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PREDICTION OF THE AERODYNAMIC CHARACTERISTICS OF WINGS OF ARBITRARY PLAN FORM

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In our present effort to fly in and through the transonic-speed range we have resorted to widely diversified plan forms. Ranges of sweep, aspect ratio, and taper ratio are being considered which extend far beyond those considered practical several years ago, and the effect of wide variations in these parameters on the subsonic aerodynamic characteristics of wings is as yet largely unknown. The multiplicity of possible plan forms precludes an investigation of each experimentally. As a result, considerable effort has been directed towards developing a theoretical method of predicting the loading over the wing since, once this loading has been determined, not only can structural loads be estimated but values of the various aerodynamic characteristics such as lift-curve slope, aerodynamic-center location, and induced drag can also be found. The investigation of several theoretical methods for the prediction of loading and the application of one of these methods to a wide range of plan forms is the subject of this paper.

The theoretical methods studied were those developed by Falkner, by Mutterperl, and by Weissinger. In each of these methods the wing is replaced by a distribution of vortices. The strength distribution of these vortices is fixed by the boundary condition which requires that the induced velocities of these vortices produce no flow through the plane of the wing. The difference among the methods lies in differences in physical location of the vortices, in the disposition of the control points where the boundary condition is applied, and in the mathematical manipulation. Figure 1 compares the layout of vortices and control points for the methods. Falkner replaced the wing with a distribution of finite horseshoe vortices, both spanwise and chordwise. He likewise distributed the control points both spanwise and chordwise, hence his method is classified as a lifting-surface method. As a result of his work Falkner has recommended a particular vortex and control point distribution which was followed in our studies. In contrast to the Falkner lifting-surface method, both the Weissinger and the Mutterperl methods are lifting-line methods; that is, the loading is concentrated on the quarter-chord line, and the control points are distributed along the three-quarter-chord line. The Weissinger and Mutterperl methods differ in the spanwise location of the control points and in the mathematical development.

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Van Dorn and DeYoung have examined each of these methods for accuracy and ease of application (reference 1). The accuracy was evaluated by comparing the predicted and the experimentally determined aerodynamic characteristics of five wings having sweep angles ranging from -45° to 45°. By keeping an account of the time required, each method was also evaluated with regard to ease of application. The results of this study are shown in figure 1. From a comparison of the predicted and experimental values of spanwise loading, lift-curve slope, and spanwise center of pressure, the Falkner method was judged to be very accurate and the Weissinger method only slightly less accurate. The Mutterperl method, while predicting with moderate accuracy the characteristics of the sweptback wings, did not give accuracy comparable to other methods in the case of the sweptforward wings. The time required per solution was least for the Weissinger method (3 hr). In contrast 28 hours was required per solution for the Mutterperl method and 30 hours for the Falkner method.

On the basis of these results it was concluded that for a detailed study of a given plan form, where a high degree of accuracy was desired and where ease of application assumed lesser importance, the Falkner method was best. However, for a general study of a variety of plan forms the Weissinger method appeared best suited since good accuracy could be had at a 90 percent saving in computing time.

Accordingly we have utilized the Weissinger method to investigate the loading and associated aerodynamic characteristics of a wide range of plan forms. Figure 2 pictures the range covered but does not indicate the number of plan forms considered. Actually the characteristics of about 200 wings were calculated. The general range of variables included sweep from 45° forward to 60° back, aspect ratios from 1.5 to 8.0, and taper ratios from 0 to 1.5. The structural feasibility of the various shapes was used as a rough guide in selecting the limiting values of the geometric parameters.

The results of this investigation are found in reference 2. Charts presented in this reference allow a rapid and simple determination of the most important aerodynamic characteristics of any wing having a plan form falling within the range of this study. Aerodynamic characteristics which can be read directly from these charts include the span-loading coefficients, spanwise center of pressure, lift-curve slope, and aerodynamic center. These parameters are given as a function of sweep for families of aspect ratio and for various taper ratios. Sufficient values of aspect ratio and taper ratio were chosen to allow rapid and accurate interpolation.

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Sample charts taken from reference 2 are shown in figure 3. For the sake of clarity in this presentation, the data shown have been limited to one taper ratio, to a few aspect ratios, and, in the case of spanwise loading, to one spanwise station. In the reference paper, of course, data are given for a complete range of these geometric parameters. The simplicity of obtaining the desired characteristics is obvious. The chart showing the characteristics is entered at the proper value of sweep of the quarter-chord line and the desired value of the characteristics obtained directly. In this manner it is possible to obtain the wing-loading coefficient at four spanwise stations, the lift-curve slope, the spanwise center of pressure, and the aerodynamic center. Aerodynamic characteristics obtained from these charts have been correlated with experimental results, and in general the agreement is good. Strictly speaking, the method applies only at zero lift. However since the aerodynamic characteristics are in general linear up to angles of attack where separation occurs, the theoretically predicted characteristics can be used with good accuracy up to this point. Specific correlations of lift-curve slope and aerodynamic center measured at zero lift will be discussed later.

Most of the qualitative effects of sweep and taper ratio on span loading are not new and hence will not be discussed in this paper. However, one of the most interesting results of this investigation showed that, for each angle of sweep there is a taper ratio for which aspect ratio has little effect on the span loading and for which the span loading is practically elliptical. This relationship is shown in figure 4. As the wing is swept forward more inverse taper is required, and as the wing is swept back more of the usual type of taper is required. Because of the elliptic loading, minimum induced drag and maximum lift-curve slope are obtained for wings on this line. For plan forms falling on this line, aspect ratio had no effect on the loading. For plan forms above the line, loading moves outboard with increasing aspect ratio, and, conversely, for plan forms below the line, loading moves inboard with increasing aspect ratio.

Two of the characteristics found directly from the Weissinger method, lift-curve slope and aerodynamic center, are of particular value because of their importance in longitudinal stability analysis and design. Since they are so important the accuracy with which the Weissinger method predicts these characteristics and the effects of plan form on these characteristics as predicted by the Weissinger method should be examined.

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In figure 5 the theoretical and experimental values of lift-curve slope are correlated for a number of random plan forms. Where sufficient clearance between points existed, the wing plan forms have been superimposed. Included in this correlation are triangular wings, highly sweptback wings, sweptforward wings, and wings with inverse taper. Most of the experimental data were taken from reference 3 and the remainder from other American papers. Deviation from the 45° line indicates the error of correlation. On the average, this deviation is less than 3 percent. Although not shown herein, we have also compared the lift-curve slopes of unswept wings as estimated by the Weissinger method and by the method employing the modified Jones' edge-velocity correction, and the agreement is nearly perfect.

A similar correlation for aerodynamic center is given in figure 6. Experiment and theory do not show as good agreement for this parameter as for the lift-curve slope. There is no clear systematic variation in the correlation with plan form, and most of the discrepancies in correlation are of the order of the accuracy with which the aerodynamic center usually can be determined by experiment. In any event, for 75 percent of the plan forms the discrepancy is less than 2 percent of the M.A.C., which discrepancy is small compared to the effects of plan form. It is our belief that the Weissinger method gives both lift-curve slope and aerodynamic center with sufficient accuracy for use in preliminary design studies.

Sample charts of lift-curve slope are shown in figure 7. Lift-curve slope is given as a function of sweep for taper ratios of 0, 0.5, and 1.5 and for aspect ratios of 1.5, 3.5, 8, and ∞. Many of the curves have again been omitted for clarity. The curve for infinite aspect ratio in each case is obtained from simple sweep theory and hence is a cosine curve. Note that in each case the effect of aspect ratio falls off with increase in sweep. Also note that for low aspect ratios, small angles of sweep have little effect on lift-curve slope. As the wing approaches a more pointed plan form, the lift-curve slope increases on sweptback wings and decreases on sweptforward wings, while inverse taper reduces lift-curve slope on sweptback wings and increases it on sweptforward wings. On highly swept wings this effect is of such magnitude that taper ratio exerts as great an influence on lift-curve slope as does aspect ratio. Thus in any theoretical approach the importance of including the effects of taper ratio is obvious.

Figure 8 presents sample charts of the aerodynamic center which is also plotted as a function of sweep for taper ratios of 0, 0.5, and 1.5 and for aspect ratios of 1.5, 3.5, and 8.0. For the plan forms investigated, the aerodynamic-center location ranged from as

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far forward as 15 percent of the M.A.C. to as far back as 40 percent of the M.A.C. In contrast, the aerodynamic center on unswept wings is seldom more than 2 or 3 percent of the M.A.C. from the 25 percent M.A.C. point. On highly tapered wings sweepback moves the aerodynamic center rearward and sweepforward moves it forward. As taper is decreased, the trend is reversed so that, on wings having inverse taper, sweepback moves the aerodynamic center forward while sweepforward moves the aerodynamic center back. The magnitude of this movement in each case is generally increased by increase in aspect ratio.

As stated earlier, lift-curve slope and aerodynamic-center location are important in longitudinal stability analysis. That the Weissinger method can predict values of these characteristics with sufficient accuracy for preliminary design has been shown. One other parameter must however be evaluated to complete the longitudinal stability analysis, and that parameter is the downwash in the location of the tail. The Weissinger method can readily be extended to compute downwash in the plane of the vortex sheet. We are at present evaluating the accuracy of the downwash results obtained in this manner by checking them with experimental data. Preliminary results of such an evaluation are given in figure 9. The maximum downwash angles as predicted and measured in a vertical plane approximately 30 percent of wing semispan out from the plan of symmetry are shown as a function of angle of attack for five swept wings tested in the Ames 40- by 80-foot tunnel. For these wings, theory predicts the variation of downwash with angle of attack within 20 percent of the measured value. Insufficient comparisons have been made to date, however, to warrant generalizations as to the accuracy of this method for a wide range of plan forms. In our present investigation we plan to establish this accuracy, improve the method where possible, and then extend the method so that downwash may be determined at points above and below the vortex sheet.

All the results obtained through use of the Weissinger method apply only to incompressible flow. However, through an application of the Prandtl-Glauert rule, it is possible to account for the effects of compressibility on span-loading characteristics for speeds below the critical speed. The method, which has been summarized in some detail in reference 4, translates the effect of compressibility into an effective change in plan form in addition to the well-known increase in section pressures.

It is apparent that, if such an approach serves to predict accurately the effects of compressibility, it can be used in conjunction with the subject paper to give a rapid estimation of the characteristics of wings throughout the Mach number range below the

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critical speed. At each Mach number the geometry of the wing would simply be distorted in the proper manner and new characteristics obtained from the charts. Only a few random experimental checks of this procedure have been made, but these comparisons have indicated moderately good agreement between theory and experiment. It is our plan to continue this study to establish the accuracy of the method, and, if necessary, search for means of improving the accuracy.

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3. Shortal, Joseph A., and Maggin, Bernard: Effect of Sweepback and Aspect Ratio on Longitudinal Stability Characteristics of Wings at Low Speed. NACA TN No. 1093, 1946.

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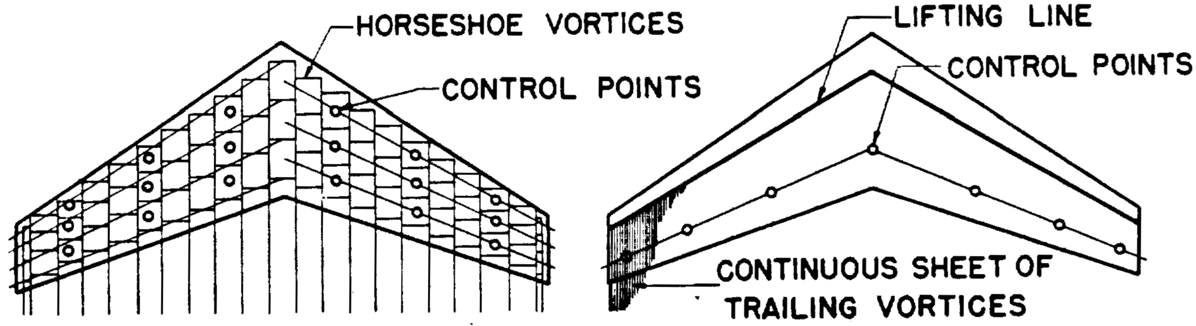

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	FALKNER	MUTTERPERL	WEISSINGER
ACCURACY	VERY GOOD	VARIABLE	GOOD
TIME	30 HR	28 HR	3 HR

Figure 1.- Comparison of theoretical methods for determining loading.

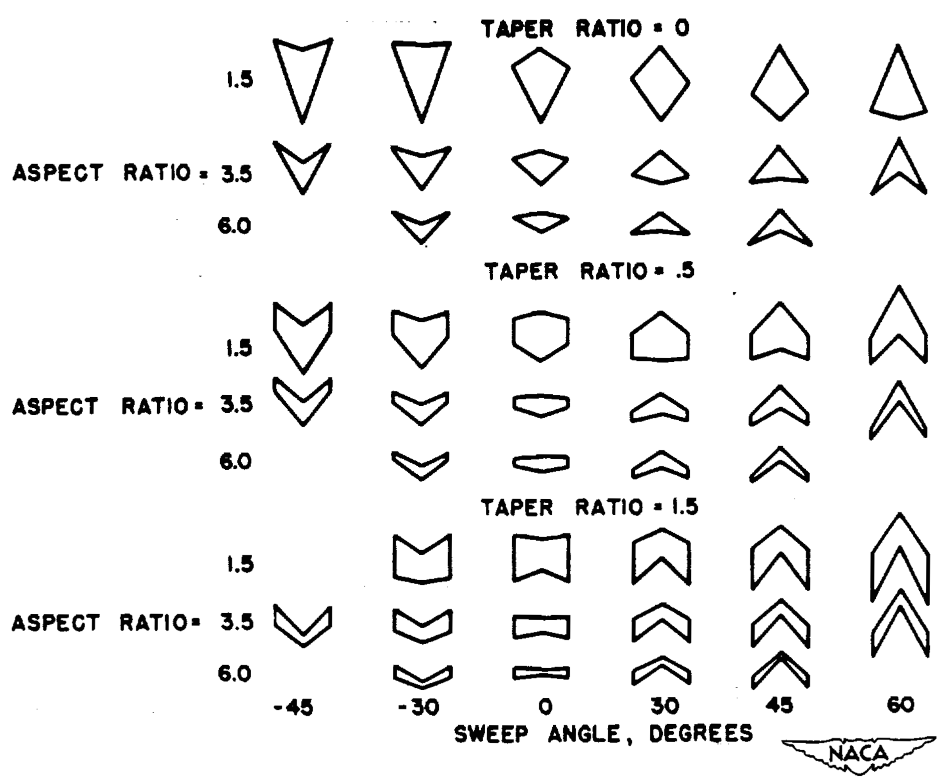

Figure 2.- Range of plan forms investigated by Weissinger method.

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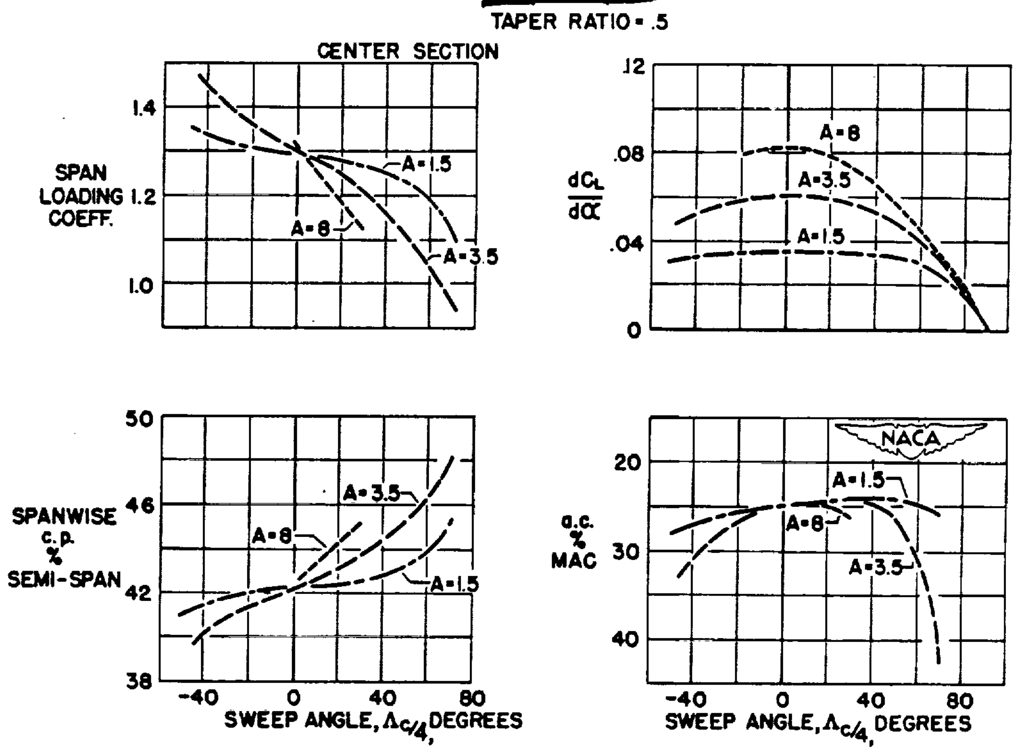

Figure 3.- Sample charts of data obtained by Weissinger method.

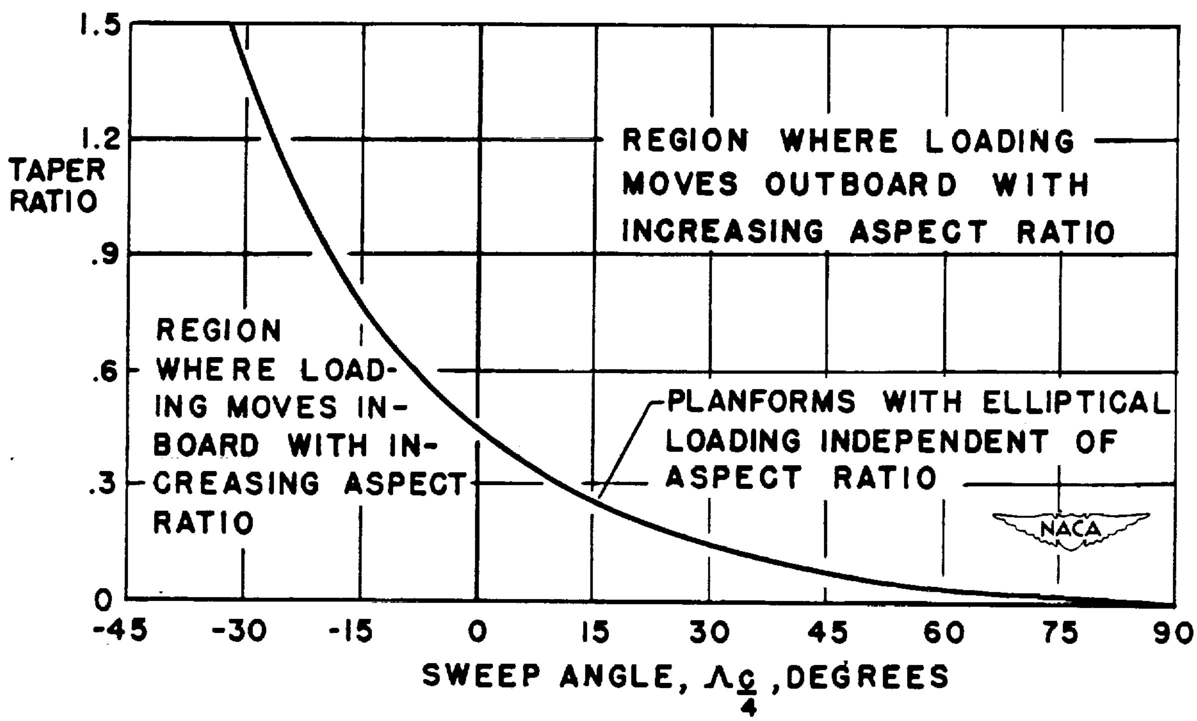

Figure 4.- Taper ratio - sweep angle relationship giving elliptic loading.

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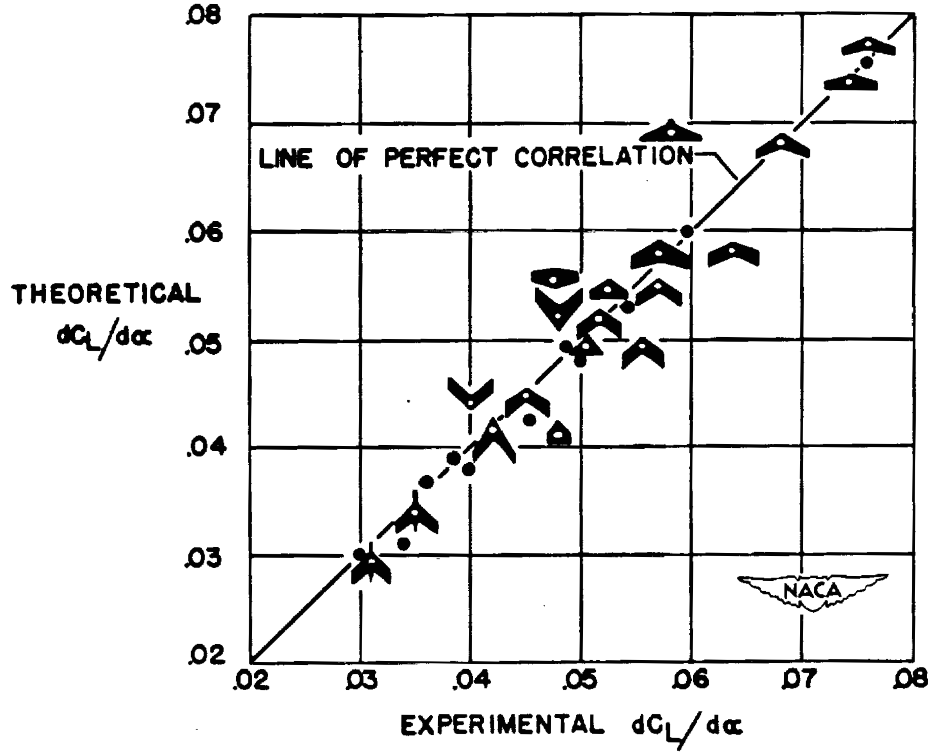

Figure 5.- Correlation of theoretical and experimental values of lift-curve slope.

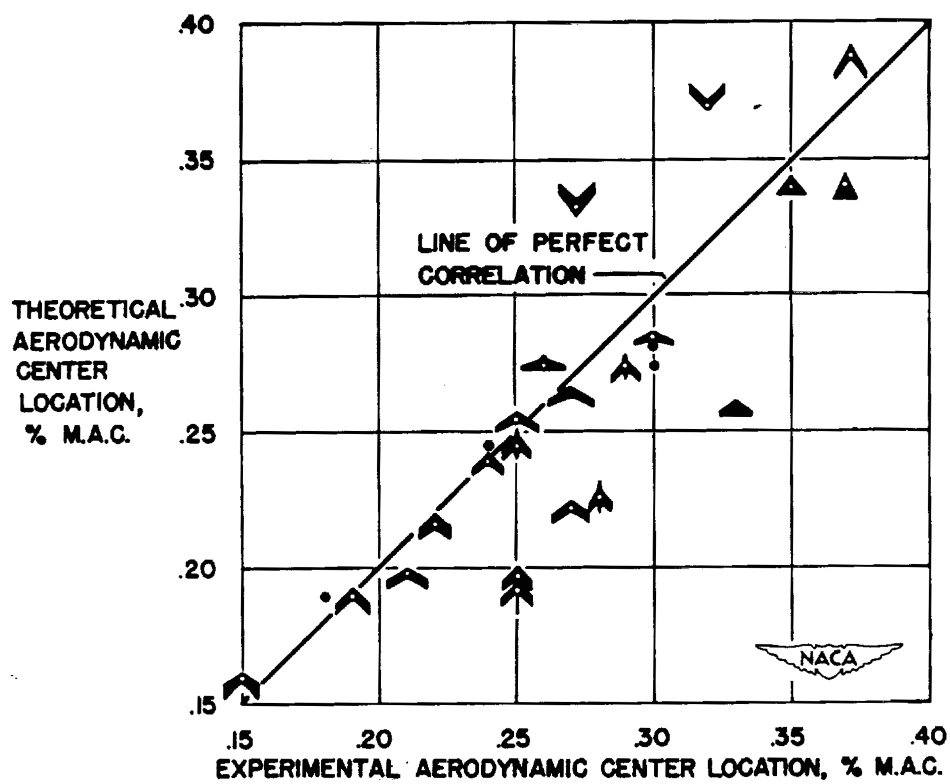

Figure 6.- Correlation of theoretical and experimental locations of the aerodynamic center.

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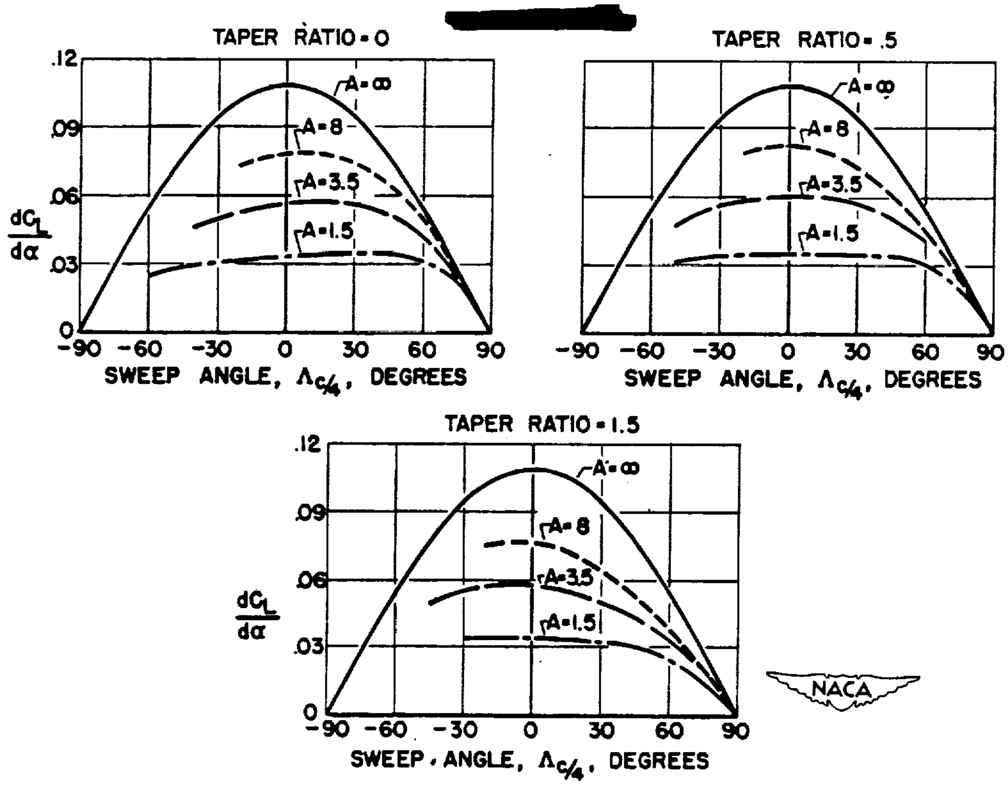

Figure 7.- Effect of sweep, taper ratio, and aspect ratio on lift-curve slope.

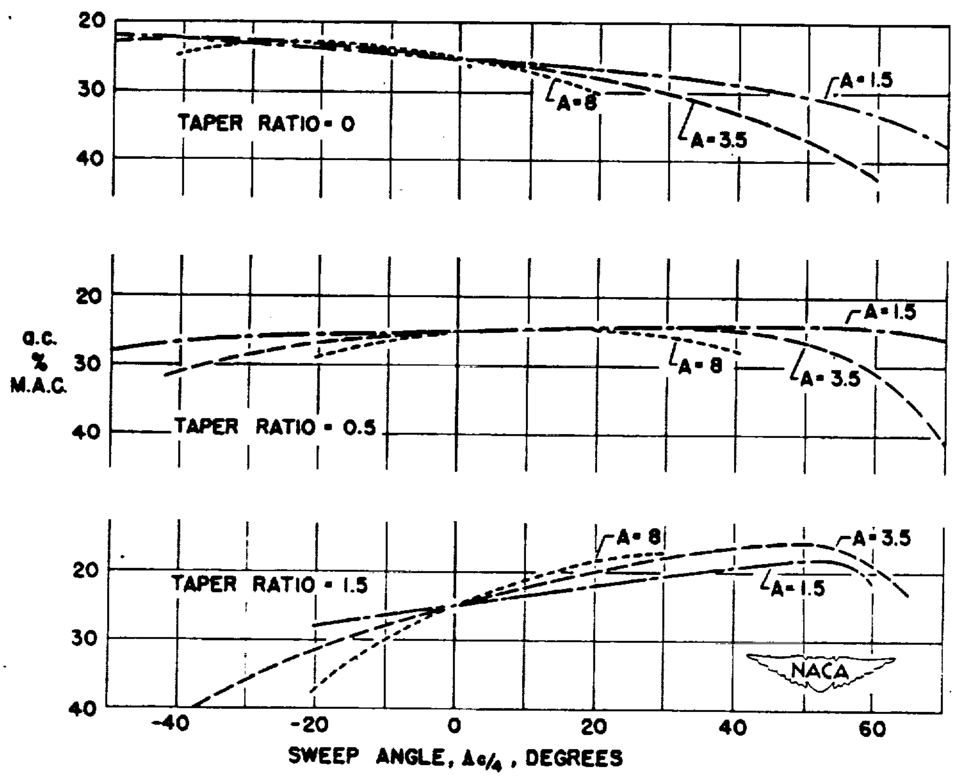

Figure 8.- Effect of sweep, taper ratio, and aspect ratio on aerodynamic-center location.

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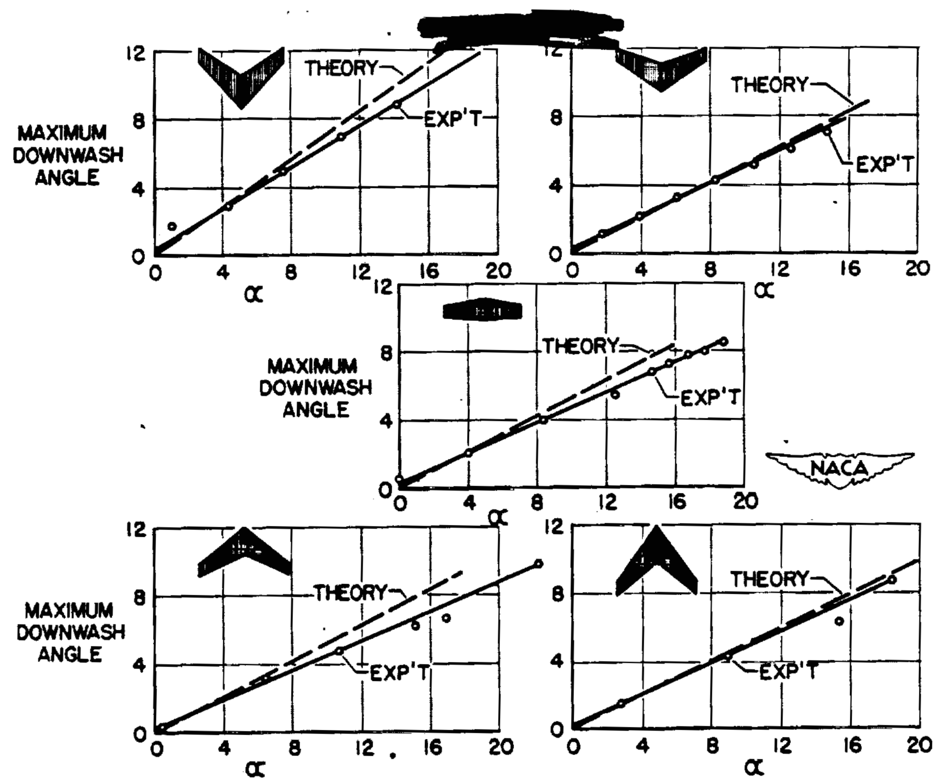

Figure 9.- Comparison of theoretical and experimental values of downwash.

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