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ON THE ACCELERATION OF THE AURORAL PLASMA OF THE EARTH DUE TO CONIC INSTABILITY

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16. Abstract

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ON THE ACCELERATION OF THE AURORAL PLASMA OF THE EARTH DUE TO CONIC INSTABILITY

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Introduction

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It is well known from experimental observation that the near-equatorial regions of the auroral zone of the earth's magnetosphere, at $4 \leq L \leq 10$ (where L is the distance in Earth radii), contain electrostatic waves of large amplitude, usually in the range of 1 to 10 MV/m [5]. Analysis in terms of linear plasma theory [2-4] has shown that the cause of the oscillation is a conic instability accompanying the electrostatic waves. A large number of observations has revealed that these waves can play an important role in the diffusion of the acceleration of, and in spilling, the low-energy plasma particles of the solar wind, at typical kinetic energies of $T_e \sim 1$ keV and $T_p \sim 10$ keV, which have been swept by convection into the magnetosphere. In the convection process charged particles which are oscillating along geomagnetic field lines are transported, together with the magnetic tubes of force, into the magnetosphere, where they experience an adiabatic acceleration in the intensified magnetic field.

The instability changes the phase and energy spectra of the particles and spills some particles into the atmosphere with additional acceleration.

The turbulent acceleration of magnetospheric plasma from the conic instability was first examined in reference [1]. This

*Numbers in the margin indicate pagination in the foreign text.

work, however, rigorously derived only the asymptotic solutions to the self-consistent problem of the quasi-linear relaxation, which are only correct at large distances from the point of the injection of the plasma into the tail of the magnetosphere.

In the present work the same question is investigated numerically with more general conditions placed on the function of particle distribution, and the treatment takes into account a number of supplementary effects, the Fermi acceleration in particular, which significantly increases the effectiveness of the acceleration. /4

1. Linear Analysis of the Conic Instability and the Initial Quasi-Linear Equations

The analysis of the acceleration of the auroral plasma by a conic instability is substantially simplified by the relatively small size of the cone of loss and convection, as a result of which the magnetospheric plasma is in a quasi-stationary state with a low level of plasma turbulence (when we consider the case of a peaceful magnetosphere).

This situation makes it possible to determine the acceleration using the kinetic equation in the drift approximation, with a quasi-linear integral for the collition of the conic instability. For a plasma at low ($T \sim 10$ keV) energies, we can ignore the drift gradient and the drift due to the curvature of the magnetic field and consider only the convection in the plane of the midnight meridian. Under the assumptions indicated the equation of the distribution function of particles (electrons will be considered later) can be written in the following way:

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial \ell} - \frac{v_\perp^2}{2B}\frac{\partial B}{\partial \ell}\frac{\partial f}{\partial v_\parallel} + \frac{v_\parallel v_\perp}{2B}\frac{\partial B}{\partial \ell}\frac{\partial f}{\partial v_\perp} - \frac{[\vec{E}\vec{B}]}{B^2}\nabla f = St(f). \qquad (1)$$

The quasi-linear integral of the collition of the electrostatic wave has the form [8]:

$$St(f) = \sum_{n=-\infty}^{\infty} \frac{e^2}{m^2}\int \frac{d\vec{k}}{(2\pi)^3}\frac{|E_{\vec{k}}|^2}{k^2}\hat{g}_{nk}\frac{i}{\omega - k_\parallel v_\parallel - n\omega_c} \times J_n^2\left(\frac{k_\perp v_\perp}{\omega_c}\right)\hat{g}_{nk} f, \qquad (2)$$

where /5

$$\hat{g}_{nk} = \frac{n\omega_c}{v_\perp}\frac{\partial}{\partial v_\perp} + k_\parallel \frac{\partial}{\partial v_\parallel},$$

and $\ell, v_\parallel$ are the coordinates and the velocity of particles along the geomagnetic field lines, $\vec{E}$ is the regular electric field transverse to the tail of the magnetosphere, and $\vec{k}, \omega_c$ are the wave vector and the cyclotron frequency.

In accordance with references [2-4] we will assume that there is a background plasma of zero temperature in addition to the accelerated auroral electrons. The dispersion equation for electrostatic oscillations will then be:

$$1 - \frac{\omega_{p0}^2}{k^2}\left(\frac{k_\perp^2}{\omega^2 - \omega_c^2} + \frac{k_\parallel^2}{\omega^2}\right) + \mathcal{D}\{f\} = 0,$$

where

$$\mathcal{D}\{f\} = \frac{\omega_p^2}{k^2}\sum_{n=-\infty}^{\infty}\int \frac{d\vec{v}\left[k_\parallel \frac{\partial f}{\partial v_\parallel} + \frac{n\omega_c}{v_\perp}\frac{\partial f}{\partial v_\perp}\right]}{\omega - n\omega_c - k_\parallel v_\parallel} J_n^2\left(\frac{k_\perp v_\perp}{\omega_c}\right),$$

and ω_p and ω_{po} are the plasma frequencies of the "fundamental" and "cold" components, respectively.

In order to provide a clear picture of the dependence of the coefficient of diffusion on the transverse velocity, we shall compare the rate of growth of the instability waves at the low-frequency ($\omega \gtrapprox \omega_c$) and high-frequency ($\omega >> \omega_c$) limits.

A detailed numerical study for the low-frequency case at various plasma parameters using a distribution function of the form

$$f = \frac{1}{\pi^{3/2} j!\, \alpha_\perp^2 \alpha_\parallel} \left(\frac{v_\perp}{\alpha_\perp}\right)^{2j} e^{-\frac{v_\parallel^2}{\alpha_\parallel^2} - \frac{v_\perp^2}{\alpha_\perp^2}},$$

$$\alpha_\parallel = \left(\frac{2T_\parallel}{m}\right)^{1/2}, \alpha_\perp = \left(\frac{2T_\perp}{m}\right)^{1/2}, \quad j = 1,2,3,\ldots$$ /6

was performed in references [3,4]. In particular, it has been shown that the instability develops at sufficiently large values of $\frac{k_\perp \alpha_\perp}{\omega_c}$ while the increment of oscillation shows sharp maxima at low values of $\frac{k_\parallel \alpha_\parallel}{\omega_c}$.

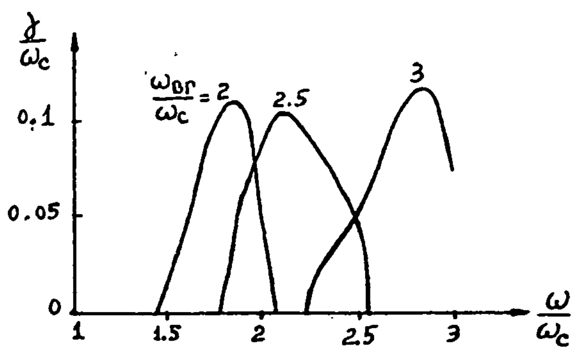

Fig. 1. Variation of the low-frequency increment with frequency [3].

The graph in figure 1 is taken from reference [3] to show the dependence of the low-frequency increment on frequency for the case j = 1 (the most realistic value for the magnetosphere [3]), $\frac{k_\parallel \alpha_\parallel}{\omega_c} = 0.4$, $\frac{T_\perp}{T_\parallel} = 1$, $\frac{n}{n_0} = 5$, $\omega_{Br} = (2 - 3)\omega_c$, where $\omega_{Br} = (\omega_{po}^2 + \omega_c^2)^{1/2}$ is the upper hybrid frequency.

Taking these aspects of the system into account, let us consider the high-frequency increment at the short-wave limit ($\frac{k_\perp \alpha_\perp}{\omega_c} \gg 1$), assuming that the oscillations are almost transverse (i.e., $\frac{\omega - n\omega_c}{k_\parallel v_\parallel} \gg 1$, $\frac{\omega}{v_\perp} \gg k_\parallel$). Using the formula

$$\sum_{n=-\infty}^{\infty} \frac{J_n^2(b)}{a-n} = \frac{1}{\sqrt{a^2-b^2}} , \quad a, b \gg 1,$$

we introduce the dispersion equation in the form

$$1 - \frac{\omega_{po}^2}{k^2}\left(\frac{k_\perp^2}{\omega^2-\omega_c^2} + \frac{k_\parallel^2}{\omega^2}\right) - \\ - \frac{2\pi\omega_p^2}{k^2}\int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} \frac{i\frac{\omega}{k_\perp}}{\sqrt{v_\perp^2 - \left(\frac{\omega}{k_\perp}\right)^2}} \frac{\partial f}{\partial v_\perp} dv_\perp . \tag{3}$$

Whence, designating $x = \left(\frac{k_\perp \alpha_\perp}{\omega_c}\right)^2$, $y = \left(\frac{\omega}{\omega_c}\right)^2$, we obtain equations defining the frequency and the increment of the oscillation in the case j = 1: /8

$$\mathrm{Re}\,\varepsilon \simeq 1 - \left(\frac{\omega_{po}}{\omega_c}\right)^2 \frac{1}{y-1} - 2\frac{n}{n_0}\frac{\omega_{po}}{\omega_c}\frac{y}{x^2} \times \\ \times \int_0^1 \frac{e^{-\frac{y}{x}u}\left(1-\frac{y}{x}u\right)}{\sqrt{1-u}} du = 0, \tag{4}$$

$$\gamma = \sqrt{\pi}\,\frac{n}{n_0}\,\omega_{po}\left(\frac{\omega_{po}}{\omega_c}\right)^3 \frac{1}{x^{3/2}} e^{-\frac{y}{x}}\left(\frac{1}{2} - \frac{y}{x}\right). \tag{5}$$

Figure 2 represents the dependence of the increment at the high-frequency approximation which is obtained from equations (4), (5) with $\frac{n}{n_0}$ = 5, $\omega_{Br} = 3\omega_c$ and $5\omega_c$. Since the increments of the high-frequency and low-frequency oscillations have the same order of magnitude, all of the terms of the sum-

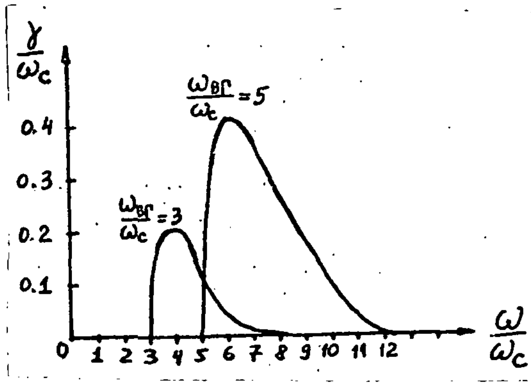

Fig. 2. Dependence of the increment at the high-frequency approximation on frequency.

mation in formula (2) must be taken into account. In this case the quasi-linear integral of the collision assumes the form[6]:

$$St(f) = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\frac{\mathcal{D}}{v_\perp}\frac{1}{v_\perp}\frac{\partial f}{\partial v_\perp}, \qquad (6)$$

where

$$\mathcal{D} = \frac{e^2}{m^2}\int \frac{d\vec{k}}{(2\pi)^3}\frac{\omega_{\vec{k}}^2 |E_{\vec{k}}|^2}{k_\perp^3}\frac{v_\perp}{\sqrt{v_\perp^2 - \left(\frac{\omega_{\vec{k}}}{k_\perp}\right)^2}}$$

As formula (5) implies, the maximum increment occurs at sufficiently small phase velocities of the perturbed wave, so that by ignoring $\frac{\omega_{\vec{k}}}{k_\perp}$ compared to $v_\perp$, we may obtain the following approximate formula for the coefficient of diffusion:

$$\mathcal{D} = \frac{e^2}{m^2}\int \frac{d\vec{k}}{(2\pi)^3}\frac{\omega_{\vec{k}}^2 |E_{\vec{k}}|^2}{k_\perp^3}. \qquad (7)$$ /9

2. Averaging of the Quasi-Linear Equation along the Tubes of Force of the Magnetic Field

Equation (1) can be simplified by using the smallness of the parameters $\tau/\tau_{\mathcal{D}}$ and $\tau_F/\tau_{\mathcal{D}}$, where $\tau = \oint \frac{dl}{v_\parallel}$ is the period of oscillation between magnetic maxima; $\tau_F = \oint \frac{dl}{v_\parallel} \Big/ \frac{V_{Dp}}{R_E}\frac{\partial}{\partial L}\oint\frac{dl}{v_\parallel}$ is the characteristic convection time; B_L is the magnetic field; $V_{Dp} = \frac{Ec}{B_L}$ is the velocity of drift at the plane of the equator; and R_E is the radius of the earth. The diffusion time $\tau_{\mathcal{D}} > \tau_{min}$ ($\tau_{min} = 2G\tau_B$) is the minimum life span of a particle within the region of strong diffusion [7], where $G = B_{max}/B_L$ is the relative ratio of magnetic maxima; and $\tau_B \simeq \frac{1}{4}\tau\,(v_\parallel, v_\perp = 0)$ is the emptying time of the cone of loss. For conditions in the magnetosphere, $\tau_{min} \sim L^4$, $\tau \sim L$, $\tau_F \sim L^{-2}$.

The presence of small parameters makes it possible to average equation (1) along the trajectory of particles in phase space with oscillations along the lines of force, so that we obtain the equation of the change of the distribution function in the equatorial plane. The possibility of averaging trajectories is obvious in regions outside the cone of loss, where paths are closed curves (for bound particles). In this case the paucity of longitudinal velocities and the diffusion create an equipartition of phase of the particles, so that in the quasi-stationary state the distribution function is constant along the trajectory, to the precision of the order of magnitude of the terms $\tau/\tau_{\mathcal{D}}$ and $\tau_F/\tau_{\mathcal{D}}$. The equations of the distribution function of particles in the equatorial plane can be obtained by considering the changes in the number of particles in the tubes of force [9].

It is appropriate at this point to change variables from /10
$v_\perp$, $v_\parallel$ to the invariants along the lines of force $\mu = \frac{B_{xb}}{B} v_\perp^2$, $E = \frac{B}{B_{xb}}\mu + v_\parallel^2$, where B_{xb} is the magnetic field in the tail of the magnetosphere at the point of injection of the plasma. Then the number of particles in the length ℓ, $\ell + d\ell$ of the tube of force with the cross-section dS_L on the equator, in the phase space μ, E, at the moment of time t will be:

$$f(\mu, E, \ell, t)\,\pi \frac{B}{B_{xb}} \frac{1}{2\sqrt{E - \frac{B}{B_{xb}}\mu}}\, d\ell\, \frac{B_L}{B}\, dS_L .$$

Correspondingly, the total number of particles in the tube in the phase space μ, E, can be written in the form:

$$F(\mu,E,L,t)=\pi\frac{B_L}{B_{xB}}\,dS_L\oint f(\mu,E,\ell,t)\frac{d\ell}{2\sqrt{E-\frac{B}{B_{xB}}\mu}}\,.$$

Assuming the equatorial cross-section of the tube of force to be unity with $L = L_{xb}$ and changing variables again to $v_{\parallel}$, μ, we obtain the following expression for the total number of particles in the tube at unit phase space $v_{\parallel}$, μ, in the central plane:

$$F(\mu,v_{\parallel L},L,t)=\pi\oint f(\mu,v_{\parallel}(\ell),\ell,t)\frac{d\ell}{v_{\parallel}(\ell)}v_{\parallel L}, \tag{8}$$

where $v_{\parallel L}$ is the longitudinal velocity of particles in the equatorial plane and the integral in (8) is preserved along the particle trajectory. It is obvious that

$$\frac{dF}{dt}=\pi v_{\parallel L}\oint\frac{df}{dt}\frac{d\ell}{v_{\parallel}(\ell)}$$

where $\frac{d}{dt}$ is the corresponding full derivative, or, in accordance with equation (1),

$$\frac{dF}{dt}=\pi v_{\parallel L}\oint St(f)\frac{d\ell}{v_{\parallel}} \tag{9}$$ /11

On the other hand, using a constant distribution function along the trajectory, we have

$$F(\mu,v_{\parallel L},L,t)\simeq\pi v_{\parallel L}\,f(\mu,v_{\parallel L},L,t)\oint\frac{d\ell}{v_{\parallel}}\,. \tag{10}$$

Equating (9) and (10) gives us an equation for the changes in the distribution function in the equatorial plane:

$$\frac{\partial f}{\partial t}+\frac{V_{DP}}{R_E}\frac{\partial f}{\partial(L_{xB}-L)}=\frac{\oint St(f)\frac{d\ell}{v_{\parallel}}}{\oint\frac{d\ell}{v_{\parallel}}}+\frac{f}{\tau_F}\,. \tag{11}$$

An analogous averaging can be performed for the unbound particles as well, in the case where the cone of loss is filled by diffusion more rapidly than it is emptied by the spilling of particles, that is, where

$$\frac{\tau_D}{G} \lesssim \tau_B . \qquad (12)$$

In this case the distribution function is also approximately constant along the trajectory. Taking into account that in the absence of diffusion the tube in the central section is emptied in the time τ_B, we have in the region of the cone of loss:

$$\frac{\partial f}{\partial t} + \frac{V_{Dp}}{R_E}\frac{\partial f}{\partial (L_{xB} - L)} = \oint \left[St(f) - \frac{f}{\tau_B} \right] \frac{d\ell}{v_\parallel} \Big/ \oint \frac{d\ell}{v_\parallel} + \frac{f}{\tau_F} . \qquad (13)$$

3. A System of Equations for the Conic Instability and the Results of Calculation

In accordance with experimental results [5] we will only consider diffusion to exist in a small region near the equator of dimension a, while setting the increment of the coefficient of diffusion equal to zero, a condition which is fulfilled when the cone of loss is filled up (as in (12)). We also will consider the period of oscillation between maxima to be at the upper limit of points of reflection, $\tau \simeq \frac{4R_E L}{v_\perp}\frac{\sqrt{2}}{3}\frac{\pi}{2}$, since the transverse velocities of the particles increase significantly during the process of convection. Finally, by considering the stationary-state problem (a stationary stat is created by a continuous injection of particles at $L = L_{xb}$), we arrive at the following system of equations for the conic instability:

$$\frac{V_{DP}}{R_E}\frac{\partial f}{\partial (L_{xB}-L)} = \frac{a}{x\,k(L)}\,\mathcal{D}\sqrt{\mu}\frac{\partial}{\partial \mu}\frac{1}{\sqrt{\mu}}\frac{\partial f}{\partial \mu} - \delta f + \frac{f}{\tau_F}, \tag{14}$$

$$\gamma \sim \int_{-\infty}^{\infty} dx \int_{0}^{\infty} \frac{1}{\sqrt{\mu}}\frac{\partial f}{\partial \mu}\, d\mu = 0, \tag{15}$$

where $\mu = \frac{B_{xB}}{B_L} v_\perp^2, \; x = v_{\parallel L}\left(\frac{L}{L_{xB}}\right)^\alpha,$

$$k(L) = R_E L \frac{\sqrt{2}}{3}\frac{\pi}{2}\left(\frac{L_{xB}}{L}\right)^\alpha \left(\frac{B_L}{B_{xB}}\right)^2,$$

$$\delta = \begin{cases} 1/\tau_B & \text{for } \mu \le \mu_c = \dfrac{\frac{B_{xB}}{B_L} v_{\parallel L}^2}{G-1} \simeq \dfrac{x^2}{G_{xB}}\left(\dfrac{L_{xB}}{L}\right)^{2\alpha} \\ 0 & \text{for } \mu > \mu_c \end{cases}$$

$$\tau_B \simeq \frac{1{,}4 R_E L}{x}\left(\frac{L}{L_{xB}}\right)^\alpha, \quad \alpha = 0;1.$$

The case $\alpha = 1$ corresponds to the conservation of the second adiabatic invariant $\oint v_\parallel dl$, and at the same time takes into account the acceleration of particles by the Fermi mechanism. We note that reference [1] investigated only the case $\alpha = 0$ and did not take into account the term f/τ_F. In addition, it solved the problem using the variables L and μ and took account of the longitudinal velocities parametrically by means of $\overline{v_{\parallel L}}$ (averaging the longitudinal velocities of the particles). /13

The system of equations (14), (15) was solved numerically under the following boundary conditions:

$$\left.\frac{\partial f}{\partial \mu}\right|_{\mu=0} = 0; \; f \to 0, \mu \to \infty; \; \left. u\frac{\partial f}{\partial x}\right|_{x=0} = 0.$$

(it was assumed that the distribution function was symmetrical with respect to x = 0). The initial distribution function at

$L = L_{xb}$ was taken to be a Maxwell distribution with $T_{\parallel} = T_{\perp} = T_{xb}$.

The resulting calculations were performed in the approximate dipole magnetic field $B_L = B_{xb}\left(\frac{L_{xB}}{L}\right)^3$ and the homogeneous electric field, for two values of the total temperature: T_{xb} = 0.5 keV and 1.0 keV. The results of the calculation are presented in Table 1 in the form of a ratio of the transverse temperature in the case of adiabatic heating, where $T_{\perp\,adiab} = T_{xb}\frac{B_L}{B_{xb}}$ for $L_{xb} = 15$, $n_{xb} = 0.2\ cm^{-3}$, $E = 10^{-5}$ V/cm, $B_{xb} = 10^{-4}$ G, and $\alpha = 0$, $\alpha = 1$.

Table 1

	$\alpha = 0$		$\alpha = I$	
L \ T_{xB}, кэВ	0,5	I	0,5	I
I5	I	I	I	I
I3	I	I,0I	I,005	I,0I
II	I,00I	I,02	I,02	I,05
9	I,02	I,07	I,I	I,2I
7	I,05	I,I8	I,3	I,8
6	I,07	I,2I	I,42	2,3
5	I,I2	I,48	2,7	4,I5
4,5	I,I5	I,6	4,32	6,I4
4	I,2I	I,8	6,5	8,6

It is clear from the table that an increase in the initial energy of the plasma at $L = L_{xb}$ leads to an increase in the effectiveness of the turbulent acceleration. The influence of the initial concentration of the plasma on the acceleration was not significant. In turn, taking the Fermi acceleration into account /14
at $\alpha = 1$ leads to a noticeable increase in the effectiveness of the acceleration.

The calculations confirmed the hypothesis about the filling of the cone of loss (stated in (12)) at all stages of convection; the figures show an anisotrpic distribution function with little spilling of particles. This differs from the result of reference [1], which supposed that at the second stage of convection, beginning at a certain value of L, the cone of loss became empty in phase space.

In order to evaluate the effectiveness of the acceleration analytically, we can make use of the self-modeling solution to equation (14), which goes to zero in the cone of loss[1]:

$$f = \mathrm{Const}\, \frac{\mu^{3/2}}{\xi^{7/4}}\, e^{-\mu^2/\xi}\, e^{\int_L^{L_{xb}} \frac{dL'}{\tau_F(L')}}, \tag{16}$$

where

$$d\xi = \frac{R_E}{V_{dp}}\, \frac{4a}{\overline{X}\, k(L)}\, \varnothing\, d(L_{xb} - L),$$

while for a rough determination of the diffusion coefficient, /15
we can use the condition of filling of the cone of loss, (12):

$$\left(\frac{L_{xb}}{L}\right)^{2\alpha} \frac{\overline{\mu^2}}{\sigma_{xb}} \Big/ \frac{4a}{\overline{X}\, k(L)}\, \varnothing = \tau_B. \tag{17}$$

Using (16), we obtain $\overline{\mu^2} = \xi$, and correspondingly

$$T_\perp = T_{\perp xb}\, \frac{B_L}{B_{xb}} \sqrt{\frac{\xi}{\xi_{xb}}} \tag{18}$$

where

$$\xi(L) = \xi_{xb} \exp\left\{ \int_L^{L_{xb}} \frac{1}{\sigma_{xb} \left(\frac{L'}{L_{xb}}\right)^{2\alpha}}\, \frac{\delta \cdot R_E}{V_{dp}}\, dL' \right\}. \tag{19}$$

The numerical calculations have a qualitative correlation with the values of (17) and (18).

All of the qualitative aspects of the acceleration which are considered above also apply to protons, although in that case the rate of loss of particles changes to:

$$\delta_p = \left(\frac{m_e}{m_p} \frac{T_p}{T_e}\right)^{1/2} \delta_e$$

The small size of the factor $\left(\frac{m_e}{m_p} \frac{T_p}{T_e}\right)^{1/2}$ reduces the effectiveness of acceleration, and the transverse energy of protons differs a little from that of adiabatic acceleration.

Conclusion

The results obtained show that the mere existence of a conic instability leads to a small amount of spilling and an isotropic distribution of particles, but it is accompanied by significant acceleration of low-energy electrons.

As the particle energy increases, the electromagnetic instability can begin to play an important role related to the perturbation of the ionic cyclotron waves and whistlers. Fundamentally accompanying this there occurs a diffusion of pitch-angle which makes the distribution function isotropic, intensifies the loss of particles, and at the same time leads to the formation of an internal boundary of the plasma layer [7]. /16

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APPENDIX

Method of Numerical Solution of the System of Quasi-Linear Equations Describing a Conic Instability

The standard method of solving the system of equations in (14), (15) is the method of fitting; that is, finding a stationary solution corresponding to the nonstationary problem:

$$\frac{\partial f}{\partial t}+\frac{V_{Dp}(L)}{R_E}\frac{\partial f}{\partial(L\,;-L)}=\frac{\sqrt{\mu}}{x}\frac{\partial}{\partial\mu}\frac{D(L,t)}{\sqrt{\mu}}\frac{\partial f}{\partial\mu}-\delta(x,\mu)f$$
$$\frac{\partial D}{\partial t}=\left(\int_0^\infty dx\int_0^\infty d\mu\,\frac{1}{\sqrt{\mu}}\frac{\partial}{\partial\mu}\right)D\equiv\gamma D.$$

The nonstationary problem is three-dimensional, with spatial coordinates L, μ, x, so its numerical solution is extremely laborious.

We can propose, however, the immediate solution of the stationary problem, a method which makes it possible to reduce the number of dimensions and to solve the two-dimensional problem in the μ, x coordinates. With this method at each step along the radius L, L + ΔL we solve the following system of equations:

$$\frac{f_{i,j}-\tilde{f}_{i,j}}{\frac{R_E\Delta L}{V_{Dp}(L)}}=D\frac{\sqrt{\mu_j}}{x_i}\,\frac{\frac{f_{i,j+1}-f_{i,j}}{\sqrt{\mu_{j+1/2}}\,\Delta\mu}-\frac{f_{i,j}-f_{i,j-1}}{\sqrt{\mu_{j-1/2}}\,\Delta\mu}}{\Delta\mu}-\delta_{i,j}\,f_{i,j}. \qquad \text{(A.1)}$$ /17

$$\gamma\simeq\sum_i\Delta x\sum_j\Delta\mu_j\frac{1}{\sqrt{\mu_{j+1/2}}}\;\frac{f_{i,j+1}-f_{i,j}}{\Delta\mu_j}=0. \qquad \text{(A.2)}$$

where, as usual, we replace the region of continuous variation of the variables μ, x with a network of discrete points with the coordinates μ_j, x_i.

$\tilde{f}_{i,j} = f_{i,j}(L + \Delta L)$ is a well-known function, but $f_{i,j} = f_{i,j}(L)$ and $\mathcal{D} = \mathcal{D}(L)$ are subject to determination.

In order to solve the system (A.1) and (A.2), we used a combined method together with a "staircase" of values. The essence of this procedure consists in that for each $\mathcal{D}$ the system of linear equations (A.1) can easily be solved by the method of a "staircase" of values, while $f_{i,j}$ depends on $\mathcal{D}$ as well as on the values of the two parameters. Correspondingly γ represents a certain function of $\mathcal{D}$. In that case, in order to find the root of the equation $\gamma(\mathcal{D}) = 0$ we can vary the iteration, by which means we can obtain any given precision in the determination of $\mathcal{D}$. Since the apparent type of dependence $\gamma(\mathcal{D})$ is unknown, the rapid iteration method, such as immediate iteration, the method of Newton and so on, is in this case difficult or inapplicable. The specific problem is solved, however, since with the transition from the $L + \Delta L$ layer to the L layer, the small size of ΔL means that the diffusion coefficient $\mathcal{D}$ does not change a great deal, so that the calculation in turn of the L layer requires only a small number of iterations, and we can /18
use the method of chords or the method of division in two in finding $\mathcal{D}(L)$, guaranteeing the agreement of the iterations.

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