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ON THE MINIMIZATION OF SPHERICAL ABERRATION
IN SPHERICAL REFLECTOR ANTENNAS

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1. SUMMARY

An expression for the phase error in the aperture plane of a spherical reflector has been developed. The nature of this phase error over the aperture plane is discussed, and, to illustrate the usage of the expression, examples are given.

2. INTRODUCTION

In radiometric and other similar applications which sometimes require very wide angle scanning along with a minimum beam degradation, spherical reflectors are being used increasingly. But a spherical reflector, unlike a parabolic reflector, is not a perfectly focusing device i.e., the reflected wavefront is not a plane. This inherent imperfection in spherical reflectors is called spherical aberration. The deterioration of the far field pattern due to phase error caused by spherical aberration, however, can be minimized by suitably picking the feed location. The purpose of this memorandum is to develop an expression for the phase error and to point out the principal characteristices of the phase error which are helpful in minimizing the spherical aberration. Knowledge of the fundamentals of reflector antennas is assumed.

3. PHASE ERROR

Consider the spherical reflector shown in Figure 1. C and R are the center and the radius of curvature of the spherical reflector. Let the feed be located on the reflector axis at F which is roughly halfway (slightly closer to the reflector) between the reflector and its center of curvature. Thus the focal length $f= OF \leq R/2$. The rays emanating from the feed strike the reflector and are then reflected such that for each ray the angle of incidence is equal to the angle of reflection. An imaginary plane located in front of the reflector such that all the reflected rays pass through this plane is called an aperture plane. In Figure 1, the aperture plane is perpendicular to the reflector axis and passes through the feed point F. The far field radiation pattern is computed by integrating the fields over this aperture plane. Referring to Figure 1, FP is an incident ray, ε is the angle of incidence and reflection, PC is normal at the point of incidence, and PQ is the corresponding reflected ray which intersects the aperture plane at point A. Similarly, a ray emanating from the feed along the axis of the reflector is reflected back in the same direction and intersects the aperture plane at F.

Let x be the distance along an incident ray between the feed and the reflector, and let y be the distance along the reflected ray between the reflector and the aperture plane. Then $(x+y-2f)\frac{2\pi}{\lambda}$ is the phase error at point A in the aperture plane with respect to point F in the aperture plane. The aim

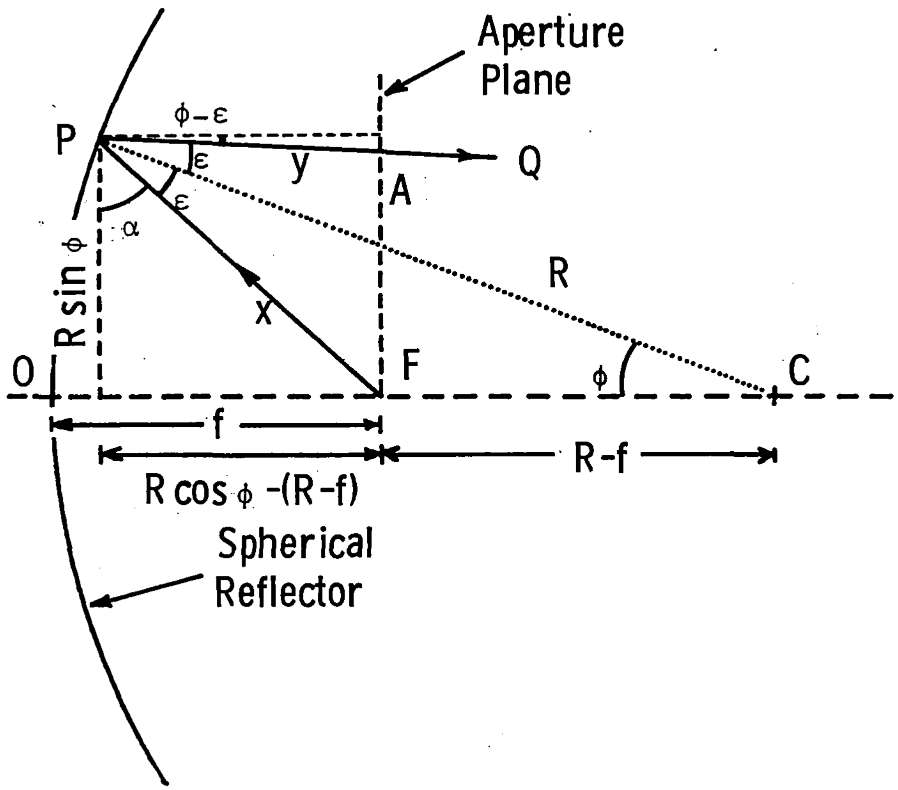

Figure 1 -- Geometry of a Spherical Reflector

here is to develop a useful expression for the path difference $(x+y-2f)$ as a function of ϕ. From Figure 1:

$$x^2 = R^2 + (R-f)^2 - 2R(R-f) \cos \phi$$

$$= (R-f)^2 \sin^2\phi + [R-(R-f) \cos \phi]^2 \quad , \tag{1}$$

$$\cos\alpha = \frac{R \sin \phi}{x} \quad , \tag{2}$$

$$\sin\alpha = \frac{R \cos \phi - (R-f)}{x} \quad , \tag{3}$$

$$\varepsilon = \frac{\pi}{2} - \phi - \alpha \text{ , and} \tag{4}$$

$$y = \frac{R \cos \phi - (R-f)}{\cos(\phi - \varepsilon)} \quad . \tag{5}$$

Now, from Eq. (4),

$$\phi - \varepsilon = 2\phi + \alpha - \frac{\pi}{2}$$

$$\therefore \quad \cos(\phi-\varepsilon) = \sin(2\phi+\alpha) = \sin 2\phi \cos\alpha + \cos 2\phi \sin \alpha ,$$

which, upon substitution from Eq. (2) and (3), becomes

$$\cos(\phi-\varepsilon) = \frac{R \cos \phi - (R-f) \cos 2 \phi}{x} \quad . \tag{6}$$

Substituting Eq. (6) into Eq. (5) gives

$$y = x \frac{R \cos\phi - (R-f)}{R \cos\phi - (R-f)\cos^2\phi + (R-f)\sin^2\phi} \tag{7}$$

From Equations (1) and (7), after some algebraic simplication, one gets

$$x+y = 2 \frac{R-(R-f)\cos\phi}{1+\tan\phi \dfrac{(R-f)\sin\phi}{R-(R-f)\cos\phi}} \sqrt{1+\left[\frac{(R-f)\sin\phi}{R-(R-f)\cos\phi}\right]^2} \tag{8}$$

Observing that

$$\tan\varepsilon = \frac{(R-f)\sin\phi}{R-(R-f)\cos\phi} \quad , \tag{9}$$

Equation (8) simplifies to

$$x+y = 2 \frac{R-(R-f)\cos\phi}{1+\tan\phi\tan\varepsilon} \sec\varepsilon \quad . \tag{10}$$

The above expression is valid for all rays leaving the feed as long as $\angle$ OFP $\leq \frac{\pi}{2}$. This limitation arises due to the location of the aperture plane. The value of ϕ corresponding to $\angle$OFP $= \frac{\pi}{2}$ is easily found from Eq. (7) by equating y = 0, i.e.

$$\cos\phi_{max} = \frac{R-f}{R} \quad . \tag{11}$$

One of the ways of studying the phase error variation over the aperture plane is to calculate and plot the values of $(x+y-2f)\frac{2\pi}{\lambda}$ for ϕ between 0 and $\cos^{-1}(\frac{R-f}{R})$ for many values of f in the vicinity of R/2. However, to get a feel for the nature of phase error variation, observe that for small ϕ, $\varepsilon \simeq \phi$. And then, using Eq. (10), the path difference can be simplified as

$$x + y - 2f = 2R \cos \phi \ (1-\cos \phi) - 2f \sin^2\phi \tag{12}$$

This is a simple expression for path difference whose properties are studied in the next section.

4. NATURE OF PHASE ERROR

a. Phase error is zero when

$$R \cos\phi\,(1-\cos\phi) = f \sin^2\phi,$$

i.e., when either $\phi=0$ or $\phi_0 = \cos^{-1}\frac{f}{R-f}$ (13)

b. Phase error is either maximum or minimum when

$$\frac{\partial(x+y-2f)}{\partial\phi} = 0. \tag{14}$$

On expanding and simplifying Eq. (14), it turns out that the phase error is zero at $\phi=0$ and is a maximum at

$$\phi_m = \cos^{-1}\frac{R}{2(R-f)} . \tag{15}$$

c. The maximum phase error for ϕ given by Eq. (15) turns out to be

$$(x+y-2f)\left.\frac{2\pi}{\lambda}\right|_{max} = \frac{\pi}{\lambda}\frac{(R-2f)^2}{(R-f)} . \tag{16}$$

Observe that for R=2f i.e., feed at exactly halfway point between the center of curvature and the reflector, the zero phase error occurs at $\phi=0$ and the magnitude of the phase error increases as ϕ increases but its sign is negative.

With the knowledge of Eqs. (13), (15), and (16), the general nature of phase error in the aperture plane for a spherical reflector can be shown by simple sketch as in Figure 2.

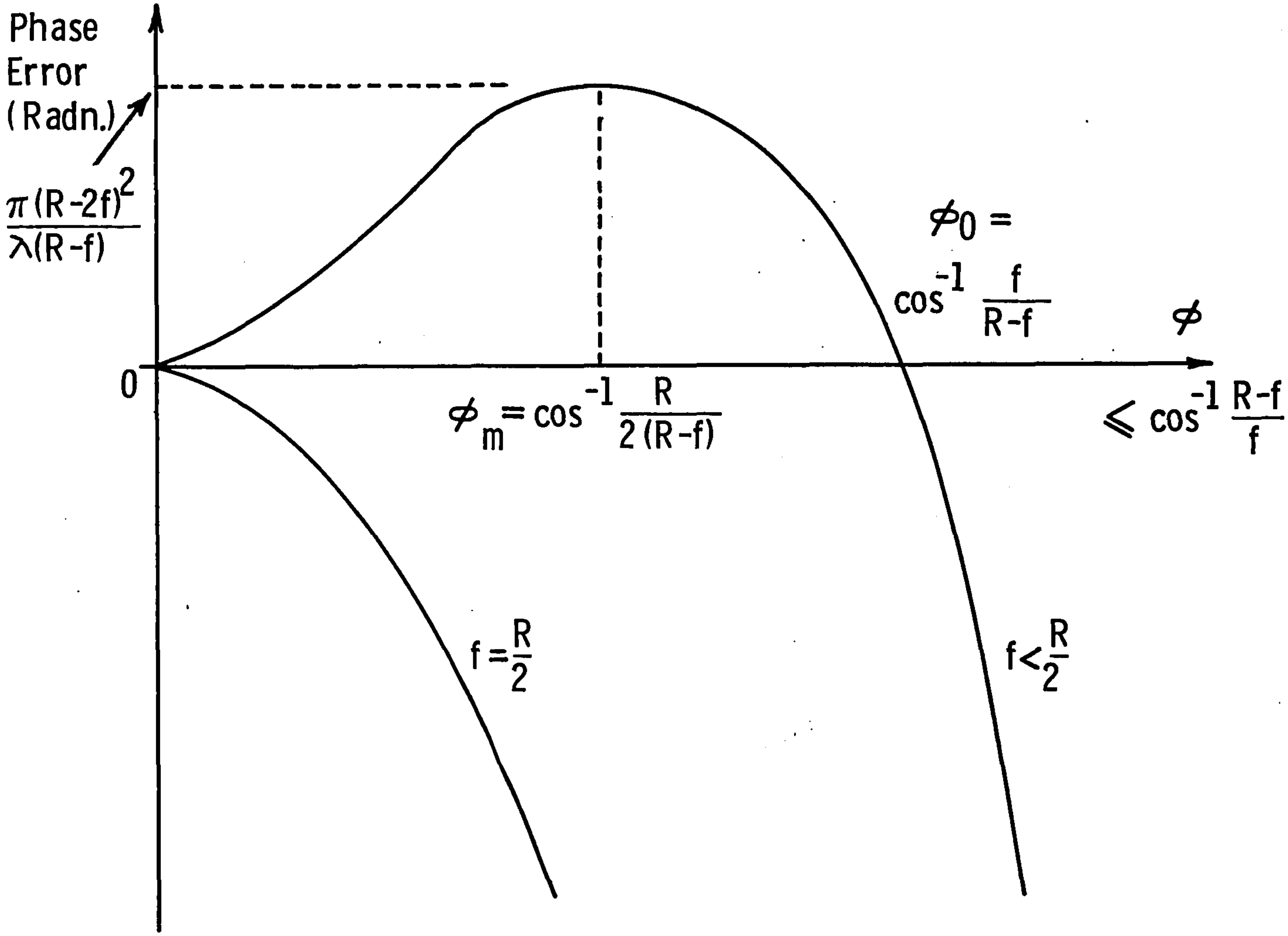

Figure 2 -- Phase Error in the Aperture Plane as a Function of Angle ϕ

Observe that as compared to the f=R/2 case, the phase error stays small for a wider range of ϕ (or over a larger aperture area) although the magnitude of maximum phase error also gets bigger. For minimum spherical aberration, the edge of the reflector should correspond to ϕ_0. The following examples illustrate this.

<u>Example 1:</u> Suppose that the spherical reflector shown in Figure 3 were to be used at 10 GHz. Then what feed position will keep the phase error over the aperture less than $\pi/9$ and always positive?

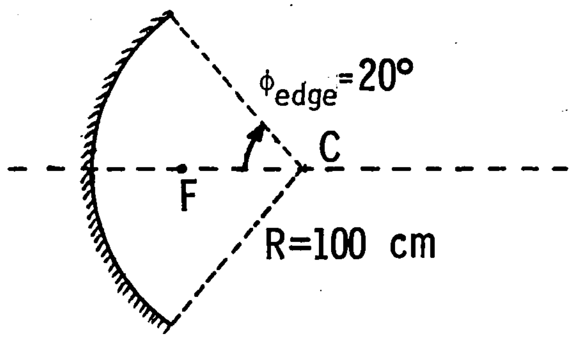

Figure 3 -- Example

<u>Solution</u>: From Eq. (16), setting R=100 cm, λ=3 cm and equating the right hand side to $\pi/9$, the focal length f turns out to be 47.92 cm. Now this will be a good feed position provided $\phi_0 \geq 20°$. And indeed from Eq. (13), ϕ_0 =23.07°. Therefore f=47.92 cm will do the job.

Example 2: In the above example since $\phi_{edge} < \phi_0$ for a maximum phase error of $\pi/9$ radians, a better feed location can be picked by making $\phi_{edge} = \phi_0$ such that the maximum phase error still remains positive and the magnitude is further reduced. $\phi_{edge}=\phi_0$ gives f=48.45 cm which means maximum phase error over the aperture, from Eq. (16), will only be 11.25°.

5. DESIGN APPLICATION

The expression for the phase error in the aperture plane of a spherical reflector developed in Section 3 can be used to pick a suitable location for the feed such that the phase error is small over the heavily illuminated part of the reflector. A lower phase error over the heavily illuminated part of the reflector (small ϕ) minimizes the effect of spherical aberration upon the far field pattern.

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16. Abstract

An expression for the phase error in the aperture plane of a spherical reflector has been developed. The nature of this phase error over the aperture plane is discussed, and, to illustrate the usage of the expression, examples are given.

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