively. Figures 1 and 2, representing the computations from tables 24 and 25, respectively, plot the GT proportion estimate (ρ) versus the computed R-value. The mean reduction coefficient (R) values (from tables 24 and 25) are as follows:

1. GT-labeled random grid — 0.718
2. AI-labeled random grid — 0.714

The standard deviations on these estimates are 0.217 and 0.182, respectively.

TABLE 24.— GT-LABELED MACHINE CLASSIFICATIONS

Segment	State	π_{10}	π_{01}	ρ	n	R	$\frac{\rho(1-\rho)}{n}$	$R\frac{\rho(1-\rho)}{n}$
1005	Colo.	0.108	0.565	0.347	60	0.859	0.00378	0.00324
1032	Kans.	0.200	0.292	0.386	59	0.744	0.00402	0.00299
1033	Kans.	.039	.667	.095	58	.881	.00148	.00131
1853	Kans.	.000	.500	.303	60	.589	.00352	.00207
1861	Kans.	.171	.500	[a].353	43	.879	.00531	.00467
1512	Minn.	0.250	0.714	0.337	59	0.999	0.00379	0.00378
1520	Minn.	.244	.133	.300	60	.667	.00350	.00233
1544	Mont.	0.043	0.703	0.383	60	0.874	0.00394	0.00344
[b]1739	Mont.	.100	.526	.284	59	.826	.00345	.00285
1582	Neb.	0.019	0.143	0.194	60	0.261	0.00261	0.00068
1604	N. Dak.	0.179	0.281	0.524	60	0.707	0.00416	0.00294
1606	N. Dak.	.346	.095	.329	47	.723	.00470	.00340
1648	N. Dak.	.175	.650	.379	60	.961	.00392	.00377
1661	N. Dak.	.094	.333	.410	53	.640	.00456	.00292
1902	N. Dak.	.057	1.00	.086	60	.995	.00131	.00130
1231	Okla.	0.417	0.022	0.741	59	0.566	0.00325	0.00184
1242	Okla.	.042	.207	.472	55	.413	.00453	.00187
1367	Okla.	.545	.324	.540	50	.982	.00497	.00488
1677	S. Dak.	0.083	0.733	0.341	51	0.939	0.00441	0.00414
1690	S. Dak.	.063	.333	.213	60	.505	.00279	.00169
1803	S. Dak.	.000	.500	.011	59	.503	.00018	.00009
[b]1805	S. Dak.	.000	.538	.158	91	.580	.00146	.00085
1056	Tex.	0.367	0.273	0.226	60	0.908	0.00292	0.00265
1059	Tex.	.226	.074	.445	57	.513	.00433	.00222
1060	Tex.	.000	.250	.231	59	.302	.00301	.00091

[a]Dot count estimate of GT was used.

[b]This segment is a mixed wheat site.

TABLE 25.— AI-LABELED MACHINE CLASSIFICATION

Segment	State	π_{10}	π_{01}	ρ	n	R	$\frac{\rho(1-\rho)}{n}$	$R\frac{\rho(1-\rho)}{n}$
1005	Colo.	0.081	0.696	0.347	60	0.915	0.00378	0.00346
1032	Kans.	0.189	0.227	0.386	59	0.667	0.00402	0.00268
1033	Kans.	.038	1.00	.095	58	.996	.00148	.00148
1853	Kans.	.217	.214	.303	60	.712	.00352	.00251
1512	Minn.	0.158	0.429	0.337	59	0.818	0.00379	0.00310
1544	Mont.	0.156	0.448	0.383	99	0.826	0.00239	0.00197
1582	Neb.	0.000	0.250	0.194	60	0.293	0.00261	0.00076
1604	N. Dak.	0.138	0.581	0.524	60	0.903	0.00416	0.00375
1606	N. Dak.	.091	.440	.329	47	.738	.00470	.00347
1648	N. Dak.	.194	.667	.379	60	.976	.00392	.00383
1661	N. Dak.	.194	.409	.410	53	.834	.00456	.00381
1902	N. Dak.	.000	1.00	.086	60	(a)		
1231	Okla.	0.364	0.021	0.741	59	0.509	0.00325	0.00166
1242	Okla.	.080	.233	.472	55	.512	.00453	.00232
1367	Okla.	.095	.172	.540	50	.466	.00497	.00232
1677	S. Dak.	0.000	0.533	0.341	51	0.634	0.00441	0.00279
1690	S. Dak.	.020	.545	.213	60	.683	.00279	.00191
1803	S. Dak.	.000	.500	.011	59	.503	.00184	.000093
[b]1805	S. Dak.	.013	.750	.158	91	.844	.00146	.00123
1056	Tex.	0.068	0.500	0.226	60	0.764	0.00292	0.00223
1059	Tex.	.259	.200	.445	57	.711	.00433	.00308
1060	Tex.	.024	.529	.231	59	.680	.00301	.00205

[a]This represents an extreme case for which the R-value does not exist.

[b]This segment is a mixed wheat site.

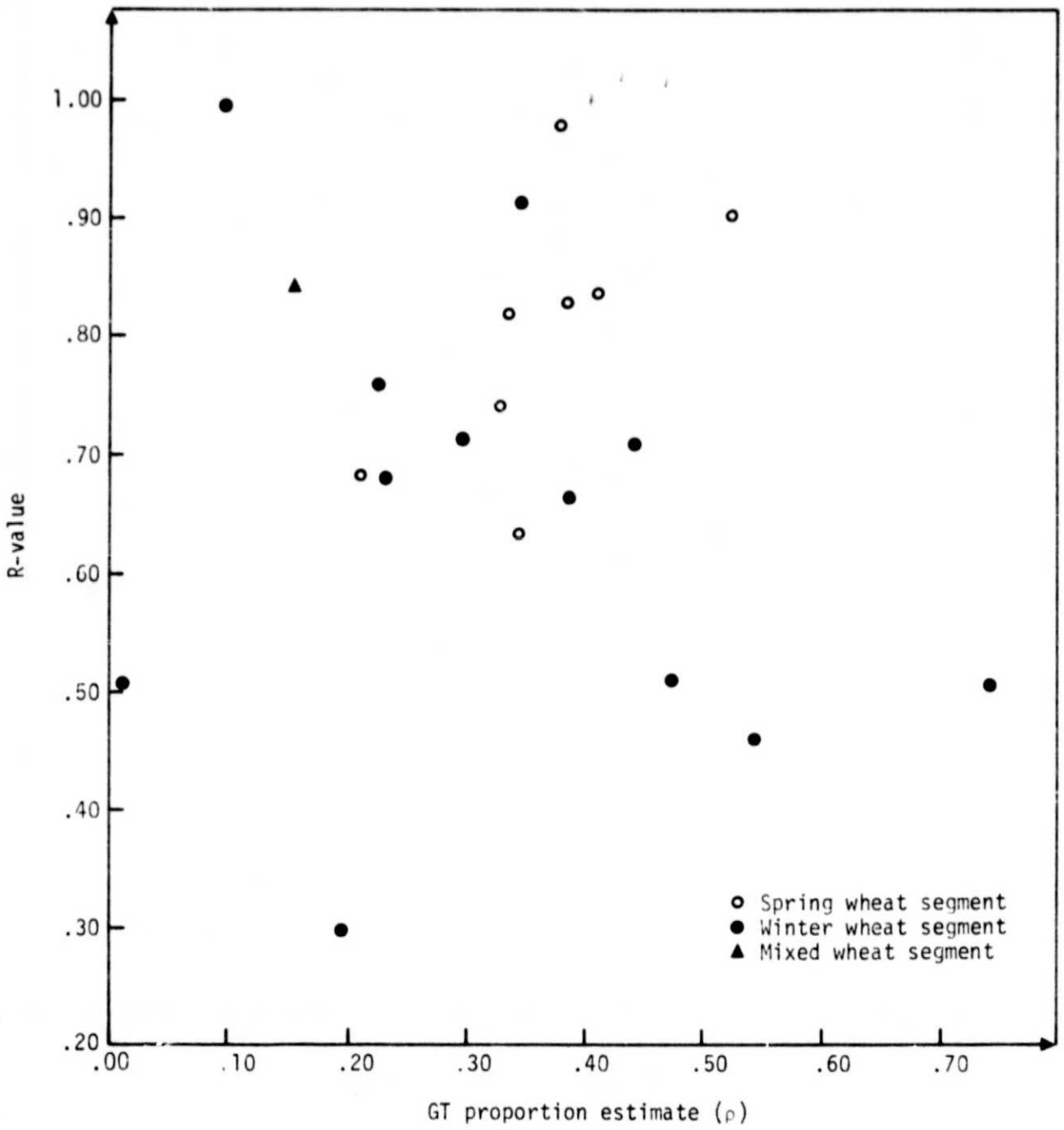

Figure 1.— GT-labeled machine classification.

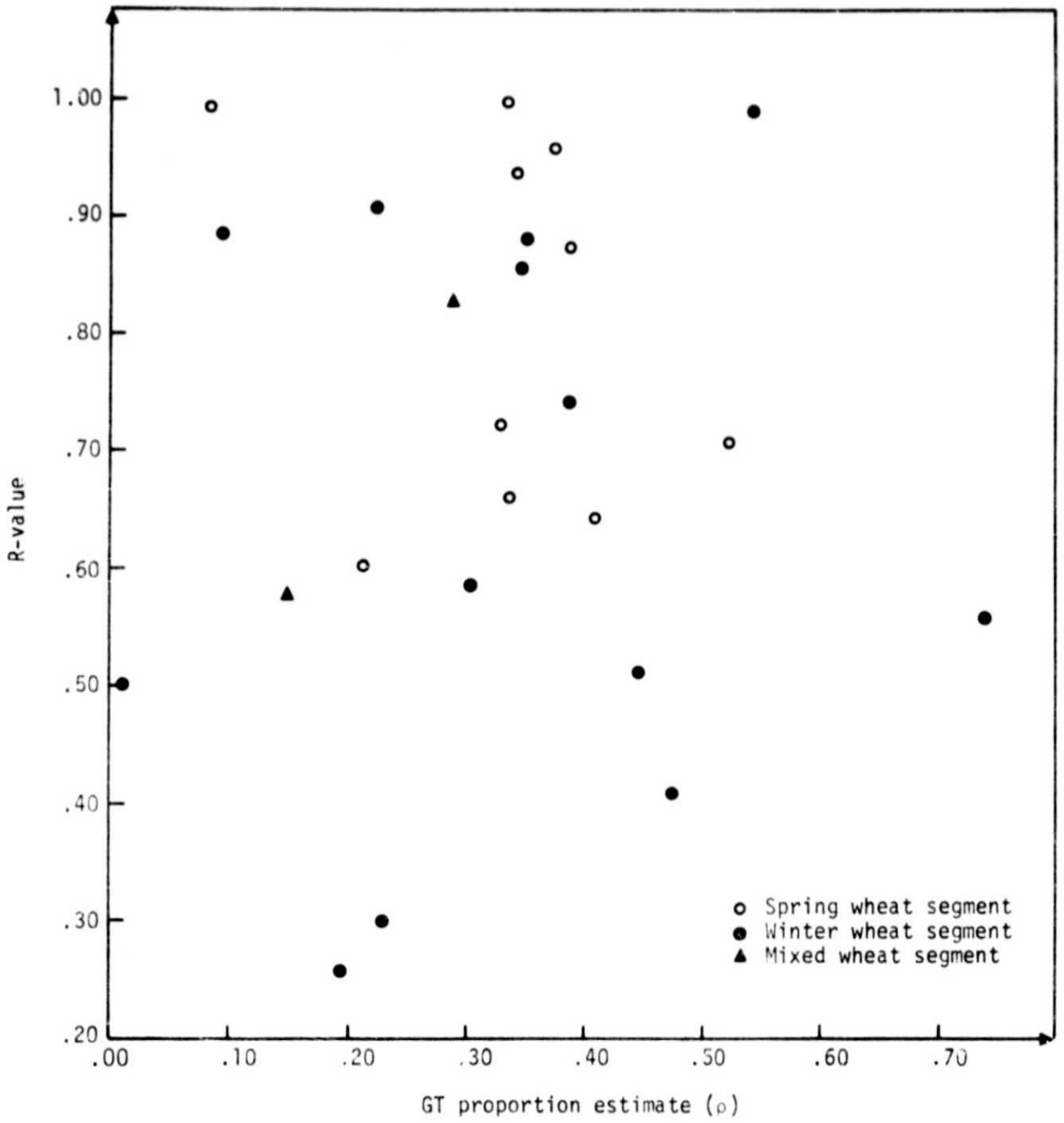

Figure 2.— AI-labeled machine classification.

5. CONCLUSIONS

A comparison of P1 proportion estimation results from the IBM study and the secondary error analysis study showed no significant differences between the two studies.

Re-evaluation of the secondary error analysis data indicated significant differences in the probabilities of correctly classifying small grains using type 1 dots (PCG1). The PCG1 for the AI-labeled random dot grid was significantly lower than both of the GT-labeled dot grids (i.e., the random and systematic dot grids). However, the two GT-labeled dot grids were not significantly different from each other. The PCG1 means were as follows:

1. GT-labeled random dot grid — 80.6 percent
2. GT-labeled uniform dot grid — 80.2 percent
3. AI-labeled random dot grid — 60.0 percent

The superior performance of the GT-labeled random grid over the GT-labeled uniform grid can probably be attributed to differences in the purity of the type 1 dots used on the two grids. The analyst selected the type 1 dots used on the GT-labeled random grid with the aid of the Landsat imagery and GT information. The type 1 dots on the GT-uniform grid were selected by inspection of GT images but without the aid of Landsat imagery to verify the purity of the type 1 dots. It is speculated that some boundary dots were inadvertently included in the type 1 GT-labeled uniform grid dots.

The analysis of the probabilities of correctly classifying small grains using type 2 dots (PCG2) showed the GT-labeled random dot grid provided significantly better performance than both the GT-labeled uniform grid and the AI-labeled random grid. No difference was noted between the PCG2's for the GT-labeled uniform grid and the AI-labeled random grid. The PCG2 means were as follows:

1. GT-labeled random dot grid — 67.9 percent
2. GT-labeled uniform dot grid — 58.4 percent
3. AI-labeled random dot grid — 53.0 percent

The analyses of the signed differences between P1 small-grain proportion estimates and GT proportions were performed separately on segments from winter wheat areas and spring wheat areas. In the winter wheat area, the proportion estimates obtained from P1 were not significantly different from GT proportions. This was true for both the GT-labeled grids and the AI-labeled grid.

In the spring wheat area, the analyses had to be performed on each state separately because of interaction of P1 proportion estimates with states. The results indicated that for Minnesota and Montana, P1 proportion estimates obtained using the GT-labeled uniform grid and the AI-labeled random grid were significantly better than the GT-labeled random grid. However, P1 proportion estimates from the GT-labeled uniform grid and the AI labeled grid were not significantly different from each other. For North Dakota and South Dakota, P1 proportion estimates from the GT-labeled random grid and the GT-labeled uniform grid were both significantly better than the AI-labeled random grid. However, the P1 proportion estimates from the two GT-labeled grids were not significantly different from each other.

The efficiency of P1 in reducing the variance of the proportion estimate obtained from bias correction using type 2 dots was computed. The mean reduction coefficient (R) for the GT-labelel random grid and the AI-labeled random grid are as follows:

1. GT-labeled random grid — 0.718
2. AI-labeled random grid — 0.714

The standard deviations on these estimates are 0.217 and 0.182, respectively.

Clearly, P1 does not provide much gain over a simple random sample proportion estimate from the type 2 dots.

6. REFERENCES

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