WIND-TUNNEL-WALL CORRECTIONS

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BOUNDARY CONDITIONS

When the wind tunnel was first developed as a practical approach to experimental aerodynamics, it was recognized that the flow about a body in a wind tunnel was not the same as the flow about the same body in flight. Since that time, mainly during the past 30 years, there has appeared a steady stream of research papers, some offering improvements in recognized corrections in keeping with the improvements in wind-tunnels, equipment, techniques, and general understanding of aerodynamics and others deriving necessary corrections for new types of aerodynamic configurations or new types of measuring techniques.

The problem arises from the fact that, although the differential equations of the flow are the same in the tunnel as in flight, the outer boundary conditions are different. In flight, the condition is simply that the flow at infinity is uniform; in the tunnel, certain other conditions, depending on the type of tunnel, must be satisfied at the tunnel boundaries. For the closed tunnel, the condition is obviously that the velocity component normal to the wall be zero. For the open tunnel, where the jet traverses a region of comparatively quiescent air, the condition is that the pressure at the boundary be uniform. By Bernoulli's law, it follows that the tunnel velocity must be uniform on the boundary. If this velocity is considered as the sum of the undisturbed tunnel velocity U and a small perturbation velocity (u, v, w) resulting from the presence of the body in the jet, the condition is then that $(U + u)^2 + v^2 + w^2 \approx U^2 + 2Uu$ be constant, from which it follows that u is constant over the entire surface. Furthermore, since u is obviously zero far in front of the body, it must be zero over the entire surface, whence it can be easily shown that the perturbation potential itself is constant over the entire surface. The somewhat obvious condition that the perturbation velocity (u, v, w) is zero far in front of the body may need special emphasis; neglect of this condition has in the past sometimes led to erroneous results (reference 1).

Some attention has been directed recently to a third case; namely, that of an open tunnel in which the body is so far forward in the jet that the presence of the closed entrance bell cannot be neglected. This case involves a mixed-boundary-value problem in which the normal velocity is zero on the closed portion of the boundary and the longitudinal perturbation velocity u is zero (or constant) over the open portion of the boundary. An interesting further boundary condition arises here, namely, that the flow velocities be continuous at the edge of the entrance bell. This condition is similar to the Kutta condition at the trailing edge of an airfoil. It arises because of the finite viscosity of air, and it provides uniqueness where otherwise an infinity of solutions would exist.

BASIC VIEWPOINT

The approach to the problem usually follows a fairly well defined pattern, although variations are sometimes necessary. In general, no effort is made to predict the complete flow and the corresponding aerodynamic characteristics for the model in the tunnel. These are normally measured by the wind-tunnel survey apparatus, the wind-tunnel balances, or other measuring equipment. The usual problem is rather to determine, primarily, by how much the presence of the tunnel boundaries modifies the "free-stream" flow at the model location and, secondly, by how much the model characteristics are altered by this flow modification.

The mathematical approach, for example, is first to assume within the model a set of singularities - sources, sinks, doublets, vortices - that, on the basis of model geometry, air-flow measurements, force measurements, and any other sources of information, are believed to effectively represent the contribution of the model to the flow field. These singularities induce a field that, in general, violates the desired condition at the tunnel boundary. An additional potential flow is now sought, having singularities only on or outside the tunnel boundary, such that when it is added to the field of the model, the desired boundary conditions will be satisfied. This additional flow is called the tunnel interference flow. Its determination and, in particular, its evaluation in the neighborhood of the model constitutes the previously mentioned primary problem. Vertical components of this additional flow are normally interpreted (after division by the main tunnel velocity) as a correction to the local flow angle; horizontal components are normally interpreted as a correction to the tunnel velocity.

In figure 1 is indicated an airplane model in a closed wind tunnel, together with several of the more important components of the interference flow. Associated with the lift of the model is a strong downflow of the air behind it; and the corresponding tunnel interference flow is essentially an upflow which neutralizes the downflow at the walls and, in the neighborhood of the model, introduces the upflow velocities indicated in the figure. The upflow velocity has a certain value near the wing, rapidly approaches twice this value behind the wing, and rapidly approaches zero in front of the wing. Since the lift of an airfoil section in a curved flow is determined roughly by the angle of attack as measured at the three-quarter-chord point, the upflow at the three-quarter-chord line is used to correct the angle of attack of the airplane. Since the lift itself (or the bound vorticity) is centered about the quarter-chord line, however, the drag correction is determined from the product of the lift and the upflow velocity at the quarter-chord line. This flow curvature is effectively an induced camber of the wing and results in a corresponding change in the wing moment and in its maximum lift coefficient. Since the upflow at the tail is greater than that at the wing three-quarter-chord line, the difference must be applied as a correction to the stabilizer setting or to the downwash angle. A correction would also be applied for the additional moment of the fuselage caused by its presence in a curved flow field.

Because the tunnel walls prevent the normal outward displacement of the streamlines about the model, there is a corresponding effective increase of the airspeed in the neighborhood of the body (constriction effect), indicated by the horizontal vector at the left of figure 1. If the drag of the model becomes fairly high, as in tests with extended flaps or at supercritical speeds, a large wake of slowly moving air exists downstream of the model, and the surrounding air of the main stream experiences a corresponding velocity increase that persists far behind the model (indicated by the horizontal vector at the right of the figure). Somewhat over half of this increase is considered to apply in the neighborhood of the model itself, in addition to the normal constriction effect due to the volume of the body; the sum is indicated by the horizontal vector near the center of the figure. Associated with the longitudinal increase of velocity along the model resulting from the wake, there is a decrease of stream static pressure toward the rear of the model. Corresponding to this effect is a longitudinal buoyancy force, roughly equal to the product of the model volume and the pressure gradient, which should be applied as a correction to the drag. Normally, however, this last correction is fairly small, and it may be noted, in any case, that if this longitudinal pressure gradient is large enough to cause a fairly large correction, it may also appreciably affect the flow phenomena, such as separation, associated with the high drag.

METHODS OF SOLUTION

Almost any interference problem for two-dimensional closed tunnels can be solved by complex-variable methods. The interference is merely the field of the system of mirror images of the model extending to infinity above and below the model. If the model can be considered as adequately represented by several simple singularities - for example, a doublet and a vortex - the interference field is simple to compute since the flow fields for infinite rows of such singularities are given by relatively simple expressions (references 2 and 3). For the exact solution of an airfoil in a closed tunnel, modern cascade theory provides applicable methods (reference 4). Corresponding solutions for an open two-dimensional tunnel (that is, a tunnel with vertical walls, but open at the top and bottom) can be similarly derived. Solutions for singularities in the open tunnel with closed entrance and exit regions are also readily possible (references 5 and 6). In all such solutions for an open tunnel, however, it is assumed that the tunnel boundary is not appreciably deformed by the singularities within the jet. Various experimental results indicate that this assumption introduces no significant error in the interference flow near the airfoil but may lead to some error in the region behind the airfoil (references 7 and 8). Exact solutions, taking into account the boundary deformation, have been obtained for special cases (reference 9); in general, however, the deformation is not considered.

For three-dimensional tunnels the problem is much more difficult. For a small-chord, unswept, and unyawed wing, however, the interference at the wing can be readily shown to reduce to a two-dimensional flow problem - that of a vortex within a contour having the shape of the tunnel cross section and on which the normal or the tangential velocity is zero for the closed or the open tunnel, respectively. Many interesting two-dimensional problems of this nature have been solved by complex-variable methods (for example, references 10 to 12). For the interference at swept or yawed wings, or for the problem of corrections to the downwash angle at the tail, no similar simplification is possible.

For rectangular tunnels with closed, open, or partly open cross sections, solutions can be obtained by the method of images in which the interference field is that due to the doubly infinite array of mirror images of the model reflected in the tunnel walls (reference 13). The infinite summation can generally be readily approximated with adequate accuracy.

For singularities within circular tunnels, solutions can be found by expansions in Bessel functions (references 14 to 16); either the open or the closed tunnel, or the open tunnel with closed entrance and exit regions, can be treated in this way (reference 17). Solutions for elliptical tunnels are found in terms of Mathieu functions (reference 14).

For tunnels of other cross-section shapes, as the NACA full-scale tunnels or the octagonal tunnels, results for the nearest rectangle or the nearest ellipse or, perhaps, an average of the results for the nearest rectangle and the nearest ellipse may be used. An indication of the accuracy of such an approximation (and also some indication of the direction in which further modification might be made) can be found by comparing the estimated interference flow at an unswept lifting line with that for the true shape (which, as previously mentioned, can be rigorously solved as a two-dimensional problem).

It may also be mentioned that solutions of the boundary-value problems that arise in the study of tunnel interference can be found by electrical-analogy methods (references 18 and 19) or by empirical comparisons between the characteristics of the model in the tunnel and those of the same model in a tunnel that is so large relative to the model that interference is negligible (reference 7).

It is not possible in the present paper to describe in further detail any of the solution procedures that have just been mentioned or the analytical studies that have been made of the reaction of the model to the interference flows (for example, reference 20). Instead, in the remainder of the paper are discussed several problems that may be of interest to those currently associated with wind-tunnel laboratories, namely, tunnel interference for swept wings, compressibility corrections, and choking.

TUNNEL INTERFERENCE FOR SWEPT WINGS

It might be supposed that, in order to be prepared with tunnel-interference calculations for any swept wing that might be proposed for test in a given tunnel, calculations would be needed for a series of wings having a range of sweep angles and a range of spans - that is, a two-parameter set of calculations. Actually, however, such extensive calculations are quite unnecessary, at least for rectangular tunnels. Consider the sweptback wing (yawed for greater generality) shown at the top center of figure 2. Associated with some point concentration of lift on the wing is a horseshoe vortex of zero span (that is, a doublet line) extending downstream to infinity from the point. The lower part of figure 2 shows the rear view of the wing in the tunnel, together with the doublet line and the image system of tunnels and doublet lines. The doublets are marked plus or minus according as they are the same as or opposite to the wing doublet. Examination of the doublet system shows that it is composed of two superimposed lattices, one of which is indicated by circles and the other, by squares. The vertical spacing in each lattice is equal to the tunnel height; the lateral spacing in each is equal to twice the tunnel width. The two lattices are thus identical and, furthermore, are determined only by the tunnel dimensions and not by the location of the lifting element in the tunnel. Accordingly, once the field of such a lattice has been calculated for the horizontal center plane of the tunnel, it can be used for determining the complete flow field regardless of the location of the lifting element. The interference flow field for the given lifting element is found by subtracting from the field of the two complete lattices the field of the single doublet that trails from the lifting element itself. Finally, by repeating the indicated procedures for a series of lifting elements on the wing, distributed according to the estimated wing lift distribution, the net tunnel interference is obtained.

Contour charts of the vertical component of the flow in the field of the lattice have been prepared for several NACA tunnels, including the 7- by 10-foot tunnels.

This procedure would not apply to nonrectangular tunnels. For circular tunnels, the NACA has published fairly complete interference fields for lifting lines of various spans and various sweep angles (reference 15). The sweep angles do not exceed 45°; however, it should be pointed out that, when necessary, interference calculations for any sweep angle can be used for any other sweep angle. This fact follows from the observation that a reasonably rough approximation to the wing loading is generally adequate for predicting tunnel interference; and the procedure is illustrated in figure 3. In the left half of the figure is shown how the loading on a

60° swept wing may be approximated by a single horseshoe vortex and two pairs of unswept horseshoe vortices, where the inner vortex of each pair has the same strength as the superimposed outer vortex but has opposite rotation. In the right half of the figure is shown similarly how a pair of horseshoe vortices and a single horseshoe vortex, all swept 45°, might be used for the same purpose.

FIRST-ORDER COMPRESSIBILITY CORRECTIONS

Consider a streamline object (fig. 4, upper left) in the (x, y, z) space, to be flown or tested at Mach number M and velocity U. It is desired to predict the perturbation velocities (u, v, w) at various points on the object or in the field about the object. According to the Glauert-Prandtl method, which takes into account only the first-order compressibility effects, the procedure for predicting the perturbation velocities involves the three following steps. (A short derivation of this procedure is given in the appendix. See also references 21 to 23.)

1. An object is constructed in the x', y', z' space that is related to the physical object according to the relations

$$x' = \frac{x}{\sqrt{1 - M^2}}$$

$$y' = y$$

$$z' = z$$

Essentially, this corresponds merely to a longitudinal stretching of the object by the factor $\frac{1}{\sqrt{1 - M^2}}$. For the model indicated in the figure, the fineness ratio of the fuselage, the wing chord, and the sweepback angle are increased by this stretching; the aspect ratio, the wing thickness ratio, and the angle of attack are reduced. If the model is in a tunnel, the cross section of the tunnel remains unchanged.

2. The incompressible flow about this elongated body is found. Specifically, the perturbation velocities u', v', w' on, or near, the object are found for an incompressible flow of stream velocity U. The problem of determining this flow may, of course, be quite difficult; however, since it is an incompressible-flow problem, it can presumably be solved by known methods.

3. The desired perturbation velocities u, v, w in the desired compressible flow are related to the perturbation velocities u', v', w' in the incompressible flow about the elongated object at corresponding points by the following equations:

$$u = \frac{u'}{1 - M^2}$$

$$v = \frac{v'}{\sqrt{1 - M^2}}$$

$$w = \frac{w'}{\sqrt{1 - M^2}}$$

To within the accuracy of the first-order approximation, this procedure applies for determining velocities on the object in flight or in the tunnel, or for determining tunnel interference velocities. In particular, constriction corrections are found by first determining the constriction effect in the x', y', z' space and then multiplying by $\frac{1}{1 - M^2}$. Angle-of-attack or downwash-angle corrections are found by first determining the correction in the x', y', z' space and then multiplying by $\frac{1}{\sqrt{1 - M^2}}$. The measured lift multiplied by $\sqrt{1 - M^2}$ gives the value of the lift that should be assumed for the incompressible flow in the x', y', z' space. Because the aerodynamic characteristics of the elongated object, in general, may bear no simple relation to those of the actual object in low-speed flight, combining the preceding three steps into a simple formula for the "compressibility effect" on tunnel interference is not possible for most cases. The constriction effect on short objects, however, does permit such a simple correction formula.

Consider an airfoil in a two-dimensional closed tunnel. It is roughly represented by a source-sink body on the left side of figure 5, where are also shown the nearest images. The constriction effect is merely the velocity contributed by these images in the region of the body. For incompressible flow, the constriction of the first upper image is indicated by the velocity vectors shown. The lower vector is due to the source at the nose of the image; the upper vector is due to the sink at the rear of the image; and the short horizontal vector is the resultant.

A similar construction applies to all the other images. Now, if the constriction effect at some Mach number M is desired, it is first necessary to construct an elongated body and determine its interference in incompressible flow. Examination of the right side of figure 5 shows that, if the body is elongated by $\frac{1}{\sqrt{1 - M^2}}$, the constriction velocity due to the first image is roughly $\frac{1}{\sqrt{1 - M^2}}$ as much as before, and similarly for the constriction velocity due to all the other images. If now, according to step 3 of the indicated procedure, this increase is multiplied by $\frac{1}{1 - M^2}$, it follows that the constriction effect for a reasonably short body in the tunnel varies as $\frac{1}{(1 - M^2)^{3/2}}$. Furthermore, although the preceding derivation was for an airfoil in a two-dimensional tunnel, it can be readily seen that the identical derivation method and final formula would apply for the open two-dimensional tunnel or for a body in a three-dimensional tunnel, either open or closed (reference 24).

From considerations of the field of the airfoil and its images in the two-dimensional closed-tunnel case, a simple rule can be derived for the body-constriction effect in the absence of an appreciable wake, namely, that the constriction effect at the airfoil is one-third the total velocity increase at the wall opposite the airfoil. This rule, which applies for both compressible and incompressible flow, provides a means of estimating the constriction effect from simple pressure measurements at the wall. For bodies in three-dimensional tunnels the factor is about one-half.

CHOKING

The choking speed of a closed tunnel containing a model is that speed for which the passage around the model serves roughly as a sonic throat and prevents further increase of the flow. Although all the flow in this minimum section may not be precisely at sonic speed, the choking speed is usually fairly accurately predicted, on the basis of the one-dimensional flow equations, from the ratio of the tunnel cross-sectional area to the minimum cross-sectional area of the passage around the model. After this condition has been reached, any further reduction of the back pressure results merely in an increase in the extent of the supersonic flow region just after the minimum without increasing the flow quantity or the upstream Mach number. Any measurements made under such conditions will obviously bear no relation to the characteristics of the model in flight. The question still remains, however, as to whether results obtained just at choking are meaningful, or, if not, what is the highest Mach number for which meaningful results can be obtained. Certain investigators have concluded that tunnel Mach numbers should not be closer than 0.02 to 0.03 to the choking Mach numbers; others, by comparing results for models of different size in the same tunnel, have concluded that the safe margin is 0.04 to 0.05, depending on the model size (reference 25); still others have concentrated on the study of constriction effects almost up to choking itself, presumably with the hope of using the measurements made under such conditions. A review of these studies seems to indicate some variations among the types of results obtained in the different tunnels. Possibly the differences are related to the differences in relative boundary-layer thicknesses on the tunnel walls; in any case, it seems desirable, for the present, that further studies be made in the different wind tunnels where the problem arises.

Figure 6 illustrates the nature of the phenomena observed. Several 5-inch-chord airfoils were mounted across the Langley 24-inch high-speed tunnel and pressures were measured on the wall opposite the models (reference 26). On the left side of the figures, these pressures, interpreted in terms of local wall Mach number, have been plotted against distance along the wall for several tunnel indicated Mach numbers. It can be seen that the constriction effect is quite small at $M_{indicated} = 0.602$ but begins to become appreciable at 0.705. At higher Mach numbers it becomes quite large and, in addition, the wake constriction effect becomes very large (indicated by the fact that the wall Mach number downstream of the model never returns to the wall Mach number upstream of the model). Finally, just before choking, the peak Mach number rises very rapidly toward 1.0. On the right side of figure 6, the peak Mach number at the wall has been plotted against tunnel indicated Mach number in order to show more clearly how rapidly the peak Mach number rises just before the tunnel chokes.

In the case of the lifting airfoil (fig. 7, left side), a variation of the choking problem arises. The stagnation streamline effectively splits the flow into two parts which pass, respectively, above and below the airfoil. The distribution of cross-sectional areas, generally, is such that choking of the upper passage, in the region just above the airfoil leading edge, occurs before choking of the lower passage. In this case, the tunnel flow quantity can continue to increase until the lower passage is also choked, although, obviously, any data obtained in this flow regime bears no relation to the true airfoil characteristics. It is therefore desirable to determine, by some means other than observation of the tunnel indicated Mach number, the existence of a choked condition in the upper passage. Pressure orifices on the wall opposite the model should be useful to detect the approach of choking, as shown in figure 6. It may also be possible to compute the streamline pattern by the method previously discussed (indicated on the right of fig. 7) - the airfoil is considered to be elongated in the stream direction by the factor $\frac{1}{\sqrt{1 - M^2}}$ and the incompressible flow pattern about this airfoil is determined. The area ratios above the stagnation streamline in this flow should apply to the compressible flow. Determination of the location of this streamline involves the solution of the flow in the infinite double cascade of airfoils consisting of the airfoil and all its mirror images. (The cascade is referred to as "double" because it consists of two superimposed cascades, one containing airfoils at a positive angle of attack and one containing airfoils at a negative angle of attack.) Although modern cascade theory can provide exact solutions to this flow, an approximate solution, such as that calculated in reference 27, should be satisfactory for this purpose.

Unsymmetrical choking of a type similar to that just discussed is a basic characteristic of any test setup in which the model supports extend below the model to the floor of the tunnel. The normal slight asymmetry introduced by such supports at low speeds becomes progressively more pronounced as the Mach number increases, and, finally, choking occurs in the region between the supports or perhaps in most of the region below the wing. Such a support system therefore becomes quite unacceptable at high speeds, and other arrangements have accordingly been developed. In one of these, a half-span model is mounted from the tunnel wall or, to avoid the thick wall boundary layer, from a plate in the center of the tunnel. In another arrangement, the complete model is supported from a sting at the rear.

The use of an open instead of a closed tunnel is also of interest with regard to choking (reference 28). At the lower speeds, the tunnel constriction effect is, in any case, about half as much as for a closed tunnel (and of opposite sign); and at very high speeds it offers the advantages that the wake constriction effect is inappreciable and that choking in the sense previously described cannot occur. The disadvantages of the open tunnel are, of course, the greater flow irregularity and the lower energy ratio, as compared with the closed tunnel.

APPENDIX

THE PRANDTL-GLAUERT METHOD FOR THREE-DIMENSIONAL FLOW

A brief derivation of a form of the Prandtl-Glauert method, correct for three dimensions, may be given as follows: A first-order approximation to the subsonic compressible flow about a thin body B, the surface of which has the equation

$$S(x, y, z) = 0$$

may be obtained by finding a solution of the linearized differential equation for the potential φ of the incremental velocities,

$$\beta^2\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \qquad \text{(A1)}$$

where the x-axis is in the stream direction and the incremental velocities φ_x, φ_y, and φ_z are small compared with the stream velocity U. At all points on the surface of B, the potential φ must satisfy the boundary condition

$$\left(U + \varphi_x\right)S_x + \varphi_y S_y + \varphi_z S_z = 0 \qquad \text{(A2)}$$

which states that the flow is tangential to B. Since B is assumed thin, S_x is small compared with S_y and S_z; consequently, the second-order term $\varphi_x S_x$ may be neglected, and the boundary condition becomes

$$US_x + \varphi_y S_y + \varphi_z S_z = 0$$

In order to solve the boundary-value problem given by equations (A1) and (A2) in terms of incompressible flow, the following transformation of variables is used

$$\left.\begin{aligned} x' &= \frac{x}{\beta} \\ \varphi' &= \beta\varphi \end{aligned}\right\} \qquad \text{(A3)}$$

Under this transformation, equations (A1) and (A2) become, respectively,

$$\varphi'_{x'x'} + \varphi'_{yy} + \varphi'_{zz} = 0 \qquad \text{(A4)}$$

$$US_{x'} + \varphi'_y S_y + \varphi'_z S_z = 0 \qquad \text{(A5)}$$

Equations (A4) and (A5) are, respectively, the differential equation and boundary condition for the potential φ' of the incremental velocities of an incompressible flow with free-stream velocity U, in the x', y, z space, about a thin body B', the surface of which has the equation

$$S(\beta x', y, z) = 0$$

The incremental velocities in the compressible flow are thus given by

$$u = \varphi_x = \frac{1}{\beta^2} \varphi'_{x'} = \frac{1}{\beta^2} u'$$

$$v = \varphi_y = \frac{1}{\beta} \varphi'_y = \frac{1}{\beta} v'$$

$$w = \varphi_z = \frac{1}{\beta} \varphi'_z = \frac{1}{\beta} w'$$

where u, v, and w and u', v', and w' are the incremental velocities at corresponding points in the compressible flow about B and the incompressible flow about B', respectively.

The foregoing analysis establishes the Prandtl-Glauert method for three-dimensional flow in the following form: The incremental velocities at a point P on the surface of a thin body B in compressible flow may be obtained in three steps:

(1) The x-coordinates of all points of B are increased by the factor $1/\beta$, where

$$\beta = \sqrt{1 - M^2}$$

and where the x-axis is in the stream direction. This transformation changes B into a stretched body B'.

(2) The incremental velocities u', v', w', in the direction of the x-, y-, and z-axes, respectively, at the point P' on B' corresponding to the point P on B are calculated as though B' were in an incompressible flow having the same free-stream velocity as the original compressible flow.

(3) The values u, v, and w of the incremental velocities at the point P on the original unstretched body B in compressible flow are then found by the equations

$$u = \frac{1}{\beta^2} u'$$

$$v = \frac{1}{\beta} v'$$

$$w = \frac{1}{\beta} w'$$

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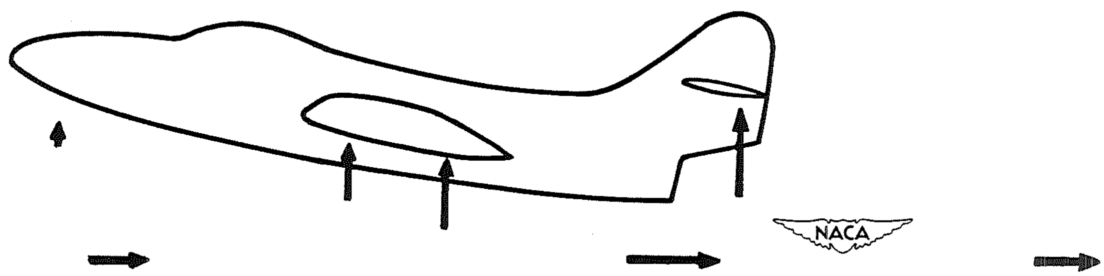

Figure 1.- Airplane model in closed wind tunnel. Several of more important components of interference flow shown.

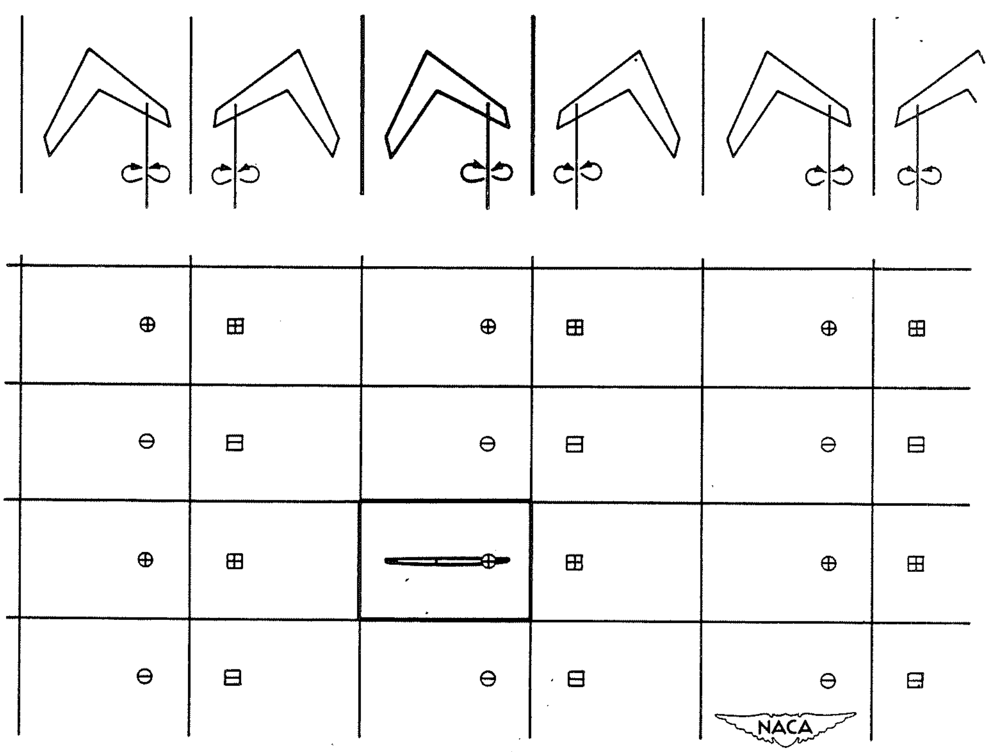

Figure 2.- Image system of doublets for a lifting element in a closed rectangular wind tunnel.

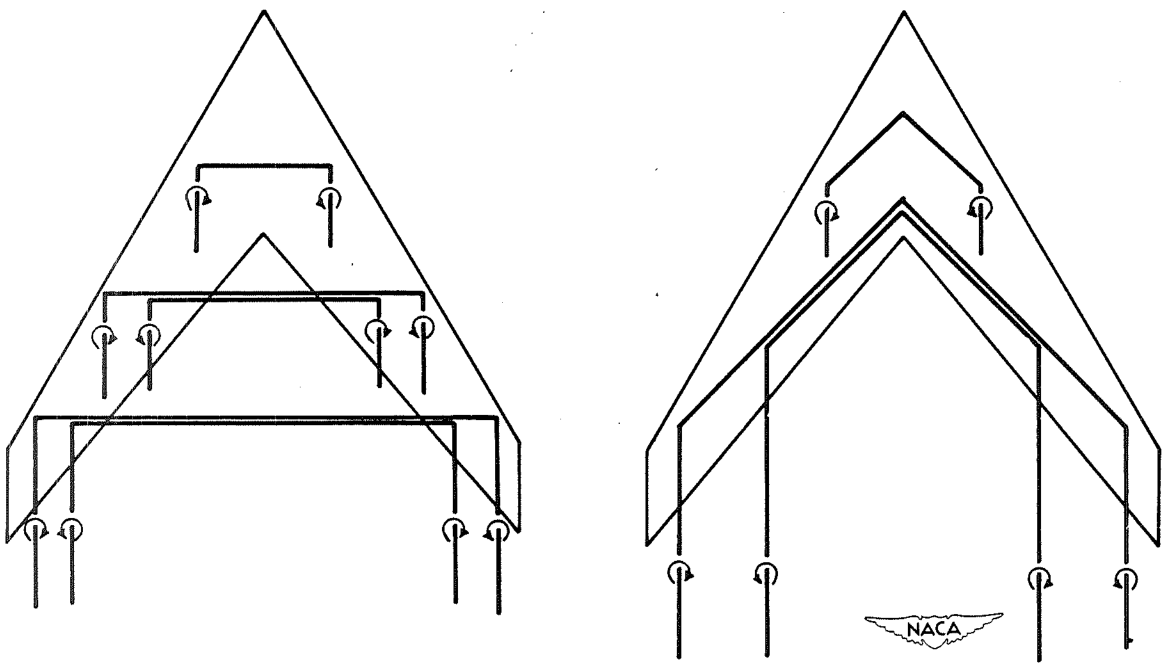

Figure 3.- Representations of the loading on a 60° swept wing by means of horseshoe vortices of other sweep angles.

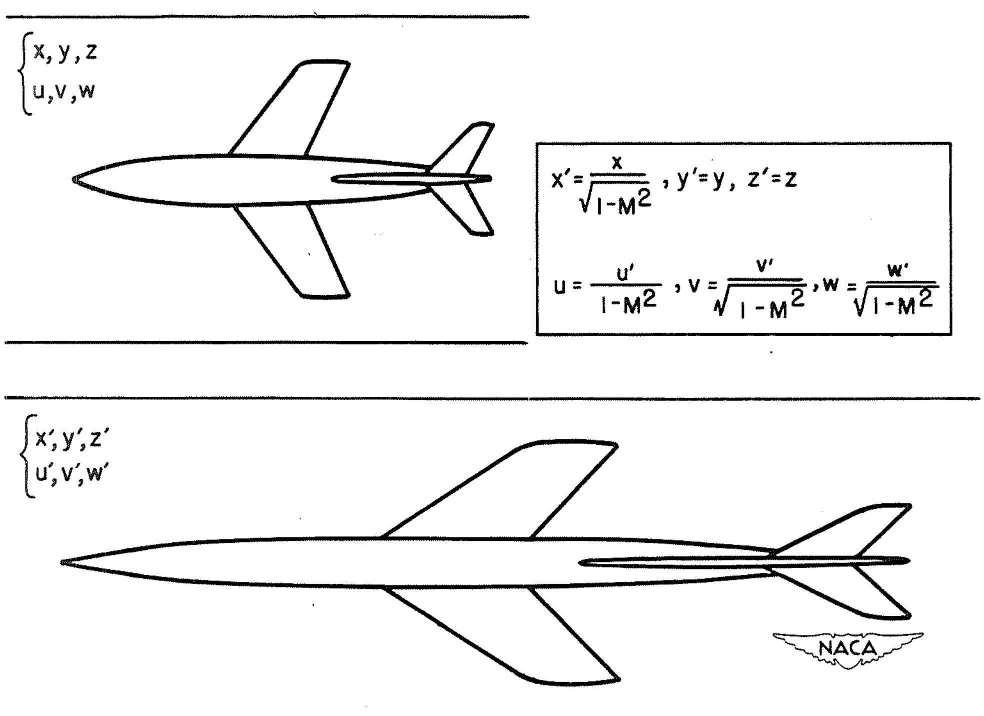

Figure 4.- Scheme for calculation of first-order compressibility effects.

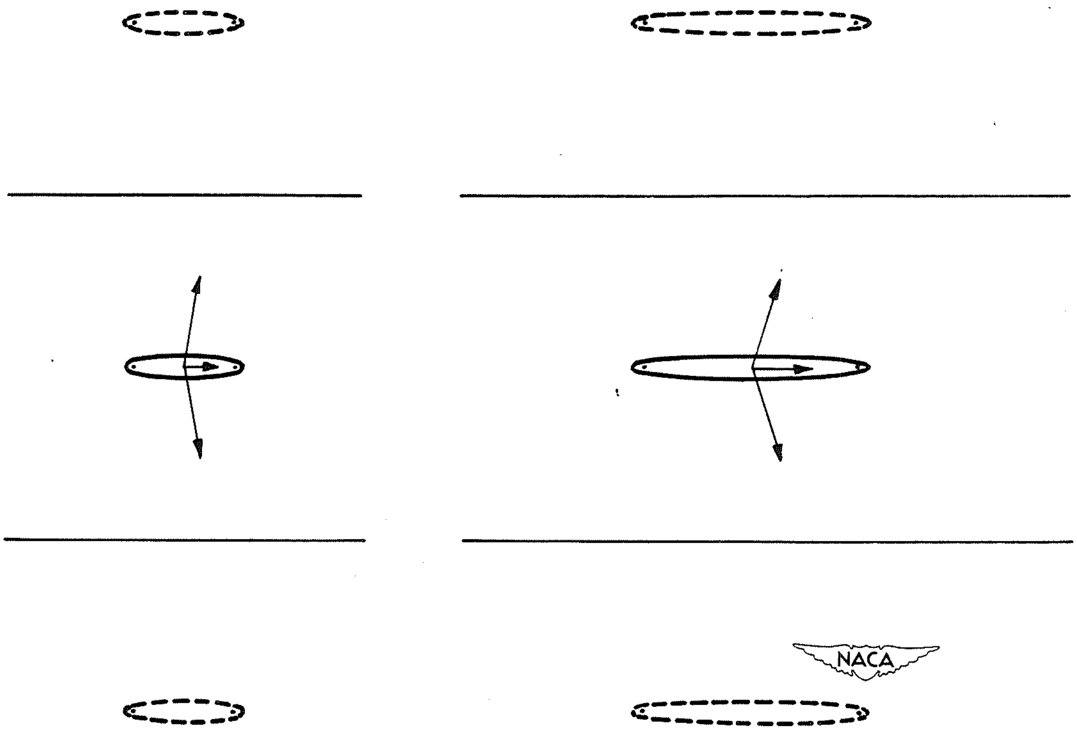

Figure 5.- Source-sink body in a two-dimensional tunnel, and its nearest images.

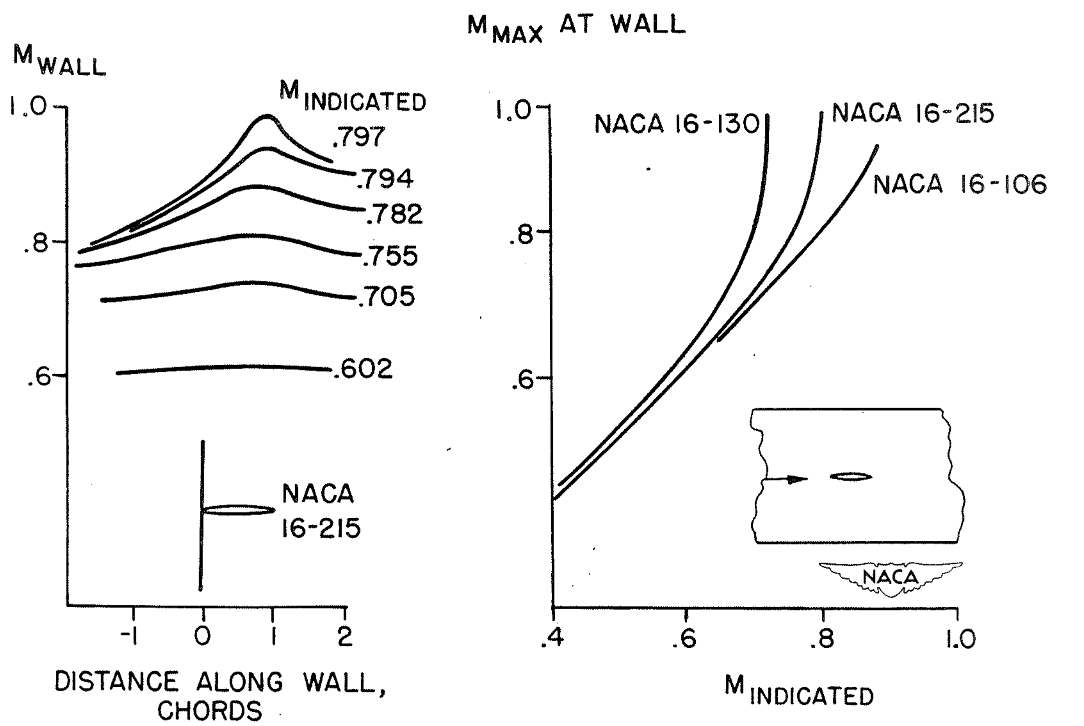

Figure 6.- Variation of wall Mach number with tunnel indicated Mach number. Five-inch chord airfoils in the Langley 24-inch high-speed tunnel.

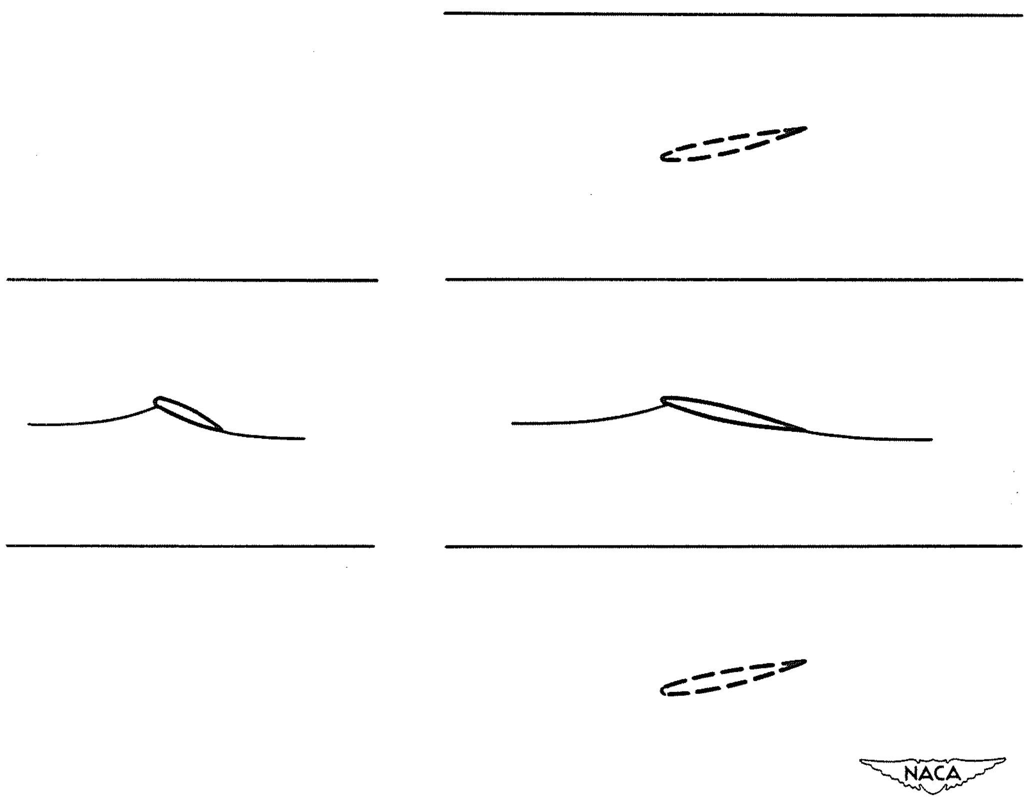

Figure 7.- Figure illustrating calculation of choking for a lifting airfoil in a closed two-dimensional tunnel.