Technical Report, Grant NGR 33-013-086

Spherical Harmonic Analysis for Verification of a Global Atmospheric Model

Zaphiris Christidis and Jerome Spar

The City College

August 1979

Introduction

A number of global atmospheric general circulation models (GCM's) have been developed during the past 20 years with the objective of accurately simulating the large-scale dynamics and physics of the atmosphere. In some GCM experiments the models have been started from a hypothetical barotropic state of rest and allowed to "spin up" to a climatology comparable to that of the present terrestrial atmosphere. In others they have been initialized with real, or realistic, data and allowed to generate a simulated forecast meteorological history. In either type of experiment, the validity of the model is tested by comparing the model-generated climatology or forecast with the real atmosphere. These comparisons have been carried out through diagnostics, such as gross energetics, transports, momenta, and hydrologies, as well as vertical-meridional cross sections of various zonal mean atmospheric properties. Model and nature have also been compared in terms of horizontal synoptic fields, using traditional measures of forecast skill, such as those commonly employed for the verification of prognostic weather maps.

(NASA-CR-158867) SPHERICAL HARMONIC ANALYSIS FOR VERFICATION OF A GLOBAL ATMOSPHERIC MODEL (City Coll. of the City Univ. of New York.) 9 p HC A02/MF A01 CSCL 04A G3/46

N79-29726

Unclas 31673

No method of verifying prognostic maps (whether applied to explicit daily forecasts, monthly mean forecasts, or simulations of climatology) is completely satisfactory. Subjective comparisons of synoptic patterns are obviously inadequate, and quantitative scalar measures of agreement (e.g., root-mean-square errors, correlation coefficients, and gradient "skill scores"), while they may be objective, convey little information about pattern agreement, except in a gross relative sense. Additional information may be provided by a comparison of the spectral components of the patterns, as represented, for example, by the expansion coefficients of a series of surface spherical harmonics.

Spherical harmonic analysis has been applied in the past to many kinds of geophysical problems (e.g., Chapman and Bartels, 1962), and is now even used operationally for global weather analysis at the National Meteorological Center (NMC). However, it has apparently not been widely adopted as a method of synoptic pattern verification. (For an example, see Leith (1974).) In this study, surface spherical harmonics are used to analyze the horizontal fields of various quantities generated by one global GCM - the GISS "climate model" (Hansen et al., 1979) - and to compare the model results with nature.

Forecast experiments have been carried out with the climate model by initializing it with global NMC "data" (actually derived from NMC operational analyses) for the first day of a month (at 00 GMT), running the model to the end of the simulated month, and computing the monthly mean fields of various predicted quantities. These monthly mean forecast fields are then compared with the corresponding observed fields for the same month. The quantities selected for analysis are the sea-level pressure (SLP) in millibars (mb), the 850 mb temperature (T8) in Kelvin degrees (K), and the 500 mb geopotential height (Z5) in meters (m).

Monthly mean (observed) climatological fields of these predictands, as well as model-generated "climatologies", are also used in the evaluation program.

Any global horizontal field (forecast, observed, or climatological) can be expanded into a finite series of spherical harmonics. (For further details on applications of spherical harmonics, see, e.g., Chapman and Bartels (1962), Spar (1950), Belousov (1962), Merilees (1973), and Blackmon (1976).) The fields may be represented by tables of the expansion coefficients, or by tables of the magnitudes and phases of the spectral components, which may also be shown, for selected wave numbers, in the form of vector diagrams ("harmonic dials"). Verification of the model output can then be expressed in terms of the relative magnitudes of the dominant harmonics, with phase angles also considered. (The practical advantages and limitations of spherical harmonics, as compared with Fourier series and Chebyshev polynomials, for the solution of atmospheric problems on the sphere are discussed by Boyd (1978).)

Spherical harmonic expansion

A horizontal field, Q, may be expanded as a function of latitude, ϕ, and longitude, λ, into a series of surface spherical harmonics,

$$\hat{Q}(\phi,\lambda) = \sum_{n=0}^{N} \sum_{m=0}^{n} \left(C_{n,m} Y_{n,m}^{(e)} + S_{n,m} Y_{n,m}^{(o)} \right) \tag{1}$$

where Yn,m are the even (e) and odd (o) normalized spherical harmonic functions (see, e.g., Blackmon, 1976),

$$\begin{Bmatrix} Y_{n,m}^{(e)} \\ Y_{n,m}^{(o)} \end{Bmatrix} = \left[\frac{2n+1}{2\pi} \cdot \frac{(n-m)!}{(n+m)!} \right]^{1/2} \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} P_{n,m}(\sin\phi), \quad n \geq m > 0$$

$$Y_{n,0} = \left[\frac{2n+1}{4\pi} \right]^{1/2} P_{n,0}(\sin\phi) \tag{2}$$

here $P_{n,o}(\sin\phi)$ are the Legendre polynomials (also sometimes called "zonal harmonics"), $P_{n,m}(\sin\phi)$ are the associated Legendre polynomials ("spherical surface harmonics"), m is the zonal (longitudinal) wave number, n-m indicates the number of modal parallels (twice the meridional wave number) from pole to pole, N is the truncation degree of the series, and $C_{n,m}$ and $S_{n,m}$ are the normalized expansion coefficients of the series, to be determined from the data, Q, by

$$\begin{Bmatrix} C_{n,m} \\ S_{n,m} \end{Bmatrix} = \int_{-\pi/2}^{\pi/2} \int_{0}^{2\pi} Q(\phi,\lambda) \begin{Bmatrix} Y_{n,m}^{(e)} \\ Y_{n,m}^{(o)} \end{Bmatrix} \cos\phi \, d\phi \, d\lambda \tag{3}$$

The orthonormality property guarantees that the area-weighted mean value of $(Y_{n,m} \cdot Y_{n',m'})$ over the sphere, for either (e) or (o), equals zero if $n \neq n'$ or $m \neq m'$ and $(4\pi)^{-1}$ if $n = n'$ and $m = m'$. This is the basis for (3) and the calculation of the coefficients. Normalization makes it possible to compare coefficients of different order, m, and degree, n, in the same series or among different series.

The magnitude, $A_{n,m}$, of any harmonic is given by $A_{n,m}^2 = C_{n,m}^2 + S_{n,m}^2$, the phase angle, $\beta_{n,m}$ by $\beta_{n,m} = \tan^{-1} S_{n,m}/C_{n,m}$, as indicated in the harmonic dial in figure 1.

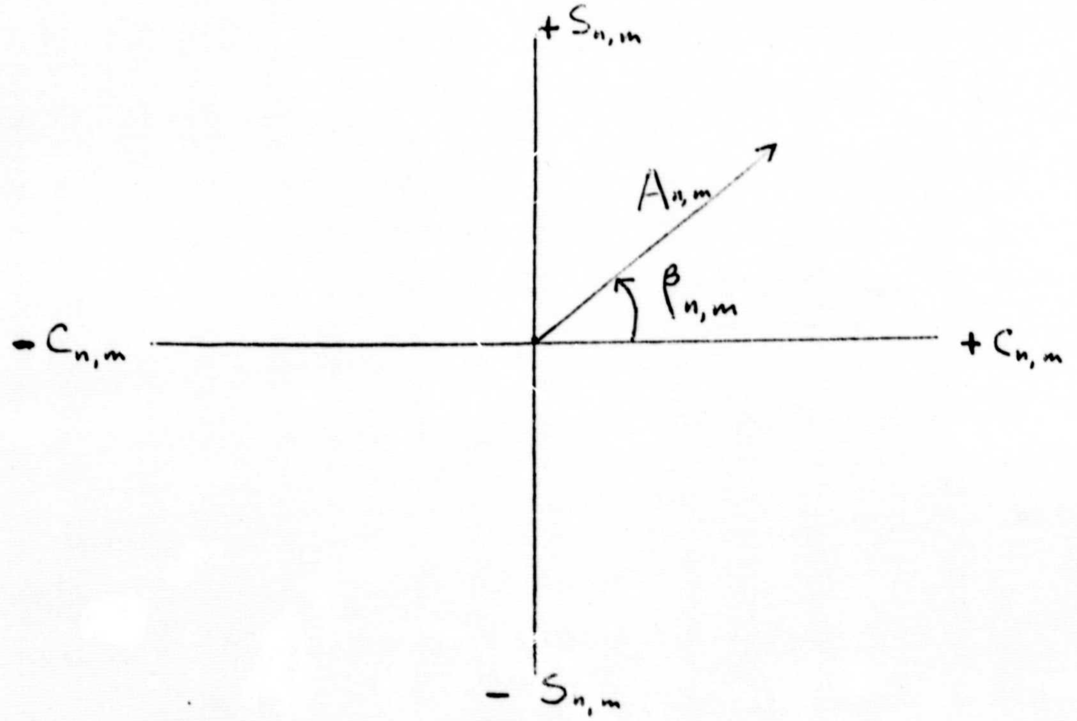


Fig. 1.

Various numerical integration schemes for the calculation of (3) were tested by checking both the orthonormality properties and the errors of reproduction of given fields. Also, tests were carried out with different degrees of truncation, the maximum being N = 18. (Alternative numerical methods for computing associated Legendre functions and expansion coefficents are discussed by Merilees, 1973.) The method finally adopted was found to be satisfactorily accurate when applied to data fields on either an 8° x 10° or 4° x 5° latitude-longitude grid.

While tables of normalized associated Legendre polynomials are available (e.g., Belousov, 1962), they generally do not correspond to the latitude interval of the GISS grid. Therefore, for the present study, the polynomials were calculated for the appropriate latitudes by means of Rodrigues' formula (Hobson, 1955), and stored on disk for use as required. A comparison of computed and tabulated polynomials for the same latitudes up to degree 18 gave perfect agreement to the 10-th decimal place.

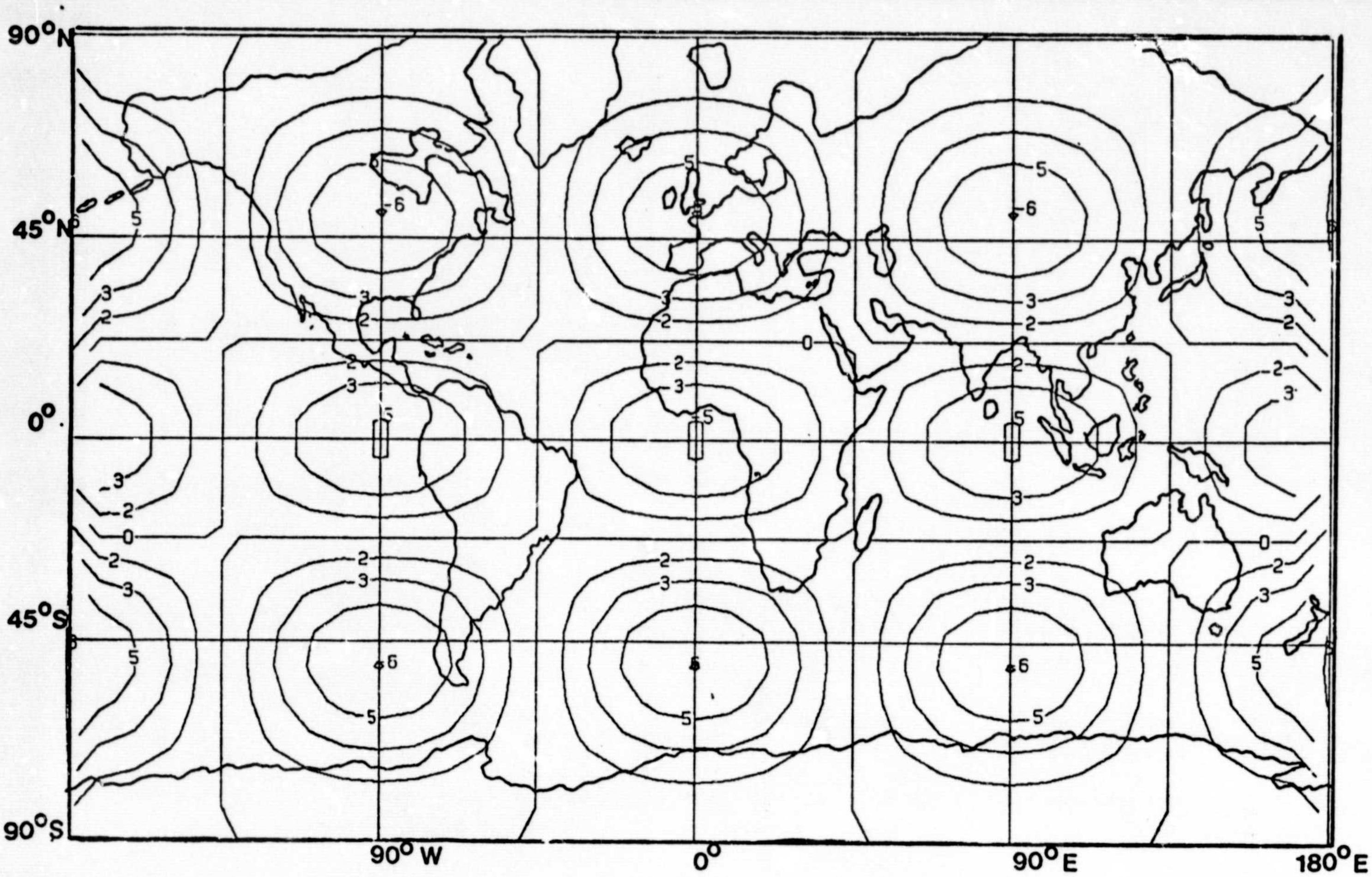
The coefficients, $C_{n,m}$ and $S_{n,m}$ are calculated from (2) and (3) in steps, integrating first over ϕ, then over λ, using a combination of the Simpson, "three-eighths", and trapezoidal rules, with a smoothing operator applied at $n > 7$, for the ϕ-integration, and Filon's rule (Davis and Rabinowitz, 1967) for the λ-integrals.

The area-weighted **mean** value of Q over the sphere is given by $(4\pi)^{-1/2} C_{o,o}$. In general, $C_{n,o}$, the coefficient of the one-dimensional spherical harmonic of degree n (a function with n nodal parallels), represents the amplitude of a zonally symmetric function of latitude only. For example, $Y_{1,o} = (\frac{4}{3}\pi)^{-1/2} \sin\phi$, $Y_{2,0} = (3.2\pi)^{-1/2} (3 \sin^2\phi - 1)$, etc. Thus, the difference between the values of Q at the North and South Poles is represented by approximately $C_{1,o}$ and the difference between the polar and equatorial mean values of Q is represented by approximately $C_{2,o}$. Polynomials of even degree are symmetric about the Equator, while those of odd degree are anti-symmetric. The coefficient $C_{6,0}$, for example, represents a component that is symmetric about the Equator, with a minimum (or maximum) at the Equator and three alternating bands of high and low values in each hemisphere (as in the case of the planetary sea-level pressure distribution).

The associated Legendre functions represent two - dimensional wave patterns. Thus, for example, $C_{4,2}$ represents the checkerboard pattern illustrated in figure 2, with 2 nodal parallels, 2 longitudinal waves, and alternating high and low values.

Some Illustrative Examples

Synoptic monthly mean maps of SLP, T8, and Z5 (observed, climatological, and forecast) are to be expanded in finite series of spherical harmonic functions up to N=18. The observed monthly mean fields (O) are derived from NMC operational analytic data obtained from the National Center for Atmospheric Research (NCAR) and interpolated to the GISS climate model grids. Monthly mean forecasts (F) are computed with the climate model, starting from 00GMT initial conditions on the first day of the month. The actual climatological (C) fields for each calendar month were provided by NCAR. (A set of monthly mean "model climatologies" (M) is also being generated by running the medium mesh 8°x10°, climate model for 5 simulated years and averaging each month's output. Evaluation of the model will be carried out in terms of both its climatology error, M-C, and its "anomaly error", (F-M) - (O-C).)



$Y^e(n,m)$, $n = 4$, $m = 2$,

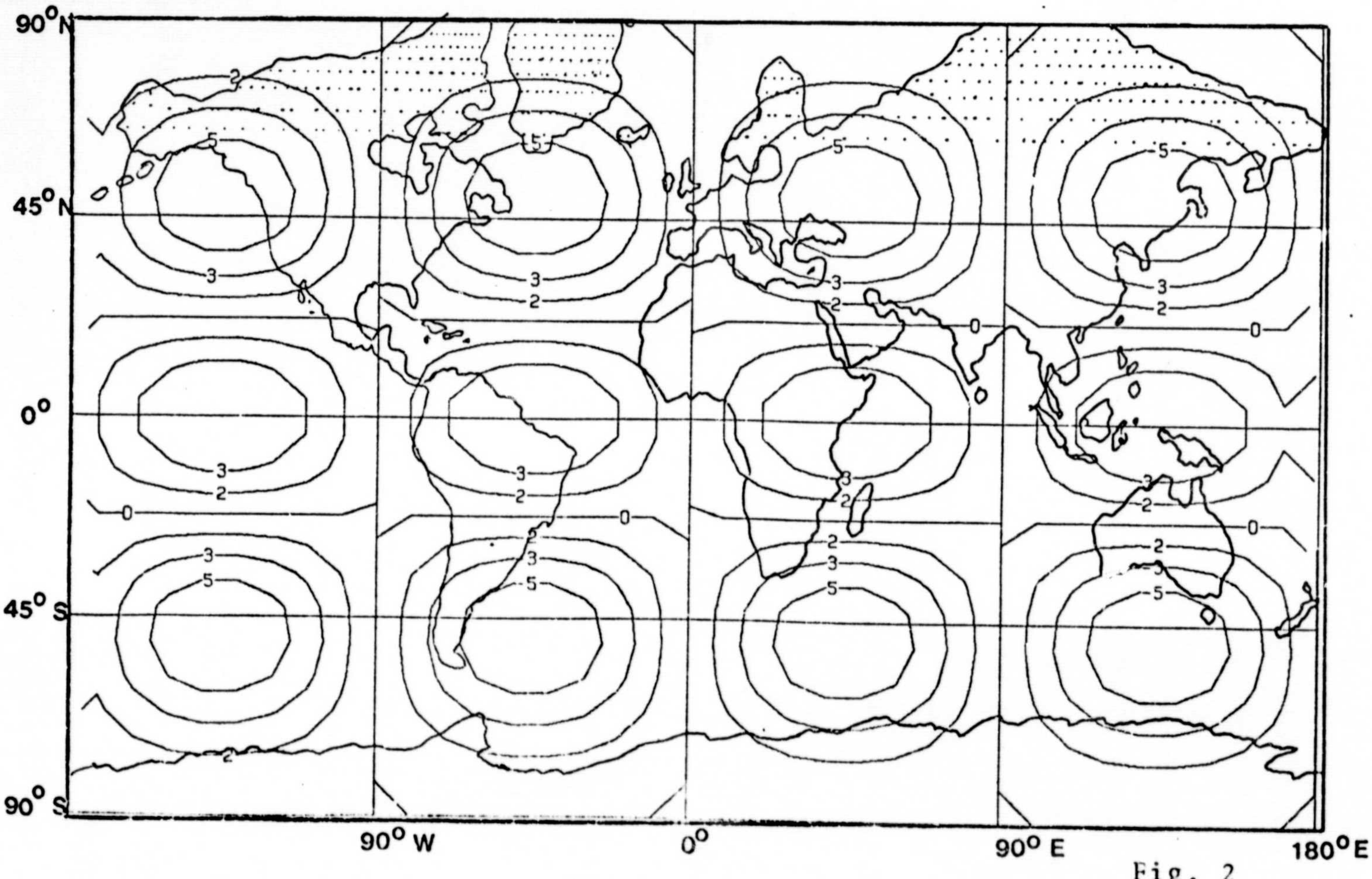


Fig. 2

$Y^o(n,m)$ $n = 4$, $m = 2$

To facilitate interpretation of the spherical harmonic expansions, the leading harmonics for each field are tabulated and ranked in descending order of the magnitudes of the amplitude coefficients An,m. Also listed, for m > o, are the phase angles. The sign of An,o in a table is actually the sign of Cn,o. Synoptic maps may then be compared in terms of the coefficients and phases of the dominant components of each field. Area-weighted global mean values (from Co,o). are also given for each field.

Table 1 shows some sample results for the December climatology (C), the December 1976 observed (O) fields, and a forecast for December 1976 (F) made with a 7-layer medium mesh (8°x10°) model (MX444M7). Only the 8 largest harmonics are listed for each field, together with the global mean values.

The data listed in Table 1 indicate that reasonable global mean values are "forecast" by the model, but with a slight cold bias. The same dominant harmonic (2,0 for T8 and Z5 and 1,0 for SLP) appears in the fields of C, O, and F. However, the model (F) overestimates the polar-equatorial difference (2,0) in T8 (41.5 vs. 37.8) and underestimates it in Z5 (800 vs. 855), while the interhemispheric difference (1,0) in SLP is grossly underestimated (11.9 vs. 22.3).

The differences between the 8 leading harmonics for C and for O indicate the character of the monthly anomaly. For example, in the expansion of T8, 6 of the 8 leading climatology harmonics are represented among the 8 leading observed harmonics. However, two outstanding observed anomalies are the (6,0) and (4,1) components, which do not appear among the first 8 climatology terms. One of these harmonics, (6,0), does appear among the leading forecast terms, albeit too large. On the other hand, the model generates some components (3,0 and 6,2)

Table 1. Global mean values and the 8 largest harmonics (An,m) for each field (T8, Z5, SLP) for December, climatology (C), and December 1976 observed (O) and Forecast (F), medium (8x10) mesh. Code: n,m / An,m/$\beta_{n,m}$

T8 (K)	Mean	1	2	3	4	5	6	7	8
C	279.4	2,0: -39.5	1,0: -9.4	2,1: 3.7/342°	3,2: 3.3/70°	3,1: 3.2/310°	5.4: 2.7/330	4,2: 2.7/55°	5,2: 2/6/51°
O	280.3	2,0: -37.8	1,0: -11.5	4,2: 4.0/56°	2,1: 3.7/334°	6,0: 3.6	4,1: 3.5/27°	3,2: 3.3/64°	5,2: 3.0/54°
F	278.9	2,0: -41.5	1,0: -11.7	6,0: 5.7	3,0: -4.8	3/2: 3.8/78°	2.1: 3.2/340°	4,2: 3.1/55°	6,2: 3.1/46°
Z5 (m)	**Mean**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
C	5634	2,0: -925	6,0: 136	4,0: -110	3,0: 60	4,1: 54/28°	2,1: 47/11°	8,1: 46/218°	7,0: -42
O	5631	2,0: -855	6,0: 159	4,1: 62/41°	2,1: 61/18°	7,4: 61/263°	6/3: 61/325°	1,0: -60	5.2: 55/110°
F	5627	2,0: -800	1,0: -127	6.0: 111	4,0: -59	3,0: -50	6,2: 43/62°	4,2: 40/92°	2,1: 40/355°
SLP (mb)	**Mean**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
C	1011.3	1,0: 17.9	2,0: -11.5	4,0: -10.7	6,0: 9.4	3,0: 5.8	2,1: 5.2/88°	4,2: 4.7/196°	5,2: 4.2/177°
O	1010.2	1,0: 22.3	6,0: 11.1	2,0: -7.2	4,2: 6.2/196°	2,1: 6.1/95°	5.2: 5.6/178	6,1: 5.4/221°	8,1: 5.0/250°
F	1010.4	1,0: 11.9	4,0: -7.8	2,0: +7.3	2,1: 4.6/98°	3,2: 4.4/251°	4,2: 4.4/194°	4,1: 4.2/114°	3,1: 4.2/150°

which do not appear among either the 8 leading observed or climatological harmonics.

Similar results are indicated for Z5 and SLP. In the case of Z5, the anomaly (difference between O and C) is represented by the last 4 harmonics in O, none of which appear among the 8 leading C coefficients. The forecast error (difference between F and O) is represented by the forecast terms, (4,0), (3,0), (6,2) and (4,2), which do not appear in the first 8 observed harmonics, and the observed coefficients, (4,1), (7,4), (6,3), and (5,2), which are not found among the first 8 forecast terms. In SLP the anomaly is represented by (6,1) and (8,1), which are present in O but not in C, and by (4,0) and (3,0), which appear in C but not in O. The forecast error is dominated by (4,0), which is not found in the observed list, by (6,0), which does not appear among the first 8 forecast components, and by (2,0), for which the forecast and observed harmonics are opposite in phase. The mismatch between the F and O harmonics is particularly evident in SLP.

References

Belousov, S. L., 1962: Tables of Normalized Associated Legendre Polynomials (translated by D.E.Brown) New York, Pergamon Press, 379 pp.

Blackmon, M.L., 1976: A climatological spectral study of the 500 mb geopotential height of the Northern Hemisphere. J.Atmos.Sci., 33, 1607-1623.

Boyd, J.P., 1978: The choice of spectral functions on a sphere for boundary and eigenvalue problems: a comparison of Chebyshev, Fourier and associated Legendre expansions. Mon.Wea. Rev., 106, 1184-1191.

Chapman, S. and J. Bartels, 1962: Geomagnetism. (Volume II. Corrected edition.) London, Oxford University Press, 1049 pp. (First edition, 1940)

Davis, P.J. and P. Rabinowitz, 1967: Numerical Integration. Waltham, Mass., Blaisdell Publishing Co., 230 pp.

Hansen, J. and Collaborators, 1979: An efficient three-dimensional global model for climate studies. Goddard Institute for Space Studies (GISS). (Unpublished)

Hobson, E.W., 1955: Theory of Spherical and Ellipsoidal Harmonics. New York, Chelsea Publishing Co., 500 pp.

Leith, C.E., 1974: Spectral statistical-dynamical forecast experiment. International Symposium on Spectral Methods of Numerical Weather Prediction, Copenhagen, August 1974. GARP Rep. No. 7.

Merilees, P.E., 1973: An alternative scheme for the summation of a series of spherical harmonics. J. Appl. Meteor, 12, 224-227.

Spar, J., 1950: On the theory of annual pressure variations. J. Meteor., 7, 167-180.