# A Computer Program <br> To Generate Equations of Motion Matrices, L217 (EOM) 

Volume I: Engineering and Usage

R. I. Kroll and R. E. Clemmons

## NASA Contractor Report 2851

# A Computer Program To Generate Equations of Motion Matrices, L217 (EOM) 

 Volume I: Engineering and UsageR. I. Kroll and R. E. Clemmons<br>Boeing Commercial Airplane Company<br>Seattle, Washington

Prepared for
Langley Research Center
under Contract NAS1-13918

## N/SN

National Aeronautics
and Space Administration
Scientific and Technical
Information Branch

## CONTENTS

Page
1.0 SUMMARY ..... 1
2.0 INTRODUCTION ..... 3
3.0 SYMBOLS AND ABBREVIATIONS ..... 4
4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION ..... 11
4.1 Formulation of the Equations of Motion ..... 11
4.2 Axis Systems and Sign Conventions ..... 12
4.3 Structural Matrices ..... 15
4.4 Inertial Coupling ..... 16
4.5 Aerodynamic Options ..... 17
4.6 FLEXSTAB AIC Option ..... 19
4.6.1 Incidence and Normalwash Calculations ..... 19
4.6.2 Aerodynamic Forces ..... 25
4.6.3 Generalized Aerodynamic Force Coefficients ..... 31
4.7 Doublet Lattice AIC Option ..... 33
4.7.1 Slender Body Aerodynamic Forces ..... 34
4.7.2 Induced Normalwash ..... 36
4.7.3 Thin and Interference Body Aerodynamic Forces ..... 37
4.7.4 Generalized Aerodynamic Force Coefficients ..... 39
4.7.5 Quasi-Steady Formulation ..... 41
4.8 Doublet Lattice Generalized Force and Pressure Option ..... 42
4.9 Coordinate Transformation and Aerodynamic Axis Shifting ..... 45
5.0 PROGRAM STRUCTURE AND DESCRIPTION ..... 48
5.1 Program L217vc ..... 48
5.2 Program Struct ..... 48
5.3 Aerodynamic Data Processing ..... 50
5.3.1 Program FLXAIC ..... 50
5.3.2 Program DLAIC ..... 51
5.3.3 Program DLPRES ..... 51
6.0 COMPUTER PROGRAM USAGE ..... 53
6.1 Control Cards ..... 53
6.2 Resource Estimates ..... 55
6.3 Card Input Data ..... 59
6.3.1 Program L217 ..... 61
6.3.2 Structural Card Input Data ..... 62
6.3.3 Aerodynamic Card Input Data ..... 71
6.3.4 Summary of Card Input Data ..... 91

## CONTENTS (Concluded)

Page
6.4 Magnetic File Input Data ..... 95
6.4.1 "GSTIFF"-Generalized Stiffness Matrix ..... 95
6.4.2 "GMASS"-Generalized Mass Matrix ..... 95
6.4.3 "SATAP"-Modal Data File ..... 97
6.4.4 "NGETP"-Doublet Lattice Geometry File ..... 98
6.4.5 "NAETP"-Doublet Lattice Aerodynamic File ..... 102
6.4.6 "GDTAPE"-FLEXSTAB Geometry File ..... 105
6.4.7 "SDINDX/SDDATA"-FLEXSTAB Aerodynamic File ..... 105
6.5 Output Data ..... 107
6.5.1 Printed Output ..... 107
6.5.2 Magnetic File Output ..... 109
6.6 Restrictions ..... 112
6.7 Diagnostics ..... 112
6.7.1 READTP Error Codes ..... 113
6.7.2 WRTETP Error Codes ..... 114
7.0 SAMPLE PROBLEM ..... 115
REFERENCES ..... 157

## FIGURES

No. Page
1 Typical Axis System Orientations ..... 13
2 Rigid Body Displacement Sign Convention ..... 14
3 Local Surface Displacement Sign Convention ..... 14
4 Surface Pressures ..... 15
5 Ordering of Response Normalwash Matrices for the FLEXSTAB AIC Option ..... 21
6. Lateral and Vertical Gust Sign Conventions ..... 22
7 Gust Rotation Matrix for FLEXSTAB ..... 24
8 Ordering Within the FLEXSTAB Pressure Coefficient Matrix ..... 26
9 Typical Slender Body ..... 27
10 Circumferential Pressure Distribution on a Slender Body Segment Due to a Y and Z Doublet ..... 27
11 Interference Body Panel Orientation Angle ..... 28
12 Slender Body-Interference Body Alignment ..... 29
13 Aerodynamic Axis Shifting ..... 47
14 L217 (EOM) Overlay Structure ..... 49
15 L217 (EOM) Communication Via External Files ..... 52
16 L217 (EOM) Card Input Data Flow ..... 60
17 L217 (EOM) Structural Card Input Data, Card Set 2.0 ..... 63
18 L217 (EOM) Aerodynamic Card Input Data, Card Set 3.0 ..... 72
19 Contents of GSTIFF ..... 96
20 Contents of GMASS ..... 96
21 Contents of a File on SATAP ..... 97
22 Contents of a File on NGETP ..... 99
23 Contents of a File on NAETP ..... 103
24 Contents of EOMTAP ..... 110
25 Contents of EOMLOD ..... 111
26 Sample Problem Configuration ..... 116

### 1.0 SUMMARY

The program L217 (EOM) is designed to formulate a set of second order linear differential equations that describe the motion of an airplane relative to its level equilibrium flight condition. The equations are formulated using the Lagrange energy approach and under the assumptions that:

- The airplane is in straight and level flight.
- All motions are small.

The option does exist to replace the structural damping with its equivalent viscous damping representation. Inertial coupling effects can also be included in the generalized inertia matrix to account for any control surface freedoms that may have been added to the basic set of structural modes.

Aerodynamic data may come from either the FLEXSTAB computer program system or the Doublet Lattice L216 aerodynamic program. The FLEXSTAB aerodynamic data can only be used with the quasi-steady aerodynamic formulation. The Doublet Lattice aerodynamic data can be used with either the quasi-steady or the full unsteady formulation.

Output from L217 (EOM) consists of:

- Equations of motion coefficients for either the quasi-steady or the full unsteady formulation.
- Aerodynamic forces for each aircraft component, such as wing, fuselage, vertical tail, etc.


## Significant program restrictions are:

- Total number of degrees of freedom cannot be greater than 100.
- Total number of aerodynamic elements must not exceed 400.
- Maximum number of interpolation points associated with any one aerodynamic surface is 100 .
- Maximum number of gust zones (regions for calculating gradual penetration effects) is 35.
- Maximum number of reduced frequencies is 20.


### 2.0 INTRODUCTION

The Equations of Motion program (L217) was developed for use as either a standalone program or as a module of the program system called DYLOFLEX (ref. 1). L217 (EOM) was designed to meet the DYLOFLEX contract requirements as defined in reference 2. These requirements specify the need for a computer program that can assemble and generate the data needed to formulate the equations of motion for a flight vehicle and to provide the necessary aerodynamic forces that can be used in the flight loads analysis of that vehicle.

The objective of this volume is to assist individuals wishing to use this program. To meet this objective, the following items are discussed:

- Engineering and mathematical equations used to formulate the problem.
- Program structure and design.
- Guidelines for the execution of the program.

A sample problem is also presented to aid the user in the execution of the program.

### 3.0 SYMBOLS AND ABBREVIATIONS

The following list contains items that appear throughout the text except for section 6.3 (card input).

Engineering notation
$[\mathrm{A}],[\mathrm{B}],[\mathrm{C}]$
$b_{T}$
$\left\{\Delta \mathrm{C}_{\mathrm{p}}\right\}$
$\left\{\Delta \mathbf{C}_{\mathbf{p}_{\mathbf{R}}}\right\},\left\{\Delta \mathrm{C}_{\mathbf{p}_{\mathbf{G}}}\right\}$
$\left\{\mathrm{C}_{\mathbf{p}_{\mathbf{R}}}\right\},\left\{\mathrm{C}_{\mathbf{p}_{G}}\right\}$
$\left\{\mathrm{C}_{\mathbf{p}}{ }^{\mathrm{Y}}\right\}_{\mathrm{SB}},\left\{\mathrm{C}_{\mathbf{p}}{ }^{\mathrm{Z}}\right\}_{\mathrm{SB}}$
$\left\{\mathrm{C}_{\mathbf{p}_{\mathbf{u}}}\right\}_{\text {IB }}$
$\left\{\mathrm{C}_{2}\right\},\left\{\mathrm{C}_{3}\right\}$
$\left[\mathrm{CP}_{\mathrm{R}}\right],\left[\mathrm{CP}_{\mathrm{R}}\right]$
$\left[\mathrm{CP}_{\mathrm{G}}\right.$ ]
[CPR]
[CPG]
[D]

Definition

Partitions of the augmented generalized inertia matrix.
Diagonal matrix of element areas.
Aerodynamic influence coefficient matrix.
Slender body projected area.
Reference semichord.
Lifting pressure coefficients.
Lifting pressure coefficients due to the response motion of the airplane and gust encounter, respectively.

Slender body segment pressure coefficients due to response motion and gust encounter.

The magnitude of the slender body segment's pressure coefficient (at the segment evaluation point) due to $Y$ and $Z$ doublets, respectively.

Upper surface pressure coefficients on interference body elements.

Generalized forcing function coefficients.
Lifting pressure coefficients for generalized coordinate displacements and rates.

Lifting pressure coefficients for gust zone velocities.
Lifting pressure coefficients for generalized coordinate responses.

Complex lifting pressure coefficients for gust zone velocities.
Doublet Lattice matrix of normalwash factors for thin and interference body boxes.

| $\mathrm{d} \delta_{\mathbf{z}} / \mathrm{dx}, \mathrm{d} \delta_{\mathbf{y}} / \mathrm{dx}$ | Derivative of surface local $z$ and $y$ displacement in the local x direction. |
| :---: | :---: |
| [F] | Doublet Lattice matrix of normalwash factors for slender body on lifting surface boxes. |
| f | Cyclic frequency (cps). |
| $\left\{\mathrm{F}_{\mathbf{A}}\right\}$ | Aerodynamic forces. |
| [FÅC] | Diagonal factor matrix that doubles the aerodynamic forces for slender bodies located on the plane of symmetry. |
| $\left\{\mathrm{f}_{\ell}\right\}$ | Distances from the gust reference point to the gust zone control points. |
| [FG] | Matrix of aerodynamic gust force coefficients for each gust zone. |
| $\mathrm{F}_{\text {IIB }}$ | Interference element aerodynamic force normal to the element surface. |
| [FR] | Matrix of aerodynamic force coefficients for generalized coordinate response. |
| $\left\{\mathrm{F}^{\mathrm{y}}\right\},\left\{\mathrm{F}^{\mathbf{z}}\right\}$ | Aerodynamic forces in the local y and z direction. |
| $\left[F 1_{\mathbf{R}}\right],\left[\mathrm{F} 2_{\mathbf{R}}\right]$ | Aerodynamic force coefficients for generalized coordinate displacements and rates. |
| [ $\mathrm{F}_{12}$ ] | Complex matrix of aerodynamic force coefficients for generalized coordinate responses. |
| \{ $\left.\mathrm{F}^{\prime} 12\right\}$ | Complex matrix of aerodynamic force coefficients due to gust. |
| [GC] | Gust correlation matrix. |
| $\left[\mathrm{g}_{\mathrm{SD}}^{\circ}\right]$ | Structural damping coefficients. |
| i | Imaginary number $=\sqrt{-1}$ |
| k | Reduced frequency $=\boldsymbol{\omega} \mathbf{b}_{\mathbf{r}} / \mathbf{V}_{\mathbf{T}}$ (radians). |
| $\mathrm{L}_{\mathrm{s}}$ | Slender body segment length. |
| $\left[\mathrm{m}_{\mathrm{CS}}^{\circ}\right]$ | Inertia matrix for control surfaces. |


| [ $\mathrm{M}_{1}$ ] | Generalized stiffness matrix. |
| :---: | :---: |
| [ $\mathrm{M}_{2}$ ] | Generalized damping matrix. |
| [ $\mathrm{M}_{3}$ ] | Generalized inertia matrix. |
| [ $\mathrm{M}_{4}$ ] | Generalized aerodynamic stiffness matrix. |
| [ $\mathrm{M}_{5}$ ] | Generalized aerodynamic damping matrix. |
| 合 | Surface unit normal. |
| NDOF | Total number of degrees of freedom. |
| NDOFCS | Number of control surface degrees of freedom. |
| $\mathrm{NDOF}_{\text {ST }}$ | Number of structural degrees of freedom. |
| NSB | Number of slender bodies. |
| N'SB | Number of thin bodies. |
| $\Delta \mathrm{n}$ | Virtual displacement in the direction of the aerodynamic force. |
| [PC] | Interference body/slender body element correlation matrix. |
| $\mathrm{P}_{\mathbf{\ell}}, \mathrm{P}_{\mathbf{u}}$ | Lower and upper surface pressure. |
| $\Delta \mathrm{P}$ | Pressure difference: $\mathrm{P}_{\boldsymbol{l}}-\mathrm{P}_{\mathbf{u}}$. |
| q | Generalized coordinate. |
| $\overline{\mathbf{q}}$ | Dynamic pressure $=1 / 2 \rho \mathrm{~V}_{\mathrm{T}}{ }^{2}$. |
| \{Q\} | Generalized aerodynamic forces. |
| [QR] | Matrix of generalized aerodynamic force coefficients fer generalized coordinate responses. |
| $\left\{\mathbf{Q}^{\prime}\right\}$ | Complex matrix of generalized aerodynamic gust force coefficients. |
| $\Delta q$ | Virtual displacement of a generalized coordinate. |
| [R] | Euler rotational transformation matrix. |
| $\left[\stackrel{0}{0}_{\mathbf{r}}\right.$ ] | Matrix of slender body segment radii. |


| [ $\mathrm{r}_{\mathbf{s}}{ }^{\prime}$ ] | Matrix of slender body segment radius slopes $=\left[\mathrm{dr}_{\mathbf{s}} / \mathrm{dx}\right]$. |
| :---: | :---: |
| $[\mathrm{RG}]_{\mathrm{S} / \mathrm{A}}$ | Gust rotation matrix for a symmetrical or antisymmetrical analysis. |
| [ $\stackrel{\circ}{\mathbf{S} F}$ ] | Matrix of symmetry factors. |
| T | Kinetic energy. |
| U | Strain energy. |
| $\mathbf{V}_{\mathbf{T}}$ | True velocity. |
| ${ }^{\mathbf{V}} \mathbf{G}$ | Gust velocity. |
| $\overline{\mathbf{v}}_{\mathbf{G} / \mathbf{L}}$ | Magnitude of a sinusodial varying gust (vertical or lateral). |
| \{w\} | Normalwash, component of the flow velocity normal to the surface. |
| $\left\{\mathbf{w}_{\mathbf{A}}\right\}$ | Augmented normalwash. |
| $\left\{-w_{B}\right\}$ | Normalwash induced on lifting and interference surfaces by the slender bodies. |
| $\Delta \mathrm{W}$ | Virtual work. |
| X, Y, Z | Rigid body linear displacements. |
| $\mathbf{X}_{\mathbf{C}}, \mathbf{Y}_{\mathbf{C}}, \mathbf{Z}_{\mathbf{C}}$ | Axis correction values (reference axis system). |
| $\mathrm{x}_{l}, \mathrm{y}_{l}, \mathrm{z}_{l}$ | Local axis system coordinates. |
| $\mathrm{X}_{\mathrm{I}}, \mathrm{Y}_{\mathrm{I}}, \mathrm{Z}_{\mathrm{I}}$ | Inertial axis system coordinates. |
| XG | Gust reference point defined in the reference axis system. |
| \{ $\mathrm{XG}_{\mathbf{C P}}$ \} | Gust control points defined in the reference axis system. |
| $\mathrm{X}_{\mathbf{O A}}, \mathrm{Y}_{\mathbf{O A}}, \mathrm{Z}_{\mathbf{O A}}$ | Coordinates of the aerodynamic axis system origin. |
| $\mathbf{X}_{\text {RAS }}, \mathrm{Y}_{\text {RAS }}, \mathrm{Z}_{\text {RAS }}$ | Reference axis system coordinates. |
| $\mathbf{X}_{\mathbf{S H}}, \mathrm{Y}_{\mathbf{S H}}, \mathrm{Z}_{\mathbf{S H}}$ | Aerodynamic axis shift values. |
| $\alpha_{G}$ | Gust angle of attach $=\mathrm{v}_{\mathrm{G}} / \mathrm{V}_{\mathrm{T}}$. |


| $\beta$ | Circumferential coordinate for slender body pressure distribution. |
| :---: | :---: |
| $\gamma$ | Element dehedral angle (Doublet Lattice). |
| $\phi, \theta, \psi$ | Rigid body rotational displacements. |
| $\theta_{\text {IB }}$ | Interference body dihedral angle (FLEXSTAB). |
| $\theta_{\text {TB }}$ | Thin body dihedral angle (FLEXSTAB). |
| $\theta_{\mathbf{X}}, \theta_{\mathbf{y}}, \theta_{\mathbf{Z}}$ | Rotational displacements in the local axis system. |
| $\rho$ | Density of air. |
| $\Phi(\mathrm{t})$ | Wagner function. |
| $\psi(t)$ | Kussner function. |
| $\omega$ | Angular frequency ( $\mathrm{rad} / \mathrm{sec}$ ) . |
| $\left[\Sigma_{y}\right],\left[\Sigma_{z}\right]$ | Interference element summation matrices for the $y$ and $z$ slender body directions. |
| $\delta_{S / A}$ | Gust delta function: <br> $=1$ for symmetric analysis. <br> $=-1$ for antisymmetric analysis. |
| $\delta_{\mathbf{X}}, \delta_{\mathbf{y}}, \boldsymbol{\delta}_{\mathbf{z}}$ | Linear displacements in the local axis system. |
| [ $\delta_{\mathrm{W}}^{\circ}$ ] | Sign function relating normalwash to flow incidence. |
| [ $\overline{\text { ] }}$ ] | Gradual penetration forcing function. |
| [ $\phi_{\delta_{\mathbf{n}}}$ ] | Mode shapes for displacement normal to the surface. |
| $\left[\phi_{\delta_{n}}^{\prime}\right]$ | Mode shape for surface streamwise slopes $=\left[\mathrm{d} \phi_{\delta_{n}} / \mathrm{dx}\right]$. |
| [1] | Unity matrix. |
| $\{\Psi\}$ | Flow incidence matrix. |
| Subscripts | Definitions |
| Aug | Augmented. |
| CP | Control point. |


| CS | Control surface. |
| :---: | :---: |
| E | Excitation. |
| FP | Force point. |
| G | Gust. |
| I | Inertial axis system. |
| IB | Interference body. |
| Im | Imaginary part. |
| L | Lateral gust. |
| $l$ | Local axis system. |
| $l_{\mathrm{s}}$ | Local structural axis system. |
| n | Normal to the surface. |
| 0 | Off the plane of symmetry. |
| p | Pitch. |
| R | Response quantity. |
| RAS | Reference axis system. |
| Re | Real part. |
| S | On the plane of symmetry. |
| S/A | Symmetric/antisymmetric. |
| SB | Slender body. |
| ST | Side translation. |
| TB | Thin body. |
| u | Upper surface. |
| Ua | Unaugmented. |
| V | Vertical gust. |

## Superscripts

Local y direction.
z
1

Matrix
Symbols

Diagonal matrix.
[1]
Rectangular matrix.
Transpose matrix.
Inverse matrix.

Column matrix.
Row matrix.

### 4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION

### 4.1 FORMULATION OF THE EQUATIONS OF MOTION

The equations describing the motion of an airplane relative to its level equilibrium flight condition are formulated using the Lagrange energy equation (ref. 3).

$$
\begin{equation*}
\frac{d}{d t}\left(\frac{\partial T}{\partial \dot{q}_{i}}\right)-\frac{\partial T}{\partial q_{i}}+\frac{\partial U}{\partial q_{i}}=q_{i} i=1, \ldots n \tag{1}
\end{equation*}
$$

where:
$\mathrm{T}=$ Kinetic energy of the system
$\mathbf{U}=$ System's internal strain energy
$\mathbf{Q}_{\mathbf{i}}=$ Set of generalized forces
$q_{i}=$ Set of generalized coordinates
The airplane's motions are specified by the array of generalized coordinates $q_{i}, i=1,2$, ..., $n$. The kinetic and strain energies are calculated in terms of the airplane's mass and stiffness properties and the generalized coordinate velocities and displacements. The generalized forces consist of forces arising from damping effects and aerodynamic forces resulting from airplane motions and atmospheric turbulance. Placing the restrictions on the analysis that:

- The airplane must initially be in straight and level flight
- All motions are small
equation (1) then leads to a set of $n$ second order linear differential equations. These equations can be either one of two forms, depending upon the type of representation used to describe the aerodynamics of the airplane.

The first form of the equations of motion comes from the use of the quasi-steady assumption in the formulation of the aerodynamic forces. Under this assumption, the aerodynamic forces are calculated by using the steady state aerodynamic characteristics with the instantaneous angle of incidence (ref. 4). Unsteady aerodynamic effects are approximated by convoluting the instantaneous incidence angle with Küssner and Wagner indicial lift growth functions. The resulting set of linear differential equations written in matrix form is:

$$
\begin{gather*}
\left(\left[\mathbf{M}_{1}\right]\{\mathbf{q}\}+\left[\mathbf{M}_{\mathbf{2}}\right]\{\dot{\mathbf{q}}\}+\left[\mathbf{M}_{\mathbf{3}}\right]\{\ddot{\mathbf{q}}\}\right)+\left(\left[\mathbf{M}_{\mathbf{4}}\right]\{\dot{\mathbf{q}}\}^{*} \boldsymbol{\Phi}(\mathbf{t})+\left[\mathbf{M}_{\mathbf{5}}\right]\{\ddot{\mathrm{q}}\}^{*} \Phi(\mathrm{t})\right)  \tag{2}\\
=\left\{\mathbf{C}_{\mathbf{3}}\right\} \dot{\boldsymbol{\alpha}}_{G^{*}}{ }^{*} \psi(\mathbf{t})
\end{gather*}
$$

where:

| $\left[\mathrm{M}_{1}\right],\left[\mathrm{M}_{2}\right],\left[\mathrm{M}_{3}\right]$ | $=$Generalized structural stiffness, damping, and inertia <br> matrices, respectively |
| :--- | :--- |
| $\left[\mathrm{M}_{4}\right],\left[\mathrm{M}_{5}\right]$ | $=$ Generalized aerodynamic stiffness and damping, respectively |
| $\left\{\mathrm{C}_{3}\right\}$ | $=$ |
| $\Phi(\mathrm{t}), \psi(\mathrm{t})$ | $=$Generalized forcing function coefficients |
| respectively |  |

All matrices are constant coefficient matrices; that is, they are not functions of frequency.

The second form of the equations of motion results from the use of a full unsteady aerodynamic formulation. This approach yields a set of matrix equations of the form:

$$
\begin{equation*}
\left(\left[\mathbf{M}_{1}\right]\{\mathbf{q}\}+\left[\mathbf{M}_{2}\right]\{\dot{\mathbf{q}}\}+\left[\mathbf{M}_{3}\right]\{\ddot{\mathbf{q}}\}\right)+\left(\left[\mathbf{M}_{\mathbf{4}}\right]\{\mathbf{q}\}+\left[\mathbf{M}_{5}\right]\{\dot{\mathbf{q}}\}\right)=\left\{\mathbf{C}_{\mathbf{3}}\right\} \boldsymbol{\alpha}_{\mathbf{g}} \tag{3}
\end{equation*}
$$

The matrix coefficients of equation (3) have the same definitions as those of equation (2). However, the aerodynamic coefficients are now of a different form. The aerodynamic stiffness and damping matrices, $\left[\mathrm{M}_{4}\right]$ and $\left[\mathrm{M}_{5}\right]$, are frequency dependent, and the generalized forcing function matrix, $\left\{\mathrm{C}_{3}\right\}$, is frequency dependent and complex.

The matrix coefficients of equations (2) and (3) will be calculated for a whole airplane. L217 (EOM) can accept structural matrices for either half or whole airplane. Matrices for half an airplane will be multiplied by two.

Detailed discussions of the formulation of the matrix coefficients will appear in sections 4.3 through 4.8. First, however, it is necessary to define the axis systems and sign conventions assumed by the program.

### 4.2 AXIS SYSTEMS AND SIGN CONVENTIONS

In the Equations of Motion (L217) program, three types of axis systems are used in the formulation of the problem. All types are right-handed systems. The basic set of axes is the reference axis system (RAS). This arbitrarily described axis system is used to describe the airplane configuration. It. is important that the same RAS be used throughout an analysis.

The second type of axis system is the local axis system (LAS). Each airplane component has a LAS associated with it. All the motion for a component is assumed to be defined in its respective local axis system. All aerodynamic calculations are performed in the LAS. All local axis systems must have an $x$-axis lying in the freestream direction with the positive $x$ direction pointing aft. For lifting surfaces, the local $x-y$ plane must lie in the plane of the surface with the positive $y$-direction pointing in the direction of
increasing span. For slender bodies (and their associated interference bodies), the z-axis orientation will be a function of the type of motion the slender body will be allowed to experience. Slender bodies that undergo vertical motion (known as z-bodies) must have their local $z$-axis parallel to the Z reference axis. Slender bodies that experience lateral motion (known as $y$-bodies) must have their local $z$-axis parallel to the $Y$ reference axis. Figure 1 illustrates typical reference and local axis system relationships.


Figure 1. - Typical Axis System Orientations

In the discussions to follow, two types of local axis systems will be encountered - the local structural axis system and the local aerodynamic axis system. These two systems are a result of the differences between the structural idealization and the aerodynamic idealization. A more thorough discussion of the differences between the two types of local axis systems will be given when local axis shifting is considered (sec. 4.9).

The last axis system that enters into the formulation is the inertial axis system. The L217 (EOM) program assumes that the interial axis system lies in the plane of symmetry of the airplane and is oriented such that $x$ is positive pointing toward the nose of the airplane and $z$ is positive pointing downward (see fig. 1). All airplane rigid body motion is assumed to be referenced to the inertial axis system.

The rigid body displacement sign convention is shown in figure 2. Note that with this convention positive plunge is down; positive side slip is to the right (facing forward); positive pitch is nose up; positive roll is right wing tip down; and positive yaw is right wing tip moving aft. Double-headed arrows indicate a right-handed rotation about a given axis.


Figure 2. - Rigid Body Displacement Sign Convention

The sign convention for the local axis system displacements is shown in figure 3, with all surface slopes shown in their positive sense.


Figure 3. - Local Surface Displacement Sign Convention

The relationship between the surface slopes and the surface rotational displacements is:

$$
\begin{align*}
& \frac{d \delta_{z}}{d x}=-\theta_{y} \\
& \frac{d \delta_{y}}{d x}=\theta_{z} \tag{4}
\end{align*}
$$

In dealing with the different aerodynamic formulations and the different types of aerodynamic bodies, two types of pressure notations will be encountered. The FLEXSTAB aerodynamic approach will give both surface pressures on slender and interference bodies and surface pressure differences (lifting pressures) on thin bodies; whereas, the Doublet Lattice approach (L216) will always give lifting pressures for all bodies. In both approaches, pressure is always compressive (see fig. 4) and the lifting pressure is defined as $\Delta P=P_{l}-P_{u}$.


Figure 4. - Surface Pressures

### 4.3 STRUCTURAL MATRICES

The structurally related matrices of equations (2) and (3) consist of the generalized structural stiffness matrix, $\left[\mathrm{M}_{1}\right]$, the damping matrix, $\left[\mathrm{M}_{2}\right]$, and the inertia matrix, [ $\mathrm{M}_{3}$ ]. Since the generalized stiffness and inertia matrices are generally formed at the time of the structural vibration analysis external to the DYLOFLEX system, no calculation of these matrices is performed within L217 (EOM).

Structural damping is generally assumed to be proportional to displacement, but in phase with velocity. This representation is accomplished by making the generalized stiffness matrix, $\left[\mathrm{M}_{1}\right]$, complex. The imaginary term of $\left[\mathrm{M}_{1}\right]$ is expressed as:

$$
\begin{equation*}
\mathrm{i}\left[\mathrm{gSS}_{\mathrm{SD}}\right]\left[\mathrm{M}_{1}^{\circ}\right]\{\mathrm{q}\} \tag{5}
\end{equation*}
$$

where:
[ $\mathrm{M}_{1}$ ] = Matrix composed of the diagonal elements of the generalized stiffness matrix, $\left[\mathrm{M}_{1}\right]$
[gSid] = Diagonal matrix of damping factors

L217 (EOM) does not calculate an [ $\mathrm{M}_{2}$ ] damping matrix when structural damping is chosen. This representation is generally associated with harmonic motion and is built into the solution program, L221 (TEV156) (ref. 5)

The option does exist to transform the structural damping into an equivalent viscous damping representation. In this instance, an [ $\mathrm{M}_{2}$ ] matrix is calculated by L217 (EOM) as:

$$
\begin{equation*}
\left[\stackrel{\circ}{\mathrm{M}_{2}}\right]=[\stackrel{\circ}{\operatorname{S} D}]\left[\mathrm{M}_{1}^{0}{ }^{1 / 2}\right]\left[\mathrm{M}_{3}^{\circ} 1 / 2\right] \tag{6}
\end{equation*}
$$

### 4.4 INERTIAL COUPLING

In the analysis of an airplane configuration, many flight conditions are usually examined. Often the only difference between some of these conditions is that in one instance control surfaces may be free to move; while, in another instance, the control surfaces remain fixed. The capability exists within L215 (INTERP) (ref. 6) to modify the basic set of mode shapes to include the added freedoms of control surface motions about their hinge lines. This being the case, the generalized inertia and stiffness matrices then become incompatible with the modified modes. Therefore, the option exists within the EOM program to augment the generalized inertial, stiffness, and damping (if calculated within this program) matrices, making them compatible with the modified modes.

The generalized stiffness matrix (and damping matrix) will be increased by adding rows and columns of zeros in order to bring the matrix to the proper size. This is shown in equation (7).

$$
\left[\mathrm{M}_{1}\right]_{\mathrm{Aug}}=\begin{align*}
& \mathrm{NDOF}_{\mathrm{ST}} \\
& \mathrm{NDOF}_{\mathrm{CS}}
\end{align*}\left[\begin{array}{l:l}
{\left[\mathrm{M}_{1}\right]_{\mathrm{ST}}} & {[0]}  \tag{7}\\
\hdashline[0] & {[0]}
\end{array}\right]
$$

where:
$\left[\mathrm{M}_{1}\right]_{\text {Aug }}=$ Augmented generalized stiffness
$\left[M_{1}\right]_{\text {Ua }}=\underset{\text { Original (unaugmented) generalized stiffness matrix obtained from the }}{\text { vibalysis }}$
$\mathrm{NDOF}_{\text {ST }}=$ Number of structural (or basic) degrees of freedom that are determined in the vibration analysis
$\mathrm{NDOF}_{\mathrm{CS}}=$ Number of control surface freedoms that were added in L215 (INTERP)
The augmented generalized mass matrix contains nonzero rows and columns that represent the inertial coupling effects resulting from the additional freedoms. The augmented $\mathrm{M}_{3}$ matrix can be written as:

$$
\left[\mathrm{M}_{3}\right]_{\mathrm{Aug}}=\begin{align*}
& \mathrm{NDOF}_{\mathrm{ST}}  \tag{8}\\
& \mathrm{NDOF}_{\mathrm{CS}}
\end{aligned} \begin{aligned}
& \mathrm{NDOF}_{\mathrm{ST}}
\end{align*} \mathrm{NDOF}_{\mathrm{CS}}
$$

where:

| $\left[\mathrm{M}_{3}\right]_{\mathrm{Aug}}$ | $=$ Augmented generalized inertia matrix |
| :--- | :--- |
| $\left[\mathrm{M}_{3}\right]_{\mathrm{Ua}}$ | $=$ Unaugmented generalized inertia |
| $[\mathrm{A}],[\mathrm{B}],[\mathrm{C}]$ | $=$ Inertial coupling matrices |

The inertial coupling matrices are formed by first assembling a diagonal inertia matrix for all mass points affected by the added control surface freedoms. This matrix, designated as the control surface inertial matrix, [ $\mathrm{m}_{\mathrm{CS}}^{\circ}$ ], will contain all the needed mass and rotational inertial properties for each node. The inertial properties input into the [ $\mathrm{m} \dot{\mathrm{C} S}$ ] matrix must meet the following requirements:

- The inertial matrix can include only those inertial properties that relate to the nodal displacements allowed.
- All inertial data must be for the whole airplane.
- All rotational inertials must be parallel and/or perpendicular to the freestream.

The mode shapes describing the displacement at each affected node are obtained by using the appropriate interpolation (SA) arrays defined in L215 (INTERP) and interpolating for any combination of surface vertical translation, the surface streamwise rotation, and the surface rotation perpendicular to the streamwise direction. The combination of nodal displacements used for a control surface is left to the discretion of the analyst. In submatrix form, the total set of mode shapes for the control surface nodes is:

$$
\begin{equation*}
[\phi]_{\text {Total }}=[\phi]_{\mathrm{ST}}{ }^{\dot{1}[\phi]_{\mathrm{CS}}} \tag{9}
\end{equation*}
$$

The ordering of $[\phi]_{\text {Total }}$ corresponds to the order of the inertial data in $\left[\mathrm{m}_{\mathrm{CS}}^{\circ}\right]$. The inertia coupling matrices can now be defined as:

$$
\begin{align*}
{[\mathrm{A}] } & =[\phi]_{\mathrm{CS}}^{\mathrm{T}}\left[\mathrm{~m}_{\mathrm{CS}}^{\circ}\right][\phi]_{\mathrm{ST}} \\
{[\mathrm{~B}] } & =[\mathrm{A}]^{\mathrm{T}}  \tag{10}\\
{[\mathrm{C}]=} & {[\phi]_{\mathrm{CS}}^{\mathrm{T}}\left[\mathrm{~m}_{\mathrm{CS}}^{\circ}\right][\phi]_{\mathrm{CS}} } \\
& \text { 4.5 AERODYNAMIC OPTIONS }
\end{align*}
$$

In L217 (EOM), the user has the capability of inputing aerodynamic data generated by either one of two aerodynamic methods, FLEXSTAB (ref. 7) or Doublet Lattice L216
(ref. 8). Together, these two methods offer the analyst four aerodynamic options in forming the equations of motion.

- FLEXSTAB AIC - quasi-steady approach.
- Doublet Lattice AIC - quasi-steady approach.
- Doublet Lattice AIC - full unsteady approach.
- Doublet Lattice Pressures and Generalized Forces - full unsteady approach.

In the FLEXSTAB AIC approach, the steady state aerodynamic influence coefficient (AIC) matrix is used to formulate the quasi-steady aerodynamics. These quasi-steady aerodynamic forces are used to form the generalized aerodynamic stiffness and damping and the generalized forcing function of equation (2). Data from the FLEXSTAB program system consists of the AIC matrix and the geometry data describing the aerodynamic idealization. Also, this approach requires the use of the modal interpolation arrays, SA arrays, from L215 (INTERP).

The Doublet Lattice AIC quasi-steady approach is similiar to that of FLEXSTAB. Here, the $\mathrm{k}=0$ (reduced frequency) aerodynamic data is used to make up the $\left[\mathrm{M}_{4}\right],\left[\mathrm{M}_{5}\right.$ ], and $\left\{\mathrm{C}_{3}\right\}$ matrices of equation (2). Data from this method consists of AIC-type matrices and the needed geometry. This approach also requires the SA arrays from L215 (INTERP).

The third option, Doublet Lattice AIC full unsteady approach, is similar to the previous Doublet Lattice option. Here, however, the AIC-type matrices are input at each $k$ value for which one wishes to define the equations of motion. This aerodynamic data is used to form the nonconstant coefficient matrices of equation (3). This approach also requires the geometry data and the SA arrays previously mentioned.

The last option is that of using the Doublet Lattice-generated pressures and generalized forces. It is a full unsteady approach that is used in forming the nonconstant coefficient matrices of equation (3). This option requires the geometry data, aerodynamic pressures, and generalized forces calculated by Doublet Lattice, but it does not require the SA arrays from L215 (INTERP).

The advantage of using the Doublet Lattice AIC option rather than the Doublet Lattice pressures and generalized forces approach is that the former can be used with any number of different sets of mode shapes. The AIC-type matrices generated by Doublet Lattice are functions of Mach number, $k$ value, and the airplane's external configuration, but are modal independent. If all these variables remain constant, and if the only difference between conditions is in the mode shapes used to define the structure's deformation, then the changes in aerodynamic forces due to the changes in the airplane's modal displacements can be calculated without having to rerun the complete aerodynamic problem. Since the majority of the cost in solving for the aerodynamic forces is the generation of the AIC-type matrices, this can provide a substantial cost savings.

The AIC option also has the added advantage of being able to account for gradual penetration of the gust. The Doublet Lattice pressure and generalized forces option does not have this advantage. The Doublet Lattice program L216, as used in the DYLOFLEX system, does not readily lend itself to the inclusion of the gust phase lag effects. Calculating gradual penetration effects in L216 (DUBFLX) would require extensive modification of the modal input to DUBFLX and the generalized aerodynamic forces output from DUBFLX. Therefore, the ability to account for gradual penetration of the gust in the DYLOFLEX system was limited to the AIC options of EOM.

Using the quasi-steady AIC approach, options one and two, offers additional savings in that the resulting aerodynamic matrices are also frequency independent. However, the user may sacrifice a certain amount of accuracy in predicting the aerodynamic forces in some instances where high frequency aerodynamics become significant.

### 4.6 FLEXSTAB AIC OPTION

The FLEXSTAB AIC option uses the geometry data and the steady state AIC matrix from FLEXSTAB and the modal interpolation arrays from L215 (INTERP) to calculate the aerodynamic forces on the airplane. These forces are then used to generate the aerodynamic-related matrices for the constant coefficient form of the equations of motion given in equation (2).

The basic equation in the formulation of the aerodynamics is that of the calculation of the pressure distribution over the different aerodynamic surfaces. Using the AIC matrix, which relates flow incidence to pressure, the pressure distribution over the different elements is given by the following equation:

$$
\begin{equation*}
\left\{\mathbf{C}_{\mathbf{p}}\right\}=[\mathrm{AIC}]\{\Psi\} \tag{11}
\end{equation*}
$$

where:
$\left\{C_{p}\right\} \quad=$ Matrix of pressure coefficients
[AIC] = Steady-state aerodynamic influence coefficients
$\{\Psi\} \quad=$ Matrix of flow incidences
The internal ordering and the size of the pressure coefficient and normalwash matrices is a function of the type of analysis performed, i.e. symmetrical or antisymmetrical, and the type of aerodynamic elements used. These factors determine the AIC matrix arrangement.

### 4.6.1 INCIDENCE AND NORMALWASH CALCULATIONS

The normalwash on a surface is defined as the component of the flow velocity normal to the surface. Dividing the normalwash by the freestream velocity gives the flow
incidence for the surface. The relationship for flow incidence in terms of normalwash is defined in FLEXSTAB as:

$$
\begin{equation*}
\{\Psi\}=\frac{1}{\mathrm{~V}_{\mathrm{T}}}\left[\delta_{\mathrm{w}}^{\circ}\right]\left\{\mathrm{w}_{\mathrm{n}}\right\} \tag{12}
\end{equation*}
$$

where:
$\left[\delta_{\mathrm{w}}^{0}\right]=$ A sign function that relates incidence flow and normalwash
$=1$ for all normalwash defined in the local z direction
$=-1$ for all normalwash defined in the local $y$ direction
$\left\{w_{n}\right\} \quad=$ Normalwash
The normalwash caused by the airplane's motion can be expressed in terms of the airplane's generalized coordinates and mode shapes.

$$
\begin{equation*}
\left\{\mathrm{w}_{\mathrm{R}}\right\}=-\left\{\mathrm{V}_{\mathrm{T}}\left[\phi_{\delta_{\mathbf{n}}^{\prime}}\right]_{\mathrm{CP}}\{\mathrm{q}\}+\left[\phi_{\delta_{\mathbf{n}}}\right]_{\mathrm{CP}}\{\dot{\mathrm{q}}\}\right\} \tag{13}
\end{equation*}
$$

where:
$\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{CP}} \quad=$ Mode shapes describing the motion normal to the surface at the surface control points
$\left[\phi^{\prime} \delta_{n}\right]_{\mathrm{CP}} \quad=$ Mode shapes describing the surface streamwise slope at the surface control points
$\{q\},\{\dot{\mathbf{q}}\} \quad=$ Generalized coordinate displacements and velocities
$R \quad=$ Normalwash due to the response of the aircraft
The FLEXSTAB AIC matrix requires this normalwash be evaluated at the control points of the slender and thin bodies. Normalwash due to interference body motions is not allowed by definition of an interference body.

The mode shapes describing surface normal displacements and streamwise slopes are calculated using the SA arrays generated in L215 (INTERP). The interpolated mode shapes describe linear displacements in the local $z$ direction of the surface and the streamwise slope of these displacements. For thin bodies, this direction is normal to the thin body surface. Slender bodies, however, may require motion in either one or both the vertical direction (z-bodies) and/or the lateral direction (y-bodies). This is a function of the slender body location (on or off the plane of symmetry) and the type of analysis performed (symmetric or antisymmetric). Since all SA arrays give motions relative to the local $z$-axis direction, motion for $y$-bodies must be determined from SA arrays whose associated local z -axes are rotated in the lateral direction. The ordering of the elements within the normalwash matrix (which corresponds to the AIC column arrangement for the symmetrical and anti-symmetrical cases) is shown in figure 5.


Figure 5. - Ordering of Response Normalwash Matrices for the FLEXSTAB AIC Option

The normalwash due to a gust is defined as the component of the gust velocity normal to the surface. In L217 (EOM), the gust velocity is defined in the inertial axis system with a vertical gust defined positive in the minus $z$ inertial direction (up gust) and a lateral gust defined positive in the positive $y$ inertial direction (see fig. 6).


Figure 6. - Lateral and Vertical Gust Sign Conventions
In a symmetric analysis where only a vertical gust is applied to the airplane, the component of the gust velocity normal to the surface of a thin body is given by:

$$
\begin{equation*}
w_{G}{ }_{T B}=v_{G} V^{\cos \theta_{\mathrm{TB}}} \tag{14}
\end{equation*}
$$

where:
$\theta_{\mathrm{TB}} \quad=$ Dihedral angle of the thin body
For slender bodies, the local $z$-direction normalwash is equal to the vertical gust velocity, ${ }^{{ }^{V}}{ }^{\mathrm{V}}$, and the local y -direction normalwash is zero. In matrix form, the normalwash due to the gust can be written as:

$$
\begin{equation*}
\left\{\mathbf{w}_{\mathbf{G}}\right\}_{\mathbf{S}}=\left[\mathrm{RG}_{\mathbf{G}}^{\circ}\right]_{\mathbf{S}}\left\{\mathbf{v}_{\mathbf{G}_{\mathbf{V}}}\right\} \tag{15}
\end{equation*}
$$

where:
$[R \mathcal{G}]_{S}$ = Diagonal gust rotation matrix for a symmetrical analysis
$\left\{\mathbf{v}_{\mathbf{G}}\right\}=\begin{aligned} & \text { Vertical gust velocity at each control point defined in the inertial axis } \\ & \text { system }\end{aligned}$
In an antisymmetrical analysis, only a lateral gust is applied to the airplane, and the component of gust velocity normal to a thin body surface is

$$
\begin{equation*}
w_{G_{T B}}=-v_{G_{L}} \sin \theta_{\mathrm{TB}} \tag{16}
\end{equation*}
$$

For slender bodies, the z-direction normalwash is now zero while the $y$-direction normalwash is equal to $\mathrm{v}_{\mathrm{G}_{\mathrm{L}}}$. In matrix form, the normalwash matrix due to a lateral gust can be written as:

$$
\begin{equation*}
\text { where }\{G\}_{A}=\left[R_{G}^{i}\right]_{A}\left\{v_{G_{L}}\right\} \tag{17}
\end{equation*}
$$

where:
$\left[\mathrm{R}_{\mathrm{G}}\right]_{\mathrm{A}} \quad=$ Diagonal gust rotation matrix for an antisymmetric analysis
$\left\{\mathbf{v}_{\mathbf{G}_{\mathbf{L}}}\right\}=$ Lateral gust velocities at each control point, defined in the inertial axis system.

By referring to figure 5 for the required ordering and using equations (14) and (16), the gust rotation matrix shown in figure 7 can be formed for either a symmetrical or an antisymmetrical analysis.

Gradual penetration of the gust can be accounted for by calculating a gust phase lag function, $\phi_{\mathrm{G}}$. The phase lag will be applied to the thin body panels and slender body segments in gust zone groupings, not to each individual panel or segment. The gust zones are defined by the X-reference coordinate of zone leading edges. Up to 35 zones may be used. Each gust zone will have a gust control point; that is, the point at which the gust phase lag for that zone will be evaluated. The gust control point is defined by the user in the reference axis system. The family of thin body panels and slender body segments whose control points fall within the same zone will experience the same gust phase lag. The point of zero phase lag is denoted as the gust reference point XG, and may be defined anywhere. The distances between the gust zone control points and the gust reference point are given in matrix form as:

$$
\begin{equation*}
\left\{\mathrm{f}_{\ell}\right\}=\left\{\mathbf{X G}_{\mathbf{C P}}\right\}-\mathbf{X G} \tag{18}
\end{equation*}
$$

For a sinusoidal gust, the gust velocity at each zone control point is written as:

$$
\begin{equation*}
\left\{v_{G_{V / L}}\right\}=\bar{v}_{G_{V / L}}\left\{e^{-i \omega \frac{f}{V_{T}}}\right\} e^{i \omega t} \tag{19}
\end{equation*}
$$

where $\overline{\mathbf{v}}_{\mathrm{G} / \mathrm{L}}$ is the magnitude of the gust in the vertical or lateral direction.
To relate the gust at each zone control point to the thin body and slender body aerodynamic control points, a gust correlation matrix of ones and zeroes can be formed, [GC]. Finally, the gust normalwash for either symmetric or antisymmetric analysis can be written as:

$$
\begin{equation*}
\left\{w_{G}\right\}_{S / A}=[R G]_{S / A}[G C]\left\{e^{-i \omega \frac{f_{\ell}}{V_{T}}}\right\} v_{G_{V / L}} e^{i \omega t} \tag{20}
\end{equation*}
$$

## Symmetrical Analysis


$[1 \%]=$ a diagonal matrix whose elements consist of alternating 1 's and 0 's with the first. element being a 1 .

Figure 7. - Gust Rotation Matrix for FLEXSTAB

### 4.6.2 AERODYNAMIC FORCES

The expression for the pressure distribution (eq. 11) can now be used with the normalwash defined in equations (13) and (20) to determine the pressure distributions due to airplane motion and atmospheric turbulence. The ordering of the elements within the pressure coefficient matrix reflects the row order of the AIC matrix. This ordering which is the same for both lateral and vertical analyses, is illustrated in figure 8.

The meaning of the elements of the pressure coefficient matrix differs with each body type. Therefore, the calculation of the aerodynamic forces for each body type will be discussed separately.

## Slender Body Aerodynamic Forces

A slender body is a body of revolution divided into a number of streamwise segments (see fig. 9). The slender body pressure coefficients are defined at the midpoint of each slender body segment. The $C_{p}{ }^{Y}$ coefficients represent the magnitude of the circumferential pressure variation due to the Y-doublet of the segment at the Y-doublet evaluation point. Likewise, the $\mathrm{C}_{p}{ }^{Z}$ coefficient is the magnitude of the circumferential pressure variation due to the Z-doublet of the segment at the Z-doublet evaluation point (see fig. 10).

The total circumferential distribution for a segment is the sum of the $Y$ and $Z$ distributions.

$$
\begin{equation*}
C_{p}(x)=-C_{p}^{Y} \cos \beta+C_{p}^{Z} \sin \beta \tag{21}
\end{equation*}
$$

where:
$\beta=$ Circumferential variable used to define the pressure distribution of the Z and Y doublets over the surface of the segment.

Forces in the local $y$ and $z$ directions are obtained by integrating the appropriate component of equation (21) over the segment area.

$$
\begin{align*}
& F_{y}=-\bar{q} \int_{L_{S}} r_{S} \int_{0}{ }^{2 \pi} C_{p}(x) \sin \beta d \beta d x  \tag{22}\\
& F_{z}=-\bar{q} \int_{L_{S}} r_{S} \int_{0}^{2 \pi} C_{p}(x) \cos \beta d \beta d x
\end{align*}
$$

where:
$\overline{\mathrm{q}} \quad=$ Dynamic pressure $=1 / 2 \rho \mathrm{~V}_{\mathrm{T}}{ }^{2}$
$L_{S} \quad=$ Length of the segment
$r_{S}=$ Radius of the segment


```
NSB \(=\) Total number of slender bodies
NIB \(=\) Total number of interference bodies
NTB \(=\) Total number of thin bodies
```

Figure 8. - Ordering Within the FLEXSTAB Pressure Coefficient Matrix


Figure 9. - Typical Slender Body


Figure 10. - Circumferential Pressure Distribution on a Slender Body Segment Due to a $Y$ and $Z$ Doublet

Performing the circumferential integration, the force expressions reduce to:

$$
\begin{align*}
& F_{y}=-\bar{q} \int_{L_{S}} r_{s}\left(-\pi C_{p}{ }^{Y}\right) d x \\
& F_{z}=-\bar{q} \int_{L_{S}} \mathbf{r}_{\mathbf{S}}\left(\pi C_{p}{ }^{Z}\right) d x \tag{23}
\end{align*}
$$

Making the assumption that $\mathrm{C}_{\mathrm{p}}{ }^{\mathrm{Y}}$ and $\mathrm{C}_{\mathrm{p}}{ }^{\mathrm{Z}}$ are constant over the length of the segment, and using the radius value at the segment midpoint, the final expressions for the $y$ and $\dot{z}$ forces for any one segment are:

$$
\begin{gather*}
F_{y}=\bar{q} F_{s} L_{s} \pi C_{p} Y \\
F_{z}=-q r_{S} L_{s} \pi C_{p} Z \tag{2}
\end{gather*}
$$

Defining the slender body segment area as:

$$
\begin{equation*}
A_{S}=2 r_{S} L_{S} \tag{25}
\end{equation*}
$$

the $y$ and $z$ segment forces for the $J^{\text {th }}$ slender body can be written in matrix form as:

$$
\begin{align*}
& \left\{\mathrm{F}_{\mathbf{y}}\right\}_{\mathrm{J}}=\overline{\mathrm{q}}[\pi / 2]\left[\mathrm{A}_{\mathbf{S}}^{\circ}\right]_{\mathrm{J}}\left\{\mathrm{C}_{\mathbf{p}} \mathrm{Y}_{\mathrm{J}}\right. \\
& \left\{\mathrm{F}_{\mathrm{Z}}\right\}_{\mathrm{J}}=\overline{\mathbf{q}}[-\stackrel{\circ}{\pi / 2}] \cdot\left[\mathrm{A}_{\mathbf{s}}^{\circ}\right]_{\mathrm{J}}\left\{\mathrm{C}_{\mathbf{p}} \mathrm{Z}_{\mathrm{J}}\right. \tag{26}
\end{align*}
$$

## Interference Body Aerodynamic Forces

The interference panel pressure coefficients represent the upper surface pressure on each panel (see fig. 11). These pressures are evaluated at the panel centroid. By definition, interference bodies do not generate their own lift, but are present to account for the interference flow about a slender body caused by thin and other slender bodies. As a result, the lower surface panel pressure is zero. Thus, the lifting pressure. coefficient for each panel is:

$$
\begin{equation*}
\Delta \mathbf{C}_{\mathbf{p}_{\mathbf{I B}}}=\mathrm{C}_{\mathbf{p}_{\boldsymbol{\ell}}}-\mathrm{C}_{\mathbf{p}_{\mathbf{u}}}=-\mathrm{C}_{\mathbf{p}_{\mathbf{u}}} \tag{27}
\end{equation*}
$$



Figure 11. - Interference Body Panel Orientation Angle

The FLEXSTAB aerodynamic theory assumes constant pressure panels. Under this assumption, the panel normal force is given by:

$$
\begin{equation*}
\mathbf{F}_{\mathbf{n}_{\mathrm{IB}}}=-\overline{\mathbf{q}} \mathrm{A}_{\mathrm{IB}} \mathbf{C}_{\mathbf{p}_{\mathbf{u}}} \tag{28}
\end{equation*}
$$

where:
$\mathbf{A}_{\text {IB }}=$ Interference panel area
Using the orientation angle of the local panel normal with respect to the local $z$-axis (see fig. 11), the $y$ and $z$ components for each interference body panel are:

$$
\begin{align*}
\mathbf{F}_{\mathbf{y}_{\mathbf{I B}}} & =-\bar{q} \sin \theta_{\mathrm{IB}} A_{\mathbf{I B}} \mathrm{C}_{\mathbf{p}_{\mathbf{u}}} \\
\mathbf{F}_{\mathbf{z}_{\mathrm{IB}}} & =-\bar{q} \cos \theta_{\mathbf{I B}} A_{\mathbf{I B}} \mathbf{C}_{\mathbf{p}_{\mathbf{u}}} \tag{29}
\end{align*}
$$

For the $K^{\text {th }}$ interference body, equation (29) for each panel can be written in matrix form as:

$$
\begin{align*}
& \left\{\mathrm{F}_{\mathbf{y}_{I B}}\right\}_{\mathrm{K}}=\overline{\mathrm{q}}\left[-\sin ^{\circ} \theta_{\mathrm{IB}}\right]_{\mathrm{K}}\left[\mathrm{~A}_{\mathrm{IB}}^{\circ}\right]_{\mathrm{K}}\left\{\mathrm{C}_{\mathbf{P}_{\mathbf{u}}}\right\}_{\mathrm{K}} \\
& \left\{\mathrm{~F}_{\mathbf{z}_{\mathrm{IB}}}\right\}_{\mathrm{K}}=\bar{q}\left[-\cos ^{\circ} \theta_{\mathrm{IB}}\right]_{\mathrm{K}}\left[\mathrm{~A}_{\mathrm{IB}}^{\circ}\right]_{\mathrm{K}}\left\{\mathrm{C}_{\mathbf{p}_{\mathbf{u}}}\right\}_{\mathrm{K}} \tag{30}
\end{align*}
$$

## Addition of Interference and Slender Body Aerodynamic Forces

To obtain the total segment forces on any slender body that has an associated interference body, the interference and slender body forces must be added together. This addition is made under the assumption that no interference panels overlap slender body segment boundaries; that is, the circumferential interference body segments must lie within the corresponding slender body segment. This requirement is illustrated in figure 12. Note that the last two circumferential segments of the interference body lie within the same slender body segment.


Figure 12. - Slender Body-Interference Body Alignment

The addition of the interference panel forces to the proper slender body segments can be accomplished by premultiplying the force expressions of equation (30) by a panel correlation matrix. This matrix is composed of ones and zeroes and is a rectangular matrix of size (number of slender body segments) by (number of interference body panels).

$$
\begin{align*}
& \left\{\mathrm{F}_{\mathbf{y}_{\mathbf{I B}}}\right\}_{\mathbf{J}}=[\mathrm{PC}]_{\mathrm{K}}^{\mathbf{J}}\left\{\mathrm{F}_{\mathbf{y}_{\mathbf{I B}}}\right\}_{\mathbf{K}} \\
& \left\{\mathrm{F}_{\mathbf{Z}_{\mathbf{I B}}}\right\}_{\mathbf{J}}=[\mathrm{PC}]_{\mathbf{K}}^{\mathbf{J}}\left\{\mathrm{F}_{\mathbf{z}_{\mathbf{I B}}}\right\}_{\mathrm{K}} \tag{3}
\end{align*}
$$

where:

$$
\begin{aligned}
& \left\{F_{\mathbf{y}_{\text {IB }}}\right\}_{J},\left\{\mathrm{~F}_{\mathrm{z}}\right\}_{\mathrm{IB}} \quad=\underset{\mathrm{J}}{ } \quad \underset{\mathrm{Jth}}{\text { Sum }} \text { of the } \mathrm{K}^{\text {th }} \text { interference body panel } \mathrm{y} \text { and } \mathrm{z} \text { forces at the } \\
& {[\mathrm{PC}]_{\mathrm{K}}^{\mathrm{J}}} \\
& \text { panels to the proper } \mathrm{J}^{\text {th }} \text { slender body segments }
\end{aligned}
$$

Combining equations (31) and (30) and defining $y$ and $z$ force summation matrices as:

$$
\begin{align*}
& {\left[\Sigma_{y}\right]_{K}=[\mathrm{PC}]_{\mathrm{K}}^{\mathrm{J}}\left[-\sin ^{\circ} \theta_{\mathrm{IB}}\right]_{\mathrm{K}}} \\
& {\left[\mathrm{\Sigma}_{\mathrm{z}}\right]_{\mathrm{K}}=[\mathrm{PC}]_{\mathrm{K}}^{\mathrm{J}}\left[-\cos ^{\circ} \theta_{\mathrm{IB}}\right]_{\mathrm{K}}} \tag{32}
\end{align*}
$$

the interference body forces summed to the respective slender body segment midpoints can then be written as:

$$
\begin{align*}
&\left\{\mathrm{F}_{\mathbf{y}_{I B}}\right\}_{J}=\bar{q}\left[\Sigma_{\mathbf{y}}\right]_{K}\left[\mathrm{~A}_{\mathrm{IB}}^{\circ}\right]_{\mathrm{K}}\left\{\mathrm{C}_{\mathbf{p}_{\mathbf{u}}}\right\}_{K}  \tag{33}\\
&\left\{\mathrm{~F}_{\mathbf{z}_{\mathrm{IB}}}\right\}_{J}=\bar{q}\left[\Sigma_{\mathrm{z}}\right]_{\mathrm{K}}\left[\mathrm{~A}_{\mathrm{IB}}^{\circ}\right]_{\mathrm{K}}\left\{\mathrm{C}_{\mathbf{p}_{\mathbf{u}}}\right\}_{\mathrm{K}}
\end{align*}
$$

The total body force now becomes the sum of equations (33) and (26). Referring to the $\mathrm{C}_{\mathbf{p}}$ matrix ordering shown in figure 7, the total slender body force distribution can be written in matrix form as:

## Thin Body Forces

The aerodynamic forces on each thin body panel are found by multiplying the lifting pressure on each panel by its area. For the $I^{\text {th }}$ thin body, this is written in matrix form as:

$$
\begin{equation*}
\left\{\mathrm{F}_{\mathbf{z}}\right\}_{\mathrm{I}}=\overline{\mathrm{q}}\left[\mathrm{~A}_{\mathbf{T B}}\right]_{\mathrm{I}}\left\{\Delta \mathrm{C}_{\mathbf{p}}\right\}_{\mathrm{I}} \tag{35}
\end{equation*}
$$

Thin body forces act in the local z axis of the respective surfaces.

## Total Aerodynamic Forces

Combining equation (34) with equation (35) for all the thin bodies, the total set of
aerodynamic forces can be written as:

Since the aerodynamic modeling in FLEXSTAB is for half an airplane, the slender body segment areas for bodies on the plane of symmetry will be half of the total area. Also, for interference bodies on the plane of symmetry, only half the interference shell is modeled. Therefore, to obtain whole body forces, the $F_{y}$ and $F_{z}$ elements for slender bodies on the plane of symmetry are multiplied by two. The final expression for the aerodynamic forces thus becomes:

$$
\begin{equation*}
\left\{\mathbf{F}_{\mathbf{A}}\right\}=\overline{\mathrm{q}}[\mathrm{FAO}][\Sigma][\AA \mathrm{A}]\left\{\mathrm{C}_{\mathbf{p}}\right\} \tag{37}
\end{equation*}
$$

where:
$[F A C]=$ Diagonal factor matrix that doubles the aerodynamic forces for slender bodies on the plane of symmetry

### 4.6.3 GENERALIZED AERODYNAMIC FORCE COEFFICIENTS

The generalized forces shown in the Laplace energy equation (1) are formed by the use of the principle of virtual work.

$$
\begin{equation*}
\mathrm{Q}_{\mathbf{i}}=\frac{\partial \Delta \mathrm{W}}{\partial \Delta \mathrm{q}_{\mathrm{i}}} \quad \mathrm{i}=1, \ldots, \mathrm{n} \tag{38}
\end{equation*}
$$

where:
$Q_{i} \quad=$ Generalized force in the $i^{\text {th }}$ freedom
$\Delta \mathrm{W}=$ Virtual work
$\Delta q_{i} \quad=$ Virtual displacement of the $i^{\text {th }}$ generalized coordinate
Virtual work is calculated by multiplying the external forces acting on the system by the virtual displacement of the structure at the point of application of the force and in the direction of the force.

$$
\begin{equation*}
\Delta W=\left\{\Delta n \jmath_{F P}\{F\}\right. \tag{39}
\end{equation*}
$$

Expressing the virtual displacement of the structure in terms of the structure's mode shapes and virtual displacement of the generalized coordinates, and performing the differentiation of equation (38), the expression for the generalized forces becomes:

$$
\begin{equation*}
\{\mathrm{Q}\}=\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{FP}}^{\mathrm{T}}\{\mathrm{~F}\} \tag{40}
\end{equation*}
$$

In the above expression, the modal displacements, $\theta_{\delta_{n}}$, describe the displacement of the structure at the point of application of the force (force point) and in the direction of the force.

## Generalized Response Aerodynamic Forces

Using the pressure coefficient equation (11) and the expression for the normalwash due to airplane motion (13), the response pressure distribution is given by:

$$
\begin{align*}
\left\{\mathbf{C}_{\mathbf{p}_{\mathbf{R}}}\right\} & =-[\mathrm{AIC}]\left[\phi_{\delta_{\mathrm{n}}}^{\prime}\right]_{\mathrm{CP}}\{\mathrm{q}\}-\frac{1}{\mathbf{V}_{\mathbf{T}}}[\mathrm{AIC}]\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{CP}}\{\dot{\mathrm{q}}\}  \tag{41}\\
& =\left[\mathrm{CP} 1_{\mathbf{R}}\right]\{\mathrm{q}\}+\frac{1}{\mathbf{V}_{\mathrm{T}}}\left[\mathrm{CP} 2_{\mathbf{R}}\right]\{\dot{\mathrm{q}}\}
\end{align*}
$$

Substituting this expression into the aerodynamic force equation (37), the response aerodynamic forces are:

$$
\begin{align*}
\left\{\mathrm{F}_{\mathrm{R}}\right\} & =\overline{\mathrm{q}}[\mathrm{FAC}][\Sigma][\AA]\left\{\left[\mathrm{CP} 1_{\mathbf{R}}\right]\{\mathrm{q}\}+\frac{1}{\mathrm{~V}_{\mathrm{T}}}\left[\mathrm{CP} 2_{\mathrm{R}}\right]\{\dot{\mathrm{q}}\}\right\} \\
& =\left[\mathrm{F} 1_{\mathbf{R}}\right]\{\mathrm{q}\}+\left[\mathrm{F} 2_{\mathbf{R}}\right]\{\dot{\mathrm{q}}\} \tag{42}
\end{align*}
$$

Finally, the force expression (eq. 42) is substituted into the generalized force equation (40) to give:

$$
\begin{equation*}
\left\{\mathrm{Q}_{\mathrm{R}}\right\}=\left[\phi_{\delta_{\mathbf{n}}}\right]_{\mathrm{FP}}^{\mathrm{T}}[\mathrm{SF}]\left\{\left[\mathrm{F} 1_{\mathbf{R}}\right]\{\mathrm{q}\}+\left[\mathrm{F} 2_{\mathbf{R}}\right]\{\dot{\mathrm{q}}\}\right\} \tag{43}
\end{equation*}
$$

where:
$[\mathrm{SF}]=$ Symmetry factor matrix that doubles the forces of the bodies off the plane of symmetry in order to calculate total airplane forces

It is important to note that the displacements at the force points are defined in the local aerodynamic axis system of each surface (see sec. 4.2). Thus a positive displacement acts in the same direction as a positive aerodynamic force, resulting in positive virtual work.

Bringing the response generalized forces to the left side of equation (1) and writing them as coefficients of generalized coordinate displacements and velocities, the $M_{4}$ and $\mathrm{M}_{3}$ matrices become:

$$
\begin{align*}
& {\left[\mathrm{M}_{4}\right]=-\left[\phi_{\delta_{n}}\right]_{\mathrm{FP}}^{\mathrm{T}}[\mathrm{SF}]\left[\mathrm{F} 1_{R}\right]}  \tag{44}\\
& {\left[\mathrm{M}_{5}\right]=-\left[\phi_{\delta_{n}}\right]_{\mathrm{FP}}^{\mathrm{T}}[\mathrm{SF}]\left[\mathrm{F} 2_{R}\right]}
\end{align*}
$$

## Generalized Excitation Aerodynamic Forces

The generalized excitation aerodynamic forces are calculated in the same manner as the response forces. Using the gust normalwash given by equation (20), the gust pressure distribution is defined as:

$$
\begin{align*}
\left\{C_{p_{E}}\right\} & =\frac{1}{V_{T}}[A I C][R G]_{S / A}[G C]\left\{e^{-i \omega \frac{f_{l}}{V_{T}}}\right\} \bar{v}_{G_{V / L}} e^{i \omega t}  \tag{45}\\
& =\frac{1}{V_{T}}\left[C P_{G}\right]\left\{e^{-i \omega \frac{f}{V_{T}}}\right\} \bar{v}_{G_{V / L}} e^{i \omega t}
\end{align*}
$$

and the excitation forces become:

$$
\begin{aligned}
\left\{F_{G}\right\} & =\frac{\bar{q}}{V_{T}}[F A C][\Sigma][\AA]\left[C_{G}\right]\left\{e^{-i \omega \frac{f_{l}}{V_{T}}}\right\}{\overline{v_{G}}}_{V / L} e^{i \omega t} \\
& =[F G]\left\{e^{-i \omega{\overline{V_{l}}}_{T}}\right\}{\overline{{ }_{v}^{G}}}^{V / L}
\end{aligned}
$$

Finally, the generalized excitation forces are:

$$
\begin{align*}
\left\{Q_{E}\right\} & =\left[\phi_{\delta_{n}}\right]_{F P}^{T}[F G]\left\{e^{-i \omega \frac{f_{\ell}}{V_{T}}}\right\} v_{G_{V / L}} e^{i \omega t} \\
& =[\tilde{\phi}]\left\{e^{-i \omega \frac{f_{\ell}}{V_{T}}}\right\} v_{G_{V / L}} e^{i \omega t}  \tag{47}\\
& =\left\{C_{3}\right\} v_{G_{V / L}} e^{i \omega t}
\end{align*}
$$

### 4.7 DOUBLET LATTICE AIC OPTION

Basically, the Doublet Lattice AIC option performs the same type of aerodynamic calculation as the FLEXSTAB AIC option discussed in section 4.6, (relating the pressure
distribution over the aircraft to the flow incidence at several aerodynamic control points). However, the differences between the two approaches lie in the method used to formulate this relationship and the fact that the Doublet Lattice approach can be used for unsteady as well as quasi-steady aerodynamics.

Unlike the FLEXSTAB approach, in which the AIC matrix contains all the interaction effects of slender, interference, and thin bodies, the Doublet Lattice formulation is separated into three distinct steps:

- First, slender body pressures are determined.
- Second, the normalwash induced on thin and interference bodies by the slender bodies is calculated.
- Finally, the thin and interference body pressures are calculated.

The Doublet Lattice AIC option uses the modal interpolation arrays from L215 (INTERP) and the geometry and aerodynamic data from L216 (DUBFLX). The aerodynamic data consists of frequency dependent matricies, [F] and [D], which relate flow incidence and lifting pressure.

In the discussion to follow, the full unsteady aerodynamic case will be examined first. The quasi-steady formulation will be presented afterward since it is a subset of the full unsteady approach.

### 4.7.1 SLENDER BODY AERODYNAMIC FORCES

The Doublet Lattice program does not pass slender body pressures to the EOM program for this option. These pressures are calculated within the EOM program using Miles' slender body theory employed in L216 (DUBFLX) (ref. 8). Based on this theory, the lifting pressure over the slender bodies is given by:

$$
\begin{equation*}
\left\{\Delta \mathrm{C}_{\mathrm{p}}\right\}_{\mathrm{SB}}=2 \pi\left[\left[\mathrm{r}_{\mathrm{S}}^{\circ}\right]+\frac{\mathrm{i} \omega}{2 \mathrm{~V}_{\mathrm{T}}}\left[\mathrm{r}_{\mathrm{s}}^{\circ}\right]\right]\left\{\frac{\mathrm{w}_{n}}{\mathrm{~V}_{\mathrm{T}}}\right\}+\frac{2 \pi}{2}\left[\stackrel{\mathrm{r}}{\mathrm{~s}}_{\circ}\right]\left\{\frac{\mathrm{w}_{n^{\prime}}}{\mathrm{V}_{\mathrm{T}}}\right\} \tag{48}
\end{equation*}
$$

where:
$\left\{\Delta \mathrm{C}_{\mathbf{p}}\right\}_{\text {SB }} \quad=$ Array of slender body segment lifting pressures in the direction of the normalwash, $w_{n}$ ( $n$ is either local $y$ or $z$ direction)
$\left[\begin{array}{rl}\circ \\ r_{S}\end{array}\right],\left[r_{S}^{i}\right]=$ Array of segment radii and streamwise slope of radii evaluated at segment midpoints ( $\mathrm{r}_{\mathrm{s}}{ }^{\prime}=\mathrm{dr}_{\mathrm{S}} / \mathrm{dx}$ )
$\left\{w_{n}\right\},\left\{w_{n}^{\prime}\right\} \quad=$ Normalwash and the streamwise derivative of the normalwash evaluated at the segment midpoints.

Slender bodies that undergo vertical motion (z-bodies) give lifting pressures in the $z$-direction. Y-bodies are slender bodies that experience lateral motion and give lifting
pressures in the $y$-direction. As discussed in section 4.6.1, different SA arrays must be used with y-bodies and z-bodies. The ordering within the pressure matrix is all lifting pressures in the $y$-direction (Y-bodies) followed by all lifting pressures in the z-direction (Z-bodies).

$$
\left\{\Delta \mathrm{C}_{\mathrm{p}}\right\}_{\mathrm{SB}}=\left\{\begin{array}{l}
\left\{\Delta \mathrm{C}_{\mathrm{p}} \mathrm{Y}\right\}_{\mathrm{SB}}  \tag{49}\\
\left\{\Delta \mathrm{C}_{\mathrm{p}} \mathrm{Z}\right\}_{\mathrm{SB}}
\end{array}\right\}
$$

## Response Aerodynamic Forces

The response pressures on the slender bodies can be calculated by using equation (48) with the response normalwash defined in equation (13). The normalwash is evaluated at the segment midpoints and is formulated in the same order as the pressure stacking shown in equation (49). Making this substitution, the slender body lifting pressures are:

$$
\begin{align*}
\left\{\Delta \mathrm{C}_{\mathbf{p}_{\mathrm{RB}}}\right\}_{\mathrm{SB}}= & -2 \pi\left[\left[\mathrm{r}_{\mathrm{s}}^{\prime}\right]+\frac{\mathrm{i} \omega}{2 \mathrm{~V}_{\mathrm{T}}}\left[\mathrm{r}_{\mathrm{S}}^{\circ}\right]\right]\left\{\left[\phi_{\delta_{\mathrm{n}}^{\prime}}^{\prime}\right]_{\mathrm{CP}}\{\mathrm{q}\}+\frac{1}{\mathrm{~V}_{\mathrm{T}}}\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{CP}}\{\dot{\mathrm{q}}\}\right\}  \tag{50}\\
& -\frac{2 \pi}{2}\left[\stackrel{\circ}{\mathrm{r}}_{\mathrm{s}}\right]\left\{\left[\phi_{\delta_{\mathrm{n}}}^{\prime \prime}\right]_{\mathrm{CP}}\{\mathrm{q}\}+\frac{1}{\mathrm{~V}_{\mathrm{T}}}\left[\phi_{\delta_{\mathrm{n}}}^{\prime}\right]_{\mathrm{CP}}\{\dot{\mathrm{q}}\}\right\}
\end{align*}
$$

For simple harmonic motion, velocity and displacement are related by the expression:

$$
\begin{equation*}
\dot{\mathbf{q}}=\mathbf{i} \omega \mathbf{q} \tag{51}
\end{equation*}
$$

Using this relationship, equation (50) can be written in complex form as:

$$
\begin{align*}
& \left\{\Delta \mathrm{C}_{\mathrm{p}_{\mathrm{R}}}\right\}_{\mathrm{SB}}=-2 \pi\left[\left[\mathrm{r}_{\mathrm{r}}^{\prime}{ }_{\mathrm{S}}\right]\left[\phi^{\prime} \delta_{\mathrm{n}}\right]_{\mathrm{CP}}+\frac{1}{2}\left[\mathrm{r}_{\mathrm{S}}^{0}\right]\left[\phi_{\delta_{\mathrm{n}}}^{\prime \prime}\right]_{\mathrm{CP}}-\frac{\omega^{2}}{2 \mathrm{~V}_{\mathrm{T}}^{2}}\left[\mathrm{r}_{\mathrm{S}}^{0}\right]\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{CP}}\right]\{q\} \\
& -\mathrm{i} \frac{2 \pi \omega}{\mathrm{~V}_{\mathrm{T}}}\left[\left[\mathrm{r}_{\mathrm{S}}\right]^{\circ}\left[\phi_{\delta_{\mathrm{n}}}^{\prime}\right]_{\mathrm{CP}}+\left[\mathrm{r}^{\prime}{ }_{\mathrm{s}}\right]\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{CP}}\right]\{\mathrm{q}\}  \tag{52}\\
& =\left[\left[\mathrm{CPR}_{\mathrm{SB}}\right]_{\mathrm{Re}}+\mathrm{i}\left[\mathrm{CPR}_{\mathrm{SB}}\right]_{\mathrm{Im}}\right]\{\mathrm{q}\}
\end{align*}
$$

The slender body forces are obtained by multiplying the pressures by the dynamic pressure and the segment projected areas.

$$
\begin{align*}
\left\{\begin{array}{l}
\left\{\mathrm{FR}^{\mathrm{Y}}\right\}_{\mathrm{SB}} \\
\left\{\mathrm{FR}^{\mathrm{Z}}\right\}_{\mathrm{SB}}
\end{array}\right\} & =\overline{\mathrm{q}}\left[\mathrm{~A}_{\mathrm{SB}}^{\circ}\right]\left[\left[\mathrm{CPR}_{\mathrm{SB}}\right]_{\mathrm{Re}}+\mathrm{i}\left[\mathrm{CPR}_{\mathrm{SB}}\right]_{\mathrm{Im}}\right]\{q\}  \tag{53}\\
& =\left[\left[\mathrm{FR}_{\mathrm{SB}}\right]_{\mathrm{Re}}+\mathrm{i}\left[\mathrm{FR}_{\mathrm{SB}}\right]_{\mathrm{Im}}\{\mathrm{q}\}\right]
\end{align*}
$$

## Excitation Aerodynamic Forces

The slender body lifting pressures due to the gust are calculated using the gust normalwash given in equation (20) with a modified version of the slender body pressure equation (48). In the modification of the pressure equation, terms dealing with the normalwash time rate of change and streamwise slope are neglected. At high frequencies, the unsteadiness produced by these terms violates the assumptions of the slender body theory. Therefore, the slender body excitation pressure will be approximated by:

$$
\begin{align*}
\left\{\Delta C_{P_{G}}\right\}_{S B} & =\frac{2 \pi}{V_{T}}\left[\stackrel{\rho}{r}_{S}^{\prime}\right]\left[R G_{S B}^{\circ}\right]_{S / A}[G C]_{S B}\left\{e^{-i \omega \frac{f}{V_{T}}}\right\}_{\bar{v}_{G_{V / L}}} e^{i \omega t} \\
& =\left[C P G_{S B}\right]\left[e^{\left.-i \omega \frac{f_{\mathcal{L}}}{V_{T}}\right\} \bar{v}_{G_{V / L}} e^{i \omega t}}\right. \tag{54}
\end{align*}
$$

The slender body gust forces become:

$$
\begin{align*}
\left\{F_{G}\right\}_{S B} & =\bar{q}\left[A_{S B}^{\circ}\right]\left[C P G_{S B}\right]\left\{e^{-i \omega \frac{f_{\ell}}{V_{T}}}\right\} v_{G_{V / L}} e^{i \omega t} \\
& =\left[F G_{S B}\right]\left\{e^{-i \omega \frac{f}{V_{T}}}\right\} \bar{v}_{G_{V / L}} e^{i \omega t} \tag{55}
\end{align*}
$$

### 4.7.2 INDUCED NORMALWASH

The second step in the formulation of the aerodynamics is the calculation of the normalwash induced by the slender bodies on the thin and interference body boxes. Note that the term "box" in the Doublet Lattice option refers to the smallest division of a thin or interference body. It is equivalent to the term "panel" used in FLEXSTAB.

The Doublet Lattice program, L216, calculates a complex matrix of normalwash factors, [F], at each frequency of interest. The matrix is rectangular, with each row relating to a thin or interference body box and each column relating to a slender body segment. The induced normalwash at the box control points is given by the product of the normalwash factors and the slender body pressures. For the induced normalwash due to the airplane response motions, the equation is:

$$
\begin{equation*}
\frac{1}{\mathrm{~V}_{\mathrm{T}}}\left\{-\mathrm{w}_{\mathbf{B}}\right\}_{\mathbf{R}}=[\mathrm{F}]\left\{\Delta \mathrm{C}_{\mathbf{p}_{\mathbf{R}}}\right\}_{\mathbf{S B}} \tag{56}
\end{equation*}
$$

and due to the gust excitation, the expression is:

$$
\begin{equation*}
\frac{1}{\mathbf{V}_{\mathbf{T}}}\left\{-\mathbf{w}_{\mathbf{B}}\right\}_{\mathbf{G}}=[\mathbf{F}]\left\{\Delta \mathbf{C}_{\mathbf{p}_{\mathbf{G}}}\right\}_{\mathbf{S B}} \tag{57}
\end{equation*}
$$

### 4.7.3 THIN AND INTERFERENCE BODY AERODYNAMIC FORCES

The relationship between the flow incidence at the thin and interference body box control points and the box lifting pressures is given by the equation:

$$
\frac{1}{\mathbf{V}_{T}}\left\{\begin{array}{l}
\left\{\mathbf{w}_{\mathrm{A}}\right\}_{\mathrm{TB}}  \tag{58}\\
\left\{\mathbf{w}_{\mathrm{A}}\right\}_{\mathrm{IB}}
\end{array}\right\}=[\mathrm{D}]\left\{\begin{array}{l}
\left\{\Delta \mathrm{C}_{\mathbf{p}}\right\}_{\mathrm{TB}} \\
\left\{\Delta \mathrm{C}_{\mathbf{p}}\right\}_{\mathrm{IB}}
\end{array}\right\}
$$

where:
[D] $\quad=A$ complex matrix of normalwash factors relating box lifting pressures to flow incidence at the box control points
$\left\{\mathbf{w}_{\mathbf{A}}\right\}_{\text {TB }},\left\{\mathbf{w}_{\mathbf{A}}\right\}_{\text {IB }} \quad=$ Augmented normalwashes at thin and interference body boxes
$\left\{\Delta \mathrm{C}_{\mathbf{p}}\right\}_{\text {TB }},\left\{\Delta \mathrm{C}_{\mathbf{p}}\right\}_{\text {IB }} \quad=$ Thin and interference body lifting pressures.
The matrix, [D], is a frequency-dependent matrix and is calculated at each frequency of interest. The augmented normalwash of equation (58) is the sum of the normalwash due to the motion of the bodies or due to the gust encounter plus the normalwash induced by the slender bodies.

## Response Aerodynamic Forces

By the definition of an interference body, that is, a body that does not generate its own lift, the normalwash due to interference body motion is set to zero. The thin body normalwash due to the airplane motions is given by equation (13). Using the induced normalwash expression of equation (56), the augmented response normalwash becomes:

$$
\begin{align*}
& \frac{1}{\bar{V}_{T}}\left\{\begin{array}{l}
\left\{w_{A_{R}}\right\}_{\mathrm{TB}} \\
\left\{\mathbf{w}_{A_{R}}\right\}_{I B}
\end{array}\right\}=\frac{1}{\bar{V}_{T}}\left\{\begin{array}{l}
\left\{w_{R}\right\}_{\mathrm{TB}} \\
\{0 \quad\}_{\mathrm{IB}}
\end{array}\right\}+[\mathrm{F}]\left\{\Delta \mathrm{C}_{\mathbf{p}_{\mathrm{R}}}\right\}_{\mathrm{SB}} \\
& \left.=\left[\begin{array}{c}
{\left[-\left[\phi_{\delta_{\mathbf{z}}}^{\prime}\right]_{\mathrm{TB}}-\frac{\mathrm{i} \omega}{\mathrm{~V}_{\mathrm{T}}}\left[\phi_{\delta_{\mathbf{Z}}}\right]_{\mathrm{TB}}\right.} \\
{[0]_{\mathrm{IB}}}
\end{array}\right]+[\mathrm{F}]\left[\mathrm{CPR}_{\mathrm{SB}}\right]\right]\{\mathrm{q}\} \tag{59}
\end{align*}
$$

Substituting equation (59) into (58) and solving for the box pressures gives:

$$
\left.\begin{array}{rl}
\left\{\begin{array}{c}
\left\{\Delta \mathrm{C}_{\mathrm{p}}\right\}_{\mathrm{TB}} \\
\left\{\Delta \mathrm{C}_{\mathbf{p}_{\mathrm{R}}}\right\}_{\mathrm{IB}}
\end{array}\right\} & =[\mathrm{D}]^{-1}\left[\begin{array}{cc}
-\left[\phi_{\delta_{\mathrm{z}}^{\prime}}^{\prime}\right] & -\frac{i \omega}{\mathrm{~V}_{\mathrm{T}}}\left[\phi_{\delta_{\mathrm{z}}}\right] \\
& {[0]_{\mathrm{TB}}}
\end{array}\right]+[\mathrm{F}]\left[\mathrm{CPR}_{\mathrm{SB}}\right] \tag{60}
\end{array}\right]\{\mathrm{q}\}
$$

Multiplying equation (60) by the matrix of box areas and the dynamic pressure gives the response aerodynamic forces for thin and interference bodies:

$$
\left\{\mathrm{F}_{\mathrm{R}}\right\}_{\mathrm{TB} / \mathrm{IB}}=\bar{q}\left[\mathrm{~A}_{\mathrm{TB} / \mathrm{IB}}\right]\left\{\begin{array}{l}
\left\{\mathrm{C}_{\mathbf{p}_{\mathbf{R}}}\right\}_{\mathrm{TB}}  \tag{61}\\
\left\{\mathrm{C}_{\mathbf{p}_{\mathrm{R}}}\right\}_{\mathrm{IB}}
\end{array}\right\}
$$

## Excitation Aerodynamic Forces

The augmented gust normalwash is obtained by adding the gust normalwash defined for the thin body boxes and given by equation (20) to the induced normalwash due to the gust slender body forces given by equation (57).

Note that by the definition of an interference body, the normalwash on the interference body boxes due to a gust encounter is set equal to zero.

Substituting equation (60) into (58) and solving for the lifting pressures, results in:

$$
\begin{align*}
& =\left[\left[\text { CPG }_{T B / I B}\right]_{R e}+i\left[\text { CPG }_{T B / I B}\right]_{I m}\right]\left\{e^{-i \omega \frac{f}{V_{T}}}\right\} \overline{\mathbf{v}}_{G_{V / L}} e^{i \omega t} \tag{63}
\end{align*}
$$

The gust forces on the boxes become:

$$
\begin{align*}
\left\{\mathrm{F}_{\mathrm{G}}\right\}_{\mathrm{TB} / \mathrm{IB}} & =\overline{\mathrm{q}}\left[\mathrm{~A}_{\mathrm{TB} / \mathrm{IB}}^{\circ}\right]\left[\mathrm{CPG}_{\mathrm{TB} / \mathrm{IB}}\right]\left\{\mathrm{e}^{\left.-\mathrm{i} \omega \frac{\mathrm{f} \ell}{\overline{\mathrm{~V}}_{\mathrm{T}}}\right\} \overline{\mathrm{v}}_{\mathrm{G}_{\mathrm{V} / \mathrm{L}}} e^{\mathrm{i} \omega \mathrm{t}}}\right.  \tag{64}\\
& =\left[\mathrm{FG}_{\mathrm{TB} / \mathrm{IB}}\right]\left\{e^{-\mathrm{i} \omega \frac{\mathrm{f}}{\mathrm{~V}_{\mathrm{T}}}}\right\} \overline{\mathrm{v}}_{\mathrm{G}_{\mathrm{V} / \mathrm{L}}} e^{\mathrm{i} \omega \mathrm{t}}
\end{align*}
$$

### 4.7.4 GENERALIZED AERODYNAMIC FORCE COEFFICIENTS

## Total Aerodynamic Forces

The total aerodynamic response and gust forces are obtained by summing the interference body box forces and the respective slender body segment forces, and then multiplying the resultant slender body forces for bodies on the plane of symmetry by two. The multiplication is needed since the modeling in the Doublet Lattice program is for half an airplane.

The total response aerodynamic forces can be written as:

$$
\begin{align*}
\left\{\mathrm{F}_{\mathrm{R}}\right\} & \left.=[\mathrm{FAC}]\left[\begin{array}{l}
{[0]} \\
{\left[\mathrm{FR}_{\mathrm{SB}}\right]}
\end{array}\right]+[\Sigma]\left[\mathrm{FR}_{\mathrm{TB} / \mathrm{IB}}\right]\right]\{q\}  \tag{65}\\
& =\left[[\mathrm{FR}]_{\mathrm{Re}}+\mathrm{i}[\mathrm{FR}]_{\mathrm{Im}}\right]\{q\}
\end{align*}
$$

Again, using the harmonic relationship between velocity and displacement, the response force of equation (65) can be written in terms of coefficients of $q$ and $q$. Thus:

$$
\begin{equation*}
\left\{\mathrm{F}_{\mathrm{R}}\right\}=[\mathrm{FR}]_{\mathrm{Re}}\{\mathrm{q}\}+\frac{1}{\omega}[\mathrm{FR}]_{\mathrm{Im}}\{\dot{\mathrm{q}}\} \tag{66}
\end{equation*}
$$

Similiarly, the total aerodynamic gust forces are:

$$
\left.\begin{array}{rl}
\left\{F_{G}\right\} & =[F \AA C]\left[\left[\begin{array}{l}
{[0]} \\
{\left[F_{S B}\right]}
\end{array}\right]+[\Sigma]\left[F G_{T B / I B}\right]\right. \tag{67}
\end{array}\right]\left\{e^{-i \omega \frac{f}{V_{T}}}\right\} \bar{v}_{G_{V / L}} e^{i \omega t}
$$

The summing matrix in equations (65) and (67) is similar to that used in the FLEXSTAB AIC option, but with an arrangement corresponding to the Doublet Lattice ordering. In submatrix form, it is shown below:

$$
[\Sigma]=\left[\begin{array}{l}
{[0][0][0]}  \tag{68}\\
{[0]\left[\Sigma_{y}\right][0]} \\
{[0][0]\left[\Sigma_{z}\right]}
\end{array}\right]
$$

where:

$$
\begin{array}{ll}
{\left[\Sigma_{y}\right]} & =[\mathrm{PC}]\left[\sin \theta_{\mathrm{IB}}\right] \\
{\left[\mathrm{\Sigma}_{\mathrm{z}}\right]} & =[\mathrm{PC}]\left[\cos \theta_{\mathrm{IB}}\right]
\end{array}
$$

The panel correlation matrix, [PC], performs the same function of adding all the interference boxes to the proper slender body segments. In terms of the Doublet Lattice box dihedral angle, $\gamma$, the box unit normal orientation is:

$$
\begin{equation*}
\theta_{\mathrm{IB}}=360^{\circ}-\gamma \tag{69}
\end{equation*}
$$



## Generalized Forces

Forming a matrix of normal displacements at the segment and box force points, the generalized response forces are given by:

$$
\begin{align*}
\left\{\mathrm{Q}_{\mathrm{R}}\right\} & =\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{TB} / \mathrm{SB}}^{\mathrm{T}}[\stackrel{\circ}{\mathrm{SF}][\mathrm{FR}]\{q\}}  \tag{70}\\
& =[\mathrm{QR}]\{\mathrm{q}\}
\end{align*}
$$

and the generalized excitation forces are given by:

$$
\begin{align*}
& \left\{Q_{E}\right\}=\left[\phi_{\delta_{n}}\right]_{T B / S B}^{T}[\stackrel{\circ}{S F}][F G]\left\{e^{-i \omega \frac{f}{V_{T}}}\right\} \bar{v}_{G_{V / L}} e^{i \omega t}  \tag{71}\\
& =[\tilde{\phi}]\left\{e^{-i \omega \frac{f^{\ell}}{V_{T}}}\right\} \overline{\bar{v}}_{G_{V / L}} e^{i \omega t}
\end{align*}
$$

Bringing the generalized response forces to the left side of the equations of motion and writing them as coefficients of $q$ and $\dot{q}$, the $M_{4}$ and $M_{5}$ matrices become:

$$
\begin{align*}
& {\left[\mathrm{M}_{4}\right]=-[\mathrm{QR}]_{\operatorname{Re}}}  \tag{72}\\
& {\left[\mathrm{M}_{5}\right]=-\frac{1}{\omega}[\mathrm{QR}]_{\mathrm{Im}}}
\end{align*}
$$

The $\mathrm{C}_{3}$ matrix is defined as:

$$
\begin{equation*}
\left\{C_{3}\right\}=[\mathscr{\phi}]\left\{e^{-i \omega^{\frac{f_{\ell}}{V_{\mathbf{T}}}}}\right\} \tag{73}
\end{equation*}
$$

### 4.7.5 QUASI-STEADY FORMULATION

The quasi-steady formulation in the Doublet Lattice AIC option is accomplished using the aerodynamic data ([D] and [F] matrices) calculated at $\omega=0$, which makes them real matrices. Also, the harmonic motion relationship between $q$ and $\dot{q}$ is not used, and the normalwash is written as real coefficients of generalized coordinate displacements and velocities.

Setting $\omega=0$ in equation (50) and grouping terms as coefficients of $q$ and $\dot{q}$, the response slender body lifting pressures are given by:

$$
\begin{align*}
& \left\{\Delta \mathbf{C}_{\mathbf{p}_{\mathbf{R}}}\right\}_{\mathrm{SB}}=-2 \pi\left[\left[\mathrm{r}_{\mathrm{S}}^{\prime}\right]_{\left[\phi_{\delta_{\mathbf{n}}}^{\prime}\right]_{\mathrm{CP}}}+\frac{1}{2}\left[\mathrm{r}_{\mathrm{S}}\right]\left[\phi_{\delta_{\mathbf{n}}}^{\prime \prime}\right]_{\mathrm{CP}}\right]\{\mathrm{q}\} \\
& -\frac{2 \pi}{\mathrm{~V}_{\mathrm{T}}}\left[\left[\mathrm{r}_{\mathrm{s}}^{\prime}\right]\left[\phi_{\delta_{\mathrm{n}}}\right]_{\mathrm{CP}}+\frac{1}{2}\left[\mathrm{r}_{\mathrm{s}}\right]\left[\phi_{\delta_{\mathrm{n}}}^{\prime}\right]_{\mathrm{CP}}\right]\{\dot{\mathrm{q}}\}  \tag{74}\\
& =\left[{ }^{\mathbf{C P} 1_{R}}\right]_{\mathbf{S B}}\{\mathrm{q}\}+\left[\mathrm{CP}_{\mathbf{R}}\right]_{\mathbf{S B}}\{\dot{\mathrm{q}}\}
\end{align*}
$$

The expression for the slender body lifting pressures due to gust is given by equation (54).

The augmented normalwash on the thin and interference boxes becomes:

$$
\begin{align*}
\frac{1}{\mathbf{V}_{\mathrm{T}}}\left\{\begin{array}{l}
\left\{\mathbf{w}_{\mathbf{A}_{\mathrm{R}}}\right\}_{\mathrm{TB}} \\
\left\{\mathbf{w}_{\mathbf{A}_{\mathbf{R}}}\right\}_{\mathrm{TB}}
\end{array}\right\}= & \left.=\left[\begin{array}{l}
-\left[\phi_{\delta_{\mathrm{Z}}}\right]_{\mathrm{TB}} \\
{[0]_{\mathrm{IB}}}
\end{array}\right]+[\mathrm{F}]\left[\mathrm{CP1}_{\mathrm{R}}\right]_{\mathrm{SB}}\right]\{\mathrm{q}\}  \tag{75}\\
& +\left[\left[\begin{array}{l}
-\frac{1}{\mathrm{~V}_{\mathrm{T}}}\left[\phi_{\delta_{\mathrm{Z}}}\right]_{\mathrm{TB}} \\
{[0]_{\mathrm{IB}}}
\end{array}\right]+[\mathrm{F}]\left[\mathrm{CP} 2_{\mathrm{R}}\right]_{\mathrm{SB}}\right]\{\dot{\mathrm{q}}\}
\end{align*}
$$

where the [F] matrix is now real.

Using equation (75) with equation (60), the response lifting pressures on thin and interference boxes are:

$$
\left\{\begin{array}{l}
\left\{\Delta \mathrm{C}_{\left.\mathbf{p}_{R}\right\}_{\mathrm{TB}}}\right.  \tag{76}\\
\left\{\Delta \mathrm{C}_{\left.\mathbf{p}_{R}\right\}_{\mathrm{IB}}}\right.
\end{array}\right\}=[\mathrm{D}]^{-1}\left[\mathrm{WA1}_{\mathrm{R}}\right]_{\mathrm{TB} / \mathrm{IB}}\{\mathrm{q}\}+[\mathrm{D}]^{-1}\left[\mathrm{WA}_{\mathrm{R}}\right]_{\mathrm{TB} / \mathrm{IB}}\{\dot{\mathrm{q}}\}
$$

With the lifting pressures established, the aerodynamic forces and generalized forces are obtained by substituting equations (74) and (76) into the proper force equation given in section 4.7-4. It should be noted that the gust forces computed by the two approaches differ in that the [D] and [F] matrices are real for the quasi-steady assumption.

### 4.8 DOUBLET LATTICE GENERALIZED FORCE AND PRESSURE OPTION

The last aerodynamic option is the Doublet Lattice generalized force and pressure option. In this option, the lifting pressures on all slender body segments, thin and interference body boxes, and the generalized response forces are calculated by the Doublet Lattice program, L216. The L217 (EOM) program uses the lifting pressures to calculate the aerodynamic forces on each aircraft component, places the generalized forces in the format required for the equations of motion, and uses the response forces to generate gust excitation forces. No modal data is required from L215 (INTERP) and the formulation of the excitation aerodynamics does not permit gradual penetration effects to be taken into account.

## Aerodynamic Forces

The complete lifting pressure matrix read from the Doublet Lattice program is a frequency-dependent complex matrix containing all slender body segment, thin and interference body box pressures. The ordering follows that used in the Doublet Lattice program, which is:

- Thin bodies on the plane of symmetry followed by thin bodies off the plane of symmetry.
- Interference bodies on the plane of symmetry followed by those off the plane of symmetry.
- All $y$-slender bodies on the plane of symmetry followed by all those off the plane of symmetry.
- All z-slender bodies on the plane of symmetry followed by those off the plane of symmetry.

$$
\left\{\Delta \mathrm{C}_{\mathbf{p}_{\mathbf{R}}}\right\}=\left[\begin{array}{l}
{\left[\mathrm{CPR}_{\mathrm{TB}}\right]}  \tag{77}\\
{\left[\mathrm{CPR}_{\mathrm{IB}}\right]} \\
{\left[\mathrm{CPR}_{\mathrm{SB}}\right]} \\
{\left[\mathrm{CPR}_{\mathrm{SB}}\right]}
\end{array}\right]\{\mathrm{q}\}
$$

Multiplying the pressure matrix by the box and segment areas, summing the interference body boxes onto the respective slender body segments, and multiplying them by the factor matrix which accounts for slender bodies on the plane of symmetry, the total set of response forces is:

$$
\begin{align*}
\left\{\mathrm{F}_{\mathrm{R}}\right\} & =\overline{\mathbf{q}}[\mathrm{FAC}][\Sigma][\AA][\mathrm{CPR}]\{\mathrm{q}\} \\
& =\overline{\mathbf{q}}\left[\mathrm{F}_{12}\right]\{\mathrm{q}\} \tag{78}
\end{align*}
$$

Using the displacement-velocity harmonic relationship shown in equation (51), the forces can be written as coefficients of $q$ and $\dot{q}$.

$$
\begin{align*}
& \left\{\mathrm{F}_{\mathbf{R}}\right\}=\overline{\mathrm{q}}\left[\mathrm{~F}_{12}\right]_{\text {Re }}\{\mathrm{q}\}  \tag{79}\\
& \left\{\mathrm{F} 2_{\mathrm{R}}\right\}=\frac{\overline{\mathrm{q}}}{\omega}\left[\mathrm{~F}_{12}\right]_{\text {Im }}\{\dot{\mathrm{q}}\}
\end{align*}
$$

The gust forces are obtained by developing the relationship between the gust velocity and the rigid body displacements and then extracting the proper rigid body columns from the force matrix to form the aerodynamic forces due to gust. The sign convention shown in figure 6 will be used along with that shown in figure 2 to establish the relationships.

For a symmetrical analysis, the airplane experiences the rigid body motions of plunge and pitch. The plunge of the airplane will give the same aerodynamic effect as a positive gust encounter. Therefore, the relationship between the vertical gust velocity and the vertical translation generalized coordinate is given by:

$$
\begin{equation*}
v_{G_{V}}=\dot{q}_{V T}=i \omega q_{V T} \tag{80}
\end{equation*}
$$

The pitch generalized coordinate is related to the vertical gust velocity by the equation:

$$
\begin{equation*}
\mathbf{v}_{\mathbf{G}_{\mathbf{V}}}=\mathbf{V}_{\mathbf{T}} \mathbf{q}_{\mathbf{p}} \tag{81}
\end{equation*}
$$

For an antisymmetric analysis, the rigid body motions of side translation and yaw are used to derive the gust relationships. The side translation generalized coordinate/lateral gust relationship is:

$$
\begin{equation*}
\mathbf{v}_{\mathbf{G}_{\mathrm{L}}}=-\dot{\mathbf{q}}_{\mathbf{S T}}=-\mathrm{i} \omega \mathrm{q}_{\mathbf{S T}} \tag{82}
\end{equation*}
$$

and the yaw/lateral gust equation is:

$$
\begin{equation*}
v_{G_{L}}=V_{T} q_{Y} \tag{83}
\end{equation*}
$$

Equations (81) and (83) are used in obtaining zero frequency gust forces, while equations (80) and (82) are used for all nonzero frequency gust forces.

Extracting the proper rigid body column from the $\left[\mathrm{F}_{12}\right]$ matrix shown in equation (78), the gust forces at $\omega=0$ are:

$$
\begin{equation*}
\left\{F_{G}\right\}_{S / A}=\bar{q}\left\{F_{12}^{\prime}\right\} \dot{\mathrm{q}}_{\mathrm{P} / \mathrm{Y}}=\frac{\overline{\mathrm{q}}}{\mathrm{~V}_{\mathrm{T}}}\left\{\mathrm{~F}^{\prime}{ }_{12}\right\} \mathrm{V}_{\mathrm{G}_{\mathrm{V} / L}} \tag{84}
\end{equation*}
$$

and for $\omega>0$ the gust forces are:

$$
\begin{equation*}
\left\{\mathrm{F}_{\mathrm{G}}\right\}_{\mathrm{S} / \mathrm{A}}=\overline{\mathrm{q}}\left\{\mathrm{~F}_{12}^{\prime}\right\} \dot{\mathrm{q}}_{\mathrm{VT} / \mathrm{ST}}=\delta_{\mathrm{S} / \mathrm{A}} \frac{\mathrm{i} \overline{\mathrm{q}}}{\omega}\left\{\mathrm{~F}^{\prime}{ }_{12}\right\} \mathrm{V}_{\mathrm{G}_{\mathrm{V} / \mathrm{L}}} \tag{85}
\end{equation*}
$$

where:
$\left\{F^{\prime}{ }_{12}\right\}=A$ complex column matrix of aerodynamic force coefficients formed by extracting the proper rigid body column from the $\left[\mathrm{F}_{12}\right]$ matrix given by equation (78)

1 for a symmetrical analysis.
$\delta_{\mathrm{S}} / \mathrm{A}=-1$ for an antisymmetrical analysis.

## Generalized Aerodynamic Force Coefficients

The generalized response force coefficients are read from the Doublet Lattice program in the form of complex matrices calculated at each $\omega$ value of interest. Relating these coefficients to the $\left[\mathrm{M}_{4}\right]$ and $\left[\mathrm{M}_{5}\right.$ ] matrices provides:

$$
\begin{align*}
& {\left[\mathrm{M}_{4}\right]=-2 \overline{\mathrm{q}}[\mathrm{QR}] \mathrm{Re}}  \tag{86}\\
& {\left[\mathrm{M}_{5}\right]=-\frac{2 \overline{\mathrm{q}}}{\omega}[\mathrm{QR}]_{\mathrm{Im}}}
\end{align*}
$$

The generalized excitation forces are formed by again extracting the proper rigid body columns from the generalized response force coefficients. Since there is no gradual penetration, the $[\tilde{\phi}]$ matrix degenerates to a $\left\{C_{3}\right\}$ matrix. For $\omega=0$, the generalized excitation is:

$$
\begin{equation*}
\left\{\mathbf{Q}_{\mathrm{E}}\right\}=\left\{\mathrm{C}_{3}\right\} \mathbf{v}_{\mathrm{G}_{\mathrm{V} / L}}=\frac{2 \overline{\mathrm{q}}}{\mathrm{~V}_{\mathrm{T}}}\left\{\mathrm{Q}^{\prime}\right\} \mathbf{v}_{\mathrm{G}_{\mathrm{V} / L}} \tag{87}
\end{equation*}
$$

and for $\omega>0$ :

$$
\begin{equation*}
\left\{Q_{\mathrm{E}}\right\}=\left\{\mathrm{C}_{3}\right\} \mathrm{v}_{\mathrm{G}_{\mathrm{V} / \mathrm{L}}}=2 \delta_{\mathrm{S} / \mathrm{A}} \frac{\mathrm{iq}}{\omega}\left\{\mathrm{Q}^{\prime}\right\} \mathrm{v}_{\mathrm{G} / \mathrm{L}} \tag{88}
\end{equation*}
$$

where:
$\left\{Q^{\prime}\right\} \quad=A$ complex column matrix of generalized aerodynamic force coefficients formed by extracting the proper rigid body column from [QR] matrix.

### 4.9 COORDINATE TRANSFORMATION AND AERODYNAMIC AXIS SHIFTING

All interpolation that is performed in L217 (EOM) using the SA arrays from L215 (INTERP) is carried out in the local structural axis system of each aircraft component (sec. 4.7 of ref. 6). The aerodynamic geometry data is input in the reference axis system and, therefore, it must be transformed into the local structural axis for interpolation. This transformation is complicated by two factors. First, in most cases the origin of the local structural axis system and the origin of the local aerodynamic axis system are not the same. Second, due to the requirements for good aerodynamic modeling, certain components may have to be placed outboard or inboard, above or below, or fore or aft, of their actual locations. Therefore, for proper interpolation, the reference axis system locations of points on such a surface must be corrected in order to reflect their true orientation.

In order to account for the difference, the transformation equation for each component has coordinate shift values which can be used to define the correct position of any surface points. The transformation equation for any point on a surface is:
where:
$\mathrm{x}_{\ell \mathrm{S}}, \mathrm{y}_{\ell \mathrm{s}}, \mathrm{z}_{\ell \mathrm{S}} \quad=$ Local structural axis system location of any point on a surface
$\mathbf{X}_{\text {RAS }}, \mathrm{Y}_{\text {RAS }}, \mathrm{Z}_{\text {RAS }} \quad=$ Reference axis system location of the point
$\mathbf{X}_{\text {OARAS }}, \mathrm{Y}_{\text {OARAS }}, \mathrm{Z}_{\text {OARAS }} \quad=$ Reference axis system location of the origin of the aerodynamic axes of the surface
 properly orient the aerodynamic axes to the structural axes
[R]
$=$ Rotation matrix that rotates the coordinates from the reference axis system into the local structural axis system

The rotation matrix is defined as:

$$
[R]=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{90}\\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{array}\right]
$$

with $\theta$ being the surface dihedral angle.
The reference axis system coordinates of the point and the origin of the aerodynamic axis system, and the surface dihedral angle are obtained from the aerodynamic geometry data read by L217 (EOM). The shifting values are input by the user for each surface.

To understand the calculation of the shifting values, consider the horizontal tail shown in figure 13. Here, a typical aft body modeling problem is illustrated. Both FLEXSTAB and Doublet Lattice require interference bodies to be of constant cross-section. Thus, in the aft portions of the fuselage where in reality the body tapers, the interference body must be kept constant. This could place the horizontal tail outboard, forward, aft, above or below its actual location. Just calculating the differences between the local aerodynamic and structural axis system origin would not reflect the true position of the aerodynamic points with respect to the interpolation axes (which are the local structural axes). Therefore, a set of correction values are needed.

Taking into account these corrections, the equation for the shift values is:

$$
\left\{\begin{array}{l}
\mathrm{x}_{\mathrm{SH}}  \tag{91}\\
\mathrm{YSH} \\
\mathrm{Z}_{\mathrm{SH}}
\end{array}\right\}=\left\{\begin{array}{l}
\mathrm{x}_{\mathrm{OA}} \\
\mathrm{Y}_{\mathrm{OA}} \\
\mathrm{Z}_{\mathrm{OA}}
\end{array}\right\}-\left\{\begin{array}{l}
\mathrm{x}_{\mathrm{OS}} \\
\mathrm{Y}_{\mathrm{OS}} \\
\mathrm{Z}_{\mathrm{OS}}
\end{array}\right\}+\left(\begin{array}{l}
\mathrm{x}_{\mathrm{C}} \\
\mathrm{Y}_{\mathrm{C}} \\
\mathrm{Z}_{\mathrm{C}}
\end{array}\right\}
$$

RAS RAS
where:
$\mathrm{X}_{\mathrm{OS}}^{\text {RAS }}, ~ \mathrm{Y}_{\mathbf{O S}}$ RAS, $\mathrm{Z}_{\mathbf{O S}} \quad=\begin{array}{r}\text { Reference axis system location of the structural } \\ \text { axis system used for the SA array. }\end{array}$
$\mathbf{X}_{\mathbf{C}_{\mathbf{R A S}}}, \mathrm{Y}_{\mathbf{C}_{\mathbf{R A S}}}, \mathrm{Z}_{\mathbf{C}_{\mathbf{R A S}}} \quad=$ Desired correction values to apply to the aerodynamic points. These values are defined in the reference axis system and are positive in the positive reference axis system directions measuring from the aerodynamic axis system to the structural axis system.
The proper calculation of the shifting values requires the user to know the location of the local structural axis system origin used in L215 (INTERP), and the local aerodynamic axis system origin established by FLEXSTAB or Doublet Lattice. This last


$$
\begin{aligned}
X_{\mathrm{C}} & =a \\
\mathrm{Y}_{\mathrm{C}} & =-\mathrm{b} \\
\mathrm{Z}_{\mathrm{C}} & =-\mathrm{c}
\end{aligned}
$$



Figure 13. - Aerodynamic Axis Shifting
value for FLEXSTAB aerodynamics is defined by the analyst when setting up the FLEXSTAB idealization. Doublet Lattice, however, defines the aerodynamic origin internally. In Doublet Lattice, the aerodynamic origins are defined in the following way:

- For slender bodies, the origin is the leading edge location of the first segment.
- For thin bodies (or panels), the origin is the $\mathrm{X} 1, \mathrm{Y} 1, \mathrm{Z} 1$ coordinates of the panel.


### 5.0 PROGRAM STRUCTURE AND DESCRIPTION

Program L217 (EOM) is structured as an overlay system consisting of a main overlay and four primary overlays listed below and displayed in figure 14.
(L217,0,0) L217vc
(L217,2,0) STRUCT
(L217,3,0) FLXAIC
(L217,4,0) DLAIC
(L217,5,0) DLPRES

### 5.1 PROGRAM L217vc

The main overlay,(L217,0,0), is named L217vc (the " $v$ " and " $c$ " are version and correction identifiers). It reads data cards that direct the execution of the primary overlays and also aids communication between the primary overlays via labelled common blocks.

### 5.2 PROGRAM STRUCT

The 2,0 primary overlay, STRUCT (L217,2,0), generates the equations of motion matrices that are dependent upon the structural data only.
[ $\left.M_{1}\right]=$ Generalized stiffness
[ $\mathrm{M}_{2}$ ] = Generalized damping (optional)
[ $\mathrm{M}_{3}$ ] = Generalized inertia
Depending on the options chosen, STRUCT may read from several magnetic files:
GSTIFF A generalized stiffness matrix $[\mathrm{K}] .\left([\mathrm{K}]=\left[\mathrm{M}_{1}\right]_{\mathrm{Ua}}\right)$
GMASS A generalized mass matrix $[\mathrm{M}] .\left([\mathrm{M}]=\left[\mathrm{M}_{3}\right]_{\mathrm{Ua}}\right.$ )
SATAP Arrays of modal data to be used in the generation of control surface freedoms to be added to $[K]$ and $[M]$.

Because certain constants ( $\overline{\mathrm{q}}$ and $\mathrm{V}_{\mathrm{T}}$ ) and defined in STRUCT, and because the matrices generated by STRUCT are required in each of the aerodynamic overlays, STRUCT must be executed prior to any of the aerodynamic overlays.


Generate aerodynamic equations of motion matrices
$\left(\left[M_{4}\right],\left[M_{5}\right]\right.$, and $\left.[\widetilde{\phi}]\right)$, plus the matrices of
aerodynamic forces per surface ( $\left[F_{1}\right],\left[F_{2}\right]$, and $\left.[F G]\right)$.

Figure 14. - L217 (EOM) Overlay Structure

### 5.3 AERODYNAMIC DATA PROCESSING

The remaining primary overlays generate the equations of motion matrices that are dependent upon the aerodynamic data. The matrices are:
[FREQM] = Array of frequencies at which the aerodynamics are defined
[ $M_{4}$ ] $=$ Generalized aerodynamic stiffness matrix
[ $\mathbf{M}_{5}$ ] = Generalized aerodynamic damping matrix
$\left\{\mathrm{f}_{\ell}\right\} \quad=$ Array distance(s) from the gust reference point to the gust control point(s) of the gust zones
[ $\overline{\boldsymbol{\sigma}}] \quad=$ Gust forcing function matrix
The matrices are written onto the file "EOMTAP" following the structural matrices generated by STRUCT.

In addition, the aerodynamic overlays produce the aerodynamic force matrices per surface and write them onto the file "EOMLOD" for subsequent use in L218 (LOADS) (ref. 9). The file includes the following matrices for each surface:
[ $\mathrm{x}, \mathrm{y}] \quad=$ Local axis coordinates of surface node points
$\{A\} \quad=$ Area of surface elements
[ $\mathrm{F}_{1}$ ] = Real part of aerodynamic response forces
[ $\mathrm{F}_{2}$ ] = Imaginary part of aerodynamic response forces
[FG] = Gust forces
The last three matrices are repeated for each frequency at which they are defined.

### 5.3.1 PROGRAM FLXAIC

The 3,0 primary overlay, FLXAIC (L217,3,0), generates the aerodynamic equations of motion and force matrices using the FLEXSTAB (ref. 7) aerodynamic influence coefficient (AIC) matrix. FLXAIC reads from the files:
"GDTAPE" FLEXSTAB geometry data.
"SDINDX" Table describing the file of aerodynamic data, SDDATA.
"SDDATA" AIC matrix [ $A_{P_{\Theta}}$ ] for a symmetric or antisymmetric case.
"SATAP" Arrays of modal data.

### 5.3.2 PROGRAM DLAIC

The 4,0 primary overlay, DLAIC (L217,4,0), generates the aerodynamic equations of motion and force matrices using the Doublet Lattice (ref. 8) AIC matrices. DLAIC reads from the files:
"NGETP" Doublet Lattice geometry data.
"NAETP" Doublet Lattice aerodynamic data ([F], [D] ${ }^{-1}$, etc.).
"SATAP" Arrays of modal data.

### 5.3.3 PROGRAM DLPRES

The 5,0 primary overlay, DLPRES (L217,5,0), generates the aerodynamic equations of motion and force matrices using the Doublet Lattice (ref. 8) generalized forces and unsteady aerodynamics. DLPRES reads from the files:
"NGETP" Doublet Lattice geometry data.
"NAETP" Doublet Lattice aerodynamic data ([Q],[ $\Delta C P]$, etc.).
Although L217 (EOM) serves as a module of the DYLOFLEX system, it can be operated as a standalone program. L217 (EOM) is heavily dependent upon data generated prior to its execution. Figure 15 displays the different files that the program may require, according to user specified options. When the program is run by itself, it becomes the user's responsibility to generate the files of input data in the format required by the program.


Figure 15. - L217 (EOM) Communication Via External Files

### 6.0 COMPUTER PROGRAM USAGE

The program was designed for use on the CDC 6600 . The machine requirements to execute L217 (EOM) are:

Card reader Read control cards and card input data.
Printer Print standard output information, optional intermediate calculations, and diagnostic messages.

Disk storage
All magnetic files not specifically defined as magnetic tapes are assumed to be disk files.

Tape drive For permanent storage of data. Magnetic files are copied to and from magnetic tapes with control cards before and after program execution.

The program L217 (EOM) is written in FORTRAN and may be compiled with either the RUN or FTN compiler. L217 (EOM) may be executed on either the KRONOS 2.1 or NOS operating system.

### 6.1 CONTROL CARDS

The following list is a typical set of control cards used to execute L217 (EOM) using the absolute binaries from the program's master tape.

Job Card
Account Card

---End-of-file

The following list is a typical set of control cards used to execute L217 (EOM) using the relocatable binaries from the program's master tape.


### 6.2 RESOURCE ESTIMATES

The computer resources used (core requirements, tapes, printed output, central processor seconds, etc.) are a function of the problem size and the program options used.

## Field Length

The field length (core) required by L217 (EOM) is dependent upon the problem size and the program module(s) used. Core must be requested for the largest module run, either the structural data overlay or one of the aerodynamic overlays. The structural and aerodynamic overlays (primaries) are broken into even smaller chunks called secondary overlays. Therefore, core must be requested for the longest secondary overlay to be executed. The equation to calculate the field length of an overlay is

$$
\text { Field Length }=\mathrm{FL}_{\mathbf{p}}+\mathrm{FL}_{\mathbf{a}}
$$

where $\mathrm{FL}_{\mathrm{p}}$ is the program length, and $\mathrm{FL}_{\mathbf{a}}$ is the array storage required.
The following pages contain $\mathrm{FL}_{\mathbf{p}}$ and a simplified formula for $\mathrm{FL}_{\mathrm{a}}$ for all L217 (EOM) secondary overlays. The user must determine which ones will be executed, which is the longest for the problem to be run, and the required field length.

It should be noted that $\mathrm{FL}_{\mathrm{p}}$ is given as an octal base number. Normally, $\mathrm{FL}_{\mathrm{a}}$ will be calculated as a decimal number and converted to octal before being added to $\mathrm{FL}_{\mathbf{p}}$.

The structural overlay is relatively short. One of the aerodynamic overlays will usually be the largest.

The following abbreviations are used in the field length formulas:
NDOF $_{C S} \quad$ Number of control surface freedoms being added (card 2.1).
NDOFST $\quad$ Number of original structural freedoms (card 2.1).
NMODES Number of modes defined (number of generalized coordinates total) $=$ $\mathrm{NDOF}_{S T}+\mathrm{NDOF}_{\mathrm{CS}}$.
$M_{\text {max }} \quad$ Minimum of 20 or NMODES.
NELE Number of control surface masses and inertias added to the structural matrices (card 2.9.1).

NGCP $\quad$ Number of gust control points (card 3.9.1).
NIBFP Number of interference body force points.
NSBFP Number of slender body force points.

NTBFP Number of thin body force points.
NAFP Number of output aerodynamic force points $=$ NTBFP + NSBFP.
NBOX Number of aerodynamic boxes $=$ NTBFP + NIBFP.
NTOTF $\quad$ Number of aerodynamic elements total $=$ NTBFP + NIBFP + NSBFP.
$\mathrm{N}_{\text {max }} \quad$ Maximum number of nodes on a surface.
Primary Overlay L217,2,0
Program Name STRUCT
If control surface freedoms are added, FORMAC will be longest.
If not, FORM13 will be the longest.

| Secondary Overlay | Program <br> Name | $\mathrm{FL}_{\mathrm{p}}$ (octal) | $\begin{aligned} & \mathrm{FL}_{\mathrm{a}} \\ & \text { (decimal) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| L217,2,1 | RSTRCT | 35000 | 6000 |
| L217,2,2 | FORMAC | 74000 | $\begin{aligned} & \text { NELE }^{*}\left(\text { NMODES }^{+1}\right)+600 \\ & +\left(\text { NDOF }_{\text {CS }}{ }^{*} \text { NDOF }_{S T}\right) \end{aligned}$ |
| L217,2,3 | FORM13 | 47000 | NMODES ${ }^{2}+$ NMODES |

Primary Overlay L217,3,0

## Program Name FLXAIC

For large problems L217,3,4-FLXFRC will be the longest.
For small problems L217,3,3-FLXMOD will be the longest.

| Secondary Overlay | Program <br> Name | $\mathrm{FL}_{\mathrm{p}}$ (octal) | $\begin{aligned} & \mathrm{FL}_{\mathbf{a}} \\ & \text { (decimal) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| L217,3,1 | FLXCDS | 36000 | 0 |
| L217,3,2 | FLXGEO | 66000 | 0 |
| L217,3,3 | FLXMOD | 73000 | 3*NAFP* ${ }^{\left(M_{\text {max }}+2\right)}$ |
| L217,3,4 | FLXFRC | 42000 | $4^{*}$ NAFP $^{*}\left(\mathrm{M}_{\text {max }}+1\right)+5 *$ NTOTF |
| L217,3,5 | FLXODR | 42000 | $\mathrm{NAFP}^{*}\left(\mathrm{M}_{\text {max }}+5\right)+$ NMODES $^{*} \mathrm{~N}_{\text {max }}$ |
| L217,3,6 | FLXEOM | 42000 | $\begin{aligned} & 2^{*} \text { NMODES }^{*}\left(\text { NMODES }^{2}\right. \text { ) } \\ & + \text { NAFP }^{*}\left(\mathrm{M}_{\max }+1\right) \end{aligned}$ |

Primary Overlay L217,4,0
Program Name DLAIC
For large problems, L217,4,6 -SOLVDW is usually the longest.
For small problems, L217,4,3 -DLAMOD is usually the longest.

| Secondary Overlay | Program <br> Name | $\mathrm{FL}_{\mathrm{p}}$ (octal) | $\begin{aligned} & \mathrm{FL}_{\mathrm{a}} \\ & \text { (decimal) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| L217,4,1 | DLACDS | 37000 | 0 |
| L217,4,2 | DLAGEO | 65000 | 0 |
| L217,4,3 | DLAMOD | 75000 | M max $^{*}$ (3*NAFP + NTBFP $)+5$ + ${ }^{\text {NAFP }}$ |
| L217,4,4 | CPANDW | 40000 | $\begin{aligned} & \text { NTBFP* }\left(\mathrm{M}_{\max }+2^{*} \text { NMODES }\right) \\ & +2^{*}(\text { NAFP }+ \text { NMODES }) \end{aligned}$ |
| L217,4,5 | FORMWS | 41000 | 2*NBOX*NMODES |
| L217,4,6 | SOLVDW | 52000 | $\mathrm{NBOX}^{*}\left(\mathrm{NBOX}+4^{*}(\mathrm{NMODES}+\mathrm{NGCP})+3\right)^{\mathbf{1}}$ |
| L217,4,7 | DLAFRC | 41000 | $\begin{aligned} & 2^{*}\left(\left(\text { NSBFP*}^{*} \text { NMODES }\right)+\right.\text { NBOX } \\ & \left.+\left(\text { NAFP* }_{\text {max }}\right)\right)+5^{*} \text { NTOTF } \end{aligned}$ |
| L217,4,8 | DLAODR | 43000 | $\begin{aligned} & 2^{*}\left(\text { NMODES }^{*} \mathrm{~N}_{\text {max }}+\text { NAFP }^{*} \mathrm{M}_{\text {max }}\right) \\ & +6^{*} \text { NAFP } \end{aligned}$ |
| L217,4,9 | DLAEOM | 43000 | $\begin{aligned} & 2^{*} \text { NMODES* }^{*}\left(\text { NMODES }^{*} \text { NGCP }\right) \\ & + \text { NAFP }^{*}\left(\mathrm{M}_{\text {max }}+1\right) \end{aligned}$ |

${ }^{1}$ The formula given above for SOLVDW is to perform a solution in core. SOLVDW will partition the problem and run with as little as $\mathrm{FL}_{\mathbf{a}}=$ NBOX*(NMODES+NGCP)

Primary Overlay L217,5,0

## Program Name DLPRES

DLPEOM will normally be the longest.

| Secondary Overlay | Program <br> Name | $\mathrm{FL}_{\mathrm{p}}$ (octal) | $\begin{aligned} & \mathrm{FL}_{\mathbf{a}} \\ & \text { (decimal) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| L217,5,1 | DLPCDS | 35000 | 0 |
| L217,5,2 | DLPGEO | 63000 | 0 |
| L217,5,4 | DLPFRC | 40000 | 7*NTOTF +3 * ${ }^{\text {NAFP }}$ |
| L217,5,5 | DLPODR | 42000 | $\mathrm{N}_{\text {max }}{ }^{*}$ (NMODES +4 ) $+8 *$ NAFP |
| L217,5,6 | DLPEOM | 41000 | 4* MMODES $^{2}+2^{*}$ NMODES |

## 'Iime Estimate

The central processing time required ( CP seconds) is dependent upon the options chosen and the problem size. The total time will be the sum of the time required by each overlay executed:

$$
\begin{array}{ll}
\text { CP }_{\text {TOT }} & =\text { CP }_{\text {STRUCT }}+\mathrm{CP}_{\text {FLXAIC }}+\mathrm{CP}_{\text {DLAIC }}+\mathrm{CP}_{\text {DLPRES }} \\
\mathrm{CP}_{\text {STRUCT }} & \leqslant 10 \\
\mathrm{CP}_{\text {FLXAIC }} & \leqslant 10 \\
\text { CP }_{\text {DLAIC }} & \leqslant\left(\frac{\text { NBOX } \left._{40}^{40}\right) * \text { NFREQM }}{}\right. \\
\text { CP }_{\text {DLPRES }} & \leqslant 5 * \text { NFREQM }
\end{array}
$$

where:
NFREQM = Number of frequencies at which the aerodynamics are defined
NBOX $=$ Number of boxes on the thin and interference bodies

## Printed Output

The amount of printed output is dependent upon card options (cards 2.4, 2.6, 2.7 and 3.11), the size of the arrays printed, and the number of frequencies at which aerodynamic data is processed. At compilation time, the print line limit is set to 40000 which should be sufficient for any single L217 (EOM) data case.

### 6.3 CARD INPUT DATA

The task(s) performed by L217 (EOM) are broken into several subtasks, each with its own section of code known as a primary overlay. The entire set of primary overlays is driven by a small program (main overlay) named L217vc.

L217ve reads program directive cards to:

- Assure that the data being read is intended for EOM
- Determine which section of code (primary overlay) of EOM is to be executed next.

Each primary overlay reads one set of card input data. A listing of the primary overlays and the card set numbers follows. The same card set (3) is read by all overlays processing aerodynamic data.

| Overlay Name | Purpose of Primary Overlay | Card Set Read |
| :---: | :---: | :---: |
| STRUCT | Generate $\left[\mathrm{M}_{1}\right],\left[\mathrm{M}_{2}\right]$ and $\left[\mathrm{M}_{3}\right]$, the equations of motion matrices based upon structural data. | 2 |
| FLXAIC | Generates the aerodynamic coefficient matrices for the equations of motion based on the quasi-steady aerodynamic assumption and using the aerodynamic data from FLEXSTAB (ref. 7) | 3 |
| DLAIC | Generates the aerodynamic coefficient matrices for the equations of motion based using Doublet Lattice AIC's (ref. 8). Either quasi-steady or full unsteady aerodynamics can be used. | 3 |
| DLPRES | Generates the aerodynamic coefficient matrices for the equations of motion using the Doublet Lattice generalized forces (ref. 8) for full unsteady aerodynamics. | 3 |

The order in which these card sets are input (and the overlays are executed) is displayed in figure 16. The overlay STRUCT must be executed before any of the aerodynamic overlays.


Figure 16. - L217 (EOM) Card Input Data Flow

## Format of Card Input Data

All card data is read in fixed fields (specific columns of the cards). On the pages that follow, the required card columns are defined next to each keyword or variable. The following conventions are used throughout the program:

- All floating point variables are read with format E10.0.
- All integer variables are read with the format I5.
- All hollerith variables (keywords, etc), are read with the format A10.

Therefore, all data fields end on a card column that is a multiple of five.
When the program is trying to recognize keywords, it checks only the first four characters.

### 6.3.1 PROGRAM L217

The driving module of L217 (EOM) (the main overlay itself is also known as L217vc) will read and check the first four characters of the input card to determine the type of execution that is to take place.

Note: All underlined capital characters contained in the KEYWORD/VARIABLE field of the input card sets must be left-justified and punched in the card columns specified in the COLS. field of the input card sets.

## L217 (EOM) Program Directive Cards

All program directive cards have a " $\$$ " in column one, the first character of the four character keywords.

Card Set. 1.0-Introduce L217 (EOM) Data

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT |
| :--- | :--- | :--- | :--- |$\quad$| DESCRIPTION |
| :--- |
| $1-10$ |
| SEOM | Al0 | Keyword introducing EOM data. This must be the first |
| :--- |
| card read by EOM. |

## Card 1.1-Title Card Labelling Printout

Optional card to be omitted or repeated as often as desired before any program directive card except \$EOM.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | STIT Ie | Al0 | Keyword indicating a title card to be read, printed, <br> and ignored. |
| $11-70$ |  |  | Available for comments |

Card 1.2-Indicate a Program Checkout Run
Optional card used only for program checkout purposes.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | SNON-stop | Al0 | Keyword telling the program to try continuing execution <br> even if fatal errors are diagnosed. |
| $11-70$ |  |  | Available for comments |

### 6.3.2 STRUCTURAL CARD INPUT DATA

After reading card 1.0, (cards 1.1 and 1.2 are optional), the next card that must be read is card 2.0 , which introduces the structural input data. Cards 2.1 through 2.3 define the problem size, the type of analysis, and the type of generalized mass and stiffness data input. Card 2.4 is optional and is used to print out intermediate data for problem checkout purposes. Card 2.5 , defining the modal input, is only required if control surface inertial coupling is to be calculated. Cards 2.6 through 2.8 define the generalized inertial, stiffness, and damping (if calculated in L217 (EOM)). Finally, card 2.9 defines the control surface inertial and nodal data needed for the inertial coupling calculations.

Once the processing of the structural data has been completed, the program reads another card to determine what is to be done next. The user may specify an aerodynamic data set, or new structural data (\$STR - card set 2), or may terminate the program's execution with card set 4 (\$QUIT).

All the cards in the set are optional except for cards 2.0 (\$STR), 2.1 (SIZE), and 2.10 (END). Card 2.1 must follow card 2.0 and card 2.10 must terminate the data set and precede the next program directive card. The flow of cards is displayed in figure 17.

For any optional card (2.2 through 2.9) that is omitted, the keywords or variables appearing on the card will assume their default values.


Figure 17. - L217 (EOM) Structural Card Input Data, Card Set 2.0

## Card Set 2.0-Introduce Structural Card Input Data

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | SSTRUCtural | Al0 | Keyword introducing the structural card input data <br> and causing the STRUCT overlay to be called into <br> execution. <br> The structural card input data (Cards 2.1-2.10) must <br> appear between \$STR and the next program directive <br> card (either \$TIT, \$CHE, or \$AER) . |
| $11-70$ |  |  | Available for comments |

Default values have been established for most of the items input in the structural data set. All cards in the set are optional, with the exception of 2.0 (\$STR), 2.1 (SIZE), and 2.10 (END). Card 2.1 must follow 2.0 , and 2.10 must terminate the structural data and precede the next program directive card. Cards 2.2 through 2.9 may be in any order.

## Card 2.1-Size of Equations of Motion Matrices

Card 2.1 must follow card 2.0.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | SIZE | Al0 | Keyword indicating a "SIZE" card. |
| $11-15$ | NDOFST | I5 | The number of structural degrees of freedom. <br> The size of the input generalized mass and stiffness <br> matrices. |
| $16-20$ | NDOFCS | I5 | The number of control surface degrees of freedom to be <br> added to the structural freedoms. |

NOTE: The equations of motion matrices generated by L217 (EOM) will contain NDOF degrees of freedom where NDOF $=$ NDOFST + NDOFCS. If the control surface freemdoms were accounted for in the generation of the structural data, then NDOFCS $=0$ and NDOFST is set equal to the total number of freedoms coming from the vibration program.

Card 2.2-Symmetric/Antisymmetric Analysis
Card 2.2 is optional. The default analysis type is symmetric.

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | ISYM | Al0 | ```One of two keywords may be used to choose the type of analysis to be performed by L217 (EOM). SYMMetric ANTI-symmetric Default: SYMM``` |
| 11-70 |  |  | Available for comments |

Card 2.3-Full/Half Airplane
Card 2.3 is optional. By default, the input generalized mass and stiffness matrices are assumed to be for a full airplane.

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | IHALF | Al0 | One of two keywords may be used to specify that the input generalized mass and stiffness are for full or half airplane. <br> FULL <br> HALF <br> Default: FULL |
| 11-70 |  |  | Available for comments |

## Card 2.4-Checkout Printout

## Card 2.4 is optional.

| COLS. | $\begin{aligned} & \text { KEYWORD/ } \\ & \text { VARIABLE } \end{aligned}$ | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | CHECkout | Al0 | Keyword requesting the printing of intermediate data calculated during the structural data processing. |
| 11-15 | ICKPRT | I5 | Option indicating the amount of intermediate data to be printed. <br> $=1$ Print message when each overlay is entered plus <br> $[\phi]$, interpolated mode shapes at control surface nodes, [MCS $]$, and the final $\left[M_{1}\right],\left[M_{2}\right]$, and $\left[M_{3}\right]$. <br> $=3$ Print all items listed under 1 plus control surface data summary, $[A]$, and $[C]$. <br> $=5$ Print all items listed under 1 and 3 plus [ $\phi_{S T}$ ], $\left[\phi_{C S}\right]$, and a message each time a VARDIM routine is called. |

## Card 2.5-File Containing the Input Mode Shapes

Card 2.5 is optional. The default file name is "SATAP."

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | MODEs | Alo | Keyword indicating a card defining the modal input data <br> file name. |
| $11-20$ | SATAP | Al0 | Name of the file (tape or disk) containing the modal <br> input data which will be used to calculate the inertia <br> coupling effects for the added control surface freedoms. |
| The name must start in column ll, begin with a letter, and |  |  |  |
| have less than seven characters. |  |  |  |
| Default: SATAP="SATAP" |  |  |  |

## Card 2.6-Generalized Stiffness Matrix Source

Card 2.6 is optional. Default values are shown below.

| cols. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | STIFfness | Al0 | Keyword indicating stiffness matrix data. |
| 11-20 | KTAPE | Al0 | Name of the file containing the generalized stiffness matrix [K] . Either "CARDS" or a file name starting in column 11, beginning with a letter, and having less than seven characters. If KTAPE="CARDS" the matrix will be read from cards (see Card 2.6.1) and the next two variables are meaningless. <br> Default: KTAPE="GSTIFF" |
| 21-25 | KFILE | I5 | ```Number of logical files to skip on KTAPE before reading [K] . Default: KFILE=0``` |
| 26-30 | KMAT | I5 | Number of matrices to skip on KTAPE before reading [K]. <br> Default: KMAT=0 |
| 31-40 | PRIN | A10 | Option requesting the printing of $[K]$ as read. Default: Do not print except when KTAPE=CARDS |
| 41-70 |  |  | Available for comments |

Card 2.6.1-Rows of Generalized Stiffness Matrix on Cards
Card 2.6.1 is input only if KTAPE = CARDS on card 2.6.
Repeat card 2.6.1 for each row of [K]; $i=1$, NDOFST.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | K (i,l) | 7E10.0 | The (i,l) element of $[K]$ the generalized stiffness matrix. |
| $11-20$ | $\begin{gathered} K(i, 2) \\ \vdots \\ K(i, j) \end{gathered}$ |  | The (i,2) element of $[K]$ The $(i, j)$ element of $[K]$ |
|  | K(i,NDOFST) |  | Repeat the card with 7 numbers per card until all elements of row i are defined. |

Card 2.7-Generalized Mass Matrix Source
Card 2.7 is optional. Default values are shown below.

| COLS | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | MASS | Al0 | Keyword indicating mass matrix data. |
| 11-20 | MTAPE | Al0 | Name of the file containing the generalized mass matrix [M] . Either "CARDS" or a file name - starting in column 11, beginning iith a letter, and having less than seven characters. If MTAPE="CARDS" the matrix will be read from cards (see Card 2.7.1) and the next two variables are meaningless. <br> Default: MTAPE="GMASS" |
| 21-25 | MFILE | I5 | Number of logical files to skip on MTAPE before reading [M]. <br> Default: MFILE=0 |
| 26-30 | MMAT | I5 | Number of matrices to skip on MTAPE before reading [M]. <br> Default: MMAT=0 |
| 31-40 | PRINt | A10 | Option requesting the printing of $[M]$ as read. Default: Do not print except when MTAPE=CARDS |
| 41-70 |  |  | Available for comments |

## Card 2.7.1-Rows of Generalized Mass Matrix of Cards

Card 2.7.1 is input only if MTAPE = CARDS in card 2.7.
Repeat card 2.7.1 for each row of [M], $i=1$, NDOFST.

| coLs. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | M (i, 1) | 7E10.0 | The ( $i, 1$ ) element of $[M]$, the generalized mass matrix. |
| $11-20$ | $\begin{gathered} M(i, 2) \\ \vdots \\ M(i, j) \end{gathered}$ |  | The (i,2) element of $[M]$ $\vdots$ The $(i, j)$ element of $[M]$ |
|  | M (i,NDOFST) |  | Repeat the card with 7 numbers per card until all elements of row $i$ are defined. |

Card 2.8-Introduce Structural Damping
Card 2.8 is optional. The default is no viscous structural damping.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT |  |
| :--- | :--- | :--- | :--- |
| $1-10$ | DAMPing | Alo | Keyword introducing the structural damping factors <br> following in Card 2.8.1 |
| $11-70$ |  |  | Available for comments |

Card 2.8.1-Structural Damping Factors
Card 2.8.1 must follow card 2.8. It is repeated until all NDOFST terms are defined.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | gSD (1) | 7E10.0 | Element 1 of $\{g S D\}$ |
| $11-20$ | gSD (2) gSD (i) |  | Element 2 of $\{g S D\}$ <br> Element i of $\{g S D\}$ |
|  | gSD (NDOFST) |  | Repeat the card with 7 numbers per card until all elements are defined. |

Card 2.9-Introduce the Addition of Control Surface Freedoms
Omit cards 2.9 and 2.9.1 if $\mathrm{NDOFCS}=0$ on card 2.1.
Repeat cards 2.9 and 2.9 .1 for each control surface affected by the additional freedoms.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | SURFace | A10 | Keyword introducing a new surface. |
| 11-15 | IS | I5 | Modal Interpolation Program surface number (Reference 5) specifying where mode shapes for this surface are to be found. |
| 16-20 | NNODEI | I5 | Number of nodes (mass points) on this control surface. |
| 21-30 | Trans | A10 | A "T" in column 21 requests a translational freedom for each mass point. |
| 31-40 | Rotl | Al0 | An "R" in column 31 requests a streamwise rotational <br> freedom, $y_{y}$, for each mass point. (local aerodynamic axes) |
| 41-50 | Rot2 | A10 | ```An "R" in column 41 requests a rotational freedom, 0 each mass point. (local aerodynamic axes)``` |
| 51-60 | Local | Al0 | An "I" in column 51 indicates that the coordinates on Card 2.9.1 are in the local structural axis system. <br> Default: Reference Axis System |

Card 2.9.1-Mass Data for Surface IS
Card 2.9.1 must follow card 2.9.
It is repeated NNODEI times, once for each node on the control surface.

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | Xi | E10.0 | The coordinates of node $i$ on surface IS. The coordinates are in the reference axis system unless specified as Local on Card 2.9. |
| 11-20 | Yi | E10.0 |  |
| 21-30 | Zi | E10.0 |  |
| 31-40 | MASSi | E10.0 | The mass at node $i$. Whole airplane. |
| 41-50 | InERTYi | E10.0 | ```The mass moments of inertia node i about the local aerodynamic y and x axes. Whole airplane. Note: INERTYi is required only if ROTl=R on Card 2.9. INERTXi is required only if ROT2=R on Card 2.9.``` |
| 51-60 | INERTXi | E10.0 |  |
| 61-70 |  |  | Available for comments |


| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | END | Al0 | Keyword indicating the end of structural card <br> input data. |
| $11-70$ |  |  | Available for comments |

### 6.3.3 AERODYNAMIC CARD INPUT DATA

After card set 2 ( 2.0 through 2.10 ) and the processing of structural data, the next card read by L217 (EOM) must introduce aerodynamic card data. Card 3.0 introduces the aerodynamic card data and indicates the source of the AIC's or generalized forces, FLEXSTAB or Doublet Lattice. Some of the cards within card set 3.0 ( 3.13 through 3.16) vary according to the source of the aerodynamic data used.

Once the processing of the aerodynamic data has been completed, the program reads another card to determine what is to be done next. The user may specify another aerodynamic data set, or new structural data (\$STR-card set 2), or may terminate the program's execution with card set 4.0 (\$QUIT).

Default values have been established for many of the items defined in the aerodynamic card set. All cards in the set are optional with the exception of 3.0 (\$AER), 3.1 (CONSTANTS), and 3.17 (END). Card 3.1 must follow 3.0 and card 3.17 must terminate the data set and precede the next program directive card. Optional cards 3.2 through 3.8 need only be present if the analyst wishes to use file names and/or spacing parameters that are different from the default values appearing in the data card write-up. The flow of cards is displayed in figure 18.


Figure 18. - L217 (EOM) Aerodynamic Card Input Data, Card Set 3.0

## Card Set 3.0-Introduce Aerodynamic Card Input Data and Choose the Aerodynamic Type

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | SAER | A10 | Keyword introducing the set of cards describing the aerodynamic data processing. Check the next two words for aerodynamic type. |
| 11-20 | ISOURC | Al0 | Keyword indicating the source of the aero data. <br> There are two possibilities: <br> FLEXstab - FLEXSTAB <br> DOUBlet - Doublet Lattice |
| 21-30 | ITYPE | Al0 | Keyword indicating the type of aero data to be used. There are two possibilities: <br> AIC - AIC matrices <br> PRESsure - Generalized forces and pressures <br> Note: If the source is FLEXSTAB the type must be <br> AIC. The PRES option is valid only with Doublet Lattice. <br> Lattice. |
| 31-70 |  |  | Available for comments |

## Card 3.1-Aerodynamic Constants

Card 3.1 must follow card 3.0.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | CONStants | Al0 | Keyword indicating a card of aerodynamic constants. |
| 11-20 | RH0 | E10.0 | $\rho$ - density of air at altitude. |
| 21-30 | VEL | E10.0 | $V_{T}=$ velocity (true airspeed) [units of length per second] Note: $\bar{q}$, the dynamic pressure, is calculated as $\bar{q}=\frac{1_{2}}{2} \rho V_{T}^{2}$ |
| 31-35 | NTDEF | I 5 | Indicates the number of the mode corresponding to the translation (vertical or sidel) freedom. That mode will be used to find the gust forces when using Doublet Lattice pressures at a nor-zero reduced frequency. |
| 36-40 | NPDEF | I5 | Indicates the number of the mode corresponding to the pitch or yaw freedom. That mode will be used to find the gust forces when using Doublet Lattice pressures at a reduced frequency of zero. |

NTDEF and NPDEF are not used, except with Doublet Lattice generalized forces and pressures.

Card 3.2-File Containing the Input Mode Shapes
Card 3.2 is optional. The default file name is "SATAP."

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT |  |
| :--- | :--- | :--- | :--- |
| $1-10$ | SATAP | Al0 | Keyword introducing replacement name for the modal <br> interpolation data file. |
| $11-20$ | ISATAP | Al0 | Name of the file (tape or disk) containing the modal <br> input data. <br> The name must start in column eleven, begin with a <br> letter, and have less than seven characters. <br> $21-70$ |

## Cards 3.3 and 3.4-FLEXSTAB Input Data Files

Omit cards 3.3 and 3.4 if Doublet Lattice aerodynamics are being used. The cards are optional even if FLEXSTAB aerodynamics are being used. Default file names are shown below.

## Card 3.3-File Containing FLEXSTAB Geometry Data

Card 3.3 is optional. The default file name and case number are "GDTAPE" and 1, respectively.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | GDTApe | Al0 | Keyword introducing replacement name for the FLEXSTAB geometry data file. |
| 11-20 | GDTAPE | Al0 | Name of the file (tape or disk) containing the FLEXSTAB geometry data. <br> The name must start in column eleven, begin with a letter, and have less than seven characters. |
| 21-25 | IGCASE | I5 | FLEXSTAB yeometry data case number from which 1217 (EOM) must extract geometry information. <br> Default: IGCASE = 1 |
| 26-70 |  |  | Available for comments |

Card 3.4-Files Containing FLEXSTAB Aerodynamic Data
Card 3.4 is optional. The default file names are "SDINDX" and "SDDATA."

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | SDSStp | Al0 | Keyword introducing replacement name for the FLEXSTAB aerodynamic data file. |
| 11-20 | SDINDX | A10 | Name of the file (tape or disk) containing the index table to the FLEXSTAB aerodynamic data. <br> The name must start in column eleven, begin with a letter, and have less than seven characters. <br> Default: SDINDX = "SDINDX" |
| 21-30 | SDDATA | A10 | Name of the file (tape or disk) containing the FLEXSTAB aerodynamic data. Note: SDINDX $\neq$ SDDATA. <br> The name must begin in column 2l, begin with a letter, and have less than seven characters. <br> Default: SDDATA = "SDDATA" |
| 31-35 | ISCASE | I5 | FLEXSTAB aerodynamic data case number from which L217 (EOM) <br> must extract $\left[\mathrm{A}_{\mathrm{P}_{\theta}}\right]$ <br> Default: ISCASE = I |
| 36-70 |  |  | Available for comments |

## Cards 3.5 and 3.6-Files Containing Doublet Lattice Data

Omit cards 3.5 and 3.6 if FLEXSTAB aerodynamics are being used.
The cards are optional even if Doublet Lattice aerodynamic data is being used. Default file names are shown below.

## Card 3.5-File Containing Doublet Lattice Geometry Data

Card 3.5 is optional. The default file name is "NGETP" and the default IGCASE $=1$.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT |  |
| :--- | :--- | :--- | :--- |
| $1-10$ | NGETP | Al0 | Keyword introducing a replacement name for the <br> input Doublet Lattice geometry file. |
| $11-20$ | NGETP | Al0 | Name of the file (tape or disk) which contains <br> the Doublet Lattice geometry data. |
| $21-25$ | IGCASE | I5 | Number of the data case (logical file) on "NGETP" <br> to be used. |
| $26-70$ |  |  | Available for comments |

## Card 3.6-File Containing Doublet Lattice Aerodynamic Data

Card 3.6 is optional. The default file name is "NAETP" and the default case number is 1 .

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | NAETP | Al0 | Keyword introducing a replacement name for the input Doublet Lattice Aerodynamic file. |
| 11-20 | NAETP | A10 | Name of the file (tape or disk) which contains the Doublet Lattice aerodynamic data. |
| 21-25 | IACASE | I5 | Number of the data case (logical file) on "NAETP" to be used. |
| 26-70 |  |  | Available for comments |

## Card 3.7-File to Contain the Equations of Motion Coefficients

Card 3.7 is optional. The default name is "EOMTAP."

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | EOMTAP | Al0 | Keyword introducing replacement name for the output <br> equations of motion file. |
| $11-20$ | EOMTAP | Al0 | Name of the file (tape of disk) to contain the <br> equations of motion matrices. <br> The name must begin in column eleven, begin with a <br> letter, and have less than seven characters. |
| $21-70$ |  | Available for comments |  |

## Card 3.8-File to Contain Aerodynamic Force Matrices

Card 3.8 is optional. The default file name is "EOMLOD."

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | EOMLOd | Al0 | Keyword introducing replacement name for the output <br> file to contain the force matrices required by L218, <br> the Load Equations program. |
| $11-20$ | EOMLOD | Al0 | Name of the file (tape or disk) to contain the force <br> matrices required by L218, the Load Equations program. <br> The name must begin in column eleven, begin with a <br> letter, and have less than seven characters. |
| $21-70$ |  |  | Available for comments |

## Card 3.9-Doublet Lattice Quasi-Steady Option

Card 3.9 is optional. The quasi-steady solution may be requested only with Doublet
Lattice AIC aerodynamic data for a reduced frequency ( $k$-value) of zero.

| COLS. | $\begin{array}{l}\text { KEYWORD/ } \\ \text { VARIABLE }\end{array}$ | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |$]$| Keyword requesting a Quasi-Steady solution for the |
| :--- |
| Doublet Lattice AIC option. |

The quasi-steady option limits processing to the first reduced frequency solution from Doublet Lattice files. Therefore, the first reduced frequency must be zero.

## Card 3.10-FLEXSTAB Thin Body Control Point Option

Card 3.10 is optional. It may be used only with FLEXSTAB aerodynamics.

| COLS | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | CENTroids | A10 | Keyword requesting 2217 (EOM) to use the centroids of thin bodies' panels for both the force points and the control points. <br> Default: Centroids are used for force points and control points will remain as defined in FLEXSTAB. |
| 11-70 |  |  | Available for comments |

Cards 3.11 and 3.11.1-Gradual Penetration
Cards 3.11 and 3.11.1 are optional. They are used only when gradual penetration is desired. Gradual penetration can not be requested when using Doublet Lattice generalized forces and pressures.

Card 3.11-Introduce Gradual Penetration Control Points

| COLS | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | GRAD. pen. | Al0 | Keyword introducing gradual penetration card input data. |
| 11-20 | XG | E10.0 | The gust reference point in the reference axis system. |
| 21-25 | NGCP | I5 | The number of gust control points. $(1 \leq N G C P \leq 35)$ |
| 26-70 |  |  | Available for comments |

## Card 3.11.1-Gradual Penetration Control Points

Card 3.11.1 is input only after card 3.11. Card 3.11.1 must be repeated for each gradual penetration control point; $\mathrm{i}=1, \mathrm{NGCP}$ (from card 3.11).

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | $\mathrm{XGCP}_{i}$ | E10.0 | The reference axis $x$-coordinate of the gust control point for zone i |
|  |  |  | $\mathrm{XGCP}_{i+1}>\mathrm{XGCP}_{i}$ |
| 11-20 | $\mathrm{XGLE}_{\mathrm{i}}$ | E10.0 | The reference axis x-coordinate of the leading edge of zone i. |
|  |  |  | $\mathrm{XGCP}_{i} \geq \mathrm{XGLE}_{i}, \quad \mathrm{XGLE}_{i+1}>\mathrm{XGLE}_{i}$ |
| 21-70 |  |  | Available for comments |

The program will group all aerodynamic elements into the zones generated by the XGLE $_{i}$ of card 3.11.1. The gust will reach each panel of zone $i$ at the time it reaches the gust control point $\mathrm{XGCP}_{\mathrm{i}} . \mathrm{XGLE}_{\mathrm{i}}$ must be forward of the first element's control point.

## Card 3.12-Request Printed Output

Card 3.12 is optional. If omitted the following items are printed by default: Card input data and interpretation of card input data. Summary of geometry data read from GDTAPE or NGETP.

Card 3.12 must be input once for each optional item to be printed.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | PRINt | Al0 | Keyword introducing optional item to be printed. |
| $11-20$ | ITEM | Al0 | Name of the item to be printed chosen from the <br> following list: |
|  |  | MODES: Modal displacements at aerodynamic and force points <br> FORCEs: Aerodynamic forces for each surface |  |

## Card 3.12.1-Select Frequencies for Printout

Card 3.12 .1 is optional. If omitted, the items requested by card 3.12 will be printed for the first reduced frequency only.

| COL. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | SELECt | A10 | Keyword introducing numbers of frequencies at which data will be printed. |
| 11-15 | $\mathrm{IFREQ}_{1}$ | 12 I 5 | Numbers of frequencies at which data will be |
| $16-20$ | $\mathrm{IFRE}_{2}$ |  | printed: <br> If more than 12 frequencies are desired use two SELEct cards. |

NOTE: The selected array of frequencies applies to all print items.

## Card 3.12.2-Checkout Printout

Card 3.12 .2 is optional and is used to generate intermediate data for problem checkout.

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-10 | CHECkout | Al0 | Keyword requesting the printing of intermediate data calculated during the aerodynamics data processing. |
| 11-15 | ICKPRT | I5 | ```Option indicating the amount of intermediate data to be printed. =1 Print message when each overlay is entered and indicate field length required for each overlay. =3 Print items under 1 above plus many of the intermediate calculations i.e. - for FLXAIC print[AIC] , [AIC][\phi],[AIC][饾] and the full [F1], [F2], [FG]. - for DLAIC print[CP SB ],[CPG SB l,[F],[w],[WG], [\triangleCP] and others. - for DLPRES print [\triangleCD] and[F12]. =5 Print items under 1 and 3 above plus a message each time a VARDIM routine is called.``` |

## Cards 3.13 through 3.16 Mode Shape Definition

The L215 program, modal interpolation (ref. 6) provides all of the mode shapes used inside L217 (EOM). The modal data is defined per surface in arrays called \{SA\} written by L 215 (ref. 6) onto the magnetic file named "SATAP."

Cards 3.13 through 3.14 .2 are used to define the correspondence between FLEXSTAB bodies and the modal interpolation SA arrays. Cards 3.15 and 3.16 serve the same purpose for Doublet Lattice bodies/surfaces. It should be noted that order of thin and slender bodies is different for the two aerodynamic programs. The correlation of modal data and aerodynamic bodies is only concerned with them and slender bodies. Interference bodies are not considered, since their effects are accounted for when dealing with their respective slender bodies (see sec. 4.0).

The thin and slender bodies in FLEXSTAB are ordered in the following manner:

- Slender bodies on the plane of symmetry.
- Slender bodies off the plane of symmetry.
- Thin bodies on the plane of symmetry.
- Thin bodies off the plane of symmetry.

Within each catagory of slender and thin bodies, the ordering must match the ordering used in the FLEXSTAB aerodynamic geometry program.

The ordering of the Doublet Lattice data must correspond to the lifting surface panel/slender body order established in the Doublet Lattice aerodynamic program (L216) which is:

- Primary vertical lifting surface panel (thin bodies) on the plane of symmetry.
- Other primary lifting surface panels (thin bodies) off the plane of symmetry.
- Y-motion slender bodies.
- Z-motion slender bodies.

Within each major catagory, the user must assure that the order matches that of the Doublet Lattice run which created the aerodynamic data. (Note: When using Doublet Lattice aerodynamics, the term "thin body" refers to a primary lifting surface panel.)

Cards 3.13 Through 3.14.2-FLEXSTAB Body Modes
Omit cards 3.13 through 3.14.2 if Doublet Lattice aerodynamics are being used.

## Card 3.13-FLEXSTAB Slender Body Modes

Repeat card 3.13 for each FLEXSTAB slender body in the order they appear on GDTAPE.

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-20 | SLENder body | Al0,10x | Keyword introducing a card defining mode shapes for a slender body. |
| 21-25 | ISY | I5 | The numbers of the Modal Interpolation Program (ref. 6) surfaces (SA arrays) to be used to determine the $Y$ and $Z$ motion, respectively, for this slender body. If $\mathrm{IS} \mathbf{n}=\mathbf{0}$, modal displacements of zero will be used. (See section 4.6.1). |
| 26-30 | ISZ | I5 |  |
| 31-35 | ISTORE | I5 | File number (a surface number) of EOMLOD in which the aero force matrices will be written. ISTORE $>0$. |
| 3.6-40 |  | 5x | Not Used |
| 41-50 | XSHIFT | El0.0 | Shifting values (defined in the reference axis system) which are used to account for the difference between the local structural axis system and the local aerodynamic axis system, and/or to account for any desired axis corrections needed. |
| 51-60 | YSHIFT | E10.0 |  |
| 61-70 | ZSHIFT | E10.0 |  |

*Normally, ISTORE will be equal to ISY or ISZ. If both ISY and ISZ are defined ISTORE should be equal to ISZ.

### 3.14-FLEXSTAB Thin Body Modes

Repeat cards 3.14 through 3.14 .2 for each FLEXSTAB thin body in the order they appear on GDTAPE.

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-20 | THIN <br> body | Al0,10x | Keyword introducing a card defining mode shapes for a thin body. |
| 21-25 | IS | I5 | Number of the Modal Interpolation Program (ref. 6) surface. (SA array ) to be used to determine the motion of this thin body. (See soction 4.6.1). |
| 26-30 |  | 5x | Not Used |
| 31-35 | ISTORE | I5 | File number (a surface number) of EOMLOD on which the aero force matrices will be written.* |
| 36-40 |  | 5x | Not Used |
| 41-50 | XSHIFT | E10.0 | Axis shifting values (defined in the reference axis |
| 51-60 | YSHIFT | E10.0 | system) which account for the difference between the |
| 61-70 | ZSHIFT | E10.0 | local structural and aerodynamic axis system and/or any axis correction desired. |

*If ISTORE is left blank ISTORE = IS is assumed. If IS = 0 (mode shapes of zero) then ISTORE must be non-zero.

Cards 3.14.1 and 3.14 .2 are optional. They are input in pairs to define mode shapes for subsurfaces (control surfaces) of the previously defined thin body.

## Card 3.14.1-FLEXSTAB Control Surface Modes

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-20 | CONTrol <br> surface | A10,10x | Keyword introducing a card defining mode shapes for a control surface. |
| 21-25 | IS | I5 | Number of the Modal Interpolation Program (Ref. 6) surface corresponding with the motion of the control surface. |
| 26-30 | NCSPAN | I5 | Number of panels on the control surface. |
| 31-35 | ICSTOR | I5 | File number (a surface number) of EOMLOD in which the aero forces will be written. <br> If ICSTOR is left blank it is set equal to ISTORE of the parent thin body. |
| 36-70 |  |  | Available for comments. |

Card 3.14.2-Panels on the FLEXSTAB Control Surface
Card 3.14.2 must follow card 3.14.1.


Cards 3.15 Through 3.16-Doublet Lattice Body Modes
Omit cards 3.15 through 3.16 if FLEXSTAB aerodynamics are being used.

Card 3.15-Doublet Lattice Thin Body (Primary Lifting Surface Panel) Modes
Repeat cards 3.15 through 3.15 .2 for each Doublet Lattice thin body in the order they appear on NGETP.

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT |  |
| :--- | :--- | :--- | :--- |
| $1-20$ | THIN <br> body | Al0,10X | Keyword introducing a card defining mode shapes <br> for a thin body. |
| $21-25$ | IS | I5 | Number of the Modal Interpolation Program (ref. 6) surface (SA <br> array) to be used to determine the motion of this thin body. |
| $26-30$ |  | $5 x$ | Not Used |
| $31-35$ | ISTORE | I5 | File number (a surface number) of EOMLOD in which <br> the aero force matrices will be stored.* |
| $36-40$ |  | $5 X$ | Not Used |
| $41-50$ | XSHIFT | El0.0 | Aerodynamic axis shifting values (defined in reference |
| $51-60$ | YSHIFT | El0.0 | axis system) which accounts for the difference between |
| $61-20$ | ZSHIFT | El0.0 | the local structural and aerodynamic axis systems and/or <br> any axis correction desired. |

* If ISTORE is left blank the program sets ISTORE = IS.

If $I S=0$ (mode shapes of zero) then ISTORE must be non-zero.

Cards 3.15.1 and 3.15.2 are optional. They are input in pairs; one pair for each control surface on the previously defined thin body requiring different mode shape definitions.

Card 3.15.1-Doublet Lattice Control Surface Modes

| COLS . | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| I-20 | CONTrol <br> surface | Al0,10x | Keyword introducing a card defining mode shapes for a control surface. |
| 21-25 | IS | I5 | Surface number specified to the Modal Interpolation Program (Ref. 5) when defining mode shapes for this control surface. |
| 26-30 | ncsbox | 15 | Number of boxes on the control surface. |
| 31-35 | ICSTOR | I5 | File number (a surface number) of EOMLOD in which the aero forces will be written. <br> If ICSTOR is left blank it is set equal to ISTORE of the parent thin body. |
| 36-70 |  |  | Available for comments. |

Card 3.15.2-Boxes on a Doublet Lattice Control Surface
Card 3.15.2 must follow card 3.15.1.

| COLS. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-5 | $\mathrm{IBOX}_{1}$ | 1415 | Numbers of the boxes of the thin body which are on the control surface.$\operatorname{IBOX}_{i} ; \quad i=1, \operatorname{NCSBOX}$ |
| 6-10 | $\mathrm{IBOX}_{2}$ |  |  |
| $11-15$ | $\mathrm{IBOX}_{3}$ |  |  |
| 66-70 | $\mathrm{IBOX}_{\text {NCSBOX }}$ |  |  |
|  | ' |  | Repeat the card with 14 numbers per card until all panels on the control surface have been specified. |

## Card 3.16-Doublet Lattice Slender Body Modes

Repeat card 3.16 for each Doublet Lattice slender body in the order they appear on NAETP.

| coLs. | KEYWORD/ VARIABLE | FORMAT | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1-20 | $\frac{\text { SLENder }}{\text { body }}$ | $\begin{aligned} & \text { Al0 } \\ & 10 \mathrm{X} \end{aligned}$ | Keyword introducing a card defining mode shapes for a slender body. |
| 21-25 | ISY | I5 | Number of the Modal Interpolation Program (ref. 6) surface (SA array) to be used to determine the $Y$ or $\mathbf{Z}$ motion, respectively, for this slender body. <br> Specify only onel (See section 4.6.1 and 4.7.1) |
| 26-30 | ISZ | I5 |  |
| 31-35 | ISTORE | I5 | File number (a surface number) of EOMLOD in which the aero force matrices will be written.* |
| 36-40 |  | 5X | Not Used |
| 41-50 | XSHIFT | E10.0 | Aerodynamic axis shifting values (defined in the reference axis system) which account for the difference between the local structural axis and the local aerodynamic axis and/or any axis corrections desired. |
| 51-60 | YSHIFT | E10.0 |  |
| 61-70 | zSHIFT | E10.0 |  |

* Normally, ISTORE will be equal to ISY or ISZ (whichever is defined).

Card 3.17-End of Aerodynamic Card Data

| COLS. | KEYWORD/ |
| :---: | :--- | :--- | :--- |
| VARIABLE |  | FORMAT $\quad$ DESCRIPTION

Card Set 4.0-Terminate L217 (EOM) Execution

| COLS. | KEYWORD/ <br> VARIABLE | FORMAT | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| $1-10$ | SQUIt | A10 | Keyword requesting that L217 (EOM) execution be <br> terminated. |
| $11-70$ |  |  | Available for comments |


| Requirements or Function | Key Words and/or Variables | Card Format | Reference Card Set (CS) |
| :---: | :---: | :---: | :---: |
|  | SFOM | Al0 | 1.0 |
| Job title card (Optional) | STitle | Al0 | 1.1 |
| Job sheckout card (Optional) | \$NON-stop | Alo | 1.2 |
| Begin input of Structural Data | SSTRuctural | Al0 | 2.0 |
| Number of degrees of freedom | SIZE NDOFST NDOFCS | Al0,2I5 | 2.1 |
| Type of analysis Optional | SYMMetric <br> ANTI-symmetric | Al0 | 2.2 |
| (Optional) | $\begin{aligned} & \text { FULL } \\ & \text { HALT } \end{aligned}$ | Al0 | 2.3 |
| (Optional) | CHECkout ICKPRT | A10, I5 | 2.4 |
| ```Modal Interpolation data to use for control surface inertial coupling (Optional)``` | MODEs SATAP | 2A10 | 2.5 |
| Source of Stiffness Data | STIFfness KTAPE KFILE KMAT PRINt | 2A10,2I5,Al0 | 2.6 |
| Omit card 2.6.1 if KTAPE $\neq$ CARDS (card 2.6) |  |  |  |
| Repeat card 2.6.1 for each row of K ; $\mathrm{i}=1$, NDOFST | $\mathbf{R}(\mathrm{i}, \mathrm{j}) \quad j \quad=1, \mathrm{NDOFST}$ | 7E10.0 | 2.6 .1 |
| Source of Mass Data | MASS MTAPE MFILE NMAT PRINt | 2Al0,2I5,Al0 | 2.7 |
| Omit card 2.7.1 if MTAPE $\neq$ CARDS (card 2.7) |  |  |  |
| Repeat card 2.7.1 for each row of $M$ $\mathrm{i}=1$, NDOFST | M $\mathrm{i}, \mathrm{j}$ ) $\quad \mathrm{j}=1, \mathrm{NDOFS}{ }^{\text {m }}$ | 7E10.0 | 2.7 .1 |



| Requirements or Function |  | Key Words and/or Variables | Card Format |
| :--- | :--- | :--- | :--- | :--- |
| Reference |  |  |  |
| Card |  |  |  |
| Set |  |  |  | (CS)



### 6.4 MAGNETIC FILE INPUT DATA

When used as a module in the DYLOFLEX, much of the data required by L217 (EOM) is prepared by upstream programs and written on magnetic files (disk or tape) to be read and used by L217. The file names and the programs that normally generate them are:

| Default <br> Name | Generating Program | References |
| :--- | :--- | :---: |
| GSTIFF | - | - |
| GMASS | - | - |
| SATAP | Modal Interpolation L215 | 6 |
| NGETP | Doublet Lattice L216 | 8 |
| NAETP | Doublet Lattice L216 | 8 |
| GDTAPE | FLEXSTAB | 7 |
| SDINDX | FLEXSTAB | 7 |
| SDDATA | FLEXSTAB | 7 |

Note that different file names may be specified via card input data (see cards 2.5 through 2.7 and 3.2 through 3.6). Also, the files may be generated by different computer programs as long as the content and format matches that described on the following pages.

### 6.4.1 "GSTIFF"-GENERALIZED STIFFNESS MATRIX

"GSTIFF," the generalized stiffness matrix, may be read by L217 (EOM) from a magnetic file (tape or disk) in the READTP/WRTETP format (see ref. 2). L217 (EOM) rewinds the file before attempting to read the stiffness matrix [K]. If requested via card input data the program will also skip "KFILE" logical files and "KMAT" matrices before reading [K] (see fig. 19).

### 6.4.2 "GMASS"-GENERALIZED MASS MATRIX

"GMASS," the generalized mass matrix, may be read by L217 (EOM) from a magnetic file (tape or disk) in the READTP/WRTETP format (see ref. 2). L217 (EOM) rewinds the file before attempting to read the mass matrix [ M ]. If requested via card input data, the program will also skip logical "MFILE" files and "MMAT" matrices before reading [M] (see fig. 20).

[K] the generalized stiffness matrix is of size
$\mathrm{NDOF}_{S T} \times \mathrm{NDOF}_{S T}$
where NDOFST is the number of structural degrees of freedom.
Figure 19. - Contents of GSTIFF


Figure 20. - Contents of GMASS

### 6.4.3 "SATAP"-MODAL DATA FILE

The mode shape data required by L 217 (EOM) is generated by the program L215 (INTERP) (ref.6) and written on a magnetic file, "SATAP", (card 2.3) in the READTP/WRTETP format described in reference 2.

L215 (INTERP) writes one logical file of data for each surface for which mode shapes are defined. The file number has a one-to- one correspondence with the surface number. Thus, when L217 (EOM) needs modal data for surface "IS," it rewinds "SATAP" and skips "IS-1" logical files before reading.

The contents of one file on "SATAP" are displayed in figure 21.


Figure 21. - Contents of a File on SATAP
$\left[\phi_{\mathrm{X}}\right],\left[\phi_{\mathrm{y}}\right]$ and $\left[\phi_{\mathrm{z}}\right]$ contain the modal deflections at the nodes in the $\mathrm{x}, \mathrm{y}$ and z directions of the local structural coordinate system.
[ $\phi_{\Theta_{\mathrm{x}}}$ ], [ $\phi \Theta_{\mathrm{y}}$ ] and [ $\phi \Theta_{\mathrm{z}}$ ] contain the modal rotations about the $\mathrm{x}, \mathrm{y}$ and z axes at the nodes in the local structural coordinate system.
[geom] contains the nodal descriptions. For each node the following six items are defined:
$\mathrm{X}=$ coordinate in the reference axis system
$\mathrm{Y}=$ coordinate in the reference axis system
Z = coordinate in the reference axis system
$\phi_{\mathbf{X}}=$ rotation about the $\mathbf{x}$-axis in radians
$\phi_{y}=$ rotation about the $y$-axis in radians
$\phi_{\mathrm{Z}}=$ rotation about the z -axis in radians
\{SA\} contains information to allow interpolation for mode shapes over the nodes. The length and contents of $\{S A\}$ vary with the interpolation type (surface spline, beam spline, motion point, motion axis, polynomial). See reference 6 for a complete description.

### 6.4.4 "NGETP"-DOUBLET LATTICE GEOMETRY FILE

When Doublet Lattice L216 (ref. 8) geometry data is required by L217 (EOM), it is read in the READTP/WRTETP format from "NGETP," a magnetic file (tape or disk) (see ref. 1). L217 (EOM) rewinds the file before attempting to read the data. If the user specifies a data case, "IGCASE," other than number one, the program will also skip "IGCASE-1" logical files before reading.

For each data case, "NGETP" contains 18 matrices followed by an end-of-file (see fig. 22). All geometry is defined in the reference axis system.

The following variables are used to define array sizes on "NGETP."
NBOD = number of primary lifting surface panels (thin bodies), plus the number of slender bodies

NBOX = number of aerodynamic boxes on all primary lifting (thin body) and interference surfaces

NTOT = number of aerodynamic elements (boxes plus slender body segments) on all surfaces (primary lifting, interference, and slender bodies)

NSBE = number of slender body segments (counting all slender bodies)
Three of the matrices on "NGETP" are described below. The others are self-explanatory.


Figure 22. - Contents of a File on NGETP
\{GCM\}-Geometry Control Matrix

| Element | Type | Description |
| :--- | :---: | :--- |
| ICASE | I | Case number. |
| ICOND | I | Condition number |
| IGEOM | I | Geometry data successfully saved if "IGEOM = 1". |
| REFCHD | R | Reference chord length. |
| REFSPN | R | Reference span length. |
| ACAP | R | Reference area. <br> FMACH |
| RTBB | R | Mach number. |
| NIBB | I | Number of thin body boxes. |
| NSBE | I | Number of interference body boxes. |
| NTB | I | Number of thin bodies (the number of lifting surface <br> panels). |
| NSB | I | Number of slender bodies. <br> NPLS |
| NPIS | I | Number of primary lifting surfaces. |
| I | Number of interference surfaces. |  |
| Not Used. |  |  |

where:
R $=$ Real
I = Integer

## [BODTAB]-Table of Thin and Slender Bodies

Note: There is one thin body for each lifting surface panel. The matrix will contain one column for each body.

where:
R = Real
I = Integer

## \{IBSB \}-Correspondence Between Interference Body Boxes and Slender Body Segments

Each interference body box will be associated with one or two slender body segments: two if the physical slender body is represented by two slender bodies, one for y motion and another for $z$ motion. Therefore, each element $i$ of the matrix $\{I B S B\}$ will contain 0 , 1 , or 2 numbers.

If element $i$ represents a thin body box or a slender body segment, then $\mathrm{IBSB}_{\mathbf{i}}=\mathbf{0}$.
If element i represents an interference body box, the first slender body segment with which it is associated, JSB1, will appear in the right-most 30 bits of IBSB $_{i}$. If there is a second segment, JSB2, it will appear in the left-most 30 bits.

$\mathrm{IBSB}_{\mathrm{I}}=$| JSB2 | JSB 1 |
| ---: | ---: |
| 1 | 30 |

### 6.4.5 "NAETP"-DOUBLET LATTICE AERODYNAMIC FILE

When Doublet Lattice L216 (ref. 8) aerodynamic data is required by L217 (EOM), it is read from "NAETP," a magnetic file (tape or disk), in the READTP/WRTETP format (see ref. 1). L217 (EOM) rewinds the file before attempting to read the data. If the user specifies a data case, "IACASE," other than number one, the program will skip "IACASE-1" logical files on "NAETP" before reading data.

The number of matrices on "NAETP" for each data case depends upon the option chosen in Doublet Lattice. The variable "IAERO" can be extracted from element 3 of the aerodynamic control matrix, the first matrix for each case. If:

| IAERO $=1$ | Matrices [D] and [F] are written onto "NAETP" for each reduced frequency |
| :---: | :---: |
| IAERO $=2$ | Matrices $\left[\Delta C_{p}\right.$ ] and [Q] are written onto "NAETP" for each reduced frequency |
| IAERO $=3$ | Matrices [D], [F], [ $\Delta \mathrm{C}_{\mathbf{p}}$ ] and [Q] are all written onto "NAETP" |
| The following variables define the size of matrices on "NAETP" displayed in figure 23. |  |
| NFREQM | $=$ Number of reduced frequencies at which the aerodynamic data is defined |
| NBOX | $=$ Number of lifting surface boxes (lifting and interference) |
| NTOT | $=$ Number of elements on all surfaces (lifting, interference, and slender) |
| NMODE | Number of modes used in Doublet Lattice |



Figure 23. - Contents of a File on NAETP

Aerodynamic Control Matrix

| Element <br> Location | Element | Type | Description |
| :---: | :---: | :---: | :---: |
| 1 | ICASE | I | Case number. |
| 2 | ICOND | I | Condition number. |
| 3 | IAERO | I | Option indicating the aerodynamic data saved. $=1[D]$ and $[F]$ saved for each frequency. <br> $=2\{\mathrm{CP}\}$ and $[\mathrm{Q}]$ saved for each frequency. <br> $=3$ both 1 and 2 above. |
| 4 | FMACH | R | Mach number. |
| 5 | NFREQM | I | Number of reduced frequencies. |
| 6 | NBOX | I | Number of lifting surface boxes. |
| 7 | NSB | I | Number of slender body elements. |
| 8 | NMODE | I | Number of modes. |
| 9 | REFCHD | R | Reference chord. |
| 10 | REFSPN | R | Reference span. |
| 11 | ACP | R | Reference area. |
| 12 | BR | R | Reference length. |
| 13 | - | - | Not used. |
| 14 | - | - | Not used. |
| 15 | RFREQ(1) | R | First reduced frequency. |
|  | RFREQ <br> (NFREQM) | R | Last reduced frequency. |

where:

$$
\begin{aligned}
& \mathbf{R}=\text { Real } \\
& \mathbf{I}=\text { Integer }
\end{aligned}
$$

### 6.4.6 "GDTAPE"-FLEXSTAB GEOMETRY FILE

Each configuration (i.e., each data case) processed by the GD program is written sequentially on "GDTAPE" as one logical file. A file begins with a 50 -word header record followed by two records for every slender, interference, and thin body in the configuration. The file ends with a 50 -word trailer record and an end-of-file (EOF) mark. A complete description of "GDTAPE" is provided in appendix A of reference 7, volume 2.

The GD program has the ability to process the geometry definitions for more than one airplane configuration; therefore, there may be more than one logical file (i.e., one file per aircaft configuration).

After the GD program terminates, the "GDTAPE" magnetic tape is generated by the job control card:
COPYBF(TAPE1,GDTAPE,n)
where $\mathrm{n}=$ number of logical files generated by the GD program.
Prior to running L217 (EOM), the data on tape must be recopied to the disk file, named "GDTAPE" by the job control card:

## COPYBF(TAPE,GDTAPE,n)

### 6.4.7 "SDINDX/SDDATA"-FLEXSTAB AERODYNAMIC FILE

SDSSTP is a binary tape generated by the FLEXSTAB SD\&SS program (ref. 7) and is composed of many logical files. In general, these files consist of:

- A master catalog of the contents of SDSSTP. This catalog is always the first file on the tape.
- Miscellaneous data required by the TH, SLOADS, and ALOADS programs. These data are stored in one file and consist of such information as moments of inertia, center of gravity location, number and size of the stability derivative matrices, etc.
- Slender body and thin body lifting pressure data. These data are stored in one logical file.
- The $\left[\mathrm{K}_{1}\right]$ and $\left[\mathrm{M}_{1}\right]$ structural matrices. These matrices (if they exist) are copied from SICTP3 onto SDSSTP.
- Stability derivative matrices.
- Displacement and camber slope matrices.
- Elastic axis load matrices (optional).
- Gust matrices (optional)
- Any user-specified matrices. An option in the SD\&SS program allows the user to specify matrices to be stored on SDSSTP, in addition to those that normally are saved.

Each matrix is stored as one logical file. The first file on SDSSTP, is the master catalog. It is composed of several data records. There are two types of data records in this catalog-matrix records and terminal records. A matrix record contains information pertaining to the data stored in one particular file of SDSSTP. Thus, for each file on SDSSTP (not counting the master catalog), there is a corresponding matrix record in the master catalog. A terminal record is used to signify the end of the catalog.

If the SD\&SS recycle capability is used (i.e., more than one data case is executed), the overall format of SDSSTP changes slightly. For each successive data case, only those matrices that are different from those of the initial case are saved (i.e., no duplication of data exists on SDSSTP). This change is also reflected in the first file (the master catalog). Additional matrix records corresponding to the files (matrices) added to SDSSTP are also included. Terminal records are used to indicate the end of the sequence of matrix records for each data case. In other words, the master catalog contains: a sequence of matrix records pertaining to the matrices saved from the initial case; a terminal record; another sequence of matrix records reflecting the new matrices added to SDSSTP from the second data case; another terminal record; and so on, for as many cases as executed. A detailed description of SDSSTP is contained in appendix $A$ of reference 7 , volume 2 .

After execution of the SD\&SS program, the SDSSTP tape is generated by the job control cards:

| COPYBF(TAPE12,SDSSTP,1) | (Master catalog) |
| :--- | :--- |
| COPY(TAPE14,SDSSTP) | (Matrices, SDSS data, and SPRES data) |
| COPYBF(EOF,SDSSTP,1) | (EOF mark on tape) |

Before executing L217 (EOM), the data on tape SDSSTP must be copied to disk by the job control cards:

COPYBF(SDSSTP,SDINDX) (Master catalog)
COPY(SDSSTP,SDDATA) (Matrices required)

### 6.5 OUTPUT DATA

L217 (EOM) will produce a printed output listing and two magnetic files (tape or disk).

### 6.5.1 PRINTED OUTPUT

The program will initialize and terminate the printed output, with special pages indicating the program name, version, and date of run.

All card input data will be printed as it is read. Where appropriate, the card images will be augmented by interpretative comments and/or diagnostics.

The geometry from the aerodynamic program will be printed in tabular form with control information based upon card input instructions.

An explanation of the SIGMA and RG arrays appearing in the geometry tables follows.
[SIGMA]
Each row is of [SIGMA] (also known as [ $\Sigma$ ]) indicates where the forces on aerodynamic element $i$ are to be stored. The primary purpose of SIGMA is to add the interference body forces to the slender body forces. However, as noted later, other factors are included in SIGMA for a FLEXSTAB AIC data case.

SIGMA(i,1) Indicates which force point will receive the SIGMA(i,2) fraction of the force of aerodynamic element $i$.

SIGMA(i,3) Indicates which force point will receive the SIGMA(i,4) fraction of the force of aerodynamic element $i$.

For a FLEXSTAB AIC data case, the following $\Delta \mathbf{C}_{\mathbf{p}}$ factors are included in the second and fourth columns of SIGMA:

- $\pi / 2$ for slender body Y-motion elements.
- $\quad-\pi / 2$ for slender body Z-motion elements.
- $\quad 1.0$ for thin body elements.
- $\quad \mathbf{1 . 0}$ for interference body elements.


## $\{\mathbf{R G}\}$

The gust rotation matrix has elements for all thin body and slender body control points. For a thin body element i

| Symmetric analysis: | $\mathrm{RG}=\cos \gamma_{\mathbf{i}}$ |
| :--- | :--- |
| Antisymmetric analysis: | $\mathrm{RG}=-\sin \gamma_{\mathbf{i}}$ | where $\gamma_{\mathrm{i}}$ is the element's dihedral angle

For a slender body element i
Symmetric analysis: $\quad$ Y-motion element $\quad \mathbf{R G}_{\mathbf{i}}=0$.

$$
\text { Z-motion element } \mathrm{RG}_{\mathbf{i}}=1
$$

Antisymmetric analysis: Y -motion element $\mathrm{RG}_{\mathrm{i}}=\mathbf{- 1}$.
Z-motion element $\quad \mathrm{RG}_{\mathrm{i}}=0$.
Note: The gust rotation matrix, which is printed, contains the sign function $\delta_{\mathrm{w}}$ shown in equation (12).

All other printed output must be requested with card input options.
During the processing of structural data, the program will optionally print:
[K] Generalized stiffness matrix read from tape (see card 2.4).
[M] Generalized mass matrix read from tape (see card 2.5).
Cards 3.12 and 3.12 .1 may be used to cause the following items to be printed during the processing of aerodynamic data:

- Interpolated mode shapes.
- Contents of the file "EOMLOD" (the forces).
- Contents of the file "EOMTAP" (the equations of motion).

Additional intermediate calculations may be requested using the checkout option, card 3.12.2.

### 6.5.2 MAGNETIC FILE OUTPUT

L217 (EOM) will generate two magnetic files, "EOMTAP" and "EOMLOD." Both are written in the READTP/WRTETP format described in reference 1.

## "EOMTAP"

The file known as "EOMTAP" will contain the equations of motion matrices generated by the L217 (EOM) program. The matrices are written onto "EOMTAP" by the subroutine WRTETP (see ref. 1 for a description of WRTETP formats). The contents of "EOMTAP" are displayed in figure 24.

## "EOMLOD"

The file known as "EOMLOD" will contain aerodynamic geometry and forces for use in the DYLOFLEX LOADS program (ref. 7). The data is written onto "EOMLOD" by the subroutine WRTETP (see ref. 1 for a description of WRTETP formats). The contents of "EOMLOD" are displayed in figure 25.

## Contents of Aerodynamic \{HEADER\} Array

The array HEADER will always contain 30 words.

```
Word Contents
    1 7HDYLOFLX.
    2 6HL2217vc.
    3 Date of run in format 10Hyr/mo/da.
    4 NDOF = number of degrees of freedom ( }1\leqslant\mathrm{ NDOF }\leqslant100)
    5 NID = 0.
    N NGCP = number of gust control points (1\leqslantNGCP}\leqslant35)
    7 NFREQM = number of reduced frequencies \geqslant1.
    8 q = dynamic pressure = 1/2
    9 V = velocity = true airspeed.
10-20 Future use = 0.
    21 1 indicates [ }\mp@subsup{M}{1}{}\mathrm{ ] is on "EOMTAP."
    22 0 or 1; a 1 indicates [M2] is on "EOMTAP."
```



Figure 24. - Contents of EOMTAP


The logical file shown above is repeated for every surface (see ISTORE on Cards 3.12-3.15). There will be empty files for imbedded surfaces not defined.

Figure 25. - Contents of EOMLOD

241 indicates [ $\mathrm{M}_{4}$ ] is on "EOMTAP."
25 1 indicates $\left[\mathrm{M}_{5}\right.$ ] is on "EOMTAP."

23

1 indicates $\left[\mathrm{M}_{3}\right]$ is on "EOMTAP." 0 indicates $\left[\mathrm{M}_{6}\right]$ is not on "EOMTAP."

0 indicates $\left\{\mathrm{C}_{2}\right\}$ is not on "ECMTAP."
0 indicates $\left\{\mathrm{C}_{3}\right\}$ is not on "EOMTAP."
1 indicates $\left\{f_{\ell}\right\}$ is on "EOMTAP."
1 indicates $[\tilde{\phi}]$ is on "EOMTAP."

### 6.6 RESTRICTIONS

L217 (EOM) problem size maximums are displayed below.

| Variable | Maximum | Variable Description |
| :--- | :---: | :--- |
| NMODES | 100 | Number of modes. |
| NTOTF | 400 | Total number of aerodynamic elements on all <br> bodies (thin, slender, and interference). <br> NGCP |
| NFREQM | 20 | Number of gust control points (zones). <br> Number of reduced frequencies at which <br> aerodynamics are defined. |
| Nmax | 100 | Number of aerodynamic elements for an <br> interpolation surface. |

### 6.7 DIAGNOSTICS

All errors detected by L217 (EOM) will result in the printing of a diagnostic message. The message will be introduced by the line,
********FATAL ERROR n IN name
where " $n$ " indicates the error number (see the list below) and "name" indicates the routine printing the message. The introductory line will be followed by other lines describing the error conditions.

## Error

Number Description of Error

1

2

3

Premature end-of-file on card input.
Keyword on data card is inappropriate or unrecognizable.
Structural data not processed correctly before processing aerodynamic card input data.

Illegal file name.
Illegal file or matrix spacing.
Variable outside allowable range.
FETAD error trying to generate a file's FIT, buffer, and RA + 2 entry.
FETDEI error trying to release a file's FIT, buffer, and RA + 2 entry.
READTP error trying to read a matrix from a file (see sec. 6.7.1 for a list of READTP error codes.)

WRTETP error trying to write a matrix on a file (see sec. 6.7.2 for a list of WRTETP error codes).

STARTR error trying to initialize blank common storage array.
INITIR error trying to generate storage for an array.
DELETR error trying to erase storage of an array.
AINTL or AINTG error trying to interpolate for modes.

### 6.7.1 READTP ERROR CODES

Error Code
$=0 \quad$ If no errors are detected during reading.
$=1000+\mathrm{I} \quad$ If an FSF (forward space file) error occurred, where I is the number of file marks remaining to be skipped when an end-of-information was encountered.
$=2$
If the number of matrices or files to be skipped, before reading starts, is less than zero.

| $=3$ | If the dimensioned number of rows in the matrix is less than or <br> equal to zero. |
| :--- | :--- |
| $=3000+\mathrm{I}$ | If an FSR (forward space record) error occurred, where I is the <br> number of records remaining to be skipped when either an <br> end-of-file or end-of-information was encountered. |
| $=4$ | Number of rows in the matrix is greater than the dimensioned row <br> size in the program. |
| $=5$ | If the name check failed. |
| $=6$ | If the number of rows in the matrix (M) times the number of <br> columns (N) is greater than the buffer size, or $M * N \leqslant 0$. |
| $=7$ | An end-of-file was read. If it occurs while reading the matrix ID, no <br> information is stored in the user's area. If it occurs while reading <br> the matrix, the ID information will be stored. Note that the records <br> will always be in pairs and an end-of-file should always be <br> encountered with the ID record. |

### 6.7.2 WRTETP ERROR CODES

## Error Code

$=0 \quad$ If no errors are detected during writing.
$=1000+\mathrm{I}$. If an FSF (forward space file) error occurred, where I is the number of file marks remaining to be skipped when an end-of-information was encountered.
$=2 \quad$ If the number of matrices or files to be skipped, before writing starts, is less than zero.
$=3 \quad$ If the dimensioned number of rows in the matrix is less than or equal to zero.
$=3000+I \quad$ If an FSR (forward space record) error occurred, where $I$ is the number of records remaining to be skipped when either an end-of-file or an end-of-information was encountered.
$=4 \quad$ If the actual number of rows is greater than the dimensioned number of rows in the matrix.
$=6$
If the number of rows in the matrix (M) times the number of columns ( N ) is greater than the buffer size.

### 7.0 SAMPLE PROBLEM

The sample problem chosen is the fuselage, wing, and horizontal tail combination shown in figure 26. One control surface is placed on the wing. Two rigid body modes (vertical trnaslation and pitch) make up the structural degrees of freedom. The control surface degree of freedom was added to the basic modes in L215 (INTERP). The FLEXSTAB aerodynamic option was chosen for this example.

Generalized stiffness and mass data is read from file GMASS. Interpolation data is on the file MODH7. There are four interpolation arrays. Two rotary inertias are input for the control surface due to the rotation of the surface about its swept hinge line.

## Boeing Commercial Airplane Company

P.O. Box 3707

Seattle, Washington 98124
May 1977


Figure 26. - Sample Problem Configuration

```
L2ITIEOMI USAGE DOCUMENT SAMPLE PRUBLEM
USING FLEXSTAB AERODYNAMIC DATA FDR HTWC.
GITLE MODESARE IN THE INERTIAL AXIS SYSTEM.
STITLE TRANSLATION IS POSITIVE DOWN. PITCHIS POSITIVE NOSE UP.
sSTRUCTURAL OATA
SI2E Z I
SYMMETRIC ANALYSIS
HALF AIRPLANE
MUDES MUUH7
STIFFNESS GMASS O IPRINT
MASS GMASS O OPRINT
OAMPING O.
SURFACE 4 LIRANS ROTY ROTX LOCAL
\begin{tabular}{llllll}
3.25 & 4.5 & 0.0 & 0.1 & 1.0 & 2.0
\end{tabular}
ENO
$AERO FLEXSTAB AIC
CANSANTS 6.11015E-g 9954.4 1 2
SATAP MOUH7
GDTAPE GOH?
SUSSTP SOIH7 SOSH7
PRINT MCDES
PEINT FORCES
PKINT EQUATIONS
SLENDER BODY 2
THIN BODY 2 
```



```
    12
THIN BOOY 0. 3 0. 1. 0.
squIT
```


## PROGRAM DIRECTIVE CARDS READ BY L2L7AO

\$EOM
(\$TITLE L2I7IEOM) USAGE DOCUMENT SAMPLE PROBLEM ..... 1MODES ARE IN THE INERTIAL AXIS SYSTEM.TRANSLATION IS POSITIVE DOWN, PITCH IS POSITIVE NOSE UP.
-- NOW IN STRUCT TO PREPARE EQUATIONS OF MOTION MATRICES IMI, M2, AND M3) DEPENDENT ONLY UPON

    STRUCTURAL INPUT DATA (MASS, STIFFNESS, ETC.I.
    
    SIRUCTURAL CARD INPUT DATA FULLOWS.
    ```
ISIZE 2 1
    TOTAL NUMBER OF DEGREES OF FREEDOM = 3
    1 21 STRUCTUKAL AND 1 1) CONTROL SURFACE.
(symmeTRIC ANALYSIS
(HALF AIRPLANE
{MODES MODH7
```

THE mODE SHAPES WIll be read from modhy
ISTIFFNESS GMASS O IPRINT 1
The generalized stiffne ss matrix will be read fron gmass
AFTER SKIPPING 1 MATRICES.
AND PRINTED AS READ.
IMASS GMASS OPRINT 1
the generalizeo mass matrix will be read from gmass
AND PRINTEO AS READ.
I DAMPING
THE DAMPING FACTORS WILL BE READ FROM CARDS

(SURFACE 4 ITRANS ROTY ROTX IOCAL
CONTROL SURFACE 4 HAS 1 NDDES:
rotation difox freedoms will be used.
ROTATION DZ/OY FREEDOMS WILL bE USED.
the coordinates are in the local aero. axis system.

| x | $r$ | $z$ | Mass | INERTIA | I NERT IA |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13.25 | 4.5 | 0.0 | 0.1 | 1.0 | 2.0 |  |

I END ..... 1


MATRIX

| LABEL | general Izeo mass |  |  |
| :---: | :---: | :---: | :---: |
| SILE |  | 2 ROWS AND | 2 COL UMN |
| COLUMN |  | 1 | 2 |
| ROW | 1 | . $1763 E+03$ | $-.1100 E+00$ |
| ROW | 2 | -. $1100 \mathrm{E}+00$ | -4800E+01 |

-- STRUCT IS FINISHED.
MAXIMUM FIELD LENGTH USED BY STRUCT WAS 073343
GETURN TO L2:7 DRIVING PROGRAM AND READ ANOTHER PROGRAM DIRECTIVE CARD.

## ( SAERO <br> FLEXSTAB AIC

## -- NOW IN FLXAIC TO PREPARE EQUATIONS OF MOTION BASED UPON FLEXSTAB AIC MATRIX.

FLEXSTAB CARD INPUT DATA FOLLOWS.
(CONSTANTS 6.11015E-8 $9954.4 \quad 1 \quad 2$
RHO $=.61102 E-07$
$V E L=.99544 E+04$
$0=-5 * R H O * V E L * * 2=-30273 E+01$

| © SATAP MODES WIL | $\begin{aligned} & \text { MOOH7 } \\ & \text { LL BE READ } \end{aligned}$ | FROM | MODH | H7 |  |  |  |  |  |  | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (GDTAPE FLEXSTAB | GDH7 GEOMETRY | WILL | READ | FROM | GDH7 |  |  | 11. |  |  | 1 |
| $\begin{aligned} & \text { CSOSSTP } \\ & \text { FLEXSTAB } \end{aligned}$ | SDIH7 <br> AERO DATA | $\begin{aligned} & \text { SDSI } \\ & \text { WILL } \end{aligned}$ | $5 H 7$ <br> BE | READ | FRUM | SDIH7 | AND | SOSH7 | ICASE | 11. | 1 |
| (PRINT | MODES |  |  |  |  |  |  |  |  |  | 1 |
| (PRINT | FORCES |  |  |  |  |  |  |  |  |  | J |
| (PRINT | ECUATIONS |  |  |  |  |  |  |  |  |  | ) |
| (SLENDER Y-MOT | $\begin{aligned} & \text { BODY } \\ & \text { ON WILL NO } \end{aligned}$ | $\text { JT } 8 \text { E }$ | RETA | $\stackrel{2}{\text { AINEO. }}$ |  |  |  |  |  |  | 1 |
| (THIN BOD | $\gamma$ |  | $1$ |  |  | . 5 |  | 1. | 0. |  | I |
| (CONTROL | SURFACE |  |  | 1 |  |  |  |  |  |  | , |
| 112 |  |  |  |  |  |  |  |  |  |  | ) |
| ITHIN BOO (END |  |  | 3 |  |  | 0. |  | 1. | 0. |  | 1 |

FLEXSTAB CASE IDENTIFICATION
THIS RUN IS a test case using the douglat test case hihc
GD-2
USER IDENTIFICATION =
CASE HTWC HAS ONE CONTROL SURFACE
GD-3
MACH NUMBER $=.8000 E+00$
UNITS ARE IN INCH

## geonetry summary for doublet lattice case i

## BOX ELEMENT COORDINATES ARE AT QUARTER CHORO.

## symmetric analysis requested.

| NO. | $\underset{X}{\text { REF }} \cdot \operatorname{AXIS}$ | COORD |  | DIHEDRAL <br> IN DEG. | Area | TYPE | POS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | . 30 | 0.00 | 0.00 | 0.00 | . 35 | 58 | ON |
| 2 | 1.50 | 0.00 | 0.00 | 0.00 | . 75 | 50 | ON |
| 3 | 2.50 | 0.00 | 0.00 | 0.00 | .05 | S H | ON |
| 4 | 3.50 | 0.00 | 0.00 | 0.00 | .93 | 5 H | ON |
| 5 | 4.50 | 0.00 | 0.00 | 0.00 | . 98 | St | ON |
| 6 | 5.50 | 0.00 | 0.00 | 0.00 | .93 | S | ON |
| 7 | 0.50 | 0.00 | 0.00 | 0.00 | . 85 | S | ON |
| 8 | 7.50 | 0.00 | 0.00 | 0.00 | . 75 | 58 | ON |
| 9 | 8.50 | 0.00 | 0.00 | 0.00 | .60 | St | ON |
| 10 | 9.50 | 0.00 | 0.00 | 0.00 | . 25 | SO | ON |
| 11 | . 50 | 0.00 | 0.00 | 0.00 | .33 | S 8 | ON |
| 12 | 1.50 | 0.00 | 0.00 | 0.00 | . 75 | 58 | ON |
| 13 | 2.50 | 0.00 | 0.00 | 0.00 | .85 | 50 | ON |
| 14 | 3.50 | 0.00 | 0.00 | 0.00 | . 93 | 50 | ON |
| 15 | 4.50 | 0.00 | 0.00 | 0.00 | . 913 | 58 | ON |
| 16 | 5.50 | 0.00 | 0.00 | 0.00 | .95 | St | ON |
| 17 | 6.50 | 0.00 | 0.00 | 0.00 | . 85 | St | ON |
| 18 | 7.50 | 0.00 | 0.00 | 0.00 | .75 | 5 U | ON |
| 19 | 8.50 | 0.00 | 0.00 | 0.00 | . 60 | $5{ }^{5}$ | ON |
| 20 | 9.50 | 0.60 | 0.00 | 0.00 | . 25 | 58 | ON |
| 21 | 2.50 | . 50 | 1,00 | 0.00 | 1.00 | 10 |  |
| 22. | 3.50 | . 50 | 1.00 | 0.30 | 1.00 | 18 |  |
| 23 | 4.50 | . 50 | 1.00 | 0.00 | 1.00 | 18 |  |
| 24 | 5.50 | - 50 | 1.00 | 0.00 | 1.00 | 10 |  |
| 25 | 6.50 | - 50 | 1.00 | 0.00 | 1.00 | 18 |  |
| 26 | 7.50 | . 50 | 1.00 | 0.00 | 1.00 | 18 |  |
| 27 | 8.50 | . 50 | 1.00 | 0.00 | 1.00 | 10 |  |
| 28 | 9.25 | . 50 | 1.00 | 0.00 | . 50 | 18 |  |
| 49 | 9.75 | . 50 | 1.00 | 0.00 | . 50 | 10 |  |
| 30 | 2.50 | 1.00 | . 50 | 90.00 | 1.00 | $1{ }^{1}$ |  |
| 31 | 3.50 | 1.CO | - 50 | 90.00 | 1.00 | 10 |  |
| 32 | 4.50 | 1.00 | - 50 | 40.00 | 1.00 | 11 |  |
| 33 | 5.50 | 1.00 | . 50 | 90.00 | 1.00 | 1 B |  |
| 3.4 | 6. 50 | 1.CO | . 50 | 90.00 | 1.00 | $1{ }^{1}$ |  |
| 35 | 7.50 | $1 . \mathrm{CO}$ | . 50 | 90.00 | 1.00 | 1 H |  |
| 36 | 8.50 | 1. CO | . 50 | 90.00 | 1.00 | 1 is |  |
| 37 | 9.25 | 1.00 | - 50 | 90.00 | . 50 | 18 |  |
| 38 | 9.15 | 1.00 | . 50 | 90.00 | . 50 | 13 |  |
| 39 | 2.50 | 1.00 | -. 50 | 90.00 | 1.00 | 18 |  |
| 40 | 3.50 | 1.00 | -. 50 | 90.00 | 1.00 | 18 |  |
| 41 | 4.50 | 1.00 | -. 50 | 40.00 | 1.00 | 18 |  |
| 42 | 5. 50 | 1.00 | -. 50 | 90.00 | 1.00 | 1 is |  |
| 43 | 6.50 | 1. CO | -. 50 | 20.00 | 1.00 | 18 |  |
| 44 | 7.50 | 1.00 | -. 50 | 90.00 | 1.00 | 10 |  |
| 45. | 0.50 | 1.00 | -. 50 | 90.00 | 1.00 | 10 |  |
| 46 | 9.15 | 1.00 | -. 50 | 90.00 | .50 | 14 |  |
| 47 | 9.15 | 1.00 | -. 30 | 90.00 | . 50 | 14 |  |
| 48 | 2.50 | . 50 | -1.00 | 180.00 | 1.00 | 11 |  |
| 49 | 3.50 | - 50 | -1.00 | 180.00 | 1.00 | 113 |  |
| . 50 | 4.80 | . 50 | -1.00 | 180.00 | 1.00 | 10 |  |

MODAL
SURFACE ----------- SIGMA
IGMA $\qquad$
FRACTION
FAC SF IN OUT ND. FRACTION
$-15708 E+01$ - $15708 E 01$ .1570 UE +01 $.15 \%$ UE +01 $.15708 E+01$ $.15708 E+01$
$.15700 E+01$ $-15700 E+01$
$-15108 E+01$ $-157085+01$ -1370 UE 01 .1570 Et 01
$.15708 E+0 L$ -.1570 EE +01
$-.1570 L E+01$ -.1370 OUE +01
$-.1370 B L+01$ $15=.1370 B L+01$
$15=-1310 U E+01$ $10-.1570$ ut + 01 $11=-1370 \mathrm{~L}(101$
$18=-1370$ 时 131 $19=-1510 \angle E+01$
$20-15700 E+01$
20 -.
30.
40.
50.
70.
$\begin{array}{ll}9 & 0 . \\ 9 & 0 .\end{array}$
90.
100.

100
10.
4
4
30.
$\begin{array}{ll}6 \\ 7 & 0 .\end{array}$
$\begin{array}{ll}0 & 0 \\ 9 & 0\end{array}$
90.
100.
100.
30.
40.
0.
40.
$\begin{aligned} & 80 . \\ & 9 \\ & 9\end{aligned} 0$.
100.
100.
30.
30.
400
30.

NNNNNNNNNNNNNNNNNNNNO000000000000000000000000000000

| 51 | 5.50 | . 50 | -1.00 | 180.00 | 1.00 | 18 | 0 | 0 | 6 | 0. | 16 | -10000E 01 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 52 | 6.50 | - 50 | -1.00 | 180.00 | 1.00 | 1 H | 0 | 0 | 7 | 0. | 17 | . 10000t +01 | 0 | 0 |
| 53 | 7.50 | - 30 | -1.00 | 180.00 | 1.00 | 18 | 0 | 0 | 8 | 0. | 18 | -10000e +01 | 0 | 0 |
| 54 | 8.50 | -50 | -1.00 | 180.00 | 1.00 | 18 | 0 | 0 | 9 | 0. | 19 | -100COE + O1 | 0 | 0 |
| 53 | 9.25 | - 50 | -1.00 | 1110.00 | . 50 | 18 | 0 | 0 | 10 | 0. | 20 | -10conto 01 | 0 | 0 |
| 36 | 9.75 | .30 | -1.00 | 160.00 | . 50 | It | 0 | 0 | 10 | 0 . | 20 | . 10000E*01 | 0 | 0 |
| 57 | 4.10 | 1.48 | 0.00 | 0.00 | . 92 | 14 OFF | 1 | 1 | 21 | -10000E+01 | 0 | 0. | 1 | 2 |
| 58 | 5.02 | 1.48 | 0.00 | 0.00 | .92 | T ${ }_{\text {d Off }}$ | 1 | 1 | 22 | . $10000 \mathrm{E}+01$ | 0 | 0. | 1 | 2 |
| 59 | 6.54 | 1.40 | 0.00 | 0.00 | - 92 | It OFF | 1 | 1 | 23 | . $10000 \mathrm{E}+01$ | 0 | 0. | 1 | 2 |
| 00 | 5.12 | 2.40 | 0.00 | 0.00 | . 75 | It OFF | 1 | 1 | 24 | . $10000 \mathrm{E}+01$ | 0 | 0. | 1 | 2 |
| 61 | 5.87 | 2.40 | c. 00 | 0.00 | . 75 | TO DFF | 1 | 1 | 25 | . 10000 E 01 | 0 | d. | 1 | 2 |
| 02 | 6.62 | 2.48 | 0.00 | 0.00 | . 75 | It OfF | 1 | 1 | 26 | . $10000 \mathrm{E}+01$ | 0 | 0. | 1 | 2 |
| 63 | 5.53 | 3.48 | 0.00 | 0.00 | . 58 | 1110 FF | 1 | 1 | 27 | . $10000 \mathrm{~F}+01$ | 0 | 0. | 1 | 2 |
| 04 | 6.12 | 3.48 | 0.00 | 0.00 | . 54 | TU OFF | 1 | 1 | 28 | . 10000E 01 | 0 | 0. | 1 | 2 |
| 6.5 | 6.71 | 3.48 | 0.00 | 0.00 | - 58 | IS UFF | 1 | 1 | 27 | -10000E +01 | 0 | 0. | 1 | 2 |
| 66 | 5.94 | 4.47 | 0.00 | 0.00 | .42 |  | 1 | 1 | 30 | -10000E +01 | 0 | 0. | 1 | 2 |
| 67 | 6.37 | 4.47 | 0.00 | 0.00 | . 42 | TB UfF | 1 | 1 | 31 | . 10000 E 01 | 0 | 0. | 1 | 2 |
| 68 | 6.79 | 4.47 | 0.00 | 0.00 | . 42 | TB UFF | 4 | 1 | 32 | -10000e 01 | 0 | 0. | 1 | 2 |
| 69 | 9.34 | 1.48 | 0.00 | 0.00 | -44 | T $B$ UFF | 3 | 3 | 33 | . $10000 \mathrm{E}-01$ | 0 | c. | 1 | 2 |
| 70 | 9.13 | 1.48 | 0.00 | 0.00 | . 44 | ia orf | 3 | 3 | 34 | . 10000 Cl 01 | 0 | 0. | 1 | 2 |
| 71 | 9.53 | 2.47 | 0.00 | 0.00 | -31 | 1H GFF | 3 | 3 | 35 | . 100000 col | 0 | 0. | 1 | 2 |
| 72 | 9.84 | 2.47 | 0.00 | 0.00 | . 31 | TB OFF | 3 | 3 | 36 | . 10000 E01 | 0 | 0. | 1 | 2 |


|  | CONTRCR PGINT GEGMEIRY REF. AXIS COORDINAIES |  |  | 800\% | $\begin{aligned} & \text { FOOAL } \\ & \text { INTERPOLATICN } \\ & \text { SURFACE(S) } \end{aligned}$ |  |  | CuS 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NO. | K | $\checkmark$ | 2 | TYPE |  |  | HG | LINE |
| 1 | - 50 | 0.10 | 0.00 | 50 | 2 | 2 | 1.00000 | 1 |
| 2 | 1.50 | 0.00 | 0.00 | SB | 2 | 2 | 1.00000 | 1 |
| 3 | 2.50 | $0 . \mathrm{CO}$ | 0.00 | St | 2 | 2 | 1.00000 | 1 |
| 4 | 3.50 | 0.10 | 0.00 | S8 | 2 | 2 | 1.00000 | 1 |
| 3 | 4.50 | 0.60 | 0.00 | SO | 2 | 2 | 1.05000 | 1 |
| 0 | 5.50 | 0.60 | 0.00 | Sis | 2 | 2 | 1.00000 | 1 |
| 1 | 6.30 | 0.60 | 0.00 | SO | 2 | 2 | 1.00000 | 1 |
| B | 7.50 | $0 . \mathrm{CO}$ | 0.00 | 54 | 2 | 2 | 1.00000 | 1 |
| 9 | B.50 | 0.60 | 0.00 | St | 2 | 2 | 1.00000 | 1 |
| 10 | 9.50 | 0.co | 0.00 | So | 2 | 2 | 1.Jvono | 1 |
| 11 | 5.02 | 1.48 | 0.00 | 16 | 1 | 1 | 1.00000 | 1 |
| 12 | 5.94 | 1.48 | 0.00 | IB | 1 | 1 | 1.00000 | 1 |
| 13 | 6.86 | 1.48 | 0.00 | 10 | 1 | 1 | 1.110000 | 1 |
| 14 | 5.38 | 2.48 | 0.00 | 10 | 1 | 1 | 1.00000 | 1 |
| 15 | 6.13 | 2.48 | 0.00 | Tu | 1 | 1 | 1.00000 | 1 |
| 16 | 6.89 | 2.48 | 0.00 | 13 | 1 | 1 | 1.00000 | 1 |
| 17 | 5.74 | 3.48 | 0.00 | 10 | 1 | 1 | 1.00000 | 1 |
| 18 | 6.32 | 3.48 | 0.00 | 10 | 1 | 1 | 1.00000 | 1 |
| 19 | 6.91 | 3.48 | 0.00 | It | 1 | 1 | 1.00000 | 1 |
| 20 | 6.09 | 4.47 | 0.00 | 10 | 1 | 1 | 1.00000 | 1 |
| 21 | 6.51 | 4.41 | 0.00 | 18 | 1 | 1 | 1.00000 | 1 |
| 22 | 0.94 | 4.47 | 0.00 | 18 | 4 | 1 | 1.00000 | 1 |
| 23 | 9.49 | 1.40 | 0.00 | 18 | 3 | 3 | 1.00000 | 1 |
| 24 | 9.93 | 1.43 | 0.00 | 18 | 3 | 3 | 1.00000 | 1 |
| 25 | 9.64 | 2.47 | 0.00 | 18 | 3 | 3 | 1.00000 | 1 |
| 26 | 9.95 | 2.47 | 0.00 | 113 | 3 | 3 | 1.00000 | 1 |



```
LABEL MDDAL DEFLECTIONS - PARTITION 1.MODES 1 THROUGH 3
SIIE 26 ROWS AND 3 COLUMNS
```

| COLUMN | 1 | 2 | 3 |  |
| :---: | :---: | :---: | :---: | :---: |
| ROW | 1 | $-.1000 E+01$ | $.4500 E+01$ | 0. |
| ROW | 2 | $-.1000 E+01$ | $.3500 E+01$ | 0. |

ROW 3 -. $1000 E+01 \quad .2500 E+01$ O.
ROW $4-.1000 E+01 \quad .1500 E+010$.
ROW $5-.1000 E+01 \quad .5000 E+000$.
ROH $6 \quad-.1000 E+01-.5000 E+00 \quad 0$.
ROW $7-.1000 E+01-.1500 E+010$.
ROH $8-.1000 E+01-.2500 E+010$.
ROW $9-.1000 E+01-.3500 E+01 \quad 0$.
ROW $10-.1000 E+01-.4500 E+010$.
ROH 11 -. $1000 E+01$-. $1831 E-010$.
ROW $12-.1000 E+01-.9432 E+00 \quad 0$.
ROW $13 \quad-.1000 E+01-.1857 E+010$.
ROW $14-.1000 E+01-.386 L E+000$.
ROW $15-.1000 \mathrm{E}+01-.1133 \mathrm{E}+010$.
ROW $16-.1000 E+01-.1 \varepsilon 86 E+010$.
ROW $17-.1000 E+01-.7319 E+00 \quad 0$.
ROW $18-.1000 E+01-.1322 E+010$.
ROW $19-.1000 E+01-.1911 E+010$ 0.
ROW $20 \rightarrow 1000 E+01-.1093 E+010$.
ROW $21-.1000 E+01-.1515 E+010$.
ROW $22-.1000 E+01-.1937 E+01-.3540 E+00$

ROW $23-.1000 E+01-.4493 E+010$ O.
ROW $24-.1000 E+01-.4934 E+010$ O.

ROW $25-.1000 E+01-.4636 E+010$.
ROW $26-.1000 E+01-.4953 E+010$.

| LABEL | MODAL | SLOPES | - | PARTITION | 1 | MODES | 1 | THROUGH | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SIZE | 26 | ROWS AND |  | COL UMNS |  |  |  |  |  |


| COL |  | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| ROW | 1 | 0. | -. $1000 \mathrm{E}+01$ | 0. |
| ROW | 2 | 0. | -. $1000 \mathrm{E}+01$ | 0. |
| ROW | 3 | 0. | -. $1000 \mathrm{e}+01$ | 0. |
| ROW | 4 | 0. | -. 1000E+01 | 0. |
| ROW | 5 | 0. | $-.1000 E+01$ | 0. |
| ROW | 6 | 0. | $-.1000 E+01$ | 0. |
| Row | 7 | 0. | -. $1000 \mathrm{E}+01$ | 0. |
| ROW | 8 | 0. | $-.1000 E+01$ | 0. |
| ROW | 9 | 0. | $-.1000 E+01$ | 0. |
| ROW | 10 | 0 - | -. $1000 \mathrm{E}+01$ | 0. |
| ROW | 11 | 0. | $-.9992 E+00$ | 0. |
| ROH | 12 | 0. | $-.9998 E+00$ | 0. |
| ROW | 13 | 0. | $-.1006 E+01$ | 0. |
| ROW | 14 | 0. | $-.1000 E+01$ | 0. |
| ROW | 15 | 0. | -. 1000E+01 | 0. |
| ROW | 16 | 0. | -.9991E+00 | 0. |
| ROW | 17 | 0 . | $-.9966 E+00$ | 0. |
| ROW | 18 | 0. | -.996 7E + 00 | 0. |
| ROW | 19 | 0. | $-.9967 E+00$ | 0. |
| ROW | 20 | 0. | $-.1001 E+01$ | 0. |
| RON | 21 | 0. | -. 1001E+02 | 0. |
| ROW | 22 | 0. | -. 1001E+01 | $-.9864 E+00$ |
| ROW | 23 | 0. | -. 1000E+01 | 0. |
| ROW | 24 | 0. | -. 1000E+01 | 0. |
| ROW | 25 | 0 - | $-.1000 \mathrm{E}+01$ | 0. |
| ROW | 26 | 0. | -. $1000 \mathrm{E}+01$ | 0. |

```
MATRIX
LABEL FDRCE PT. MODAL DEFLECTIONS - PART. 1 MODES 1 THRU 3
SI2E 36 ROWS AND 3 COLUMNS
```

| COL |  | 1 | 2 |  |
| :---: | :---: | :---: | :---: | :---: |
| ROW | 1 | 0. | 0. | 0. |
| ROW | 2 | 0. | 0. | 0. |
| ROW | 3 | 0. | 0 . | 0. |
| ROW | 4 | 0. | 0. | 0. |
| ROW | 5 | 0. | 0. | 0. |
| ROH | 6 | 0. | 0. | 0. |
| ROW | 7 | 0. | 0. | 0. |
| ROW | 8 | 0. | 0. | 0. |
| ROW | 9 | 0. | 0 . | 0. |
| ROW | 10 | 0. | 0. | 0. |
| ROW | 11 | -. 1000E+01 | -4500E + 01 | 0. |
| ROW | 12 | $-.1000 E+01$ | . $3500 \mathrm{E}+01$ | 0. |
| ROW | 13 | -. 1000E+01 | . $25000+01$ | 0. |
| Raw | 14 | $-.1000 E+01$ | -1500E+01 | 0. |
| ROW | 15 | $-.1000 E+01$ | . $5000 \mathrm{E}+00$ | 0. |
| ROW | 16 | -. 1000E+01 | -. $5000 \mathrm{E}+00$ | 0. |
| ROW | 17 | -. 1000E+01 | $-.1500 E+01$ | 0. |
| ROW | 18 | $-.1000 E+01$ | -. $2500 \mathrm{E}+01$ | 0. |
| RON | 19 | -. $1000 \mathrm{E}+01$ | -. $3500 E+01$ | 0. |
| ROW | 20 | -. $1000 \mathrm{E}+01$ | -.4500E+01 | 0. |
| ROW | 21 | $-.1000 E+01$ | . 304 TE+00 | 0. |
| ROK | 22 | $-.1000 E+01$ | $-.6209 E+00$ | 0. |
| ROW | 23 | $-.1000 E+01$ | -. $1538 \mathrm{E}+01$ | 0. |
| ROW | 24 | $-.1000 E+01$ | -. $1264 \mathrm{E}+00$ | 0. |
| ROW | 25 | -. $2000 \mathrm{E}+01$ | $-.8708 E+00$ | 0. |


| ROW | 26 | $-.1000 E+01$ | $-.1622 E+01$ | 0. |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 27 | $-.1000 E+01$ | $-.5252 E+00$ | 0. |
| ROW | 28 | $-.1000 E+01$ | $-.1116 E+01$ | 0. |
| ROW | 29 | $-.1000 E+01$ | $-.1705 E+01$ | 0. |
| ROW | 30 | $-.1000 E+01$ | $-.9453 E+00$ | 0. |
| ROW | 31 | $-.1000 E+01$ | $-.1367 E+01$ | 0. |
| ROW | 32 | $-.1000 E+01$ | $-.1789 E+01$ | $-.2082 E+00$ |
| ROW | 33 | $-.1000 E+01$ | $-.4339 E+01$ | 0. |
| ROW | 34 | $-.1000 E+01$ | $-.4780 E+01$ | 0. |
| ROW | 35 | $-.1000 E+01$ | $-.4525 E+01$ | 0. |
| ROW | 36 | $-.1000 E+01$ | $-.4842 E+01$ | 0. |

The matrices printeo below are mritten onto the file nameo eumlod

## matrix

LaBEL FREQUENCIES AI WHICH THE AERO FORCES ARE OEFINED IN CYCLES / SECOND.

## MATRIX

LABEL ELEMENT LOCAL COORD. ( $X, Y$ I
SURFACE 1
SIZE

| COLUMN | 1 | 2 |  |
| :--- | :---: | :---: | :---: |
| ROW | 1 | $.1202 E+01$ | $.1485 E+01$ |
| ROW | 2 | $.2121 E+01$ | $.1485 E+01$ |
| ROW | 3 | $.3040 E+01$ | $.1485 E+01$ |
| ROW | 4 | $.1617 E+01$ | $.2481 E+01$ |
| ROW | 5 | $.2370 E+01$ | $.2481 E+01$ |
| ROW | 6 | $.3123 E+01$ | $.2481 E+01$ |
| ROW | 7 | $.2032 E+01$ | $.3476 E+01$ |
| ROW | 8 | $.2619 E+01$ | $.3476 E+01$ |
| ROW | 9 | $.3206 E+01$ | $.3476 E+01$ |
| ROW | 10 | $.2444 E+01$ | $.4467 E+01$ |
| ROW | 11 | $.2867 E+01$ | $.4467 E+01$ |
| ROW | 12 | $.3289 E+01$ | $.4467 E+01$ |

## MATRIX

LABEL ELEMENT AREAS SURFACE 1

SI2E 12 ROWS AND 1 COLUMNS

| COLUMN |  | 1 |
| :--- | ---: | :--- |
| ROW | 1 | $.9167 E+00$ |
| ROW | 2 | $.9167 E+00$ |
| ROW | 3 | $.9167 E+00$ |
| ROW | 4 | $.7500 E+00$ |
| ROW | 5 | $.7500 E+00$ |
| ROW | 6 | $.7500 E+00$ |
| ROW | 7 | $.5833 E+00$ |
| ROW | 8 | $.5833 E+00$ |
| ROW | 9 | $.5833 E+00$ |
| ROW | 10 | $.4167 E+00$ |
| ROW | 11 | $.4167 E+00$ |
| ROW | 12 | $.4167 E+00$ |

MATRIX
LABEL FL-RESPONSE AR FORCES 10 SURFACE, I AT FREQUENCY 1 COEF.I
$S I 2 E-12 R O W S$ AND 3 COLUMNS

| COLUMN |  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 1 | 0. | $.2865 E+02$ | $.5213 E+00$ |
| ROW | 2 | 0. | $.9162 E+01$ | $.4511 E+00$ |
| ROW | 3 | 0. | $.4115 E+01$ | $.4035 E+00$ |
| ROW | 4 | 0. | $.2691 E+02$ | $.6343 E+00$ |
| ROW | 5 | 0. | $.7455 E+01$ | $.5074 E+00$ |
| ROW | 6 | 0. | $.3616 E+01$ | $.4981 E+00$ |
| ROW | 7 | 0. | $.2319 E+02$ | $.8238 E+00$ |
| ROW | 8 | 0. | $.6002 E+01$ | $.7589 E+00$ |
| ROW | 9 | 0. | $.2863 E+01$ | $.9914 E+00$ |
| ROW | 10 | 0. | $.1736 E+02$ | $.1169 E+01$ |
| ROW | 11 | 0. | $.3635 E+01$ | $.1534 E+01$ |
| ROW | 12 | 0. | $.1654 E+01$ | $.5895 E+01$ |


| LABEL |  | f2-RESPONSE AIR FORCES (Q dot coef.) SURFACE, 1 AT FREQUENCY 1 |  |  |
| :---: | :---: | :---: | :---: | :---: |
| SI2E |  | 12 RONS ANO | 3 COL |  |
| COLUMM |  | 1 | 2 | 3 |
| ROW | 1 | . 2879E-02 | .1873E-02 | . 1879E-04 |
| ROW | 2 | .9187E-03 | -2196E-02 | .1626E-04 |
| ROW | 3 | .4065E-03 | . 2249 E -02 | . 1455E-04 |
| ROW | 4 | . 2703E-02 | .2445E-02 | . 2287E-04 |
| ROW | 5 | . 7486E-03 | . $2156 \mathrm{E}-02$ | . 1829E-04 |
| ROW | 6 | . 363 OE-03 | . 1678E-02 | .1790E-04 |
| ROW | 7 | . 233 2E-02 | . $26498-02$ | .2970E-04 |
| ROH | 8 | .6029E-03 | . $1569 \mathrm{E}-02$ | . $2736 \mathrm{E}-04$ |
| ROW | 9 | -2875E-03 | .1136E-02 | . $3574 \mathrm{E}-04$ |
| ROW | 10 | . $1744 \mathrm{E}-02$ | . $2340 \mathrm{E}-02$ | . $4215 \mathrm{E}-04$ |
| ROW | 11 | . 3652E-03 | .9041E-03 | . $5530 \mathrm{E}-04$ |
| ROW | 12 | . 1661 E-03 | .6086E-03 | . 2125 E-03 |

## MATRIX

## LABEL FGUST-GUST AIR FORCES (COMPLEX)

SI2E
(COMPLEX) 12 ROWS ANO 1 COLUMNS

| COLUMN |  | 1 |  |
| :--- | :--- | :--- | :--- |
| ROW | 1 | $.2879 E-02$ | 0. |
| ROW | 2 | $.9187 E-03$ | 0. |
| ROW | 3 | $.4065 E-03$ | 0. |
| ROW | 4 | $.2703 E-02$ | 0. |
| ROW | 5 | $.7486 E-03$ | 0. |
| ROW | 6 | $.3630 E-03$ | 0. |
| ROW | 7 | $.2332 E-02$ | 0. |
| ROW | 8 | $.6029 E-03$ | 0. |
| ROW | 9 | $.2875 E-03$ | 0. |
| ROW | 10 | $.1744 E-02$ | 0. |
| ROW | 11 | $.3852 E-03$ | 0. |
| ROW | 12 | $.1661 E-03$ | 0. |

```
MATRIX
LABEL ELEMENT LOCAL COORD. (X,Y)
    SURFACE 2
SI2E
    10 RONS ANO 2 COLUMNS
```

COLUMN 1
ROW 1 . $1000 E+00$ 0.
ROW 2 . 1500E+01 0 .
ROW $3 \quad .2500 E+010$.
ROW $4 \quad .3500 E+010$.
ROW 5 .4500E +010 .
ROW 6 .5500E+01 0 .
ROW $7 \quad .6500 E+010$.
ROW 8 .7500E+01 0 .
ROW $9.8500 E+\mathrm{O}_{1} 0$.
ROW 10 .9500E+01 0 .

## MATRIX

LABEL ELEMENT AREAS
SI2E 10 RONS ANO 1 COLUMNS

| COLUAN | 1 |  |
| :--- | :---: | :---: |
| ROW | 1 | $.3500 E+00$ |
| ROW | 2 | $.7500 E+00$ |
| ROW | 3 | $.8500 E+00$ |
| ROW | 4 | $.9250 E+00$ |
| ROW | 5 | $.9750 E+00$ |
| ROW | 6 | $.9500 E+00$ |
| ROW | 7 | $.8500 E+00$ |
| ROW | 8 | $.7500 E+00$ |
| ROW | 9 | $.6000 E+00$ |
| ROW | 10 | $.2500 E+C 0$ |



Label
F2-RESPONSE A.IR FORCES 10 DOT COEF. 1
SURFACE, 2 AT FREUUENCY

SIZE

| COLUMN | 1 |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 1 | 0. | 0. | 0. |
| ROW | 2 | 0. | 0. | 0. |
| ROW | 3 | 0. | 0. | 0. |
| ROW | 4 | 0. | 0. | 0. |
| ROW | 5 | 0. | 0. | 0. |
| ROW | 6 | 0. | 0. | 0. |
| ROW | 7 | 0. | 0. | 0. |
| ROW | 8 | 0. | 0. | 0. |
| ROW | 9 | 0. | 0. | 0. |
| ROW | 10 | 0. | 0. | 0. |
| ROW | 11 | $.5760 E-03$ | $-.2041 E-02$ | 0. |
| ROW | 12 | $.5554 E-03$ | $-.1119 E-02$ | 0. |
| ROW | 13 | $.7142 E-03$ | $.1331 E-02$ | $.8944 E-05$ |
| ROW | 14 | $.8879 E-03$ | $.1553 E-02$ | $.1003 E-04$ |
| ROW | 15 | $.1272 E-02$ | $.3145 E-02$ | $.1841 E-04$ |
| ROW | 16 | $.9224 E-03$ | $.3475 E-02$ | $.2376 E-04$ |
| ROW | 17 | $.4298 E-03$ | $.2662 E-02$ | $.2232 E-04$ |
| ROW | 18 | $.5801 E-04$ | $.1535 E-02$ | $.1575 E-04$ |
| ROW | 19 | $-.2812 E-03$ | $.4383 E-03$ | $.1220 E-04$ |
| ROW | 20 | $-.2405 E-03$ | $.2435 E-03$ | $.9451 E-05$ |

MATRIX
LABEL FGUST- GUST AIR FORCES (COMPLEX) SURFACE, 2 AT FREQUENCY 1
SIZE 20 ROWS AND 1 COLUMNS
(COMPLEX)

| COLU |  |  | 1 |
| :---: | :---: | :---: | :---: |
| ROW | 1 | 0. | 0. |
| ROW | 2 | 0. | 0. |
| ROW | 3 | 0. | 0. |
| ROW | 4 | 0. | 0. |
| ROW | 5 | 0. | 0. |
| ROW | 6 | 0. | 0. |
| ROW | 7 | 0. | 0. |
| ROW | 8 | 0. | 0. |
| ROW | 9 | 0. | 0. |
| ROW | 10 | 0. | 0. |
| ROW | 11 | . $5760 \mathrm{E}-03$ | 0. |
| ROW | 12 | . $5554 \mathrm{E}-03$ | 0. |
| ROW | 13 | . $7142 \mathrm{E}-03$ | 0. |
| ROW | 14 | . 8879E-03 | 0. |
| ROW | 15 | . 1212E-02 | 0. |
| ROW | 16 | .9224E-03 | 0. |
| ROW | 17 | . $4298 \mathrm{E}-03$ | 0. |
| ROW | 18 | . 580 1E-04 | 0 - |
| ROW | 19 | -. $2812 \mathrm{E}-03$ | 0. |
| ROW | 20 | -. 2405E-03 | 0. |



| COLUMN | 1 | 2 |  |
| :--- | :---: | :---: | :---: |
| ROW | 1 | $.3393 E+00$ | $.1476 E+01$ |
| ROW | 2 | $.7798 E+00$ | $.1476 E+01$ |
| ROW | 3 | $.5250 E+00$ | $.2467 E+01$ |
| ROW | 4 | $.8417 E+00$ | $.2467 E+01$ |

MATRIX
LABEL ELEMENT AREAS SURFACE 3

SIIE 4 ROWS AND 1 COLUMNS

| COLUMA |  | 1 |
| :--- | :--- | :--- |
| ROW | 1 | $.4375 E+00$ |
| ROW | 2 | $.4375 E+00$ |
| ROW | 3 | $.3125 E+00$ |
| ROW | 4 | $.3125 E+00$ |


matrix
LABEL FZ-RESPONSE AIR FORCES IO DOT COEF. 1
SIZE
4 KOWS AND 3 COLUMNS

| COLUMN |  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 1 | $.6043 E-04$ | $.4 C 59 E-02$ | $.1573 E-04$ |
| ROW | 2 | $.1089 E-04$ | $.1154 E-02$ | $.3946 \mathrm{E}-05$ |
| ROW | 3 | $.1409 E-03$ | $.3504 E-02$ | $.1523 E-04$ |
| ROW | 4 | $.2219 E-04$ | $.8214 E-03$ | $.2961 E-05$ |

MATRIX

LABEL FGUST-GUST AIR FORCES (COMPLEXI SURFACE, 3 AT FREQUENCY 1

```
SIZE 4 ROWS AND l COLUMNS
```

| COLUMN |  | 1 |  |
| :--- | :--- | :--- | :--- |
| ROW | 1 | $.6043 E-04$ | 0. |
| ROW | 2 | $.1089 E-04$ | 0. |
| ROW | 3 | $.1409 E-03$ | 0. |
| ROW | 4 | $.2219 E-04$ | 0. |

the matrices printed beloh are mritten onto the file named eomtap

## MATRIX

LABEL MI
SIZE 3 ROWS ANO 3 COLUMNS

| COLUMN |  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 1 | 0. | 0. | 0. |
| ROW | 2 | 0. | 0. | 0. |
| ROW | 3 | 0. | 0. | 0. |

MATRIX

LABEL M2
SILE 3 ROWS AND 3 COLUMNS

| COL UMN |  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 1 | 0. | 0. | 0. |
| ROW | 2 | 0. | 0. | 0. |
| ROW | 3 | 0. | 0. | 0. |

## MATRIX

## LABEL M3

SILE 3 ROWS AND 3 COLUMNS

| COLUMN | 1 | 2 | 3 |  |
| :--- | :---: | :---: | :---: | :---: |
| ROW | 1 | $.3526 E+03$ | $-.2200 E+00$ | $.1644 E-01$ |
| ROW | 2 | $-.2200 E+00$ | $.9600 E+01$ | $.1014 E+01$ |
| ROW | 3 | $.1644 E-01$ | $.1014 E+01$ | $.1030 E+01$ |

SIZE 1 ROWS AND 1 COLUMNS
COL UMN
1

## MATRIX

## LABEL M4

SIZE 3 ROWS AND 3 COLUMNS

| COLUMN |  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 1 | 0. | $.3225 E+03$ | $.3383 E+02$ |
| ROW | 2 | 0. | $.6407 E+02$ | $.5046 E+02$ |
| ROW | 3 | 0. | $.6887 E+00$ | $.2455 E+01$ |

## LABEL. <br> M5

SIZE 3 ROWS AND 3 COLUMNS

| COLUMN | 1 | 2 | 3 |  |
| :--- | :--- | :--- | :--- | :--- |
| ROW | 1 | $.3239 E-01$ | $.7551 E-01$ | $.1220 E-02$ |
| ROW | 2 | $.6482 E-02$ | $.1417 E+00$ | $.1819 E-02$ |
| ROW | 3 | $.6918 E-04$ | $.2535 E-03$ | $.8851 E-04$ |

LABEL DISTANCES FROM GUST REF. PT. TO ZONES.
SILE 1 ROWS AND 1 COLUMNS

COLUMN $\quad 0.1$

MATRIX

## LABEL PHIT

SIIEMPLEX) 3 ROWS AND 1 COLUMNS

```
COLUMN I
ROH 1 -.3239E-01 0.
ROW 2 -.6482E-02 0.
ROW 3 -.6918E-04 0.
--- FLXXIC IS FINISHEDDIN USED BY FLXAIC WAS O7255I
    REIURN TO L2I7 TO READ ANOTHER DIRECTIVE CARD.
```

PROGRAM DIRECTIVE CAROS READ BY L2ITAO

IsOUIT

## REFERENCES

1. Miller, R. D.; Kroll, R. I.; and Clemmons, R. E.: Dynamic Loads Analysis System (DYLOFLEXX) Summary. NASA CR-2846, 1979.
2. Miller, R. D.; Richard, M, and Rogers, J. T.: Feasibility of Implementing Unsteady Aerodynamics Into the FLEXSTAB Computer Program System. NASA CR-I32530, October 1974.
3. Thomson, W. T.: Vibration Theory and Applications. Prentice-Hall, Inc., 1965.
4. Fung, Y. C.: An Introduction to the Theory of Aeroelasticity. Dover Publications, Inc., 1969.
5. Miller, R. D, ; and Graham, M. L.: Random Harmonic Analysis Program, L221 (TEV156), Volume I: Engineering and Usage. NASA CR-2857, 1979.
6. Kroll, R. I.; and Hirayama, M, Y.: Modal Interpolation Program, L215 (INTERP) - Volume I: Engineering and Usage. NASA CR-2847, 1979.
7. Dusto, A. R.; Hink, G. R.; et al: A Method for Predicting the Stability Characteristics of an Elastic Airplane. NASA CR-114712 through 114715, Volumes I through 4, 1974.

| Volume 1 | FLEXSTAB | Theoretical Description | NASA CR-114712 |
| :--- | :--- | :--- | :--- |
| Volume 2 | FLEXSTAB 1.02.00 | User's Manual | NASA CR-114713 |
| Volume 3 | FLEXSTAB 1.02.00 | Program Description | NASA CR-114714 |
| Volume 4 | FLEXSTAB 1.02.00 | Demonstration Cases | NASA CR-114715 | and Results

8. Richard M.; and Harrison, B. A.: A Program to Compute Three-Dimensional Subsonic Unsteady Aerodynamic Characteristics Using the Doublet Lattice Method, L216 (DUBFLX) - Volume 1: Engineering and Usage. NASA CR-2849, 1979.
9. Miller, R. D.; and Anderson, L. R.: A Program for Calculating Load Coefficient Matrices Utilizing the Force Summation Method, L218 (LOADS) Volume 1: Engineering and Usage, NASA CR-2853, 1979.
