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DYNAMICS OF THE ACCUMULATION PROCESS OF THE EARTH GROUP OF PLANETS

FORMATION OF THE REVERSE ROTATION OF VENUS

N. N. Koslov and T. M. Eneyev

Translation of "Dinamika Protesessa Akkumulyatsii Planet Zemnoy Gruppy. Formirovaniye Obratnogo Vrashcheniya Venery", Academy of Sciences, USSR, Institute of Applied Mathematics, Moscow, Preprint No. 135, 1978, pp. 1-43.

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N. N. Koslov and T. M. Eneyev

Abstract. /2*

A numerical simulation of the process of formation of the terrestrial planets is carried within the framework of a new theory for the accumulation of planetary and satellite systems [1]. The numerical simulation permitted determining the parameters of the protoplanetary disk for which the three planets (Mercury, Venus and the Earth) are formed as a result of the evolution. The acquisition of a slow retrograde rotation for Venus was discovered during the course of investigation, whereas Mercury and the Earth acquired direct rotation about their axes. Deviations of the semi-major axes of these three planets as well as the masses of the Earth and Venus from the true values are small as a rule (< 10%). It is shown that during the accumulation of the terrestrial planets, there existed a profound relationship between the process of formation of the orbits and masses of the planet and the process of formation of their rotation about their axes. Estimates are presented for the radii of the initial effective bodies and the time of evolution for the terrestrial accumulation zone.

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1. INTRODUCTION /5

This work presents the results of a numerical simulation of the process of accumulation of the terrestrial planets, which is carried out within the framework of a new theory for the formation of planetary and satellite systems [1]. This theory arose during the course of investigating a new mathematical model of the protoplanetary cloud, and its principle results were confirmed by two independent methods: a qualitative investigation of the integral-differential equation of coagulation of bodies into orbits and a computer simulation of the process of accumulation of the protoplanets. A correlation was established between the results obtained by the two essentially different methods. A similarity theory for the evolutionary processes in protoplanetary clouds was constructed during the course of the qualitative analysis of the solution of the coagulation equation and with the help of the comparison of the results of this analysis with the results obtained by the numerical simulation.

An important effect was discovered as a result of the investigation — annular compression of the material arising during the evolution of the plane protoplanetary disk. The most significant features of this effect are its invariant properties with respect to the sizes and masses of the initial bodies of the cloud. This effect and the constructed similarity theory made it possible to formulate a general law for the formation of planetary systems within the framework of which was understood the meaning and nature of the Titius-Bode law. A very signficant factor is that the annular compression of material occurs for very small eccentricities of the orbits of the combining bodies. Thus, the processes of planetary accumulation investigated in [1] differ significantly from the analogous processes of the majority of preceeding works using the scheme of the exhaustion by "nuclei" — nucleation /6
centers of the planets of the material of the protoplanetary cloud, whose particles had very significant eccentricities $(e \sim 0.2)$.

Another very important result of work [1] is the discovery during the course of the investigation of a possible mechanism for the formation of prograde and retrograde rotation of planets during their accumulation. A clear and simple explanation in terms of a system of frontal and glancing contacts of the interacting bodies is given for the mechanism for the formation of the predominately prograde rotation of planets. It was also shown that the frontal contacts do not predominate over the glancing contacts. In some cases, the opposite pattern can occur and the planet acquires retrograde rotation completely naturally as a result.

The results of this work were obtained on the basis of analysing the operation of the particular (liminting) model of the protoplanetary cloud, in which the gravitational coupling is always concluded by the combining of the effective bodies. In contrast to [1], where the simplified drop model of the protoplanetary cloud was investigated, the more complex gravitational model is investigated here. It is very essential that the experiments with the drop model and the similarity theory permitted establishing a relationship between the parameters of the mathematical model and the parameters of the actual protoplanetary cloud on the basis of the kinetic characteristics of the rotation of the planets. The sizes of the initial effective bodies and the evolution time for some characteristic regions of the actual protoplanetary cloud could be calculated with the hlep of this relation. Moreover, a correspondance was discovered between the obtained sizes of the initial effective bodies and the sizes of the condensations into which the proto- /7
planetary cloud disintegrated as a result of the gravitational instability arising in it.[2]. This correspondance should certainly be considered as a serious argument in favor of the new accumulation theory.

We note that the numerical experiments with the gravitational model of the cloud along with the annular compression of material also displayed a number of troublesome effects. This is above all the final effect of exhaustion of material and also the effect of the collapse of the orbit of the combining bodies because of the transition of a significant fraction of the orbital momentum to the rotational momentum. These effects are a consequence of the coarseness of the mathematical model and, above all, are due to the relatively small number of proto-bodies of the model (N - 25600). Special procedures were specified in the algorithm to neutralize these side effects.

2. GRAVITATIONAL MODEL OF THE PROTOPLANETARY CLOUD

As was shown in work [1], numerical experiments with the drop model of the cloud permitted obtaining sufficiently reliable data for calculating some important parameters of the gravitational model. These parameters should include, above all, the total effective cross sections area of the initial bodies of the cloud per unit area of the protoplanetary disk S_0. The necessity of a very high compactness in the distribution of the effective bodies of the cloud at the beginning of the evolutionary process evidently finds a completely convincing basis in the theory of gravitational instability of the preplanetary cloud [2]. This theory permitted finding another important characteristic of the structure of the initial protoplanetary cloud — the average radius of protoplanets per unit area of the protoplanetary disk $\hat{R}$. In contrast to the effective area /8
S_0, which can be considered in the first approximation independent of the radial distance from the central body z, the average radius $\hat{R}$, according to formula (II.2) from [1], can depend very significantly on this distance. It is obvious that knowing S_0 and $\hat{R}$ in itself does not give comprehensive information about the initial structure of the protoplanetary cloud. Such information is contained in the initial distribution function of the effective bodies $n(\zeta, z, 0)$ (cf. [1]). However, on the basis of knowing the parameter S_0 and the function $\hat{R}(z)$, one can nevertheless construct a completely acceptable model in the first approximation for the initial structure of the protoplanetary cloud. The construction of such a model is based on the natural and quite plausible (within the framework of the theory of gravitational instability of the preplanetary cloud) assumption of the insignificant dispersion of the relative masses of the bodies ζ at a given distance ρ and at

t = 0. Then developing this point of view, one can use the corresponding analog of the delta-function for describing the structure of the distribution function $n(\zeta, r, 0)$. In fact, as in the case of the drop model of the cloud, we assume that the initial distribution function of the bodies $n(\zeta, r, 0)$ can be represented in the form:

$$\begin{cases} n=0 & \text{for} \quad \zeta \neq \zeta_*(r) \\ n=\infty & \text{for} \quad \zeta = \zeta_*(r), \end{cases} \tag{2.1}$$

Then, as previously, one must satisfy the relation:

$$\int_0^\infty n(\zeta, r, 0)\, d\zeta = \tilde{n}(r), \tag{2.2}$$

where: /9

$$\tilde{n}(r) = \begin{cases} n_0(r) & \text{for} \quad a_{min} \leq r \leq a_{max} \\ 0 & \text{for} \quad a_{min} > r > a_{max}. \end{cases} \tag{2.3}$$

The parameters α_{min} and α_{max} represent respectively the outer and inner boundaries of the protoplanetary disk. The definition of the function $\zeta_*(r)$ will be given below.

According to the results obtained in [1], the radius of the effective bodies of the initial protoplanets can be determined from the formula:

$$R = \gamma \sigma_0 r^3. \tag{2.4}$$

Keeping in mind that $\sigma_0(r)$ is an average value of the surface density of material for a given r (under conditions of a discrete structure of the protoplanetary disk), it is not difficult to obtain the following formula for the masses of the initial protoplanets:

$$m = \frac{\pi \gamma^2}{S_0} \sigma_0^3 r^6. \tag{2.5}$$

It follows from (2.5), finally, that:

$$\zeta_*(r) = \frac{\pi\gamma^2}{m_0 S_0}\sigma_0^3 r^6 \tag{2.6}$$

where m_0 is a characteristic mass. We note further that under the conditions of the special form of the distribution function $n(\zeta, r, 0)$ represented by equations (2.1) and (2.2) (the delta-function), formula (4.19) from [1] is not applicable for determining the value of the characteristic mass m_0. However, one can show that any value is acceptable in this case for the parameter m_0.

It is necessary to derive a formula for the parameter γ to complete the description of the initial structure of the protoplanetary disk. The formula for γ obtained in [2] refers to the real protoplanetary cloud and is unsuitable for the generalized gravitational model. Moreover, it is necessary to derive a formula for determining the parameter λ. This parameter appears in formula (3.2) from [1], with the help of which the radii of the effective bodies of the protoplanets in the process of operation of the cloud model are determined: $R = \lambda r \sqrt[3]{\frac{m}{M_0}}$. /10

One can derive the following asymptotic formula ($R \to 0$) for the total number of bodies in the protoplanetary cloud N:

$$N = 2 S_0 \int_{a_{min}}^{a_{max}} \frac{\zeta}{R^2}\, dr \ . \tag{2.7}$$

Keeping in mind (2.4) and using (2.7), it is not difficult to derive the following formula for determining γ:

$$\gamma^2 = \frac{2 S_0}{N} \int_{a_{min}}^{a_{max}} \frac{dr}{\sigma_0^2 r^5} \ . \tag{2.8}$$

Further, substituting (2.5) into (3.2) from [1] and comparing the formula obtained as a result of this substitution with (2.4), it is easy to derive the following formula for λ:

$$\lambda = \sqrt[3]{\frac{S_0\gamma}{\pi} M_0} \tag{2.9}$$

Thus, if S_0, N and $\sigma_0(r)$ are specified, then the parameters γ and $\varkappa$ are determined successively from equations (2.8) and (2.9).

Equations (2.5), (2.6), (2.8), (2.9) are the basis of the mathematical apparatus with which specific discrete models of the protoplanetary cloud are formulated. This mathematical apparatus contains, in particular, the algorithm for calculating the random (more precisely, the "pseudorandom") numbers with which the random element is introduced into the initial distribution of the effective bodies of the cloud (at t = 0). The effective bodies located /11
initially in more or less "correct" order are shifted randomly within insignificant and allowable limits along their radial coordinate r. The orbits of the bodies maintain circular form (i.e., the condition $e = 0$ is maintained), while the average surface density of material in the disk $\bar{\sigma}_0(r)$ deviates from the specified initial law $\sigma_0(r)$ also within very small limits. The dynamic operation of the gravitational model of the protoplanetary cloud begins after this "stochastic" procedure.

3. FORMATION OF THE TERRESTRIAL PLANETS

The gravitational model constructed above for the protoplanetary cloud was used to study the problem of the formation of the terrestrial planets. It should be noted that this study encountered, from the very beginning, serious difficulties caused, on the one hand, by insufficiently clear understanding of the physical conditions existing in the accumulation zone of Mars during formation of the terrestrial planets and, on the other hand, by the coarseness of the mathematical model of the cloud.

There is justification for assuming that the accumulation zone of Mars was subjected to a more or less strong effect from the intense accumulation zone of Jupiter [3]. This, in particular, can explain that Mars has a relatively small mass as compared to the Earth or Venus. However, the small mass of Mars can also be explained by an insufficiently high surface density of material in its accumulation zone at the beginning of the evolutionary process. The latter hypothesis, at first glance, permits using the constructed model of the protoplanetary cloud for studying the process of formation of the terrestrial planets. However, this utilization is hindered by the coarseness of the mathematical model of the cloud caused by the insufficiently large number of initial bodies contained in it (25,600). Actually, it is not difficult to show that in this model, using formula (2.4) to calculate the radii of the bodies, an overwhelming majority of the initial bodies will be concentrated in the accumulation zones of Mars and Mercury. For the relatively small number of initial bodies of the cloud (25,600), this leads to the accumulation zones of Venus and, particularly, the earth being described so roughly that simulation of the evolutionary process of the formation of the planets becomes practically impossible. /12

All these circumstances indicated above led to using for simulation of the process of formation of the terrestrial planets a limited model of the protoplanetary cloud containing accumulation zones for three planets: Mercury, Venus and the Earth. The possibility and the advisability of using this limited model can be justified by the following consideration.

The relatively large rotational momentum of the earth is evidence in favor of its accumulation zone belonging to the "strong" type. This type of zone, as was already indicated in [1], is characterized in some degree by the autonomy of its evolutionary development. This autonomy permitted, in particular, carrying out the isolated simulation of the process of formation of a number of planets, which was described in [1]. Thus, isolation of the outer part of the accumulation zone of the Earth from the accumulation zone of Mars is completely acceptable in the first approximation. Moreover, another serious argument can be presented in favor of the acceptability of this isolation. Independent of the causes leading to the formation of Mars with a comparatively small mass, the very fact of the insignificance of this mass serves as a sufficiently strong, although indirect, argument that the total effect of the zone of Mars on the formation of the Earth /13 could not have been large. We also note that isolated simulation of the process of formation of Venus cannot be carried out since its accumulation zone evidently belongs to the "weak" type. Thus, the limited model of the protoplanetary cloud indicated above, in making it possible to obtain a complete picture of the formation of the three planets, is completely necessary at the same time to simulate the formation of the middle planet — Venus.

It follows from the material presented above that it is necessary to specify numberical values of the parameters s_0, α_{min}, α_{max} and the function $\sigma_0 = \sigma_0(r)$ to completely describe the limited model of the protoplanetary cloud. It is very desirable that the function $\sigma_0(r)$, at least in the first numerical experiments, be sufficiently simple. An exponential dependence of σ_0 on r was taken as this function:

$$\sigma_0 = \sigma_{00}\, e^{-\lambda_1 r}, \tag{3.1}$$

where σ_{00} and λ_1 are the constants to be determined after α_{min}, α_{max} are specified and also the total mass M and the total orbital momentum K of the protoplanetary cloud.

The following values were taken in the first series of numerical experiments for α_{min} and α_{max}:

$$\alpha_{min} = 0.36, \qquad \alpha_{max} = 1.21 \tag{3.2}$$

The values of the parameters α_{min} and α_{max} were obtained with the /14 help of approximate estimates based on experience in carrying out experiments before this with the drop model of the cloud. The values of the mass M and angular momentum K were taken equal to the present total values respectively of the mass and orbital momenta of Mercury, Venus and the Earth:

$$M_\Sigma = 1.871, \qquad K_\Sigma = 10.86. \tag{3.3}$$

Here and in all the subsequent examples, the numerical results are given in the following system of units: unit of distance — the astronomical unit, unit of time — the terrestrial year, unit of mass — the mass of the earth. The following approximate values were obtained on the basis of the data of (3.2) and (3.3) for σ_{00} and λ_1:

$$\sigma_{00} = 0.55 \,, \quad \lambda_1 = 0.28 \,. \tag{3.4}$$

The following construction of the discrete model of the protoplanetary cloud with the previously described random shift of the effective bodies along the radial coordinate led to some change in the parameters M_Σ and K_Σ:

$$M_\Sigma = 1.870 \,, \quad K_\Sigma = 10.82 \,. \tag{3.5}$$

However, keeping in mind the coarseness of the mathematical model of the cloud and the related degree of approximation in describing real processes in the formation of the planets, one should recognize these changes as very insignificant and completely acceptable (deviation from the specified values is less than 0.5%).

The use of formula (2.8) based on (3.1) permitted calculating the values of the parameter γ for different S_0. Knowing the parameter γ and the function $\sigma_0(r)$ made it possible to construct graphs of the dependence of the radius of the initial effective bodies R on the distance r. Such a graph is presented in Figure 1. Figure 1 shows the significant dependence of the radius of the effective bodies R on the distance r: the radii of the bodies at the inner and outer boundaries of the protoplanetary disk differ by almost forty times. This fact led to the distribution of the initial bodies over the accumulation zones being very nonuniform. Of the 25,600 bodies of the model of the protoplanetary cloud, almost 13,000 belong to the zone of Mercury, about 12,000 to the zone of Venus and less than 1000 (∿ 800) to the zone of the Earth. This nonuniformity in itself is natural and is closely related to the essence of the problem under study. However, this nonuniformity was a source of serious difficulties in simulating the process of formation /15 of the planets because of the coarse description of the accumulation zone of the Earth. In this case, the final effect of exhaustion of material [1] appeared in very sharp form at the concluding stage of evolution in the outer zone of the protoplanetary cloud. To counter the consequences of this effect, a special procedure for isolating the accumulation of the earth before the end of the evolutionary process had to be introduced into the algorithm for solving the problem.

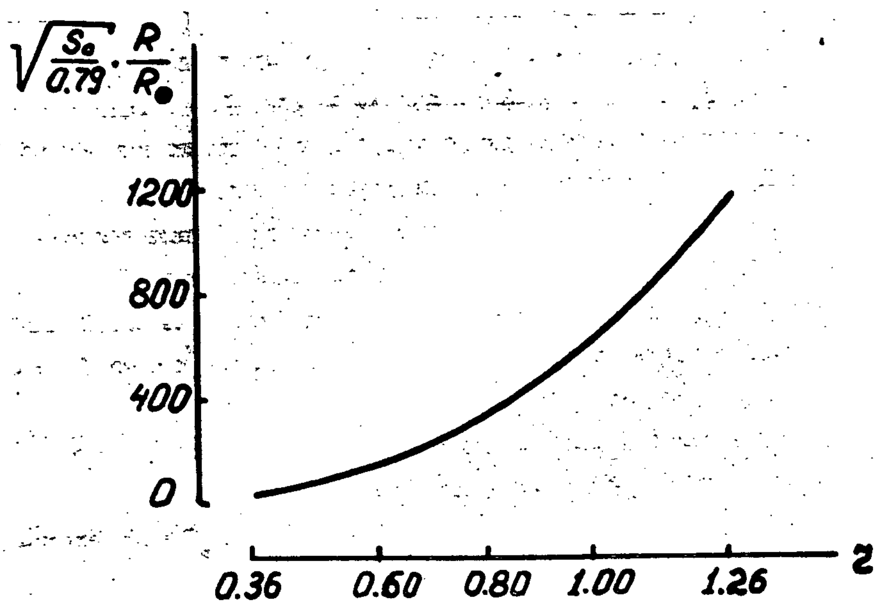

Figure 1. Dependence of the initial radii of the effective bodies on distance for versions of 4.0 - 4.4 of the gravitational model of the protoplanetary cloud. Version 4.0 - S_0 = 0,79 ; version 4.I - S_0 = 0,74 ; version 4.2 - S_0 = 0,68 ; version 4.3 - S_0 = 0,62 ; version 4.4 - S_0 = 0,57.

The effective cross section area of the initial bodies S_0, as in the drop model of the cloud, was the main variable parameter in this series of numerical experiments. The choice of the range of variations of the parameter S_0 was made on the basis of analyzing the results of the numerical experiments with the drop model of the cloud. This analysis indicated two possible regions of values of the parameter S_0, which would correspond to the real geometry of the orbits of the terrestrial planets. In fact, keeping in mind that $a_{max}/a_{min} \simeq 3.8$ in this case, it follows from analysis of Figure

5 of work [1] that the required value of S_0 is advisably sought in the range $0.4 < S_0 < 0.45$ (cf. version 1.2 and 1.3). On the other hand, the analysis presented in [1] for the final mechanism of exhaustion of material in the cloud (cf. versions 2.1 -2.4) indicated the possibility of the existance of an additional planet in this interval ε $(0.36 < \varepsilon < 1.2)$ for S_0 close to their limiting values (~ 0.8). In connection with this, it wad decided to investigate, above all, the cases of the limiting values of the parameter S_0 belonging to the previously indicated important range of its variation $(0.4 < S_0 < 0.8)$. Three typical versions were calculated, whose parameters are presented in Table I. /17

TABLE I. VALUES OF THEPPARAMETER S_0 FOR VERSIONS 3.1, 3.2, AND 4.0

Version	3.1	3.2	4.0
S_0	0.41	0.45	0.79

Numerical results of the calculations of versions 3.1 and 3.2 are presented in Table II. Table II presents in order the parameters of the forming planetary systems: the planet number n, the semimajor axis of the planets of the model α, the so-called reduced semimajor axis $\tilde{\alpha}$*, the mass of the planets m, the rotational momentum of the planets of the model K_{sm}, the eccentricity of the orbits e. Analysis of Table II unambiguously shows that the relatively small values of the parameter S_0 used for versions 3.1 and 3.2 cannot provide the formation of the required number of planets in the given range of distances ε. Instead of three planets in the indicated range of ε, no fewer than four planets are formed, of which two should evidently belong to the accumulation zone of Venus. /18

* $\tilde{\alpha}=\left(\sqrt{\alpha}+\frac{K_{sm}}{\sqrt{\mu}}\right)^2$, where μ is the gravitational constant of the sun. The parameter $\tilde{\alpha}$ indicates the value of the semimajor axis of the planet in the case when the entire angular momentum (including rotational) is concentrated in the orbital motion of the planet.

TABLE II. PARAMETER OF THE PLANETARY SYSTEM FOR VERSIONS 3.1 AND 3.2. GRAVITATIONAL MODEL OF THE PROTOPLANETARY CLOUD

N ver.	S_o	n	α	$\tilde{\alpha}$	m	K_{sm}	e
3.I	0.4I	I	0.40	0.40	0.08	0.0I3	0.006
		2	0.50	0.5I	0.2I	0.0I3	0.007
		3	0.64	0.66	0.29	0.057	0.0I5
		4	0.95	I.00	I.27	0.I45	0.0I7
		5	I,2I	I.22 x)	0.02	0.0I5	0.002
3.2	0.45	I	0.4I	0.4I	0.I2	0.0I7	0.003
		2	0.55	0.56	0.32	0.038	0.005
		3	0.75	0.78	0.46	0.I05	0.0I5
		4	0.98	I.06	0.97	0.262	0.035

x) The appearance of particles with $\tilde{\alpha} > \alpha_{max}$ in this table and in other tables and figures is related to the procedure of initial random shifts of the bodies along the radial component (cf. above).

Version 4.0 was more successful. However, before presenting the final results of the calculations for it, we first discuss an analysis of the concluding stage of formation of the planets of this version. Figures 2.1 - 2.6 present the dynamics of the concluding stage of the evolutionary process expanded in time. Figures 2.1 - 2.2 and 2.3 - 2.4 present respectively histograms of the distribution of mass and rotational momentum of the accumulating bodies along the radial coordinate of the protoplanetary disk for various times. Figures 2.5 - 2.6 present for the same intervals of the histograms in the scale of the radial coordinate of the disk the sizes of the largest bodies, as a rule, one from each interval of the histogram. The number of bodies existing at the given time in the cloud is indicated on these figures for several phases of evolution.

Analysis of the mass histograms shows that the time $t_m^{**)}$ =I.5 (N =80)

**t_m is the time of the evolutionary process for the generalized model of the cloud [1]; calculation of the true evolution time is presented below.

three groups of bodies are rather clearly formed, belonging respectively to the zones of Mercury, Venus and the Earth. /19

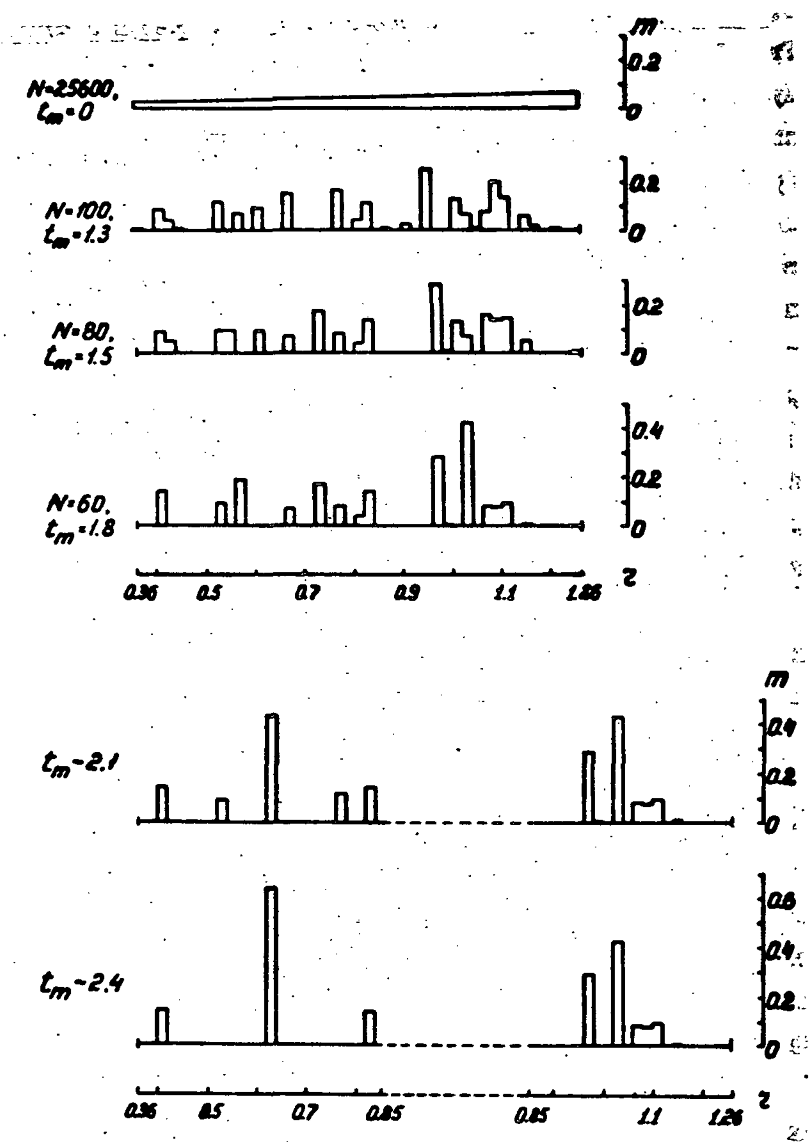

Figure 2.1. Dynamics of the accumulation process of formation of the terrestrial planets in version 4.0 (S_0 =0.79) . Mass histograms.

Figure 2.5 shows that at the given time, the group of bodies of the zone of Mercury was completely separate from the group of bodies of the zone of Venus. However, such a division did not occur with the groups of bodies of the zones of Venus and the Earth.

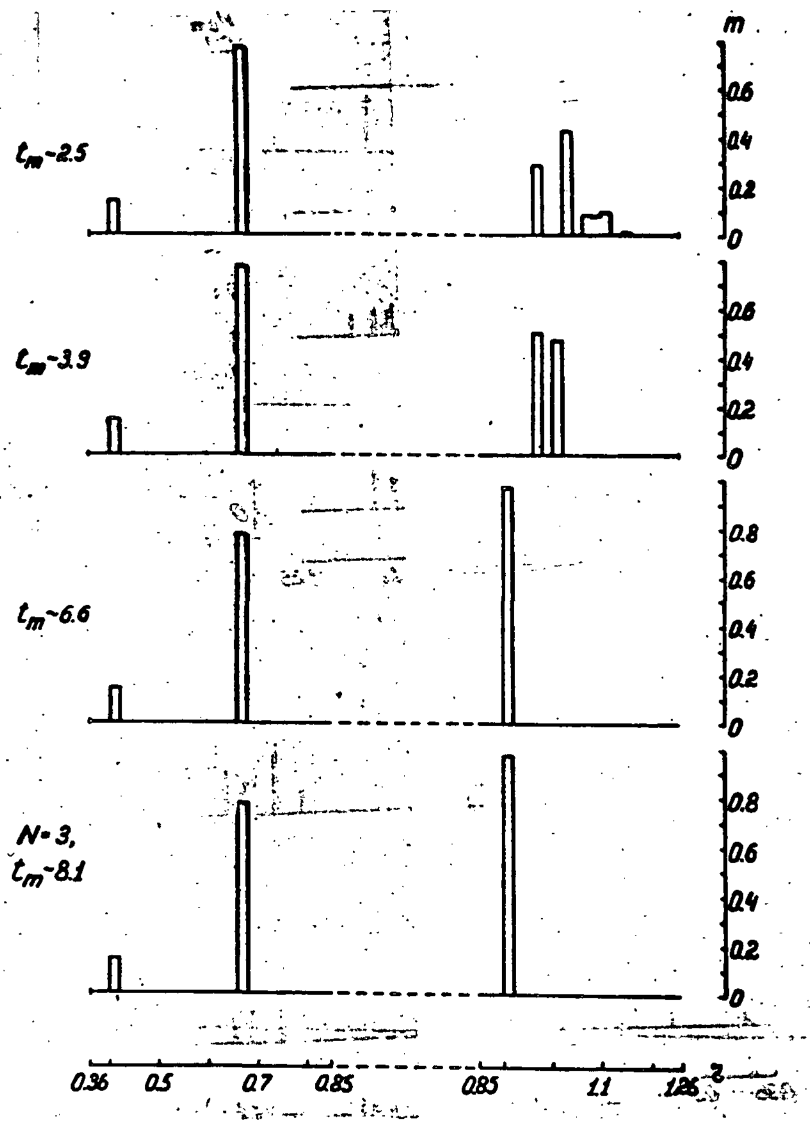

Figure 2.2. Continuation of Figure 2.1. Venus is formed at t_m = 2.5, the earth is formed at t_m = 6.6, Mercury is formed at t_m = 8.1

The cause of this was the above noted coarseness in the description of the accumulation zone of the Earth, caused by the insufficiently large number of bodies in the cloud model. At the concluding stage of the evolutionary process, the effective bodies of the zone of the Earth became so large that contact continued to be maintained with the zone of Venus in spite of the fact that separation of the accumulation zones had actually already occurred (according to the analysis of the mass histograms). Continuation of the simulation of the evolutionary process inevitably caused the appearance of the final mechanism of exhaustion of material and the related significant distortion of the pattern of formation of the planets. In connection with this, the accumulation zone of the earth was isolated from the zone of Venus from the time $t_m = 1.8$ (N = 60) and its further evolution was considered independently of the evolution of the other zones. This isolation is noted by dashed lines on the presented figures.

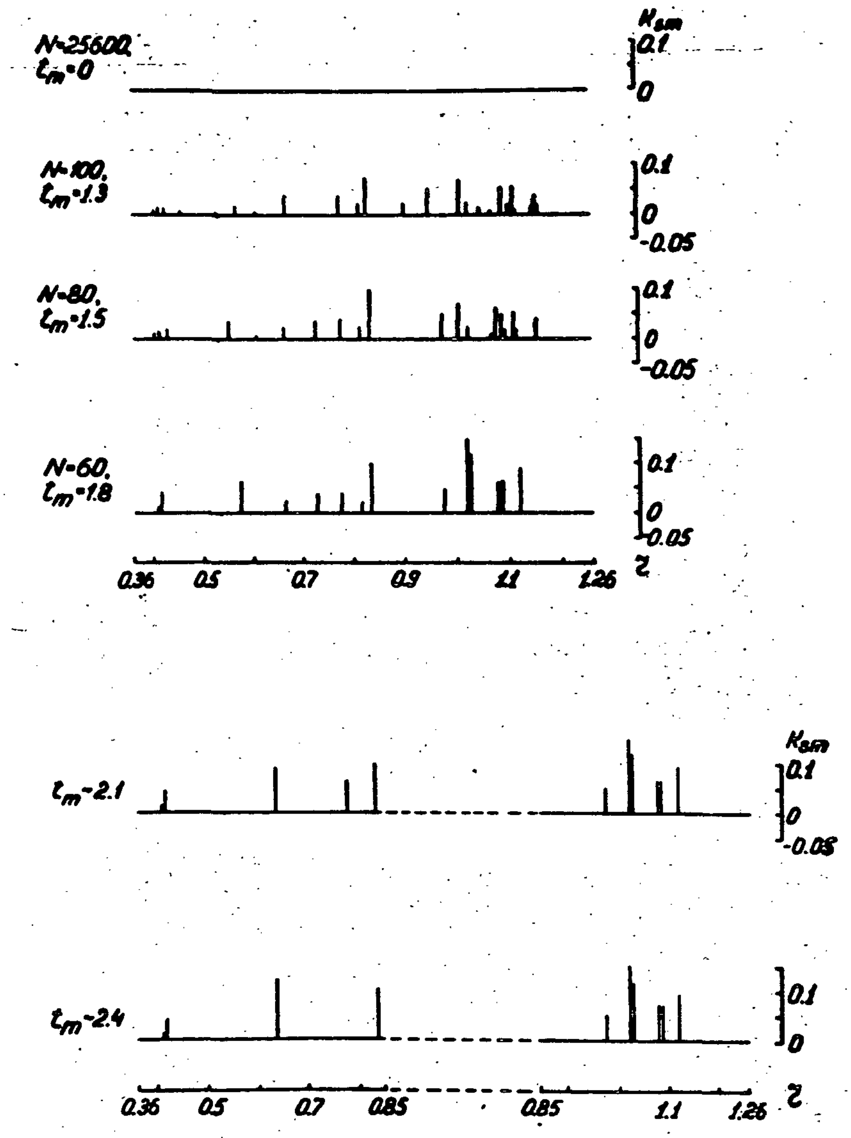

Figure 2.3. Dynamics of the accumulation process of formation of the terrestrial planets in version 4,0 (S_0 =0,79) . Rotational momentum for the largest bodies.

Sequential formation of the planets occurred in the course of simulating the evolutionary process. In this model, Venus was formed first (T_m = 2.5), the earth was then formed (T_m = 6.6), Mercury

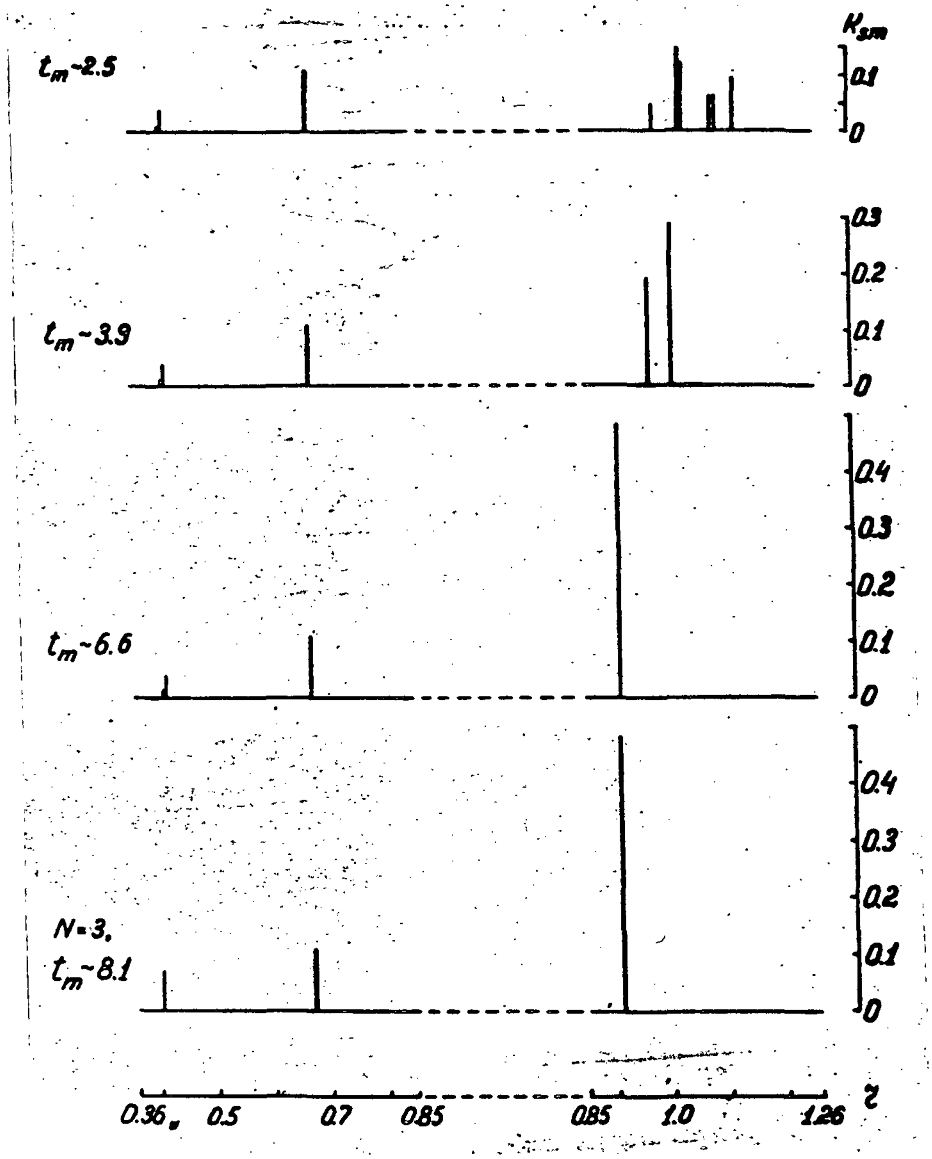

Figure 2.4. Continuation of Figure 2.3.

was formed last (T_m = 8.1). It is interesting to note that at the concluding stage of evolution, two bodies of approximately the same mass moved for a long time in the zone of Mercury in very close orbits — the pattern is typical for the evolution of "strong" accumulation zones. An analogous pattern also occurred in the zone of formation of the Earth.

The final results of the simulation are presented in Table III, where the parameters of the real planetary system are presented along with the parameters of the model under consideration.

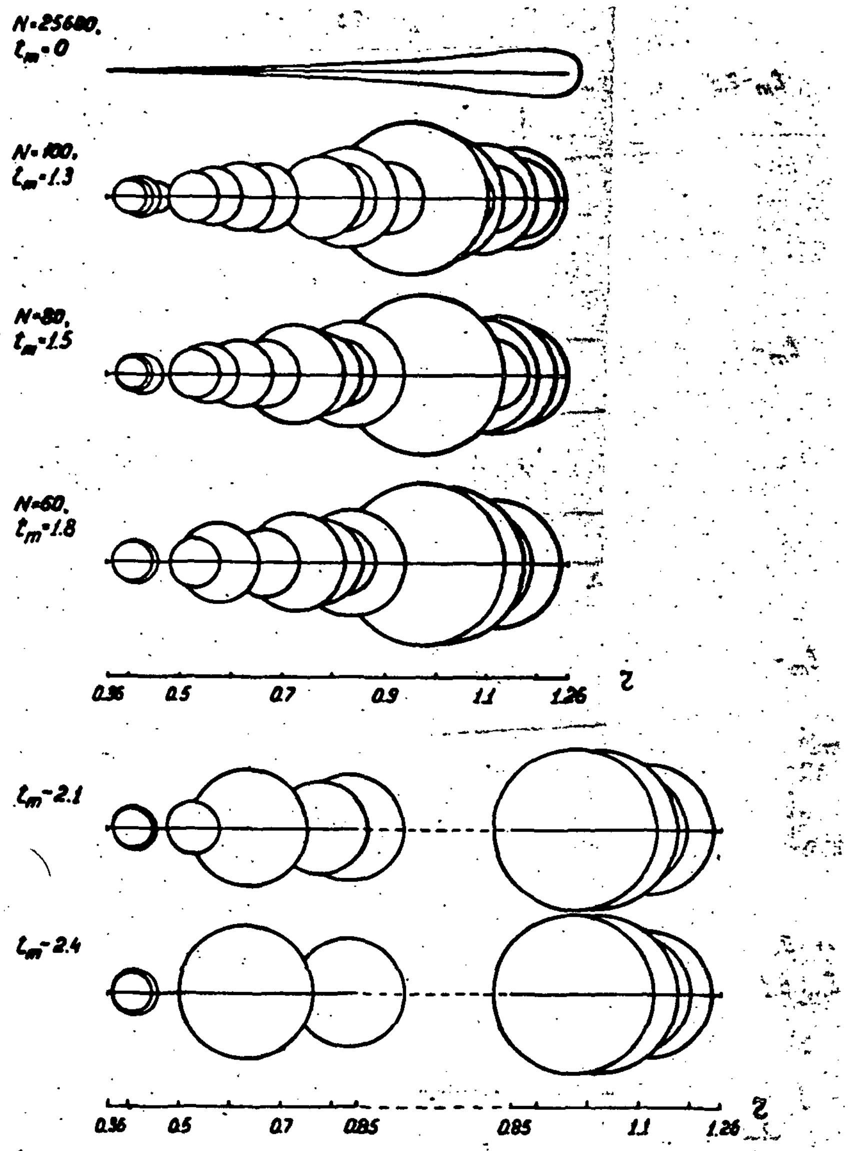

Figure 2.5. Dynamics of the accumulation process of formation of the terrestrial planets in version 4.0 (S_0 =0,79) . Sizes of the largest bodies for the corresponding intervals of the mass histograms.

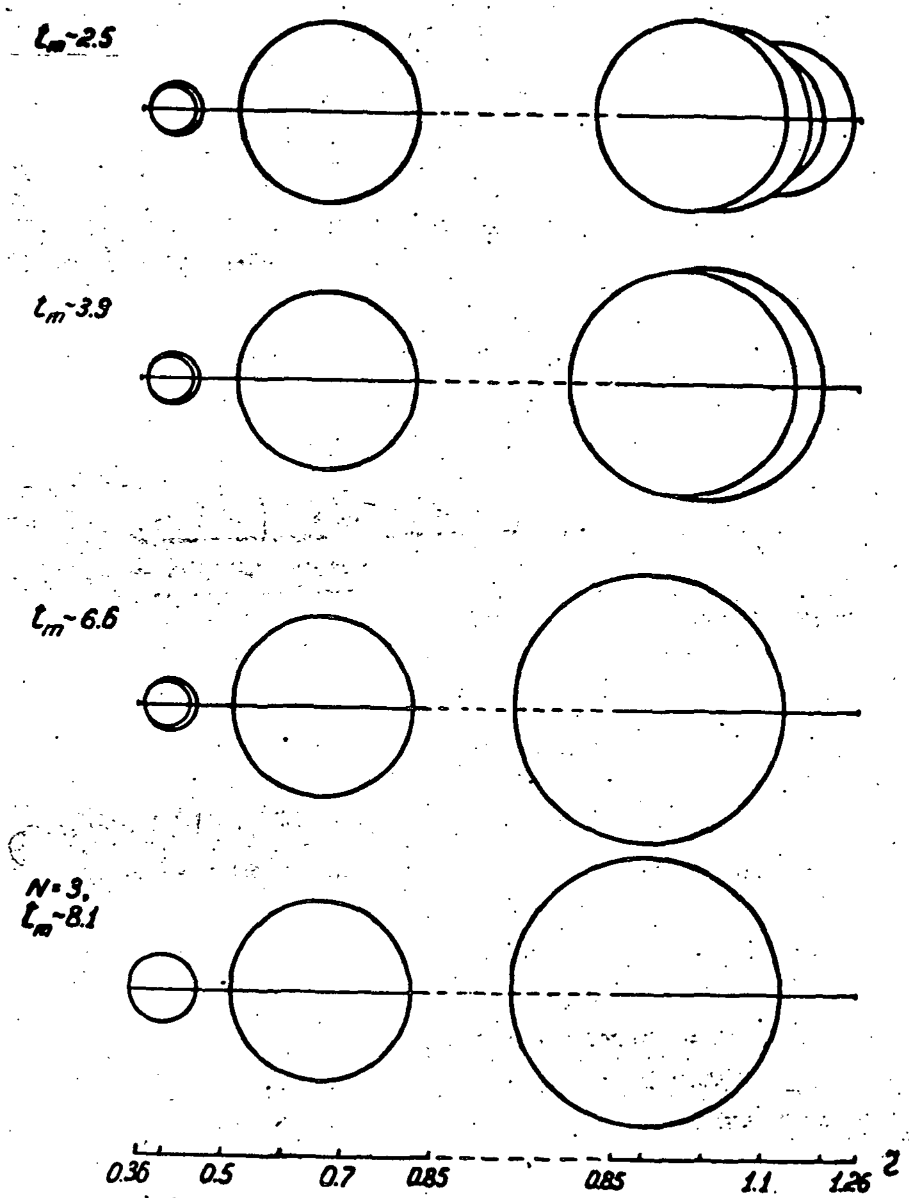

Figure 2.6. Continuation of Figure 2.5.

TABLE III. PARAMETERS OF THE PLANETARY SYSTEM FOR VERSION 4.0 GRAVITATIONAL MODEL OF THE PROTOPLANETARY CLOUD. PRESENTED ARE: TRUE VALUES OF THE SEMIMAJOR AXIS α, MASS m, ROTATIONAL MOMENTUM K_s FOR MERCURY, VENUS, THE EARTH, $\Delta\alpha$, Δm — DEVIATIONS IN THE SEMIMAJOR AXIS AND THE MASS OF THE SOLUTION OF VERSION 4.0 FROM THE TRUE VALUES OF THESE PARAMETERS (IN %)

n	a	$\tilde{a}$	m	K_{sm}
I	0.408	0.4I9	0.I39	0.055
2	0.670	0.699	0.766	0.I09
3	0.907	I.060	0.959	0.484
4	I.246	I.249	0.006	0.009

	a	m	K_s	$\Delta a\%$	$\Delta m\%$
Mercury	0.387	0.055	$0.42 \cdot 10^{-8}$	8	I53
Venus	0.723	0.8I6	$-0.62 \cdot 10^{-8}$	3	6
Earth	I.000	I.000	$0.72 \cdot 10^{-5}$ x)	6	4

x) K_s for the earth-moon system.

4. RETROGRADE ROTATION OF VENUS /26

Table III of the preceeding section along with Figure 2.1 - 2.6 demonstrate the rather good qualitative, and quite satisfactory quantitative in some parameters, description of the process of formation of the terrestrial planets. In fact, the deviations of the semimajor axes of the orbits of all the planets and also the masses of the Earth and Venus from the true values, considering the coarseness of the mathematical model of the cloud, are within more or less acceptable limits (less than 8% of the true values).

The pattern of the distribution of the rotational momentum /27
over the planets is very important for the problem under study. Both the model K_{sm} and the true K_s values of this momentum are presented in Table III. It is seen from the table that in contrast to the actual planetary system, the rotational momenta of all the planets in the model have positive values. At first glance, there is significant contrast in the pattern of the momentum distribution in the real and model systems, i.e., the planet Venus has negative

rotational momentum in the real system. However, this contrast is significantly moderated after a detailed analysis of Table III and particularly after analysis of the dynamics of the evolutionary process of the formation of the planetary system shown in Figures 2.1 - 2.6. Actually, it is easily discovered with a more detailed analysis of Table III that the rotational momentum of Venus is relatively insignificant as compared to the rotational momentum of the Earth, in spite of the relative closeness in mass of the two planets. Detailed analysis of the dynamics of the evolutionary process of formation of the planets shown in Figures 2.1 - 2.6 permits discovering two important peculiarities of this process.

Firstly, at the concluding stage of evolution, the group of protoplanets of the accumulation zone of Venus is significantly less compact than the analogous group of the zones of Mercury and the Earth.

Secondly, at the concluding contacts of these protoplanets (beginning at $t_m = 1.8$), their maximum angular momentum is practically unchanged.

Both these peculiarities are typical for accumulation zones of the "weak" type.

In connection with all the indicated facts, the assumption naturally arises that for a somewhat more fortunate choice of the basic parameters of the protoplanetary cloud, the rotational momentum of Venus may take a negative value. Since the evolutionary process of formation of the planets is most sensitive to the initial effective area of the cross sections of the protoplanets S_0, this parameter was primarily subjected to variation. A series of numerical experiments was carried out with decreasing values of the parameter S_0. The parameters α_{max}, σ_{00} and λ_1 were maintained as before, while the parameter S_0 was varied in the range from $S_0 = 0.79$ to $S_0 = 0.57$. /28

Table IV presents a summary of the results of the numerical

TABLE IV. PARAMETERS OF THE PLANETARY SYSTEM FOR VERSIONS 4.0 - 4.4. GRAVITATIONAL MODEL OF THE PROTOPLANETARY CLOUD. PARAMETERS OF THE EXPONENTIAL DISTRIBUTION OF THE SURFACE DENSITY: σ_{00} = 0.54, λ_1 = 0.28; SIZE OF THE DISK 1.21).

		Mercury				Venus				Earth			
N_{ver}	S_0	a	$\tilde{a}$	m	K_{sm}	a	$\tilde{a}$	m	K_{sm}	a	$\tilde{a}$	m	K_{sm}
4.0	0.79	0.4I	0.42	0.I4	0.055	0.67	0.70	0.77	0.II	0.9I	I.06	0.96	0.48
4.I	0.74	0.38	0.39	0.048	0.037	0.64	0.64	0.75	0.0064	0.88	I.04	I.08	0.52
4.2	0.68	0.4I	0.42	0.I3	0.039	0.66	0.70	0.84	0.I3	0.96	I.08	0.90	0.39
4.3	0.62	0.44	0.44	0.20	0.030	0.67	0.7I	0,67	0.I2	0.95	I.06	I.00	0.33
4.4	0.57	0.43	0.44	0.I9	0.035	0.66	0.69	0.65	0.I0	0.94	I.06	I.03	0.34

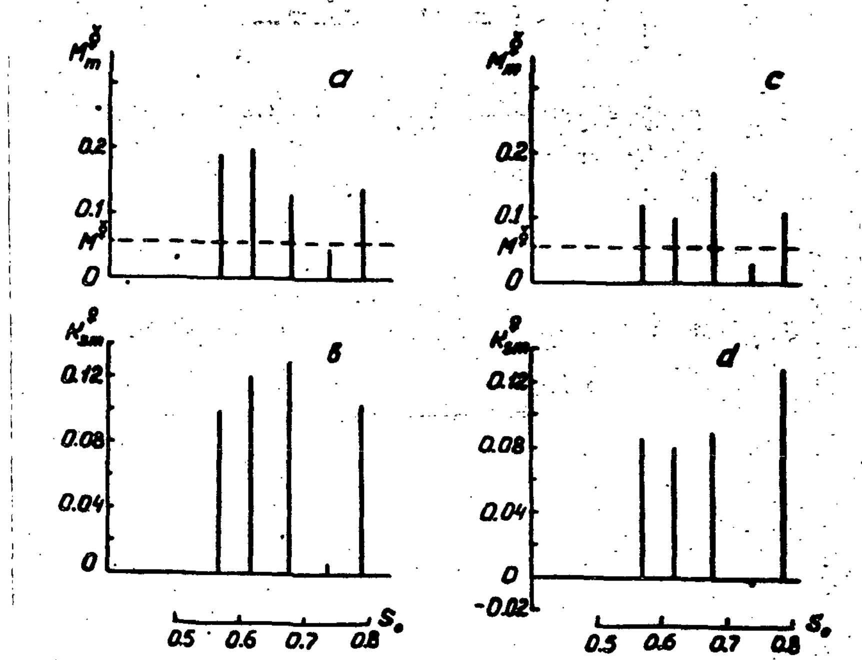

Figure 3. Dependence of the mass of Mercury $M_m^{☿}$ and the rotational momentum of Venus $K_{sm}^{♀}$ on the parameter S_0: a, b — versions 4.0 - 4.4; c, d — versions 5.0 - 5.4.

experiments for all the indicated versions. Graphs of the dependence of the rotational momentum of Venus $K_{sm}^{♀}$ and the mass of Mercury $M_m^{☿}$ on the initial effective area of the cross sections of the protoplanets S_0 are presented for these same versions in Figure 3.

We see from Table IV and Figure 3 that a decrease in the parameter S_0 from S_0 = 0.79 (version 4.0) to S_0 = 0.74 (version 4.1) leads to a significant decrease in the mass of Mercury and rotational momentum of Venus. However, a further decrease in the parameter S_0 leads to a very significant increase in the mass of Mercury and positive rotational momentum of Venus.

The outer boundary of the model of the protoplanetary cloud α_{max} was decreased in another series of numerical experiments. The new value α_{max} = 1.11 was taken. In order to maintain the values of the total mass M and total angular momentum K of the protoplanetary cloud as before, i.e., corresponding to (3.5), it was also necessary to change somewhat the parameters σ_{00} and λ_1. The new values σ_{00} =0.66, λ_1 =0.31 were taken for them. Five versions (versions 5.0 - 5.4) were calculated for the same values of the parameter S_0 used in calculating versions 4.0 - 4.4. The results of these calculations are presented in Table V and in Figure 3 (graphs c and d). /31

TABLE V. PARAMETERS OF THE PLANETARY SYSTEM FOR VERSIONS 5.0 - 5.4. GRAVITATIONAL MODEL OF THE PROTOPLANETARY CLOUD:
σ_{00}=0.66, λ_1=0.31, $r \in$ (0.36, 1.11).

		Mercury				Venus				Earth			
$N_{ver.}$	S_0	α	$\tilde{\alpha}$	m	K_{sm}	α	$\tilde{\alpha}$	m	K_{sm}	α	$\tilde{\alpha}$	m	K_{sm}
5.0	0.79	0.39	0.40	0.11	0.062	0.62	0.65	0.81	0.13	0.88	0.98	0.96	0.33
5.1	0.74	0.38	0.39	0.032	0.018	0.62	0.62	0.86	-0.0029	0.83	0.97	0.97	0.49
5.2	0.68	0.41	0.42	0.17	0.047	0.64	0.66	0.76	0.090	0.88	0.98	0.95	0.34
5.3	0.62	0.39	0.40	0.10	0.042	0.62	0.65	0.84	0.079	0.86	0.99	0.93	0.45
5.4	0.57	0.38	0.40	0.12	0.039	0.64	0.65	0.84	0.073	0.92	0.99	0.90	0.24

Comparison of both series of numerical experiments indicates a definite similarity between them (the decrease in mass of Mercury and momentum of Venus at $S_0 = 0.74$). However, at $S_0 = 0.74$ in version 5.1, in contrast to version 4.1, Venus acquires a small negative rotational momentum, while Mercury has a very small mass as compared to the other versions. In view of the fundamental importance of this version, a more or less detailed picture of the concluding stage of the evolutionary process of the formation of the planetary system is presented for it in Figures 4.1 - 4.6. The dynamics of the formation of the retrograde rotation of Venus in the example under consideration has some features similar to the dynamics of the formation of the rotation of the first planet in version 1.5 from [1] of the drop model of the cloud. In this case, as in version 1.5, there appears near the inner boundary of the accumulation zone a relatively large body — the "nucleus" of the future planet which, in experiencing a series of glancing collisions, gradually recedes from the central body. Table Va illustrates the evolution of the leading body ("nucleus") of the accumulation zone of Venus (version 5.1).

To correctly evaluate the above presented results of the numerical experiment, it is necessary to make one important remark. The indicated results are qualitative in nature to a significant degree and one should refer to their quantitative side with great care. This refers above all to the determination of the mass of Mercury and the rotational momentum of Venus. The final effect of exhaustion of material caused by the coarseness of the model of the cloud, /40 in spite of the measures taken to neutralize it (isolation at the concluding stage of evolution of the acumulation zone of the earth), often led to significant distortion in the pattern of the evolutionary process in the zone of Venus. This distortion led in turn to instability of the results of the calculation expressed, in particular, in a "rough" dependence of the parameters of the simulated planetary systems on the parameters S_0 and α_{max}. It is possible that the instability of the results of the calculation is also related to other causes not completely clear as yet. It still has

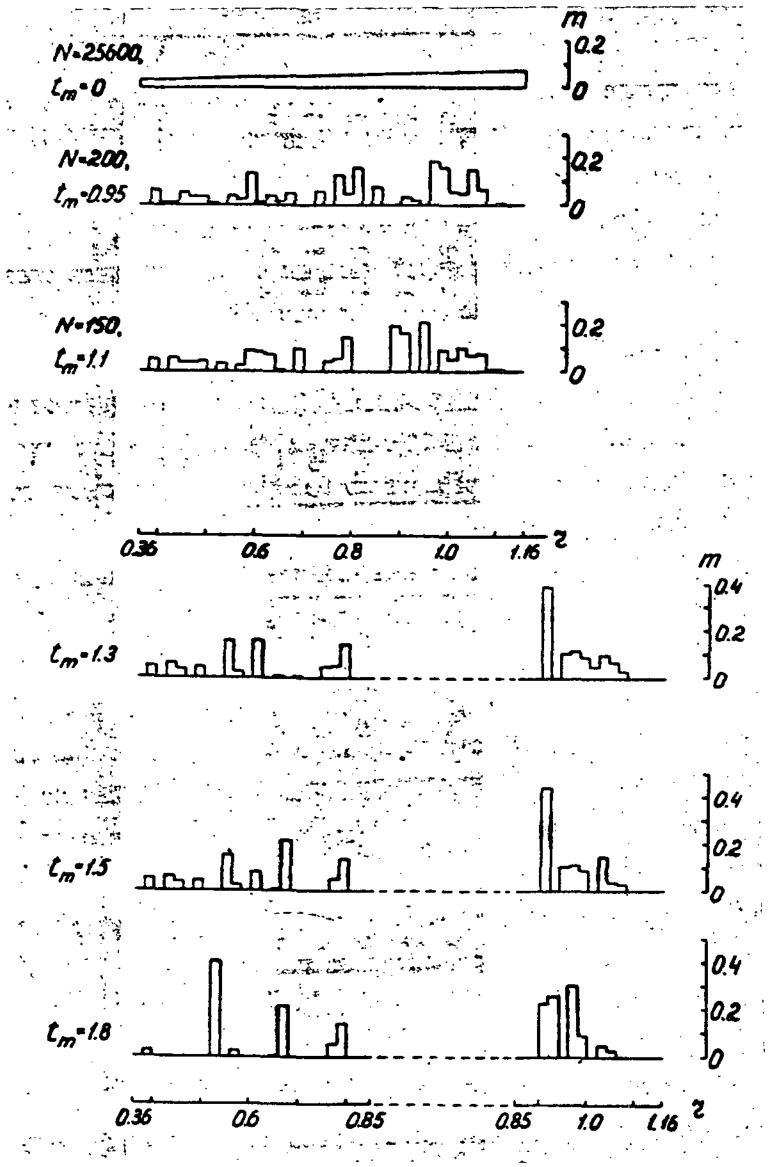

Figure 4.1. Dynamics of the accumulation process of formation of the terrestrial planets in version 5.1 (S_0 = 0.74). Mass histograms.

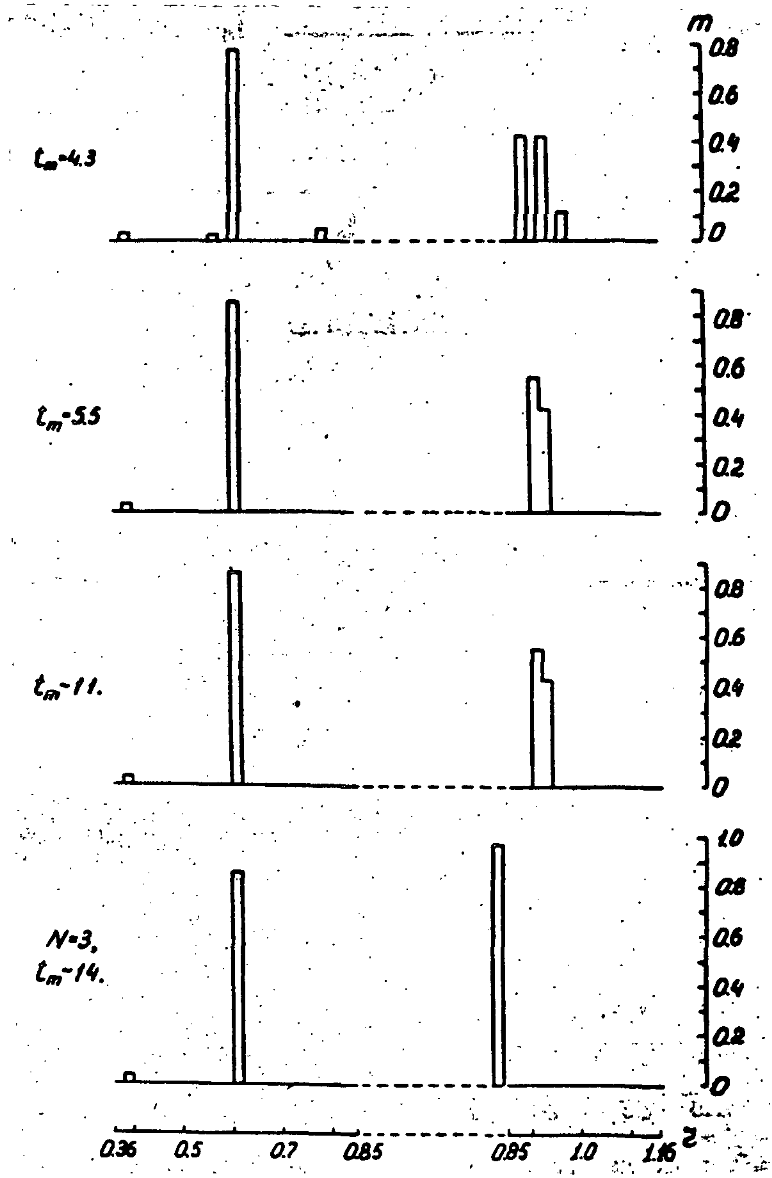

Figure 4.2. Continuation of Figure 4.1. Venus formed at t_m = 5.5, Mercury is formed at t_m = 11, the earth is formed at t_m = 14.

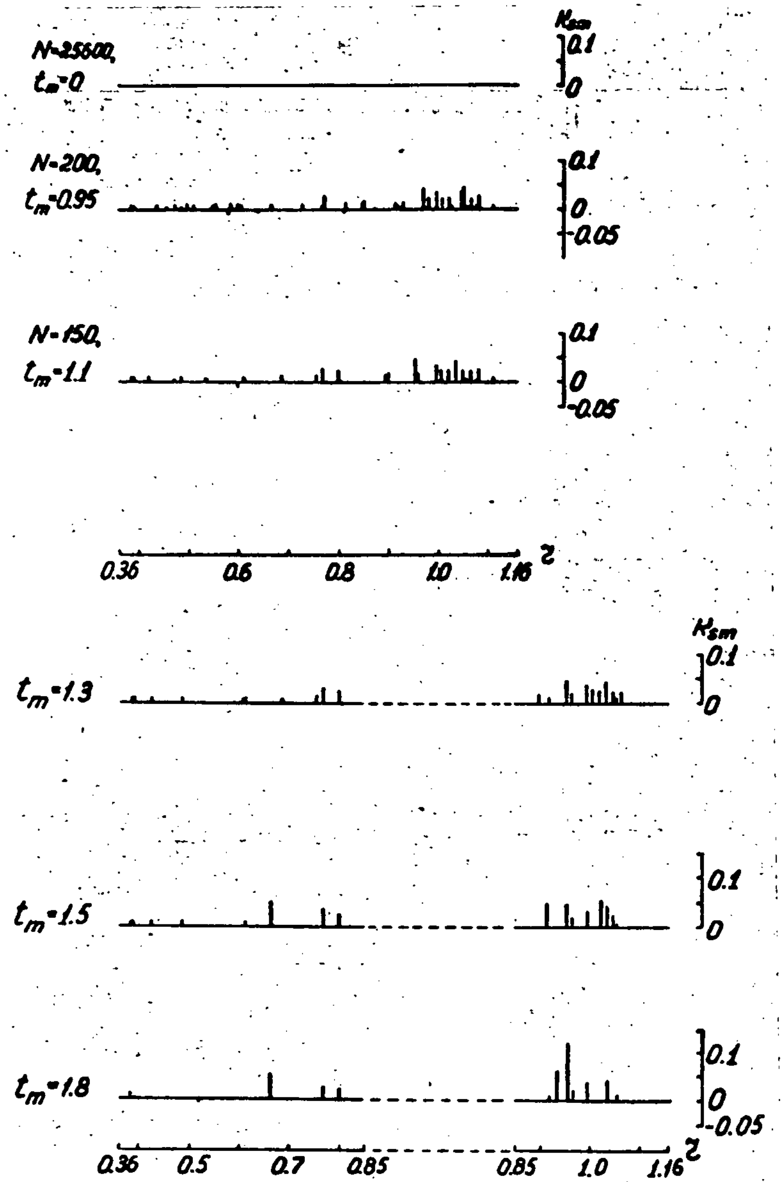

Figure 4.3. Dynamics of the accumulation process of formation of the terrestrial planets in version 5.1 (S_0 = 0.74). Rotational momentum for the largets bodies.

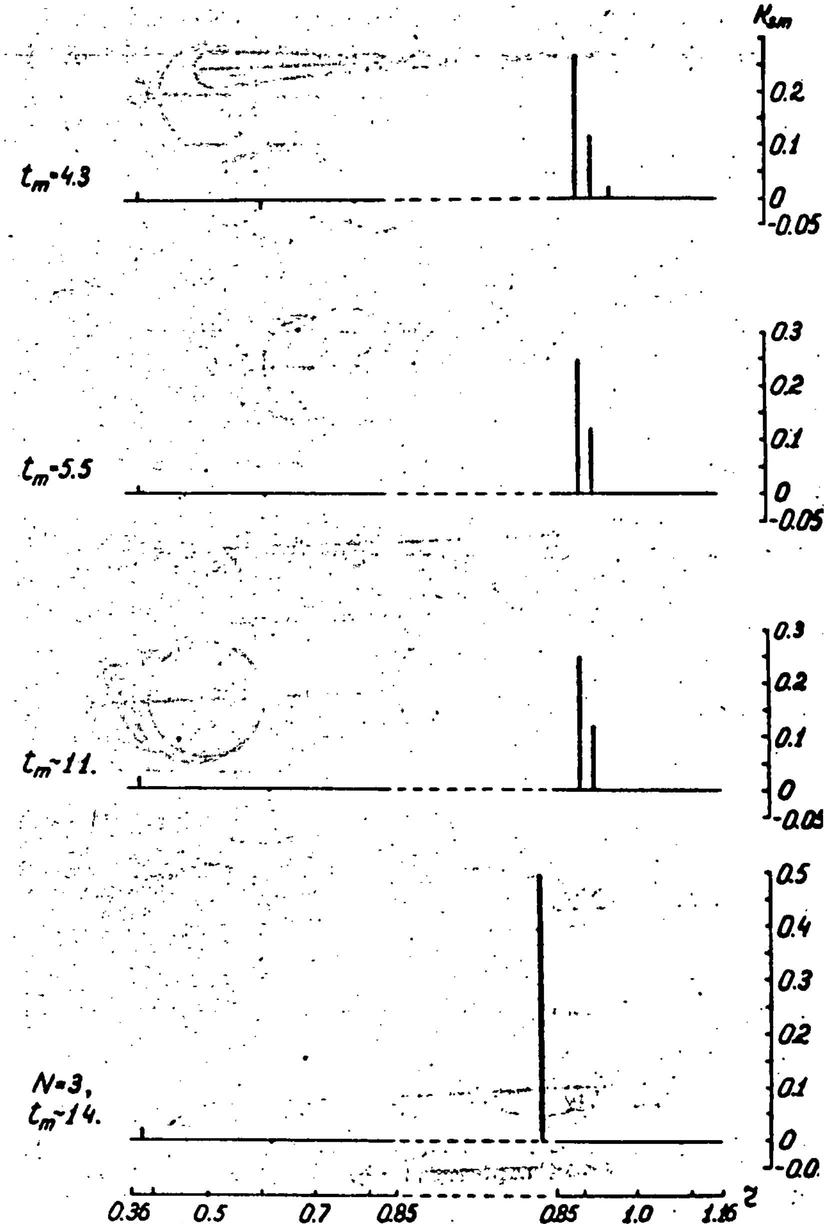

Figure 4.4. Continuation of Figure 4.3. Final stage of formation of the retrograde rotation of Venus.

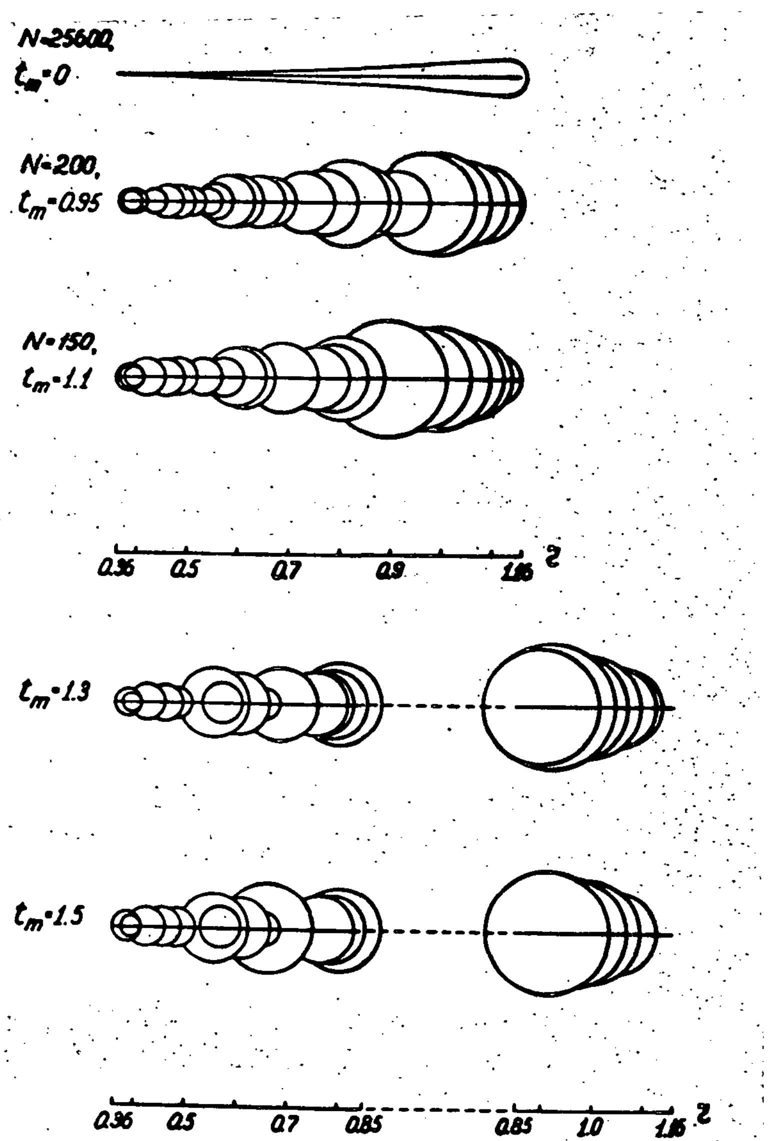

Figure 4.5. Dynamics of the accumulation process of formation of the terrestrial planets in version 5.1 (S_0 = 0.74). Sizes of the largets bodies for the corresponding intervals of the mass histograms.

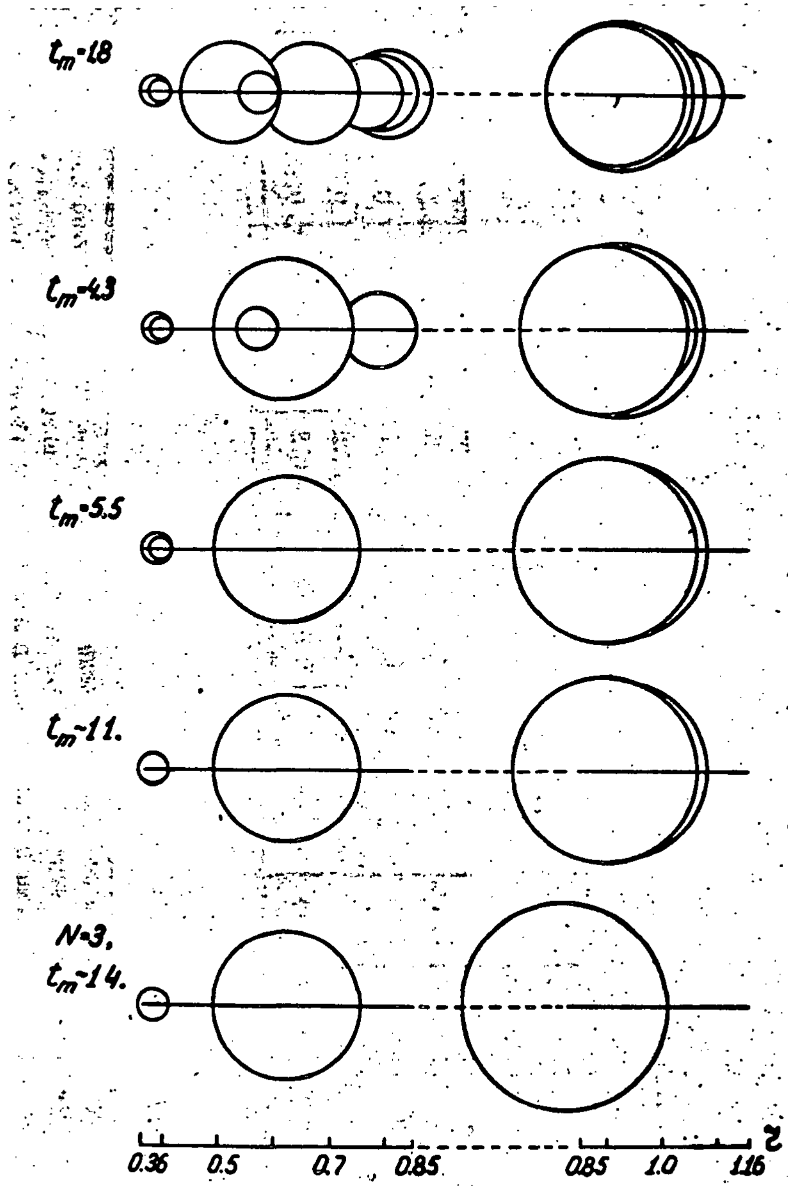

Figure 4.6. Continuation of Figure 4.5.

TABLE Va. DYNAMICS OF THE FORMATION OF VENUS IN VERSION 5.1. GRAVITATIONAL MODEL OF THE PROTOPLANETARY CLOUD. t_m — TIME OF THE EVOLUTIONARY PROCESS FOR THE GENERALIZED MODEL; ALSO PRESENTED: MASS OF THE LEADING BODY OF THE ZONE OF VENUS — $m^{♀}$; SEMIMAJOR AXIS OF THE BODY — $a^{♀}$; ROTATIONAL MOMENTUM OF THE BODY — $K_{sm}^{♀}$; ECCENTRICITY — $e^{♀}$; EFFECTIVE RADIUS OF THE BODY $R^{♀}$:

t_m	$m^{♀}$	$a^{♀}$	$K_{sm}^{♀}$	$e^{♀}$	$R^{♀}$
I.I	0.06	0.42	+0.0082	0.002	0.037
I.4	0.I5	0.56	-0.0038	0.007	0.069
I.8	0.4I	0.52	-0.0060	0.009	0.089
3.4	0.63	0.57	+0.0085	0.0I4	0.II3
4.3	0.77	0.62	-0.0I55	0.0I5	0.I30
4.5	0.83	0.63	-0.0I68	0.0I7	0.I35
5.5	0.86	0.62	-0.0029	0.0I7	0.I36

to be determined whether this instability is a consequence only of the coarseness of the mathematical model of the protoplanetary cloud or, besides that, it reflects some important physical peculiarities of the process of formation of planetary systems.

In spite of the remarks presented above, one can evidently consider as established the fundamental possibility of Venus acquiring retrograde rotation during the annular compression of material due to the regular mechanism of glancing collisions.

At the end of this section, Table VI presents for versions 4.0, 4.1, and 5.1 estimates of the radii of the initial effective bodies $\bar{R}_0$ for $z = 1$ and the time of evolution of the accumulation zone of the Earth T calculated with the help of the similarity theory [1]. Comparison of the data of TAble VI with the data for the Earth of Table 3 from [1] ($T \sim 14000$ yr, $\bar{R}_0 \sim 720$ km) shows satisfactory agreement in orders of magnitude of these data, in spite of the difference in models of protoplanetary clouds.

TABLE VI. SOME CHARACTERISTICS OF THE EVOLUTIONARY PROCESS OF FORMATION OF THE EARTH. PRESENTED ON THE BASIS OF SIMILARITY THEORY AND DATA OF CALCULATIONS FROM VERSIONS 4.0, 4.1 AND 5.1, GRAVITATIONAL MODEL: REDUCED MODEL ROTATIONAL MOMENTUM $\bar{K}_{sm}$, SIMILARITY PARAMETER p, TIME OF EVOLUTIONARY PROCESS OF FORMATION OF THE EARTH $\bar{T}$, RADII OF THE INITIAL EFFECTIVE BODIES OF THE PROTOPLANETARY CLOUD IN THE ACCUMULATION ZONE OF THE EARTH $\bar{R}_0$. CALCULATION CARRIED OUT FOR $K_S = 0.72 \cdot 10^{-5}$.

$N_{ver.}$	$\bar{K}_{sm}$	p	T (yr)	$\bar{R}_0$ (km)
4.0	0.47	4060	24000	970
4.I	0.5I	4300	26000	890
5.I	0.50	4260	62000	855

CONCLUSION /42

The process of formation of the terrestrial planets has been investigated in this work. The gravitational model of the protoplanetary cloud was used for mathematical simulation of this process. In the course of the investigation, several examples displayed the effect of Venus acquiring a slow retrograde rotation, whereas Mercury and the Earth acquired prograde rotation around their axes.

The numerical experiments with the help of similarity theory perimtted finding the sizes of the initial protobodies in the zone of the Earth and the time of formation of the Earth. It was found that the time for the formation of the Earth corresponds to the so-called "short-scale" — 10^4-10^5 years.

A "rough" dependence of the parameters of the simulated planetary systems on the parameters S_0 and α_{max} is found. However, it is not yet clear whether this effect is a consequence of the coarseness of the mathematical model being used or whether it reflects the possibility of instability in the process of formation of the terrestrial planets. It is significant that, in spite of this "roughness", there occurred in all the numerical experiments a clear correlation between the small mass of Mercury ($m < 0.05$) and the very small rotational momentum of Venus (positive or negative). Thus, there is found as a result of the performed investigation a previously unknown [4] close relationship between the process of formation of the orbits and masses of the planets and the process of formation of their rotation about their axes.

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