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July 1979

Prepared for
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I. INTRODUCTION

The intent of this report is to make explicit the general applicability of performability modeling concepts and techniques. Earlier reports and publications on the subject have emphasized applications where the performance variable takes the values in a discrete (and usually finite) accomplishment set A. Although we have alluded to the fact that A may also be continuous (see [1], for example), the definitions of basic concepts such as performability and capability have heretofore been formulated for the discrete case only.

In the discussion that follows, the continuous case is treated explicitly, where we find that certain issues must be dealt with more carefully to insure that a given base model X_S and capability function Y_S comprise a legitimate performability model (X_S, Y_S).

The development of these ideas follows the format of [1] and much of the commentary there regarding motivation and justification has been retained in this report. The report thus serves as a self-contained description of the basic concepts and constructs of performability modeling. This includes discussion of a hierarchical modeling framework, whose purpose is to aid the solution process. On the other hand, solution techniques per se, are not discussed. The latter is a subject in itself (in both the continuous and discrete cases) and constitutes an important area of ongoing research. Traditionally, modeling of computer performance (see [1] - [3], for example) has stressed the need to

represent the probabilistic nature of user demands (workload) and internal state behavior, under the assumption that the computer's structure is fixed, that is, there are no permanent changes in structure due to faults. On the other hand, modeling of computer reliability (beginning with the pioneering work of Bouricius, Carter, and Schneider [4]) has stressed representation of the probabilistic nature of structural changes caused by transient and permanent faults of the computer.

In the face of these traditional modeling distinctions, we consider an important class of computing systems wherein system performance is "degradable," that is, depending on the history of the computer's structure, internal state, and environment during some specified "utilization period" T, the system can exhibit one of several worthwhile levels of performance (as viewed by the user throughout T). In this case we find that performance evaluations (of the fault-free system) will generally not suffice since structural changes, due to faults, may be the cause of degraded performance. By the same token, traditional views of reliability (probability of success, mean time to failure, etc.) no longer suffice since "success" can take on various meanings and, in particular, it need not be identified with "absence of system failure."

Modeling needs for (gracefully) degradable systems were first investigated by Borgerson and Frietas [5] in connection with their analysis of the PRIME system [6]. Although they recognized the need to formulate the probability of each possible level of performance, that is, the probability of k "crashes" during T for k = 0, 1, 2, ..., their evaluation effort dealt mainly with the question of reliability (the probability of no

crashes during T_j. Other studies employing Markov models have likewise emphasized the evaluation of reliability oriented measures (see [7] - [9], for example).

Some recent investigations, on the other hand, have dealt with measures aimed at quantifying performance as well as reliability. In particular, Beaudry [10] has introduced a number of computation related measures for degradable computing systems and has shown how to formulate these measures in terms of a transformed Markov model. In examining reconfiguration strategies for degradable systems, Troy [11] has distinguished levels of performance according to "workpower" and has formulated system effectiveness (referred to as "operational efficiency") as expected workpower. In another recent study, Losq [12] has investigated degradable systems in terms of degradable resources, where each resource is modeled by an irreducible, recurrent, finite-state Markov process.

In the discussion that follows, we describe a general modeling framework that permits the definition, formulation, and evaluation of a unified performance-reliability measure referred to as "performability." It is shown that performability relates directly to system effectiveness and is a proper generalization of both performance and reliability. A critical step in performability modeling is the introduction of the concept of a "capability function" which relates low-level system behavior to user-oriented levels of performance. A hierarchical modeling scheme is used to formulate the capability function, and capability is used, in turn, to evaluate performability.

II. SYSTEM MODELS

A computing system, as it operates in its use environment, may be viewed at several levels. At a low level, there is a detailed view of how various components of the computer's hardware and software behave throughout the utilization period. At this level, there is also a detailed view of the behavior of the computer's "environment," where by this term we mean both man-made components (user input, peripheral subsystems, etc.) and natural components (radiation, weather, etc.) which can influence the computer's effectiveness. The computer, together with its environment, will be referred to as the "total system." A second view of the total system is the user's view of how the system behaves during utilization, that is, what the system accomplishes for the user during the utilization period. A third, even less detailed view, is the economic benefit derived from using the system, that is, the computing system's worth (as measured, say in dollars) when operated in its use environment.

To formalize these views, we postulate the existence of a probability space (Ω, E, P) that underlies the total system, where Ω is the sample space, E is a set of events (measurable subsets of Ω), and $P: E \to [0,1]$ is the probability measure (see [13], for example). This probability space represents all that needs to be known about the total system in order to describe the probabilistic nature of its behavior at the various levels described above. It thus provides a hypothetical basis for defining higher level (i.e., less detailed) models. In general, however, it will neither be possible nor desirable to completely specify Ω, E and P.

In the discussion that follows, let S denote the total system, where S is comprised of a computing system C and its environment E. At the most detailed level, the behavior of S is formally viewed as a stochastic process [14], [15]

$$X_S = \{X_t | t \in T\} \tag{1}$$

where

T = a set of real numbers (observation times) called the utilization period

and, for all $t \in T$, X_t is a random variable

$$X_t : \Omega \to Q$$

defined on the underlying description space and taking values in the state space Q of the total system. Depending on the application, the utilization period T may be discrete (countable) or continuous and, in cases where one is interested in the long-run behavior, it may be unbounded (e.g., $T = \mathbb{R}_+ = [0,\infty)$). The state space Q embodies the state sets of both the computer and its environment, i.e.,

$$Q = Q_C \times Q_E$$

where Q_C and Q_E can, in turn, be decomposed to represent the local state sets of computer and environmental subsystems. For our purposes, it suffices to assume that Q is discrete and, hence, for all $t \in T$ and $q \in Q$, "$X_t = q$" has a probability (i.e., $\{\omega | X_t(\omega) = q\} \in E$). The stochastic process X_S is referred to as the base model of S. An instance of the base model's behavior for a fixed $\omega \in \Omega$ is a state trajectory $u_\omega : T \to Q$ where

$$u_\omega(t) = X_t(\omega), \quad \forall t \in T. \tag{2}$$

(The term "state trajectory" derives from modern usage in the theory of modeling [16]; synonyms in the more specific context of stochastic processes are "sample function," "sample path," and "realization" [14], [15].) Thus, corresponding to an underlying outcome $\omega\epsilon\Omega$, u_ω describes how the state of the total system changes as a function of time throughout the utilization period T. Accordingly, the "description space" for the base model is the set

$$U = \{u_\omega | \omega\epsilon\Omega\} \tag{3}$$

which is referred to as the (state) trajectory space of S.

As generally defined, the concept of a base model thus includes the type of queueing models used in computer performance evaluation [3] and the kind of Markov or semi-Markov models employed in reliability evaluation [7]-[9]. The intent of the definition, however, is the inclusion of less restricted base models which can represent simultaneous variations in the computer's structure and internal state (via the state set Q_C) and environment (via the state set Q_E). In other words, the emphasis here is on the modeling of degradable computing systems where changes in structure, internal state, and environment can all have an influence on the system's ability to perform. Accordingly, these base models may be regarded as generalized performance models, where structure is allowed to vary, or equivalently as generalized reliability models where variations in internal state and/or the computational environment are taken into account.

In formal terms, the user-oriented view of system behavior is likewise defined in terms of the underlying probability space (Ω, E, P). Here we assume that the user is interested in distinguishing a number of different "levels of accomplishment" when judging how well

the system has performed throughout the utilization period (one such level may be total system failure). The user's "description space" is thus identified with an accomplishment set A whose elements are referred to alternatively as accomplishment levels or (user-visible) performance levels. A may be finite, countably infinite, or uncountable (in the last case, A is assumed to be an interval of real numbers). Thus, for example, the accomplishment set associated with a nondegradable system is $A = \{a_0, a_1\}$ where a_0 = "system success" and a_1 = "system failure." In their modeling of the PRIME system, Borgerson and Freitas [5] viewed accomplishment as the set $A = \{a_0, a_1, a_2, \ldots\}$ where a_k = "k crashes during the utilization period T." If the user is primarily concerned with system "throughput," a continuous accomplishment set might be appropriate, i.e, $A = \mathbb{R}_+ = [0,\infty)$, where a number $a \in A$ is the "throughput averaged over the utilization period T."

In terms of the accomplishment set, system performance is formally viewed as a random variable

$$Y_S: \Omega \to A \tag{4}$$

where $Y_S(\omega)$ is the accomplishment level corresponding to outcome ω in the underlying description space. Similarly, assuming that the economic gain (or loss) derived from using the system is represented by a real number r (interpreted, say, as r dollars), system worth is a random variable defined as

$$W_S: \Omega \to \mathbb{R} \text{ (the real numbers)} \tag{5}$$

where $W_S(\omega)$ is the worth associated with outcome ω.

The terminology and notation defined above is summarized in Figure 1. Note that, at this point in the development, there are no implied relationships between these three views; their only common bond so far is that they are representations of the same system or, more formally, that they are defined on the same underlying probability space. To be useful, however, the base model X_S should support the performance variable Y_S in an appropriate manner (indeed, the term "base" is suggestive of this need) and, in turn, Y_S should support the worth variable W_S. The precise nature of these connections, as they relate to the system's effectiveness, is developed in the section that follows.

III. EFFECTIVENESS, PERFORMABILITY AND CAPABILITY

When applied to computing systems, "system effectiveness" (see [2], for example) is a measure of the extent to which the user can expect to benefit from the tasks accomplished by a computer in its use environment. More precisely, if benefit is identified with the worth W_S of the system then system effectiveness is expected worth, i.e., the expectation (expected value) of the random variable W_S; in short

$$\text{Eff}(S) = E[W_S]. \tag{6}$$

(An implicit assumption here is that W_S is defined such that $E[W_S]$ exists; see [13], for example.) Because a direct evaluation of Eff(S), using the definition of W_S, is generally not feasible (cf. our earlier remarks concerning the hypothetical nature of the underlying probability space), we wish to establish connections among the base model X_S, system performance Y_S and system worth W_S which can be used in the process of evaluating Eff(S).

To express system effectiveness in terms of system performance, the user's view of system worth must be compatible with system performance to the extent that W_S can be formulated as a function of Y_S. More precisely, we assume there exists a worth function $w: A \to \mathbb{R}$ such that, for all $\omega \in \Omega$,

$$W_S(\omega) = w(Y_S(\omega)). \tag{7}$$

If $a \in A$, $w(a)$ is interpreted as the "worth of performance level a." As for the performance variable Y_S, a natural measure that quantifies both system performance and reliability (i.e., the ability to perform) is the probability measure induced by Y_S. We refer to this unified performance-reliability measure as the "performability of S" which, in terms of our modeling framework, can be generally defined as follows:

Definition 1: If S is a total system with performance Y_S taking values in accomplishment set A, then the performability of S is the function p_S where, for each measurable set B of accomplishment levels ($B \subseteq A$),

$$p_S(B) = P(\{\omega | Y_S(\omega) \in B\}).$$

Since P is the probability measure of the underlying probability space, the interpretation of performability is straightforward, that is, for a designated set B of accomplishment levels, $p_S(B)$ is the probability that S performs at a level in B. The requirement that B be "measurable" says simply that the corresponding event $\{\omega | Y_S(\omega) \in B\}$ must lie in the underlying event space, insuring that the right-hand probability is defined.

If the performance variable Y_S is continuous then A must be continuous and, hence (by an earlier assumption), A is some interval of real numbers, including the possibility that $A = \mathbf{R} = (-\infty, \infty)$. In this

case (or if Y_S happens to be discrete and yet real-valued), we know from probability theory that the induced measure p_S is uniquely determined by the <u>probability distribution function</u> of Y_S (see [13], for example), i.e., by the function F_{Y_S} (which we write simply as F_S) where, for all $b \varepsilon A$,

$$F_S(b) = P(\{\omega | Y_S(\omega) \leq b\}). \tag{8}$$

Moreover, p_S can then be expressed as the Lebesgue-Stieltjes measure induced by F_S (cf. [13], Sec. 4.5), that is, for any (measurable) set B of accomplishment levels, the performability value of B is given by

$$p_S(B) = \int_B dF_S(b). \tag{9}$$

In particular, if B is a single interval $B = (b_0, b_1]$ where $b_0 < b_1$, then

$$p_S(B) = F_S(b_1) - F_S(b_0).$$

This special case has practical significance since it quantifies the ability of S to perform within the specified limits b_0 and b_1.

If, on the other hand, Y_S is a discrete random variable then each singleton set $B = \{a\}$ $(a \varepsilon A)$is measurable and p_S is uniquely determined by the <u>probability distribution</u> of Y_S, i.e., by the set of values

$$\{p_S(a) | a \varepsilon A\} \tag{10}$$

where $p_S(a)$ denotes $p_S(\{a\})$. Given this distribution, if B is a set of accomplishment levels then $p_S(B)$ can be written as the sum

$$p_S(B) = \sum_{b \varepsilon B} p_S(b). \tag{11}$$

Hence, the probability distribution of Y_S or, equivalently, the restriction of p_S to single accomplishment levels, suffices to determine the performability. For this reason, when Y_S is discrete the performability of S can be alternatively defined as follows:

Definition 1a: If S is a total system with performance Y_S taking values in accomplishment set A and, moreover, Y_S is a discrete random variable, then the performability of S is the function p_S where, for each accomplishment level a ($a \in A$),

$$p_S(a) = P(\{\omega | Y_S(\omega) = a\}).$$

Note that Definition 1a is essentially the restriction of Definition 1 to single accomplishment levels (which have probabilities defined when X_S is discrete). Conversely, given Definition 1a, its extension to Definition 1 can be obtained (at least conceptually) by applications of equation (11).

To justify the notion of performability in the context of system effectiveness, if we assume the existence of a worth function w (see (7)), then the real-valued random variable W_S is a function w of the random variable Y_S. Moreover, we know that w is a "measurable" function (e.g., see [13], Sec. 3.8) since, prior to (7), we assumed that W_S was a random variable. (Indeed, condition (7) is actually stronger than needed; its advantages, however, are its simplicity and the fact that it serves the purpose of the present discussion.) Hence, we are able to appeal to the well developed theory of functions of a random variable and, particularly, expectations where, again, it is convenient to distinguish two cases. If the performance variable Y_S is continuous (and thus real-valued) with probability distribution function F_S (8) then (cf. [13], Sec. 5.1).

$$E[w(Y_S)] = \int_A w(a)\, dF_S(a) \qquad (12)$$

where the integral is a Lebesgue-Stieltjes integral. In case Y_S is discrete, then (12) still applies provided the levels in A are represented by real numbers. However, independent of whether Y_S is real-valued, a simpler and more familiar formulation holds in this case where, if $p_S(a)$ is as defined in (10), then

$$E[w(Y_S)] = \sum_{a \in A} w(a)p_S(a) . \qquad (13)$$

By the definition of system effectiveness (6) and the fact that $W_S = w(Y_S)$ (7), we have

$$Eff(S) = E[W_S] = E[w(Y_S)]$$

and, accordingly, (12) and (13) are formulations of the effectiveness of S. Moreover, we see that each formula involves the worth function w and the performability p_S (although p_S does not occur explicitly in (12), recall that F_S characterizes p_S (9)). In other words, relative to the system-user interface delineated by the accomplishment set A, effectiveness evaluation may be decomposed into worth evaluation (on the user side of the interface) and performability evaluation (on the system side). Consequently, looking in toward the system, performability emerges as a key measure with regard to evaluations of system effectiveness.

To further justify the concept of performability, we note that traditional evaluations of computer performance and computer reliability are concerned with special types of performability. Performance evaluation is concerned with evaluating p_S under the assumption that the computer part of S is fixed (i.e., its structure does not change as the consequence of internal faults). Reliability evaluation is concerned with evaluating $p_S(B)$ where B is a designated subset of accomplishment levels associated with system "success." If A is finite, a performability evaluation can alternatively be regarded as $|A|$ reliability evaluations, one for each singleton success set $B = \{a\}$, and the evaluation may actually be carried out in this manner. As this process is generally more complex than a typical reliability evaluation procedure (in particular, it involves distinguishing all the performance levels as well as determining their probabilities),

we reserve the term "reliability evaluation" to mean the evaluation of "probability of success" for some specified success criterion B. Moreover, even when $|A| = 2$, we find (as discussed later in this section) that performability models represent a proper extension of models typically employed in reliability evaluation.

As a final remark regarding justification, we have found that when system performance is not degradable (as in the case, for example, with fault-tolerant architectures which employ standby sparing [4], N modular redundancy [17], or combinations thereof), it is possible to treat performance and reliability as separate issues in the process of evaluating system effectiveness. On the other hand, if performance is degradable, it can be shown (see [18]) that the more general concept of performability must typically be invoked (as in equations (12) and (13), for example) when evaluating system effectiveness.

With performability established as the object of the evaluation process, we are now in a position to specify how the base model process X_S (1) must relate to the performance variable Y_S (4) if it is to support an evaluation of the performability p_S. To precisely state this relationship, we suppose that X_S is specified by its finite-dimensional probability distributions (or by information that determines these distributions) and we let Pr denote the probability measure (defined on a σ-algebra of subsets of U) which is uniquely determined by these finite-dimensional distributions (see [14], for example). If Pr is defined for a trajectory set V ($V \subseteq U$) then, relative to the underlying measure P,

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$$\Pr(V) = P(\{\omega | u_\omega \varepsilon V\}), \tag{14}$$

i.e., Pr(V) is the probability that an observed state trajectory u_ω (see (2)) lies in the set V. In practice, however, Pr(V) will be calculated directly from the finite-dimensional distributions that determine Pr. The measure Pr thus serves to formally describe the

probabilistic nature of the base model X_S.

For X_S to support Y_S, we now impose the following restrictions. We assume first that the base model is refined enough to distinguish the levels of accomplishment perceived by the user, that is, for all $\omega,\omega' \varepsilon \Omega$,

$$Y_S(\omega) \neq Y_S(\omega') \text{ implies } u_\omega \neq u_{\omega'} \tag{15}$$

where u_ω and $u_{\omega'}$ are the state trajectories associated with outcomes ω and ω'. This implies that each trajectory $u \varepsilon U$ is related to a unique accomplishment level $a \varepsilon A$. In addition, we assume that the probabilistic nature of Y_S is determinable from that of X_S. More precisely, if B is a measurable set of accomplishment levels, i.e., the set $\{\omega | Y_S(\omega) \varepsilon B\}$ is in the domain of the underlying measure P, then we require that the corresponding trajectory set

$$U_B = \{u_\omega | Y_S(\omega) \varepsilon B\}$$

lie in the domain of the base model measure Pr; in short

If B is measurable then Pr is defined for U_B. (16)

Given that conditions (15) and (16) are satisfied, we can establish a link between X_S and Y_S which, in the context of effectiveness modeling [18], is generally referred to as the "capability" of S. Adopting this terminology, we have

Definition 2: If S is a system with trajectory space U and accomplishment set A then the capability function of S is the function $\gamma_S: U \to A$ where $\gamma_S(u)$ is the level of accomplishment resulting from state trajectory u, that is,

$\gamma_S(u) = a$ if, for some $\omega \varepsilon \Omega$, $u_\omega = u$ and $Y_S(\omega) = a$.

Condition (15) insures that the capability function γ_S is well-defined (i.e., it deserves the name "function"), for if $u_\omega = u_{\omega'}$ then

$\gamma_S(u_\omega) = \gamma_S(u_{\omega'})$. Condition (16) guarantees that the inverse γ_S^{-1} of the capability function (γ_S^{-1} is a relation between A and U but generally not a function) will carry sets that are measurable with respect to Y_S into sets that are measurable with respect to X_S. To substantiate this fact, suppose that B is a measurable set of accomplishment levels. Then the inverse image of B is the trajectory set

$$\gamma_S^{-1}(B) = \{u | \gamma_S(u) \varepsilon B\}$$

or, equivalently, by the definition of γ_S (Definition 2),

$$\gamma_S^{-1}(B) = \{u_\omega | Y_S(\omega) \varepsilon B\}$$

$$= U_B.$$

But condition (16) insures that Pr is defined for U_B and, hence, $\gamma_S^{-1}(B)$ is measurable where

$$Pr(\gamma_S^{-1}(B)) = Pr(U_B). \tag{17}$$

In effect, therefore, conditions (15) and (16) say that γ_S can be viewed as a random variable defined on the probability space (with measure Pr) induced by the base model X_S. Of more practical significance, however, is the fact that, under these conditions, X_S and γ_S suffice to determine the performability of S. To argue the latter, if B is measurable then, by (14),

$$Pr(U_B) = P(\{\omega | u_\omega \varepsilon U_B\})$$

$$= P(\{\omega | Y_S(\omega) \varepsilon B\})$$

which, by Definition 1, is just the performability of S for accomplishment levels B, i.e.,

$$Pr(U_B) = p_S(B). \tag{18}$$

Combining equations (17) and (18), we conclude that

$$p_S(B) = Pr(\gamma^{-1}(B)), \tag{19}$$

substantiating the fact that X_S and γ_S (which determine P_r and γ_S^{-1}, respectively) suffice to support an evaluation of the performability p_S.

In view of what has just been observed, if X_S and Y_S admit to the definition of a capability function γ_S (in which case we presume that conditions (15) and (16) are satisfied), the pair (X_S,γ_S) is said to constitute a performability model of S. If B is a (measurable) set of accomplishment levels, the inverse image $\gamma_S^{-1}(B)=U_B$ is referred to as the trajectory set of B, where its determination requires an analysis of how levels in B relate back down via γ_S^{-1} to trajectories of the base model. $p_S(B)$ is then determined by a probability analysis of $\gamma_S^{-1}(B)$. In case Y_S is discrete (Def. 1a), it suffices to consider inverse images of the form $\gamma_S^{-1}(a)$ where $a\in A$. Methods of implementing this process in the discrete case are discussed in Section IV.

The role of a capability function in performability evaluation is similar to that of a "structure function" [19] in reliability evlauation. However, even when performability is restricted to reliability, the concept of a capability function is more general. The special class which corresponds to the use of structure functions in "phased mission" analysis (see [20], for example) may be characterized as follows. Let S be a system where Q is the state space of the base model and A = {0,1} is the accomplishment set (here, 1 denotes "success" and 0 denotes "failure"). Then a capability function γ_S is structure-based if there exists a decomposition of T into k consecutive time periods T_1, T_2,..., T_k and there exist functions φ_1, φ_2,..., φ_k with $\varphi_i: Q \to \{0,1\}$ such that, for all $u\in U$,

$$\gamma_S(u) = 1 \text{ iff } \varphi_i(u(t)) = 1, \qquad (20)$$

for all $i \in \{1, 2, \ldots, k\}$ and for all $t \in T_i$. In the context of "phased mission" analysis, T_i is referred to as the i^{th} phase (of the mission) and φ_i is the structure function of the i^{th} phase. For each function φ_i, the inverse image $\varphi_i^{-1}(1)$ can be interpreted as the set of "success states" of the i^{th} phase and, accordingly, (20) says that S performs successfully ($\gamma_S(u) = 1$) if and only if u(t) is a success state throughout each phase. Thus, the advantage of a structure-based formulation is that each phase may be treated independently when determining the set $\gamma_S^{-1}(1)$ of all successful state trajectories.

If system success is viewed in structural terms, as is the case in most reliability studies, a structure-based capability function will usually suffice. On the other hand, when success relates to system performance we find that capability may no longer be expressible in terms of locally defined success criteria as specified by the structure functions φ_i. The following example serves to demonstrate this fact.

Let S = (C,E) where C represents a distributed computer comprised of n subsystems, and E represents the computer's workload. Suppose further that system "throughput" (i.e., the user-visible work rate of C in E) varies as a function of the number of faulty subsystems. For our purposes here, it suffices to assume that the workload E is constant and, hence, the operational states of S can be represented by the state space $Q = \{q_0, q_1, \ldots, q_n\}$ where state q_i corresponds to "i faulty subsystems." The variation in throughput is described by a function $\tau: Q \to \mathbb{R}_+$ where $\tau(i)$ = the throughput of S in state q_i.

Assuming S is used continuously throughout a utilization period $T = [0,h]$ of duration $h > 0$, the base model of S is a stochastic process $X_S = \{X_t | t\varepsilon[0,h]\}$ where each X_t is a random variable taking values in Q. (The probabilistic nature of X_S is not an issue here.) As for performance, suppose that the user is interested in the average throughput of the system, where the average is taken over the utilization period T. Suppose further that system "success" is identified with a minimum average throughput $\bar{\tau}$. Then the capability function of S is the function $\gamma_S: U \to \{0,1\}$ where

$$\gamma_S(u) = \begin{cases} 1 \text{ if } \frac{1}{h}\int_0^h \tau(u(t))dt \geq \bar{\tau} \\ 0 \text{ otherwise.} \end{cases} \qquad (21)$$

Due to the inherent memory of the integration operation, we find that γ_S does not admit to a structure-based formulation. To verify this fact with a simple 2-state example, suppose $Q = \{q_0,q_1\}$, $\tau(q_0) > \tau(q_1)$, and $\bar{\tau} = (\tau(q_0)-\tau(q_1))/2$. Then, according to (21), $\gamma_S(u) = 1$ iff the total time for which $u(t) = q_0$ is at least $h/2$. In particular, this says that more than one trajectory results in success, i.e., $|\gamma_S^{-1}(1)| > 1$. To prove that γ_S is not structure-based let us suppose to the contrary, that is, there exist phases $T_1, T_2, \ldots, T_k$ and structure functions $\varphi_1, \varphi_2, \ldots, \varphi_k$ such that (20) is satisfied. If we let R_i denote the success states of phase i, i.e., $R_i = \varphi_i^{-1}(1)$, then $R_i \neq \phi$, for all i, or otherwise no trajectory results in success. It must also be the case that $R_i \neq \{q_0,q_1\}$ for all i, for if $R_i = \{q_0,q_1\}$ (all states are success states during phase i) then the condition $\varphi_i(u(t)) = 1$, $\forall t\varepsilon T_i$ (see

(20)) is always satisfied, that is, phase i can be ignored when determining whether u spends at least half its time in state 0. This is clearly impossible if the duration of T_i is at least h/2. If the duration of T_i is less than h/2, trajectories u and v can be found such that $u(t) = v(t)$, $\forall t \in (T-T_i)$, and yet $\gamma_S(u) \neq \gamma_S(v)$ contradicting the ability to ignore phase i. The only remaining alternative is that $|R_i| = 1$, for all i, that is, each phase has exactly one success state which, in turn, implies that there is exactly one success trajectory u. This contradicts our initial observation that $|\gamma_S^{-1}(1)| > 1$ and proves that γ_S is not structure-based.

We can conclude, therefore, that even in the case of two accomplishment levels, the concept of a capability function (Def. 2) represents a proper extension of relations between state behavior and system performance that are typically assumed in the theory of reliability. Moreover, we have found that this extension permits the phases of a utilization period to be "functionally dependent" in a precisely defined sense, whereas the phases associated with a structure-based capability function must be functionally independent. The reader is referred to [21] for a more complete discussion of functional dependence and its implications.

IV. PERFORMABILITY EVALUATION

As established in the previous section (see (19)), if (X_S, γ_S) is a performability model then the performability of S for a set of accomplishment levels B may be expressed as $p_S(B) = Pr(\gamma_S^{-1}(B))$. Accordingly, one method of evaluating a particular $p_S(B)$ is to (i) determine $\gamma_S^{-1}(B)$ and then (ii) evaluate $Pr(\gamma_S^{-1}(B))$. Since the "distance" between the base model X_S and the accomplishment set A may be considerable, step (i) can be facilitated by introducing additional models between X_S and A.

In general, each intermediate model is defined in a manner similar to that of the base model. More precisely, if there are m+1 levels in the hierarchy, the <u>level-i model</u> (i = 0, 1, ..., m, where level-0 is the least detailed model at the "top" of the hierarchy) is a stochastic process

$$X^i = \{X_t^i | t \in T^i\}, \; T^i \subseteq T$$

where, for a fixed $t \in T^i$, X_t^i is a random variable taking values in a set Q^i, the <u>state space</u> of X^i. The state space Q^i is generally composed of two components, i.e.,

$$Q^i = Q_c^i \times Q_b^i$$

where Q_c^i is the <u>composite state set</u> and Q_b^i is the <u>basic state set</u> (at level-i). States in the composite part Q_c^i represent a less detailed view of the operational status of the system than do states in Q^{i+1}, such that the state behavior at level-(i+1) uniquely determines the composite state behavior at level-i (this will be made more precise in a moment). States in Q_b^i, on

the other hand, represent basic information not conveyed by states in Q^{i+1}, i.e., Q_b^i is a coordinate set of the base model state s. :e Q. In case there is no composite (alternatively, basic) part at level-i, $Q_c^i(Q_b^i)$ is simply deleted, that is, $Q^i = Q_b^i$ ($Q^i = Q_c^i$). In particular, the above definition precludes a composite state set at level-m (the "bottom" level of the hierarchy) and, hence, $Q^m = Q_b^m$.

In specifying the model hierarchy, it is convenient to view X^i as a pair of processes which determine the projections on Q_c^i and Q_b^i, respectively. (If one of Q_c^i or Q_b^i does not exist, this pair reduces to a single process.) More precisely, given Q_c^i, the <u>composite process</u> (at level-i) is the stochastic process

$$X_c^i = \{X_{c,t}^i | t \varepsilon T_c^i\},\ T_c^i \subseteq T^i$$

where the random variables $X_{c,t}^i$ take values in Q_c^i. For a fixed outcome ω in the underlying sample space Ω, a <u>composite state trajectory</u> is a function $u_{c,\omega}: T_c^i \to Q_c^i$ where $u_{c,\omega}(t) = X_{c,t}^i(\omega)$; the <u>composite trajectory space</u> is the set $U_c^i = \{u_{c,\omega} | \omega \varepsilon \Omega\}$. Similar definitions, terminology, and notation apply to the <u>basic process</u> X_b^i. To permit extension of either X_c^i or X_b^i to larger time bases, a fictitious state $\not\in$ is adjoined to each of Q_c^i and Q_b^i so that if $t \notin T_c^i$ (similar remarks apply to the basic part) then $X_{c,t}^i$ is defined to be a degenerate random variable that always assumes the value $\not\in$, i.e.,

$$X_{c,t}^i(\omega) = \not\in, \text{ for all } \omega \varepsilon \Omega. \tag{22}$$

If X_c^i and X_b^i are so extended to T^i, and we take X^i to be the process whose projections on Q_c^i and Q_b^i are X_c^i and X_b^i, respectively, then X^i is uniquely determined by X_c^i and X_b^i. (Note that the processes X_c^i and X_b^i may be statistically dependent.)

By the above observation, we can alternatively regard the level-i model as the pair of processes

$$X^i = (X^i_c, X^i_b)$$

which is a convenient view for the purpose of specifying interlevel relationships. With this identification, a state trajectory of X^i is viewed as a pair of trajectories, i.e., the <u>trajectory space</u> U^i (at level-i) is taken to be the set $U^i_c \otimes U^i_b$ where

$$U^i_c \otimes U^i_b = \{(u_{c,\omega}, u_{b,\omega}) | \omega \in \Omega\}.$$

In case there is no composite (alternatively, basic) state set at level -i, the above representations of X^i and U^i are understood to be their appropriate single component versions.

The required relationship of these models to the base model, the accomplishment set, and the capability function is prescribed by the following definition.

<u>Definition 3</u>: If S is a total system with base model X_S and capability function γ_S, the collection $\{X^0, X^1, \ldots, X^m\}$ of level-0 to level-m models is a <u>model hierarchy for S</u> if the following conditions are satisfied.

(i) $X^m = X^m_b$, that is, all variables of the bottom model are basic.

(ii) If each model X^i is extended to the utilization period T, the base model X_S is the stochastic process $X_S = \{X_t | t \in T\}$ where $X_S = (X^m_{b,t}, X^{m-1}_{b,t}, \ldots, X^0_{b,t})$. Accordingly, the state space of X_S is $Q = Q^m_b \times Q^{m-1}_b \times \ldots \times Q^0_b$ and the trajectory space U is represented by the set $U^m_b \otimes U^{m-1}_b \otimes \ldots \otimes U^0_b$.

(iii) For each level i, there exists an <u>interlevel translation</u> κ_i where

$$\kappa_0 : U_c^0 \otimes U_b^0 \to A$$

$$\kappa_i : U_c^i \otimes U_b^i \to U_c^{i-1} \quad (1<i<m)$$

$$\kappa_m : U_b^m \to U_c^{m-1}$$

such that the capability function γ_S can be decomposed as follows. If $u \in U$ where $u = (u_m, u_{m-1}, \ldots, u_0)$ with $u_i \in U_b^i$, then

$$\gamma_S(u) = \kappa_0(\kappa_1(\ldots \kappa_{m-1}(\kappa_m(u_m), u_{m-1})\ldots), u_0). \quad (23)$$

The terminology and notation of Definition 3 is summarized in Figure 2 where a) is the original model and b) is the hierarchical model.

A model hierarchy thus provides a step-by-step formulation of the capability function in terms of interlevel translations of state trajectories, beginning with a translation of the bottom model. It also permits the expression of capability relative to higher level (less detailed) views of total system behavior. More precisely, let $\overline{U}^i$ denote the level-i trajectory space, along with all the basic trajectory spaces of higher level models, i.e.,

$$\overline{U}^i = U^i \otimes U_b^{i-1} \otimes \ldots \otimes U_b^0.$$

(Note that, at the extremes, $\overline{U}^0 = U^0$ and $\overline{U}^m = U$.) Then the <u>level-i based capability function</u> is the function

$$\gamma_i : \overline{U}^i \to A$$

defined inductively as follows. If $i = 0$ and $u \in \overline{U}^0$, then

$$\gamma_0(u) = \kappa_0(u). \quad (24)$$

If $i > 0$ and $(u,u') \in \overline{U}^i$ where $u \in U^i$ and $u' \in U_b^{i-1} \otimes \ldots \otimes U_b^0$, then

$$\gamma_i(u,u') = \gamma_{i-1}(\kappa_i(u),u'). \quad (25)$$

It is easily shown that γ_i has its intended interpretation,

i.e., if u and u' correspond to a base model trajectory v then $\gamma_i(u,u') = \gamma_S(v)$. In particular, if i = m then $\gamma_m = \gamma_S$.

The practical significance of the model hierarchy, however, is the ability to formulate the inverse of γ_S via the inverses of the γ_i, thereby providing a step-by-step, top-down method of elaborating a set of accomplishment levels B. Beginning with level-0-based capability, by (24) we have

$$\gamma_0^{-1}(B) = \kappa_0^{-1}(B). \tag{26}$$

Assuming that $\gamma_{i-1}^{-1}(B)$ has been determined, by (25) it follows that

$$\gamma_i^{-1}(B) = \bigcup_{(u,u')\in\gamma_{i-1}^{-1}(B)} (\kappa_i^{-1}(u),u'), \tag{27}$$

where $(\kappa_i^{-1}(u),u') = \{(v,u') | \kappa_i(v) = u\}$. This process is iterated until i = m, yielding $\gamma_m^{-1}(B) = \gamma_S^{-1}(B)$.

V. CONCLUSION

In conclusion, we see that performability modeling with continuous accomplishment sets can proceed in much the same manner as modeling with discrete accomplishment sets. The essential differences are that measurability conditions must be observed more closely in the continuous case when defining performability (Definition 1) and capability (see condition 1b)). Also methods of actually specifying γ_S or, in case a hierarchy is used, the interlevel translations κ_i, differ in the two cases: in the continuous case, values of these functions must be specified by formulas for computing their values; in the discrete case there are situations (e.g., when A is finite) where γ_S or the κ_i can be tabulated completely.

Finally, regarding solutions (although our subject here is not solution techniques, per se), if A is continuous it may often be the case that a complete knowledge of performability, i.e., the ability to compute $p_S(B)$ for <u>every</u> measurable set B, is not required. For example, intervals of the form

(i) $B = (b_o, b_1]$, $(b_o < b_1)$

or

(ii) $B = (B_o, \infty)$

may suffice where, in case (i), $p_S(B)$ is the probability that S performs within specified limits b_o and b_1 and, in case (ii) $p_S(B)$ is the probability that the performance of S is greater than a specified minimum b_o. In other cases, even less information may satisfy the evaluation needs of the user, e.g., the mean $E[Y_S]$ and variance var $[Y_S]$ of the performance variable Y_S. Although these are not performability measures in the strict sense of Definition 1, they are "performability-based" in the sense that the performability p_S uniquely determines $E[Y_S]$ and var $[Y_S]$. Moreover, the latter should be easier to evaluate since generally, as experienced in the performance evaluation of fault-free systems [2], [3], full knowledge of p_S is not required in the process of determining $E[Y_S]$ and var $[Y_S]$.

Model	Description Space
Base model X_S	Trajectory space U
System performance Y_S	Accomplishment set A
System worth W_S	The real numbers $\mathbb{R}$

Figure 1

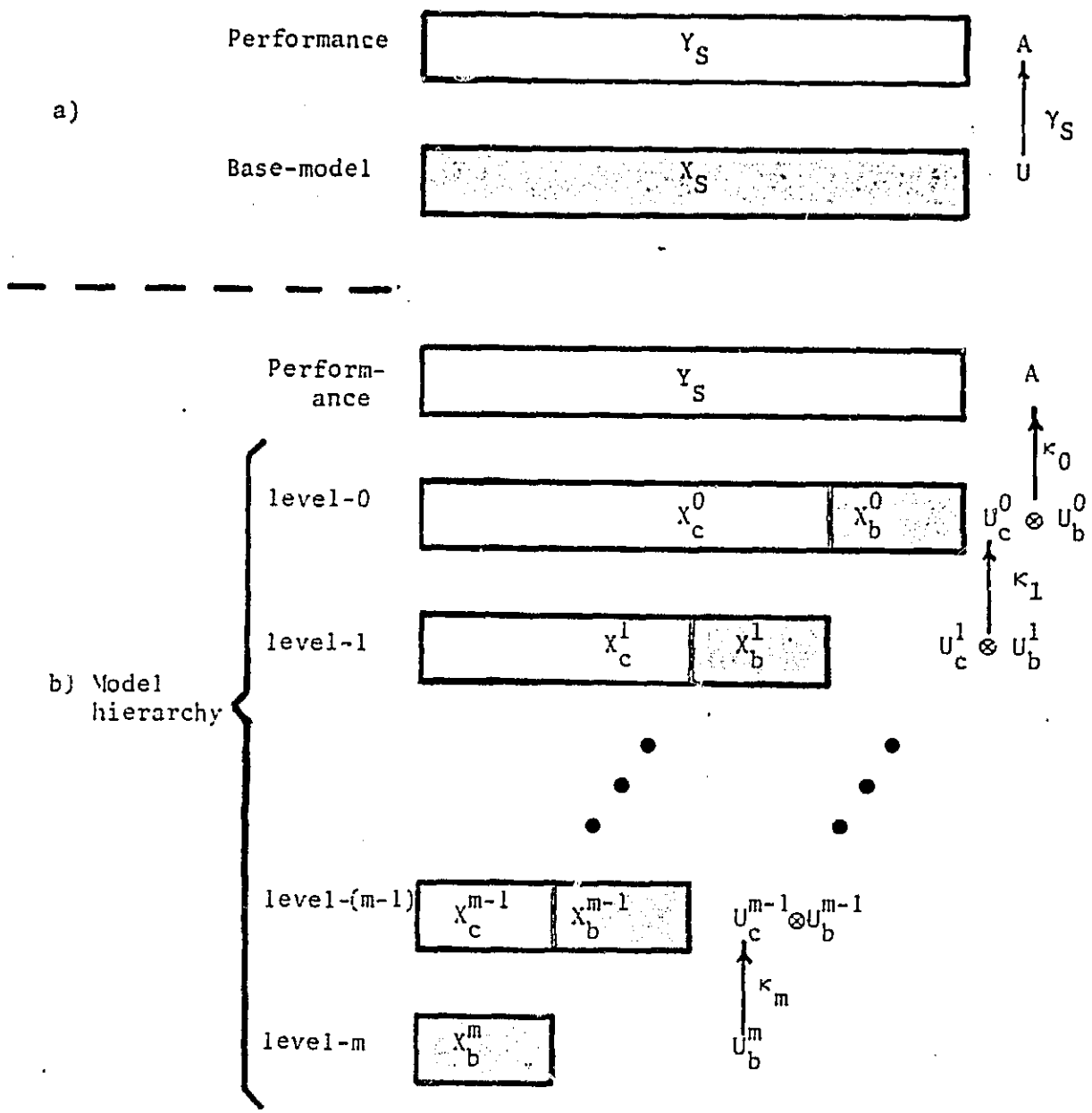

Performance
Y_S
A
Y_S
U
a)
Base-model
X_S
Perform-
ance
Y_S
A
κ_0
level-0
X_c^0
X_b^0
$U_c^0 \otimes U_b^0$
κ_1
level-1
X_c^1
X_b^1
$U_c^1 \otimes U_b^1$
b) Model
hierarchy
level-(m-1)
X_c^{m-1}
X_b^{m-1}
$U_c^{m-1} \otimes U_b^{m-1}$
κ_m
level-m
X_b^m
U_b^m

Figure 2

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