A SURVEY OF FLUTTER

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The field of flutter is concerned largely with a study of the circumstances whereby a complicated elastic structure such as an aircraft, or the components of an aircraft, can interact with the surrounding air stream and spontaneously extract energy to an extent that may cause damage or destruction. The problems of the flutter field have expanded with modern aircraft developments so that they involve and overlap very large parts of aerodynamics and mechanics. It is the purpose of this paper to dwell on various aspects and concepts of this broad field.

A particularly simple example of flutter is the "fish tail" motion due to mass unbalance of a control surface (common during World War I). Suppose that a very heavy mass is placed at the trailing end of a control surface. If the motion of the wing is, say, upwards the inertia of the mass tends to create a fixed point at the control-surface trailing edge. Hence the control surface deforms in such manner as to tend to increase the lift, that is, the unbalanced mass brings into being an aerodynamic force tending to increase the motion. In the downward part of the cycle, similarly, there is a force tending to increase the motion. This "tail wagging the dog" type of control-surface flutter is satisfactorily eliminated by proper dynamic mass balancing.

Modern aircraft are subject to various types of flutter troubles. There is the classical type of flutter associated with a clean efficient flow pattern, which usually, though not necessarily, involves the coupling of several degrees of freedom of the structure. And there is another type that is difficult to analyze which may involve separated flows, periodic breakaways and reattachment of the flow, stalling conditions, shocks, and various hysteresis or time-lag effects between the flow pattern and the motion. In this type of flutter only a single degree of freedom of the structure may be prominently involved (example, stall flutter). There is also a possible merging of the types.

Common cures and remedies for flutter troubles are increased stiffnesses (particularly torsional), decreased coupling as, for example, mass balancing of control surfaces, and increased damping. Because of the great number of structural parameters, however, and of the various kinds of modes and types of flutter, there is no field in which it is more true that exceptions can be found for every rule of thumb. This means that (along with usual statistical, empirical, and experimental studies) the problem should be examined analytically along fundamental lines. Since the primary source of the self-excited motion is the (uniform) air stream itself, it will perhaps be worthwhile to examine first the basis of the nonstationary potential air forces.

For the purpose of classifying the aerodynamic problems at both low and high speeds, it is desirable to look at the general nonstationary flow equations for irrotational potential flow of a compressible fluid. The governing differential equation for the velocity potential follows from Euler's equations of motion and from the equation of continuity, with the assumption that the pressure is a function of density only and with the use of the local speed of sound as $c^2 = \frac{dp}{d\rho}$.

The general equation satisfied by the velocity potential may be put into a very pretty invariant form,

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + \overline{v}.\nabla\right)^2 \phi = \nabla^2\phi \tag{1}$$

where $\frac{\partial}{\partial t}$ and $\overline{v}.\nabla$ (which is $v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$ in rectangular Cartesian coordinates) operate only on ϕ, not on $\overline{v}$, and where, for example, for the adiabatic pressure-density relation, the local variable speed of sound is given by

$$c^2 = c_o^2 - \frac{\gamma - 1}{2} v^2 \tag{2}$$

Here γ is the adiabatic index (ratio of specific heats), and c_o is the velocity of sound corresponding to $v = 0$. The invariant "wave equation" form shows that the potential is propagated in the manner of a wave disturbance of finite amplitude. (For a derivation of equation (1) see appendix B of reference 1.)

Equation (1) serves to unify the discussion of the whole compressible-potential-flow picture and it shows up the difficulties inherent in the nonstationary, nonviscous flow problem in its unrestricted form. For example, when $\frac{\partial}{\partial t}$ is absent and the flow disturbances are not necessarily small, the equation becomes one treated by Rayleigh, Janzen, Poggi, and Kaplan. In a space of one dimension, for example, it reduces to the equation of Riemann for aerial plane waves of finite amplitudes.

$$\phi_{tt} + 2\phi_x \, \phi_{xt} - \left(\frac{dp}{d\rho} - \phi_x^2\right) \phi_{xx} = 0 \tag{3}$$

It is known that aerial plane waves of finite amplitudes cannot preserve their forms (reference 2) but that compression wave fronts steepen and rarefactions become less steep. Just as a shock condition awaits a compression front so too the past history of a rarefaction wave cannot be indefinitely prolonged without encountering discontinuities. Thus the existence of the continuous shockless pattern is for only a finite time. In a certain physical sense we must question the existence or creation of continuous steady flow patterns in the whole of space. The steep front must be treated then by Rankine–Hugoniot relations as a shock condition. The temptation to assign similar phenomena to the general flow, of which the one-dimensional unsteady case is a special one, is very great. Then, it should not be surprising if the potential-flow equations for continuous flow impose conditions impossible of physical fulfillment just as in the Riemann case. Physical phenomena such as shocks and instabilities and mathematical phenomena such as "limiting lines" can then arise. Precise discussions of these phenomena in relation to boundary conditions are matters of great difficulty. This subject must be considered to be in an incomplete state.

If the velocity of propagation of disturbances is assumed infinite ($c = \infty$), equation (1) reduces to Laplace's equation for the incompressible fluid:

$$\nabla^2\phi = 0 \tag{4}$$

Even this deceptively simple-looking equation leads to recondite matters both of a physical and a mathematical nature, for it embraces the whole of two- and three-dimensional incompressible potential flow, stationary or nonstationary, for small or large disturbances.

Before discussing certain aspects of the physical picture, it is of special interest to look at the small-disturbance linear equation to which the original nonlinear equation may be reduced. For stability studies the main interest is often precisely in the small-disturbance form of the equations. For small disturbances from a main-stream velocity V in the x-direction and with c now regarded as a constant, the disturbance velocity potential satisfies the equation

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)^2 \phi = \nabla^2\phi \tag{5}$$

This equation contains, for $V = 0$, the equation for the propagation of sound. For steady flow, $\frac{\partial\phi}{\partial t} = 0$, it is the general equation for linearized subsonic and supersonic flow which leads, for example, to the Prandtl–Glauert rule for subsonic flow and to the Ackeret rule for supersonic flow. In the general nonstationary case it is the theoretical basis for much of the existing work on the aerodynamical background of flutter, both at subsonic and supersonic speeds. This is perhaps the proper place to mention that equation (5) can also be associated with the purely acoustical problem of moving sources of sound.

In the near-sonic region, however, the linearized theoretical basis clearly requires modification as indicated by the Prandtl–Glauert and

Ackeret rules leading to infinite slopes of the lift curve at $M = 1$. It is likely that in this region it is necessary to employ iterative methods and to take into account second-order and other effects including viscosity and shape factors, but even the small-disturbance equation appears differently. Thus if all velocities are only slightly different from the critical local velocity of sound c^* $\left(c^* = \sqrt{\frac{2}{\gamma + 1}}\, c_o\right)$ and if the main stream is in the x-direction, there is obtained for the equation satisfied by the velocity potential

$$\frac{1}{c^{*2}}\left(\frac{\partial}{\partial t} + c^*\frac{\partial}{\partial x}\right)^2 \phi = \nabla^2\phi + (\gamma + 1)\,\frac{\partial^2\phi}{\partial x^2}\left(1 - \frac{1}{c^*}\frac{\partial\phi}{\partial x}\right) \tag{6}$$

or

$$\phi_{tt} + 2c^*\phi_{xt} - (\gamma + 1)(c^{*2} - c^*\phi_x)\,\phi_{xx} - c^{*2}\left(\phi_{yy} + \phi_{zz}\right) = 0$$

This equation reduces in the steady case to a nonlinear equation leading to the transonic similarity rules discussed recently by Von Kármán and others. Its use in conjunction with boundary conditions for solving flow problems has not as yet been attempted. Clearly, difficulties of mathematical and physical conceptions arise here too. There is a noteworthy similarity of the structure of this equation with that of equation (3).

Physical conceptions bearing on the origin of lift and the genesis of flow patterns are of special interest for nonstationary flows. The role of the trailing edge in subsonic aerodynamics in distinguishing an airfoil from a nonlifting body cannot be overstated. It is remarkable that the nature of the Kutta-Joukowski condition for smooth flow at the trailing edge has not been more deeply studied but rests only on descriptive and plausible grounds. A fuller study of the flow mechanism must of course involve, in some measure, dissipation, the boundary layer, and the wake.

The trailing edge may be considered the means for separating a zero circulation into equal positive and negative parts, one part being left bound to the airfoil, the other free floating in the wake. It may be recalled that the total circulation – the bound circulation over the airfoil and the free circulation over the surface of discontinuity which the airfoil has left behind – satisfies the Helmholtz-Kelvin theorem and vanishes. It is instructive to describe this key mechanism in another way. Consider the nature of the disturbance flow pattern of a flat-plate airfoil of infinite span undergoing a vertically downward motion; the main features are the acceleration of the fluid downward on the top and bottom sides and the spilling of fluid upwards along both edges, the whole pattern akin to that of two equal and opposite vortices. This flow pattern superposed with a uniform stream yields the noncirculatory flow pattern past a straight-line airfoil at an angle of attack. Then the further effect of the forward motion and of the trailing edge consists in effectively "sliding ahead and slicing off" the back part of the disturbance flow pattern thus leaving a bound circulation on the airfoil and a countercirculation in the wake. Of course, the influence of the field of floating vortices left behind in the wake must also be taken into account in analyzing the resulting pattern at the airfoil.

These conceptions are not self-evident but have since Lanchester slowly evolved and are still crystallizing. It is of some interest to note that without these concepts everyday natural phenomena such as the dynamic nature of the flight of birds, which supposedly first stirred man's imagination to attempt imitation of flight, remain imperfectly understood. (See reference 3.)

The potential type of flow is actually more nearly realized physically in the nonstationary than in the steady case. Thus for quick movements, very high lift coefficients and more nearly theoretical values of the slope of the lift curve can be realized. All this appears to be related to the nonstationary processes in the boundary layer which "effectively" yield a thinner boundary layer for higher frequencies; though to bring in these effects directly is a highly complex affair. A basic nondimensional parameter for comparing similar flows in the harmonically oscillating case, directly relating in a significant manner frequency, size, and velocity, is the "reduced" frequency defined by the ratio of the circular frequency times the half-chord to the main-stream velocity:

$$\left(k = \frac{\omega b}{V}\right)$$

In the incompressible nonstationary case there are two basic procedures which turn out to be completely equivalent: (a) the Birnbaum method followed up in particular by Cicala and Küssner and (b) the Wagner method followed up in particular by Glauert and Theodorsen. (See references 4 to 10.)

In the Birnbaum method a distribution of vorticity over the mean chord is assumed in a particular form of an infinite series (implicitly going to zero at the trailing edge); relations between the bound vorticity and the free vorticity are evaluated with consideration of the conditions at the trailing edge and the boundary condition that the main flow plus the induced flow at the airfoil surface corresponds with the actual motion of the airfoil, so that the resulting combined flow is at all times tangential to the airfoil surface. Knowledge of the local pressures is obtained directly from the vorticity distribution. It is of considerable mathematical interest that an explicit solution can be obtained in the two-dimensional incompressible case. (For example, see reference 1.)

In the Wagner method (which conveniently utilizes the principles of conformal mapping) the trailing-edge condition plays a more explicit role. The flow pattern may be thought of as built up by superposition of many elementary flows, each elementary flow being that of the straight-line airfoil with an infinitesimal segment locally deflected; each elementary flow is composed of two parts, a noncirculatory flow pattern corresponding to a source-sink or doublet distribution in the presence of a finite line and the wake flow pattern corresponding to a distribution of vortices generated at the trailing edge during the past history of the motion. Again, from the local pressures, the integrated forces and moments follow by integration.

It is instructive to point out that in the Wagner approach to the problem, the velocity potential or response to a sudden change of angle of attack plays an important part while in the Theodorsen developments the steady-state response for the harmonically oscillating airfoil is significant, and that these two functions are mated ones in the sense of the Laplace or Fourier integral transforms. (See reference 12.) This observation is an aid to the further application of the superposition theorem and to the treatment of gusts and other transient conditions. (See reference 13.)

The extension of the procedures to higher Mach numbers has been the objective of much of the more recent work. Solutions of the original linearized compressible-flow equations (equation (5)) are sought which can serve to solve the boundary problem. Main references in the subsonic case are the original paper by Possio, a subsequent general formulation by Küssner, and a calculation procedure by Dietze (references 14 to 16). It is noteworthy that the methods of the acceleration potential have found prominent application in these subsonic-flow studies. Difficulties, even in the plane case, arise: (1) The elementary solutions corresponding to sources and doublets have a different structure and (2) the boundary conditions lead to an integral equation with a highly complicated kernel function. (It should be remarked that the problem has also been treated by utilizing directly the velocity potential. (See reference 17.) A simpler kernel function occurs, but certain Mathieu functions are required for further practical developments.)

Several procedures have been tried to obtain numerical solutions of the integral boundary equation. Frazer and Skan (references 18 and 19) give a method of collocation in which boundary conditions are satisfied at a set of points, leading to n equations in n unknowns. Another procedure, a more flexible one, is the iteration procedure of Dietze which in contrast with the other procedures also lends itself to aileron calculations. Applications to flutter problems have been made in several papers (for example, references 19 and 20). Of practical interest are the facts that the Prandtl-Glauert rule appears as a limiting case for static instabilities and for low "reduced" frequency cases corresponding to high-density wings and high altitudes and that, while in general the compressibility effects are very complicated, the magnitudes of the effects are not large in the range of validity of the linearized theory (approximately $M < 0.75$) for structural parameters of normal and practical concern.

Of much interest too is the study of nonstationary air forces at supersonic speeds. (See reference 21.) There is a peculiar reversal of the role of the leading and trailing edges as compared with subsonic conditions. Thus, there are the conditions necessary for an attached shock at the leading edge that require a sufficiently sharp leading-edge angle. Otherwise a detached shock ahead of the body and a mixed supersonic-subsonic type of flow are involved. The trailing edge need play no determining role as it does in the subsonic case and, in fact, a compression-expansion wave mechanism is involved in the generation of lift. In general the flow pattern must be pieced together (as in method of characteristics) of several regions with various edge conditions and various conditions at the Mach lines.

In the small-disturbance linearized treatment of oscillating air forces (with no strong shocks or other large disturbances assumed present in the underlying steady flow pattern) elementary source-type solutions play a key role. The elementary source effect may be associated with a locally deflected flow pattern and, in accordance with the similarity of the acoustical and hydrodynamical problem as already observed, behaves as a source of spherical sound waves in motion uniformly through the medium with supersonic speed.

The moving-source solutions have a considerable interest in themselves. Historically, they are involved in the Doppler effect and were encountered also in electrodynamics at the turn of the century (reference 22), in somewhat disguised fashion from present forms, in the study of electrons moving at speeds both above and below that of light. (The doctrine of special relativity was still young.)

In order to illustrate briefly the source effects at supersonic speeds there are presented figure 1 and the velocity potential relation (reference 1):

$$\phi = \frac{1}{r}\left[f(t - \tau_1) + f(t - \tau_2)\right]$$

where

$$r = \frac{1}{M^2 - 1}\sqrt{(x - \xi)^2 - (M^2 - 1)\left[(y - \eta)^2 + (z - \zeta)^2\right]}$$

$$\tau_{1,2} = \frac{M}{c}\,\frac{x - \xi}{M^2 - 1} \mp \frac{r}{c}$$

The field point (x,y,z) at any time t is influenced by two waves which originated at times τ_1 and τ_2 earlier. A given field point at successive times $T + \tau_1$ and $T + \tau_2$ experiences, respectively, the effect of penetration into the spherical wave front of a pulse created at the origin at time T and the emergence out of the same wave front. At penetration of the wave front for a positive source there is a compression and subsequent equal expansion and, at emergence, the opposite effect. The distance r occurring in the velocity-potential relation, which in the case of a fixed source is the actual distance from the source to the field point, is shown geometrically in figure 1. At subsonic speeds there is only the single effect of penetration into the wave front because the field point never emerges from within the wave front.

The synthesis of solutions of boundary problems in terms of the source solution (and its normal derivative corresponding to a doublet solution) is of considerable general scope and validity. The applications form a wide field of research activity and it is regretted that they must be passed by with so few words at this time. It is of interest to mention that there are many papers now appearing in Russian dealing with similar problems. (See for example, reference 23.)

These aerodynamic considerations have been dwelt on because the motivating source of energy for flutter is the air stream itself and it is necessary to have some ideas of the nature of the oscillating air forces and moments which act, and their relative phases and amplitudes, in order to assess or analyze flutter effects.

Attention is now diverted to the mechanical nature of the flutter problem. For simplicity a configuration as in figure 2, an idealized wing on springs, is first considered. Corresponding to the two degrees of freedom, vertical deflection h and rotation α, there are two simultaneous differential equations, representing the equilibrium of vertical forces and of moments about the axis of rotation:

$$\ddot{h}A + \ddot{\alpha}B + hC = P$$

$$\ddot{\alpha}D + \ddot{h}B + \alpha E = M_o$$

where A and D are structural inertia terms, B is the coupling term due to mass unbalance about the axis of rotation, C and E are elastic restoring terms, and P and M_o are terms of aerodynamic origin.

If the air forces appropriate to small sinusoidal motions are employed, the flutter solution appears as a certain determinant put equal to zero, (which represents the condition for a nontrivial solution of the algebraic equations in h and α):

$$\begin{vmatrix} A_{a\alpha} & A_{ah} \\ A_{c\alpha} & A_{ch} \end{vmatrix} = 0$$

The individual terms are combinations of the inertia, elastic, and aerodynamic effects. This solution states that mechanical equilibrium is possible, that is, the laws of motion are satisfied, in the border sinusoidal case at a certain airspeed with a certain frequency and with certain amplitude and phase relations between the degrees of freedom. The question of whether the border stability condition, corresponding to a vanishing of the damping for the particular sinusoidal motion, separates a damped oscillation from a growing (negatively damped) oscillation, or vice versa, or is merely a resonance condition, is answered by other considerations – for example, by further study of the effects of the parameters, particularly structural damping, at the border condition, or by physical arguments.

The flutter determinantal equation (which contains complex elements, and hence is really two simultaneous equations) yields information on both the flutter frequency and the flutter speed. Several procedures, numerical, graphical, algebraic, and vectorial, for obtaining its solution, or for varying the parameters in the neighborhood of a definite solution have been developed. This phase of the flutter problem is a popular one and is the subject of many papers in the literature. One procedure which deserves special mention is the plotting of structural damping against airspeed as in reference 24 which treats directly the complex roots of the equation. The imaginary parts can be interpreted as the damping needed to obtain a flutter condition, negative damping then meaning that external energy must be added, stability thus being indicated. The plots of the imaginary parts of the complex roots against airspeed serve to measure the nearness to flutter and to give an indication of the violence and the type of flutter involved. (Of course after the flutter condition is encountered and small disturbance limits are exceeded, nonlinear effects may take over to limit the amplitude of oscillation, provided the structure holds together.) It should be briefly mentioned at this point that in addition to the dynamic instability conditions, the determinantal equation also contains the static instability conditions corresponding to wing divergence or control reversal. As pointed out previously, in these static cases in particular, the theoretical values need modifications to represent more closely experimental values for example, of the slope of the lift curve, center-of-pressure location, and hinge-moment coefficients.

In order to improve the foregoing idealized simple picture it is necessary to take into account a larger number of degrees of freedom and to bring in three-dimensional structural considerations. (See references 24 to 28.) This end is readily accomplished by the classical methods of Lagrange in which each degree of freedom may represent a spanwise mode of vibration (generalized coordinate) and the kinetic energy and the potential energy of the mechanical system play a central role. The terms representing the aerodynamic energy are obtained from the work done by the air forces in each coordinate.

The Lagrangian equations of motion representing the equilibrium in the chosen degrees of freedom then lead, as before in the sinusoidal case, to a characteristic flutter-stability equation in which the spanwise-mode effect is properly weighted and, conveniently, the mechanical potential energy (as in the Rayleigh vibration-mode methods) may involve the natural uncoupled frequencies of the structure. In this approach, matrix methods arise in a very natural manner. In recent years the matrix methods have become increasingly popular even with "practical" vibration people and it is believed this trend should be fostered rather than feared. It is however always a matter of taste and judgment and often very difficult to choose the degrees of freedom and their number to compromise properly between time, labor, physical grasp, and accuracy.

The problem of a continuous wing structure can also be set up as an integro-partial differential equation (instead of a system of simultaneous ordinary differential equations) in which the modes of vibration in the flutter condition are solved for rather than assumed. It is recognized however that, in general, the problem involves elastic problems which are too complex to be exactly handled even without consideration of the air forces and includes aerodynamic problems which are complicated enough even in the steady case and for rigid structures. In practice the procedures are iterative or approximate. (See reference 29.) The uniform cantilever wing has recently been given such a treatment (reference 30) with two-dimensional air forces assumed.

In fact in most flutter treatments two-dimensional air forces have been employed, frequently with over-all corrections for finite span inserted. Appropriate corrections for finite-span effects have occupied the attention of several authors. (See references 31 to 35.) The subject, however, is not in a too satisfactory state mainly because of complexity. The nonstationary effects attributed to aspect ratio are, in general, fairly small for moderate aspect ratios. There is room for both theoretical and experimental contributions in this field for wings of small aspect ratio.

A few words should perhaps be devoted to the subject of flutter of sweptback wings, a study which has been only lightly touched on by several writers. With sweepback the problem is complicated in both its structural and its aerodynamic aspects. Structurally, there exists a greater degree of coupling between bending and torsion as, for example, for a curved or bent-back elastic axis. Even the conception of an elastic axis, commonly used for unswept wings without large cut-outs, may, because of cross-stiffness effects, need to be replaced by the more general conception of influence-coefficients. In its aerodynamic aspect there is a greater degree of coupling in the air forces; for example, the bending deformation (dihedral effect) enters into the angle of attack of a wing section. Thus a small dihedral leads to second-order effects for unswept wings and to first-order effects for highly swept wings.

For an infinite uniform yawed wing (yawed at an angle not near 90°) two-dimensional (low speed) considerations indicate that the flutter speed increases by a factor of one over the cosine of the angle of yaw or sweep. A finite yawed wing, mounted on springs permitting it to move vertically and to rotate about an axis, would be expected to have a flutter speed with a factor of sweep higher than one over the cosine. However, for a finite sweptback wing clamped at its root, the combined effect of the elastic and aerodynamic coupling adversely affects the flutter speed so that, in general, the factor is considerably lowered.

There are many indications, however, that the static instability aileron reversal (in which the rolling power vanishes at a certain airspeed) rather than the dynamic instability may impose more severe design requirements for sweptback wings (for example, reference 35) at high speeds.

It has been possible to present here only a selection of aspects of the flutter field. The whole story of modern experimental techniques and research has had to be omitted. It is clear that measurement of aerodynamic coefficients for nonstationary flow throughout the subsonic, near-sonic, and supersonic speed ranges requires very exacting experimental techniques and critical tests. In testing for flutter in some of these speed ranges, it has been found convenient to employ, in addition to wind-tunnel research, techniques utilizing bomb drops and rocket missiles. Also required are the modern developments in pressure cells, strain gages, and electronic, telemeter, and vibration equipment.

In closing this survey of flutter, it is again emphasized that the physical classification of the flutter problem of a given structure is not easy for an attempt must be made to recognize which of the abundant sources of modes may be significantly involved and whether the type of flow is primarily of the potential classical type or includes a merging with other types of flow. In the near-sonic range, in particular, there is a clash between the potential and separated flows and a susceptibility to both kinds of flutter troubles. It is believed that refinements made in the aerodynamic and mechanical aspects of the flutter problem to be significant should to an extent keep in step with each other. It is hoped that some of the many facets and challenges of the flutter problem have been indicated.

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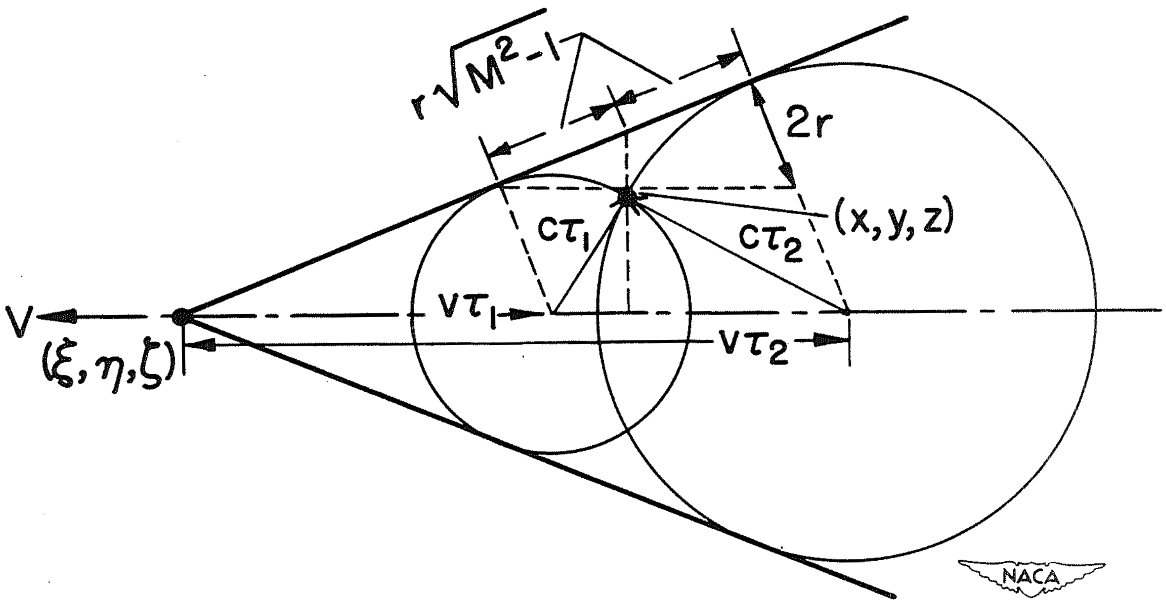

Figure 1.- Field of influence of a spherical source moving at a constant supersonic velocity.

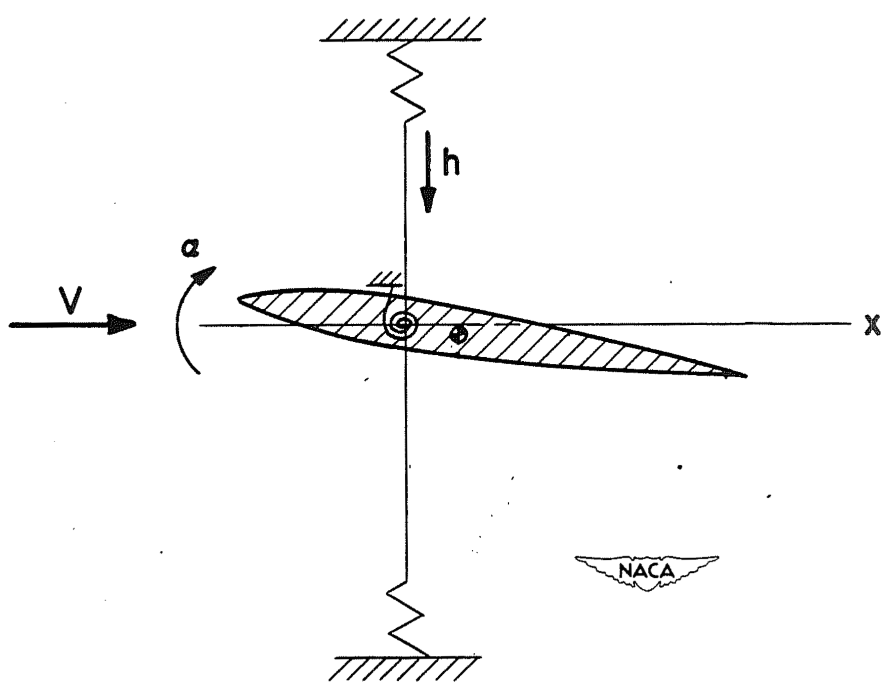

Figure 2.- Idealized wing configuration with two degrees of freedom.