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ATLAS – An Integrated Structural Analysis and Design System

Design Module Theory

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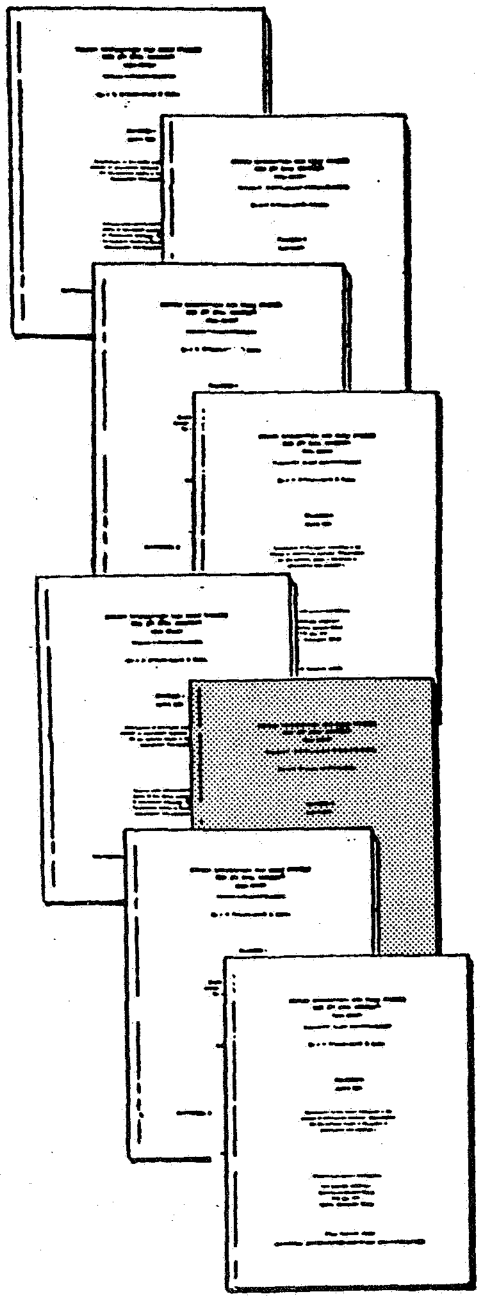
ATLAS SYSTEM DOCUMENTATION



VOLUME I
ATLAS User's Guide
NASA CR-159041

VOLUME II
System Design Document
NASA CR-159042

VOLUME III
User's Manual-Input and Execution Data
NASA CR-159043

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VOLUME VIII
SNARK User's Manual
Boeing Computer Services
BCS-G0686

FOREWORD

Development of the ATLAS integrated structural analysis and design system was initiated by The Boeing Commercial Airplane Company in 1969. Continued development efforts have resulted in the release and application of several extended versions of the system to aerospace and civilian structures. Those capabilities of the current ATLAS version developed under the NASA Langley Contract No. NAS1-12911 include the following: geometry control, thermal stress, fuel generation/management, payload management, loadability curve generation, flutter solution, residual flexibility, strength design of composites, thermal fully stressed design, and interactive graphics. The NASA monitor of this contract was G. L. Giles. The inertia loading capability was developed under the Army Contract No. DAAG46-75-C-0072.

This document is one volume of a series of documents describing the ATLAS System. The remaining documents present details regarding the input data and program execution, data management, system design, the engineering method used by the computational modules, and system-demonstration problems.

The key responsibilities for development of ATLAS have been within the Integrated Analysis/Design Systems Group of the Structures Research Unit of BCAC and the ATLAS System Group of the BCS Integrated Systems and Systems Technology Unit. R. E. Miller, Jr. was the Program Manager of ATLAS up to 1976 after which K. H. Dickenson assumed this position. The current ATLAS System is the result of the combined efforts of many Boeing engineering and programming personnel. Those who contributed directly to the current version of ATLAS are as follows:

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ABSTRACT

This document describes the automated design theory underlying the operation of the ATLAS Design Module. The methods, applications and limitations associated with the fully stressed design, the thermal fully stressed design and a regional optimization algorithm are presented herein. A discussion of the convergence characteristics of the fully stressed design is also included. Derivations and concepts specific to the ATLAS design theory are shown, while conventional terminology and established methods are identified by references.

CONTENTS

FIGURES

NOTATION

This list of notation complements the explanations provided in the text. Subscripted variables are only indicated with a dot in the following list and are explained in their proper context.

$A.$	Design variable; area
b	Distance
b	Design variation coefficient
B	Design variation matrix
C	Cosine of fiber direction (α)
C_k	Value of equality constraint
d	Value of inequality constraint
$D, D.$	Laminate constitutive matrix
D'_i	Lamina constitutive matrix
D_n	Design variation matrix
e	Subscript
$e.$	Exponent
$E, E.$	Moduli of elasticity
E'	Derivative of modulus of elasticity with respect to stress
$E.$	Element subsets (see ref. [10])
$f, f.$	Applied stress
$f.$	Factor
$F, F., \vec{F}$	Allowable stress; (vector)
$F.$	Derivative of allowable stress with respect to thickness
g_ℓ	Constraint function

G, G_i	Shear modulus
G, H	Sum of inverted squares of allowable axial stress
i	Subscript
I	Unit matrix
j	Subscript
k	Subscript; buckling coefficient
K_1	Correction term
l, m	Subscript
L, M	Reciprocal of squared shear allowable stress
MS, MS_i	Margin of safety
n	Subscript
N	Reciprocal of squared shear allowable stress
N_i	Endload
o	Subscript
P_i	Force
Q'_{ik}	Lamina compliance matrix
R, R_i	Stress ratio
R, R_i	Interaction function
R'	Derivative of interaction function with respect to thickness
S	Sine of fiber direction (α)
t, t_i	Thickness; design variable
T	Allowable stress matrix
$w_i, \vec{w}$	Weight factor; (vector)
W	Weight

x	Subscript; vector
y	Subscript
z	Subscript
α	Fiber direction; reciprocal of squared axial allowable stress
β, γ	Reciprocal of squared allowable stress
δ_{ij}	Kronecker delta
Δ	Difference operator
$\varepsilon, \varepsilon_., \vec{\varepsilon}$	Strain; (vector)
∇	Gradient
$\lambda, \lambda_., \vec{\lambda}$	Lagrange multipliers; (vector)
ν	Poisson's ratio
ρ	Density
$\sigma, \sigma_., \vec{\sigma}$	Stress; (vector)

1.0 INTRODUCTION

The analysis segment of the structural design process has, for a **reasonably** long time, been supported by finite element based software. This support has resulted in practical and economical stress analysis of complex structures, like supersonic transport airplanes. As the design process, however, also requires repeated improvements and parametric changes of alternative structures, especially during the preliminary design phase, there is a great need for automated resize (design) methods.

The ATLAS Design Module has been developed to satisfy these needs as far as strength and stability considerations are concerned. The methods have been developed and selected for application to large problems, (≥1000 design variables). Some recent applications can be found in appendix C.

It should be noted that the methods presented here are only intended as <u>computational aids</u> in the design process. Engineering judgement is a required foundation for successful usage. This applies to the preparation of input data for the Design Module as well as interpretation and evaluation of the results.

A judicious use, however, has been shown to produce both practical and economical benefits. It relieves the engineer of time consuming numerical tasks and makes him available for other design related activities (see refs. [1]-[3]).

2.0 OVERVIEW OF THEORY

The general problem being addressed in this document involves the minimization of the structural weight (weight of the stiffness finite elements),

$$W = W(t_1, \ldots, t_n) \tag{2.1}$$

where t_i represents "n" design variables subject to both the equality constraints

$$f_k(t_1, \ldots, t_n) = c_k \tag{2.2}$$

where $k=1, \ldots, m \leq n$,
and the inequality constraints

$$g_1(t_1, \ldots, t_n) \leq d_1 \tag{2.3}$$

The constraints are of stress type which involve strength and stability requirements that must be satisfied simultaneously.

The problem identified above is clearly of optimization type. The solution of this problem will be approached from two viewpoints. One involves optimality criteria methods, whereas the other one concerns itself with "math-programming" and local optimization.

2.1 OPTIMALITY VERSUS DIRECT SEARCH

The optimality criteria and the direct search method can produce minimum weight designs. Caution is advised, however, as this is not always the case.

The "math-programming" methods (ref. [4]) are of direct search type which normally produce a "local" minimum. The computational state-of-the-art is, however, such that problems involving thousands of design variables are not practically solvable by these methods because of the cost and time factors. The "math-programming" methods are, therefore, limited to small and moderate size problems for which they work quite well. Recent developments involving the conjugate gradient method (ref. [5]) have shown promise for moderately large problems.

The optimality criteria methods (ref, [6]) are of indirect type, i.e. no explicit minimization is required. Instead, a set of criteria is defined in such a manner that, when satisfied, a minimum or "almost" minimum is produced. Intuitive arguments are often used in selection of the criteria and consequently there will not be any guarantees that a "low-weight-design" has been achieved. Caution is clearly needed in interpretation of results. Experience shows, however, that optimality criteria quite successfully can be used to establish good resize procedures, and that for large problems these methods are the only feasible ones.

2.2 FULLY STRESSED DESIGN (F.S.D.)

The "fully stressed design" method (F.S.D.) is of optimality criterion type. It represents the traditional way of sizing aerospace structures. That is to say, areas, thicknesses, etc. are chosen so that the applied loads yield stresses that are equal to the allowables. The redistribution of internal loads due to changes in structural properties is not considered until the next "stress-analysis," which consequently defines an iterative method based on repeated reanalyses.

The method could be defined by the equations (2.2) when m=n. This yields a system of "n" equations in "n" unknowns. Solutions are, therefore, possible without any formal minimization steps. This is, however, in general a system of nonlinear equations for which convergence, uniqueness and a starting point must be evaluated separately for each application.

The literature on automated design has an abundance of examples of the F.S.D. method producing undesirable results. The example illustrated herein is a square plate with a central circular hole (figs. 1-5). This example is selected because the F.S.D. method is known to give poor results for this kind of problem. The first three figures show the change in thickness distribution. The fourth shows how the presence of several load cases tend to dampen the changes. Finally, figure 5 shows how stable the weight can be despite significant local changes.

It is a well-known fact that the F.S.D. method works well for statically determinate structures. The main reason for this success is that a change in area or thickness for one member only changes the stress in that member and nowhere else. A matrix containing the derivatives of member stresses with respect to design variable would be diagonal for this case.

It is also true that for large structural models, like the ones used in the aerospace industry, the effect of changing a size variable is localized. This causes the matrix of the derivatives with respect to the size variables to be strongly diagonal. As shown in references [7] and [8], this improves the chances of the F.S.D. to be optimum and to converge rapidly. Practical structures are quite often designed by multiple load conditions, a fact which also tends to improve the convergence behavior of the F.S.D. approach (see e.g. ref. [9]).

The F.S.D. method implemented in ATLAS is designed to be controlled by the user through input constraints and during execution by convergence specifications. The user can specify upper and lower bounds, input margins of safety, fixed values and regions to be excluded from resizing, (see ref. [10]) and by so doing influence the convergence characteristics.

It is finally noted that the "fully stressed design" method must be used with discretion and that results require judicious interpretation. When used in the preliminary design process it must be supported by experienced engineering personnel. However, it is still quite feasible to use it as a computational aid in the large problem environment. It is also quite frequent that the method produces consecutive designs with very minor weight changes even when member sizes exhibit significant changes (see figs. 4 and 5). In conclusion, it is noted that the "fully stressed design" method is presently one of the few practical approaches to large problems.

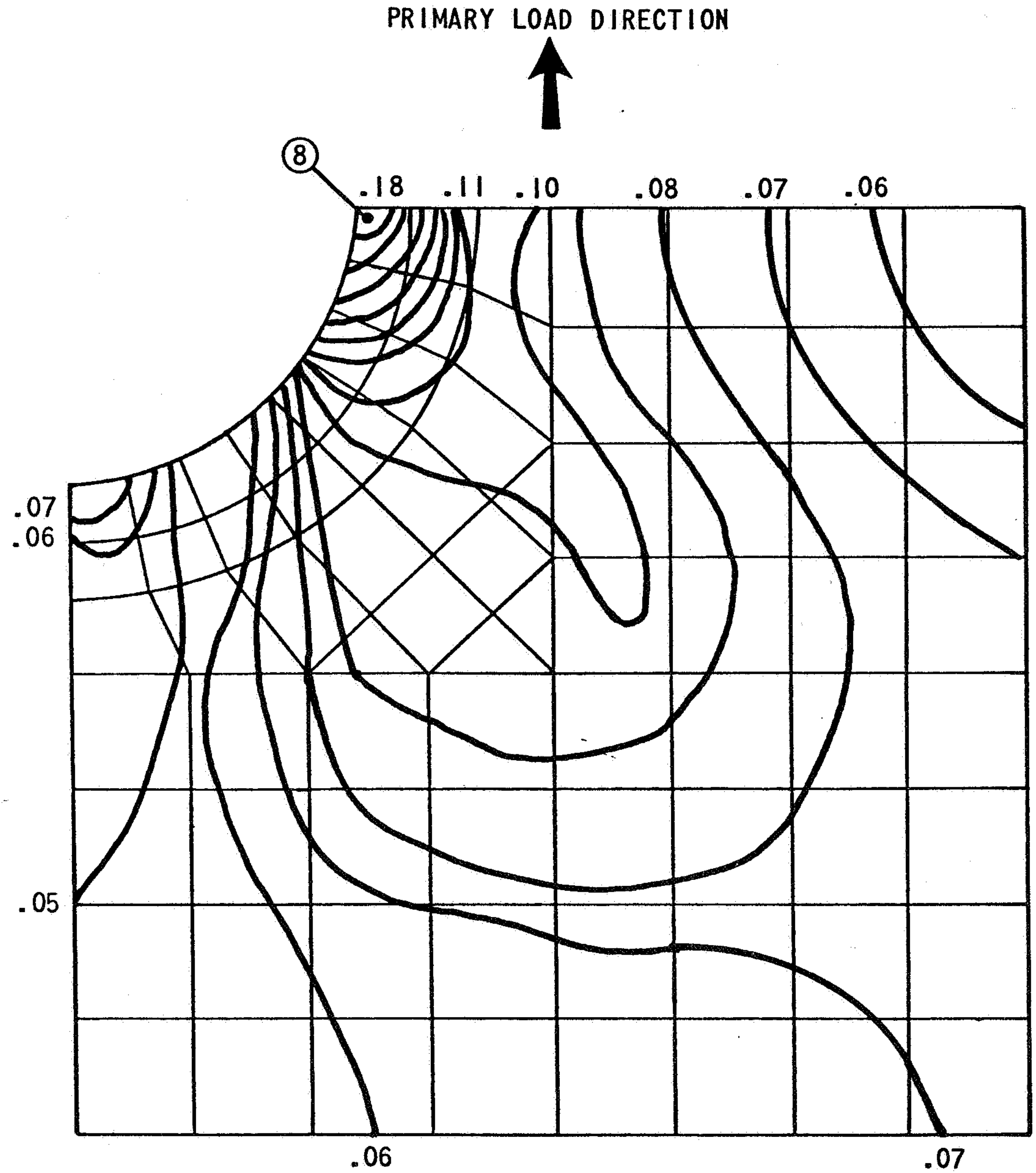

Figure 1. Thickness after One Cycle
(Initial constant thickness)

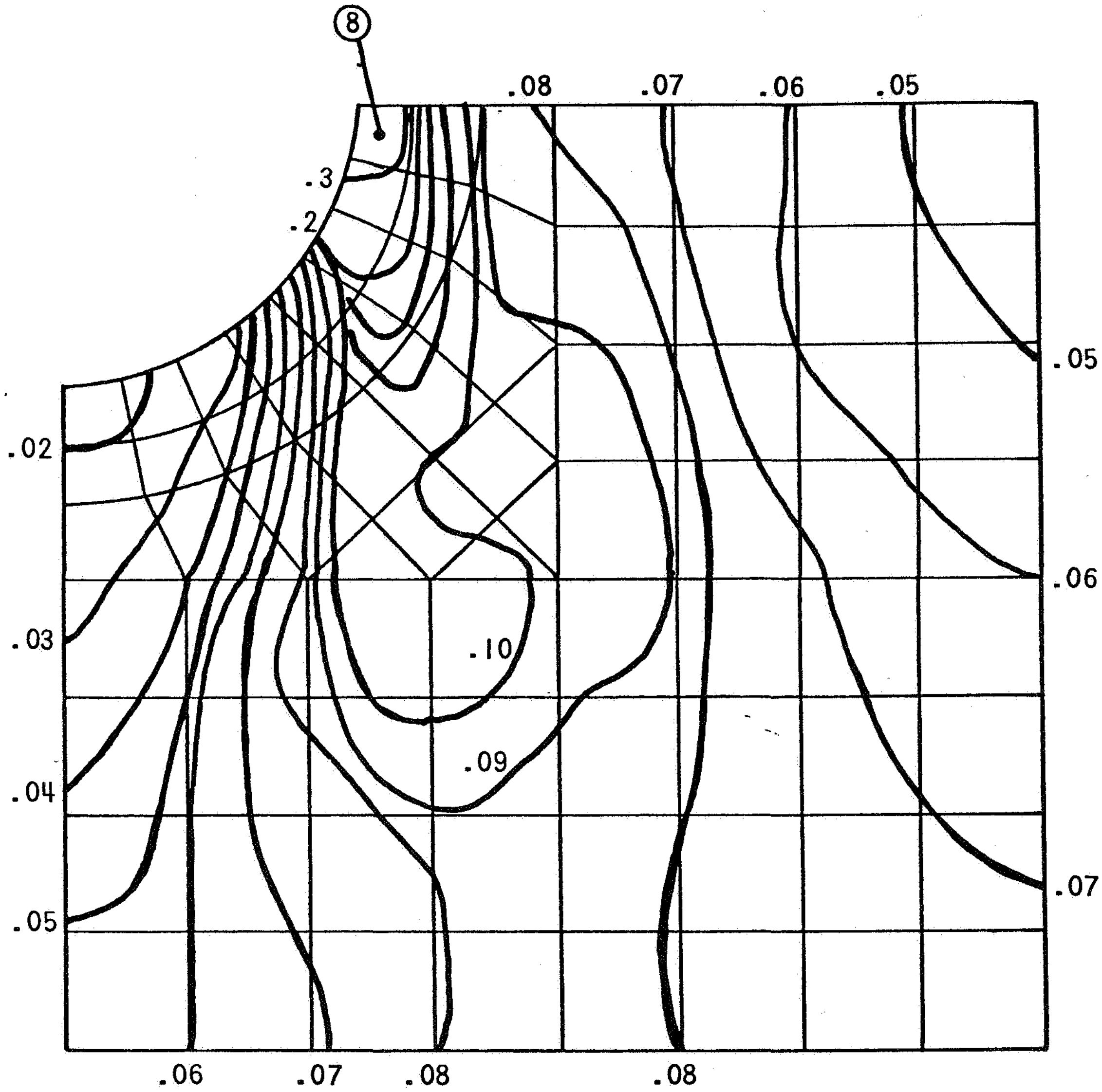

Figure 2. Thickness after Five Cycles
(Initial constant thickness)

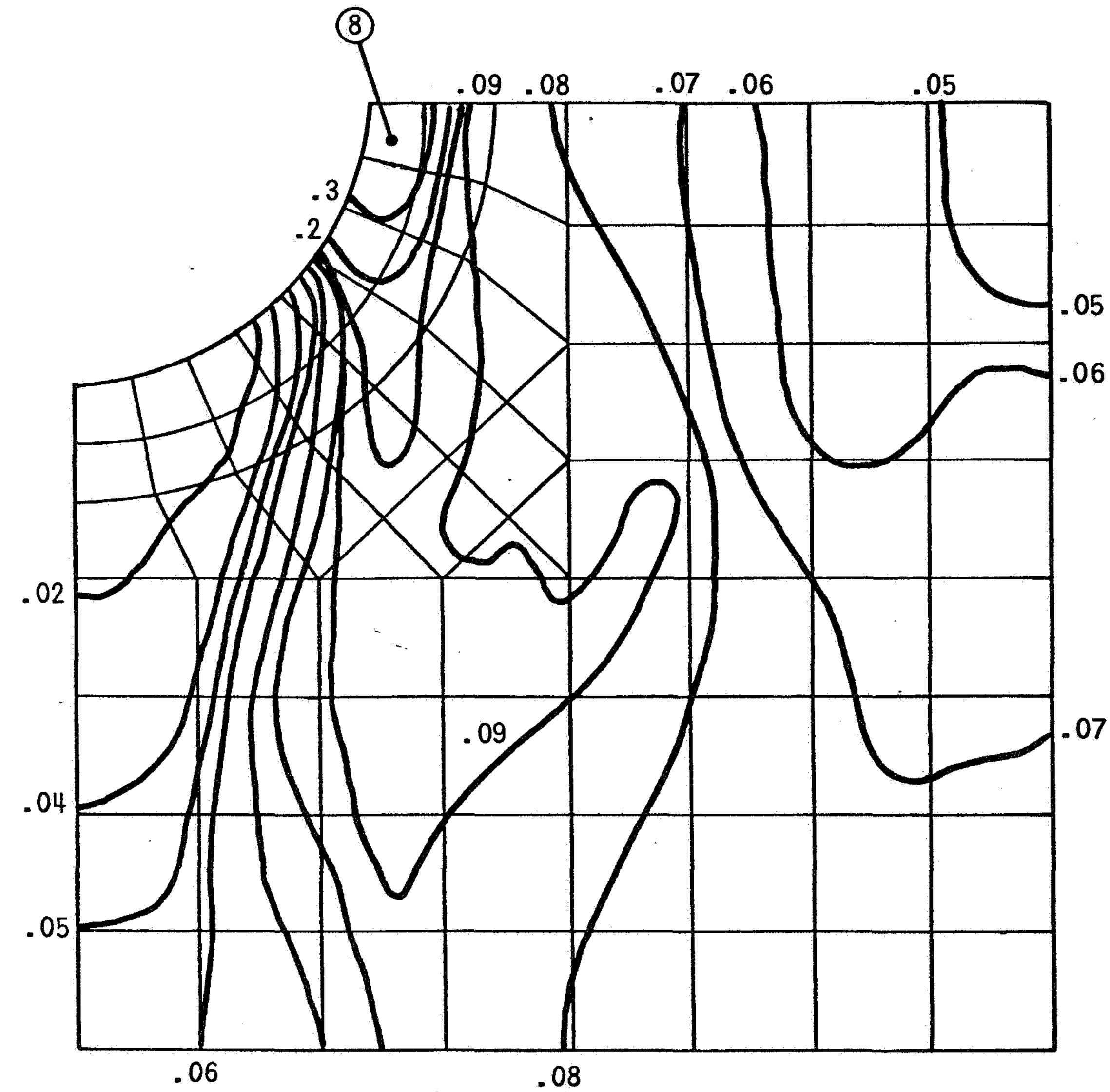

Figure 3. Thickness after Ten Cycles
(Initial constant thickness)

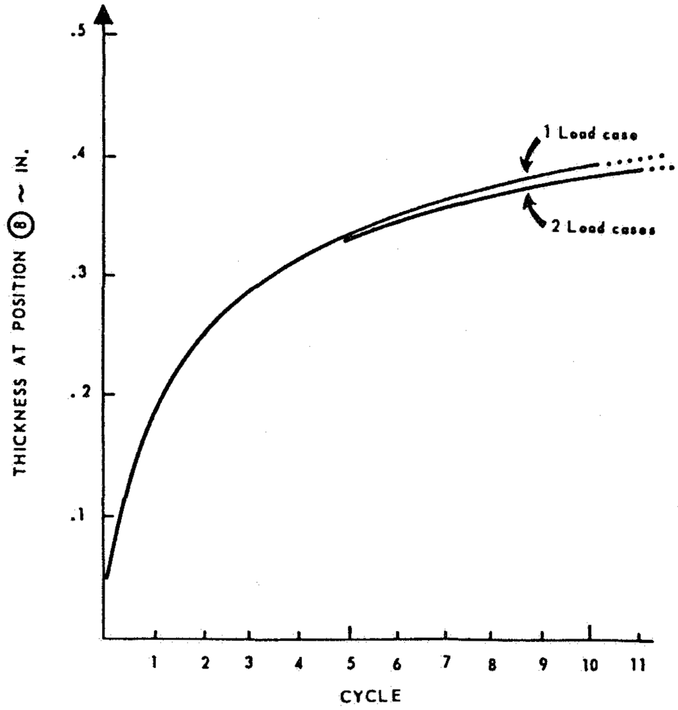

Figure 4. Change of Thickness at the Edge of the Circular Hole after Different Design Cycles

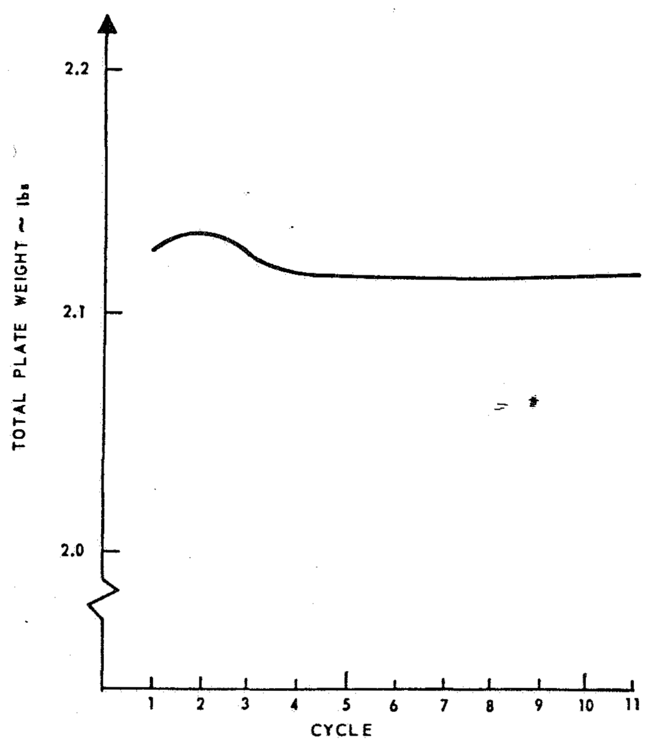

Figure 5. Total Weight for Different Design Cycles

2.3 THERMAL FULLY STRESSED DESIGN (T.F.S.D.)

The "fully stressed design" method is based on the assumption that internal element loads remain constant during resizing. A consequence of this assumption is that the following resize algorithm can be formulated

$$A_{new} = \frac{f}{F} A_{old} \tag{2.4}$$

where A_{new} is the "new" value of the design variable, f is the applied stress, F is the allowable stress and A_{old} the design variable value used when "f" was calculated.

The algorithm shown by equation (2.4) has often been deemed adequate for resizing of structures subjected to mechanical loads. However, for structures with significant thermal influences it can lead to very slow convergence. The main reason for this is that stresses due to thermal effects quite often remain reasonably constant while the mechanical stresses change with changing size variables. An algorithm intended for situations involving significant thermal stresses has been developed and tested. Results and conclusions are shown in reference [11]. A combination of thermal and mechanical stresses is considered and the resize algorithm is,

$$A_{new} = \frac{f}{F-f_{th}} A_{old} \tag{2.5}$$

where the notation is the same as in equation (2.4) and "f_{th}" represents the applied thermal stress. It is demonstrated in reference [11] how the use of this algorithm results in faster convergence than produced by the traditional "fully stressed design."

2.4 REGIONAL OPTIMIZATION

The successful application of "math-programming" methods in the large problem environment is contingent on the use of strategies that reduce a large problem to a set of smaller ones. That philosophy has been applied to the "composite optimization." This optimization is based on the method of feasible direction and is only used for structural models containing the composite elements CPLATE and CCOVER (see sec. 4 and ref. [10]).

Local optimization is used and total loads on the composite elements are assumed constant. The change in internal load distribution is, as for the F.S.D. method, introduced through the repetition of the stress analysis.

The regional concept is introduced in such a manner that results, produced in the optimization, are valid for all elements in the region. These regions (subsets) are defined in the input data.

The number of design variables available within the region is determined by the number of element variables. (All elements in a region have the same number of variables). The number of constraints are, however, dependent on both the number of elements included in the subregion (see sec. 4) and the number of load cases involved.

Criticality (relative value of constraints) of all the load cases is established during the initial phase of the optimization. This criticality is used to reduce the number of constraints, applied in the numerical search, to a number that is the same as the number of design variables.

The problem defined in this manner is thus considerably reduced in size and quite feasibly solved by "math-programming" methods.

3.0 OPTIMALITY CRITERIA

The role of optimality criteria in automated design has been discussed extensively in the technical literature. It seems to be a common conclusion (see e.g. ref. [9]) that these kinds of methods, in the case of large structural systems, appear to be the most practical. Appendix C contains problems which presently cannot be solved directly by "math-programming" methods, and are consequently natural candidates for optimality criteria-based resizings. It is, however, also clear that for the size-range where "direct search" methods are applicable they constitute a quite natural complement to the optimality criteria approaches.

The ATLAS Design Module contains two optimality criteria methods, the "fully stressed design" (F.S.D.) and the "thermal fully stressed design" (T.F.S.D.). Both methods are based on repeated reanalysis, and convergence criteria can be of both weight and variable type.

Both the F.S.D and the T.F.S.D. produce "margins of safety" that are used to update finite element properties. The resize algorithm used in both cases is,

$$A_{new} = \frac{A_{old}}{1+MS} (1+MS_{inp}) \tag{3.1}$$

where "MS" is a calculated margin of safety and "MS_{inp}" an input one. The design variables are denoted "A." It should be noted that all design variables are of "area" and "thickness" type. The details for each element are presented in appendix A.

The "margin of safety" defined here is, as the attentive reader already noticed, of an equivalent nature. It actually describes how a design variable should be changed so that the corresponding property value is just adequate for the applied loads. The more traditional definition of margin, on the other hand, results in a description of how the applied loads should be changed, so that applied and allowable stresses agree.

The equivalent margins of safety, as calculated according to both the F.S.D. and T.F.S.D., are screened and the minima are used to establish an envelope of margins of safety. This envelope is used in equation (3.1).

3.1 FULLY STRESSED DESIGN THEORY

The "fully stressed design" resize is based on three criteria. The Hill's criterion for strength (ref. [12]), an interaction criterion for panel buckling using input table allowables, and an interaction criterion for local buckling which uses calculated buckling allowables.

The "fully stressed design" uses four kinds of allowable stresses. There are material allowables, average buckling allowables, gage-dependent table buckling allowables and local buckling allowables. The latter kind is calculated based on input "modulus tables" and "spacing" data. The buckling allowables are only applicable to plate-like elements.

The calculation of "margins of safety" for buckling using "gage dependent allowables" involves two options. Both table allowables and local allowables are gage dependent and the margins can be calculated either as a first order approximation or by iteration to convergence. The change in allowable stress due to change in thickness is considered in both cases.

3.1.1 Strength Criterion

Hill's criterion (ref. [12]) is used to define failure for the "non-uniaxial" case. This criterion can be used for orthotropic materials and can, in its general form, be written as,

$$F(\sigma_y-\sigma_x)^2+G(\sigma_z-\sigma_x)^2+H(\sigma_x-\sigma_y)^2+ \; 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 \leq 1 \qquad (3.2)$$

where "$\sigma_.$" represents axial stress and "$\tau_{..}$" shear stresses. The allowables-related variables are defined in the following manner,

$$\begin{aligned} 2F &= \frac{1}{F_y^2} + \frac{1}{F_z^2} - \frac{1}{F_x^2} \\ 2G &= \frac{1}{F_z^2} + \frac{1}{F_x^2} - \frac{1}{F_y^2} \\ 2H &= \frac{1}{F_x^2} + \frac{1}{F_y^2} - \frac{1}{F_z^2} \end{aligned} \qquad (3.3)$$

where "F." represents allowable axial stress. The shear related definitions are,

$$2L = \frac{1}{F_{yz}^2}, \; 2M = \frac{1}{F_{zx}^2}, \; 2N = \frac{1}{F_{xy}^2} \qquad (3.4)$$

The variables "F.." represent allowable shear stress. The plane case is traditionally established by an assumption of transverse isotropy; i.e. Fy=Fz. The resulting two-dimensional criterion becomes,

$$R = \left(\frac{\sigma_x}{F_x}\right)^2 + \left(\frac{\sigma_y}{F_y}\right)^2 + \left(\frac{\sigma_{xy}}{F_{xy}}\right)^2 - \frac{\sigma_x \sigma_y}{F_x^2} \qquad (3.5)$$

where R ≤1.0 when the criterion is satisfied.

Extensive discussions of this type of criteria can be found in the literature (see e.g. refs. [12]-[16]). The advantage with this criterion is that it can be shown to reduce to the classical von Mises form for isotropic materials. It should be noted that only **empirical** arguments are available as far as explaining the analogy between yielding and failure.

3.1.2 Buckling Criterion

Buckling interaction criteria are used for the plate-like elements in the ATLAS system. Details can be found in appendix A. Three types of allowables are considered, average buckling allowables, gage-dependent table allowables and calculated local buckling allowables. A discussion of the former two is presented in reference [10] in the DESIGN input section.

The local buckling allowables are of the form,

$$F = kE\left(\frac{t}{b}\right)^2 \tag{3.6}$$

where "b" and the plate aspect ratios are defined in the "DETAIL DATA" in reference [10]. The assumption is that the geometry input defines rectangular plates on simple supports. The "k" values are calculated in accordance with the theory described in reference [17], (pages 348-406).

The general interaction criterion is,

$$R = f_1 R_x^{e1} + f_2 R_y^{e2} + f_3 R_{xy}^{e3} \leq 1 \tag{3.7}$$

where f_1 to f_3 and e_1 to e_3 are user specified parameters and "R." represents ratios between applied and allowable stress. If the parameters are not input, the following default interaction expression is used.

$$R = \left(\frac{R_x}{1-R_{xy}^2}\right)^4 + \frac{R_y}{1-R_{xy}^4} \tag{3.8}$$

This expression represents a parametric fit to results produced by the classical theory of buckling of simply supported, thin plates (see e.g. ref. [19]). A discussion of results using this type of interaction can be found in reference [18].

3.1.3 Equivalent Margin of Safety

It was shown in section 3.0 how the equivalent margins of safety **determine** how the pertinent design variable should be changed in order to make the applied and allowable stresses equal. The general interaction expression can be written as,

$$R_o = R(t_o) \tag{3.9}$$

and the margin of safety is defined as,

$$t_o + \Delta t = \frac{t_o}{1+MS} \tag{3.10}$$

The intention is to change the design variable "t" so that,

$$R = 1 = R(t_o + \Delta t) \tag{3.11}$$

A Taylor expansion yields,

$$1 = R(t_o + \Delta t) = R(t_o) + \Delta t \, R'(t_o) + 0\left((\Delta t)^2\right) \tag{3.12}$$

A first order approximation and a substitution of the value for Δt yields,

$$1 \approx R_o - \frac{MS}{1+MS} t_o R' \tag{3.13}$$

and the equivalent margin of safety therefore becomes,

$$MS \approx \frac{1-R_o}{R_o-1-t_o R'_o} \tag{3.14}$$

where R'_0 is the first order derivative evaluated at $t=t_0$.

This is quite clearly an approximation that, for converging or stabilizing design, becomes increasingly good. It is quite obvious that there are ranges for which equation (3.14) is meaningless. If that happens during the design, the module will revert back to the strength requirements and use those instead. Equation (3.14) can be interpreted as a predictor that tries to correct for changes in, for example, allowable stress due to changes in plate thicknesses.

Both of the interaction equations (3.7) and (3.8) can be written in the form,

$$R = R(R_x, R_y, R_z) \tag{3.15}$$

and derivatives of the type required in (3.14) have to include differentiation of R_x, R_y and R_z with respect to "t". The stress ratio R_x is,

$$R_x = \frac{N_x}{tF_x(t)} \tag{3.16}$$

where N_x is endload and $F_x(t)$ represents the input table allowables. Differentiation yields,

$$\frac{dR_x}{dt} = -\frac{R_x}{t} \cdot \left(1 + \frac{F_x' t}{F_x}\right) \tag{3.17}$$

The variable F_x' symbolizes the derivative of the allowable stress with respect to thickness. These derivatives are obviously not available and the module, therefore, uses calculated "differences" to establish numerical "slopes" of the values in the tables.

When calculated local buckling allowables are used, the typical stress ratio R_x becomes,

$$R_x = \frac{N_x b^2}{k} \cdot \frac{1}{t^3 E} \tag{3.18}$$

The same terminology as in equation (3.6) is used, and "N_x" is the endload in the x-direction. The modulus of elasticity can be written as a function of applied (σ) stress in the following manner,

$$E = E(\sigma) = E\left(\frac{N_x}{t}\right) \tag{3.19}$$

Differentiation with respect to the design variable (plate thickness) yields,

$$\frac{dE}{dt} = -E' \frac{N_x}{t^2} \tag{3.20}$$

Finally, differentiation of equation (3.18) with respect to "t" gives,

$$\frac{dR_x}{dt} = \frac{R_x}{t}\left(3 - \frac{N_x E'}{tE}\right) \tag{3.21}$$

This equation can be rewritten as

$$\frac{dR_x}{dt} = \frac{R_x}{t}\left(3 - R_x \frac{k t^2 E'}{b^2}\right) \tag{3.22}$$

where E', as before, is the derivative of the modulus of elasticity with respect to applied stress. The value of this derivative is established from calculated differences of the data in the user supplied modulus tables. Allowable local shear stress and shear modulus tables are treated in an analogous manner.

Expressions like equations (3.17) and (3.22) must be evaluated in order to calculate the margin of safety (MS) in equation (3.14). This equation represents, as already discussed, a first order approximation. In order to improve on this equation, a local iteration is used as shown in section 3.1.4.

The concept of "equivalent margin of safety" will finally be compared with the traditional approach. The interaction expression can again be written as,

$$R = R(R_x, R_y, R_{xy}) \tag{3.23}$$

which, with "increasing" load, becomes

$$1 = R(R_x + \Delta R_x, R_y + \Delta R_y, R_{xy} + \Delta R_{xy}) \tag{3.24}$$

which up to a first order approximation is,

$$1 = R + MS\left(R_x \frac{\partial R}{\partial R_x} + R_y \frac{\partial R}{\partial R_y} + R_{xy} \frac{\partial R}{\partial R_{xy}}\right) \tag{3.25}$$

If the "equivalent margin of safety" is used, the following expression is obtained.

$$1 = R + \left(-\frac{MS_e}{1+MS_e} \cdot t\right) \frac{dR}{dt} \tag{3.26}$$

where "MS_e" represents "equivalent margin of safety." The relation between the two kinds of margins becomes, from comparison of equations (3.25) and (3.26),

$$MS = \frac{MS_e}{1+MS_e}(1 + K_1) \tag{3.27}$$

where "K_1" is zero for the case of constant variables. For small values for MS_e,

$$MS \approx MS_e \tag{3.28}$$

3.1.4 Local Iteration

Determination of design variables, when "gage dependent allowables" are included, involves the solution of the nonlinear equation,

$$R(t) - 1 = 0. \tag{3.29}$$

This form immediately brings the "Newton-Raphson method" to mind (see ref. [20]). The indicated method leads to a solution of the above equation by using the recurrence relation,

$$t_{i+1} = t_i - \frac{R(t_i)-1}{R'(t_i)} \tag{3.30}$$

where "i" is the iterative index. A comparison shows that equation (3.30) is analogous to equation (3.14); or in other words the calculation of the "equivalent margin of safety" represents a first step in a "Newton-Raphson-like" iteration.

Problems relating to divergence of the scheme suggested by equation (3.30) must be avoided. The convergence requirements will now be investigated by pursuing the analogy with "equivalent margins of safety." Equation (3.14) can be rewritten as,

$$MS = -1 - \frac{tR'}{R-1-tR'} \tag{3.31}$$

The general definition of the interaction equations yields the following relation,

$$R'=R'(t) < 0 \tag{3.32}$$

which results in the final form

$$MS = -1 + \frac{|tR'|}{R-1+|tR'|} \tag{3.33}$$

The definition of margin of safety leads to the requirement,

$$MS > -1 \tag{3.34}$$

which gives,

$$R - 1 + |tR'| > 0 \tag{3.35}$$

The default interaction is defined by equation (3.8) which can be written as,

$$R = R_1 + R_2 \tag{3.36}$$

Differentiation with respect to "t" yields

$$|R't| \geq 4R_1+R_2 \geq R \tag{3.37}$$

Substitution into (3.36) yields

$$|t_iR'(t_i)| > \frac{1}{2} \tag{3.38}$$

This is a necessary requirement that is enforced during the iteration. The general interaction criterion in equation (3.7) leads to the same requirement shown by equation (3.38). Numerical experimentation has not produced a single case of divergence, once the said requirement is enforced.

3.2 THERMAL FULLY STRESSED DESIGN THEORY

The "thermal fully stressed design" algorithm is developed for structures subjected to both mechanical and thermal stresses at the same time. The thermal stresses are assumed to be significant compared to the mechanical ones, but are not allowed to exceed material allowable stresses. It is, in addition, assumed that mechanical internal loads and thermal stresses remain constant during resizing. It is, therefore, obvious that the thermal stresses cannot be allowed to be larger than the allowables, as that leads to an impossible situation. In a practical design situation one would expect that either a new material would have to be selected, or that a less severe thermal environment be found.

The uni-axial case leads to the following expression,

$$\sigma_n = \frac{\sigma_M A_o}{A_n} + \sigma_{th} \tag{3.39}$$

where

σ_n ="new" total stress
σ_M =mechanical stress (old)
σ_{th} =thermal stress
A_o ="old" area
A_n ="new" area

which results in the "thermal fully stressed design" resize algorithm,

$$A_n = \frac{\sigma_M}{F-\sigma_{th}} \cdot A_o \tag{3.40}$$

Here the total stress has been equated with the allowable. This expression illustrates quite clearly what happens when the thermal stress approaches the allowable.

It is "tempting" at this point to extend the approach by using ratios between mechanical stresses and "modified allowables" in the interaction expressions used for the plane stress case. The analogy with equation (3.40) is intuitively quite pleasing and has been explored extensively in the aerospace industry. However, a comparison between the traditional measure of criticality and the one indirectly suggested here, shows that in order for these to be equal the following equation must be satisfied,

$$\frac{\sigma_M+\sigma_{th}}{F} = \frac{\sigma_M}{F-\sigma_{th}} \tag{3.41}$$

which is the case only when,

$$\sigma_M = F-\sigma_{th} \tag{3.42}$$

As this obviously is true only when the ratios in equation (3.41) are equal to one, we find that the interaction expressions for "plate-like" elements require special attention.

A comparison between the results produced by the T.F.S.D. method and the F.S.D. method are shown in figure 6. Different ratios for areas for different stress levels are plotted according to equation,

$$\frac{A_n^1}{A_n^2} = \frac{\sigma_M F}{(F-\sigma_{th})(\sigma_M+\sigma_{th})} \tag{3.43}$$

It is easily seen that quite drastic differences can be experienced.

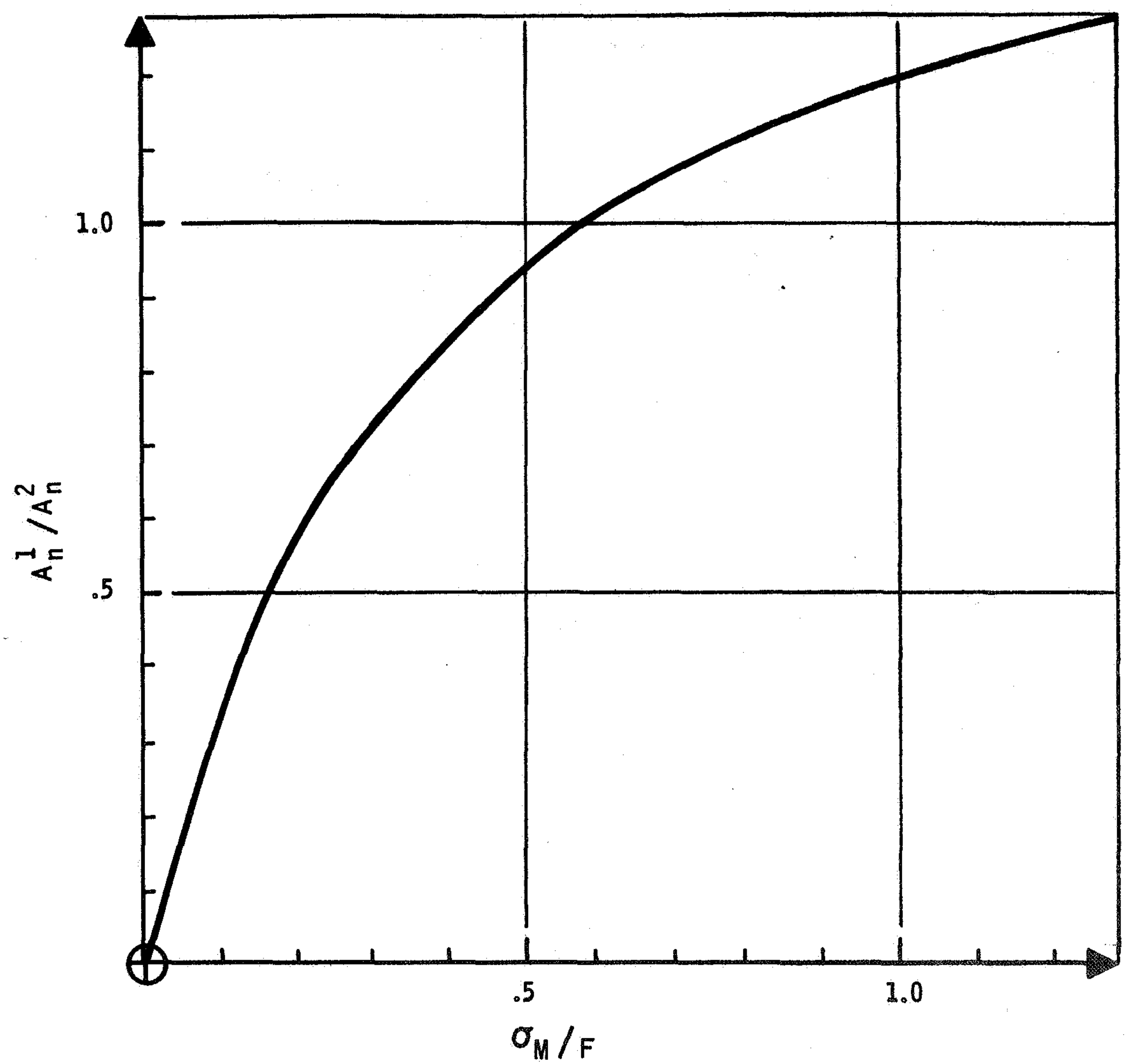

Figure 6. Ratio of Areas Produced by the T.F.S.D. and F.S.D. Methods as a Function of Applied Mechanical Stress; $\sigma_{th} = .4F$

3.2.1 Strength Criterion

The strength criterion used for the T.F.S.D. is the same as used for the F.S.D., (see eq. (3.5)), namely Hill's criterion. If mechanical and thermal effects are kept separate, the criterion yields,

$$R = \left(\frac{\frac{N_x}{t}+\sigma_x}{F_x}\right)^2 + \left(\frac{\frac{N_y}{t}+\sigma_y}{F_y}\right)^2 + \left(\frac{\frac{N_{xy}}{t}+\sigma_{xy}}{F_{xy}}\right)^2 - \frac{\left(\frac{N_x}{t}+\sigma_x\right)\left(\frac{N_y}{t}+\sigma_y\right)}{F_x^2} \qquad (3.44)$$

The following simplified terminology has been used,

$N\bullet$ = endload due to mechanical loads
$\sigma_\bullet$ = thermal stress
$F_\bullet$ = allowable stress
t = plate thickness (design variable)

The resizing is now based on the requirement of zero margin of safety, (i.e. R=1.0). The following second order equation results,

$$t^2\left[\alpha\sigma_x^2\left(1-\frac{\sigma_y}{\sigma_x}\right) + \beta\sigma_y^2 + \gamma\sigma_{xy}^2 - 1\right]$$

$$+ 2t\left[\alpha N_x\sigma_x\left[\frac{1}{2}\left(1-\frac{\sigma_y}{\sigma_x}\right)+\frac{1}{2}\left(1-\frac{N_y}{N_x}\right)\right] + \beta N_y\sigma_y + \gamma N_{xy}\sigma_{xy}\right]$$

$$+\alpha N_x^2\left(1-\frac{N_y}{N_x}\right)+\beta N_y^2+\gamma N_{xy}^2 = 0 \qquad (3.45)$$

The Greek letters are functions of allowables in the following manner,

$$\alpha = \frac{1}{F_x^{\,2}}, \quad \beta = \frac{1}{F_y^{\,2}}, \quad \gamma = \frac{1}{F_{xy}^{\,2}}$$

and the "new" thickness is calculated as the lowest positive root of equation (3.46).

It is interesting to note how the traditional criticality approach compares to the one established for mechanical stresses and "modified allowables." The special case with one axial stress and shear will be investigated. The traditional way yields the ellipse,

$$\frac{(f_x+\sigma_x)^2}{F_x^{\,2}} + \frac{(f_{xy}+\sigma_{xy})^2}{F_{xy}^{\,2}} = 1 \qquad (3.46)$$

and the "modified allowables" approach gives,

$$\frac{f_x^{\,2}}{(F_x-\sigma_x)^2} + \frac{f_{xy}^{\,2}}{(F_{xy}-\sigma_{xy})^2} = 1 \qquad (3.47)$$

A comparison between the two ellipses in figure 7 shows that the difference (the shaded region) changes with thermal stress and that quite unconservative designs can result. The "modified" curve consists of four different ellipses which, for small thermal stresses, would be a quite good approximation. The modified criticality ratio can be written as,

$$\frac{f_x}{F_x-\sigma_x} \approx \frac{f_x}{F_x} + \frac{f_x\sigma_x}{F_x^{\,2}} \qquad (3.48)$$

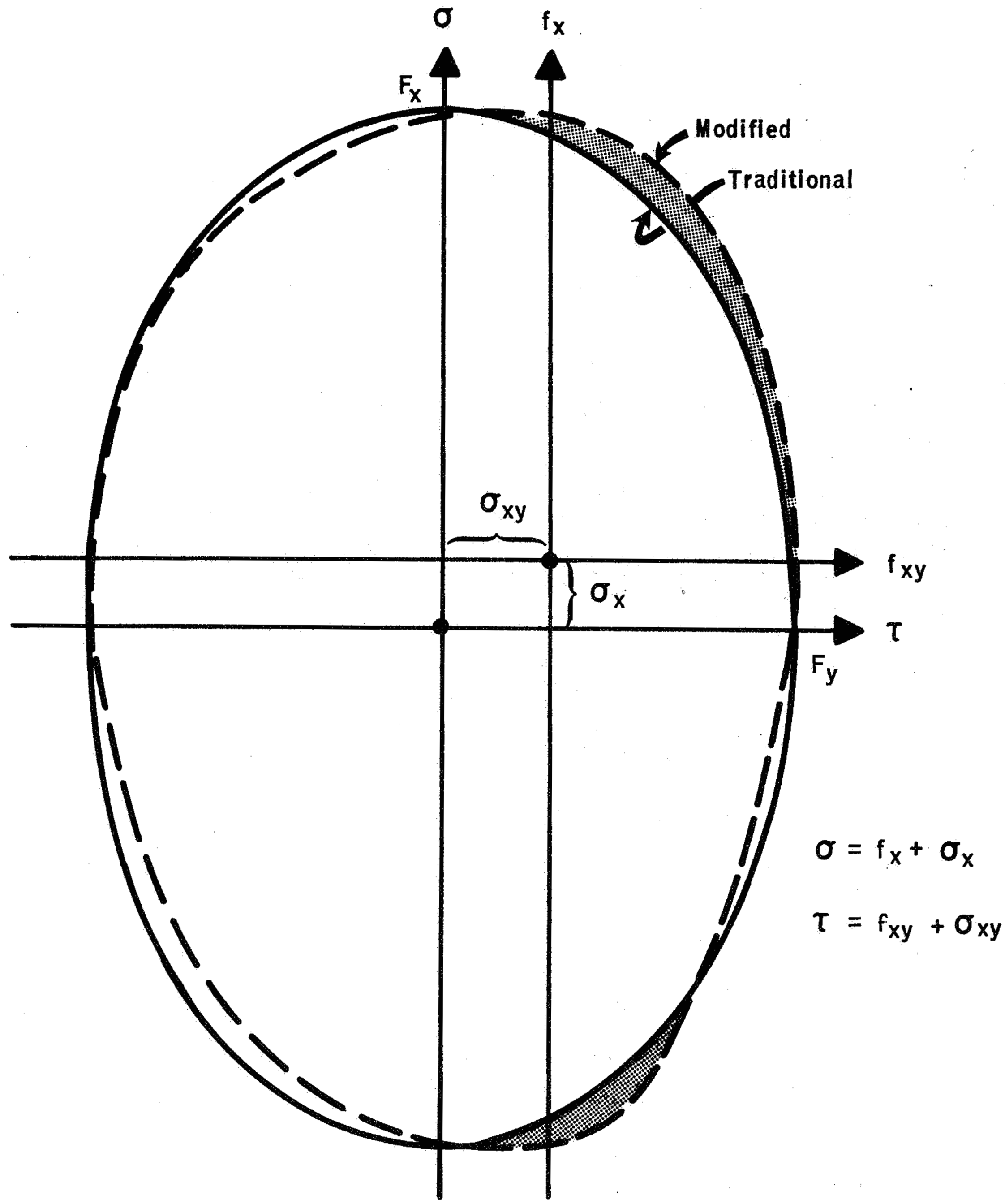

Figure 7. Criticality Comparison; T.F.S.D. (axial plus shear)

which, for small thermal stresses, becomes,

$$\frac{f_x}{F_x - \sigma_x} = \frac{f_x}{F_x}\left[1 + \frac{\sigma_x}{F_x} + \frac{\sigma_x{}^2}{F_x{}^2} + \ \cdots\right] \qquad (3.49)$$

It is furthermore expected that applied mechanical and allowable stresses will be reasonably close for this case which yields,

$$\frac{f_x}{F_x - \sigma_x} \approx \frac{f_x}{F_x} + \frac{f_x}{F_x} \cdot \frac{\sigma_x}{F_x} \approx \frac{f_x}{F_x} + \frac{\sigma_x}{F_x} \qquad (3.50)$$

which could be interpreted as a good approximation.

4.0 COMPOSITE OPTIMIZATION

The composite optimization in ATLAS is of a "local" type and is devised for the large problem environment. The method was developed for application to a composite SST with the number of design variables exceeding 6000. The composite membrane elements, CPLATE and CCOVER, are considered in the optimization. The CPLATE finite element has up to ten laminas of orthotropic material with user-defined fixed directions (see ref. [10]). The CCOVER is built up from two CPLATE elements. (Terminology can be found in appendix D)

The structure to be optimized is considered to be divided into a number of regions (optimization problems) which are treated separately. These regions are defined by the input data and can be anything from one element to the whole structure. The design of a structure consequently involves the repeated solution of weight optimization problems, each of which concerns itself with a small portion of the structure.

4.1 REGIONAL OPTIMIZATION METHOD

The regional optimization method involves the solution of the following problem. For a given region (design subset of elements) E_k and an associated subregion (optimization subset) E_{ks} we have

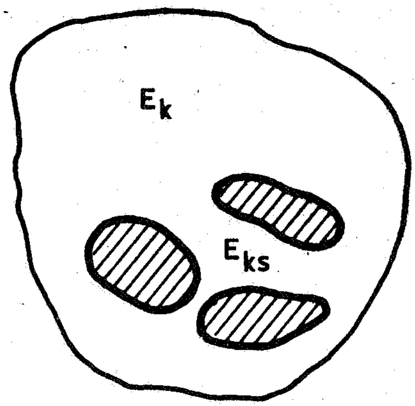

Figure 8. Regional Definition (E_k is a small part of the structure, and E_{ks} can be several subregions)

laminate strains and a set of initial lamina thicknesses. Based on this we want to find the lamina thicknesses "t_i", $i=1,\ldots,n$ where $n \leq 10$, (analogously upper and lower surface for CCOVER) for all composite elements (of the same type and with the same number of laminas) in E_{ks} assuming regionally constant results. This is done in such a manner that minimum weight is produced without violation of any of the strength constraints in E_{ks}.

4.1.1 Constraints

The composite optimization is of strength type for which two optional types of constraints are considered; "Hill's criterion" and "the maximum strain criterion" (see refs. [14],[15] and [16]).

The maximum strain criterion, which simply involves the comparison of applied strain to allowable strain for each component (2 axial and one shear) separately, uses the most severe one as the representation of criticality. The allowable strains are produced from input allowable stresses in the following manner,

$$\vec{\varepsilon}^{\,*}_{ik} = Q'_{ik}\vec{F} \tag{4.1}$$

where "i" represents the lamina and "k" the laminate. The matrix " Q_{ik}" (in lamina reference frame) is defined as,

$$Q'_{ik} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \tag{4.2}$$

(see ref. [14], p 19). The allowable stress vector "$\vec{F}$" is defined as,

$$\vec{F} = \begin{bmatrix} F_{xik} \\ F_{yik} \\ F_{xyik} \end{bmatrix} \tag{4.3}$$

Tension allowable stress is used for both "F_{xik}" and "F_{yik}" when tension allowable strain is to be produced, and analogously for compression allowable strain.

The alternative to the maximum strain criterion is Hill's criterion. This criterion has already been identified in equation (3.5), but will be shown here in an alternative form. The criterion can be expressed as,

$$\vec{\sigma}'^{T}_{ik}\, T_{ik}\, \vec{\sigma}'_{ik} \leq 1.0 \tag{4.4}$$

where "σ_{ik}" represents the stress vector for the "i:th" lamina of the "k:th" element. The allowables matrix "T_{ik}" is defined as,

$$T_{ik} = \begin{bmatrix} \frac{1}{F_{xik}^{2}} & -\frac{1}{2F_{xik}^{2}} & 0 \\ -\frac{1}{2F_{xik}^{2}} & \frac{1}{F_{yik}^{2}} & 0 \\ 0 & 0 & \frac{1}{F_{xyik}^{2}} \end{bmatrix} \tag{4.5}$$

The notation used here and in equation (4.3) is

F_{xik} = axial allowable stress in x-direction;

F_{yik} = axial allowable stress in y-direction;

F_{xyik} = shear allowable stress;

in all cases for lamina "i" of element "k". Tension or compression allowable is selected in accordance with the sign of the comparable applied stresses. In order to use this criterion as a basis for resizing, we find that certain requirements with regard to allowable stresses must be satisfied.

The quadratic form in equation (4.4) must be positive definite. If that would not be the case, it would be possible to have stress fields for which the expression (4.4) remained negative, independent of how lamina thicknesses were changed. The determinant of the matrix T_{ik}, equation (4.5), is,

$$\det T_{ik} = \frac{1}{F_{xyik}^2} \left[\frac{1}{F_{xik}^2} \left(\frac{1}{F_{yik}^2} - \frac{1}{4F_{xik}^2} \right) \right] \tag{4.6}$$

It is easily seen from this expression that T_{ik} is not positive definite when $F_{yik} > 2F_{xik}$. In order to avoid this problem, it will be required that the larger axial allowable stress always defines the primary direction.

The two criteria discussed here constitute the baseline for the constraints used in the optimization. The general form of the criteria for the maximum strain type is,

$$R_{ik\ell} = \max_{j} \left\{ \frac{\varepsilon'ikj}{\varepsilon^*ikj} \; ; \; j=1,\ldots,3 \right\}_{\ell} \tag{4.7}$$

and for Hill's,

$$R_{ik\ell} = \left(\vec{\sigma}_{ik}^{\,'T} \, T_{ik} \, \vec{\sigma}_{ik}^{\,'} \right)_{\ell} \tag{4.8}$$

where the index "ℓ" refers to load case and "i" and "k" according to what was defined earlier.

A detailed discussion of failure criteria for composites can be found in references [12]-[16].

4.1.2 Criticality

Each optimization problem is solved in an iterative manner. Each cycle in this iteration contains a screening phase and a solution phase.

The screening phase consists of a search of subset E_{ks}. This search establishes the critical element and load case for each lamina "i", identifying the following quantities.

$$R_i = \max_{k,\ell} \left\{ R_{ik\ell}, k=1,\ldots,ne; \ell=1,\ldots,nl \right\} \qquad (4.9)$$

where "ne" denotes the number of elements in E_{ks} and "nl" is the number of load cases to be considered. The "$R_{ik\ell}$" s represent the value produced by the strength criterion used (Hill's criterion" or "the maximum strain criterion": see equations (4.7) and (4.8)).

The screening is performed with the objective of establishing the strength constraints to be used during the solution phase. This procedure obviously requires all elements in E_{ks} to have the same number of variables (laminas), and subsets containing CCOVER will be treated as two independent problems.

The solution phase involves the minimization of weight with strength constraints defined by the screening. Thus, the defined optimization problem contains "n" design variables and "n" constraints. Here "n" is the number of laminas for any plate in E_{ks} (upper or lower surface for CCOVER).

The screening is repeated after the local optimization is completed. If this does not result in "new" critical elements or load cases, the solution is complete, otherwise an additional cycle of optimization is performed and is repeated until the criticality order has stabilized. This is repeated a maximum of ten times for each problem.

4.2 LOCAL OPTIMIZATION

The local optimization is of "math-programming" type that is concerned with the determination of a minimum weight design for a laminate. The lamina thicknesses (number of layers in each lamina) are the design variables, and there is one strength constraint present for each lamina.

The starting point for the optimization is a set of laminate strains and an initial set of lamina thicknesses. The program establishes a starting point such that at least one lamina constraint is active. The starting point is determined from the information gained during the screening for criticality. The initial thicknesses are determined from the input values, if Hill's criterion is used as,

$$t_{is} = \sqrt{R_{imax}} \cdot t_{io} \tag{4.10}$$

and if the maximum strain criterion is used as,

$$t_{is} = R_{imax} \cdot t_{io} \tag{4.11}$$

where t_{is} = starting point thickness,

t_{io} = input thickness,

and

$$R_{imax} = \max_{i} \left\{ R_i ; i=1,...,n \right\} \tag{4.12}$$

which establishes a starting point that is an acceptable design.

The optimization is local in the sense that the total load on a laminate is assumed constant during the search. The redistribution of internal loads due to changes in laminate stiffnesses is introduced through reanalysis.

4.2.1 Laminate Constitutive Relations

A laminate, which in this context represents a CPLATE or an upper or lower surface of a CCOVER element, has in the general case anisotropic "membrane" properties. Each laminate is built-up of a maximum of ten directional laminas, each one with orthotropic elastic properties.

For a laminate of total thickness "t" the following constitutive relation is applicable,

$$\vec{\sigma} = D\vec{\varepsilon} \qquad (4.13)$$

where $\vec{\sigma}$ represents gross "element" stresses and $\vec{\varepsilon}$ the gross "element" strains. (Element subscripts are dropped for convenience).

Each lamina has, in the local (lamina) reference frame, the following constitutive equation,

$$\vec{\sigma}_i{}' = D_i'\vec{\varepsilon}_i{}' \qquad (4.14)$$

The theory is based on engineering strains and the strain transformation is, as easily can be verified in any elementary textbook,

$$\vec{\varepsilon}_i{}' = A_i\vec{\varepsilon} \qquad (4.15)$$

where the transformation matrix "A " is,

$$A_i = \begin{bmatrix} C^2 & S^2 & SC \\ S^2 & C^2 & -SC \\ -2SC & 2SC & C^2-S^2 \end{bmatrix} \qquad (4.16)$$

Here "C" and "S" are, respectively, the cosine and sine of the angle (α, see appendix D) between the laminate reference direction and the lamina primary material direction. The transformation of stresses becomes consequently,

$$\vec{\sigma}_i{}' = (A_i{}^{-1})^T \vec{\sigma}_i \qquad (4.17)$$

where "σ_i" represents the stresses in lamina "i" referred to the laminate reference frame, and "σ_i" is referred to the lamina (local) frame. The total endload in the laminate, referred to the laminate frame can be written as,

$$\vec{\sigma} t = \sum_{i=1}^{n} t_i A_i^T D_i' A_i \vec{\varepsilon} = tD\vec{\varepsilon} \qquad (4.18)$$

which gives the following constitutive matrix for the laminate,

$$D = \frac{1}{t} \sum_{i=1}^{n} t_i A_i^T D_i' A_i \qquad (4.19)$$

where "D " is the orthotropic constitutive matrix for lamina "i". The coefficients in this matrix are well-known and can, for example, be found in reference [14] page 18, or in appendix D.

4.2.2 Lamina Stress and Strain

The input to the optimization procedure consists of a set of initial thicknesses and laminate strains. The optimization requires the repeated evaluation of stresses or strains as the design variables change. The lamina thicknesses "t_i" are perpetually compared to the initial values "t_{i0}", and stresses and strains are based on the assumption that total laminate load stays constant,

$$\vec{N} = t\vec{\sigma} = \text{constant} \tag{4.20}$$

This assumption provides the following expression for the laminate stresses,

$$\vec{\sigma}_n = \frac{t_o}{t_n}\vec{\sigma}_o \tag{4.21}$$

where "n" indicates new values and "o" initial values. In addition, total strain compatibility is assumed in the laminate, yielding the lamina strains,

$$\vec{\varepsilon}\,'_{in} = \frac{t_o}{t_n} A_i D_n^{-1} D_o \vec{\varepsilon}_o \tag{4.22}$$

and the lamina stresses,

$$\vec{\sigma}\,'_{in} = \frac{t_o}{t_n} D'_i A_i D_n^{-1} D_o \vec{\varepsilon}_o \tag{4.23}$$

Here only "t_n" and "D_n^{-1}" are variables. It should be noted that, in the general case, a different element is "critical" for each lamina (compare the screening phase in sec. 4.1.2). It is quite clear from this situation that all the elements in the "design subset" E_{ks} must have the same number of laminas and identical orientations.

In order to illustrate conditions surrounding a stable criticality situation, sufficiency in a special case will be shown. It is known that the initial lamina strains for two elements with the same orientations are related in the following manner,

$$A_i \vec{\varepsilon}_{ok} > A_i \vec{\varepsilon}_{ol} \tag{4.24}$$

A change in design variables results in the following difference between lamina strains,

$$\vec{\varepsilon}'_{ik} - \vec{\varepsilon}'_{il} = \frac{1}{t_n} A_i D_n^{-1} (t_{ok} D_{ok} \vec{\varepsilon}_{ok} - t_{ol} D_{ol} \vec{\varepsilon}_{ol})$$

If both elements have the same starting point,

$$\vec{\varepsilon}'_{ik} - \vec{\varepsilon}'_{il} = \frac{t_{ok}}{t_n} A_i D_n^{-1} D_{ok} (\vec{\varepsilon}_{ok} - \vec{\varepsilon}_{ol}) \tag{4.25}$$

which results in the same criticality as for the starting point. It is quite clear that this represents a very special case. However, it illustrates the phenomenon considered in the repeated optimization procedure, and indications are that initial criticality quite often is preserved.

4.2.3 Math-Programming Method

The optimization is based on the method of feasible direction (see refs. [5] and [21]). The function to be minimized is the structural weight,

$$W = \sum_{i=1}^{n} \rho_i t_i \tag{4.26}$$

where, ρ_i = lamina density

t_i = lamina thickness

n = number of laminas

and the constraints are,

$$g_i = R_i - 1 \leq 0, i=1,\ldots,n. \tag{4.27}$$

These constraints are of strength type, as discussed previously, and defined by equation (4.9).

The method of feasible direction (Zoutendijk's method described in ref. [4]) establishes a direction along which a step can be taken without violating the constraints, starting from a specific point in the design space. The feasible direction is, in this method, found by solving a linear programming problem in which the decrease in the structural weight "W" is maximized subject to constraints which insure feasibility; i.e. do not violate the constraints (4.27).

The design variables "t_i" are modified prior to the optimization, so that the largest constraint is equal to zero. The same normalization also takes place after the optimization is completed. Derivatives with respect to design variables, both for the objective function and the constraints, are established by using finite differences.

Convergence of the optimization is considered to have been attained, if, in three consecutive iterations the relative and absolute change in the value of "W" is less than .001. The maximum number of iterations allowed is ten.

Constraints can also be imposed directly on the design variables "t_i". Upper and lower bounds can be defined for each design variable, and the smallest lower bound is one layer thickness. No default exists for the upper bounds.

It is, in addition, possible to define constraints that equate thicknesses for different laminas. These constraints are of the type,

$$t_i = t_{n_k}; \quad n_k = n_1, n_2, \ldots, n_m$$

where $m \leq n-1$. The resulting minimization problem involves less than "n" variables and "n" strength constraints of the type shown in equation (4.27).

APPENDIX A

FINITE ELEMENT DESIGN

This appendix contains a condensed description of each element type as seen from the standpoint of design. A detailed description of the finite elements is presented in reference [10].

Design variables, applied loads, allowables and criteria and algorithm information are presented for each element type. Additional information with regard to constraints (upper bounds, lower bounds, fixed data, etc.), allowables used, defaults and available options is described under "Design Input Data" in reference [10].

ROD

DESIGN VARIABLES:	ONE AREA AT EACH END (2)
APPLIED LOAD:	AXIAL FORCE
ALLOWABLES:	MATERIAL ALLOWABLE STRESSES
DESIGN PROPERTIES:	NONE
REMAINING STIFFNESS PROPERTIES:	NONE
DESIGN ALGORITHM:	F.S.D. AND T.F.S.D.

BEAM

DESIGN VARIABLES:	ONE AREA AND TWO SHEAR AREAS AT EACH END. (6)
APPLIED LOAD:	TWO BENDING MOMENTS AND ONE AXIAL LOAD AT EACH END FOR THE AREA. TWO SHEAR FORCES AT EACH END, ONE FOR EACH SHEAR AREA.
ALLOWABLES:	MATERIAL ALLOWABLE STRESSES
DESIGN PROPERTIES:	SECTION MODULI, TWO AT EACH END; SHEAR STRESS CONCENTRATION FACTORS, TWO AT EACH END.
REMAINING STIFFNESS PROPERTIES:	AREA MOMENTS OF INERTIA, THREE AT EACH END; CHANGED IN THE SAME PROPORTIONS AS THE AREA AT THE SAME END.
DESIGN ALGORITHM:	F.S.D.

SPAR

DESIGN VARIABLES:	TWO AREAS FOR EACH CHORD PLUS WEB THICKNESS. (5)
APPLIED LOAD:	AVERAGE LOAD FOR EACH CHORD, AND EQUIVALENT SHEAR FLOW FOR SPAR-WEB. 1
ALLOWABLES:	MATERIAL ALLOWABLE STRESSES, PLUS SHEAR BUCKLING ALLOWABLES FOR THE WEB.
DESIGN PROPERTIES:	STIFFENER AREA AND SPACING
REMAINING STIFFNESS PROPERTIES:	LUMPED CAP AREAS, CHANGED BY THE SAME FACTOR AS THE WEB.
DESIGN ALGORITHM:	F.S.D. AND T.S.F.D.

1 STIFFENER AREA CHECKED FOR SHEAR CARRIED AS DIAGONAL TENSION.

PLATE

DESIGN VARIABLES:	PLATE THICKNESS AND TWO SMEARED STIFFENER THICKNESSES, (3)
APPLIED LOAD:	MEMBRANE ENDLOAD, TWO AXIAL AND ONE SHEAR COMPONENT.
ALLOWABLES:	MATERIAL ALLOWABLE STRESS AXIAL BUCKLING ALLOWABLE STRESS } [1] SHEAR BUCKLING ALLOWABLE STRESS } LOCAL AXIAL BUCKLING STRESS } [2] LOCAL SHEAR BUCKLING STRESS }
DESIGN PROPERTIES:	2 STIFFENER RATIOS
REMAINING STIFFNESS PROPERTIES:	NONE
DESIGN ALGORITHM:	F.S.D. AND T.F.S.D.
DESIGN INTERACTION:	HILL'S CRITERION FOR STRENGTH USER DEFINED INTERACTION FOR BUCKLING.
SIZING:	PREDICTOR APPROXIMATION, LOCAL ITERATION; BOTH FOR GAGE DEPENDENT ALLOWABLES
ASSUMPTIONS:	TENSION IS NEGLECTED IN BUCKLING INTERACTION - STIFFENERS USE THE SAME ALLOWABLES AS THE PLATE

[1] INPUT GAGE DEPENDENT TABLES, [2] CALCULATED.

GPLATE

DESIGN VARIABLES:	UP TO FIVE THICKNESSES; ONE AT EACH CORNER AND ONE IN THE MIDDLE. (5)
APPLIED LOAD:	MEMBRANE ENDLOADS ONLY, TWO AXIAL AND ONE SHEAR COMPONENT.
ALLOWABLES:	MATERIAL ALLOWABLE STRESS AXIAL BUCKLING ALLOWABLE STRESS [1] [3] SHEAR BUCKLING ALLOWABLE STRESS [1] [3] LOCAL AXIAL BUCKLING STRESS [2] [3] LOCAL SHEAR BUCKLING STRESS [2] [3]
DESIGN PROPERTIES:	NONE
REMAINING STIFFNESS PROPERTIES:	BENDING THICKNESSES, UNCHANGED.
DESIGN ALGORITHM:	F.S.D.
DESIGN INTERACTION:	HILL'S CRITERION FOR STRENGTH. USER DEFINED FOR BUCKLING.
SIZING:	LOCAL ITERATION FOR GAGE DEPENDENT ALLOWABLES
ASSUMPTIONS:	AVERAGE MEMBRANE STRESSES ARE REPRESENTATIVE; BENDING STRESSES ARE SMALL AND NOT DESIGNING.

[1] INPUT GAGE DEPENDENT TABLES. [2] CALCULATED. [3] BASED ON AVERAGE GAGE

SPLATE

DESIGN VARIABLE:	ONE THICKNESS (1)
APPLIED LOAD:	EQUIVALENT SHEAR FLOW
ALLOWABLES:	MATERIAL ALLOWABLE STRESS SHEAR BUCKLING ALLOWABLE STRESS [1]
DESIGN PROPERTIES:	NONE
REMAINING STIFFNESS PROPERTIES:	NONE
DESIGN ALGORITHM:	F.S.D. AND T.F.S.D.
SIZING:	PREDICTOR APPROXIMIATION

[1] INPUT GAGE DEPENDENT TABLES.

CPLATE

DESIGN VARIABLES:	NUMBER OF LAYERS FOR EACH LAMINA (MAX. 10)
APPLIED LOAD:	LAMINATE STRAIN, THREE COMPONENTS
ALLOWABLES:	COMPOSITE MATERIAL ALLOWABLES
DESIGN PROPERTIES:	NONE
REMAINING STIFFNESS PROPERTIES:	NONE
DESIGN ALGORITHM:	COMPOSITE OPTIMIZATION
DESIGN CRITERIA:	HILL'S CRITERION OR THE MAXIMUM STRAIN CRITERION
SIZING:	MATH-PROGRAMMING; FEASIBLE DIRECTION
ASSUMPTIONS:	TOTAL LOAD IN LAMINATE REMAINS CONSTANT DURING RESIZING

REMAINING ELEMENTS

COVER:	Analogous to two PLATE:s.
BRICK:	Margin of safety calculated according to Hill's criterion
SCALAR:	Not designed
SROD:	The same as ROD
CCOVER:	Analogous to two CPLATE:s

APPENDIX B

CONVERGENCE OF THE FULLY STRESSED DESIGN

Application of the "fully stressed design" method to practical structures normally leads to concern with regard to the convergence characteristics and "closeness" to minimum weight. Section 2.2 contains illustrations of the convergence behavior under adverse conditions.

A discussion of the fully stressed design method is presented in references [7] and [8]. Both papers discuss the problems relating to convergence. Some attention has been given to the question of verifying optimality for practical problems. No criterion has, however, been developed for applications to structures with many design variables, but some of the analytical aspects of the formulation are quite important even from a practical standpoint.

The fully stressed design method is of iterative type and its application in a practical design situation requires determination of the following:

- Is the method converging?
- When should the iteration be terminated?
- If it does not converge, what should be done from a constraint standpoint, so that the modified problem will converge?
- How close to minimum weight is the solution?

The fully stressed design, at the final point of convergence, satisfies the following set of equations,

$$A_i - \frac{P_i}{F_i} = 0 \qquad (B.1)$$

where i=1,...,n and "n" is the number of design variables present. The variables are,

A_i = design variable,

P_i = equivalent design load,

F_i = allowable stress.

The general design problem, on the otherhand, is,

minimize
$$W = \sum_{i=1}^{n} w_i A_i \qquad (B.2)$$

subject to the constraints:

$$g_i = A_i - \frac{P_i}{F_i} \geq 0 \qquad (B.3)$$

where i=1, ..., n. Before the conditions surrounding optimality are described a few concepts are defined.

The problem of convergence and optimality of the fully stressed design will be studied in a range where the changes in the design variables are reasonably small. A linear expansion of the critical forces yields,

$$P_i^{(k)} = P_i^{(k-1)} + \sum_{j=1}^{n} \frac{\partial P_i}{\partial A_j}^{(k-1)} \Delta A_j \qquad (B.4)$$

where the superscript denotes the iteration number. The design variation matrix, $B=[b_{ij}]$ is defined by differentiation of equation (B.1) yielding,

$$b_{ij} = \frac{\partial A_i}{\partial A_j} = \frac{1}{F_i} \cdot \frac{\partial P_i}{\partial A_j} \qquad (B.5)$$

The design vector $A^{(k)}$ has the design variables as components and the superscript indicates the iteration number. The "design modification" vector is,

$$A^{(k)} = A^{(k+1)} - A^{(k)} \tag{B.6}$$

which also can be written as,

$$\Delta A^{(n)} = B_1 B_2 \ldots B_n \; \Delta A^{(o)} \tag{B.7}$$

where B_n should be evaluated at a point between k-1 and k. The iterative process will converge if,

$$D_n = B_1 B_2 \ldots B_n \rightarrow 0, \text{ when } n \rightarrow \infty \tag{B.8}$$

A sufficient condition for convergence is that for the spectral norm of D_n the following holds,

$$|D_n| \rightarrow 0, \text{ when } n \rightarrow \infty \tag{B.9}$$

which is the case if,

$$|B_i| < 1, \text{ for } i=1,\ldots,n \tag{B.10}$$

The spectral norm is defined as

$$|B| = \sup_{x \neq 0} \frac{|Bx|}{|x|} \tag{B.11}$$

where $|x|$ is the Euclidian norm of x. It is quite clear from (B.11) that all the coefficients of B_i must be less than zero. The Euclidian norm for every column vector in B_i must also be less than zero. The relations described here do not constitute a practical method for determination of convergence requirements, but provide a background for better understanding of the convergence problems enountered with the fully stressed design method.

The relation between the "fully stressed design" and the minimum weight design is of great concern. When these two designs coincide, they will both be solutions to equations (B.2) and (B.3). It is a well-known fact that for statically determinate structures equation (B.5) becomes,

$$b_{ij} = 0 \qquad \text{(B.12)}$$

in which case the right hand side of equation (B.3) becomes a constant and the minimization becomes a linear programming problem. The solution is always at the vertex of "n" of the "constraints". As there are only "n" constraints, we find that the fully stressed design represents a minimum.

For indeterminate structures the "internal loads" are nonlinear functions of the design variables, and the minimization becomes a nonlinear programming problem. The solution is not necessarily a vertex, and the fully stressed design will consequently not always be a minimum.

It is of great importance, in the practical situation to determine how close a fully stressed design is to the optimum. The Kuhn-Tucker optimality condition (ref. [5]) is,

$$\nabla W = \sum_{j=1}^{n} \lambda_j \nabla g_j \qquad \text{(B.13)}$$

where ∇ is the "gradient" symbol, and n is the number of active constraints. Equation (B.13) must be satisfied at the optimum point, which if coincident with the fully stressed design, would have to be a vertex. In that case,

$$\sum_{k=1}^{n} \lambda_k \frac{\partial g_k}{\partial A_i} = w_i \tag{B.14}$$

and

$$g_i = 0 \text{ for } i=1,\dots,n \tag{B.15}$$

Differentiation of the constraint equation yields,

$$\frac{\partial g_j}{\partial A_i} = \delta_{ij} - \frac{1}{F_j}\frac{\partial P_i}{\partial A_j} \tag{B.16}$$

where δ_{ij} is the Kronecker delta, and equation (B.14) can be written as,

$$(I-B^T)\vec{\lambda} = \vec{w} \tag{B.17}$$

where I is a unit matrix, B^T is the transpose of the matrix defined in equation (B.5). The variable $\vec{\lambda}$ is a vector with the Lagrange multipliers as components and $\vec{w}$ is a vector containing the right hand sides of all the equations similar to (B.14). The optimality condition is,

$$\vec{\lambda} = (I-B^T)^{-1}\vec{w} > 0 \tag{B.18}$$

which should be interpreted as every component having to be positive. The requirement of positive Lagrangian multipliers is part of the Kuhn-Tucker conditions.

It is known that for a converging fully stressed design, the matrix $I-B^T$ is nonsingular and the optimality condition can be written as,

$$\vec{\lambda} = \left[I+B^T+B^{T2}+ \ldots .\right]\vec{w} > 0 \qquad \text{(B.19)}$$

The series inside the brackets converges rapidly when the coefficient b_{ij} are much smaller than unity. Finally, if the matrix in brackets becomes diagonally dominant with positive diagonal values, the fully stressed design is optimum.

Equation (B.18) shows that verification of the optimality of the fully stressed design requires computation of matrix B at the point of convergence. An approximation of this matrix can be produced through the use of finite differences. The coefficients would be calculated in the following manner,

$$b_{ji} \approx \frac{A_j^{(k+1)} - A_j^{(k)}}{\Delta A_i} \qquad \text{(B.20)}$$

This approach is presently the only one available, and it can be seen that it would be feasible only for very small problems, as up to "n" reanalyses could be required.

The state-of-the-art is presently such that the convergence and optimality questions can be addressed only from inituitive standpoints. Experience seems to indicate that for rapidly converging fully stressed designs, the outcome most likely is an optimum. For slowly converging designs, however, optimality must be investigated.

APPENDIX C

SIGNIFICANT AUTOMATED DESIGN APPLICATIONS

This appendix describes a few applications from the field of preliminary design. All examples have been analysed and designed using ATLAS. These structures are dominantly of SST-like character, with one exception. All belong to a size-range that presently renders math-programming global optimization methods impractical.

Representative size and computing requirements are shown in figure C.3. A more detailed description of the three airplanes presented here can be found in references [1]-[3].

The examples are shown because of their sizes and because there are no implications with regard to convergence. The weight for each cycle is illustrated in figures C.1 and C.2 as a representation of what can be expected.

The two SST models, the Arrow Wing and the NST, as well as the "freighter" require further study in order to establish convergence and minimum weight.

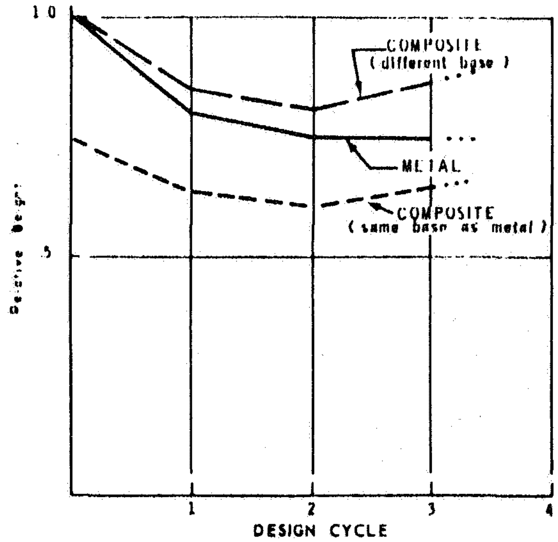

Figure C.1. Arrow Wing Structural Weight

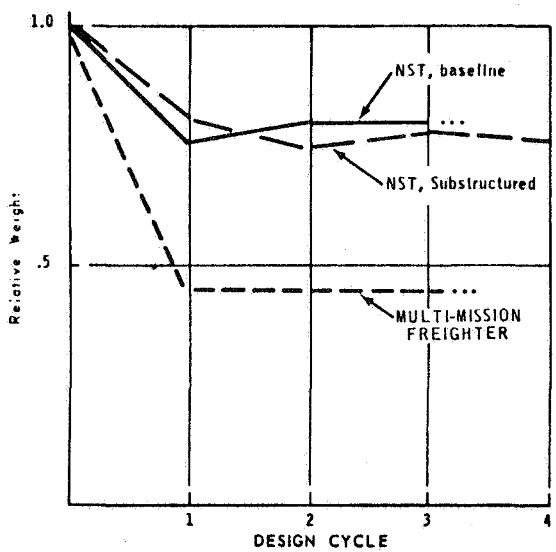

Figure C.2. NST Structural Weight (with comparison to Multi-Mission Freighter)

NO.	IDENTIFICATION NAME	SIZE NUMBER DOF	SIZE NUMBER D.V.
1	METAL; ARROW WING	9000	20000
2	NST; base-line	8000	16000
3	NST; substructure	8000	10000
4	MULTI-MISSION FREIGHTER	9000	20000
5	COMPOSITE; ARROW WING	9000	6000

ORDER OF MAGNITUDE: Degrees of Freedom (DOF)
Design Variables (D.V.)

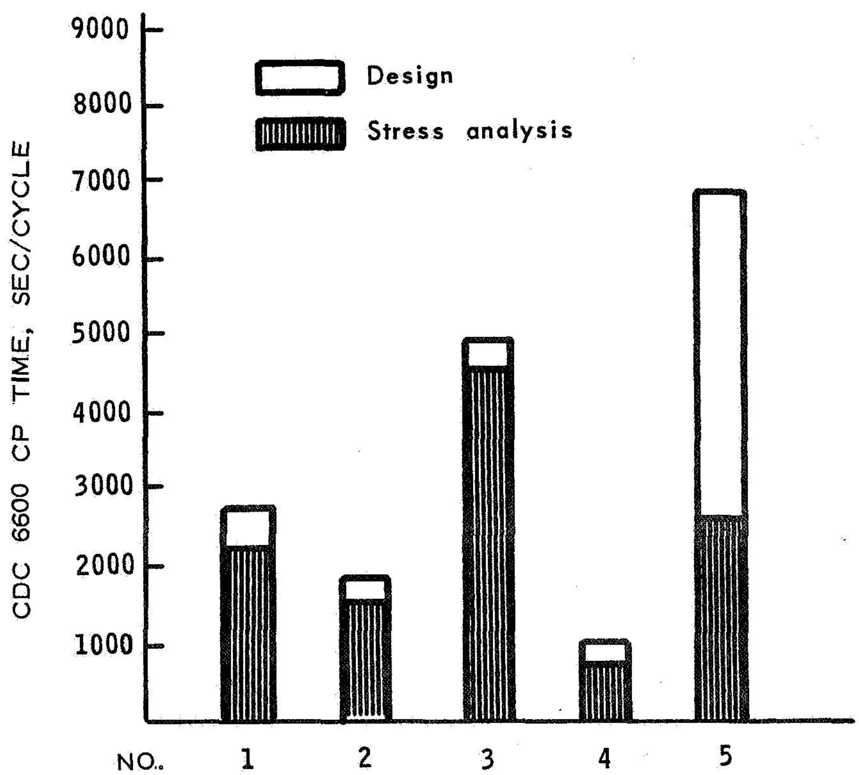

Figure C.3. Computer Time Requirements for One Cycle

APPENDIX D

COMPOSITE TERMINOLOGY

The ATLAS system has two composite elements that are identified by the names CPLATE and CCOVER.

The CPLATE is a triangular or quadrilateral laminated constant-strain membrane plate. Each plate element (laminate) is comprised of up to 10 orthotropic laminas. Each lamina is defined by,

- fiber direction (α)
- material properties
- temperature
- number of layers

The layer thickness is part of the composite material definition (see ref. [10]).

The CCOVER element is built up from two CPLATE elements separated by rigid posts.

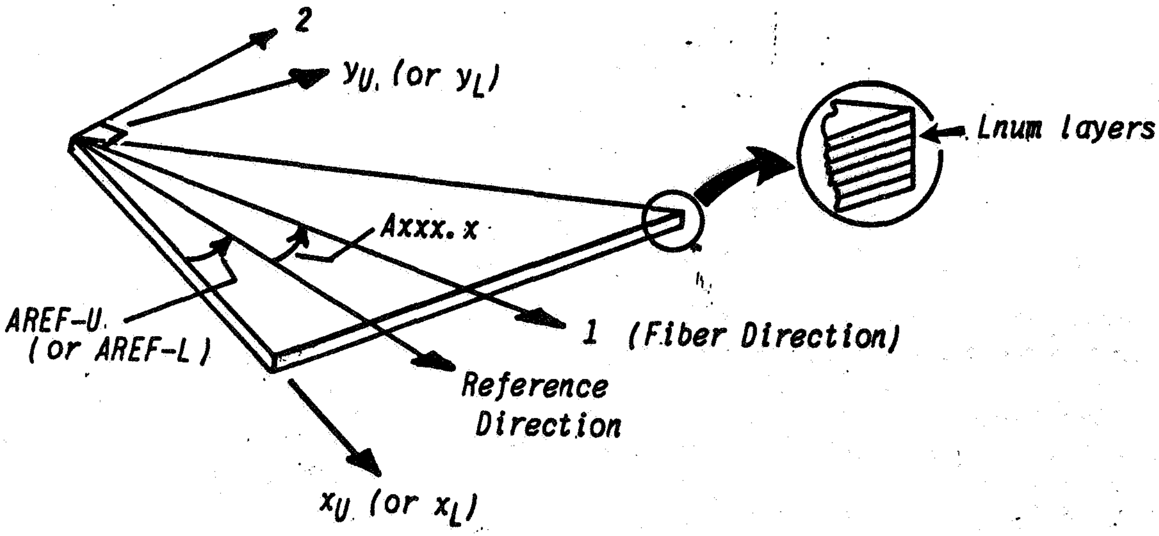

Figure D.1. Typical Lamina "i"

The orthotropic constitutive matrix for lamina "i" is,

$$D_i = \begin{bmatrix} E_{11}/(1-\nu_{12}\nu_{21}) & \nu_{12}E_{22}/(1-\nu_{12}\nu_{21}) & 0 \\ \nu_{12}E_{22}/(1-\nu_{12}\nu_{21}) & E_{22}/(1-\nu_{12}\nu_{21}) & 0 \\ 0 & 0 & G_{12} \end{bmatrix}$$

where engineering strains are used and where,

$$\nu_{21}E_{11} = \nu_{12}E_{22}$$

APPENDIX E

DESIGN THEORY SUMMARY

The ATLAS Design Module contains four different functions:

- Fully stressed design, F.S.D.;
- Thermal fully stressed design, T.F.S.D.;
- Composite optimization;
- User specified changes.

Each of these automated design methods is of reanalysis type; i.e. changes in internal loads due to changes in thicknesses and areas are considered through a "new" stress analysis. The Design Module execution is consequently a part of the design process which, in addition, involves the stress analysis modules. It is primarily intended to be a part of the design process which involves the stress analysis modules. It is primarily intended for preliminary design applications. The first three functions above are all of iterative type.

The "closeness" to minimum weight and the convergence characteristics are, of course, of great importance. It is, however, presently not possible to give general rules for the convergence behavior. The successful application of these methods will, therefore, ultimately have to depend on engineering judgement.

The Design Module input data has been structured in such a manner that the convergence behavior of the structure can be changed during the design process. The input data contain the following constraints:

- upper bounds
- lower bounds
- fixed data
- input margins of safety
- regional exclusion from resizing

All these data sets can be modified or changed through an update capability that allows for partial input data sets. This feature, in addition to the fourth function specified above, allows the user to constrain the process or to change the direction in which a "design" is developing.

The F.S.D. and the T.F.S.D. methods produce equivalent margins of safety that are based on the resize algorithm,

$$A_{new} = \frac{A_{old}}{(1+MS)}(1+MS_{inp}) \tag{E.1}$$

where MS is calculated and MS_{inp} is an input margin of safety. The design variables involved are thicknesses and areas. Additional finite element properties are changed in the same proportions as the associated design variables are modified.

The F.S.D. method considers both strength and buckling requirements. Hill's criterion is used for strength, and user specified interaction criteria are applied for buckling. These two criteria are described in sections 3.1.1 and 3.1.2. Margins of safety produced by these criteria are based on the assumption of constant internal loads. The buckling criterion, however, can contain gage dependent allowable stresses. These can be of input type, table allowables, or they can be calculated local allowables. Both cases lead to a resize requirement that involves solutions of nonlinear equations. The principle of the solution of these equations shown in section 3.1.3.

The T.F.S.D. method considers only strength requirements which are based on Hill's criterion. The method is intended for preliminary design of structures that are subjected to both mechanical and thermal loads at the same time. The method is described in section 3.2. It is based on the assumption that mechanical loads and thermal stresses remain unchanged during resizing. The thermal stresses are considered to be of the same order of magnitude as the mechanical stresses.

The composite optimization is intended for finite element models containing CPLATE and CCOVER elements. This optimization is of local type, and lamina thicknesses are design variables. Regions containing the same type of elements with the same number of laminas and user-specified fixed fiber directions can be defined in the input data. Section 4.1 contains a description of these definitions and reference [10] describes the details.

The optimization is performed for minimum weight. The method of feasible direction is used and the constraints are of strength type. These strength constraints can be based on either Hill's criterion or on the "maximum strain criterion." The <u>local</u> optimization is based on the assumption that total laminate loads remain constant. The changes in internal loads are introduced in the "next" stress analysis.

The function that changes the finite element properties according to user specifications is intended as a means of introducing design requirements other than strength and stability based element criteria. This function consists of two algorithms, one that allows factoring of element subsets (flutter changes, e.g.) and one that allows individual changes to the finite element properties. This provides a capability for model changes that does not require "new" finite element input data.

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16. Abstract

This document describes the automated design theory underlying the operation of the ATLAS DESIGN Module. The methods, applications and limitations associated with the fully stressed design, the thermal fully stressed design, and a regional optimization algorithm are presented herein. A discussion of the convergence characteristics of the fully stressed design is also included. Derivations and concepts specific to the ATLAS structural-resizing theory are shown, whereas conventional terminology and established methods are identified by references.

17. Key Words (Suggested by Author(s)) Optimization, automated design, optimality criterion, fully stressed design, equivalent margin of safety, strength, composites, math-programming, convergence, thermal stress, allowables, finite elements, buckling

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