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THE ECONOMICS OF PROJECT ANALYSIS: OPTIMAL INVESTMENT CRITERIA AND METHODS OF STUDY

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NOMENCLATURE

Symbols	Definitions
a	value of project investment fixed costs
b_i	nonlabor technical coefficient
B	matrix of nonlabor technical coefficients
C_I	project variable investment expense
C_k	project capital expense
C_v	present value of project total investment outlay
C_x	total project cost
d	static depreciation rate
D	market value of debt finance
E	market value of equity finance
f	function
F	function
F_k	project investment fixed costs
g	inverse function
h	proportion of project investment cost financed by debt
i_a	average internal rate of return
i_m	marginal internal rate of return
l_i	labor technical coefficient
l	labor input coefficient vector
m	interval over which project productive capacity will exceed product demand
mc_v	marginal present value of project investment expense
mpp	marginal productivity
mp_v	marginal present value of project net income
n	project life span; number of elements in Leontief analysis

NOMENCLATURE (Continued)

Symbols	Definitions
p	price vector
p_1	unit cost (i.e., price) of z_1
p_2	unit "flow" cost of capital input
p_k	"stock" price of capital input
p_L	price of labor
P_v	present value of project
P_{v_a}	present value of net income expected over the interval from t_o to t_m (i.e., excess capacity interval)
P_{v_b}	present value of income expected over the interval from t_m to t_n (i.e., the full capacity interval)
q	net benefit-cost ratio
R	net income
r,	market rate of interest
r_b	bond, or debt, rate of interest
r_e	expected yield per share of common stock
r_k	capitalization rate appropriate to the project's market classification
S	sinking fund contribution rate
t	time
v	capital-income coefficient
W	goodwill of the project
x	quantity of output; a vector in the Leontief analysis
x_o	given output level
x_i^d	final product of process "i"
y	expected net income
y_o	base value of net income expected from product sales
y_m	expected project net income at full capacity production and sales

NOMENCLATURE (Concluded)

Symbols	Definitions
z_1	quantity of current variable input
z_2	quantity of capital input; measure of project scale
γ	percentage rate of growth
π	net profit
σ	profit rate
$\frac{\partial R_r}{\partial R_s}$	marginal rate of return over cost

TECHNICAL MEMORANDUM

THE ECONOMICS OF PROJECT ANALYSIS: OPTIMAL INVESTMENT CRITERIA AND METHODS OF STUDY

INTRODUCTION AND OBJECTIVES

The present study represents an endeavor to outline considerations of correct project investment analysis. The subject obviously requires treatment beyond the limits of a relatively short research period. This report reveals shortcomings in traditionally accepted investment analysis methods and suggests either ways to overcome these shortcomings or alternative methods of analysis.

In project analysis, two of the major challenges to the analyst are (1) identifying criteria of selection for choosing among alternative project options in a consistent way, and (2) formulating a statement of project characteristics that will enable an application of selection criteria in a way compatible with accepted principles of economic analysis. The problem is one of investment analysis which draws on the principles of economic capital theory. Yet the technical characteristics of many projects require also the expertise of the engineering analyst. This has led to the development of the field of engineering economy, which plays a major role in project analysis.

ENGINEERING ECONOMY IN PROJECT ANALYSIS

Reliance on economic principles involves a body of doctrine which encompasses a set of well-established methods that are inherently systems oriented. Engineering economy generally has developed independently of the discipline of economics. It has tended to follow a highly case-oriented approach to analysis, making only limited use of key economic principles. Given the specialized focus of engineering economy, integration of its findings with larger aspects of project planning and development is achieved through systems engineering and management controls [1,2,3].

This is not meant as an indictment of engineering economy. Indeed, without formally drawing on such economic concepts as the production function and marginal productivity, we find these concepts implicit in the "economic balance" method of analysis followed by engineers in project analysis. This method is of use in investment analysis where under static conditions the minimization of the total cost composed of the interest charge on capital facility and the operating cost of noncapital variable factors of production can lead to an optimal investment decision as regards project cost projections. The projected revenue or benefit flow of a project must be analyzed in relation to costs to complete the investment decision.

COORDINATING ROLE OF THE RATE OF INTEREST

The interest rate previously mentioned is often an enigma to the analyst who tends to treat it uncertainly as an instrument of discount or compounding without clear understanding of the rationale therefor. In part, this may be the result of specialization of function resulting from an hierarchical structure of decision-making responsibility. Here the task of analysis and decision making is broken into subgroup chunks with each group expected to achieve an optimization of its subproblem [4]. In a well-designed and managed system, subgroup endeavors can be coordinated to achieve an overall optimization in consequence of the sub-optimization efforts of the various operating subgroups. However, the very real possibility of inconsistency of subgroup decisions with higher level criteria exists in this system of responsibility. This is evident in the case of investment analysis where it has been held that given capital rationing, the market rate of interest is irrelevant to project analysis [5].

Under the systems framework of general economic analysis, the rate of interest may correctly be viewed in either of two ways [5]:

1) Agio concept — a premium on the value of current claims in comparison with claims of specified future dates.

2) Price concept — the price of current funds measured in terms of the equivalent level perpetuity.

In either case, the rate of interest, like a relative price, is the quotient of two quantities. In general equilibrium analysis it coordinates [5,6]:

1) The marginal rate of substitution between present and future consumption (i.e., time preference).

2) The marginal rate of substitution between present and future production (i.e., technical substitution).

3) The marginal rate of return to saving or investment (i.e., marginal efficiency of capital).

4) The marginal rate of liquidity preference (i.e., portfolio composition).

In general equilibrium, a balance is achieved between each individual's rate of time preference, liquidity preference, yield on investment, as well as between each firm's rate of technical substitution. The coordinating role is played by the market rate of interest. In the market economy then, the rate of interest does tend to signal society's preference for present versus future goods. Does this mean it is the only arbiter in the realm of capital analysis where the two key questions are (1) to what extent is it in society's best interest to postpone today's consumption in return for an increase in future goods, and (2) of the investment options currently available, how should they be ranked?

Some have argued that in the realm of government project analysis, the rate of interest is not an appropriate instrument of analysis. Indeed, in the presence of capital rationing and given a total lack of concern for the market's evaluation of new projects, the analyst would have no reason to use the market rate of interest [4]. This view may have some merit in the case of suboptimization analysis [4,5]. However, while a consistent ranking of projects may be possible without use of the market rate of interest in some instances, the procedure is one which may invite abuse. For example, such would be the case where a budget surplus has been spent on a project of dubious private or social benefit so the budget unit, whether in the private or public sector, may save itself the embarassment of returning unused funds at the end of the fiscal year.

PANELS OF EXPERTS

To help guard against such abuses while at the same time to assure the fullest possible accounting of benefits and costs, it is sometimes recommended that proposed projects be analyzed by a panel of experts. This approach has much to recommend it and is in essence the method followed in engineering economy where specialists in the various engineering aspects of a proposed project develop in conjunction with cost analysts a full detailing of project costs which are then combined with revenue estimates provided by the marketing and sales departments of private firms or their appropriate counterparts in the public sphere in arriving at a decision on the merit of the investment proposal. To bring order to the detailed elements of engineering economics analysis, someone or some group must be specifically charged with the systems responsibility of coordination and integration of the results of contributing specialists.

The panel of expert's approach to project analysis is not without its drawbacks [7]. Not least of these is the failure of experts to fully count the costs of a program as more than once has been evidenced by public estoppment of highway, water and nuclear projects, as well as other public undertakings. In a similar vein do we find cases exampled by the California Department of Highways engineeringly sound but aesthetically monstrous proposal for a new bridge crossing for the south end of San Francisco Bay. In an appeal for recommendations from the public for a more acceptable bridge design, the San Francisco Chronicle was soon able to report a plan for a prestressed concrete structure far more flattering to the Bay's skyline without sacrifice of either structural integrity or cost effectiveness of design [7].

A thoroughgoing analysis of all values (including the whole range of human values) pertinent to, or measurably impacted on by, any project under consideration, is mandatory. Equally important is the realization that rational decision making often requires quantification of so-called nonmeasurable values [7].

CHOICE OF CAPITALIZATION RATE

As a rule, benefit-cost information developed is to be evaluated in reference to the market rate of interest for the reason we have already noted — that it is the market rate of interest that signifies society's determination of the urgency of present versus future benefits [7]. If there be a difference between project evaluation by government as opposed to private industry, let it be because government calculates the net social productivity of the investment including an accounting of nonpecuniary social benefits which are likely never to accrue to a private firm undertaking the same investments [7].

To permit the government to evaluate projects at a lower rate of capitalization than set by the market (Some would set a capitalization rate of zero!) simply because the government can borrow at lower rates than private industry or because (some) government projects are funded incrementally is to calculate uneconomically [7]. The capitalization rate used in analysis to determine the cutoff point for attractive investments, like any price, should as accurately as possible reflect the opportunity cost of employing scarce resources in the projects under consideration. If, in the face of a full reckoning of all opportunity costs bearing on a proposed project, the net benefit expected to occur therefrom is nil or negative, it is in society's best interest, regardless of whether the initiator be private industry or the government, that further consideration of the project be dropped until such time that changed circumstance may suggest its favorable reconsideration.

Having settled on the market rate of interest as a key instrument in project analysis does not end the matter of defining the role of the rate of interest. For there are many rates of interest and there are alternative ways in which they may be applied in analysis. There is no simple answer to either problem. For example, in private industry, if a firm utilizes retained earnings as its only source of finance, the proper rate of interest for it to use in investment analysis would be the rate it could get on the next best allocation of its funds, including the possibility of loaning them out at the market loan rate of interest.

A company may choose, or find it necessary, to seek funds on the market through issue of bonds or stocks or both. The composition of the firm's loan portfolio in fact is a matter of preference analyses which calls for the establishment of a management utility function incorporating such variables as the firm's growth potential, liquidity requirements, and so on. In much the same way can the government agency form a utility function in regard to finance requirements for its endeavors, though one may expect its function to be described in terms of social variables involving employment, price stability, income growth, and the like, in lieu of or in addition to the arguments of the preference function of a private concern [8,9].

PROJECT VALUE IS INDEPENDENT OF THE METHOD OF FINANCE

One reasonably plausible argument is that the market value of a project is independent of its "capital structure" (i.e., mix of debt and equity finance). That is, the argument states that the value of a project, determined by its prospective net receipts stream, is the value to which the sum of the market-determined values of the debt and equity claims issued to finance the undertaking must adjust. On the basis of this we may write:

$$P_{v_o} = D + E \quad . \tag{1}$$

The assumption of "complete markets" (i.e., a system in which every contingency corresponds to a distinct marketable commodity) is essential to this result as is the absence of transactions costs and other external drains in the form of corporate and personal income taxes, etc. [5].

Though the assumptions specified are seldom fulfilled in practice, the simplicity of the value relation has rendered it irresistible as a basis for approximating the expected yield on a share of stock issued jointly with debt in the financing of some given project. Here the expected yield is equal to the appropriate capitalization rate, r_k, for a pure equity stream in the class of the project being evaluated, plus a premium related to the financial risk equal to the debt-to-equity ratio times the spread between r_k and the bond rate of interest, r_b. (Reference 10 discusses the effect of taxation as well.)

$$r_e = r_k + (r_k - r_b)(D/E) \tag{2}$$

Alternatively, the preceding expression could be used to determine a project's capitalization rate given the prior specification of its payout on debt and equity finance:

$$r_k = \frac{r_e + r_b\,(D/E)}{(1 + D/E)}\ ^{1/} \quad . \tag{3}$$

To apply this analysis to the formulation of a project cost equation is appropriate. Here, capital costs, net of depreciation, would be

$$C_k = \{(1-h)r_e + hr_b\} \cdot C_I \quad . \tag{4}$$

Upon substitution of equation (2) into the above expression and simplification of that result, the net cost of capital to the project is seen to be nothing more than the capitalization rate, r_k, appropriate to the class of project under consideration times the planned level of project investment expenditure:

$$C_K = r_k \cdot C_I\ ^{2/} \quad . \tag{5}$$

THE ENGINEER'S BALANCE METHOD OF ANALYSIS

From here it is but a short step to the engineer's solution to a typical economic balance problem wherein capital (or so-called) "fixed" costs are shown rising linearly and variable costs are shown falling at a

1/ For example, if bonds are to pay 9 percent interest and a stock is to yield 15 percent return on equity, then given a debt equity ratio equal to one, the capitalization rate of the project would be 12 percent.

2/ With depreciation, we would have

$$C_k = \{(1-h)r_e + hr_b + d\}\ C_I = (r_k+d)C_I \quad . \tag{6}$$

diminishing rate with respect to some common design variable which is frequently a physical measure of the capital input to the project. The economic balance or minimum cost (i.e., saddle) point occurs where the rate of increase in project investment cost is equal to, or just "balanced" by, the rate of decline in the variable costs [3].

To relate the engineer's analysis to the production and cost analysis of the economist, we identify the production function, $x = f(z_1, z_2)$, in which the output, x, is rendered by the application of the current variable input, z_1, and a capital input, z_2. Expressing the current input to production as a function of a given level of output and variable level of capital input yields the expression

$$z_1 = g(x_o, z_2) \quad . \tag{7}$$

Define p_1 as the unit cost of z_1 and p_2 as the unit "flow" cost of the capital input, where

$$p_2 = (r_k + d)p_k \quad . \tag{8}$$

Total project cost is then

$$C_x = p_1 z_1 + p_2 z_2 \quad . \tag{9}$$

Substituting from equation (7) into equation (9) yields the project cost function

$$C_x = p_1 g(x_o, z_2) + p_2 z_2 \quad . \tag{10}$$

The point of "economic balance" is achieved when the quantity of capital input to the project leads to the fulfillment of the condition,

$$\frac{\partial C_x}{\partial z_2} = 0, \quad \text{or} \quad p_1 g_1 + p_2 = 0 \quad . \tag{11}$$

That is, an economic balance (i.e., equilibrium) is achieved when "marginal" variable cost, $-p_1 g_2$ reaches equality with "marginal" investment cost, p_2.

Given that g_2 is equal to the negative of the ratio of the marginal productivity of capital to the marginal productivity of current variable input,

$$g_2 = -\frac{mpp_2}{mpp_1} , \tag{12}$$

we find the economic balance condition in equation (10) is equivalent to the economist's solution to the least cost problem in production theory. In the latter case, for a given level of output, input quantities are varied until a least cost combination of inputs is achieved. This state is characterized by the condition that the marginal productivity of the last dollar spent on each input is equal:

$$\frac{p_1}{mpp_1} = \frac{p_2}{mpp_2} . \tag{13}$$

An advantage of the engineer's economic balance method of analysis is that miscellaneous costs of installation, service, and operating labor complementary to the specified design variable (or capital input measure) are easily incorporated into a cost function similar to equation (10).

The shortcomings of the method are generally attributable to a lack of integration of the methods of engineering economy due in part at least to the engineer not normally being concerned with price and sales decisions so much as with choice of technique.

More specifically, the balance method fails to provide a conceptual distinction between economic and engineering data and hence fails to facilitate the investigation of consequences of changes in design technology or input prices.

From a systems point of view, the analyses may fail to provide for the determination of minimum total cost over the entire input space in relation to different levels of output. A related problem is the determination of scaling requirements in relation to project costs and revenue opportunities [11].

In terms of investment criteria, the economic balance method of analysis does make rational use of the market rate of interest in determining the long run marginal cost of capital. However, to the extent the method fails to take account of the entire field of project costs, its usefulness is severely limited. Another drawback to the method is the assumption implied in its application of an unchanging time profile of project costs. For these reasons, it is recommended that the results from the engineer's economic balance method of analysis be used only as an aid to, rather than as the sole basis for, the project investment decision process.

VARIABLE NET INCOME STREAM OPTIMIZATION

It is typically assumed that the flow of net receipts accruing to a project is not only given, but constant through time. Yet upon scrutiny we find that while this assumption may be appropriate to bond analysis, it is not generally correct in project analysis. For it is the rule that the yield on a project may vary from period to period; moreover, diminishing returns will be experienced with respect to the effort to expand any period's net income relative to the next period's income. This is due to several reasons:

1) An increased rate of activity causes an increase in the rate of depreciation.

2) As full utilization of resources is reached, the quality of remaining resources diminishes.

3) As the output for any period is considered for increase, management becomes increasingly uncertain of market prospects for the increased output.

4) An increased rate of activity ultimately will result in the occurrence of bottlenecks in the production process.

Following Fisher, the optimal time stream of net income is relatively easily determined. Let us assume the intraperiod maximization of net income already has been achieved. The present value of the resulting income stream is given by

$$P_v = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \ldots + \frac{R_n}{(1+r)^n} \quad \text{3/} \quad . \tag{14}$$

3/ Throughout this presentation a constant rate of interest is assumed. The analysis is easily adjusted to handle the case of a variable rate of interest.

The condition that this expression shall be a maximum is that the differential quotient thereof shall be zero,

$$dP_v = \frac{dR_1}{(1+r)^1} + \frac{dR_2}{(1+r)^2} + \ldots + \frac{dR_n}{(1+r)^n} = 0 \quad . \tag{15}$$

This condition contains within itself a number of subsidiary conditions. To derive them, we may consider a slight variation in the net income stream affecting only the income items pertaining to the first two periods, R_1 and R_2 (the remaining income items, $R_3, \ldots, R_n$, being regarded for now as constant), and we may denote the magnitude of dR_1 and dR_2, under the assumption of restricted variations, by ∂R_1 and ∂R_2. Then, under the assumed constancy of $R_3, \ldots, R_n$, it follows that $dR_3, \ldots, dR_n$, are equal to zero, and equation (15) becomes

$$\frac{\partial R_1}{(1+r)^1} + \frac{\partial R_2}{(1+r)^2} = 0 \quad . \tag{16}$$

From this it follows directly that

$$\frac{\partial R_2}{\partial R_1} = (1+r) \quad . \tag{17}$$

By definition, the left-hand member of this equation is one plus Fisher's marginal rate of return over cost, which, it is easily shown, measures the same thing as the marginal internal rate of return (discussed below), so that the two terms may be used interchangeably. Now, in equilibrium, net income in period two will be increased at the "expense" of period one net income until the marginal internal rate of return, i_m, has achieved equality with the market rate of interest, r.

$$(1+i_m) = (1+r) \;, \qquad \text{or} \qquad i_m = r \quad . \tag{18}$$

That is, the condition that the marginal internal rate of return for the period under consideration achieve equality with the corresponding market rate of interest follows as a consequence of the general condition that the present value of the net income stream must be a maximum.

This same reasoning may be applied to any pair of successive years. Thus if we assume variations in R_2 and R_3 without any variations in the other elements in the net income stream, R_1, R_4, ..., R_n, the original differential equation becomes

$$\frac{\partial R_2}{(1+r)^2} + \frac{\partial R_3}{(1+r)^3} = 0 \tag{19}$$

from which we again obtain equation (18) [5].

The upshot of this analysis is the determination of a maximum present value of net income function expressed in terms of the level of investment expenditure,

$$P_v = f(C_I) \quad . \tag{20}$$

NECESSARY AND SUFFICIENT CONDITIONS OF PRESENT VALUE ANALYSIS

If the time configuration of net income is technologically, or otherwise, given then equation (20) may be established without recourse to the above analysis. In any event, following the establishment of this last equation, the task of determining the optimum level of project investment expenditure remains.

This determination may be carried out in terms of the marginal present value of net income and investment expense. The rate of change of the present value of net income with respect to variable investment expense yields the project's marginal value of income:

$$mp_v = f'(C_I) \quad . \tag{21}$$

The marginal present value of project investment expense, which is a measure of scale of undertaking, is simply the variation of investment expense in terms of itself. It is thus identically equal to one:

$$mc_v = 1 \quad . \tag{22}$$

Setting the marginal value of income equal to the marginal value of investment expense,

$$f'(C_I) = 1 \quad , \tag{23}$$

it generally will be possible to solve for the optimal scale of project undertaking, a conclusion wholly in agreement with classical analysis.

$$C_I^* = F(1) \quad . \tag{24}$$

While by the present value approach to investment analysis we have been following, this is a necessary and sufficient condition to determination of the optimal scale of project investment, it does not guarantee the overall profitability of the project.

Thus for example, in the building of a facility, there will typically be fixed preparation costs in addition to expenses that vary with respect to scale of undertaking. By way of illustration, consider the New York World Trade Center as an investment project. Site preparation costs, definitely a part of the overall project expenses, were largely composed of items insensitive to building girth, height, interior structure, etc. Surely this condition must apply to investment projects in general.

Accordingly, a second condition for determination of optimality of an investment undertaking is that the average present value of project net income, AP_v, must equal or exceed the average present value of project total investment outlay, or equivalently that the total present value of income must equal or exceed the total present value of total investment outlays. Going a step further, we may suppose that in the absence of capital rationing, project development will be pushed to the point where the present value of the income stream equals the present value of project total investment cost. Hence, the second condition for optimality in project selection may be written as an equality:

$$P_v = C_v \;\; \underline{4}/ \quad . \tag{25}$$

4/ This is the same condition assumed to hold in analysis based on the average internal rate of return criteria, i_a, which is discussed later. Suffice it to say for now that present value is measured in terms of the market rate of interest, r, or some other appropriately chosen capitalization rate, r_k, whereas in average internal rate of return based analysis, i_a is treated as an unknown.

In terms of present value analysis then, we are led to the conclusion that relations (24) and (25) express the necessary and sufficient conditions for optimality in project selection.

To illustrate the principles of the present value analysis previously outlined, consider the case of a product whose demand is expected to grow through time at the constant percentage rate, γ.

DYNAMIC CASE OF DESIGN FOR EXCESS CAPACITY

From the vantage of point in time, t_o, the project analyst will realize the necessity of choosing some level of product demand, expected to be realized "m" periods hence, in regard to which to scale the capacity of the proposed production facility. Until time t_m is reached, the project will have excess capacity.

The first part of the present value of project income will be based on the expected growth in product sales over the interval "m":

$$P_{v_a} = y_o \sum_{j=1}^{m} \lambda^j = \lambda(1 - \lambda^m)Y_o/(1 - \lambda) \tag{26}$$

$$\lambda = (1 + \gamma)/(1+r) \quad .$$

The second part of the present value calculation of project income will derive from receipts expected from the sale of capacity output of the firm over the interval from t_m to the end of the venture at time t_n:

$$\begin{aligned} P_{v_b} &= y_m \sum_{j=1}^{n} \frac{1}{(1+r)^j} \\ &= \left[\frac{1}{r(1+r)^m} - \frac{1}{r(1+r)^n}\right] \cdot y_{m+1} \quad . \end{aligned} \tag{27}$$

Where the project is intended to continue in perpetuity (i.e., $n = \infty$), the second part of the present value of income calculation simp'.fies to

$$P_{v_b} = y_{m+1}/r(1+r)^m \quad . \tag{28}$$

The total present value of project income, viewed from point in time, t_o, over the interval "n" and measured in regard to the scale limit set by demand "m" periods hence, is given by the sum of the parts defined. (For convenience, we shall take the sum of equations (26) and (28).)

$$\begin{aligned} P_v &= P_{v_a} + P_{v_b} \\ &= \lambda(1-\lambda^m)y_o/(1-\lambda) + y_o(1+\gamma)^{m+1}/r(1+r)^m \qquad (29) \\ &= \lambda y_o/(1-\lambda) + [(1+\gamma)/r - \lambda(1-\lambda)]\ y_o\lambda^m \quad . \end{aligned}$$

The breakdown of project investment outlay into fixed and variable components now comes into play. Fixed costs are assumed to be known:

$$F_k = a \quad . \tag{30}$$

Project investment variable costs were treated as a given value in the introduction to the present value analysis section. Now a more involved situation is being modeled in which these costs are defined in reference to expected income data that in turn are a function of time.

Investment variable costs, C_I, are equal to the price of capital inputs, p_k, times the quantity of capital, z_2, necessary to fulfill project capital requirements.

$$C_I = p_k \cdot z_2 \quad . \tag{31}$$

Capital input, which is also a measure of project scale, is dependent on the rate of production which is here expressed in terms of expected net income, y:

$$z_2 = v \cdot y \quad . \tag{32}$$

Expected income can be expressed in terms of base period income, y_o, times the expected growth of product demand, $(1+\gamma)^m$:

$$y = y_o(1 + \gamma)^m \quad . \tag{33}$$

Total investment cost, C_v, is given by the sum of fixed and variable outlays:

$$C_v = F_k + C_I \quad . \tag{34}$$

Consolidating the content of equations (30) through (34), we obtain the following expression of the present value of project total investment outlay:

$$C_v = a + p_k v y_o (1+\gamma)^m \quad . \tag{35}$$

The "goodwill," W, of the project is determined by the difference between the present value of expected income, P_v, and the present value of project investment cost, C_v [12]:

$$\begin{aligned} W &= P_v - C_v \\ &= \lambda y_o/(1-\lambda) + [(1+\gamma)/r - \lambda/(1-\lambda)] y_o \lambda^m - a - p_k v y_o (1+\gamma)^m \quad . \end{aligned}$$

Maximum goodwill is achieved by establishing that scale of enterprise for which the marginal present values of income and investment cost are equal. Since time is the driving force in this case, the determination of maximum goodwill evidently is achieved by varying goodwill with respect to the time variable, m, and setting the result equal to zero:

$$\begin{aligned} \frac{\partial W}{\partial m} &= mp_v - mc_v = 0 \qquad (37) \\ &= [(1+\gamma)/r - \lambda/(1-\lambda)] y_o \lambda^m \ln \lambda - p_k v y_o (1+\gamma)^m \ln(1+\gamma) = 0 \quad . \end{aligned}$$

From this equation, it is possible to solve for the optimal value of "m" which will be recalled indicates the interval of project expected excess capacity:

$$m = \frac{1}{\ln(1+r)} \cdot \ln \left\{ \frac{\left[\frac{(1+\gamma)}{r} - \frac{\lambda}{1-\lambda}\right]\ln\lambda}{p_k v \ln(1+\gamma)} \right\} \quad . \tag{38}$$

It will be observed that the solution for m is independent of the base period income value, y_o. Accordingly, it is possible to solve for the value of the latter variable in terms of m, and thereby determine the optimal scale of the project.

For this purpose, let us define the net benefit-cost ratio, q, of a project as the ratio of goodwill to the present value of project investment costs:

$$q = W/C_v \quad . \tag{39}$$

In the absence of capital rationing, the scale of a project will normally be expanded to the point that the net benefit-cost ratio is zero:

$$q = 0 \quad . \tag{40}$$

Proceeding in terms of equation (39), the expression for determination of base period income may be stated as follows:

$$y_o = \frac{(1+q)a}{\left\{\frac{\lambda}{1-\lambda} + \left[\frac{(1+\gamma)}{r} - \frac{\lambda}{1-\lambda}\right]\lambda^m - (1+q)p_k v(1+\gamma)^m\right\}} \quad . \tag{41}$$

Given the fulfillment of equation (40), it follows that the present value of income equals the present value of investment cost, so that via equations (37) and (40) the necessary and sufficient conditions for optimal investment policy in the absence of capital rationing are satisfied. [See equations (24) and (25).]

Present value analysis is capable of considerable flexibility in application. For this reason it presents a powerful method upon which to base the investment decision.

AVERAGE AND MARGINAL INTERNAL RATES OF RETURN

An alternative procedure to ranking investment options by their contribution to present value is that of ranking them according to their internal rate of return. Indeed, this method has attracted considerable attention from analysts, though they seldom specify just which internal rate of return they have in mind. Generally, however, they have in mind the average internal rate of return. The definitions of the average and marginal internal rates of return are as follows [13]:

1) The average internal rate of return (or "average efficiency of capital") is the rate of discount which, if used to discount back to the present all the revenues from the project, makes their present value equal to the total project investment cost.

2) The marginal internal rate of return (or "marginal efficiency of capital") is the rate which if used to discount back to the present the marginal net income, due to an additional unit of funds invested, makes the present value of that income equal to unity.

Under ideal conditions the internal rate of return approach will give results consistent with the present value method of project analysis. Unfortunately, these conditions are seldom realized in project analysis, making the internal rate of return criteria generally unsatisfactory as a basis for investment decision making. However, it is important to note that the method is not irredeemable.

It is averred that the internal rate of return indicates the rate of net profitability of a project without requiring the separate calculation of a schedule for depreciation [14]. In the absence of a theory of optimal depreciation, it could be a considerable boon to project analysis for the analyst to be able to sidestep the need to directly reckon with determination of a depreciation reserve program. However, unless funds recovered from the project can be invested at the computed internal rate of return, then the depreciation reserve problem must be reckoned with directly. Otherwise, the internal rate of return criterion will give results inconsistent with the present value criterion for ranking projects.

To deal with the depreciation reserve problem, let a sinking fund be established on the convention of a constant contribution rate geared to the market rate of interest. In regard to this setting, determine the minimum contribution rate required to accumulate the value of the project's replacement cost "n" periods hence. The net profit stream of the project will then be given by the difference between the net income elements, R, and the corresponding contribution to the depreciation reserve, S. The rate of discount that equates this stream (which includes in the nth period the accumulated sum of sinking fund contributions) to the initial project investment cost, C_I, will correctly measure the average internal rate of return to the project.

If the constant rate sinking fund contribution is,

$$S = \{r/(1+r)^n - 1\}C_I \quad , \tag{42}$$

then the net profit for the "jth" period will be

$$\pi_j = R_j - S_j = R_j - \{r/(1+r)^n - 1\}C_I \quad . \tag{43}$$

The present value of the net profit stream plus recovery in period "n" of the replacement cost of the project yields the expression:

$$\begin{aligned} P_v &= \sum_{j=1}^{n} \frac{R_j - S_j}{(1+i_a)^j} + \frac{C_I}{(1+i_a)^n} \\ &= \sum_{j=1}^{n} \frac{R_j}{(1+i_a)^j} + \frac{C_I}{(1+i_a)^n} \\ &\quad - \left\{ \frac{(1+i_a)^n - 1}{i_a(1+i_a)^n} \right\} \left\{ \frac{r}{(1+r)^n - 1} \right\} C_I \quad . \end{aligned} \tag{44}$$

Clearly, if it happens that the market rate of interest (or its equivalent) equals the average internal rate of return, then the present value calculation in equation (44) simplifies to

$$P_v = \sum_{j=1}^{n} \frac{R_j}{(1+i_a)^j} \quad , \tag{45}$$

which when set equal to the project investment cost, C_I, yields the average internal rate of return of traditional description.

The sinking fund adjustment to the internal rate of return method also will help to assure consistent rankings of mutually exclusive investment options such as occur when variations in scale are being considered in determination of optimal size of facility.

Another problem with the internal rate of return approach is the occurrence of multiple roots to the polynomial equation

$$\sum_{j=1}^{n} \frac{R_j}{(1+i_a)^j} - C_I = 0 \tag{46}$$

used in solving for the average internal rate of return, i_a. That is, depending on the time configuration of the net receipt stream which may pass several times between the ranges of negative and positive values, more than one internal rate of return may occur which will satisfy the above equality.

We must not forget the lack of clarity by analysts in specifying what internal rate of return they have in mind to serve as a basis to the investment decision. We have previously discussed the role of the marginal internal rate of return in the determination of the optimal time configuration of a variable net income stream. Where project options vary continuously, the marginal internal rate of return again is recommended for consideration subject to the warnings noted. When project options are few in number, the average internal rate of return approach, modified as noted, may be used. It should be mentioned that the relationship of the average to the marginal internal rate of return is akin to the relationship of the marginal to the average present value of an investment option.

PRICING OF PRODUCT IN A MULTIPLE PRODUCT PROJECT

Under fairly liberal conditions, the method of Leontief's input-output analysis effectively may be used in the determination of output prices in the multiple product project [11,15,16]. Price information combined with appropriate knowledge of output and cost levels can then be organized in terms of present value analysis for the purpose of ranking project options. Any shortcut to this procedure likely will result in inconsistent results. For example, it is not recommended that projects be ranked by technical coefficients which only reflect direct effects of activity when the indirect effects of project interrela ·)nships may be significant. The author is aware of the analyst's desire .or simple rules to analysis, but to proceed in terms of faulty method is to achieve unreliable results. A brief sketch follows of the application of the Leontief method to the problem at hand.

In the case of multistage or multiprocess projects, one may proceed to carry out the input-output analysis in terms of products, since clearly it is the product of each process which serves as an input to some other (or perhaps the same) activity. Establishing a scheme of process interrelationships in a simple example we might have the following:

Feeder process → Boiler process → Turbine process → Generator process → Transmission process.

Conversion of this to a product based statement might yield the following sequence:

Fuel (coal) → Superheated steam (heat) → Mechanical energy → Electrical energy → Transported energy.

For simplicity, the ensuing analysis is limited to three products. Here, let x_i measure the gross output of process "i;" x_{ij} the provision of process "i" to "j;" and x_i^d, the final product of process "i." Assuming these values to be determined for each process, it will be possible to form an input-output table:

	Process 1	Process 2	Process 3	Final Use	Gross Output
Process 1	x_{11}	x_{12}	x_{13}	x_1^d	x_1
Process 2	x_{21}	x_{22}	x_{23}	x_2^d	x_2
Process 3	x_{31}	x_{32}	x_{33}	x_3^d	x_3

(Subscripts (i,j) indicate (row, column) designations.)

To develop the corresponding statement of technical coefficients, the various input quantities to each process are divided by the gross output of that process.

Let b_{ij} be the technical coefficient indicating the amount of product "i" needed to produce one unit of "j":

$$b_{ij} = \frac{x_{ij}}{x_j} \quad . \tag{47}$$

Then from the table we obtain the matrix of technical coefficients:

$$B = (b_{ij}) = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad .$$

On the basis of the general relation,

$$x_i = x_{i1} + x_{i2} + x_{i3} + x_i^d \quad , \tag{48}$$

which underlies the construction of the input-output table, taken together with the matrix of technical coefficients, B, it is possible to write the following matrix statement:

$$x = Bx + x^d \quad . \tag{49}$$

Solving for the gross supply of product, x, in terms of the given levels of final product demand, x^d, yields the result (in matrix form):

$$x = (I-B)^{-1}x^d \quad . \tag{50}$$

Now when physical units of product are used in the Leontief open input-output system, such as we have done above, we may also treat product prices as variables and calculate equilibrium prices as well as equilibrium output levels. To do this, however, it is appropriate to assume a single profit rate prevails in all phases of the project. Then each process will have its product priced in an amount just equal to average cost plus profit. The price of the "ith" product will be:

$$p_i = \underbrace{p_1 b_{1i} + \cdots p_n b_{ni}}_{\text{costs of inputs from other project processes}} + \underbrace{p_L l_i}_{\text{cost of labor}} + \underbrace{p_i \sigma}_{\text{amount of profit}} \quad . \tag{51}$$

Denote the price vector by p, including n product prices, l as the labor input vector including the direct labor input in the "n" processes. Then we may form the equation,

$$p = B'p + p_L l + \sigma p \quad , \tag{52}$$

which yields the solution for the price vector,

$$p = (I = B' - \sigma I)^{-1} p_L l \quad . \tag{53}$$

Clearly, to solve for p, it is necessary that the analyst has already determined both the profit rate and the unit price of labor (as well as the prices and technical coefficients of other material inputs to the project).

CONCLUSION

The report displays a clear preference, based on analytical considerations, for present value criteria as a basis for project investment analysis. However, the internal rate of return criteria can be rendered serviceable in some instances.

In the case of multiproduct project proposals, the application of the Leontief input-output analysis to determination of project prices has been shown to be feasible.

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APPROVAL

THE ECONOMICS OF PROJECT ANALYSIS: OPTIMAL INVESTMENT CRITERIA AND METHODS OF STUDY

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