NASA TECHNICAL MEMORANDUM NASA TM-75648

EVALUATION OF THE EFFECT OF ELASTIC JOINTS ON THE AUTO-OSCILLATION OF SPACECRAFT WITH GAS-REACTIVE DIRECTION SYSTEMS

G. G. Sasin

Translation of "Otsenka Vliyaniya Uprugosti Prisoedinennykh Elementov Konstruktsii na Avtokolebaniya Kosmicheskogo Letatel'nogo Apparata s Gasoreaktivnoi Sistemoi Upravleniya," USSR Academy of Sciences Institute of Space Research, Moscow, Report Pr-407, 1978, pp 1-52

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JULY 1979

STANDARD TITLE PAGE

1. Report No. NASA TM-75648	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle EVALUATION OF THE EFFECT OF ELASTIC JOINTS ON THE AUTO-OSCILLATION OF SPACECRAFT WITH GAS-REACTIVE DIRECTION SYSTEMS		5. Report Date July 1979
		6. Performing Organization Code
7. Author(s) G.G. Sasin USSR Academy of Sciences Institute of Space Research		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASW - 3199
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Otsenka Vliyaniya Uprugosti Prisoedinennykh Elementov Konstruktsii na Avtokolebaniya Kosmicheskogo Letatel'nogo Apparata s Gasoreaktivnoi Sistemoi Upravleniya," USSR Academy of Sciences Institute of Space Research, Moscow, Report Pr-407, 1978, pp 1-52

16. Abstract

Research was conducted on the effect of elasticity in the construction joints on the auto-oscillation of spacecraft with gas-reactive direction systems; a mathematical model was obtained, on the basis of the method of mixed coordinates, of a generalized flexible spacecraft at one end of which was appended the directive action of a system of gas-reactive nozzles. Various structural forms were obtained functionally describing flexible spacecraft, as systems consisting of a solid central body with flexible structural elements joined to it.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified-Unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 53	22. Price

PREFACE

Research was conducted on the effect of elasticity in the construction joints on the auto-oscillation of spacecraft with gas-reactive direction systems; a mathematical model was obtained, on the basis of the method of mixed coordinates, of a generalized flexible spacecraft at one end of which was appended the directive action of a system of gas-reactive nozzles. Various structural forms were obtained functionally describing flexible spacecraft, as systems consisting of a solid central body with flexible structural elements joined to it. Studies of the auto-oscillatory processes were conducted on the basis of a method of point-by-point transformation. The work derives the equations of the correspondence function and the equations of velocity at the limiting cycle, taking into account the delays in the relay and command elements. These equations were studied using analog and digital computers.

LIST OF SYMBOLS

$\bar{F}^i$, $\bar{T}^i$ - force and moment, respectively, acting on an element of the structure

m_i, $\bar{H}^i$ - mass and kinetic moment, respectively, of an element of the structure

$\bar{a}^{p_i}$ - inertial acceleration of A_i, an element of the structure

O - point located on body B in the center of mass system when the system is undeformed

$\bar{C}$ - determines the displacement of the center of mass

Q - any point on the boundary between bodies A and B

Q_i - the point occupied by the element A_i when the structure is undeformed

p_i - the point occupied by the element A_i when the structure is displaced

$\bar{R}$, $\bar{z}^i$ - vectors connecting the body B to A_i

$\bar{X}$ - the vector which determines the position of the center of mass in inertial space

$\bar{a}_1, \bar{a}_2, \bar{a}_3$ - unit vectors

$\bar{b}_1, \bar{b}_2, \bar{b}_3$ - unit vectors

$\bar{\omega}$ - vector of the inertial angular velocity of the body B

$\bar{\omega}^i$ - inertial angular velocity of the element A_i

$\bar{\lambda}^i$ - linear vector: a function of the inertia of element A_i

$\bar{\beta}^i$; β^i - vector and B-based matrix of small incremental motions of A_i with respect to B

θ - matrix of directional cosines

I^i - matrix of the moments of inertia of element A_i

M - total mass of the spacecraft

K	- matrix of rigidity of the structure
Λ	- matrix of external forces and moments, 6n x I
$\bar{I}$	- linear vector: a function of the inertia of the entire system relative to point O
dm	- infinitesimal mass of the element A_i
$\bar{P}$	- the vector joining the elementary mass dm of element A_i to the point O at the center of the system of mass
$\bar{\rho}_i$	- vector of the position of element A_i relative to its own center of mass ρ_i
θ	- matrix of the angles of orientation of B
$\theta_1, \theta_2, \theta_3$	- elements of θ
η	- matrix, with 6n x I elements, of normal coordinates
Φ	- matrix of vectors
ζ_i	- damping coefficient in the i^{th} normal mode of oscillation
σ_i	- true frequency of the i^{th} normal mode of oscillation
t	- time
t^0	- dimensionless time
S	- Laplace operator
$G_\alpha(S)$	- transmission function of the unsubstituted circuit
χ_0	- effectiveness of the direction
δ	- zone of insensitivity of the relay element
H	- hysteresis of the relay element
$K_{0}\delta$	- coefficient of amplification of an aperiodic reverse signal (ARS)
τ_o	- time constant for switching on the ARS circuit
τ_B	- time constant for switching off the ARS circuit
F	- level of thrust of the gasreactive system

L	- moment arm
U(t)	- directional signal
Z(t)	- signal at the output of an ARS
$\pm f$	- signal at the output of a relay element
$\dot{\theta}_a$	- velocity in the limit cycle
$(\ )^T$	- transposition of a matrix
$(\tilde{\ })$	- diagonally symmetric matrix
$(\ast)$	- derivative with time or differentiation of a vector with time in inertial space
$(^0)$	- differentiation of a vector with time in the relative system of coordinates: a partial derivative
$(\hat{\ })$	- dimensionless quantity
$(\vec{\ })$	- vectorial quantity

EVALUATION OF THE EFFECT OF ELASTIC JOINTS ON THE AUTO-OSCILLATION OF SPACECRAFT WITH GAS-REACTIVE DIRECTION SYSTEMS

G. G. Sasin,
USSR Academy of Sciences Institute of Space Research

Both the scientific and experimental requirements of contemporary artificial earth satellites (AES) have become more rigorous every year, as well as those with respect to the accuracy of stabilization. So, for example, the AES placed in geostationary orbits must be light and compact but at the same time must have a substantial lifetime. The greater and greater energy consumption of such AES has necessitated the use of large panels of solar cells (SC). For the triaxial stabilization of a similar spacecraft at least one type has used systems of reaction motors of both hot and cold gases and will soon use electroreactive motors (ERM). The wide frequency range of the spectrum of these engines will generate a variety of modes of oscillation. The effect of these high resonance frequencies on the characteristics of the roll regulator, the requirement of relatively sensitive reception in the 0-10 Hz band, the achievement of a high precision of orientation and stabilization (from several minutes down to seconds of arc), can all lead to substantial difficulty in setting up such systems. /5* /6

Experience has shown that, with the growing requirements for accuracy of orientation and the high cost of mockups, there is an increasing value in many projected developments in creating a mathematical model of the spacecraft with heightened flexibility and complicated dynamics of construction,

---

*Numbers in the margin refer to pagination in the foreign text.

since the performance of tests on the ground of such structures is rather expensive and often simply impossible. At the present time there already exists [1] an entire group of mathematical models describing flexible spacecraft, the most general outlines of which are that they are formed of an essentially solid body with elastic consoles attached to it, which in turn can be composed of a continuous medium [2] or of a discrete collection of adjoining solid bodies joined by means of massless elastic elements called finite elements [3]. Depending on whether the initial model of the panels is continuous or discrete, its displacement with respect to the parent body is characterized by either distributive or modal coordinates, while the orientation of the basic body of the spacecraft in inertial space is always described by the three angles of its spatial position. Reference [4] proposed a method, combining the advantages of the discrete and distributive coordinates, called "the method of mixed (hybrid) coordinates." This method makes it possible to provide a mathematical description of the basic solid body with attached flexible panels in such a manner that the oscillation of the basic solid body is described with the usual differential equations and the oscillations of the flexible panels with equations in partial derivatives.

In this work the method of mixed coordinates is used to obtain a mathematical model of a generalized flexible spacecraft, to the basic body of which the directing action of a system of gas-reactive nozzles is added. The various structural forms of the transmission function are derived, as well as the relationships among them. A method of pointwise transformation is used to study auto-oscillatory processes in the spacecraft, whose direction circuit is pictured as an aperiodic reverse signal (ARS) with a double time constant which includes a relay element with a band of insensitivity. The equations of correspondence which are obtained as well as the equation /7 of velocity in the limit cycle take into account the delays in the relay element and control console. The research made use of an analog computer of middling size (AVM-MN-18M) and a small digital machine (type MIR-2) for which programs were developed which made it possible not only to study the transitional and established processes from the initial system of equations, to construct the König-Lamery diagrams, and to obtain the value of the velocity in the limit cycle, but also to maintain a double control on the correctness of the solution.

Description of the Mathematical Model /8

The dynamical model of the system is sketched in Figure 1. It consists of a solid, rigid body B and, attached to it, the flexible structures A, consisting in turn of the elementary bodies A_i. Assuming the elastic deformation to be negligibly small, let us first derive the equations of the superstructure.

The Newton-Euler equations describing the motion of the element A_i can be written as

$$\bar{F}^i = m_i \bar{a}^{\rho_i} \tag{1}$$

$$\bar{T}^i = \dot{\bar{H}}^i \tag{2}$$

In the inertial coordinate system the acceleration $\bar{a}^{\rho_i}$ can be obtained as twice the differentiated sum of the displacement vectors $\bar{X} + \bar{C} + \bar{R} + \bar{\tau}^i + \bar{U}^i$, which determine the relation between the immovable point O' in the inertial space and ρ_i (see Figure 1).

Let the orthonormal system of vectors $\bar{b}_1, \bar{b}_2, \bar{b}_3$ refer to body B, and $\bar{a}_1, \bar{a}_2, \bar{a}_3$ to body A. Then the transition from the system $\bar{a}_\alpha$ to the system $\bar{b}_\alpha$ (where α = 1,2,3) can be performed by means of the time-varying matrix of directional cosines, C. That is,

$$\begin{Bmatrix} \bar a_1 \\ \bar a_2 \\ \bar a_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \bar b_1 \\ \bar b_2 \\ \bar b_3 \end{Bmatrix} \tag{3}$$

or

$$\{\bar a\} = C\{\bar b\} \tag{4}$$

Geometrically, the coefficient C_{ik} is the cosine of the angle /9
between the base vectors $\bar b_i$ and the rotated base vector $\bar a_k = C\bar b_k = \sum_{i=1}^{3} C_{ik}\bar b_i$. We then write

$$\bar a^{p_i} = \frac{{}^i d^2}{dt^2}(\bar X + \bar C + \bar R + \bar r^i + \bar U^i) \tag{5}$$

The relation between the inertial base vectors $\bar j_1, \bar j_2, \bar j_3$ and the base vectors $\bar b_1, \bar b_2, \bar b_3$ is determined by the matrix of directional cosines B , or

$$\{\bar b\} = B\{\bar j\} \tag{6}$$

Realizing that $\quad {}^i d/dt\,\bar c = {}^b d/dt\,\bar c + \bar\omega^{bi} \times \bar c$

we obtain from equations (5) and (1)

$$\bar F^i = m_i\{\ddot{\bar X} + \overset{\circ\circ}{\bar C} + \overset{\circ\circ}{\bar U}{}^i + 2\bar\omega\times(\overset{\circ}{\bar C} + \overset{\circ}{\bar U}{}^i) + \dot{\bar\omega}\times(\bar C + \bar R + \bar r^i + \bar U^i) + \bar\omega\times[\bar\omega\times(\bar C + \bar R + \bar r^i + \bar U^i)] , \tag{7}$$

where (^{0}) is the vectorial derivative in the system at rest with respect to the body B.

The equation of rotational motion of element A_i is

$$\bar T^i = d/dt(\lambda^i\cdot\bar\omega^i) - \lambda^i\cdot\dot{\bar\omega}^i + \bar\omega^i\times\lambda^i\cdot\bar\omega^i . \tag{8}$$

Assuming a condition exists of small values of the orthogonal displacements $\beta_1^i, \beta_2^i, \beta_3^i$ of the flexible structure A relative to the body B, we obtain the inertial angular velocity of the element A_i:

$$\bar{\omega}^i = \bar{\omega} + \mathring{\bar{\beta}}^i. \tag{9}$$

The vector $\mathring{\bar{\beta}}^i$ lies within the base vectors $\bar{b}_1, \bar{b}_2, \bar{b}_3$. Substituting equation (9) into equation (8), we obtain

$$\boxed{\bar{T}^i = \lambda^i(\dot{\bar{\omega}} + \ddot{\bar{\beta}}^i + \bar{\omega} \times \mathring{\bar{\beta}}^i) + \bar{\omega} \times \lambda^i \cdot \bar{\omega} + \bar{\omega} \times \lambda^i \mathring{\bar{\beta}}^i + \mathring{\bar{\beta}}^i \times \lambda^i \cdot \bar{\omega} + \mathring{\bar{\beta}}^i \times \lambda^i \cdot \mathring{\bar{\beta}}^i.} \tag{10}$$

Replacing $\mathring{\bar{\beta}}^i \times \lambda^i \cdot \bar{\omega}$ with $(\lambda^i \bar{\omega}) \times \mathring{\bar{\beta}}^i$ and $\mathring{\bar{\beta}}^i \times \lambda^i \cdot \mathring{\bar{\beta}}^i$ with $\lambda^i \cdot \mathring{\bar{\beta}}^i \times \mathring{\bar{\beta}}^i$ in equation (10) and ignoring $\lambda^i \cdot \mathring{\bar{\beta}}^i \times \mathring{\bar{\beta}}^i$, we obtain at last

$$\bar{T}^i = \lambda^i \cdot (\dot{\bar{\omega}} + \ddot{\bar{\beta}}^i + \bar{\omega} \times \mathring{\bar{\beta}}^i) + \bar{\omega} \times \lambda^i \cdot \bar{\omega} - (\lambda^i \cdot \bar{\omega}) \times \mathring{\bar{\beta}}^i + \bar{\omega} \times \lambda \cdot \mathring{\bar{\beta}}^i \tag{11}$$ /10

The vectorial product of two vectors with the same coordinate bases is easily expressed in matrix form using a diagonally symmetric operator. For example, if $\tilde{\omega}$ is any diagonally symmetric linear operator in the three-dimensional Euclidean vector space $\mathcal{U}$, it is represented in the orthonormal base coordinates $\bar{b}_1, \bar{b}_2, \bar{b}_3$ by a diagonally symmetric matrix. Thus,

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{12}$$

and for each vector $\bar{c}$ within $\mathcal{U}$, the relationship

$$\bar{\omega} \times \bar{c} = \tilde{\omega} \cdot \bar{c} \tag{13}$$

is fulfilled.

Equations (7) and (11) can be rewritten in matrix form:

$$F^i = m^i\{\theta\ddot{X} + \ddot{c} + \ddot{u}^i + 2\tilde{\omega}[\dot{c} + \dot{u}^i] + \dot{\tilde{\omega}}[c + R + r^i + u^i] + \tilde{\omega}\tilde{\omega}[c + R + r^i + u^i]\} \quad (14)$$

$$T^i = I^i(\dot{\omega} + \ddot{\beta}^i + \tilde{\omega}\dot{\beta}^i) + \tilde{\omega}I^i\omega - I^i\tilde{\omega}\dot{\beta}^i + \tilde{\omega}I^i\dot{\beta}^i + [I^i\dot{\tilde{\omega}} - [I^i\dot{\omega}]^{\sim} - \tilde{\omega}[I^i\omega]^{\sim} + \tilde{\omega}I^i\tilde{\omega}]\beta^i. \quad (15)$$

It is obvious that the matrix C which characterizes motions in the CM system depends on the displacements of the elements A_i of the structure A, which are expressed in the matrix U^i. But since, generally speaking, the matrix C also depends on other variables (the deformation of the other superstructure, external disturbances, directional moments, etc.) as well, the substitution for C should be written in the form /11

$$c = \frac{-1}{M}\sum_{i=1}^{n} m^i u^i + e \quad (16)$$

where e is any variation in the CM independent of the variation of the structure under consideration.

Let us join the matrices in equations (14) and (15). They have dimensions of 3x1 for each element $A_1, \ldots, A_n$ in the simple matrix equation of the dimensionality 6n x 1. A total displacement motion (translational and rotational) of the structure is described by means of the matrix coordinates q (a matrix column of dimensions 6n x 1).

$$q \equiv [u_1^1 u_2^1 u_3^1 \beta_1^1 \beta_2^1 \beta_3^1 u_1^2 u_2^2 u_3^2 \ldots \beta_3^n]^T \quad (17)$$

The inertia matrix μ, which is zero except for the matrices of the mass and the moments of inertia of the elements of the superstructure distributed along the main diagonal, has the form:

$$\mathcal{M} = \begin{bmatrix} m^1 & & & & 0 \\ & I^1 & & & \\ & & m^2 & & \\ & & & \ddots & \\ 0 & & & & I^n \end{bmatrix} \tag{18}$$

Since the total matrix equation must contain elements which are characterized by two parameters (u^i , β^i and m^i , I^i), it is appropriate to introduce the matrix operators

$$\Sigma_{EO} \equiv [EOEO \ldots EO]^T \tag{19}$$

and

$$\Sigma_{OE} \equiv [OEOE \ldots OE]^T \tag{20}$$

where E and O are the unit and zero matrices, respectively, of dimensions 3x3. /12

The matrix C in equation (16) now becomes

$$C = \frac{-1}{M} \Sigma_{EO}^T \mathcal{M} \ddot{q} + e \tag{21}$$

We obtain the equation of motion of the structure, assuming the system is without friction or plastic deformation, from equations (14), (15) and (21):

$$\begin{aligned} &\mathcal{M}(E - \Sigma_{EO}\Sigma_{EO}^T \mathcal{M}/M)\ddot{q} + [2\mathcal{M}(\tilde{\Omega}_1 - \Sigma_{EO}\tilde{\omega}\Sigma_{EO}^T \mathcal{M}/M) + \mathcal{M}\tilde{\Omega}_2 + \Omega_2\mathcal{M} - \mathcal{M}\tilde{\Omega}]\dot{q} + \\ &+ [\mathcal{M}(\tilde{\Omega}_1 + \tilde{\Omega}_1\tilde{\Omega}_1) - \mathcal{M}\Sigma_{EO}(\tilde{\dot{\omega}} + \tilde{\omega}\tilde{\omega})\Sigma_{EO}^T \mathcal{M}/M + \mathcal{M}\tilde{\dot{\Omega}}_2 - (\mathcal{M}\dot{\Omega}_2)^{\sim} - \tilde{\Omega}_2(\mathcal{M}\Omega_2)^{\sim} + \\ &+ \tilde{\Omega}_2\mathcal{M}\tilde{\Omega}_2 + K]q = -\mathcal{M}\Sigma_{EO}[\Theta\ddot{X} + \ddot{e} - 2\tilde{\dot{e}}\omega - (\tilde{e} + \tilde{R})\dot{\omega} + \tilde{\omega}\tilde{\omega}(e + R)] + \\ &+ \mathcal{M}(\tilde{\imath}\Sigma_{EO}\dot{\omega} - \tilde{\Omega}_1\tilde{\Omega}_1\imath) + \Lambda - \mathcal{M}\dot{\Omega}_2 - \tilde{\Omega}_2\mathcal{M}\Omega_2 , \end{aligned} \tag{22}$$

where K is the matrix of the rigidities of the structure, Λ is the matrix of external forces and moments (of dimensions 6n x 1), and the new matrices Ω_1 , Ω_2 and $\imath$ are defined

$$\left.\begin{aligned} \Omega_1 &\equiv \Sigma_{EO}\,\omega = [\omega O \omega \ldots O]^T \\ \Omega_2 &\equiv \Sigma_{OE}\,\omega = [O \omega O \ldots \omega]^T \\ \imath &\equiv [\imath^1 O \imath^2 \ldots O]^T \end{aligned}\right\} \tag{23}$$

(The tilda sumbol ($\tilde{}$) above a matrix indicates dimensions of 6n x 6.)

The extended matrix has elements of 3 x 3 dimensions and is zero off the diagonal, where the elements are the corresponding elements of diagonally symmetric matrices obtained in a manner similar to (12) and correspond to the matrices of dimensions 6n x 1. For example,   obtained from equation (23) is

$$\tilde{\imath} \equiv \begin{bmatrix} \tilde{\imath}^1 & & & 0 \\ & \tilde{\imath}^2 & & \\ & & \ddots & \\ 0 & & & \tilde{\imath}^n \end{bmatrix} \tag{24}$$ /13

and $\tilde{\Omega}_1$, $\dot{\tilde{\Omega}}_1$, $\tilde{\Omega}_2$, $\dot{\tilde{\Omega}}_2$ and $\int \widetilde{N\Omega_2}$ are similar.

The Equation of Motion of the Spacecraft

The equations of motion of the spacecraft are written with consideration of the fact that the flexible superstructures are contained in the structure of the spacecraft. Thus

$$\bar{T} = \dot{\bar{H}} \tag{25}$$

$$\bar{H} = \bar{I}\bar{\omega} + M\dot{\bar{c}} \times \bar{c} \times \int \bar{\rho} \times \dot{\bar{\rho}}\, dm \tag{26}$$

We rewrite (25) taking (26) into account:

$$\bar{T} = \bar{J}\cdot\dot{\bar{\omega}} + \bar{\omega} \times \bar{I}\omega + \dot{\bar{I}}\cdot\bar{\omega} + M[\ddot{\bar{c}} + 2\bar{\omega}\times\dot{\bar{c}} + \bar{\omega}\times(\bar{\omega}\times\bar{c}) + \dot{\bar{\omega}}\times\bar{c}]\times\bar{c} + \frac{d}{dt}\int \bar{\rho}\times\dot{\bar{\rho}}\, dm \tag{27}$$

where $\bar{\rho} = \bar{R} + \bar{\imath}^i + \bar{u}^i + \bar{\rho}^i$

We now rewrite (27) in matrix form:

$$
\begin{aligned}
T = {} & I\dot{\omega} + \tilde{\omega} I \omega + \{2[M(\Sigma_{EO} R + \tau)]^T q - R q^T M \Sigma_{EO} - \Sigma_{EO}^T M q R^T + \\
& + \Sigma_{OE}^T (\tilde{q} M - M \tilde{q}) \Sigma_{OE}\} \dot{\omega} + M(\ddot{e} + 2\tilde{\omega} e + \tilde{\omega}\tilde{\omega} e + \dot{\tilde{\omega}} e)^{\sim} e + (\omega R)^{\sim} \Sigma_{EO}^T M \dot{q} + \\
& + \tilde{R} \Sigma_{EO}^T M \ddot{q} + \tilde{R} \tilde{\omega} \Sigma_{EO}^T M \dot{q} + \Sigma_{EO}^T \tilde{\tau} M \ddot{q} + \tilde{\omega} \Sigma_{OE}^T M \dot{q} - \tilde{\omega} \Sigma_{EO}^T \tilde{\tau} M + \\
& + \Sigma_{OE}^T M (\Sigma_{OE} \omega)^{\sim} \dot{q} - \Sigma_{EO}^T (\tilde{\tau} \Sigma_{EO} \omega)^{\sim} M \dot{q} + \Sigma_{OE}^T \dot{\tilde{q}} M \Sigma_{OE} \omega + \\
& + \tilde{\omega} \Sigma_{OE}^T \tilde{q} M \Sigma_{OE} \omega - \tilde{\omega} \Sigma_{OE}^T M \tilde{q} \Sigma_{OE} \omega + \tilde{\omega} \{2[M(\Sigma_{EO} R + \tau)]^T q - \\
& - R q^T M \Sigma_{EO} - \Sigma_{EO}^T M q R^T\} \omega + \{2[M(\Sigma_{EO} R + \tau)]^T \dot{q} - \\
& - R \dot{q}^T M \Sigma_{EO} - \Sigma_{EO}^T M \dot{q} R^T\} \omega .
\end{aligned}
\tag{28}
$$

Linearization of the Equation of Motion /14

With a wide variety of spacecraft which have flexible structures attached and are stabilized in 3 dimensions, it can be assumed that their angular velocity is small enough to ignore expressions of the second order and their derivatives.

$$
\begin{aligned}
\omega &\longrightarrow \dot{\theta} \\
\dot{\omega} &\longrightarrow \ddot{\theta} \\
\theta &\longrightarrow (E - \tilde{\theta})
\end{aligned}
$$

where $\theta = [\theta_1, \theta_2, \theta_3]^T$ of the base vectors b_1, b_2, b_3 connected to the body B. Under these conditions the linearized equation corresponding to (22) is

$$
\begin{aligned}
& M(E - \Sigma_{OE} \Sigma_{EO}^T M/M) \ddot{q} + Kq = -M \Sigma_{OE} \ddot{\theta} + M \Sigma_{EO} [\ddot{X} + \ddot{e} - 2\tilde{e} \dot{\theta} - \\
& - (\tilde{e} + \tilde{R}) \ddot{\theta}] + M \tilde{\tau} \Sigma_{EO} \ddot{\theta} + \Lambda + M \Sigma_{EO} \tilde{\theta} \ddot{X}
\end{aligned}
\tag{29}
$$

We transform (29) according to the form of oscillation of the craft itself,

$$
q = \Phi \eta \tag{30}
$$

and, rewriting (29) taking (30) into consideration,

$$
\begin{aligned}
& \Phi^T M (E - \Sigma_{EO} \Sigma_{EO}^T M/M) \Phi \ddot{\eta} + \Phi^T K \Phi \eta = -\Phi^T M \{[\Sigma_{EO} - \\
& - \Sigma_{EO} (\tilde{e} + \tilde{R}) - \tilde{\tau} \Sigma_{EO}] \ddot{\theta} + \Sigma_{EO} [(E - \tilde{\theta}) \ddot{X} + \ddot{e} - 2\tilde{e} \dot{\theta}]\} + \Phi^T \Lambda
\end{aligned}
\tag{31}
$$

Normalization of the proper vectors can be performed in the following manner. Let us assume that $\phi^T M(E - \Sigma_{\epsilon o}\Sigma_{\epsilon o}^T M/M)\phi = E$ (where E is the unit matrix). In that case $\phi^T K \phi$ becomes the matrix σ^2, determining the proper values. These operators, in combination with equation (31), make it possible to obtain the equations of motion of a modular element in a form which /15 in its homogeneous parts corresponds to the classical formulation of the equation of oscillation in the functions of normalized coordinates.

In developments of the model of structures with discrete parameters the damping is usually ignored, but the diagonal damping matrix $2\zeta\sigma$ with the diagonal elements $2\zeta_i\sigma_i$ is included in equation (22) after transformation according to (30).

In finite form the equation of motion of the flexible structure can be written in the form

$$\ddot{\eta} + 2\zeta\sigma\dot{\eta} + \sigma^2\eta = -\phi^T M\{[(\Sigma_{\omega} - \Sigma_{\epsilon\omega})(\tilde{e} + \tilde{R}) - \tilde{l}\Sigma_{\epsilon\omega}]\ddot{\theta} + \Sigma_{\omega}[(E - \tilde{\theta})\ddot{l} + \ddot{e} - 2\tilde{e}\dot{\theta}]\} + \phi^T\Lambda. \tag{32}$$

The linearized equation corresponding to equation (28) can be rewritten in view of the above premises, equation (30), and the fact that in small displacements of the structure the values of C are small enough to ignore expressions of the second order and their derivatives:

$$T = I\ddot{\theta} + M\Phi\ddot{\eta}(\tilde{R}\Sigma_{\epsilon o}^T + \Sigma_{\epsilon o}^T\tilde{l} + \Sigma_{\omega}^T) \tag{33}$$

In order to choose the N modes of oscillation which will be appropriate and to represent with a sufficient degree of precision the dynamics of the reaction of the system which has been modeled by means of 6n modes of oscillation, certain criteria are necessary. For this criterion it is appropriate to select the lowest modes or those modes in which the corresponding frequency will be located in the neighborhood of the assumed "forced frequency."

Transmission Functions

The equations of motion of the spacecraft which include the attached structures, (32) and (33), after excluding the effects of external forces and displacements, and independent of internal displacements of the structures (that is $e = 0$, $\varphi^T \Lambda = 0$), can be given in the following form:

$$T = I\ddot{\theta} + \Delta^T \ddot{\underline{\eta}} \tag{34}$$ /16

$$\ddot{\underline{\eta}} + 2\zeta\sigma\dot{\underline{\eta}} + \sigma^2\underline{\eta} = \Delta\ddot{\theta}, \tag{35}$$

where $\Delta = \varphi^T M(\Sigma_{0E} - \Sigma_{E0}\tilde{R} - \tilde{r}\Sigma_{E0})$.

Here and subsequently we regard the matrix of modal coordinates η of dimensions 6n x 1 as truncated to the form $\underline{\eta}$ with dimensions N + 1, and the underline (_) will be omitted.

By using a Laplace transformation we exclude from consideration the values of the displaced superstructure η. Then equations (34) and (35) become

$$T(S) = S^2 I\theta(S) - S^2\Delta^T\eta(S) \tag{36}$$

$$S^2\eta(S) + 2S\sigma\zeta\eta(S) + \sigma^2\eta(S) = S^2\Delta\theta(S) \tag{37}$$

Solving equation (37) for $\eta(S)$ and substituting this into equation (36), we obtain

$$T(S) = (S^2 I - S^4\Delta^T\mathcal{D}\Delta)\theta(S) \tag{38}$$

The matrix $\mathcal{D}_i = (S^2 + 2\zeta_i\sigma_i S + \sigma_i^2)^{-1}$ is diagonal. In accordance with the principle of superposition in the linearized system under consideration, we can rewrite the matrix of the transmission function of equation (38) in the form

$$\theta(S) = \left[I S^2 - S^4 \sum_{i=1}^{N} \frac{\Delta_i^T \Delta_i}{S^2 + 2\zeta_i \sigma_i S + \sigma_i^2} \right]^{-1} T(S) \tag{39}$$

If the matrix of the transmission function is diagonal, the reaction of the spacecraft can be represented in the following system of equations:

$$\begin{cases} \ddot{\eta}_i + 2\zeta_i \sigma_i \dot{\eta}_i + \sigma_i^2 \eta_i = \Delta_{i1}\ddot{\theta}_1 + \Delta_{i2}\ddot{\theta}_2 + \Delta_{i3}\ddot{\theta}_3 \\ I_1\ddot{\theta}_1 + I_{12}\ddot{\theta}_2 + I_{13}\ddot{\theta}_3 + \sum_{i=1}^{N} \Delta_{i1}\ddot{\eta}_i = T_1 \\ I_2\ddot{\theta}_2 + I_{21}\ddot{\theta}_1 + I_{23}\ddot{\theta}_3 + \sum_{i=1}^{N} \Delta_{i2}\ddot{\eta}_i = T_2 \\ I_3\ddot{\theta}_3 + I_{31}\ddot{\theta}_1 + I_{32}\ddot{\theta}_2 + \sum_{i=1}^{N} \Delta_{i3}\ddot{\eta}_i = T_3 \end{cases} \tag{40}$$ /17

A structural format corresponding to equation (40) is laid out in Figure 2.

The scalar transmission function of the open contour $G_\alpha(S)$ for the α axis (α = 1,2,3) is obtained from equation (39). When $I_{12} = I_{13} = I_{21} = I_{23} = I_{31} = I_{32} = 0$ the transmission function becomes

$$G_\alpha(S) = \frac{\theta_\alpha(S)}{T_\alpha(S)} = \frac{S^2 + 2\zeta_i \sigma_i S + \sigma_i^2}{I_\alpha S^2 [S^2(1 - \Delta_{i\alpha}\Delta_{\alpha i}/I_\alpha) + 2\zeta_i \sigma_i S + \sigma_i^2]} \tag{41}$$

The value in the round brackets is called the "normalized reduced inertia" for the α axis and the i^{th} mode of oscillation. It is designated R_α^i or simply R.

$$1 + \Delta_{\alpha i} \cdot \Delta_{\alpha i}/I_\alpha = R_\alpha^i \tag{42}$$

The physical significance of R for the scalar values $\Delta_{\alpha i}\,\Delta_{\alpha i}$ or in a more general view for the (3 x 3)-dimensional matrix

$\Delta^T\Delta = \sum_{i=1}^{\nu} \Delta_i^T \Delta_i$ in equation (39) proceeds from a consideration of the limiting case.

If the superstructure approaches rigidity, the transmission function of equation (39) is reduced to the transmission function of a rigid spacecraft

$$G(S) = 1/I_\alpha S^2 \tag{43}$$

If, on the other hand, the superstructure becomes elastic to the limit, we can assume that the transmission function will be based only on the transmission function of the solid body, while the superstructure will have essentially separated, since the reaction under consideration will be lacking on the part of the superstructure. Therefore, in that case, the proper frequencies of the structure A in equation (39) will all tend toward zero, and the equation becomes /18

$$\theta(S) = [(I - \sum_{i=1}^{\nu} \Delta_i^T \Delta_i) S^2]^{-1} T(S) \tag{44}$$

That is, the 3 x 3 matrix $I - \sum_{i=1}^{\nu} \Delta_i^T \Delta_i = I - \Delta^T\Delta$ should be the matrix of the moments of inertia of the spacecraft relative to its center of mass, without the moments of inertia of the structures. This value varies between zero and one, and the value R_α^1 from equation (42) can vary from zero (for the limit of large structures which are very sensitive to deviations in θ_α) up to unity (for small structures or those not sensitive to changes in θ_α). Thus, $0 \leq R_\alpha^1 \leq 1$.

In many cases it is more acceptable to write the transmission function describing the dynamics of the rigid spacecraft in some other form than (41).

Let us write equations (34) and (35) in scalar form, taking into account the fact that cross-connections between channels are lacking:

$$\ddot{\theta}_\alpha = -\chi_0^\alpha + \sum_{i=1}^{N} \Delta_i^{T'} \ddot{\eta}_i \tag{45}$$

$$\ddot{\eta}_i + 2\zeta_i \sigma_i \dot{\eta}_i + \sigma_i^2 \eta_i = \Delta_i \ddot{\theta}_\alpha, \tag{46}$$

where $\chi_0^\alpha = T_\alpha / I_\alpha$, $\Delta_i^{T'} = \Delta_i^T / I_\alpha$.

From equations (45) and (46) we find

$$\frac{1-\Delta_i^{T'}\Delta_i}{\Delta_i}\ddot{\eta}_i + \frac{2\zeta_i\sigma_i}{\Delta_i}\dot{\eta}_i + \frac{\sigma_i^2}{\Delta_i}\eta_i = -\chi_0^\alpha . \tag{47}$$

Designating

$$\frac{1-\Delta_i^{T'}\Delta_i}{\Delta_i}\eta_i = \eta_i' ; \quad \frac{\zeta_i}{\sqrt{1-\Delta_i^{T'}\Delta_i}} = \zeta_i' ; \quad \frac{\sigma_i}{\sqrt{1-\Delta_i^{T'}\Delta_i}} = \sigma_i' ; \quad \frac{\Delta_i^{T'}\Delta_i}{1-\Delta_i^{T'}\Delta_i} = A_i,$$ /19

we obtain

$$\ddot{\theta}_\alpha = -\chi_0^\alpha + \sum_{i=1}^{N} A_i \ddot{\eta}_i' \tag{48}$$

$$\ddot{\eta}_i' + 2\zeta_i'\sigma_i'\dot{\eta}_i' + \sigma_i'^2\eta_i' = -\chi_0 \tag{49}$$

The transmission function of this system will be

$$G_\alpha'(S) = \frac{\theta_\alpha(S)}{T_\alpha(S)} = \frac{S^2(1+A_i)+2\zeta_i'\sigma_i'S+\sigma_i'^2}{I_\alpha S^2(S^2+2\zeta_i'\sigma_i'S+\sigma_i'^2)} \tag{50}$$

The structural flow diagram corresponding to the transmission function of (49) and (50) is sketched in figure 3. To obtain equations (48) and (49) in dimensionless form, let us introduce the dimensionless time t° :

$$\begin{cases} t^\circ = \dfrac{\sigma_i}{\sqrt{1-\Delta_i^{T'}\Delta_i}}\,t = \sigma_i' t \\ dt^\circ = \dfrac{\sigma_i}{\sqrt{1-\Delta_i^{T'}\Delta_i}}\,dt = \sigma_i' dt \\ dt^{\circ 2} = \dfrac{\sigma_i}{\sqrt{1-\Delta_i^{T'}\Delta_i}}\,dt^2 = \sigma_i' dt^2 \end{cases} \tag{51}$$

Using (51) equation (47) becomes

$$\frac{\sigma_i^2}{\Delta_i}\frac{d^2\eta_i}{dt^{\circ 2}} + \frac{\sigma_i^2}{\Delta_i\sqrt{1-\Delta_i^T\Delta_i}}\frac{d\eta_i}{dt^\circ} + \frac{\sigma_i^2}{\Delta_i}\eta_i = -\chi_0^* . \tag{52}$$

Introducing the new variable

$$\hat{\eta}_i = \frac{\sigma_i^2}{\Delta_i \chi_0^*}\eta_i \tag{53}$$

we obtain finally for the flexible element

$$\ddot{\hat{\eta}}_i + 2\mathcal{D}_i^\circ \dot{\hat{\eta}}_i + \hat{\eta}_i = -\hat{\chi} \tag{54}$$

where /20

$$\mathcal{D}_i^\circ = \zeta_i / \sqrt{1-\Delta_i^T\Delta_i}$$

Let us determine the expression for the coordinates of the solid body. Passing to dimensionless time t° in equation (45), we obtain

$$\frac{d^2\theta_\alpha}{dt^{\circ 2}} = -\sum_{i=1}^{\nu}\frac{1-\Delta_i^T\Delta_i}{\sigma_i^2}\chi_0^* + \sum_{i=1}^{\nu}\Delta_i^{T}\frac{d^2\eta_i}{dt^{\circ 2}} \tag{55}$$

Transforming this expression and using the relation (53), we obtain:

$$\frac{d^2\theta_\alpha}{\chi_0^* dt^{\circ 2}}\cdot\sum_{i=1}^{\nu}\frac{\sigma_i^2}{1-\Delta_i^T\Delta_i} = -\hat{\chi} + \sum_{i=1}^{\nu}\frac{\Delta_i^T\Delta_i}{1-\Delta_i^T\Delta_i}\cdot\ddot{\hat{\eta}}_i \tag{56}$$

We introduce the new variable

$$\hat{\theta} = \sum_{i=1}^{\nu}\frac{\sigma_i^2}{1-\Delta_i^T\Delta_i}\,\frac{\theta}{\chi_0} \tag{57}$$

In final form the equation (55) becomes

$$\ddot{\hat{\theta}} = -\hat{\chi} + \sum_{i=1}^{\nu} A_i \ddot{\hat{\eta}}_i . \tag{58}$$

We will study the auto-oscillations of a nonrigid spacecraft in the direction circuit of which we will include aperiodic feedback.

Description of the System

The system (see figure 4) includes a standard regulator with aperiodic feedback with a double time constant, a pair of gas-reactive nozzles, a position guage, and the dynamics of the nonrigid spacecraft. The regulator consists of a relay element, including aperiodic feedback with a double time constant (τ_0, when $|y(t)=\pm\delta$, τ_B when $|y(t)=0$) and an amplification coefficient $-K_0\delta$. The zone of insensitivity of the relay element is δ, the hysteresis in H. The gas-reactive nozzles have a thrust level F and a moment arm L. The dynamics /21 of the spacecraft include the oscillations of the spacecraft and the leastic weakly-damped oscillations of the attached structural elements.

For the class of spacecraft under consideration -- apparati with elastically attached structural elements -- the initial synthesis and classification of the parameters of the circuits with aperiodic feedback is based on the premise that the spacecraft is basically governed by the law of rigid bodies. The regulator should be designed in such a manner that the frequency of the proper oscillations of the superstucture will be above the frequency band of the channel of the direction system. It is however impossible to ignore the nonrigidities of the system: the damping of the proper modes of the elastic oscillations is very small, there are time delays in the circuit which are comparable to the periods of elastic modes of oscillation, the position gauge follows not only the low grquency of the motion of the solid part of the spacecraft but also the higher frequencies of the attached elements.

Retruning to the previous section, we have obtained the equations of planar motion of a nonrigid spacecraft in dimensionless form, namely:

$$\ddot{\hat{\theta}} = -\hat{\chi} + \sum_{i=1}^{N} A_i \ddot{\hat{\eta}}_i \tag{58}$$

$$\ddot{\hat{\eta}}_i + 2 D_i \dot{\hat{\eta}}_i + \hat{\eta}_i = \hat{\chi} \tag{54}$$

$$(K_0 \hat{\delta}) f = \hat{z} + (T_0, \ell) \dot{\hat{z}}, \tag{59}$$

where

$$f = \begin{cases} \left.\begin{array}{ll} +\hat{\chi} & npu \;\; U > \hat{\delta} \\ 0 & npu \;\; -\ell\hat{\delta} < U < \hat{\delta} \\ -\hat{\chi} & npu \;\; U < \ell\hat{\delta} \end{array}\right\} \text{for } \dot{U} > 0 \\ \left.\begin{array}{ll} -\hat{\chi} & npu \;\; U < -\hat{\delta} \\ 0 & npu \;\; -\hat{\delta} < U < \ell\hat{\delta} \\ +\hat{\chi} & npu \;\; U > \ell\hat{\delta} \end{array}\right\} \text{for } \dot{U} < 0 \end{cases} \tag{60}$$ /22

$$t^{\circ} = \begin{cases} t^{\circ}_{cp} \; npu & \begin{cases} U = \hat{\delta} \; u \; \dot{U} > 0 \\ U = -\hat{\delta} \; u \; \dot{U} < 0 \end{cases} \\ t^{\circ}_{otn} \; npu & \begin{cases} U = \ell\hat{\delta} \; u \; \dot{U} < 0 \\ U = -\ell\hat{\delta} \; u \; \dot{U} > 0 \end{cases} \end{cases} \tag{61}$$

Equations (59), (60), (61) describe the action of the regulator.

The system has delays in the relay element t_j^{o} and in the command element t_j'. A flow diagram of the stabilization circuit of a nonrigid spacecraft is presented in figure 4. At the input of the relay element (1) the directing signal U(t) arrives; at the output of the relay element the signal f(t) enters the command element (2). The command element generates the stabilizing moment T(t) which acts on the spacecraft in the desired manner. This same signal f(t) includes feedback (4). The feedback signal Z(t) and the output signal of the angle gauge (5) enter the summing device (6), in which the directing signal

U(t) is formed.

We assume that disturbing moments do not act on the system, that the angle gauge is ideal, and that the command and relay elements used are ideal circuit elements with pure time delays. The nonrigidity of the system is incorporated into the model as an externally attached elastic structure (7).

The real motion of the dynamical system corresponds to the motion of a representative point in phase space with the coordinates θ, $\dot{\theta}$, z. Let us project the trajectory of the motion of the representative point onto the phase plane θ, $\dot{\theta}$, passing from the consideration of spatial motion to planar motion. /23

We find the solutions of equations (58), (54), (59) in the mth section of the trajectory, where $\hat{\chi} \neq 0$:

a) for the successively integrated equation (58), we obtain:

$$\begin{cases} \dot{\theta} = -\hat{\chi} t^\circ + \sum_{i=1}^{N} A_i \dot{\hat{\eta}}_i + \dot{\theta}_0 - \sum_{i=1}^{N} A_i \dot{\hat{\eta}}_{i0} \\ \theta = -\chi (t^\circ)^2/2 + \sum_{i=1}^{N} A_i \hat{\eta}_i + (\dot{\theta}_0 - \sum_{i=1}^{N} A_i \dot{\hat{\eta}}_{i0}) t^\circ + \theta_0 - \sum_{i=1}^{N} A_i \hat{\eta}_i \end{cases} \tag{62}$$

b) for equation (54) the solution has the form:

$$\begin{cases} \hat{\eta}_i = -\hat{\chi} + e^{-D_i t^\circ} \left[(\hat{\chi} + \hat{\eta}_{i0}) \cos \nu_i t^\circ + \frac{(\hat{\chi} + \hat{\eta}_{i0}) D_i + \dot{\hat{\eta}}_{i0}}{\nu_i} \sin \nu_i t^\circ \right] \\ \dot{\hat{\eta}}_i = e^{-D_i t^\circ} \left[\dot{\hat{\eta}}_{i0} \cos \nu_i t^\circ - \frac{(\hat{\chi} + \hat{\eta}_{i0}) + D_i \dot{\hat{\eta}}_{i0}}{\nu_i} \sin \nu_i t^\circ \right] \end{cases} \tag{63}$$

where $\nu_i = \sqrt{1-(D_i)^2}$ and $\dot{\hat{\eta}}_{i0}$, $\hat{\eta}_{i0}$ are the initial conditions.

c) the solution to the third equation has the form:

$$z(t^\circ) = -K_0 \delta (1 - e^{-t^\circ/T_0}) \tag{64}$$

We begin consideration of the motion at the moment $t^\circ = 0$, when

$$\hat{\theta}|_{t^\circ=0} = \hat{\delta}\,, \quad \dot{\hat{\theta}}|_{t^\circ=0} = \dot{\hat{\theta}}_0\,, \quad \hat{z}|_{t^\circ=0} = 0.$$

The command element is switched on at the moment $t^\circ = t^\circ_{j1}$. At this moment (see figure 4)

$$\hat{\theta}_{bx1} = \hat{\delta} + \dot{\hat{\theta}}_0 t^\circ_{j1} \tag{65}$$

Beginning from the moment $t^\circ = t^\circ_{j1}$, the representative point moves along a phase trajectory intersecting the axis $\dot{\hat{\theta}} = 0$. At the moment in time $t^\circ = t^\circ_1 + t^{\circ 1}_{j0}$, when

$$\hat{\theta}(t^\circ_1 + t^{\circ\prime}_{j0}) + \hat{z}(t^\circ_1 - t^{\circ\prime}_{j1} + t^{\circ\prime}_{j0}) = \hat{\delta}\ell - \hat{\chi}(t^{\circ\prime}_{j0})^2/2, \tag{66}$$ /24

the signal for switching off the command element goes from the ouput of the relay element to the electropneumatic valve (EPV) of the command element.

The equations of motion remain as in (62) and (64), but taking into account the delays they assume the form

$$\begin{cases} \hat{z}(t^\circ_1 - t^{\circ\prime}_{j1} + t^{\circ\prime}_{j0}) = -K_0 \hat{\delta}\left(1 - e^{-\frac{t^\circ_1 - t^{\circ\prime}_{j1} + t^{\circ\prime}_{j0}}{\tau^\circ}}\right) \\ \dot{\hat{\theta}}_1 = -\hat{\chi}(t^\circ_1 - t^\circ_{j1}) + \sum_{i=1}^{\nu} A_i \dot{\hat{\eta}}_i + \dot{\hat{\theta}}_0 - \sum_{i=1}^{\nu} A_i \dot{\hat{\eta}}_{i0} \\ \hat{\theta}_1 = -\chi(t^\circ_1 - t^\circ_{j1})^2/2 + \sum_{i=1}^{\nu} A_i \hat{\eta}_i + (\dot{\hat{\theta}}_0 - \sum_{i=1}^{\nu} A_i \dot{\hat{\eta}}_{i0})(t^\circ_1 - t^\circ_{j1}) + \hat{\theta}_{bx1} - \sum_{i=1}^{\nu} A_i \hat{\eta}_{i0} \end{cases} \tag{67}$$

Note I

Taking into account the fact that at the moment of each subsequent switching off of the command element the elastic oscillations of the flexible structures diminish, that is, that $\hat{\eta}_{i0} = \dot{\hat{\eta}}_{i0} = 0$, the latter two equations of system (67) can be rewritten, taking (65) and (66) into consideration, where $\hat{\theta}(t^\circ_1 - t^\circ_{j1}) = \hat{\theta}_1$, $\dot{\hat{\theta}}(t^\circ_1 - t^\circ_{j1}) = \dot{\hat{\theta}}_1$. We have

$$\begin{cases} \dot{\hat{\theta}}_1 = -\hat{\chi}(t^\circ_1 - t^\circ_{j1}) + \sum_{i=1}^{\nu} A_i \dot{\hat{\eta}}_i + \dot{\hat{\theta}}_0 \\ K_0 \hat{\delta}\left(1 - e^{-\frac{t^\circ_1 - t^\circ_{j1} + t^\circ_{j0}}{\tau^\circ}}\right) + \delta\ell = -\chi(t^\circ_1 - t^\circ_{j1})^2/2 + \sum_{i=1}^{\nu} A_i \hat{\eta}_i + \dot{\hat{\theta}}_0 t^\circ_{j1} + \delta + \dot{\hat{\theta}}(t^\circ_1 - t^\circ_{j1}) \end{cases} \tag{68}$$

Switching off the command element takes place at the moment in time $t^{\circ} = t_i^{\circ} + t_{j\text{I}}^{\circ}$. At this moment

$$\mathring{\hat{\theta}}(t_i^{\circ} + t_{j\text{I}}^{\circ}) = \mathring{\hat{\theta}}_2 = \mathring{\hat{\theta}}_1 - \hat{\chi}\, t_{j\text{I}}^{\circ} . \tag{69}$$

Taking (69) into account and assuming that $t_{j\text{I}}^{\circ} = t_{j\text{I}}^{\circ}$, $t_{j\text{I}}^{\circ\prime} = t_{j\text{I}}^{\circ\prime}$, (that is, that the delays of the switching on and switching off are the same) the system (68) can be rewritten

$$\begin{cases} \mathring{\hat{\theta}}_2 = -\chi t_i^{\circ} + \sum_{i=1}^{N} A_i \mathring{\hat{\eta}}_i + \mathring{\hat{\theta}}_0 \\ K_0 \hat{\delta}(1 - e^{-t_i^{\circ}/T_i^{\circ}}) - \hat{H} = -\chi (t_i^{\circ} - t_j^{\circ})^2/2 + \mathring{\hat{\theta}}_0 t_i^{\circ} + \sum_{i=1}^{N} A_i \hat{\eta}_i , \end{cases} \tag{70}$$ /25

where

$\hat{\delta} = \hat{\delta}_\ell - \hat{H}$

Noting that $\mathring{\hat{\theta}}_2 = -S'$ and $\mathring{\hat{\theta}}_0 = S$ in the system of equations (70) and solving this system for S and S', we obtain

$$\begin{cases} S = \dfrac{K_0 \hat{\delta}(1 - e^{-t_i^{\circ}/T_i^{\circ}}) + \hat{\chi}/2\,(t_i^{\circ 2} + 2t_i^{\circ} t_j^{\circ} + t_j^{\circ 2}) - \hat{H} - \sum_{i=1}^{N} A_i \hat{\eta}_i}{t_i^{\circ}} \\ S' = \dfrac{-K_0 \hat{\delta}(1 - e^{-t_i^{\circ}/T_i^{\circ}}) + \hat{\chi}/2\,(t_i^{\circ 2} + 2t_i^{\circ} t_j^{\circ} - t_j^{\circ 2}) + \hat{H} + \sum_{i=1}^{N} A_i \hat{\eta}_i - \sum_{i=1}^{N} A_i \mathring{\hat{\eta}}_i}{t_i^{\circ}} \end{cases} \tag{71}$$

where

$$\begin{cases} \hat{\eta}_i = -\hat{\chi}\left[1 - e^{-\mathcal{D}_i^{\circ} t_i^{\circ}}\left(\cos \nu_i t_i^{\circ} + \dfrac{\mathcal{D}_i^{\circ}}{\nu_i} \sin \nu_i t_i^{\circ}\right)\right] \\ \mathring{\hat{\eta}}_i = -\hat{\chi}/\nu_i \cdot e^{-\mathcal{D}_i^{\circ} t_i^{\circ}} \sin \nu_i t_i^{\circ} \end{cases} \tag{72}$$

Beginning with the moment $t^{\circ} = t_i^{\circ} + t_j^{\circ}$ the representative point intersects in its trajectory the $\mathring{\theta}$ axis, and at the moment in time $t^{\circ} = t_2^{\circ} + t_j^{\circ}$, the signal to switch off the command element goes from the relay element to the EPV of the command element, which ceases to operate at the moment t° = $= t_2^{\circ} + t_j^{\circ}$.

Then for the segment where the directing moment disappears

($\hat{\ddot{\chi}} = 0$), the equation for the velocity of motion of the system becomes

$$\dot{\hat{\theta}}(t_2^{\circ}) = \sum_{i=1}^{N} A_i \dot{\hat{\eta}}_i' + \dot{\hat{\theta}}_2 - \sum_{i=1}^{N} A_i \dot{\hat{\eta}}_{i0}' \tag{73}$$

where $\dot{\hat{\eta}}_{i0}' = \dot{\hat{\eta}}_i$ from the second equation of the system (72).

Because the system under consideration has no unsymmetrical limit cycle, since in the idealized version presented here no disturbing moments act on it, the function of resenblance can be examined in the sections S, S', and S". In the final variation, taking equation (73) into account, the function of correspondance can be written /26

$$\begin{cases} S = \dfrac{\hat{\ddot{\chi}}/2(t_1^{\circ 2} - 2t_1^{\circ} t_3^{\circ} + t_3^{\circ 2}) + K_0 \hat{\delta}(1 - e^{-t_1^{\circ}/T_0^{\circ}}) - \hat{H} - \sum_{i=1}^{N} A_i \hat{\eta}_i}{t_1^{\circ}} \\ S' = \dfrac{\hat{\ddot{\chi}}/2(t_1^{\circ 2} + 2t_1^{\circ} t_3^{\circ} - t_3^{\circ 2}) - K_0 \hat{\delta}(1 - e^{-t_1^{\circ}/T_0^{\circ}}) + \hat{H} + \sum_{i=1}^{N} A_i \hat{\eta} - \sum_{i=n}^{N} A_i \dot{\hat{\eta}}_i}{t_1^{\circ}} \\ S'' = S' + \sum_{i=1}^{N} A_i \dot{\hat{\eta}}_i \end{cases} \tag{74}$$

where $\dot{\hat{\eta}}_i' = e^{-\mathcal{D}_i^{\circ} t_2^{\circ}} [\dot{\hat{\eta}}_{i0} \cos \nu_i t_2^{\circ} - \dfrac{\hat{\eta}_{i0} + \mathcal{D}_i^{\circ} \dot{\hat{\eta}}_{i0}}{\nu_i} \sin \nu_i t_2^{\circ}] = 0$.

are the conditions of attenuation.

<u>Determination of the Velocity in the Limit Cycle</u>

From the first equation of the system (70) we have

$$t_1^{\circ} = \frac{\dot{\hat{\theta}}_0 - \dot{\hat{\theta}}_2 + \sum A_i \dot{\hat{\eta}}_i}{\hat{\ddot{\chi}}} = \frac{S + S''}{\hat{\ddot{\chi}}} \tag{75}$$

Substituting (75) into the second equation of system (70) and taking account of the fact that $S = \dot{\hat{\theta}}_0$, $S'' = -\dot{\hat{\theta}}_2 + \sum_{i=1}^{N} A_i \dot{\hat{\eta}}_i$, we obtain

$$0 = K_0 \hat{\delta}(1 - e^{-\frac{S + S''}{\hat{\ddot{\chi}} T_0}}) + \frac{(S'')^2 - S^2}{2\hat{\ddot{\chi}}} - (S + S'') t_3^{\circ} + \frac{\hat{\ddot{\chi}} t_3^{\circ 2}}{2} - \hat{H} - \sum_{i=1}^{N} A_i \hat{\eta}_i \tag{76}$$

From equation (76) we find the velocity at the limit cycle $\dot{\hat{\theta}}_a$

for which $S'' = S = \hat{\dot{\theta}}_a$.

$$K_0\hat{\delta}(1-e^{-2\hat{\dot{\theta}}_a/\hat{\chi}\tau_i})-2\hat{\dot{\theta}}_a t_j^0+\frac{\hat{\chi}t_j^{0 2}}{2}-\hat{H}-\sum_{i=1}^{N}A_i\hat{\eta}_i=0 \tag{77}$$

or

$$e^{-\frac{2\hat{\dot{\theta}}_a}{\hat{\chi}\tau_i}}=1-\hat{H}/K_0\hat{\delta}-2\hat{\dot{\theta}}_a/K_0\hat{\delta}+\hat{\chi}t_j^{0 2}/2K_0\hat{\delta}-\sum_{i=1}^{N}A_i\hat{\eta}_i/K_0\hat{\delta}, \tag{78}$$

whence

$$\hat{\dot{\theta}}_a=\frac{\hat{\chi}\tau_i}{2}\ln\left(1-\frac{\hat{H}}{K_0\hat{\delta}}-\frac{2\hat{\dot{\theta}}_a t_j^0}{K_0\hat{\delta}}+\frac{\hat{\chi}t_j^{0 2}}{2K_0\hat{\delta}}-\frac{\sum_{i=1}^{N}A_i}{K_0\hat{\delta}}\right). \tag{79}$$

Results of the Modeling

The analysis described above was conducted for the stabilization of a spacecraft along a single axis. Conducting an analytical study of the stailization of the spatial motions of a spacecraft represented a task of great complexity, so at the design stage the real direction devices were designed to be replaced with modeling, using an electronic computer. In addition, at the planning stage the interactions determined by the values of I_{12}, I_{13}, I_{21}, I_{32}, I_{31}, (see figure 2) could frequently be ignored, due to the symmetries of the system and the small angular displacements in the area of stabilization.

The modeling problem involved the study of the dynamical behavior of the system for one total moment of inertia I of the system but different variants of the value of the moment of inertia I_K of the solid body and I_n of the attached elastic structural elements (panels of solar cells); for different values of the frequency of proper oscillation of the solar cell panels; with and without time delays. We also verified the possibility of performing similar studies both on an analog and on a small digital computer (of the Mir-2 type), for which programs were developed. For the solving of the equations on the analog computer (type MN-18M) the structural model of the transmission function was obtained.

Equations (58), (54), (59), and the corresponding flow diagram in figure 4 were modeled in the analog computer in the form of the chart in figure 5. The delay was modeled according to the Pade arrangement (using amplifiers 10, 11, 12, 13). The elasticity, including the fundamental frequency of elastic oscillations, was modeled either by the amplifiers 6,7,8,9 (which corresponds to the structural arrangement in figure 3a) or by amplifiers 14, 15,16,17 (which corresponds to figure 3b). The selection of the elements R_1, R_2, C_2 of the aperiodic feedback with a double time constant was made in the following manner. When the circuit is turned on, the signal Z (with zero initial conditions) at the output is

$$Z(t) = K_0 \delta [1 - e^{-(R_1+R_2)t/R_1 R_2 C_2}]. \tag{80}$$

At the switching off with Z_0 initial conditions the signal Z becomes

$$Z(t) = Z_0 e^{-t/R_1 C_2} \tag{81}$$

So that $\qquad \tau_8 = R_1 C_2, \quad \tau_0 = R_1 R_2 C_2/(R_1 + R_2)$

or $\qquad R_1 = \tau_8/C_2, \quad R_2 = R_1 \tau_0/(R_1 C_2 - \tau_0).$

In the determination of the velocity $\dot{\theta}_a$ in the limit cycle equation (77) is used, transformed to the form

$$K_0 \delta (1 - e^{-t_1/\tau_8}) - \chi_0 t_1 t_3 + \frac{\chi_0 t_3^2}{2} - H - \sum_{i=1}^{d} A_i \eta_i, \tag{82}$$

where

$$\eta_i = -G[1 - e^{-\zeta_0 t_1}(\cos \sigma_0 t_1 + \zeta_0/\sigma_0 \sin \sigma_0 t_1)], \tag{83}$$

and $\qquad \sigma_0 = \sqrt{\sigma_i'^2 - (\sigma_i' \zeta_i')^2}, \quad \zeta_0 = \sigma_i' \zeta_i', \quad G = \chi/\sigma_i'^2.$

The given equations were solved in the digital computer by means of the program in figure 8. In order to solve the same equation on the analog computer, a Laplace transformation was performed, obtaining

$$Y(s)=\frac{K_0\delta}{1+\tau_0 s}-\frac{\chi_0 t_1}{s}+\frac{\chi_0 t_1^2}{2}-H-\sum_{i=1}^{2}\frac{A_i}{s^2+2\zeta_i\sigma_i s+\sigma_i^2} \qquad (84)$$

The model with the transmission function (84) is shown in figure 6.

In order to construct the Königs-Lamery parametric equation, the correspondance fuction (74) is rewritten in the form

$$\begin{cases} S(t)=\dfrac{0.5\chi_0(t_1^2-2t_1t_2+t_2^2)+K_0\delta(1-e^{-t_1/\tau_0})-H-\sum_{i=1}^{2}A_i\eta_i}{t_1} \\ S'(t)=\dfrac{0.5\chi_0(t_1^2+2t_1t_2-t_2^2)-K_0\delta(1-e^{-t_1/\tau_0})+H+\sum_{i=1}^{2}A_i\eta_i}{t_1} \end{cases} \qquad (85)$$ /29

This equation was solved on the digital computer by means of the program in figure 9. The printout produced the values of t_1, $S(t_1)$, $S'(t_1)$, $\eta(t_1)$, $\dot{\eta}(t_1)$, where

$$\dot{\eta}(t_1)=G\left(\sigma_0+\frac{\zeta_0^2}{\sigma_0}\right)e^{-\zeta_0 t_1}\sin\sigma_0 t_1 . \qquad (86)$$

The structural model of the system (85) for the use of the analog computer was obtained in a manner analogous to (84) and is diagrammed in figure 7.

The parameters of the system of equations and the dynamical characteristics of the model under study are produced for various values of I_K and I_n in Table 1 and presented in figure 9. The same system was studied with weak damping of the proper modes of elastic oscillation with the following parameters:

$K_0\delta$ = 162.9 arc min.; H = 0.9 arc min.; τ_0 = 2.1 sec; τ_1 =39.6 sec
T = 0.682 N ; I = 1115 Nm sec^2 ; χ_0 = 2.1 arc min/sec^2
A_1 = 6.39 ; $\Delta_1\Delta_1$ = 803.98 $Nmsec^2$; σ_1 =0.1 Hz
A_2 = 0.853 ; $\Delta_2\Delta_2$ = 107.3 Nm sec^2; σ_2 = 0.18 Hz
A_3 = 0.457 ; $\Delta_3\Delta_3$ = 57.56 Nm sec^2; σ_3 = 0.224 Hz
$\zeta_1 - \zeta_2 - \zeta_3$ = 3.5×10^{-3}

It was shown in the modeling process that it is possible to obtain solutions to equations (84) and (85) on both an analog and a digital computer.

The solution to equation (82) for the 20th variant of Table 1 is given in figure 10.

The numerical solution to the system (85) is given both selectively in Table 2 for the variants of the 1st, 5th, 7th, /30 9th, and 15th modes of the Königs-Lamery diagram including delays (t_3 = 0.1 sec for "1","5","9","15" and t_3 = 0.2 sec for "1") and in figure 11 without them.

Figure 12 shows the dependence of the velocity in the limit cycle $\dot{\theta}_a$ on the inertial coefficient characterizing the flexible superstructure A with time delay.

We should note that upon an increase in A the velocity $\dot{\theta}_a$ decreases, and in the presence of time delays it declines particularly rapidly as A > 10. As A → 0 the effect of a time delay on the change in $\dot{\theta}_a$ is insignificant. As is clear from an examination of figure 10 and Table 2, the change in value of the frequencies of the proper oscillations of the superstructure, σ with a single value of A, are practically unaffected by $\dot{\theta}_a$, especially for A < 10. By means of the modeling on [illegible], shown in figure 5, confirmed the complete identity of the structures modeling the dynamics of a spacecraft with attached flexible structures in Figure 3. The research conducted on the analog computer showed that the largest contribution to the dynamical processes of the system belonged to the primary (first) harmonic of the elastic oscillations of the structure and that at the initial design stages of the system, higher harmonics can be neglected.

REFERENCES

1. Sasin, G.G., Dinamika kosmicheskikh letatel'nykh apparatov s prisoedinennymi uprugimi elementami konstruktsii. perspektivnyi analiticheskii obzor ("Dynamics of a Spacecraft with Attached Elastic Structural Elements. Brief Analytic Outline"), Moscow, VINITI, 1977.

2. Hughes, P.C., "Attitude Dynamics of a Three-Axis Stabilized Satellite with a Large Flexible Solar Array," Journal of Astronautical Science 20/3 (Nov.-Dec. 1972).

3. Likins, P.W., "Finite Element Appendage Equations for Hybrid Coordinate Dynamic Analysis," Internat. J. Solids and Structures 8 (1972).

4. Likins, P.W., "Dynamics and Control of Flexible Space Vehicles," Jet Propulsion Laboratories, Report TR-32-1329, Rev. 1, 1970.

5. Neimark, Yu.M., Metod tochenykh otobrazhenii v teorii nelineinykh kolebanii ("The Method of Pointwise Representation in the Theory of Nonlinear Oscillations"), Moscow, Nauka, 1972.

6. Gaushus, E.V., Issledovanie dinamicheskikh sistem metodom tochenykh preobrazovanii ("Research on Dynamical Systems by the Method of Pointwise Transformation"), Moscow, Nauka, 1976.

7. Vertan', A.F., and V.F. Evdogimov, Elektronnoe modelirovanie peredatochnykh funktsii ("Electric Modeling of Transmission Functions"), Kiev, Tekhnika, 1971.

TABLE 1.*

	I	I_A	I_K	χ_0	A	σ_1	ζ_1'	σ_1	ζ_1	σ_0	ζ_0	G_0	$K_0\delta$	τ_0	H	$t_з$
	нм·с^2	нм·с^2	нм·с^2	угл.мин / с	-	$\frac{1}{с}$	-	$\frac{1}{с}$	-	$\frac{1}{с}$	$\frac{1}{с}$	угл.мин	угл.мин	с	угл.мин	с
I / 2	I5000	I4600	200	35	74	0,26 / 2,6	0,54	0,03 / 0,3	0,062	0,22 / 2,2	0,I4 / I,4	5I7,75 / 5,I775	90	I,5	3	0, 0,I / 0,2
3 / 4	I5000	I4500	500	35	23	0,I64 / I,64	0,34	0,03 / 0,3	0,062	0,I54 / I,54	0,056 / 0,56	I30I,I / I3,0II	90	I,5	3	0, 0I / 0,2
5 / 6	I5000	I4000	I000	35	I4	0,II6 / I,I6	0,24	0,03 / 0,3	0,062	0,II3 / I,I3	0,028 / 0,28	260I,I / 26,0II	90	I,5	3	0, 0I / 0,2
7 / 8	I5000	I3500	I500	35	9	0,I / I	0,I9	0,03 / 0,3	0,062	0,098 / 0,98	0,0I9 / 0,I9	3500 / 35	90	I,5	3	0, 0,I / 0,2
9 / I0	I5000	I3000	2000	35	6,5	0,082 / 0,82	0,I7	0,03 / 0,3	0,062	0,08I / 0,8I	0,0I4 / 0,I4	5205 / 52,05	90	I,5	3	0, 0,I / 0,2
II / I2	I5000	I2500	2500	35	5	0,073 / 0,73	0,I5	0,03 / 0,3	0,062	0,072 / 0,72	0,0II / 0,II	6567,8 / 65,678	90	I,5	3	0, 0,I / 0,2
I3 / I4	I5000	I2000	3000	35	4	0,067 / 0,67	0,I4	0,03 / 0,3	0,062	0,066 / 0,66	0,009 / 0.09	7796,8 / 77,968	90	I,5	3	0,I / 0,2
I5 / I6	I5000	I0000	5000	35	2	0.052 / 0,52	0,II	0,03 / 0,3	0,062	0,052 / 0,52	0,006 / 0,06	I2943 / I29,44	90	I,5	3	0, 0,I / 0,2
I7 / I8	I5000	7500	7500	35	I	0,042 / 0,42	0,09	0,03 / 0,3	0,062	0,042 / 0,42	0,004 / 0,04	I9469 / I94,69	90	I,5	3	0, 0,I / 0,2
I9 / 20	I5000	5000	I0000	35	0,5	0,03I / 0,3I	0,06	0,03 / 0,3	0,062	0,03I / 0.3I	0,002 / 0,02	37624 / 376,24	90	I,5	3	0, 0,I / 0,2

[*Commas in tabulated material are equivalent to decimal points.]

TABLE 2.

I=10
"ТАБЛИЦА"1

T	S	S2	Э1	ЭН
.3000$_{10}$-1	-.1100$_{10}$ 1	.2150$_{10}$ 1	-.1579$_{10}$-1	.1051$_{10}$ 10
.3020$_{10}$-1	-.1797$_{10}$ 0	.1236$_{10}$ 1	-.1600$_{10}$-1	.1058$_{10}$ 10
.3040$_{10}$-1	.7323$_{10}$ 0	.3316$_{10}$ 0	-.1622$_{10}$-1	.1065$_{10}$ 10
.3060$_{10}$-1	.1635$_{10}$ 1	-.5649$_{10}$ 0	-.1643$_{10}$-1	.1072$_{10}$ 10
.3080$_{10}$-1	.2531$_{10}$ 1	-.1453$_{10}$ 1	-.1665$_{10}$-1	.1079$_{10}$ 10
.3100$_{10}$-1	.3418$_{10}$ 1	-.2333$_{10}$ 1	-.1686$_{10}$-1	.1086$_{10}$ 10
.3120$_{10}$-1	.4297$_{10}$ 1	-.3205$_{10}$ 1	-.1708$_{10}$-1	.1093$_{10}$ 10
.3140$_{10}$-1	.5168$_{10}$ 1	-.4069$_{10}$ 1	-.1730$_{10}$-1	.1100$_{10}$ 10
.3160$_{10}$-1	.6031$_{10}$ 1	-.4925$_{10}$ 1	-.1752$_{10}$-1	.1107$_{10}$ 10
.3180$_{10}$-1	.6886$_{10}$ 1	-.5773$_{10}$ 1	-.1774$_{10}$-1	.1114$_{10}$ 10
.3200$_{10}$-1	.7735$_{10}$ 1	-.6615$_{10}$ 1	-.1797$_{10}$-1	.1121$_{10}$ 1

I=20
"ТАБЛИЦА"1

T	S	S2	Э1	ЭН
.3000$_{10}$-1	-.2082$_{10}$ 1	.3132$_{10}$ 1	-.1540$_{10}$-1	.1012$_{10}$ 10
.3020$_{10}$-1	-.1174$_{10}$ 1	.2231$_{10}$ 1	-.1560$_{10}$-1	.1018$_{10}$ 10
.3040$_{10}$-1	-.2754$_{10}$ 0	.1339$_{10}$ 1	-.1580$_{10}$-1	.1024$_{10}$ 10
.3060$_{10}$-1	.6148$_{10}$ 0	.4561$_{10}$ 0	-.1601$_{10}$-1	.1031$_{10}$ 10
.3080$_{10}$-1	.1496$_{10}$ 1	-.4187$_{10}$ 0	-.1622$_{10}$-1	.1037$_{10}$ 10
.3100$_{10}$-1	.2370$_{10}$ 1	-.1285$_{10}$ 1	-.1642$_{10}$-1	.1044$_{10}$ 10
.3120$_{10}$-1	.3235$_{10}$ 1	-.2143$_{10}$ 1	-.1663$_{10}$-1	.1050$_{10}$ 10
.3140$_{10}$-1	.4093$_{10}$ 1	-.2994$_{10}$ 1	-.1684$_{10}$-1	.1057$_{10}$ 10
.3160$_{10}$-1	.4942$_{10}$ 1	-.3836$_{10}$ 1	-.1706$_{10}$-1	.1063$_{10}$ 10
.3180$_{10}$-1	.5784$_{10}$ 1	-.4671$_{10}$ 1	-.1727$_{10}$-1	.1069$_{10}$ 10
.3200$_{10}$-1	.6618$_{10}$ 1	-.5498$_{10}$ 1	-.1748$_{10}$-1	.1076$_{10}$ 1

I=30

.3900$_{10}$-1	.2717$_{10}$ 1	-.1352$_{10}$ 1	-.2653$_{10}$-1	.1359$_{10}$ 10
.3920$_{10}$-1	.3210$_{10}$ 1	-.1830$_{10}$ 1	-.2680$_{10}$-1	.1366$_{10}$ 10
.3940$_{10}$-1	.3699$_{10}$ 1	-.2320$_{10}$ 1	-.2708$_{10}$-1	.1373$_{10}$ 10
.3960$_{10}$-1	.4184$_{10}$ 1	-.2798$_{10}$ 1	-.2735$_{10}$-1	.1380$_{10}$ 10
.3980$_{10}$-1	.4666$_{10}$ 1	-.3273$_{10}$ 1	-.2763$_{10}$-1	.1387$_{10}$ 10
.4000$_{10}$-1	.5143$_{10}$ 1	-.3743$_{10}$ 1	-.2791$_{10}$-1	.1394$_{10}$ 10
.4020$_{10}$-1	.5617$_{10}$ 1	-.4210$_{10}$ 1	-.2819$_{10}$-1	.1401$_{10}$ 10
.4040$_{10}$-1	.6087$_{10}$ 1	-.4673$_{10}$ 1	-.2847$_{10}$-1	.1408$_{10}$ 10
.4060$_{10}$-1	.6553$_{10}$ 1	-.5132$_{10}$ 1	-.2875$_{10}$-1	.1415$_{10}$ 10
.4080$_{10}$-1	.7016$_{10}$ 1	-.5588$_{10}$ 1	-.2903$_{10}$-1	.1422$_{10}$ 10
.4100$_{10}$-1	.7475$_{10}$ 1	-.6040$_{10}$ 1	-.2932$_{10}$-1	.1429$_{10}$ 1

TABLE 2 (continued)

I=40				
.3900$_{10}$-1	.2455$_{10}$ 1	-.1090$_{10}$ 1	-.2618$_{10}$-1	.1332$_{10}$ 10
.3920$_{10}$-1	.2945$_{10}$ 1	-.1573$_{10}$ 1	-.2644$_{10}$-1	.1339$_{10}$ 10
.3940$_{10}$-1	.3431$_{10}$ 1	-.2052$_{10}$ 1	-.2671$_{10}$-1	.1345$_{10}$ 10
.3960$_{10}$-1	.3914$_{10}$ 1	-.2528$_{10}$ 1	-.2698$_{10}$-1	.1352$_{10}$ 10
.3980$_{10}$-1	.4392$_{10}$ 1	-.2999$_{10}$ 1	-.2725$_{10}$-1	.1359$_{10}$ 10
.4000$_{10}$-1	.4867$_{10}$ 1	-.3467$_{10}$ 1	-.2753$_{10}$-1	.1365$_{10}$ 10
.4020$_{10}$-1	.5338$_{10}$ 1	-.3931$_{10}$ 1	-.2780$_{10}$-1	.1372$_{10}$ 10
.4040$_{10}$-1	.5805$_{10}$ 1	-.4391$_{10}$ 1	-.2808$_{10}$-1	.1379$_{10}$ 10
.4060$_{10}$-1	.6269$_{10}$ 1	-.4848$_{10}$ 1	-.2835$_{10}$-1	.1385$_{10}$ 10
.4080$_{10}$-1	.6729$_{10}$ 1	-.5301$_{10}$ 1	-.2863$_{10}$-1	.1392$_{10}$ 10
.4100$_{10}$-1	.7185$_{10}$ 1	-.5750$_{10}$ 1	-.2891$_{10}$-1	.1399$_{10}$ 1
I=50				
.4300$_{10}$-1	.7270$_{10}$ 0	.7779$_{10}$ 0	-.3253$_{10}$-1	.1512$_{10}$ 10
.4310$_{10}$-1	.9133$_{10}$ 0	.5951$_{10}$ 0	-.3268$_{10}$-1	.1516$_{10}$ 10
.4320$_{10}$-1	.1098$_{10}$ 1	.4131$_{10}$ 0	-.3284$_{10}$-1	.1519$_{10}$ 10
.4330$_{10}$-1	.1283$_{10}$ 1	.2318$_{10}$ 0	-.3299$_{10}$-1	.1523$_{10}$ 10
.4340$_{10}$-1	.1467$_{10}$ 1	.5128$_{10}$-1	-.3314$_{10}$-1	.1526$_{10}$ 10
.4350$_{10}$-1	.1651$_{10}$ 1	-.1285$_{10}$ 0	-.3329$_{10}$-1	.1530$_{10}$ 10
.4360$_{10}$-1	.1833$_{10}$ 1	-.3076$_{10}$ 0	-.3345$_{10}$-1	.1533$_{10}$ 10
.4370$_{10}$-1	.2015$_{10}$ 1	-.4859$_{10}$ 0	-.3360$_{10}$-1	.1537$_{10}$ 10
.4380$_{10}$-1	.2196$_{10}$ 1	-.6636$_{10}$ 0	-.3375$_{10}$-1	.1540$_{10}$ 10
.4390$_{10}$-1	.2377$_{10}$ 1	-.8405$_{10}$ 0	-.3391$_{10}$-1	.1544$_{10}$ 10
.4400$_{10}$-1	.2556$_{10}$ 1	-.1016$_{10}$ 1	-.3406$_{10}$-1	.1547$_{10}$ 1
I=60				
.4300$_{10}$-1	.6493$_{10}$ 0	.8556$_{10}$ 0	-.3229$_{10}$-1	.1496$_{10}$ 10
.4310$_{10}$-1	.8352$_{10}$ 0	.6732$_{10}$ 0	-.3244$_{10}$-1	.1499$_{10}$ 10
.4320$_{10}$-1	.1020$_{10}$ 1	.4915$_{10}$ 0	-.3259$_{10}$-1	.1502$_{10}$ 10
.4330$_{10}$-1	.1204$_{10}$ 1	.3106$_{10}$ 0	-.3274$_{10}$-1	.1506$_{10}$ 10
.4340$_{10}$-1	.1388$_{10}$ 1	.1304$_{10}$ 0	-.3290$_{10}$-1	.1509$_{10}$ 10
.4350$_{10}$-1	.1571$_{10}$ 1	-.4897$_{10}$-1	-.3305$_{10}$-1	.1513$_{10}$ 10
.4360$_{10}$-1	.1753$_{10}$ 1	-.2276$_{10}$ 0	-.3320$_{10}$-1	.1516$_{10}$ 10
.4370$_{10}$-1	.1935$_{10}$ 1	-.4056$_{10}$ 0	-.3335$_{10}$-1	.1520$_{10}$ 10
.4380$_{10}$-1	.2115$_{10}$ 1	-.5829$_{10}$ 0	-.3350$_{10}$-1	.1523$_{10}$ 10
.4390$_{10}$-1	.2295$_{10}$ 1	-.7594$_{10}$ 0	-.3365$_{10}$-1	.1526$_{10}$ 10
.4400$_{10}$-1	.2475$_{10}$ 1	-.9353$_{10}$ 0	-.3381$_{10}$-1	.1530$_{10}$ 1

TABLE 2 (continued)

I=70				
.4400$_{10}$-1	-.1351$_{10}$ 1	2891$_{10}$ 1	-.3388$_{10}$-1	.1539$_{10}$ 10
.4420$_{10}$-1	-.1012$_{10}$ 1	.2559$_{10}$ 1	-.3419$_{10}$-1	.1546$_{10}$ 10
.4440$_{10}$-1	-.6755$_{10}$ 0	.2229$_{10}$ 1	-.3450$_{10}$-1	.1553$_{10}$ 10
.4460$_{10}$-1	-.3414$_{10}$ 0	.1902$_{10}$ 1	-.3481$_{10}$-1	.1560$_{10}$ 10
.4480$_{10}$-1	-.1009$_{10}$-1	.1578$_{10}$ 1	-.3512$_{10}$-1	.1567$_{10}$ 10
.4500$_{10}$-1	.3185$_{10}$ 0	.1256$_{10}$ 1	-.3544$_{10}$-1	.1574$_{10}$ 10
.4520$_{10}$-1	.6446$_{10}$ 0	.9373$_{10}$ 0	-.3575$_{10}$-1	.1581$_{10}$ 10
.4540$_{10}$-1	.9680$_{10}$ 0	.6209$_{10}$ 0	-.3607$_{10}$-1	.1588$_{10}$ 10
.4560$_{10}$-1	.1288$_{10}$ 1	.3070$_{10}$ 0	-.3639$_{10}$-1	.1595$_{10}$ 10
.4580$_{10}$-1	.1607$_{10}$ 1	-.4342$_{10}$-2	-.3671$_{10}$-1	.1602$_{10}$ 10
.4600$_{10}$-1	.1923$_{10}$ 1	-.3132$_{10}$ 0	-.3703$_{10}$-1	.1609$_{10}$ 1
I=80				
.4400$_{10}$-1	-.1388$_{10}$ 1	.2928$_{10}$ 1	-.3370$_{10}$-1	.1527$_{10}$ 10
.4420$_{10}$-1	-.1049$_{10}$ 1	.2596$_{10}$ 1	-.3401$_{10}$-1	.1534$_{10}$ 10
.4440$_{10}$-1	-.7130$_{10}$ 0	.2267$_{10}$ 1	-.3432$_{10}$-1	.1541$_{10}$ 10
.4460$_{10}$-1	-.3792$_{10}$ 0	.1940$_{10}$ 1	-.3462$_{10}$-1	.1548$_{10}$ 10
.4480$_{10}$-1	-.4827$_{10}$-1	.1616$_{10}$ 1	-.3493$_{10}$-1	.1554$_{10}$ 10
.4500$_{10}$-1	.2800$_{10}$ 0	.1294$_{10}$ 1	-.3525$_{10}$-1	.1561$_{10}$ 10
.4520$_{10}$-1	.6057$_{10}$ 0	.9762$_{10}$ 0	-.3556$_{10}$-1	.1568$_{10}$ 10
.4540$_{10}$-1	.9288$_{10}$ 0	.6601$_{10}$ 0	-.3587$_{10}$-1	.1575$_{10}$ 10
.4560$_{10}$-1	.1249$_{10}$ 1	.3465$_{10}$ 0	-.3619$_{10}$-1	.1582$_{10}$ 10
.4580$_{10}$-1	.1567$_{10}$ 1	.3558$_{10}$-1	-.3651$_{10}$-1	.1589$_{10}$ 10
.4600$_{10}$-1	.1882$_{10}$ 1	-.2729$_{10}$ 0	-.3683$_{10}$-1	.1596$_{10}$ 1
I=90				
.4500$_{10}$-1	-.1653$_{10}$ 1	.3228$_{10}$ 1	-.3542$_{10}$-1	.1574$_{10}$ 10
.4520$_{10}$-1	-.1336$_{10}$ 1	.2918$_{10}$ 1	-.3573$_{10}$-1	.1581$_{10}$ 10
.4540$_{10}$-1	-.1021$_{10}$ 1	.2610$_{10}$ 1	-.3605$_{10}$-1	.1588$_{10}$ 10
.4560$_{10}$-1	-.7092$_{10}$ 0	.2305$_{10}$ 1	-.3637$_{10}$-1	.1595$_{10}$ 10
.4580$_{10}$-1	-.3996$_{10}$ 0	.2002$_{10}$ 1	-.3669$_{10}$-1	.1602$_{10}$ 10
.4600$_{10}$-1	-.9254$_{10}$-1	.1702$_{10}$ 1	-.3701$_{10}$-1	.1609$_{10}$ 10
.4620$_{10}$-1	.2120$_{10}$ 0	.1404$_{10}$ 1	-.3733$_{10}$-1	.161[illegible]$_{10}$ 10
.4640$_{10}$-1	.5142$_{10}$ 0	.1109$_{10}$ 1	-.3766$_{10}$-1	.1623$_{10}$ 10
.4660$_{10}$-1	.8140$_{10}$ 0	.8169$_{10}$ 0	-.3798$_{10}$-1	.1630$_{10}$ 10
.4680$_{10}$-1	.1111$_{10}$ 1	.5264$_{10}$ 0	-.3831$_{10}$-1	.1637$_{10}$ 10
.4700$_{10}$-1	.1406$_{10}$ 1	.2383$_{10}$ 0	-.3864$_{10}$-1	.1643$_{10}$ 1

TABLE 2 (continued)

I=100				
.4500$_{10}$-1	-.1673$_{10}$ 1	.3248$_{10}$ 1	-.3528$_{10}$-1	.1564$_{10}$ 10
.4520$_{10}$-1	-.1356$_{10}$ 1	.2938$_{10}$ 1	-.3560$_{10}$-1	.1571$_{10}$ 10
.4540$_{10}$-1	-.1041$_{10}$ 1	.2630$_{10}$ 1	-.3591$_{10}$-1	.1578$_{10}$ 10
.4560$_{10}$-1	-.7296$_{10}$ 0	.2325$_{10}$ 1	-.3623$_{10}$-1	.1585$_{10}$ 10
.4580$_{10}$-1	-.4202$_{10}$ 0	.2023$_{10}$ 1	-.3654$_{10}$-1	.1592$_{10}$ 10
.4600$_{10}$-1	-.1133$_{10}$ 0	.1723$_{10}$ 1	-.3686$_{10}$-1	.1599$_{10}$ 10
.4620$_{10}$-1	.1911$_{10}$ 0	.1425$_{10}$ 1	-.3718$_{10}$-1	.1606$_{10}$ 10
.4640$_{10}$-1	.4931$_{10}$ 0	.1130$_{10}$ 1	-.3751$_{10}$-1	.1613$_{10}$ 10
.4660$_{10}$-1	.7927$_{10}$ 0	.8382$_{10}$ 0	-.3783$_{10}$-1	.1620$_{10}$ 10
.4680$_{10}$-1	.1090$_{10}$ 1	.5479$_{10}$ 0	-.3815$_{10}$-1	.1626$_{10}$ 10
.4700$_{10}$-1	.1384$_{10}$ 1	.2600$_{10}$ 0	-.3848$_{10}$-1	.1633$_{10}$ 1
I=110				
.4600$_{10}$-1	-.1291$_{10}$ 1	.2901$_{10}$ 1	-.3708$_{10}$-1	.1612$_{10}$ 10
.4620$_{10}$-1	-.9925$_{10}$ 0	.2609$_{10}$ 1	-.3740$_{10}$-1	.1619$_{10}$ 10
.4640$_{10}$-1	-.6955$_{10}$ 0	.2319$_{10}$ 1	-.3773$_{10}$-1	.1626$_{10}$ 10
.4660$_{10}$-1	-.4009$_{10}$ 0	.2031$_{10}$ 1	-.3805$_{10}$-1	.1633$_{10}$ 10
.4680$_{10}$-1	-.1087$_{10}$ 0	.1746$_{10}$ 1	-.3838$_{10}$-1	.1640$_{10}$ 10
.4700$_{10}$-1	.1811$_{10}$ 0	.1463$_{10}$ 1	-.3871$_{10}$-1	.1647$_{10}$ 10
.4720$_{10}$-1	.4687$_{10}$ 0	.1183$_{10}$ 1	-.3904$_{10}$-1	.1654$_{10}$ 10
.4740$_{10}$-1	.7540$_{10}$ 0	.9049$_{10}$ 0	-.3937$_{10}$-1	.1661$_{10}$ 10
.4760$_{10}$-1	.1037$_{10}$ 1	.6289$_{10}$ 0	-.3971$_{10}$-1	.1668$_{10}$ 10
.4780$_{10}$-1	.1317$_{10}$ 1	.3551$_{10}$ 0	-.4004$_{10}$-1	.1675$_{10}$ 10
.4800$_{10}$-1	.1596$_{10}$ 1	.8353$_{10}$-1	-.4038$_{10}$-1	.1682$_{10}$ 1
I=120				
.4600$_{10}$-1	-.1306$_{10}$ 1	.2916$_{10}$ 1	-.3694$_{10}$-1	.1603$_{10}$ 10
.4620$_{10}$-1	-.1007$_{10}$ 1	.2624$_{10}$ 1	-.3727$_{10}$-1	.1610$_{10}$ 10
.4640$_{10}$-1	-.7106$_{10}$ 0	.2334$_{10}$ 1	-.3759$_{10}$-1	.1617$_{10}$ 10
.4660$_{10}$-1	-.4161$_{10}$ 0	.2047$_{10}$ 1	-.3791$_{10}$-1	.1624$_{10}$ 10
.4680$_{10}$-1	-.1240$_{10}$ 0	.1762$_{10}$ 1	-.3824$_{10}$-1	.1631$_{10}$ 10
.4700$_{10}$-1	.1656$_{10}$ 0	.1479$_{10}$ 1	-.3857$_{10}$-1	.1638$_{10}$ 10
.4720$_{10}$-1	.4531$_{10}$ 0	.1198$_{10}$ 1	-.3889$_{10}$-1	.1645$_{10}$ 10
.4740$_{10}$-1	.7302$_{10}$ 0	.9207$_{10}$ 0	-.3922$_{10}$-1	.1652$_{10}$ 10
.4760$_{10}$-1	.1021$_{10}$ 1	.6448$_{10}$ 0	-.3955$_{10}$-1	.1659$_{10}$ 10
.4780$_{10}$-1	.1301$_{10}$ 1	.3711$_{10}$ 0	-.3989$_{10}$-1	.1666$_{10}$ 10
.4800$_{10}$-1	.1580$_{10}$ 1	.9963$_{10}$-1	-.4022$_{10}$-1	.1672$_{10}$ 1

TABLE 2 (continued)

I=130				
$.4700_{10}$-1	$-.6535_{10}$ 0	$.2298_{10}$ 1	$-.3858_{10}$-1	$.1641_{10}$ 10
$.4720_{10}$-1	$-.3695_{10}$ 0	$.2021_{10}$ 1	$-.3891_{10}$-1	$.1648_{10}$ 10
$.4740_{10}$-1	$-.8776_{10}$-1	$.1746_{10}$ 1	$-.3924_{10}$-1	$.1655_{10}$ 10
$.4760_{10}$-1	$.1917_{10}$ 0	$.1474_{10}$ 1	$-.3957_{10}$-1	$.1662_{10}$ 10
$.4780_{10}$-1	$.4609_{10}$ 0	$.1204_{10}$ 1	$-.3991_{10}$-1	$.1669_{10}$ 10
$.4800_{10}$-1	$.7440_{10}$ 0	$.9359_{10}$ 0	$-.4024_{10}$-1	$.1676_{10}$ 10
$.4820_{10}$-1	$.1016_{10}$ 1	$.6700_{10}$ 0	$-.4058_{10}$-1	$.1683_{10}$ 10
$.4840_{10}$-1	$.1287_{10}$ 1	$.4063_{10}$ 0	$-.4092_{10}$-1	$.1690_{10}$ 10
$.4860_{10}$-1	$.1556_{10}$ 1	$.1446_{10}$ 0	$-.4125_{10}$-1	$.1697_{10}$ 10
$.4880_{10}$-1	$.1822_{10}$ 1	$-.1148_{10}$ 0	$-.4159_{10}$-1	$.1704_{10}$ 10
$.4900_{10}$-1	$.2087_{10}$ 1	$-.3723_{10}$ 0	$-.4194_{10}$-1	$.1711_{10}$ 1
I=140				
$.4700_{10}$-1	$-.6623_{10}$ 0	$.2307_{10}$ 1	$-.3848_{10}$-1	$.1635_{10}$ 10
$.4720_{10}$-1	$-.3784_{10}$ 0	$.2030_{10}$ 1	$-.3881_{10}$-1	$.1641_{10}$ 10
$.4740_{10}$-1	$-.9678_{10}$-1	$.1755_{10}$ 1	$-.3913_{10}$-1	$.1648_{10}$ 10
$.4760_{10}$-1	$.1826_{10}$ 0	$.1483_{10}$ 1	$-.3947_{10}$-1	$.1655_{10}$ 10
$.4780_{10}$-1	$.4598_{10}$ 0	$.1213_{10}$ 1	$-.3980_{10}$-1	$.1662_{10}$ 10
$.4800_{10}$-1	$.7347_{10}$ 0	$.9452_{10}$ 0	$-.4013_{10}$-1	$.1669_{10}$ 10
$.4820_{10}$-1	$.1007_{10}$ 1	$.6793_{10}$ 0	$-.4047_{10}$-1	$.1676_{10}$ 10
$.4840_{10}$-1	$.1278_{10}$ 1	$.4157_{10}$ 0	$-.4080_{10}$-1	$.1683_{10}$ 10
$.4860_{10}$-1	$.1546_{10}$ 1	$.1541_{10}$ 0	$-.4114_{10}$-1	$.1690_{10}$ 10
$.4880_{10}$-1	$.1813_{10}$ 1	$-.1052_{10}$ 0	$-.4148_{10}$-1	$.1697_{10}$ 10
$.4900_{10}$-1	$.2077_{10}$ 1	$-.3626_{10}$ 0	$-.4182_{10}$-1	$.1704_{10}$ 1
I=150				
$.4800_{10}$-1	$-.9292_{10}$ 0	$.2609_{10}$ 1	$-.4033_{10}$-1	$.1680_{10}$ 10
$.4820_{10}$-1	$-.6633_{10}$ 0	$.2350_{10}$ 1	$-.4067_{10}$-1	$.1687_{10}$ 10
$.4840_{10}$-1	$-.3995_{10}$ 0	$.2093_{10}$ 1	$-.4100_{10}$-1	$.1694_{10}$ 10
$.4860_{10}$-1	$-.1379_{10}$ 0	$.1838_{10}$ 1	$-.4134_{10}$-1	$.1701_{10}$ 10
$.4880_{10}$-1	$.1216_{10}$ 0	$.1586_{10}$ 1	$-.4168_{10}$-1	$.1708_{10}$ 10
$.4900_{10}$-1	$.3791_{10}$ 0	$.1335_{10}$ 1	$-.4203_{10}$-1	$.1715_{10}$ 10
$.4920_{10}$-1	$.6346_{10}$ 0	$.1087_{10}$ 1	$-.4237_{10}$-1	$.1722_{10}$ 10
$.4940_{10}$-1	$.8880_{10}$ 0	$.8409_{10}$ 0	$-.4272_{10}$-1	$.1729_{10}$ 10
$.4960_{10}$-1	$.1139_{10}$ 1	$.5964_{10}$ 0	$-.4306_{10}$-1	$.1736_{10}$ 10
$.4980_{10}$-1	$.1389_{10}$ 1	$.3539_{10}$ 0	$-.4341_{10}$-1	$.1743_{10}$ 10
$.5000_{10}$-1	$.1636_{10}$ 1	$.1134_{10}$ 0	$-.4376_{10}$-1	$.1750_{10}$ 1

TABLE 2 (continued)

I=160				
.4800⏨-1	-.9321⏨ 0	.2612⏨ 1	-.4026⏨-1	.1676⏨ 10
.4820⏨-1	-.6662⏨ 0	.2353⏨ 1	-.4060⏨-1	.1683⏨ 10
.4840⏨-1	-.4024⏨ 0	.2096⏨ 1	-.4093⏨-1	.1690⏨ 10
.4860⏨-1	-.1408⏨ 0	.1841⏨ 1	-.4127⏨-1	.1696⏨ 10
.4880⏨-1	.1187⏨ 0	.1589⏨ 1	-.4161⏨-1	.1703⏨ 10
.4900⏨-1	.3761⏨ 0	.1338⏨ 1	-.4195⏨-1	.1710⏨ 10
.4920⏨-1	.6316⏨ 0	.1090⏨ 1	-.4230⏨-1	.1717⏨ 10
.4940⏨-1	.8850⏨ 0	.8439⏨ 0	-.4264⏨-1	.1724⏨ 10
.4960⏨-1	.1136⏨ 1	.5995⏨ 0	-.4299⏨-1	.1731⏨ 10
.4980⏨-1	.1385⏨ 1	.3570⏨ 0	-.4333⏨-1	.1738⏨ 10
.5000⏨-1	.1633⏨ 1	.1165⏨ 0	-.4368⏨-1	.1745⏨ 1
I=170				
.4900⏨-1	-.4805⏨ 0	.2195⏨ 1	-.4193⏨-1	.1711⏨ 10
.4920⏨-1	-.2285⏨ 0	.1950⏨ 1	-.4227⏨-1	.1718⏨ 10
.4940⏨-1	.2135⏨-1	.1707⏨ 1	-.4262⏨-1	.1725⏨ 10
.4960⏨-1	.2693⏨ 0	.1466⏨ 1	-.4297⏨-1	.1732⏨ 10
.4980⏨-1	.5152⏨ 0	.1227⏨ 1	-.4331⏨-1	.1739⏨ 10
.5000⏨-1	.7593⏨ 0	.9906⏨ 0	-.4366⏨-1	.1746⏨ 10
.5020⏨-1	.1001⏨ 1	.7555⏨ 0	-.4401⏨-1	.1753⏨ 10
.5040⏨-1	.1241⏨ 1	.5223⏨ 0	-.4436⏨-1	.1760⏨ 10
.5060⏨-1	.1480⏨ 1	.2909⏨ 0	-.4472⏨-1	.1767⏨ 10
.5080⏨-1	.1716⏨ 1	.6145⏨-1	-.4507⏨-1	.1774⏨ 10
.5100⏨-1	.1951⏨ 1	-.1662⏨ 0	-.4542⏨-1	.1781⏨ 1
I=180				
.4900⏨-1	-.4815⏨ 0	.2196⏨ 1	-.4188⏨-1	.1708⏨ 10
.4920⏨-1	-.2295⏨ 0	.1951⏨ 1	-.4223⏨-1	.1715⏨ 10
.4940⏨-1	.2037⏨-1	.1708⏨ 1	-.4257⏨-1	.1722⏨ 10
.4960⏨-1	.2683⏨ 0	.1467⏨ 1	-.4292⏨-1	.1729⏨ 10
.4980⏨-1	.5142⏨ 0	.1228⏨ 1	-.4326⏨-1	.1736⏨ 10
.5000⏨-1	.7583⏨ 0	.9916⏨ 0	-.4361⏨-1	.1743⏨ 10
.5020⏨-1	.1000⏨ 1	.7565⏨ 0	-.4396⏨-1	.1750⏨ 10
.5040⏨-1	.1240⏨ 1	.5233⏨ 0	-.4431⏨-1	.1757⏨ 10
.5060⏨-1	.1479⏨ 1	.2919⏨ 0	-.4466⏨-1	.1764⏨ 10
.5080⏨-1	.1715⏨ 1	.6249⏨-1	-.4502⏨-1	.1771⏨ 10
.5100⏨-1	.1950⏨ 1	-.1651⏨ 0	-.4537⏨-1	.1778⏨ 1

TABLE 2 (continued)

I=1.90				
.4900$_{10}$-1	-.9060$_{10}$ 0	.2621$_{10}$ 1	-.4217$_{10}$-1	.1721$_{10}$ 10
.4920$_{10}$-1	-.6557$_{10}$ 0	.2377$_{10}$ 1	-.4252$_{10}$-1	.1728$_{10}$ 10
.4940$_{10}$-1	-.4075$_{10}$ 0	.2136$_{10}$ 1	-.4286$_{10}$-1	.1735$_{10}$ 10
.4960$_{10}$-1	-.1613$_{10}$ 0	.1897$_{10}$ 1	-.4321$_{10}$-1	.1742$_{10}$ 10
.4980$_{10}$-1	.8288$_{10}$-1	.1660$_{10}$ 1	-.4356$_{10}$-1	.1749$_{10}$ 10
.5000$_{10}$-1	.3251$_{10}$ 0	.1424$_{10}$ 1	-.4391$_{10}$-1	.1756$_{10}$ 10
.5020$_{10}$-1	.5655$_{10}$ 0	.1191$_{10}$ 1	-.4426$_{10}$-1	.1763$_{10}$ 10
.5040$_{10}$-1	.8040$_{10}$ 0	.9599$_{10}$ 0	-.4462$_{10}$-1	.1770$_{10}$ 10
.5060$_{10}$-1	.1040$_{10}$ 1	.7303$_{10}$ 0	-.4497$_{10}$-1	.1777$_{10}$ 10
.5080$_{10}$-1	.1275$_{10}$ 1	.5025$_{10}$ 0	-.4533$_{10}$-1	.1784$_{10}$ 10
.5100$_{10}$-1	.1508$_{10}$ 1	.2766$_{10}$ 0	-.4569$_{10}$-1	.1791$_{10}$ 1
I=2.00				
.4900$_{10}$-1	-.9405$_{10}$ 0	.2655$_{10}$ 1	-.3879$_{10}$-1	.1582$_{10}$ 10
.4920$_{10}$-1	-.6904$_{10}$ 0	.2412$_{10}$ 1	-.3910$_{10}$-1	.1589$_{10}$ 10
.4940$_{10}$-1	-.4424$_{10}$ 0	.2171$_{10}$ 1	-.3942$_{10}$-1	.1595$_{10}$ 10
.4960$_{10}$-1	-.1963$_{10}$ 0	.1932$_{10}$ 1	-.3974$_{10}$-1	.1602$_{10}$ 10
.4980$_{10}$-1	.4776$_{10}$-1	.1695$_{10}$ 1	-.4006$_{10}$-1	.1608$_{10}$ 10
.5000$_{10}$-1	.2899$_{10}$ 0	.1460$_{10}$ 1	-.4039$_{10}$-1	.1615$_{10}$ 10
.5020$_{10}$-1	.5301$_{10}$ 0	.1226$_{10}$ 1	-.4071$_{10}$-1	.1621$_{10}$ 10
.5040$_{10}$-1	.7685$_{10}$ 0	.9954$_{10}$ 0	-.4104$_{10}$-1	.1628$_{10}$ 10
.5060$_{10}$-1	.1005$_{10}$ 1	.7659$_{10}$ 0	-.4136$_{10}$-1	.1634$_{10}$ 10
.5080$_{10}$-1	.1239$_{10}$ 1	.5383$_{10}$ 0	-.4169$_{10}$-1	.1640$_{10}$ 10
.5100$_{10}$-1	.1472$_{10}$ 1	.3125$_{10}$ 0	-.4202$_{10}$-1	.1647$_{10}$ 10

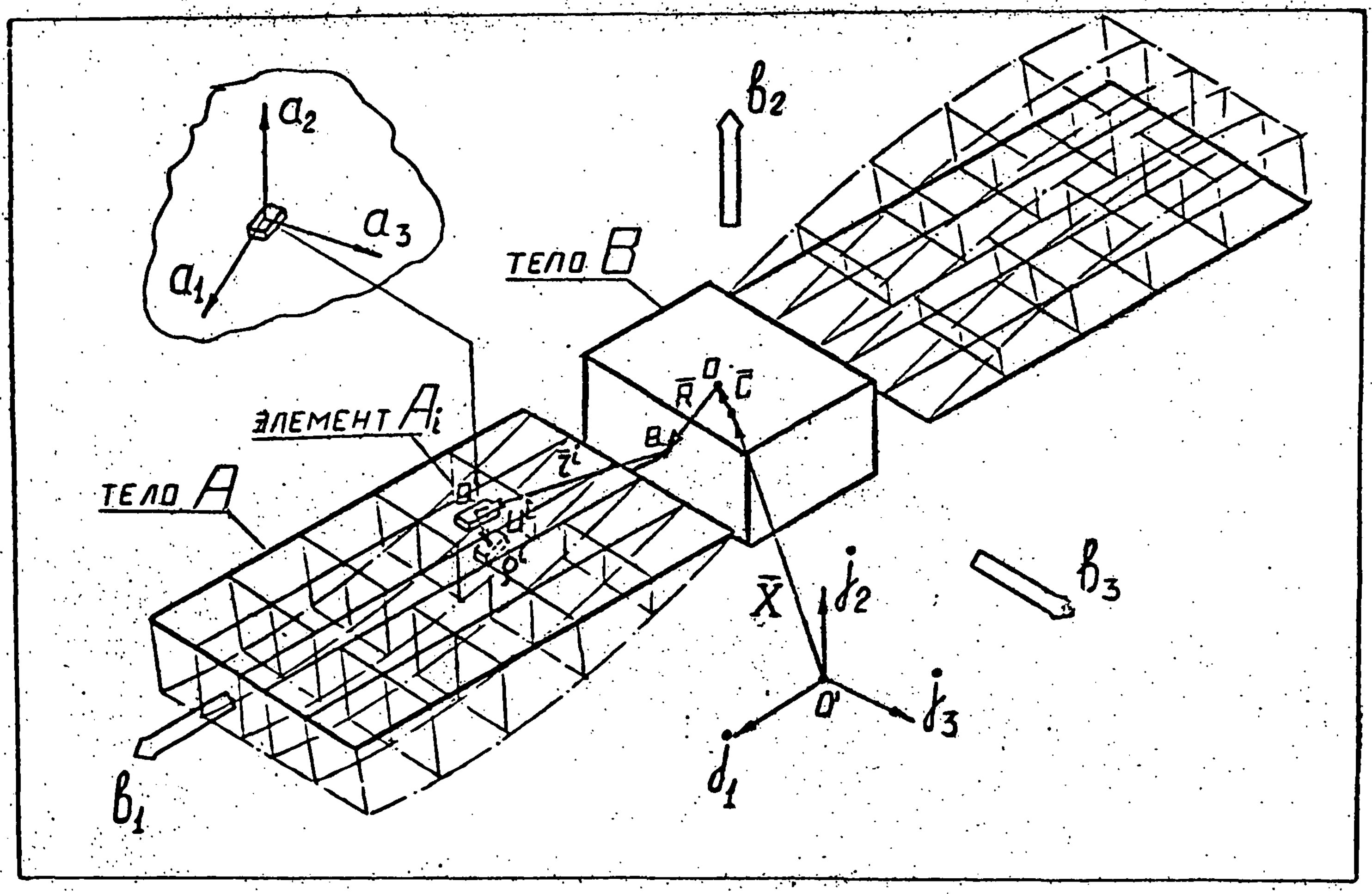


FIGURE 1.

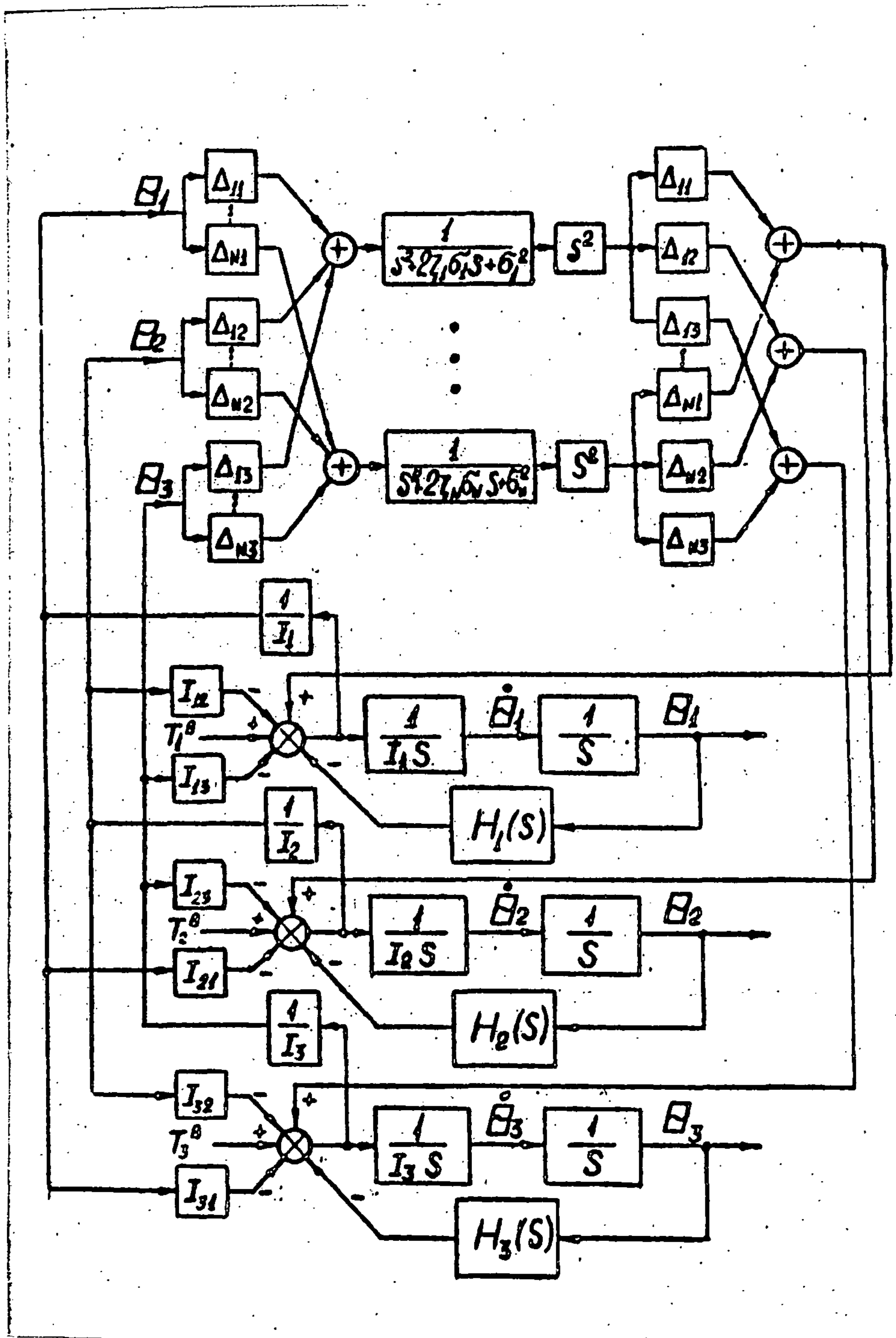

Θ_1
Θ_2
Θ_3
Δ_{11}
Δ_{N1}
Δ_{12}
Δ_{N2}
Δ_{13}
Δ_{N3}
S^2
$\frac{1}{I_1}$
$\frac{1}{I_2}$
$\frac{1}{I_3}$
I_{12}
I_{13}
I_{23}
I_{21}
I_{32}
I_{31}
T_1^B
T_2^B
T_3^B
$\frac{1}{I_1 S}$
$\frac{1}{I_2 S}$
$\frac{1}{I_3 S}$
$\frac{1}{S}$
$\dot{\Theta}_1$
$\dot{\Theta}_2$
$\dot{\Theta}_3$
$H_1(S)$
$H_2(S)$
$H_3(S)$

FIGURE 2.

$$A_i = \frac{\Delta_i^{T'}\Delta_i}{1-\Delta_i^{T'}\Delta_i} \; ; \quad \Delta_i^{T'}\Delta_i = \frac{\Delta_i^{T}\Delta_i}{I} \; ; \quad \zeta_i' = \frac{\zeta_i}{\sqrt{1-\Delta_i^{T'}\Delta_i}} \; ; \quad \sigma_i' = \frac{\sigma_i}{\sqrt{1-\Delta_i^{T'}\Delta_i}}$$

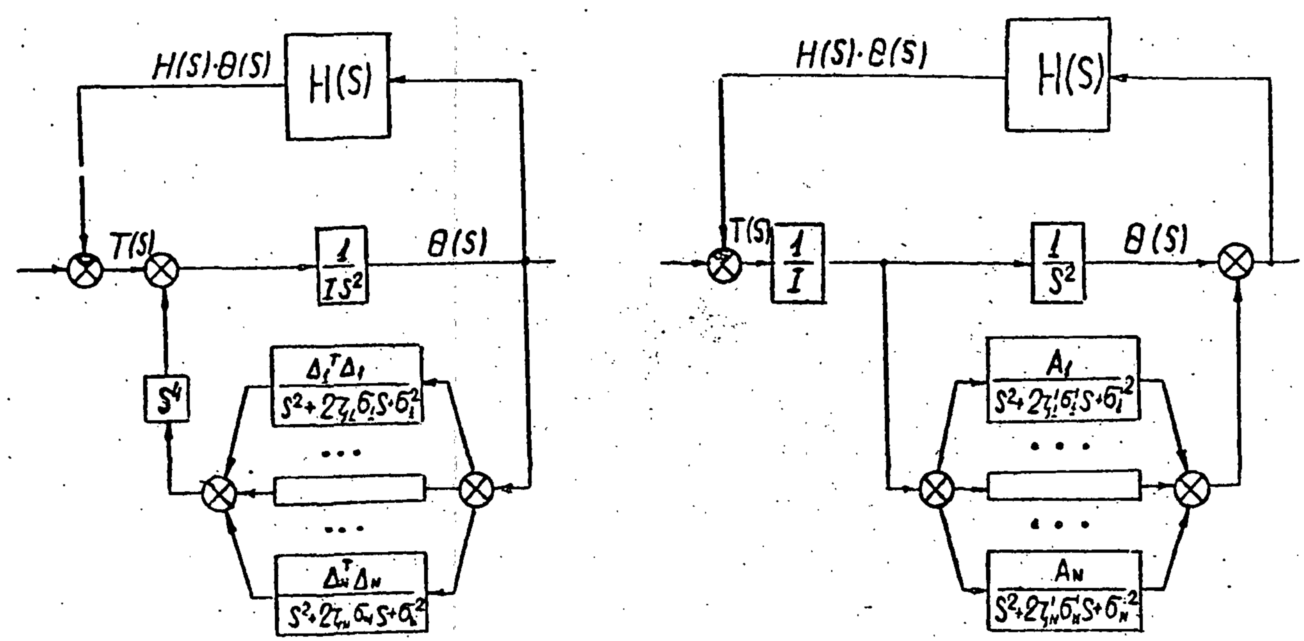


FIGURE 3.

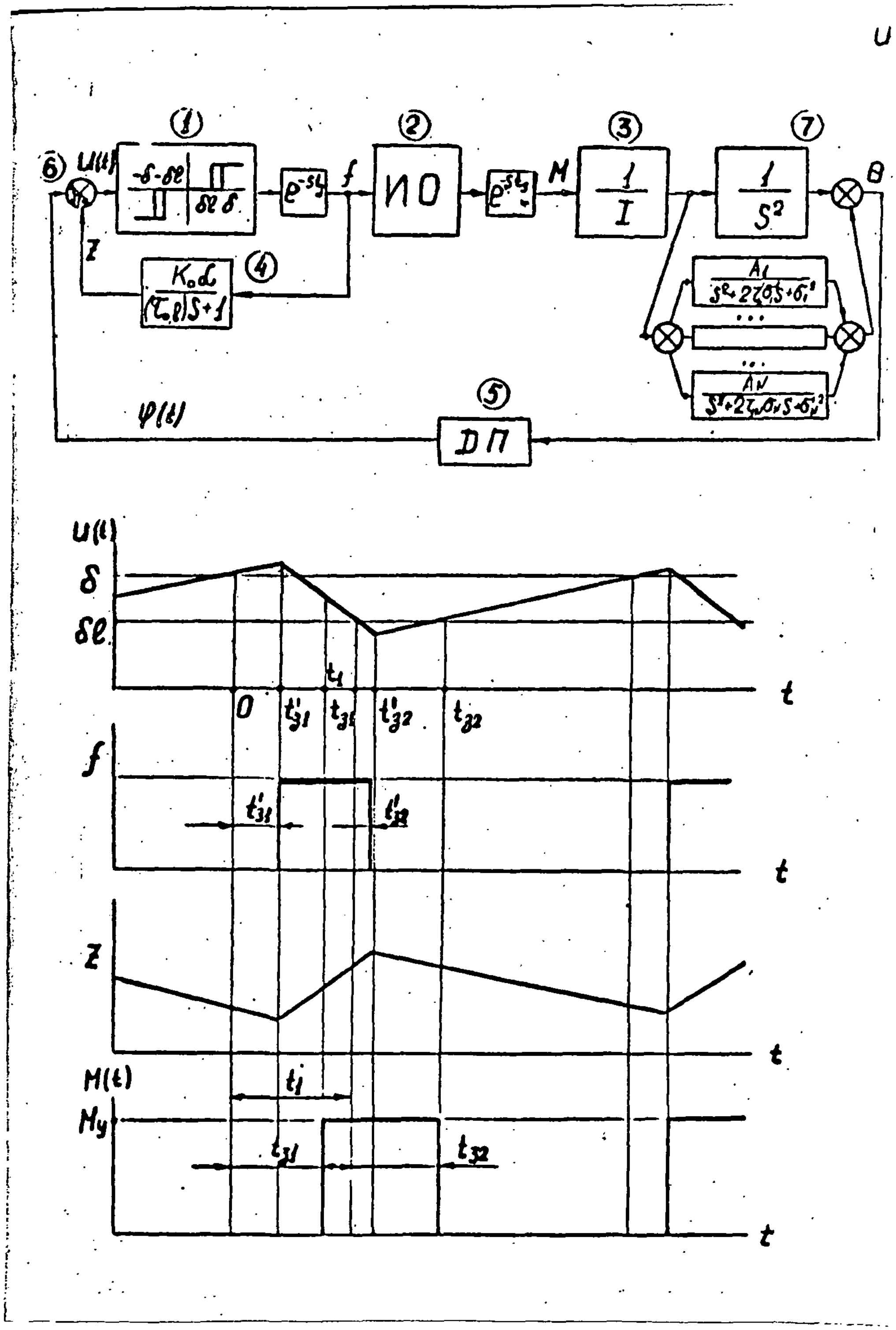


FIGURE 4.

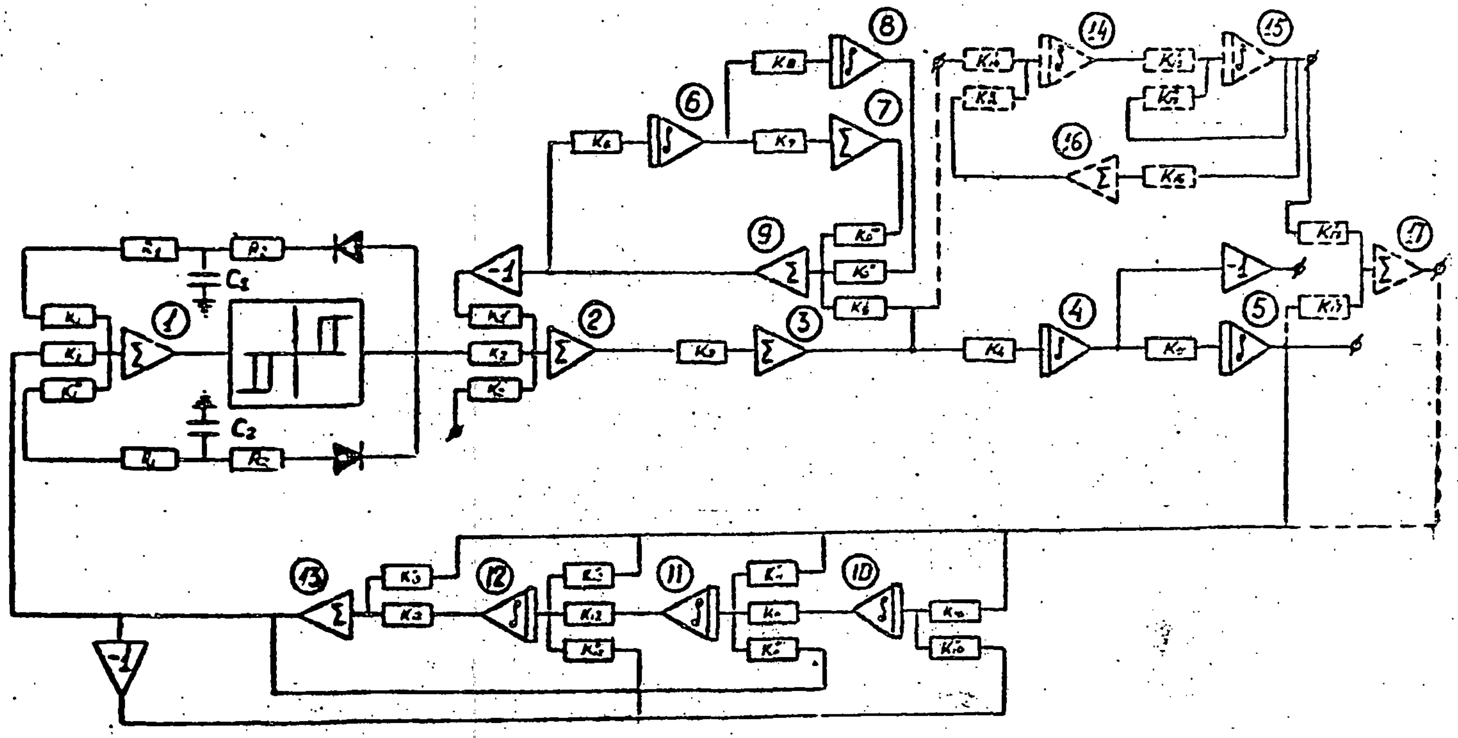


FIGURE 5.

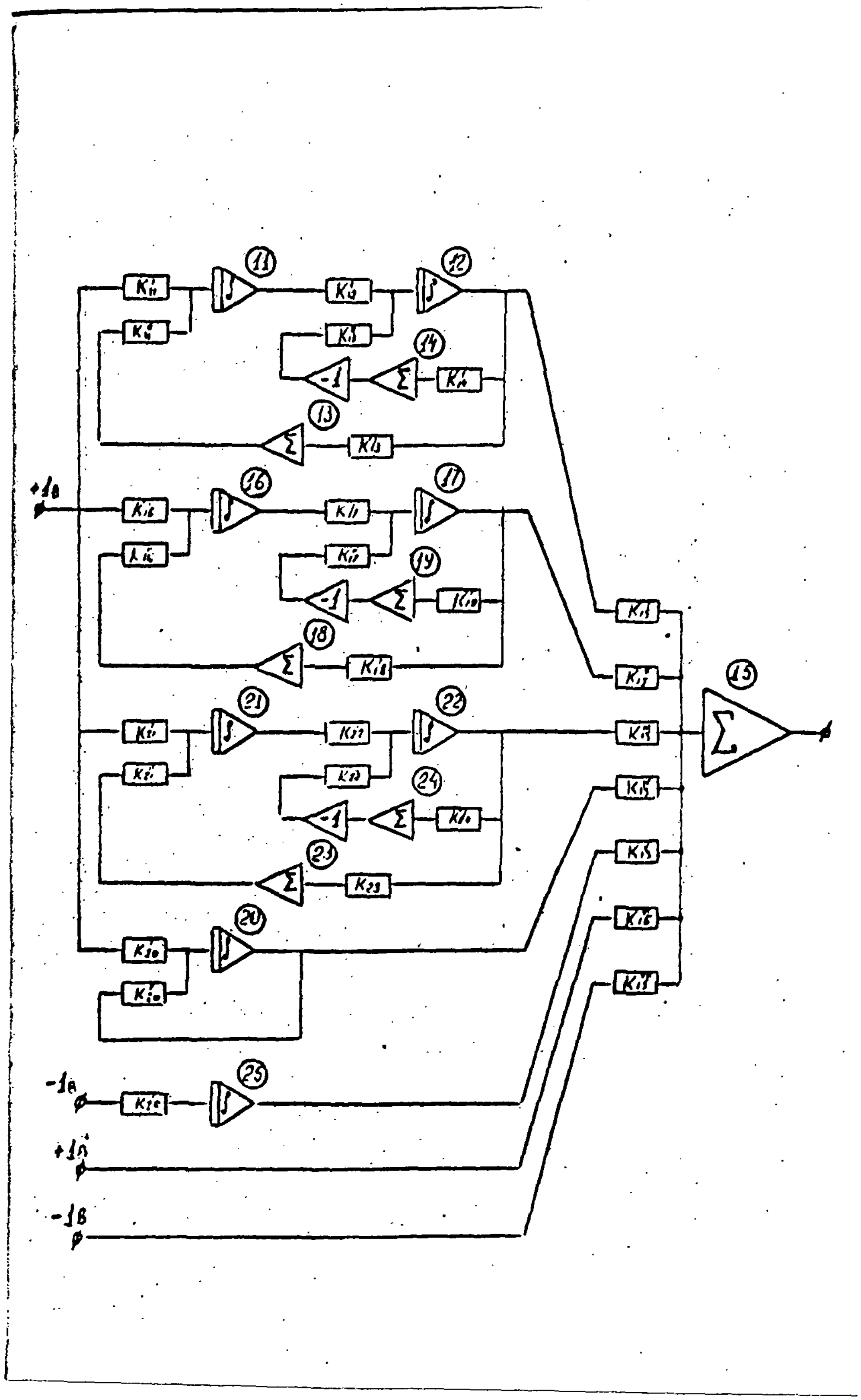


FIGURE 6.

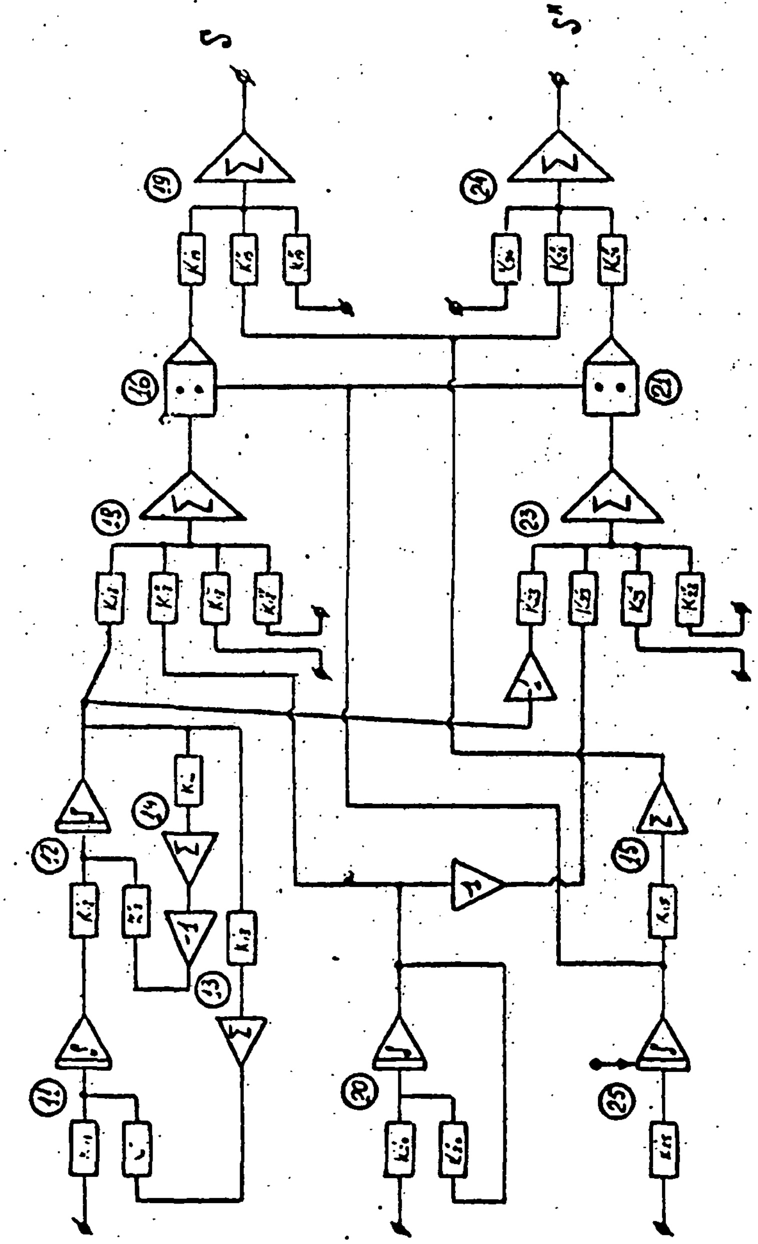


FIGURE 7.

```
"•ПУСТ"Н.Е=₃-3;"ДЛ"I=1!•"Ш"1"ДО"20"ЗАП"("ВЫВ""СТРО",I;А1=МТН[I];В=МТК[I];"РАЗР"18;
У1=F(А1,I);У3=F(В,I);"Е"У1×У3>•••У3>0"ТО""НА"Я1;Я.Х=А1+В)/(А1+В)/2;У2=F(Х,I);"Е"АВS(У2)>Е"
ТО"("Е"У1×У2>0"ТО"А1=Х"ИНА"В=Х;"НА"Я);"РАЗР"4;"ВЫВ""ПРОБ"4,Х;"НА"Я2;Я1."ВЫВ"'НЕ+•Т';Я2.Х=0)
"ТДЕ"F(Т,N)=МКА[N]×(1-ЕХР(-Т/1.5))-3-МА[I]×(-МG[I]×(1-ЕХР(-МD3[I]×Т)×(СОS(МСН[I]×Т)+МD3[I]
//•МСН[I]×SIN(МСН[I]×Т)))"КОН"◇"ВМ"МG[I]"ЗАП"МG[N])◇"ВМ"I•МА[I]"ЗАП"МА[N]"ВМ"МD3[I]"ЗАП"
МD3[N]"ВМ"МСН[I]"ЗАП"МСН[N]◇"ВМ"МD3[I]"ЗАП"МD3[N]"ВМ"МСН[I]"ЗАП"МСН[N]"ВМ"[I]"•×Т"ЗАП"
[N]×ТО"ЗАП""НА"М"КОН"◇
I=10
```

FIGURE 8.

```
"ПУСТ"МА[20]*]=74,74,29,29,14,14,9,9,6.5,6.5,5,5,4,4,2,2,1,1,.5,.5;М[*J*—*G[20]=517.75,
5.1775,1301.31,13.0131,2601.1,26.011,3500,35,5205,52.05,6567.8,65.678,7696***796.8,77.968,12943.8,
29.438,19468.6,194.686,37624.3,346.243,*;МСН[20]=.22,22,.154,1.54,.113,1.13,.0981,.981,.0808,.808,
07222,.722,.0663,.663,.0517,.517,.0422,.422,.0305,.305,*;МD3[20]=.14,1.4,.056,.56,.0278,.278,.0196,
196,.014,.14,.0111,.111,.0093,.093,.0057,.047,***57,.0037,.037,.0019,.019;МТН[20]=.024,.024,.035,.035,
041,.041,.044,.044,.045,.045,.046,.046,.047,.047,.048,.048,.049,.049,.049,.049;МТК[20]=.026,.026,
027,.037,.043,.043,.046,.046,.047,.047,.048,.048,.049,.049[*,.05,.05,.051,.051,.051,.051"КОН"◊

"ПУСТ"СА1."ДЛ"I=1"Ш"Х"d(**ДО"*****1"ДО"20"ВЫП"("ВЫВ""СТРС",I;A=МА[I];G=МG[I];СН=МСН[I];
D3=МD3[I];КА=90;ТА=1.5;Н=3;Т3=0;МИ=35;ТН=МТН[I];ТК=МТК[I];ДТ=(ТК-ТН)/10;"ДЛ"Т=ТН"Ш"ДТ"ДО
ТК"ВЫП"("РАЗР"6;Э1=-G×(1-EXP(-D3×Т)×(COS(СН×Т)+D3/СН×SIN(СН×Т)));S=(.5×МИ×(Т↑2-2×Т×Т3+Т3↑2)
(1-EXP(-Т/ТА))-Н-А×Э1)/Т;S2=(.5×МИ×(Т↑2×**+У*2×Т×Т3-Т3↑2)-КА×(1-Э1√***EXP(-Т/ТА))+Н+А×Э1)/Т;
ЭН=G×(СН+D3↑2/СН)×EXP(-D3×Т)×SIN(СН×Т);"РАЗР4*"4;"ВЫВ""ТАБЛ"1,Т,3*S,S1,Э1,ЭН))"КОН"◊

"ВЫП""НА"СА1"КОН"◊
```

FIGURE 9.

```
                           "ВМ"П"ЗАП""ПО"ВЫП"КА=90;"НА"Ы"КОН"0
I=10     X=.3035⏨-1
I=20     X=.3057⏨-1  !"ВМ"I=1"ЗАП"I=30"ВЫПИШ".
I=30     X=.3818⏨-1    ****П"А1=.03;В=.05;КА=90;"НА"Ы"КОН"0
I=50     X=.4301⏨-1
I=60     X=.4306⏨-1
I=70     X=.4528⏨-1
I=80     X=.4531⏨-1
I=90     X=.4659⏨-1
I=100    X=.4662⏨-1
I=110    X=.4746⏨-1
I=120    X=.4746⏨-1
I=130    X=.4606⏨-1
I=140    X=.4607⏨-1
I=150    X=.4937⏨-1
I=160    X=.4937⏨-1
I=170    X=.5009⏨-1
I=180    X=.5009⏨-1
I=190    X=.5046⏨-1
I=200    X=.5050⏨-10
```

FIGURE 10.

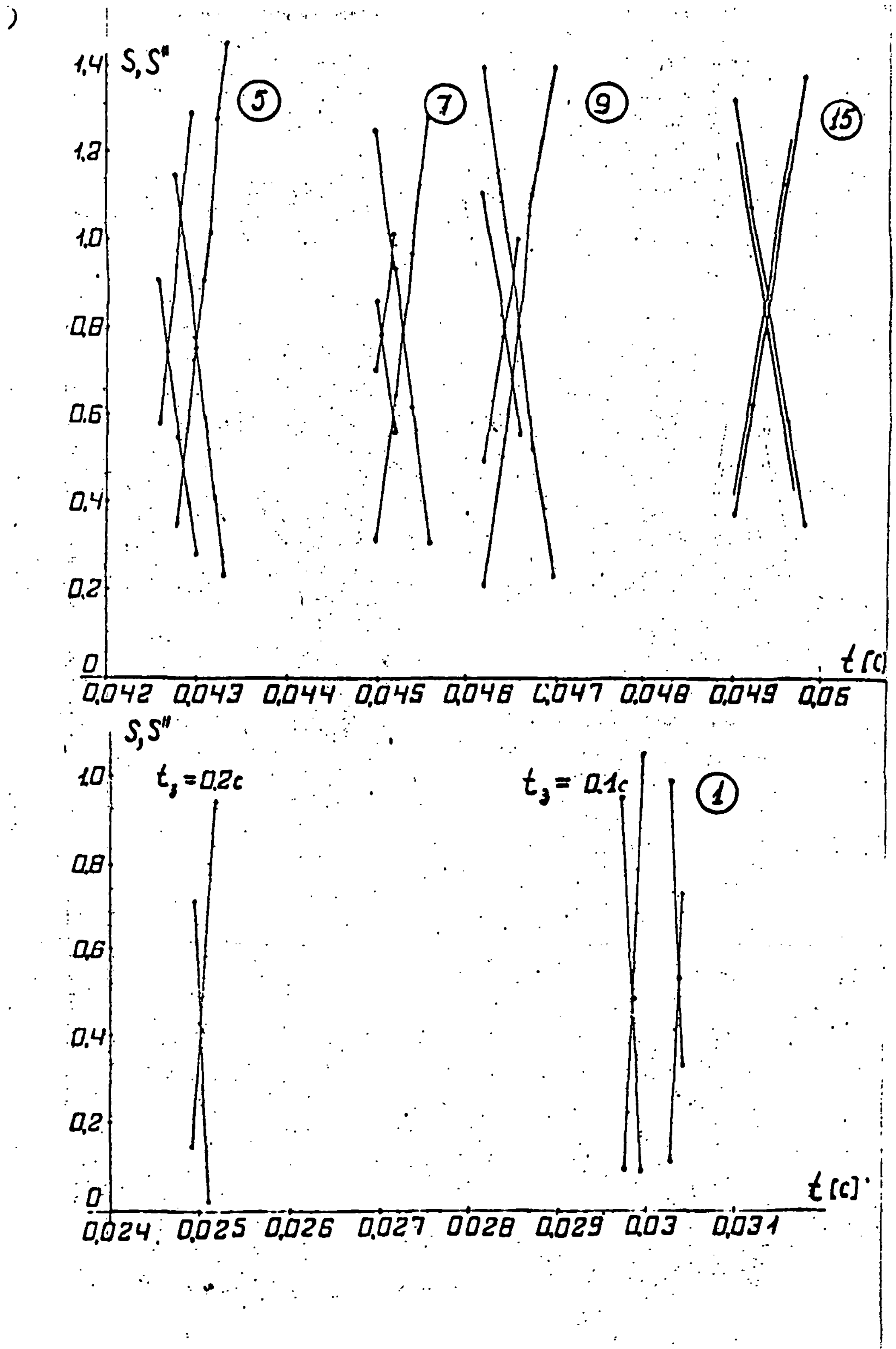

S, S''
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0
5
7
9
15
t [c]
0,042
0,043
0,044
0,045
0,046
0,047
0,048
0,049
0,05
S, S''
1,0
0,8
0,6
0,4
0,2
0
t_1 = 0,2c
t_3 = 0,1c
1
t [c]
0,024
0,025
0,026
0,027
0,028
0,029
0,03
0,031

FIGURE 11.

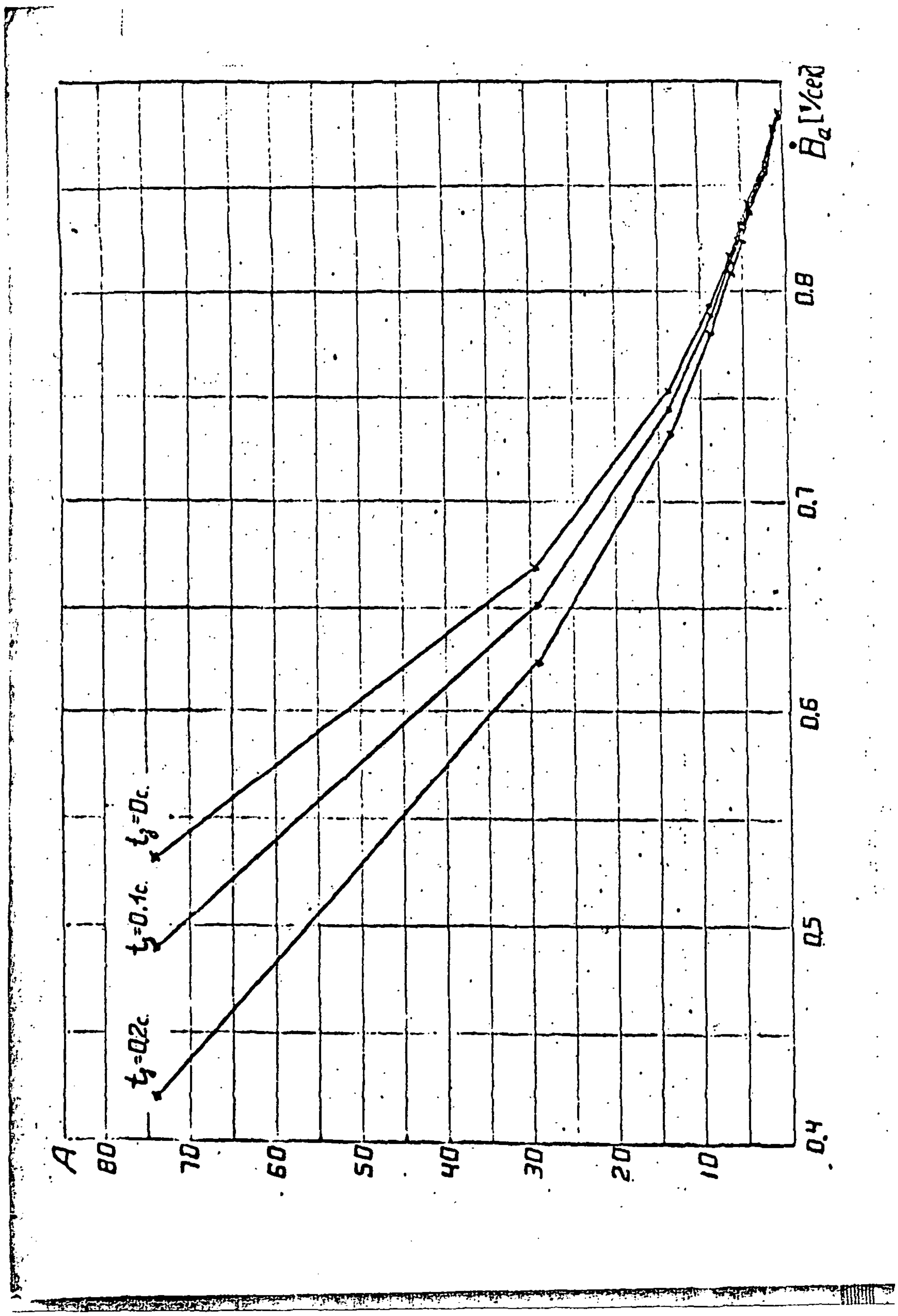
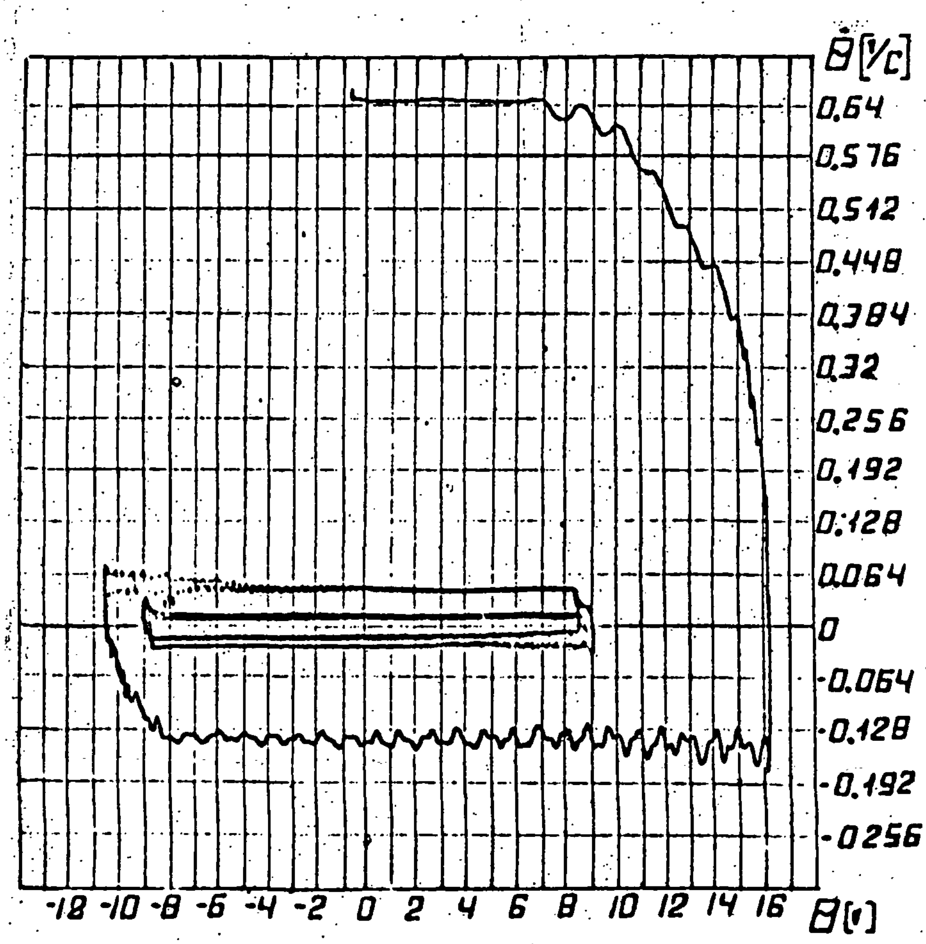


FIGURE 12.



$\dot{\theta}$ [°/c]
0.64
0.576
0.512
0.448
0.384
0.32
0.256
0.192
0.128
0.064
0
-0.064
-0.128
-0.192
-0.256
-18 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16
θ [°]

FIGURE 13.