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Microscale Instabilities in Stream Interaction Regions

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AUGUST 1979

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MICROSCALE INSTABILITIES IN STREAM INTERACTION REGIONS

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Submitted to: The Journal of Geophysical Research
ABSTRACT

A theoretical investigation of the microstructure of solar wind stream interaction regions is presented. We discuss the role of several electrostatic kinetic instabilities which may be important within the stream interface and the compression region. Inside of 1 AU the interface is likely to be stable against the electrostatic streaming instabilities considered. Between 1 and 2 AU we argue that the interface will excite the magnetized ion-ion instability. The compression region is also found to be unstable beyond 1 AU where the modified two-stream instability, beam-cyclotron instability, and ion-acoustic instability will be important in determining the structure of the compressive pulses as they evolve into forward and reverse shocks. We conclude that the modified two-stream instability and beam-cyclotron instability predominately play a role in heating the electrons to the threshold for the ion-acoustic instability. Various electrostatic plasma waves, ranging in frequency from the lower-hybrid to harmonics of the electron cyclotron frequency, would be produced by these instabilities. Their signature should also be seen by high time resolution measurements of the temperature of the various plasma species.
1. INTRODUCTION

Two prominent features of the interaction regions of high and low speed solar wind streams are the stream interface and the surrounding compression region which, at distances beyond 1 AU, often evolves into forward and reverse shock pairs. In both these regions there is observational and theoretical evidence suggesting that strong gradients form in solar wind velocity, plasma density and/or magnetic field. In this paper, we examine the consequences of assuming that as the scale lengths of the gradients decrease both the stream interface and compression region will excite several electrostatic instabilities.

In the following section we first investigate the stream interface, and argue that ions from the fast stream are likely to penetrate at least an ion Larmor radius into the slow stream, thereby exciting the magnetized ion-ion instability. The region in the heliosphere where this instability can be excited is restricted by the necessity for there to be a fairly large (>20 km s\(^{-1}\)) discontinuous jump in the component of solar wind velocity orthogonal to the interplanetary magnetic field. This is most likely to first occur near 1 AU or slightly beyond. The wave modes excited, their frequency, and the effect of the instability on the temperatures of the electrons, protons and helium are discussed.

In §3 a similar discussion is developed for the compression region. There too, we argue that the plasma is likely to become unstable as the forward and reverse shocks begin to form. The electron-ion instabilities
excited there will produce a wide range of electrostatic waves, ranging in frequency from the lower-hybrid to harmonics of the electron gyrofrequency. Estimates are given for the saturation amplitudes of these instabilities, and their role in limiting the thickness of the developing shock structure.
2. STREAM INTERFACES

Observations. A stream interface is a sharp transition in velocity, density, temperature and flow angle of the wind often observed within the stream interaction region. It separates the cool dense slow stream from the hot tenuous fast stream and is associated with a local maximum in thermal pressure [for a review see Burlaga, 1975]. The existence of a tangential discontinuity separating the fast and slow streams was suggested as early as 1963 [v., e.g. Dessler and Fejer, 1963].

Interfaces were first reported by Belcher and Davis [1971] based on three-hour average data. Subsequently Burlaga [1974] showed that interfaces could be very thin although he also included fairly broad interfaces where the transitions in flow velocity and density took ~30 minutes, but these will not concern us. The existence of large jumps in flow velocity at stream interfaces appears confined to observations near 1 AU. Beyond 2 AU Pioneers 10 and 11 did not usually observe such discontinuities [see, e.g. Smith and Wolfe, 1977]. The kinetic effects discussed below can only be important at those interfaces which exhibit sharp, discontinuous changes in flow velocity over the fastest time scales available to plasma instruments. One should keep in mind, however, that the time interval over which those measurements have traditionally been made is still long compared with the few tenths of a second that it takes a region of a few ion Larmor radii to be convected past a spacecraft. Additional observations have been reported by Gosling et al. [1978] who analyzed the properties of the flow near 23 discontinuous interfaces with thicknesses less than $4 \times 10^4$ km. These were
observed only when the solar wind speed was less than 450 km s\(^{-1}\). They further found evidence of a sharp shear flow in solar wind velocity at interfaces, and noted that at interfaces the electron temperature rose sharply there by about 40%. We will argue below that several of these features are a natural consequence of kinetic instabilities operating in regions of sharp gradients in plasma velocity and density.

Because of the very high conductivity of the solar wind it is difficult for material on neighboring magnetic flux tubes to interpenetrate. Consequently, as fast plasma overtakes slow plasma, a compression region forms which tends to steepen with increasing heliocentric distance, eventually forming a tangential discontinuity separating the compressed ambient plasma from the fast stream [Hundhausen, 1972, p. 132ff; Hundhausen and Burlaga, 1975]. If this boundary becomes as thin as a few proton Larmor radii, the faster stream can begin to penetrate the slower one. It is at this stage that kinetic effects become important. Inside of \(-0.3\) AU, where adjoining flux tubes are nearly radial, the only kinetic interaction allowed is a viscous shear [e.g. Eviatar and Wolf, 1968]. By the time the streams have reached \(0.5\) AU, however, the component of the velocity jump across the interface perpendicular to \(R\) can exceed the thermal velocity of solar wind protons. This situation can then be unstable to excitation of an electrostatic instability known as the magnetized ion-ion instability (MII), provided that \(2V_1 < U \cdot \sin(\Gamma) < 2.5(1 + \xi_e) V_A\), where \(U\) is the magnitude of the velocity change at the interface, \(\Gamma = \pi R/V_{SW}\) is the (garden-hose) angle at heliocentric distance \(R\) between the radius vector and the magnetic line of force, \(R_s = 2.7 \times 10^{-6}\) is the angular rotation rate of the sun, \(\xi_e = 8 \pi N e / B^2\) is the ratio of electron thermal energy density to magnetic energy density, \(T_e\)
is the electron temperature, $V_A = B(4\pi NM)^{-1/2}$ is the Alfvén speed, $N$ is the total density in the interpenetration region of the two streams, $V_{SW}$ is the solar wind velocity in the slow stream, $M$ is the ion mass, and $V_i$ is the ion thermal velocity [Papadopoulos et al., 1971]. The geometry of the interaction is shown in Figure 1.

The Magnetized Ion-Ion Instability. This instability has been investigated in a somewhat different context with regard to the stream interaction in papers by Papadopoulos [1973a] and Papadopoulos et al. [1974]. Our application of the magnetized ion-ion instability to the stream interaction problem differs in several respects. Papadopoulos [1973a], following Papadopoulos et al. [1971], simplified the dispersion relation for the MII by assuming that the ions were cold, i.e., $V_i \ll \omega/k \sim U$. In addition, as noted above, $U$ must be less than a few times $V_A$. In the solar wind $V_i$ is often of order $V_A$ and these two conditions are difficult to satisfy simultaneously. This has necessitated a return to the more general dispersion relation in which the ions are warm. From the studies of Burlaga [1974] and Gosling et al. [1978], we know that discontinuous jumps in the solar wind velocity observed at 1 AU are typically only 15-50 km s$^{-1}$; and even with a jump of 50 km s$^{-1}$ the component directed across $B$ is reduced by a factor of $\sin \gamma$. In this situation the plasma is very close to marginal stability and the warm ion dispersion relation must be used.

Furthermore, Papadopoulos [1973a] and Papadopoulos et al. [1974] assumed that even close to the sun the plasma flowed across $B$ [cf. Figure 1a in Papadopoulos, 1973a]. But to date there is no observational evidence in support of such an initial condition. It is interesting to note, however, that the interaction regions of streams do appear sharper at smaller helio-
centric distances [Rosenbauer et al., 1977], although the scale of the velocity gradients remains larger than required for interpenetration. The assumption that gradients associated with stream interactions should be steeper near the sun than at 1 AU was used in the multifluid simulations reported by Papadopoulois et al. [1974], and was implicit in the kinetic calculations of Goldstein and Evistar [1973] and Papadopoulos [1973a].

Recently, evidence has been presented indicating that the interface itself is not necessarily a discontinuity inside ~0.6 AU [Schwenn, Mühlhäuser, and Marsch, 1978]. In the following discussion we adopt the picture illustrated in Figure 1 in which the fast stream originates in regions of the corona adjacent to those giving rise to slow flow; as would occur at coronal holes. Thus close to the corona the flow is initially characterized by a shear in velocity, and only at distances greater than 0.6 AU will the component of velocity across $\mathbf{B}$ exceed $2V_i$.

In deriving the dispersion relation for the MII we have retained terms which arise from electromagnetic and finite $\mathbf{B}$ effects because in the solar wind those terms can be important [Wagner et al., 1971].

The properties of the magnetized ion-ion instability can be found from the general dispersion relation for waves propagating at a large angle to $\mathbf{B}$ [Stix, 1962]. In the appropriate regime of unmagnetized ions but magnetized electrons, the general dispersion relation simplifies greatly. These simplifications are justified so long as the growth rate, $\gamma$, exceeds $\Omega_i eB/Mc$, the thermal ion Larmor frequency ($M$ is the proton mass), which in turn requires that the wavenumber, $k$, satisfy $L_i^{-1} \ll k \ll L_e^{-1}$, where $1/L_i = \Omega_i e/V_i$, and where $L_i$ ($L_e$) is the thermal ion (electron) Larmor radius, and $\Omega_i = |eB/mc|$ ($m$ is the electron mass). The sums over Bessel
functions appearing in the general dispersion relation can then be eliminated. The frequency range is thus restricted to \(\omega^2 \ll |\omega|^2 \ll \omega_0^2\), where \(\omega\) is the complex frequency of the oscillation. We retain the warm electron term in order to investigate the validity of the usual assumption that the waves propagate so nearly perpendicular to \(B\) that \(\omega/(kV_0 \sin \theta) \gg 1\), where \(\tan \theta = k_\perp/k_\parallel\).

With these assumptions the dispersion relation for the magnetized ion-ion instability in the frame of the slow plasma becomes (Appendix A)

\[
\frac{\omega^2_i}{2k^2V_i^2} Z^\prime \left( \frac{\omega-k_\parallel U \sin(\theta) \cos(\theta)}{\sqrt{2}kV_i} \right) + (1-\eta)\frac{\omega^2_i}{2k^2V_i^2} Z^\prime \left( \frac{\omega}{\sqrt{2}kV_i} \right) = 0
\]

\[
\frac{\omega^2_e}{2k^2V_e^2} Z^\prime \left( \frac{\omega}{\sqrt{2}kV_e \sin(\theta)} \right) = 1 + \frac{\omega^2_e}{\sqrt{2}kV_e} \left[ 1 + \frac{\omega^2}{k^2c^2(1+\beta_e)} \right]
\]

where \(\omega^2_i, e = 4\pi ne^2/(M,m), T_e = mV_e^2\), and \(\alpha\) is the fractional density in each stream. As pointed out by McBride and Ott [1972], in reference to the related modified two-stream instability, electromagnetic effects \((\omega_e/k^2c^2\alpha \neq 0)\) can sometimes have a stabilizing influence. In the solar wind, because \(L_i/L_e \approx 20\) one has \(\omega_e/k^2c^2\alpha \approx 1\) and the electromagnetic corrections cannot be ignored. Observations indicate [Burlaga, 1974 and Gosling et al., 1978] that at stream interfaces the faster plasma is about half as dense as the slow material, and so we have taken \(\alpha = 1/3\). Equation (2.1) can then be solved numerically. The existence of unstable roots depends sensitively on
the variation of solar wind parameters throughout the interplanetary medium.

Close to the sun, where \( U \cdot \sin \theta / V_i \ll 1 \), no instability is found. As one moves out in heliocentric distance to about 0.8 AU, \( U \cdot \sin \theta / V_i \gg 1 \) and excitation of the magnetized ion-ion instability depends primarily on the local values of \( U_i \), \( V_i \), and \( V_{SW} \). We have investigated solutions to (2.1) extending from 0.6 AU to 5 AU using a range of values for solar wind parameters. Because the MII excites waves propagating perpendicular to \( \mathbf{B} \), it is \( T_i \bot \) which controls the onset of the instability, and so we have used \( T_i = (T_i \bot) = 2 \text{ eV} \ (2 \times 10^4 \text{ K}) \) in (2.1) at 1 AU. For \( N \) and \( B \) (also at 1 AU) we took \( N=7 \text{ cm}^{-3} \) and \( B=5 \times 10^{-5} \text{ G} \). To evaluate (2.1) at heliocentric distances both larger and smaller than 1 AU, we adopted the following scaling laws: \( N(R) \sim 1/R^2 \), \( T_i(R) \sim R^{-(4/3)} \), and \( T_e(R) \sim R^{-(1/3)} \) [see, e.g., the discussion in Hundhausen, 1972 and the recent measurements by Ogilvie and Scudder, 1979].

To determine the variation in the threshold conditions of the magnetized ion-ion instability we varied \( V_{SW} \) from 250 km s\(^{-1}\) to 450 km s\(^{-1}\) using the \( T-V \) relation [Burlaga and Ogilvie, 1973], \( T_i = V_{SW} \). This was normalized so that \( (T_i \bot) = 2 \text{ eV} \) for \( V_{SW} = 350 \text{ km s}^{-1} \).

As an example, consider the solution to (2.1) at 1.5 AU with \( V_{SW} = 300 \text{ km s}^{-1} \). Using the 1 AU values and the scaling relations defined above, we found unstable roots to (2.1) when the discontinuous jump in velocity at the interface was \( U = 40 \text{ km s}^{-1} \). [Recall that \( U \) denotes the total jump in \( V_{SW} \) at the interface—the component perpendicular to \( \mathbf{B} \) is \( U \cdot \sin \theta(R) \).] In Figure 2, we have plotted the real and imaginary roots, \( \omega(k, \theta) \) and \( \gamma(k, \theta) \) first against \( \theta \) using the value of \( k \) which maximizes \( \gamma \) at each \( \theta \) (denoted \( k_m \)) and then against \( k \) using the value of \( \theta \) which maximizes \( \gamma \) at each \( k \) (denoted \( \theta_m \)). \( \gamma \) is plotted for two different values of \( \theta_c \). Figure 2a illustrates
the well known property of the MII that it is most efficiently excited across the magnetic field. Note that for \( \theta < 1^\circ \), we could have assumed that the electrons were "cold" and expanded the electron plasma dispersion function [Papadopoulos, 1973a]. The effect of making such a cold electron approximation is illustrated in Figure 3 (using somewhat different parameters). The cold electron approximation is seen to be valid only so long as \( \theta < 1^\circ \).

From Figure 2, we can define \( \gamma_m = \gamma(k_m, \theta_m) \) as the maximum value of \( \gamma \), maximized with respect to both \( k \) and \( \theta \). In Figure 4, we have plotted \( \gamma_m \) and \( \omega_m \) [the value of \( \omega(k, \theta) \) associated with \( \gamma_m \)] against \( R \) using two different values of \( U \) and a wind velocity \( V_{SW} = 300 \text{ km s}^{-1} \). Also plotted in the Figure is \( n_i \) and \( \omega_0 \) defined by

\[
\omega_0^2 = \frac{\omega_i^2}{1 + (\omega_i^2/n_e^2)}
\]

(2.2)

In the solar wind because \( \omega_e^2/n_e^2 \gg 1 \), \( \omega_0 = \omega_{LH}/n_e \). Papadopoulos et al. [1971] found that \( \gamma_m \) was as large as \( \omega_0 \). However, because of the stabilizing effects of the electromagnetic contributions to (2.1), together with the fact that \( U \) is not much greater than \( V_i \), we find that \( \gamma_m \) is systematically smaller than \( \omega_0 \), but that \( \omega_m = \omega_0 \), as it is in the cold ion approximation. (Note that our definition of \( \omega_0 \) differs by a factor of \( \sqrt{2} \) from that of Papadopoulos et al. [1971].) For \( U = 50 \text{ km s}^{-1} \) the magnetized ion-ion instability is first excited at 1 AU, while for \( U = 40 \text{ km s}^{-1} \) threshold for the MII is not reached until 1.5 AU. This reflects the distance at which \( U \sin \theta \) first exceeds \( V_i \) by an amount sufficient to excite the instability. Our assumption that the ions are demagnetized is verified \textit{a posteriori} because \( \gamma_m > \Omega_i \).
The heliocentric distance at which the MII is likely to reach threshold is a sensitive function of $U$ and $V_{SW}$. This relationship is shown in Figure 5, where $U$ is plotted against $R$ for various values of $V_{SW}$. The curves represent the values of $R$ and $U$ for which $\gamma_m$ exceeds $\omega_1$, and suggest the heliocentric distance at which the MII is likely to become important. For example, for discontinuous velocity jumps of $40 \text{ km s}^{-1}$ in a $250 \text{ km s}^{-1}$ solar wind (recall that $V_{SW}$ refers to the slow stream), the MII can be excited at 1 AU and beyond. However, if the "slow" wind has a velocity of $450 \text{ km s}^{-1}$, then with $U=40 \text{ km s}^{-1}$ the MII is not likely to be excited inside 2 AU. In general, the smaller the velocity jump and the higher the wind speed, the further out one must go before the conditions for excitation of the MII can be satisfied. However, even $U=20 \text{ km s}^{-1}$ is adequate to excite the MII near 2 AU with $V_{SW}=250-300 \text{ km s}^{-1}$. Conversely, velocity jumps in excess of $40 \text{ km s}^{-1}$ are necessary in order to excite the MII inside 1 AU. Thus this instability is likely to be important where $V_{SW}$ is low and $U \sim 20-40 \text{ km s}^{-1}$.

Saturation Effects. The nonlinear stages of the magnetized ion-ion instability have been intensively investigated both theoretically and by means of computer simulations [Papadopoulos et al., 1971]. The waves first evolve as described by quasilinear theory until ion trapping becomes important. Stabilization then occurs after a time $\tau_M \approx (M/eE^*k)^{1/2} = 1/\gamma_m$ ($E^*$ is the rms value of the electric field of the growing wave [Manheimer, 1971]). Subsequently, the ions are reflected in the potential wells of the waves and the waves will stop growing after being further amplified by another factor of $e = 2.7$ [Papadopoulos et al., 1971]. From the definition of $\tau_M$ the maximum value of the electric field is
In Figure 6 we have plotted $E_M$ between 1 and 5 AU for $V_{SW}=300$ km s$^{-1}$ and $U=50$ km s$^{-1}$. Values lie are between 100-400 $\mu$V/m with the peak occurring near 2 AU. Observation of these waves has thus far not been reported, perhaps because the frequency range is below that of most experiments that have been flown. For example, on Helios 1 and 2 where $\omega_{LH}/2\pi < 10$ Hz the plasma wave experiment was insensitive below 31 Hz [Gurnett and Anderson, 1977]. The waves may be difficult to detect because the instability occurs over only a few ion Larmor radii ($L_i \sim 50$ km at 1 AU), a distance which is convected past a spacecraft in less than a second.

Stabilization by ion trapping suggests that within the interface the ion temperature should be significantly enhanced over its value on either side. In essence, within the stabilization region the thermal velocity will have increased to $\sqrt{V_i + U \cdot \sin(\gamma)}$, so that as long as $U \cdot \sin\gamma > 2V_i$ the increase in temperature can exceed a factor of two. Computer simulations in which $U \cdot \sin\gamma >> V_i$ have found that the ion temperature at stabilization is $T_i \sim 0.3V_i [U \cdot \sin\gamma]^2$ [Papadopoulos, 1973a], with the increase being primarily in $(T_i)_\perp$. Gosling et al. [1978] using one-hour averages reported an increase of nearly a factor of two in $(T_i)_\perp$ at the interface (cf. their Figure 4). With five-minute averages only the change in $(T_i)_\parallel$ was reported [v. Figure 2 of Gosling et al., 1978]. Of course, enhancements in $T_i$ caused by kinetic interactions are in addition to those expected due to the compression of the solar wind fluid at the interface and those resulting from the different boundary conditions in the coronal source regions of the fast and slow streams.
Alpha particles will also be affected by the MII. Once thermalized, the alpha particle temperature will be of order $T_a = \left(\frac{M_a}{2}\right) \cdot \left(U \cdot \sin \gamma\right)^2$, so that $\left(\frac{T_a}{T_i}\right) = \left(\frac{M_a}{M}\right) = 4$. Gosling et al. [1978] only report one-hour averages of alpha particle parameters near interfaces, and so their observations may primarily reflect fluid, as opposed to kinetic, characteristics of the stream interaction. Nonetheless, they found that at interfaces the alpha to proton temperature ratio increased by more than 30%, and that $T_a$ itself rose by nearly a factor of three.

One additional consequence of this instability is that at the interface the amount of shear at the tangential discontinuity will be reduced because the primary effect of the ion trapping is to thermalize that component of the flow directed across $B$, i.e. the fraction $\left(\frac{1}{2}\right)M(U \cdot \sin \gamma)^2$. It is important to realize that the instability does not convert all of the streaming energy into electrostatic waves. In fact, for the values of $E_m$ shown in Figure 6, only a very small fraction of the streaming energy ($< 10^{-6}MU^2$) is converted into waves in the process of thermalizing the ions and producing the increase in proton temperature at the interface. Because the component of the flow parallel to $B$ is unaffected, the directed flow of the fast plasma appears to change from radial to shear at an interface. At greater heliocentric distances a larger fraction of the flow energy will be thermalized by the instability, $E_m$ will increase, and the shear should decrease until the instability has eroded the velocity gradients to the point where it cannot be excited. In this context it is not surprising that discontinuous jumps in velocity are often absent in interfaces observed beyond 1 AU.

**Electron Heating.** At first glance the magnetized ion-ion instability
would not appear to be a very efficient means of heating electrons. However, the electrons will respond to the electrostatic fields produced by the ion-ion interaction by drifting in the direction of $E \times B$ [Papadopoulos et al., 1971]. For electric fields approaching 1 mV/m, the drift velocity can exceed 100 km s$^{-1}$, which is adequate, as we shall discuss in more detail in the next section, to excite the modified two-stream instability. This instability both heats the electrons, increasing $(T_e)_{||}$, and lowers $U_e$, their mean drift velocity. The primary source of free energy is the electron drift energy $(1/2)mU_e^2$ [Lampe et al., 1975]. Assuming that $U_e = 200$ km s$^{-1}$, and $V_e = 1000$ km s$^{-1}$, the increase in perpendicular electron temperature at the interface will be -5%. Gosling et al. [1978] utilizing five-minute averaged data do report an increase in $(T_e)_{||}$ of some 10% at interfaces (v. their Figure 3).

In the following section we examine several kinetic interactions which may be important in the formation of the forward and reverse shocks often observed in stream interaction regions beyond 1 AU.
3. FORWARD AND REVERSE SHOCKS

The compression region on either side of the stream interface steepens with heliocentric distance until a forward and reverse shock pair forms (see Fig. 1). These corotating shock pairs have not been observed inside 1 AU [Rosenbauer et al., 1977], and are only rarely observed at 1 AU [Ogilvie, 1972]. However, when Pioneer 10 and 11 encountered corotating stream interaction regions beyond 1 AU, observation of shock pairs became common [Smith and Wolfe, 1977]. Pizzo [1978a] and [1978b], using multidimensional fluid models of corotating streams, has shown that formation of forward and reverse shock pairs is expected to occur only near and beyond 1 AU. Nonradial flows in these models transport mass, energy and momentum away from the compression region and that tends to delay the formation of shocks to beyond 1 AU.

In this section we discuss several microinstabilities likely to be excited as the forward and reverse shocks form in the compression region. The instabilities considered are not meant to exhaust all possibilities, and for example we consider neither the closely related lower-hybrid drift instability [Davidson et al., 1977], nor the drift-cyclotron instability [Gladd and Huba, 1979]. The ones treated do, however, illustrate the range of frequencies and classes of waves that one might observe with appropriate instrumentation.

Modified Two-Stream Instability (MTSI). As the compressive MHD waves steepen, the local magnetic gradients within the waves produce streaming of electrons through ions. This situation was first described by Fredricks
[1969]. In a plasma in which $T_e < 10T_i$ this strutting is known to excite the modified two-stream instability [Ott et al., 1972 and McBride et al., 1972]. This instability is similar to the magnetized ion-ion instability discussed above; the major distinction is that instead of having two ion distributions flowing through each other in the presence of electrons, one now has a single ion distribution streaming through electrons. The MTSI can be excited so long as $V_d$, the relative velocity of electrons and ions, exceeds both $V_i$ and the sound speed, $c_s^2(T_e/M)^{1/2}$. Again we retain the electromagnetic corrections to the dispersion relation, allow both the ion and electrons distributions to be warm, and restrict our attention to the wavenumber and frequency ranges $k^{-1} < k < k^{-1}$ and $\omega^2 < | \omega |^2 < \omega e$, respectively. The resulting dispersion relation becomes (Appendix A)

$$\frac{\omega^2}{2k^2V_i^2} - \frac{\omega^2}{2k^2V_e^2} = 1 + \omega^2 \left[ 1 + \frac{\omega^2}{k^2c^2(1+\beta_e)} \right]$$

Excitation of the instability is determined by the magnitude of $V_d$, which in turn is determined by the drift current, $j_e$, within the MHD wave. Thus [Fredricks, 1969]

$$\alpha(\Delta B / \Delta x) = 4\pi j_e = 4\pi NeV_d$$

(3.2)

We solve (3.1) using $V_d$ as a free parameter. Once having found the
value of \( V_d \) which maximizes the growth rate, (2.2) can be used to estimate the scale size of the MHD wave required to excite the MTSI. Because the growth rates are relatively insensitive to \( V_{SW} \) and \( R \), we first solve (3.1) using parameters typical of 1 AU; a choice which permits comparison with the results of Lemons and Gary [1977], who have used a more general formulation of the MTSI to investigate the formation and structure of the earth's bow shock.

In Figure 7 we plot the real and imaginary roots of (3.1) first versus \( k \) [at the value of \( \theta \) which maximizes \( \gamma(k) \)], and then versus \( \theta \) [at the value of \( k \) which maximizes \( \gamma(\theta) \)]. The computations were performed using \( V_{SW} = 450 \text{ km s}^{-1} \), \( R = 1 \text{ AU} \), \( T_i = (T_i)_{1} = 3.3 \text{ eV} \), \( B = 5 \), \( N = 7 \text{ cm}^{-3} \), and \( \beta_e = 0.6 \) and 1.1. In this example, \( V_d = 5V_i \). As illustrated in the Figure, as \( \beta_e \) decreases, \( \gamma_m \) increases as more free energy becomes available to drive the instability, i.e. \( V_d - C_s \) increases. In deriving (3.1) it was necessary to assume that \( k_1/k < V_d/V_e \) in order to reduce to a single term the summations over electron Bessel functions which appear in the full dispersion relation. This implies that the validity of (2.1) is restricted to the domain \( \theta < 4^\circ \) (with \( \beta_e = 1.1 \)) or \( \theta < 5^\circ \) (for \( \beta_e = 0.6 \)). In contrast, Lemons and Gary [1977] retained the sums over Bessel functions and found that although \( \gamma(k_m,\theta) \) was still maximum at small values of \( \theta \), it remained relatively constant to beyond \( \theta < 20^\circ \). (Note that our definition of \( \theta \) is the complement of the definition used by Lemons and Gary [1977].) Figure 8 shows \( \gamma_m \) and \( \omega_m \) for the modified two-stream instability as a function of \( V_d/V_i \) for \( \beta_e = 1.1 \) and 0.6 \((T_e = 10 \text{ and } 5 \text{ eV}, \text{ respectively})\). At threshold \( V_d = 3V_i \) and \( \omega_m < \omega_{LH} \). As \( V_d \) increases, so does \( \gamma_m \), while \( \omega_m \) increases more rapidly, eventually exceeding \( \omega_{LH} \).
As mentioned above, the scale of the MHD pulse can be estimated from (3.2), now rewritten in a more convenient form:

\[
\Delta x = \left(\frac{n_e}{\omega_p}\right) \cdot \left(\frac{c}{V_d}\right) \cdot \left(\frac{\Delta B}{B}\right) \cdot \left(\frac{c}{\omega_p}\right)
\]  

At 1 AU we have \(n_e/\omega_p = 5.9 \times 10^{-3}\). Assuming \(V_d/V_i = 3\) at threshold, \(c/V_d = 8.4 \times 10^3\), so that \(\Delta x = 150(c/\omega_p) = 300\) km for \(\Delta B/B = 3\). This value for \(\Delta x\) is consistent with observational and theoretical evidence of thicknesses of large amplitude MHD waves and shocks [see, e.g. Fredricks and Coleman, 1969 and Manheimer and Boris, 1972].

The situation at 2 AU is similar, and is also illustrated in Figure 8. Again the instability threshold is reached at \(V_d/V_i = 3-4\), and \(\Delta x = 92(c/\omega_p) = 370\) km, where now \(n_e/\omega_p = 5 \times 10^{-3}\), \(\Delta B/B = 3\), \(N=1.75\ \text{cm}^{-3}\), \((T_i)_{\perp} = 1.3\ \text{eV}\) and \(V_{SW} = 450\ \text{km s}^{-1}\). This value for \(\Delta x\) is close to the thicknesses of \(-1000\) km reported by Smith and Wolfe [1977] for forward and reverse shock pairs using 453 km s\(^{-1}\) as the average propagation speed.

**Saturation Effects.** The modified two-stream instability results in heating both ions and electrons. The unmagnetized ions are primarily heated by raising \((T_i)_{\perp}\), while the electrons, constrained as they are to move along \(B\), are primarily heated in \((T_e)_{\parallel}\) [Ott et al., 1972]. Stabilization of the MTSI has been studied using numerical simulations [Ott et al., 1972 and McBride et al., 1972] where trapping was found to be the stabilization mechanism. Waves propagating at angles \(\theta > (m/M)^{1/2} = 1.3^\circ\) trapped electrons, while ion trapping occurred first if \(\theta < (m/M)^{1/2}\). From Figure 7, significant growth is obtained only for \(\theta_m < 1.3^\circ\), so that ion trapping is
likely to be the more important effect. Consequently, (2.3) can be used to estimate the magnitude of the electric fields associated with this instability, and we find that for \( V_d/V_i = 4 \) at both 1 and 2 AU, and \( E = 100 \ \mu \text{V/m} \) which increases to \( \approx 300 \ \mu \text{V/m} \) when \( V_d/V_i = 8-10 \). If the MTSI evolves to saturation, \( V_d \) would be reduced to the marginally stable level \( (V_d/V_i = 3-4) \) and an amount of free energy equal to \( (1/2)Nm\cdot(\Delta V_d)^2 \) could be extracted from the current and converted to wave energy and heating [Lampe et al., 1975]. The amount of electron heating that actually occurs depends sensitively on \( \Delta V_d \); ranging from as much as a 10% increase in \( (T_e)_{||} \) if \( \Delta V_d = 8 V_i \), to a negligible change in \( (T_e)_{||} \) if \( \Delta V_d = V_i \).

The modified two-stream instability is not the only cross-field instability that can be excited in MHD pulses. Several others produce electrostatic waves at frequencies well above \( \omega_{\text{LH}} \) and thus may be more easily observable by plasma-wave experiments. We will briefly consider two such instabilities; the beam-cyclotron and the ion-acoustic instabilities.

**Beam-cyclotron instability.** Like the modified two-stream, the beam cyclotron instability (also known as the electron cyclotron drift instability) is driven by a relative drift of electrons and ions. In this case the instability arises from a coupling of Bernstein and ion acoustic waves [Forslund et al., 1970 and Gary and Sanderson, 1970]. The resulting unstable waves are confined to a narrow cone about the normal to \( B \) (as are Bernstein waves) with frequencies close to harmonics of \( \omega_e \), and \( kL_e \gg 1 \) [Gary, 1971]. To excite the instability, \( V_d \) must exceed \( C_s \). Significant growth occurs even with \( T_e = T_i \) and \( \beta_e = 1 \) [Lashmore-Davies, 1971]. This instability has been extensively studied both analytically and in numerical simulations by Lampe et al. [1972].
The dispersion relation for $\mathbf{k} \cdot \mathbf{B} = 0$ and $kL_e > 1$ has been derived by Lampe et al. [1972], and is given by

\begin{equation}
1 - (k\lambda_e)^{-2} (T_e/T_i) \cdot Z \left( \frac{\omega - kV_d}{\sqrt{2kV_i}} \right) + (k\lambda_e)^{-2} + (k\lambda_e)^{-2} \frac{\omega}{2/2kV_e} \cdot \left[ Z \left( \frac{\omega}{\sqrt{2kV_e}} \right) - Z \left( \frac{-\omega}{\sqrt{2kV_e}} \right) + i \cdot \cot(\pi\omega/n_e) \cdot [Z \left( \frac{\omega}{\sqrt{2kV_e}} \right) + Z \left( \frac{-\omega}{\sqrt{2kV_e}} \right)] \right]
\end{equation}

(3.4)

where $\lambda_e = v_e/\omega_e$ is the electron Debye length, and $V_d$ is assumed parallel to $\mathbf{k}$. The neglect of electromagnetic effects in (3.4) is well justified for the parameter range of interest [Lampe et al., 1972]. Because of the strong Landau damping of Bernstein waves propagating at $\theta \neq 0$, the unstable waves are similarly confined to angles $\theta < n_e/(2\pi\omega_k\lambda_e)$ [Kamimura et al., 1978]. At 1 AU, $\omega_e/n_e = 170$, so that $\theta < 1^\circ$ for $k\lambda_e = 0.1$. For a given frequency, unstable solutions to (3.4) are somewhat sensitive to the ratio of $V_d/V_i$--large $V_d$ results in smaller $k\lambda_e$, and hence a wider propagation cone.

To compare properties of this instability with the MTSI we solved (3.4) using solar wind parameters typical of 1 AU with $\beta = 1.1$. As in Figure 8, $V_{SW} = 450$ km s$^{-1}$, $N = 7$ cm$^{-3}$, $B = 5$ $\gamma$, and $T_e/T_i = 3$. The results are shown in Figure 9 for $V_d/V_i = 10$, where $\gamma$ and $\omega$ are plotted as functions of $k$ for the first 8 harmonics of $n_e$. The maxima in $\gamma$, denoted $\gamma_m$, fall approximately at

\begin{equation}
k = \frac{n\eta_e}{V_d - C_s/(1 + k^2\lambda_e^2)^{1/2}}
\end{equation}

(3.5)

a relationship derived by Lampe et al. [1972] for the case $T_e >> T_i$. Although
the ratio $\gamma_m/\omega$ is rather small, ranging from $5\times10^{-3}$ - $5\times10^{-2}$, $\gamma_m$ is 5-6 times greater than found for the MTSI (cf. Figure 8). On the other hand, the beam-cyclotron instability requires a slightly higher threshold for $V_d$ (5-6 $V_i$, compared to 3-4 $V_i$ for the MTSI). Thus it is not altogether clear which instability will dominate, although one can imagine a situation in which the MTSI is first excited, but is not immediately able to dissipate the available free energy. The MHD waves will then continue to steepen, and $V_d$ will increase until the beam-cyclotron instability is excited. Once excited, because of its large growth rate, the beam-cyclotron instability will saturate before the MTSI. The quasilinear and nonlinear stages of evolution have been examined by Lampe et al. [1972] who found that initially the beam-cyclotron instability saturated at a relatively low level via resonance-broadening. Once this happened, they found that the plasma could be further unstable to the ion-acoustic instability because the nonlinear dispersion relation for the beam-cyclotron instability when saturated through resonance-broadening has the form of the linear zero magnetic field ion-acoustic dispersion relation. In the solar wind, where $T_e/T_i$ is usually too low to excite the ion-acoustic instability, the beam-cyclotron instability will saturate, allowing the modified two-stream instability to evolve until ion or electron trapping has reduced $V_d$ to a marginally stable state. However, because both the MTSI and beam cyclotron instabilities can heat electrons at least to some extent, it is quite possible that $T_e$ will increase by just enough to excite the ion-acoustic instability. We explore this possibility below.

In spite of the consensus that saturation of the beam-cyclotron instability via resonance broadening occurs at a low level of electric field
turbulence [Lampe et al., 1972; Biskamp, 1973 and Lemons and Gary, 1978], it is not clear that this will be true for solar wind parameters. The critical amplitude of the turbulent fields when resonance broadening becomes important was found by Lampe et al. [1972] to be $E = 96 \cdot \left( N T_e / \omega_e \right)^{(1/k \lambda_e)}^{1/2}$ (mV/m), with $T_e$ measured in eV. Assuming $N = 7 \text{ cm}^{-3}$, $k \lambda_e = 5 \times 10^{-2}$, and $\omega_e / \omega_e = 170$, we find $E = 1 \text{ mV/m}$, which is actually larger than the value of $\approx 100 \mu\text{V/m}$ found above for the saturation level of the modified two-stream instability.

From our numerical solutions of (3.4), we know that the instability can be excited up to at least the eleventh harmonic of $\omega_e$, the exact maximum being a function of $V_d / V_e$ [Lampe et al., 1972]. These high harmonics have relatively large values of $k \lambda_e$, and are thus confined to a very narrow cone about $\theta = 0$ of much less than 1°. This may in turn limit the growth of the highest harmonics because of the necessity of the magnetic field to remain constant in direction over a growth time [Lemons and Gary, 1978]. These waves have frequencies in the range 100-1000 Hz, depending on the amount of Doppler shift, and should be easily observable. In fact, Wu and Fredricks [1972] have argued that this instability may have been observed in the earth's bow shock where amplitudes of 1-20 mV/m have been reported at frequencies near 1 kHz [Fredricks et al., 1970; Fredricks and Coleman, 1969]. Biskamp's [1973] suggestion that the observed turbulence levels were too high to be explained by the beam-cyclotron instability should be reexamined, at least for space plasmas.

The quasilinear analysis and numerical simulations carried out by Lampe et al. [1972] indicate that as long as $V_d >> C_s$, the bulk of the energy taken out of the relative streaming between the protons and electrons goes into heating electrons. Thus the beam-cyclotron instability appears to be
more efficient at heating electrons than is the modified two-stream instability, and with \( V_d = 10V_i = 6C_s \), electron heating should be fairly efficient. At that drift velocity, only a small fraction of the total available free energy (equal to \( MV_d^2 \)) need go into electron heating in order that the electron temperature increase substantially. For example, \( T_e \) would double if an amount of energy equal to \( MV_d^2/30 \) were removed from the stream. Again, based on the results of Lampe et al. [1972], this is probably the maximum amount of energy that could be extracted before saturation by ion trapping occurs. Even this amount is unlikely unless the beam-cyclotron instability in turn excites the ion-acoustic instability. Therefore it is of interest to see just how much it is necessary to heat the electrons before excitation of the ion-acoustic instability becomes possible.

**Ion-Acoustic Instability.** The threshold for the ion-acoustic instability can be found from the dispersion relation discussed by Fried and Gould (1961) and Stringer [1964]. In our notation:

\[
1 - \frac{1}{2k^2T_e} \left( \frac{\omega - kV_d}{2kV_i} \right)^2 \left( \frac{\omega}{2kV_i} \right)^2 = 0 \quad (3.6)
\]

For \( V_d/V_i = 10 \), unstable roots first appear at \( T_e/T_i = 10 \). Recall we have assumed that initially \( (T_e^i)/(T_i^i) = 3 \). As we have seen, an increase in \( T_e/T_i \) by more than a factor of two is highly unlikely. If the ion-acoustic instability is not excited, relatively little streaming energy will be removed from the plasma. Therefore, the MHD pulse, or shock will continue to evolve, causing \( V_d \) to increase still further. As \( V_d/V_i \) approaches 20, it is then possible to excite the ion-acoustic instability with \( (T_e^i)/(T_i^i) = 6 \).
The ion-acoustic instability saturates by ion trapping [Biskamp and Chodura, 1971 and Lampe et al., 1972], and is capable of reducing the relative streaming to marginally stable levels. As an example, in Figure 10 we have plotted ω and γ against k for the case V_d/V_i = 20 and T_e/T_i = 7. These waves too will be subject to a Doppler shift in the solar wind, but should be observable by plasma wave instruments currently flown. [Note that because we have assumed that the protons are streaming through stationary electrons, the values of ω in Fig. 9 can exceed ω_i and ω/k>C_s.] Saturated turbulent field intensities of order 1 mV/m can be expected. Furthermore, the ion-acoustic instability produces waves propagating into a wide cone about θ=0. In fact, for V_d/V_i=20, Δθ=80°.
4. CONCLUSIONS

In the preceding sections we have discussed several kinetic instabilities which we argue are important in determining the microscale structure of the solar wind stream interaction region. The selected instabilities are not meant to comprise an exhaustive list. We have ignored, for example, all of the lower-hybrid drift instabilities driven by strong gradients in density and magnetic field, primarily because their physics is similar in many respects to the magnetized ion-ion and modified two-stream instabilities [v., e.g. Lemons and Gary, 1978]. Another example of a related instability we did not discuss is the cross field, current driven ion-acoustic instability [Barrett et al., 1972] because this shares many characteristics with the field free instability and MTSI.

Perhaps the best way to summarize our discussion is to imagine following a stream interaction region as it evolves outward in the interplanetary medium. If one assumes that the fast stream originates in a coronal hole, while the slower material comes from the adjoining region, then close to the sun a shear interaction will predominate. Although we have not discussed any kinetic instabilities in this region, one can imagine several dynamical effects which might be important. For example, if the gradients in the shear become so large that the fast and slow ions are separated by only a few Larmor radii, then an electromagnetic ion cyclotron instability would be excited if the shear velocity exceeded $\sim 2.7V_A$ [Eviatar and Wolf, 1968 and Goldstein and Eviatar, 1973]. On a larger scale the interface could become Kelvin-Helmholtz unstable. Eventually, however, the interface between the two streams will steepen as the faster flow begins to overtake the slower
At the interface the flow can be thought of as having two components, one along the interface, parallel to the magnetic field, the other across the field. Because of their large gyroradii, the ions will be the first component of the fast stream to penetrate the slow plasma. Depending primarily on the relative velocities of the two streams and the velocity of the solar wind in the slow flow, this situation will become unstable to the magnetized ion-ion instability somewhere between 1 and 2 AU. The magnetized ion-ion instability will heat the ions (proportional to their mass). The amplified electric fields produced by the interpenetration of the two streams can then give rise to electric fields which excite either the modified two-stream instability or one of the other instabilities discussed in §2. Excitation of these kinetic instabilities will quickly erode any sharp gradients at the interface, leaving it in a state of marginal stability.

Beyond one AU the compression region, which has gradually steepened as the streams evolve, forms pulses within which strong gradients in $\mathbf{B}$ can excite various electron-ion cross field instabilities. The possible range of frequencies excited extends from the lower-hybrid (due to the modified two-stream instability) through the ion-acoustic frequency range, to harmonics of the electron cyclotron frequency. It appears unlikely that either the modified two-stream instability or the beam-cyclotron instability would be able to reduce the streaming to a marginally stable state. Should that be the case, the compression regions would continue to evolve until those two instabilities had heated the electrons sufficiently to drive ion-acoustic waves unstable. Once excited, the observed shock thickness and electron temperature would remain close to their marginally stable level.
[Manheimer and Boris, 1972]. As yet, insufficient research has been conducted on the detailed structure of forward and reverse shocks in stream interaction regions to know whether the structure is consistent with this scenario. However, Morse and Greenstadt [1976], in an analysis of the earth's bow shock, did conclude that under certain conditions the thickness of the shock appear to be controlled by the conditions for marginal stability against the ion-acoustic instability. Present and future plasma wave experiments on deep space probes should be able to answer many of these questions.
APPENDIX

The dispersion relation can be written in the form:

\[ \mathbf{E}^* \cdot \mathbf{D} \cdot \mathbf{E} = 0 \]  \hspace{1cm} (A1)

where \( \mathbf{D} \) is the dyadic given by Montgomery and Tidman [1961 eq. 10.35]. It is convenient to write the electric field as the sum of electrostatic and electromagnetic waves:

\[ \mathbf{E} = -i[\phi \mathbf{k} + (\omega/c) \mathbf{A}] \]  \hspace{1cm} (A2)

where \( \phi \) and \( \mathbf{A} \) are the scalar electrostatic and vector electromagnetic vector potentials, respectively. We take \( \mathbf{B}_0 \) in the \( \mathbf{z} \) direction, \( \mathbf{k} = (k_\perp, 0, k_\parallel) \), and \( \mathbf{A} = \mathbf{A}\mathbf{\hat{z}} \). Expansion of (A1) gives

\[ \phi^2 [k_\perp^2 \mathbf{D}_{xx} + k_\parallel k_\perp (\mathbf{D}_{xz} + \mathbf{D}_{zx}) + k_\parallel^2 \mathbf{D}_{zz}] + (\omega/c)^2 \mathbf{A}^2 \partial_{yy} \]

\[ + \phi \mathbf{A} (\omega/c) k_\parallel (\mathbf{D}_{yz} + \mathbf{D}_{zy}) = 0 \]  \hspace{1cm} (A3)

where we have used \( \mathbf{D}_{xy} = -\mathbf{D}_{yx} \).

Following McBride et al. [1972], we assume \( \omega_i << |\omega| << |\omega_e| \), \( k_\parallel << k_\perp \), \( k_\perp^2 L_e << 1 \), and \( k_\parallel^2 L_i \gg 1 \), which implies that the ion trajectories are well approximated by straight lines and the electron trajectories by helicies. We also assume that the temperatures of each species are the same in both
streams and are isotropic. However, the ion and electron temperatures may differ from one another.

For the magnetized ion-ion instability we take the following distribution functions

\[ f_i = \frac{1}{(2\pi)^{3/2} V_i^3} a \exp\left[-\frac{(v - \mathbf{U})^2}{2V_i^2}\right] + (1-a) \exp\left[-\frac{v^2}{2V_i^2}\right] \]

\[ f_e = \frac{1}{(2\pi)^{3/2} V_e^3} \exp\left[-\frac{v^2}{2V_e^2}\right] \]

where \( \mathbf{U} = (U \sin \gamma, 0, U \cos \gamma) \) is the drift velocity and \( \gamma \) is the garden-hose angle.

Let \( \psi \) be the angle between \( \mathbf{k} \) and \( \mathbf{U} \), and \( \theta \) the angle between \( \mathbf{k} \) and the normal to \( \mathbf{B} \). Then the ion component of the electrostatic dispersion relation, which is the part of (A3) multiplying \( \psi^2 \), becomes

\[ 1 - \frac{\omega_i^2}{2k^2 V_i^2} a Z\left(\frac{\omega - \mathbf{k} \cdot \mathbf{U}}{\sqrt{2kV_i}}\right) + (1-a) Z\left(\frac{\omega}{\sqrt{2kV_i}}\right) \]

where \( Z(\tau) \equiv (1/\pi) \int_{-\infty}^{\infty} dy \exp(-y^2)/(y - \tau) \) is the plasma dispersion function. Using \( \psi + \theta + \gamma = \pi/2 \), and \( \sin \theta = 0 \), we have

\[ k^*U = k U \cos \psi = k U (\sin \gamma \cos \theta + \cos \gamma \sin \theta) \]

\[ = k U \sin(\gamma + \theta) = k U \sin \gamma \cos \theta. \]

This provides the ion terms of equation (2.1).
The electrons, on the other hand, are highly magnetized. In the strong magnetic field limit, the electrostatic electron terms become

\[
\frac{\omega_e^2}{kV_e^2} \left(1 - \exp(-k^2L_e^2) I_0(k^2L_e^2) \left[1 + \frac{1}{2} \frac{Z'(\omega/2kV_e \sin \theta)}{\omega_e/kc} \right] \right)
\]

\(I_0\) is the modified Bessel function.

The electromagnetic and mixed terms proportional to \(A^2\) and \(\phi A\) in (A3) tend to stabilize these instabilities, and so must be kept [McBride et al., 1972]. However, in the range of frequencies of interest here, it is possible to further simplify these terms by taking the fluid limit in which all resonances are ignored. Thus we take the arguments of the Bessel functions to be small, the arguments of the plasma dispersion functions to be large (except for \(n = 0\)) and expand accordingly. We also use \(I_0' = I_1, I_0 = 1 + (1/4)(kV_e/n_e)^2,\) and \(I_1 = (1/2)(kV_e/n_e)^2.\) We retain the warm electron terms for reasons discussed in the text and obtain terms of the type

\[
1 + \frac{(\omega_e/kc)^2}{1 + \beta_e + \omega_e/kc} \frac{Z'(\omega/2kV_e \sin \theta) + (\omega_e/n_e)^2}{1 + \beta_e} + \frac{\omega_e/kc}{1 + \beta_e}
\]

Combining these terms gives (2.1). If we replace the ion distribution function given in (A4) by

\[
f_i = \frac{1}{(2\pi)^{3/2}v_i^3} \exp[-(y - v_D)^2/2v_i^2]
\]

we obtain the dispersion relation for the modified two-stream instability given in (3.1).
Acknowledgements. We would like to thank Drs. J. S. Scudder, L. Burlaga, A. Klimas and K. Ogilvie for many stimulating discussions. One of us (A. E.) acknowledges the support provided by the Department of Physics and Astronomy of the University of Maryland and the Laboratory for Extraterrestrial Physics of the Goddard Space Flight Center during the summers of 1977 and 1978, when this work was performed.
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FIGURE CAPTIONS

Fig. 1. Schematic view of the stream interaction region showing the compression (shaded) and rarefaction produced as high velocity plasma (long arrows) overtakes the low velocity plasma. The view is onto the ecliptic plane from above the north pole. The spiral pattern of the magnetic field is also shown. For further details see Pizzo [1979a], from which this figure is adapted.

Fig. 2. Growth rate $\gamma$, and real frequency $\omega$, of the magnetized ion-ion instability (eq. 1.1) at 1.5 AU. In panel (a), $\omega$ and $\gamma$ are plotted against $\theta$, the angle between $k$ and the normal to $\mathbf{B}$, with $|k|$ held constant, and equal to $k_m$, the value which maximizes $\gamma(k)$. In panel (b), the wavenumber variation of $\omega$ and $\gamma$ are shown with $\theta = \theta_m$, the value which maximizes $\gamma(\theta)$. Note that the maximum value of $\gamma$ is relatively insensitive to variations in $\beta_e$, although the angular spread of $\gamma$ is greatly reduced as $\beta_e$ decreases. Because $\omega$ is relatively insensitive to variations in $\beta_e$, only $\omega(\beta_e=0.9)$ is plotted. In computing these values of $\omega$ and $\gamma$ we used $V_{SW}=300\text{ km s}^{-1}$ and 1 AU values of $T_i$ and $N$ equal to 2 eV and 7 cm$^{-3}$, respectively. The scaling laws defined in the text were employed to find $T_i$ and $N$ at 1.5 AU. In addition, we used $U=40 \text{ km s}^{-1}$, where $U \sin \gamma$ is the component of the velocity jump in $V_{SW}$ at the interface that is perpendicular to $\mathbf{B}$.

Fig. 3. The dashed curve represents a solution to (1.1) for the magnetized ion-ion instability in which the electron plasma dispersion function was approximated using the asymptotic expansion valid for large argument, or
equivalently, small electron thermal velocity. That approximation is seen to be valid only for very small angles ($\xi \approx 1^\circ$), at least for parameters typical of the solar wind. In this example, the 1 AU values of $U$, $V_{SW}$, $T_i$ and $\beta_e$ were 50 km s$^{-1}$, 300 km s$^{-1}$, 1.5 eV and 1.9, respectively. The solution was obtained at $R=2$ AU. $T_i(1$ AU) and $\beta_e(1$ AU) were found from the assumed radial dependence discussed in the text.

Fig. 4. A plot of $\omega_m$ and $\gamma_m$ as functions of heliocentric distance, with $V_{SW}=300$ km s$^{-1}$. $\gamma_m$ is the maximum growth rate of the magnetized ion-ion instability, maximized with respect to both $k$ and $\theta$; $\omega_m$ is the accompanying value of $\omega(k_m, \theta_m)$. Results are plotted using both $U=40$ and 50 km s$^{-1}$. Also plotted is $\omega_0(R)$, given by (1.2), and $n_i(R)$. Note that $\gamma_m > n_i$, justifying the assumption that the ions are unmagnetized. The largest values of $\gamma_m$ are found between 1 and 2 AU.

Fig. 5. The threshold conditions of the magnetized ion-ion instability plotted as a function of $U$ and $R$ for three different values of $V_{SW}$. The magnetized ion-ion instability is unstable for values of $U$ which lie above the curve corresponding to the choice of $V_{SW}$.

Fig. 6. The maximum value of the electric field in lower-hybrid waves excited by the magnetized ion-ion instability. Stabilization is assumed to occur by ion trapping. The largest values of $E$ are found between 1.5-2.5 AU. The curve was obtained using $V_{SW}=300$ km s$^{-1}$, and $U=50$ km s$^{-1}$.

Fig. 7. In panel (a), $\gamma(k, \theta_m)$, the growth rate of the modified two-stream
instability, is plotted as a function of k for $\beta_e=0.6$ and 1.1 at $\theta=\theta_m$. In panel (b), k is held constant at $k=k_m$, and $\gamma$ is plotted against $\theta$. In both panels $V_{SW}=450$ km s$^{-1}$, $R=1$ AU, $T_i=3.3$ eV, $B=5$ $\gamma$, and $N=7$ cm$^{-3}$. In addition, $\omega(k,\theta_m)$ and $\omega(k_m,\theta)$ are shown for $\beta_e=1.1$.

Fig. 8. Real and imaginary frequencies of the modified two-stream instability as a function of $V_d/V_i$. Solid lines represent the solution at 1 AU using $N=7$ cm$^{-3}$, and $(T_i)_\perp=3.3$ eV. Dashed lines represent the solution at 2 AU using $N=1.75$ cm$^{-3}$, and $(T_i)_\perp=1.3$ eV. In both cases $V_{SW}=450$ km s$^{-1}$. The solution for $\omega_m$ is shown for $\beta_e=1.1$ at 1 AU and $\beta_e=1.5$ at 2 AU, while $\gamma_m$ is also plotted for $\beta_e(1$ AU$)=0.6$ and $\beta_e(2$ AU$)=0.8$. The variation in $N$, $T_i$, and $\beta_e$ between 1 and 2 AU is a result of the scaling with heliocentric distance discussed in §1.

Fig. 9. Real and imaginary roots of the dispersion relation for the beam-cyclotron instability as functions of $kL_e$ (bottom scale), and $k\lambda_e$ (top scale). The parameters are characteristic of 1 AU ($N=7$ cm$^{-3}$, $B=5$ $\gamma$, $T_e/T_i=3$, $\beta_e=1.1$). In addition, $V_{SW}=450$ km s$^{-1}$ and $V_d/V_i=10$.

Fig. 10. Solution of the ion-acoustic dispersion relation assuming $V_d/V_i=20$ and $T_e/T_i=7$. As before $N=7$ cm$^{-3}$ and $V_{SW}=450$ km s$^{-1}$. 
STREAM INTERACTION

Figure 1
Figure 2
Figure 3
Figure 4

$u = 50 \text{ km/s}$

$u = 40 \text{ km/s}$

$\Omega_i$

$R(\text{AU})$
Figure 7
Figure 8