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APPLICATION OF ABSTRACT HARMONIC ANALYSIS TO THE HIGH-SPEED RECOGNITION OF IMAGES

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16. Abstract

By means of the theory of abstract harmonic analysis methods are constructed for rapidly computing correlation functions. The theory being developed includes as a particular case the familiar Fourier transform method for a correlation function which makes it possible to find images which are independent of their translation in the plane. Described are two examples of the application of the general theory: the search for images, independent of their rotation and scale, and the search for images which are independent of their translations and rotations in the plane.

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Foreward /2*

By means of the theory of abstract harmonic analysis methods are constructed for rapidly computing correlation functions. The theory being developed includes as a particular case the familiar Fourier transform method for a correlation function which makes it possible to find images which are independent of their translation in the plane. In the paper we describe two examples of the application of the general theory: the search for images, independent of their rotation and scale, and the search for images which are independent of their translations and rotations in the plane.

INTRODUCTION /3

Denote by f(x) a positive bounded function in the plane, e.g., the image of a part of a printed text, and by v(x) the standard image of any letter from the text, say the letter "A". Let us calculate the correlation function

$$K(x) = \int_{-\infty}^{+\infty} v(x'-x)\, f(x')\, dx'. \tag{1}$$

(Here x is a vector with components x_1, x_2). The coordinates of x which attain the absolute maximum K(x), indicate the position of the letter "A" in the text: the letter "A" in the text f(x) is translated towards the vector x from the energy center of the standard v(x) [1].

*Numbers in the margin indicate pagination of original foreign text.

It is well-known that the holographic method of reading printed texts is based on this principle [1, 2]. By means of a monochromatic light source and a system of lenses the Fourier transform of the image is performed; in its turn the correlation function (1) can be expressed in terms of a Fourier transform:

$$K(t) = \bar{\nu}(t) f(t). \tag{2}$$

(The stroke denotes the complex conjugate).

Computation of the correlation function (1) on digital computers with the help of Fourier analysis (2) has found especially widespread use after the invention of the RFT -- the rapid Fourier transformation method [3]. If the standard and the image are given on the network N x N, then when the RFT methods are applied to calculate the correlation function, only arithmetic operations on the order of $N^2 \log_2 N$ are required, as against $\sim N^4$ operations if the integral (1) is computed directly. On a network of ~ 500 x 500 elements the payoff in terms of high speed response is approximately 10^4 fold.

However, if the standard letter is turned around relative to the letters in the text, or if its size is different from /4
that in the text, correlational analysis by means of formulas (1) and (2) becomes impossible. Computation of correlations by means of formula (1) makes it possible to detect only those images which are invariant with respect to translations, but not with respect to rotations or scale transformations.

1. Correlational analysis for arbitrary groups.

In the present paper it is emphasized that it is possible to construct a correlation function, expand it in a Fourier series, and after certain natural approximations, to apply the RFT methods in the general case when an arbitrary group of transformations is assumed. As an example we shall consider groups of scale and rotational transformations, and a group of motions of the plane (translations and rotations). The application of abstract harmonic analysis, as in the example with the group of translations (1), makes it possible to achieve a substantial economy in computations.

Let G be a group which maps x into x, and let g be the elements of the group. The correlation function has the form:

$$K(g) = \int v(g^{-1}x)\, f(x)\, dx \tag{3}$$

The basis of the correlational analysis is the Schwartz inequality [1]. For any element g of the group:

$$\int v(g^{-1}x)\, f(x)\, dx \leq \left(\int v^2(g^{-1}x)\,dx \int f^2(x)\,dx \right)^{1/2}. \tag{4}$$

The search for the standard v(x) in the image f(x) reduces to the search for an element of the group g for which one of the local maxima of the ratio $K(g)/\left(\int v^2(g^{-1}x)\,dx\right)^{1/2}$ is attained, or what is the same thing, such that the maximum

$$K'(g) = K(g)\, J^{-1/2}(g), \tag{5}$$

is attained, where T(g) is the Jacobian of the change to the variable x' = gx.

For the correlation function (1), due to the invariance of the measure dx on the group of translations, $\int v^2(g^{-1}x)\,dx$ does not depend on g.

Spectral analysis of the correlation function (3) does not /5
actually differ from the classical spectral analysis on convolutions [4]. Recall that during the spectral analysis of convolutions, the space X is decomposed into homogeneous spaces relative to the group G. Each homogeneous space is identified with the

factor group G/H, where H is a stationary subgroup of some point x (hx = x for all $h \in H$), and the invariant measure on the factor group G/H is obtained from the left- (or right-) invariant measure on the group G. (It is assumed that the group is locally compact). The Fourier coefficients are found when expanding the functions v(x) and f(x) with respect to the matrix elements of the irreducible unitary representations of the group.

In practice the classical approach is not always convenient. For example, if only the group of rotations is given, then the "radial" part of the measure in the correlation integral can not be determined from the measure on the group. For any continuous group it is convenient to start with the measure dx and to construct a relatively invariant measure on the group [5]. What has been said will be illustrated below in concrete examples.

Computations on digital computers can be performed only on finite bases and networks; therefore, the initial group (as a rule, non-compact) is "compactified". For example, in a case involving a non-compact group of translations we convert to a compact group of additions modulo 2T, where T is the interval of definition of the functions v(x) and f(x). Next the compact group obtained is replaced by a discrete group. In the example under consideration, involving a group of two-dimensional translations, the direct sum of two groups of whole numbers multiplied modulo N_1 and N_2, where N_1 and N_2 are the number of nodes of the decomposition with respect to the axes X_1 and X_2, respectively, /6
appears as a discrete group. After discretization the calculations can be performed on a digital computer. Analogous transformations are necessary in the case of other continuous or non-compact groups. The possibility of using RFT with arbitrary groups arises following the discretization procedure.

2. High speed methods for calculating the Mellin and Fourier-Bessel transforms.

We shall denote the initial functions and their images by the same letter, but the argument symbols will vary.

1. The continuous Fourier transform in Cartesian coordinates:

$$f(x) = \Phi_{x,y} f(y) = \int_{-\infty}^{\infty} e^{i(x,y)} f(y)\,dy. \tag{6}$$

where (x,y) is the scalar product.

2. The continuous Fourier transform in polar coordinates:

$$f(r,\psi) = \Phi_{r,\psi,\rho,\varphi} f(\rho,\varphi) = \int_0^{\infty} \rho\,d\rho \int_0^{2\pi} d\varphi\, e^{ir\rho\cos(\varphi-\psi)} f(\rho,\varphi). \tag{7}$$

3. The discrete Fourier transform:

$$f(n) = \Phi_{n,\varphi} f(\varphi) = \int^{2\pi} e^{in\varphi} f(\varphi)\,d\varphi. \tag{8}$$

4. The inverse discrete Fourier transform:

$$f(\varphi) = \Phi^{-1}_{\varphi,n} f(n) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} f(n)\, e^{-in\varphi}. \tag{9}$$

5. The Mellin transform:

$$f(R) = \mathcal{M}_{R,\rho} f(\rho) = \int_0^{\infty} \rho^{iR-1} f(\rho)\,d\rho. \tag{10}$$

6. The Fourier-Bessel transform: /7

$$f(R) = B^{n}_{R,\rho} f(\rho) = \int_0^{\infty} J_n(\rho R)\,\rho\,d\rho. \tag{11}$$

The complex conjugate transforms are obtained by changing the sign of the argument:

$$\bar{f}(x) = \bar{\varphi}_{x,y} f(y) = \varphi_{-x,y} f(y) \; ,$$
$$\bar{f}(n) = \bar{\varphi}_{n,\varphi} f(\varphi) = \varphi_{-n,\varphi} f(\varphi) \; , \tag{12}$$
$$\bar{f}(R) = \bar{\mathcal{M}}_{R,\rho} f(\rho) = \mathcal{M}_{-R,\rho} f(\rho) \; .$$

In the correlation analysis of the group M(2) use is made of the fact that

$$\bar{\varphi}_{n,\psi} \varphi_{\tau,\psi,\rho,\varphi} f(\rho,\varphi) = 2\pi i^{n} \varphi_{n,\varphi} B^{n}_{\tau,\rho} f(\rho,\varphi) . \tag{13}$$

In the proof of (13) the following integral representation of Bessel functions is used [6]:

$$J_n(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{ix\sin\theta - in\theta} d\theta = \frac{1}{2\pi} \bar{\varphi}_{n,\theta} e^{ix\sin\theta} . \tag{14}$$

The Mellin and Fourier-Bessel transforms reduce to a continuous Fourier transform:

$$\mathcal{M}_{R,\rho} f(\rho) = \int_{-\infty}^{\infty} e^{itR} f(e^t) dt = \varphi_{R,t} f(e^t) . \tag{15}$$

The Fourier-Bessel transform (11) has the form of a correlation function on the group of scale transformations; therefore the harmonic expansion of this transform can be carried out with the help of the Mellin transform:

$$\mathcal{M}_{\tau,R}\, B^{n}_{R,\rho}\, f(\rho) = \int_0^\infty R^{i\tau-1}\, dR \int_0^\infty J_n(R\rho)\,\rho\, f(\rho)\, d\rho =$$

$$= \int_0^\infty f(\rho)\,\rho^{-i\tau+1}\, d\rho \int_0^\infty J_n(x)\, x^{i\tau-1}\, dx = \left[\bar{\mathcal{M}}_{\tau,\rho}\left(f(\rho)\rho^2\right)\right]\left[\mathcal{M}_{\tau,x} J_n(x)\right]. \tag{16}$$

It is well known [7] that the Mellin transform of Bessel /8
functions is expressed in terms of gamma functions:

$$\int_0^\infty x^{a-n} J_n(x)\, x^{s-1}\, dx = 2^{a+s-n-1}\,\Gamma\!\left(\frac{a+s}{2}\right)\Big/\Gamma\!\left(n-\frac{a+s}{2}+1\right),$$

$\left(a>0,\ n>a-\frac{1}{2}\right)$. For the case n = a:

$$\mathcal{M}_{\tau,x} J_n(x) = \int_0^\infty J_n(x)\, x^{i\tau-1}\, dx = 2^{i\tau-1}\,\Gamma\!\left(\frac{n+i\tau}{2}\right)\Big/\Gamma\!\left(\frac{n-i\tau}{2}+1\right). \tag{17}$$

Due to the connection between the Mellin and Fourier transforms (15), all the transforms (16) can be performed rapidly, and therefore the same is true of the Fourier-Bessel transform.

The functions (17) can also be computed rapidly; therefore it is not necessary to tabulate them in advance. To compute the functions (17) rapidly, the Bessel functions are represented with the aid of the Fourier transform (14).

3. Scale-rotational transformations.

Let us carry out the harmonic expansion of the correlation function used in the recognition of images independent of scale transformations and rotation. The correlation function expressed in polar coordinates has the form:

$$K(R,\Psi) = \int_0^\infty \rho\, d\rho \int_0^{2\pi} d\varphi\, v\left(\frac{\rho}{R}, \varphi-\Psi\right) f(\rho,\varphi). \tag{18}$$

Parameters of the group g: R is the scale factor; Ψ is the angle of rotation.

The measure $\rho\, d\rho$ is not invariant relative to the scale transformations; therefore, according to (5), during the correlational analysis it is necessary to find the maximum of the normalized correlation function:

$$K'(R,\Psi) = \int_0^\infty \rho\, d\rho \int_0^{2\pi} d\varphi\, \frac{1}{R} v\left(\frac{\rho}{R}, \varphi-\Psi\right) f(\rho,\varphi). \tag{19}$$

Since the group G is the direct sum of a multiplicative and additive group, in order to perform a harmonic analysis it is necessary to apply the discrete Fourier transform in the case of the argument Ψ, and the Mellin transform in the case of the argument R. On applying the discrete Fourier transform to parts of (19) we obtain: /9

$$K'(R,n) = \int_0^\infty \frac{1}{R} \bar{v}\left(\frac{\rho}{R}, n\right) f(\rho,n)\, \rho\, d\rho. \tag{20}$$

After application of the Mellin transform to (20) we have:

$$K'(\tau,n) = \left[M_{\tau,\rho}(\rho\, \bar{v}(\rho,n))\right]\left[M_{\tau,\rho}(\rho\, f(\rho,n))\right]. \tag{21}$$

The representation (21) shows that the harmonic expansion makes it possible when calculating the function $K'(\tau,n)$ to get by with the order $N^2 \log_2 N$ of the operations (if the region of variation of Ψ is broken up into N nodes and the region of variation of ρ is broken up into N nodes), as against $\sim N^4$ operations, if (19) is computed directly.

It is especially important to note that harmonic analysis makes it possible to perform all calculations without interpolation, i.e., using the values of the functions $\vartheta(\rho,\varphi)$ and $f(\rho,\varphi)$ only in the case of the given set of nodes. In particular in the example under consideration the domain of variation φ $(0, 2\pi)$ is decomposed into N equivalent nodes, and the domain ρ $(-T, T)$, into the nodes $\rho_1 = e^{-T}$, $\rho_2 = e^{-T+\Delta T}$, ..., $\rho_N = e^{T}$. $(\Delta T = \frac{2T}{N-1})$.

3. The group of plane motions M(2).

Next let us make a correlational analysis for the recognition of images which are independent of translations and rotations. The correlation function has the form:

$$K(a,\alpha) = \int_{-\infty}^{\infty} \vartheta((x-a)_{-\alpha})\, f(x)\, dx. \qquad (22)$$

where a is the translation vector, α is the angle of rotation, x_α denotes the rotation of the vector x through the angle α. In [6] an irreducible representation of the group M(2) is constructed and the leading role of the Fourier-Bessel transform is shown. The harmonic analysis of the correlation function (22) is carried out as follows. To begin with, we apply the continuous Fourier transform: $K(y,\alpha) = \Phi_{y,a} K(a,\alpha)$, which decomposes (22) into a product: /10

$$K(y,\alpha) = \bar{\vartheta}(y_{-\alpha})\, f(y). \qquad (23)$$

Next, in (23) the argument y is written in polar coordinates (r, ψ):

$$K(r,\psi,\alpha) = \bar{\vartheta}(r, \psi-\alpha)\, f(r,\psi). \qquad (24)$$

The application of two discrete Fourier transforms $\varphi_{n,\alpha}\varphi_{m,\psi}K(\tau,\psi,\alpha)$ yields:

$$K(\tau,m,n)=\vartheta(\tau,n)f(\tau,n+m) \tag{25}$$

Using the ratio (13), we finally obtain the following harmonic decomposition:

$$K'(\tau,m,n)=2\pi(-1)^n\vartheta'(\tau,n)f'(\tau,m+n).$$

or

$$K'(\tau,l-n,n)=2\pi(-1)^n\vartheta'(\tau,n)f'(\tau,l),\quad (l=m+n).$$

where

$$\begin{aligned}
K'(\tau,m,n)&=\varphi_{n,\alpha}\varphi_{m,\psi}B^m_{\tau,\rho}K(\rho,\psi,\alpha);\\
\vartheta'(\tau,n)&=\varphi_{n,\psi}B^n_{\tau,\rho}\vartheta(\rho,\psi);\\
g'(\tau,m+n)&=\varphi_{m+n,\psi}B^{m+n}_{\tau,\rho}f(\rho,\psi).
\end{aligned} \tag{26}$$

All the transforms involved in (26) are carried out rapidly. /11 The computation of the correlation function by means of harmonic analysis requires on the order of $N^3\log_2 N$ arithmetic operations (it is assumed that the domains of change of the variables ρ, ψ, α are divided into N nodes each), as against $\sim N^5$ operations if (22) is calculated directly.

Conclusion

Let us mention a number of problems whose solution would be facilitated by further progress in the applications of the methods of abstract harmonic analysis to the recognition of images.

1. In the performance of harmonic analysis with respect to RFT exclusively, among the most interesting groups in the practice of the development of images are: motion of the plane simultaneously with a scale transformation; projective transformations; groups acting in three-dimensional space, and Lie groups in the plane for the study of non-linear distortions, e.g., in the problem of reading a handwritten text.

2. Construction of general methods for finding RFT for arbitrary groups. In connection with this problem it is interesting to consider the possibility of approximating a compact and discrete initial group with the help of RFT functions which serve as a representation of finite Abelian groups [8]. Analysis of RFT for these groups has been partially carried out in [9].

3. Investigation of the behavior of a correction function near local maxima for different kinds of images.

4. Construction of analogous systems which realize spectral analysis in groups similar to what was done in the case of the Walsh transform and the discrete Fourier transform.

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