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ALGORITHM FOR CONSTRUCTING THE PROGRAMMED MOTION OF A BOUNDING VEHICLE FOR THE FLIGHT PHASE

V. V. Lapshin

Translation of "Algoritm postroeniya programmnogo dvizheniya prygayushchego apparata dlya fazy poleta", Academy of Sciences of the USSR, Moscow, Institute of Applied Mathematics, Named for M. V. Keldysh, Preprint No. 26, 1979, pp. 1-53.

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16. Abstract

The study examines the construction of the programmed motion of a multi-leg bounding vehicle in the flight phase. An algorithm is constructed for solving the boundary value problem for construction of this programmed motion. If the motion is shown to be symmetrical, a simplified use of the algorithm can be applied. A method is also proposed for non-impact of the legs during lift-off of the vehicle, and softness of touchdown. Tables are also utilized to construct this programmed motion, for a broad set of standard motion conditions.

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ALGORITHM FOR CONSTRUCTING THE PROGRAMMED MOTION OF A BOUNDING VEHICLE FOR THE FLIGHT PHASE

V. V. Lapshin

Annotation /3*

Examined in the study is the problem of constructing the programmed motion of a multi-leg bounding vehicle in the flight phase. An algorithm is constructed for solving the boundary value problem of constructing the programmed motion for the flight phase. Also shown is the possibility of the substantial simplification of the work of this algorithm for symmetrical motion of a vehicle. A method is proposed for constructing the translational motion of the legs, which provides no impact during lift-off and softness of placement of the legs on the supporting surface. The algorithms were developed by the method of mathematical simulation on a computer. The results of the calculations are given. An algorithm is realized for constructing the programmed motion utilizing tables for a sufficiently broad set of standard motion conditions.

Key Words: multi-leg vehicle, bound, dynamics, motion control system, mathematical simulation.

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Introduction

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The given study is a continuation of the investigations, begun in [3], on the creation of a mathematical model of motion and a motion control system for a bounding vehicle. Examined in study [3] was the supported phase of motion of the vehicle. Constructed in the present study is a mathematical model of motion of a vehicle in the flight phase, and the problem of construction of programmed motion in the unsupported phase is solved.

A vehicle is examined which consists of a body and four or six two-section legs, which have three degrees of freedom each. The total weight of the legs comprises an appreciable portion of the weight of the body. Motion of the vehicle consists of the alternation of two phases: supported, when all of the vehicle's legs stand on a supporting surface and quasistatic stability takes place, and unsupported, or the flight phase, in the course of which the center of mass moves along a ballistic trajectory. We request that the vehicle leave the supporting surface without impact, and, at the moment of touchdown, it should set down sufficiently softly, with a small absolute stop velocity (zero, if possible). The requirement for softness of placement of the legs, instead of impactless, ensures reliability of contact of the legs with the supporting surface at the moment of touchdown, with the presence of errors in execution and informational errors on the actual supporting area.

In the problem of constructing the programmed motion of the vehicle in the flight phase, it is necessary to find those values of the linear and angular velocity of the vehicle which provide movement from a given initial position to a given final position, with the adopted method of shifting of the legs. /6

Obtained in § 1 are the primary integrals of the equations of motion of the vehicle (law of motion of the vehicle's center of mass and law of conservation of angular momentum relative to the center of mass), and a mathematical model of the spatial motion of the vehicle in the flight phase is constructed. Formulated in § 2 is the boundary value problem for constructing the programmed motion of a bounding vehicle in the flight phase, and an economical algorithm is obtained for its solution. A method is proposed for constructing the translational motion of the legs, which ensures non-impact of lift-off and the required softness of placement of the legs on the supporting surface. Also examined is the possibility of substantial simplification of the working of the algorithm for constructing the programmed motion in the case of symmetrical motion of the vehicle. The results of the calculations are given in § 3.

For standard motion conditions, which encompass a broad class of the most frequently encountered motion conditions of the vehicle, it is proposed that one recall the dependence of the initial angular velocity of the body on the parameters of the impending flight phase. The volume of stored information is relatively small. The value of the initial linear velocity of the body is calculated. Such an approach sharply shortens the time for calculations, and makes it possible to solve the problem of constructing the programmed motion for the flight phase during motion of the vehicle.

The algorithm for constructing the programmed motion and the mathematical model of spatial motion of the bounding vehicle in the flight phase were realized in FORTRAN language on the BESI-6 computer.

The constructed programmed motion can serve in the capacity of supported motion for construction of the algorithm of stabilization of the angular motion of the body during flight.

The author expresses his deep appreciation to D. Ye. Okhotsimskiy for posing of the problem and attention to the study.

§ 1. Mathematical Model of Motion of Bounding Vehicle in the Flight Phase /7

1.1. Kinematics of Vehicle and Designations

We will examine a vehicle which consists of a body and $N(N=4,6)$ two-section legs. The motion of the vehicle occurs in the absolute system of coordinates $O_1\xi\eta\zeta$. The coordinate system O_{xyz} is closely associated with the body of the vehicle, and its axes are directed along the main axes of inertia of the body. The position of the body of the vehicle we will determine by the coordinates of the center of mass of the body ξ, η, ζ and the angles ψ, θ, γ (fig. 1), where ψ is the angle between the axis $O_1\eta$ and the projection of O_y on the plane $O_1\xi\eta$ (yaw angle), θ is the angle between the axis O_y and its projection on the plane $O_1\xi\eta$ (pitch angle), and γ is the angle of rotation around the axis O_y (bank angle). The plane $O_1\xi\eta$ is horizontal.

Let each leg consist of two sections (thigh and shank), and have two degrees of freedom at the point of support of the thigh and body, and one degree of freedom at the knee. The angle between the axis O_x and the projection of the leg on the plane O_{xy} will be designated by α, the angle between the negative direction of the axis O_z and the thigh by β, and the angle between the thigh and the shank by q (fig. 2). The plane of the leg, formed by the thigh and the shank, is perpendicular to the plane O_{xy}.

The contact of the leg with the supporting surface has a point nature. We will call the end of the leg the foot. The center of mass of the thigh lies on the axis which connects the joints at the point of support leg and body and at the knee, while the center of mass of the shank lies on the axis which connects the knee joint to the foot.

We will introduce the following designations:

M - mass of the body,

τ_x, τ_y, τ_z - main moments of inertia of the body, /9

m_{ji} - mass of the j section of the i leg (here, and subsequently, $j=1$ corresponds to the shank, and $j=2$ to the thigh),

$$\tau_{ji} = \begin{pmatrix} J_{ji}^{11} & J_{ji}^{12} & J_{ji}^{13} \\ J_{ji}^{12} & J_{ji}^{22} & J_{ji}^{23} \\ J_{ji}^{13} & J_{ji}^{23} & J_{ji}^{33} \end{pmatrix}$$ - tensor of inertia of the j section of the i leg,

ℓ_{ji} - length of the j section of the i leg,

ℓ_{ji}^{y} - distance from the point of support of the j section of the i leg to its center of mass.

$\bar{r}_{ni} = (x_{ni}, y_{ni}, z_{ni})$ - radius-vector of the point of support of the i leg and body,

$\bar{r}_{ji} = (x_{ji}, y_{ji}, z_{ji})$ - radius-vector of the center of mass of the j section of the i leg,

$\bar{r}_{ci} = (x_{ci}, y_{ci}, z_{ci})$ - radius-vector of the foot of the i leg,

A - matrix of transition from absolute system of coordinates $O_1 \xi \eta \zeta$ to system of coordinates O_{xyz}, associated with the body of the vehicle,

$\bar{\rho} = (x_m, y_m, z_m)$ - vector directed from the center of mass of the vehicle towards the center of mass of the body,

$\bar{R} = (\xi, \eta, \zeta)$ - absolute coordinates of the center of mass of the body

$\bar{R}_a = (\xi_a, \eta_a, \zeta_a)$ - absolute coordinates of the center of mass of the vehicle,

$\bar{\omega}$ - angular velocity of the body of the vehicle.

We will write out some kinematic relationships:

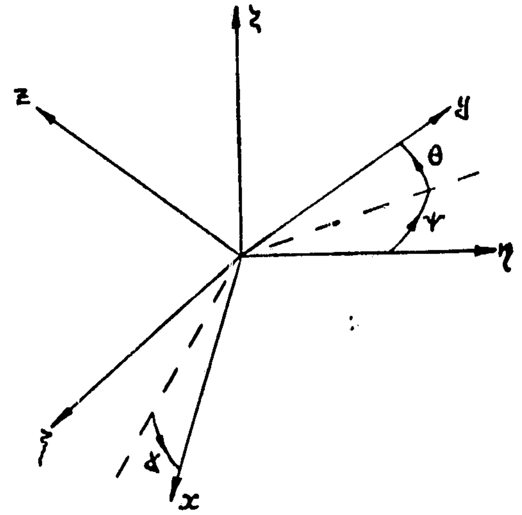

Fig. 1

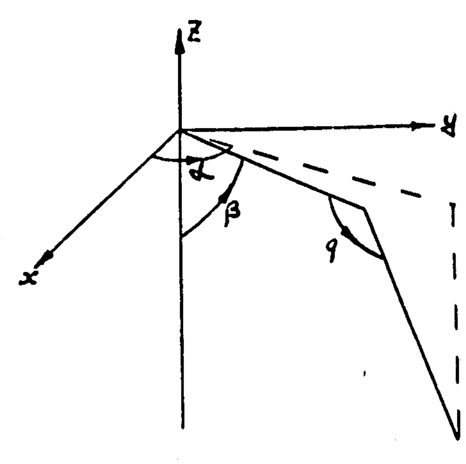

Fig. 2

$$
\begin{aligned}
x_{c_i} &= x_{n_i} + (l_{1i}\sin\varphi_i + l_{2i}\sin\beta_i)\cos\alpha_i, \\
y_{c_i} &= y_{n_i} + (l_{1i}\sin\varphi_i + l_{2i}\sin\beta_i)\sin\alpha_i, \\
z_{c_i} &= z_{n_i} - l_{2i}\cos\varphi_i - l_{2i}\cos\beta_i,
\end{aligned}
\tag{I.1}
$$

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$$
\begin{aligned}
x_{1i} &= x_{n_i} + (l_{1i}^{*}\sin\varphi_i + l_{2i}\sin\beta_i)\cos\alpha_i, & x_{2i} &= x_{n_i} + l_{2i}^{*}\sin\beta_i\cos\alpha_i, \\
y_{1i} &= y_{n_i} + (l_{1i}^{*}\sin\varphi_i + l_{2i}\sin\beta_i)\sin\alpha_i, & y_{2i} &= y_{n_i} + l_{2i}^{*}\sin\beta_i\sin\alpha_i, \\
z_{1i} &= z_{n_i} - l_{1i}^{*}\cos\varphi_i - l_{2i}\cos\beta_i, & z_{2i} &= z_{n_i} - l_{2i}^{*}\cos\beta_i,
\end{aligned}
$$

where

$$\varphi_i = \beta_i + q_i - \pi.$$

$$\bar{\rho} = -\frac{\sum_{j=1}^{2}\sum_{i=1}^{N} m_{ji}\bar{r}_{ji}}{M + \sum_{j=1}^{2}\sum_{i=1}^{N} m_{ji}} \tag{I.2}$$

$$
A = \begin{pmatrix}
\cos\psi\cos\gamma - \sin\psi\sin\theta\sin\gamma & \sin\psi\cos\gamma + \cos\psi\sin\theta\sin\gamma & -\cos\theta\sin\gamma \\
-\sin\psi\cos\theta & \cos\psi\cos\theta & \sin\theta \\
\cos\psi\sin\gamma + \sin\psi\sin\theta\cos\gamma & \sin\psi\sin\gamma - \cos\psi\sin\theta\cos\gamma & \cos\theta\cos\gamma
\end{pmatrix}
\tag{I.3}
$$

1.2. Primary Integrals of Equations of Motion in the Flight Phase

We will examine the motion of the vehicle with weighable legs. During the flight phase, a single external force acts on the vehicle—the force of gravity, directed along the axis $O_1\zeta$ and applied at the center of mass of the vehicle.

From the theorem on motion of the center of mass of a mechanical system, it follows that the center of mass of the vehicle moves along a ballistic trajectory [2]

$$\bar{R}_a = \bar{R}_a^{\circ} + \bar{V}_a^{\circ} t - \bar{g}\frac{t^2}{2},$$

where $\bar{R}_a = \bar{R} - \bar{\rho}$ - radius-vector of the center of mass of the vehicle,

$\bar{V}_a = \dot{\bar{R}} - \bar{\omega} \times \bar{\rho} - \dot{\bar{\rho}}$ - velocity of the center of mass of the vehicle,

$\bar{\omega}$ - angular velocity of the body.

According to the theorem on the change in the vector of the kinetic moment of a mechanical system, relative to the center of mass, we obtain [2] /11

$$\bar{G}_a = const, \qquad (I.5)$$

where $\bar{G}_a$ - vector of the kinetic moment of the vehicle in the absolute system of coordinates $O_1\xi\eta\zeta$.

According to Koenig's theorem [2], the value of the vector of kinetic moment of a random solid body is equal to

$$\bar{G}_T = J_T \bar{\omega}_T + M_T [\bar{R}_T \times \bar{V}_T], \qquad (I.6)$$

where J_T - tensor of inertia of the body relative to its center of mass,

M_T - mass of the body,

$\bar{R}_T$ - radius-vector of its center of mass,

V_T - velocity of the center of mass,

$\bar{\omega}_T$ - angular velocity of the body.

We will make use of formula (1.6) for the calculation of the values of the kinetic moment of the body and the sections of the legs relative to the center of mass of the vehicle.

As a result, we obtain the following expression for the vector of the kinetic moment of the body:

$$\bar{G}_K = J\bar{\omega} + M\bar{\omega}(\bar{\rho}, \bar{\rho}) - M\bar{\rho}(\bar{\omega}, \bar{\rho}) + M\bar{\rho} \times \dot{\bar{\rho}}, \quad (I.7)$$

where J - tensor of inertia of the body.

We will associate a system of coordinates, designating its unit vectors $\bar{e}^1_{ji}$, $\bar{e}^2_{ji}$, $\bar{e}^3_{ji}$, with the $\underline{j}$ section of the $\underline{i}$ leg. The vector $\bar{e}^1_{ji}$ is directed along this section of the leg, and

$\bar{e}^2_{ji}$ is perpendicular to the plane of the leg, $\bar{e}^3_{ji} = [\bar{e}^1_{ji} \times \bar{e}^2_{ji}]$. The origin of the system of coordinates will be placed at the center of mass of the j section of the i leg. In this case, the matrix of transition from the system of coordinates O_{xyz} to the system of coordinates associated with the j section of the i leg has the form [1] /12

$$A_{ji} = \begin{pmatrix} \sin\alpha_i & -\cos\alpha_i & 0 \\ \sin\varphi_{ji}\cos\alpha_i & \sin\varphi_{ji}\sin\alpha_i & -\cos\varphi_{ji} \\ \cos\varphi_{ji}\cos\alpha_i & \cos\varphi_{ji}\sin\alpha_i & \sin\varphi_{ji} \end{pmatrix}, \qquad \text{(I.8)}$$

where

$$\varphi_{ji} = \begin{cases} \beta_i, & \text{if } j=2 \\ \beta_i + \gamma_i - \pi, & \text{if } j=1. \end{cases}$$

The angular velocity of the j section of the i leg, in the projection on the axes associated with this section, is equal to [1]

$$\bar{\omega}_{ji} = A_{ji}\bar{\omega} + \begin{pmatrix} \dot{\varphi}_{ji} \\ -\dot{\alpha}_i \cos\varphi_{ji} \\ \dot{\alpha}_i \sin\varphi_{ji} \end{pmatrix}. \qquad \text{(I.9)}$$

The velocity of the center of mass of this section we will calculate according to Euler's formula [2]

$$\bar{V}_{ji} = \dot{\bar{\rho}} + \dot{\bar{z}}_{ji} + \bar{\omega} \times (\bar{\rho} + \bar{z}_{ji}). \qquad \text{(I.10)}$$

Utilizing (1.6), (1.9), and (1.10), we obtain the final expression for the vector of the kinetic moment of the j section of the i leg, relative to the center of mass of the vehicle

$$\bar{G}_{ji} = J_{ji}\bar{\omega}_{ji} + m_{ji}\bar{\omega}(\bar{\rho}+\bar{\tau}_{ji}, \bar{\rho}+\bar{\tau}_{ji}) - m_{ji}(\bar{\rho}+\bar{\tau}_{ji})(\bar{\omega},\bar{\rho}+\bar{\tau}_{ji}) + \\ + m_{ji}[(\bar{\rho}+\bar{\tau}_{ji})\times(\dot{\bar{\rho}}+\dot{\bar{\tau}}_{ji})]. \qquad (1.11)$$

The vector of the kinetic moment of the vehicle, relative to the center of mass, is equal to the sum of the kinetic moments of the body and the sections of the legs

$$\bar{G}_a = \bar{G}_k + \sum_{j=1}^{2}\sum_{i=1}^{N} G_{ji}.$$

From (1.1)-(1.3) and (1.6)-(1.12), it follows that

$$\bar{G}_a = I\,\bar{\omega} + \sum_{i=1}^{N} I_i\,\dot{\bar{p}}_i\,, \qquad (1.13)$$

where $\bar{p}_i=(\alpha_i,\beta_i,q_i)^T$,

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$$I=\begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix}, \qquad I_i=\begin{pmatrix} I_{x\alpha_i} & I_{x\beta_i} & I_{xq_i} \\ I_{y\alpha_i} & I_{y\beta_i} & I_{yq_i} \\ I_{z\alpha_i} & I_{z\beta_i} & I_{zq_i} \end{pmatrix}.$$

In that case when the legs of the vehicle are immobile, relative to the body, the kinetic moment of the vehicle is equal to the first term $I\omega$ in formula (1.13), and the second term of $\sum_{i=1}^{N} I_i\dot{\bar{p}}_i$ is associated with motion of the legs relative to the body. The matrix I is the tensor of inertia of the vehicle with a given position of the legs in a relative system of coordinates. The elements of the matrices I, I_i $(i=1,2,\dots N)$ depend solely on the angles in the leg joints α_i, β_i, q_i $(i=1,2,\dots N)$. The values of the elements I, I_i are written out in explicit form in the appendix.

Let control of shifting of the legs in the flight phase be accomplished according to position, i.e., α_i, β_i, q_i $(i=1,2,\dots N)$ are prescribed by functions of time. We will construct a mathematical model of motion of the vehicle, utilizing the theorem on the motion of the center of mass and the theorem on the change in the vector of the kinetic moment relative to the center of mass. The equations of motion of the vehicle in projection onto absolute coordinates will be written in the foll..'.ng manner [2]

$$\ddot{\bar{R}}_a = -\bar{g}, \qquad (1.14)$$

$$\bar{G}_a = \bar{G}_a^{\circ}. \qquad (1.15)$$

In view of the nonsingularity of the matrix I (I is the tensor of inertia of the vehicle), it follows from (1.5), (1.13), and (1.15) that

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = I^{-1}\left\{ A \begin{pmatrix} G_{a\xi}^{\circ} \\ G_{a\eta}^{\circ} \\ G_{a\zeta}^{\circ} \end{pmatrix} - \sum_{i=1}^{N} I_i \begin{pmatrix} \dot{\alpha}_i \\ \dot{\beta}_i \\ \dot{q}_i \end{pmatrix} \right\}. \qquad (1.16)$$

The right-hand portions of the equations (1.16) are functions of time and the angular coordinates of the body, since the elements of the matrices I, I_i depend solely on the angles in the leg joints α_i, β_i, q_i ($i=1,2,\ldots N$), which are known functions of time. /14

We will add the kinematic equations of Euler [1,5] to (1.16)

$$\begin{aligned} \dot{\psi} &= (\omega_z \cos\gamma - \omega_x \sin\gamma)/\cos\theta, \\ \dot{\theta} &= \omega_x \cos\gamma + \omega_z \sin\gamma, \qquad (1.17) \\ \dot{\gamma} &= \omega_y - \dot{\psi}\sin\theta. \end{aligned}$$

By integrating equations (1.16)-(1.17) on a computer by the Runge-Kutt method, we obtain motion of the body of the vehicle near the center of mass, i.e., the dependence of the angles ψ, θ, γ on time.

We would note that equations (1.17) deteriorate with $\theta=\pi/2$. Geometric deterioration takes place. In that case when this may take place, it is sufficient to change the direction of the axes $O_1\xi\eta\zeta$ in an appropriate manner.

Motion of the center of mass of the vehicle will be determined from equations (1.14), determined for a parabolic trajectory.

The motion of the center of mass of the body will be found, utilizing the relationships:

$$\bar{R} = \bar{R}_a + \bar{\rho}, \qquad \bar{V} = \bar{V}_a + \bar{\omega} \times \bar{\rho} + \dot{\bar{\rho}}. \qquad (1.18)$$

In the given paragraph, we first obtained the integrals of the equations of motion of a bounding vehicle in the flight phase. Based on the primary integrals, we constructed a mathematical model of motion of the vehicle in the flight phase.

§ 2. Construction of Programmed Motion of the Vehicle for the Flight Phase /15

2.1. Posing of Problem

The system of construction of the programmed motion of the vehicle in the flight phase determines the velocities of the body of the vehicle at the moment of lift-off from the supporting surface, necessary for realization of the impending flight phase, and creates the nominal translational motion of the legs. The initial and final position of the vehicle, determined by the algorithm of formation of the attitude of the bounding vehicle at the moments of lift-off and touchdown [3], are the input parameters of the algorithm of construction of the programmed motion. In the flight phase, the vehicle should move so as to satisfy the boundary conditions:

$$\bar{\xi}(t_0) = \bar{\xi}^0, \quad \bar{R}_{ci}(t_0) = \bar{R}_{ci}^0, \qquad \bar{\xi}(t_1) = \bar{\xi}^1, \quad \bar{R}_{ci}(t_1) = \bar{R}_{ci}^1 \quad (i = 1, 2, \ldots N), \qquad (2.1)$$

where $\bar{\xi} = (\xi, \eta, \zeta, \psi, \theta, \gamma)$; $\bar{R}_{ci} = (\xi_{ci}, \eta_{ci}, \zeta_{ci})$; t_0, t_1 are the moments of beginning and ending of the flight phase. [1]Positioning and lift-off from the supporting surface of all of the legs of the vehicle are accomplished simultaneously.

In order to determine the initial velocity of the body of the vehicle at the moment of lift-off from the supporting surface $\dot{\bar{r}}(t_*) = \dot{\bar{r}}^0$, necessary for realization of the impending flight phase, and to construct the nominal translational motion of the legs, we will utilize the following calculation scheme:

1. Utilizing the boundary conditions (2.1), we will obtain the value of the velocity of the center of mass at the moment of lift-off from the supporting surface and the duration of the flight phase from the law of motion of the center of mass (1.4).

2. The translational motion of the legs in the flight phase leads to a change in the angular coordinates of the body, in view of the law of conservation of the vector of the kinetic moment (1.5). The problem of determining the initial angular velocity of the body amounts to a boundary value problem, which is solved by the iteration method. In each step of this iteration process, it is necessary to know the corresponding translational motion of the legs. /16

The velocity of the center of mass of the body of the vehicle is determined according to Euler's formula [2], utilizing the known values of the velocity of the center of mass of the vehicle and the angular velocity of the body.

3. Shifting of the legs will be accomplished in a relative system of coordinates $Oxyz$, thus ensuring non-impact of lift-off and sufficiently soft placement of the legs on the supporting surface with a small absolute velocity, which comprises a given part of the velocity of the center of mass of the vehicle at this moment. At the moment of touchdown, the requirements for softness of placement of the legs, instead of non-impact, ensures reliability of contact of the legs of the vehicle with the supporting surface in real motion, with the presence of errors in execution and informational errors on the real support area.

The boundary values of the velocities at the leg joints depend on the initial velocity of the body of the vehicle. Consequently, construction of the translational motion of the legs must be accomplished in each step of the iteration process of the solution of the boundary value problem by determining the initial angular velocity of the body.

We will switch to the description of the operation of the basic blocks of the algorithm of construction of the programmed motion.

2.2. Determination of Duration of Flight Phase and Parameters of Trajectory of Motion of Vehicle's Center of Mass /17

From the boundary conditions (2.1), which determine the initial and final position of the body and feet of the vehicle in the absolute system of coordinates $O_1\xi\eta\zeta$, we will find the matrix A, the relative coordinates of the feet and angles in the leg joints, and the value of the vector $\bar{\rho}$, directed from the center of mass of the vehicle towards the center of mass of the body, at the moments of lift-off and touchdown. We will then obtain the values of the coordinates of the center of mass of the vehicle at the moments of lift-off and touchdown

$$\bar{R}_a^0 = \bar{R}^0 - \bar{\rho}^0, \quad \bar{R}_a^1 = \bar{R} - \bar{\rho}^1, \tag{2.2}$$

Let the tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal plane $O_1\xi\eta$, equal to λ, be given.

From the law of motion of the center of mass of the vehicle (1.4) and the boundary conditions (2.2), we will find the duration of the unsupported phase of motion and the initial velocity of the center of mass of the vehicle

$$\begin{aligned} &T = t_1 - t_0 = \sqrt{\frac{2[\lambda\sqrt{(\xi_a^1-\xi_a^0)^2+(\eta_a^1-\eta_a^0)^2} - (\zeta_a^1-\zeta_a^0)]}{g}} \\ &V_{a\xi}^0 = (\xi_a^1-\xi_a^0)/T, \quad V_{a\eta}^0 = (\eta_a^1-\eta_a^0)/T, \\ &V_{a\zeta}^0 = \lambda\sqrt{(V_{a\xi}^0)^2+(V_{a\eta}^0)^2}. \end{aligned} \tag{2.3}$$

Equations (1.4) and (2.3) fully determine the motion of the center of mass of the vehicle in the flight phase.

2.3. Shifting of the Legs

According to the absolute coordinates of the feet and body of the vehicle, known from the boundary conditions (2.1), we /18
will calculate the coordinates of the feet in the relative system of coordinates and, transforming the third group of equations (1.1), we will obtain the values of the angles at the leg joints at the moments of touchdown and lift-off from the supporting surface, according to the formulas:

$$\varphi_i = \arccos\left\{\frac{1}{2l_{2i}l_{3i}}[(x_{1i}-x_{ni})^2+(y_{1i}-y_{ni})^2+(z_{1i}-z_{ni})^2-l_{2i}^2-l_{3i}^2]\right\},$$

$$\beta_i = \arccos\left\{\frac{-\sqrt{(x_{ci}-x_{ni})^2+(y_{ci}-y_{ni})^2}\,\ell_{2i}\sin q_i-(z_{ci}-z_{ni})(\ell_{2i}-\ell_{1i}\cos q_i)}{\sqrt{(x_{ci}-x_{ni})^2+(y_{ci}-y_{ni})^2+(z_{ci}-z_{ni})^2}}\right\}, \quad (2.4)$$

$$\alpha_i = \arcsin\left\{\frac{y_{ci}-y_{ni}}{\ell_{1i}\sin\varphi_i+\ell_{2i}\sin\beta_i}\right\} \quad (i=1,2,\dots N).$$

Formulas (2.4) deteriorate when the projection of the foot on the plane O_{xy} coincides with the projection of the point of support of the leg on this plane, <u>i.e.</u>, $\ell_{1i}\cdot\sin \ell_1 + \ell_{2i}$ $\ell_{2i}\cdot\sin\beta_i=0$. With the formation of the initial and final attitude of the vehicle, we require that, at these moments in time, the projections of the foot and the point of support of the leg on the plane Oxy do not coincide.

The values of the velocities at the leg joints at the moments of lift-off and touchdown are determined from the conditions of non-impact of lift-off of the legs from the supporting surface and the conditions of sufficiently soft placement of the legs with a small absolute velocity (zero, if possible), which comprises a given part of the velocity of the center of mass of the body at the moment of touchdown

$$\bar{V}'_{ci} = (C_\xi V'_{a\xi},\ C_\eta V'_{a\eta},\ C_\zeta V'_{a\zeta})^T, \quad (2.5)$$

where $C_\xi, C_\eta, C_\zeta \ll 1$ are the given constants.

The velocity of the center of mass of the vehicle is known from (1.4) and (2.3); thus, the absolute velocity of the feet at the moments of lift-off and touchdown are known. We will write out the expressions for the absolute velocity of the feet, according to Euler's formula [2]

$$\bar{V}_{ci} = \bar{V}_a + \bar{\omega}\times(\bar{\rho}+\bar{z}_{ci}) + \dot{\bar{\rho}} + \dot{\bar{z}}_{ci}, \quad (2.6)$$

We will substitute in (2.6) the values of $\dot{\bar{\rho}}, \dot{\bar{\eta}}_{ci}$ $(i=1,2,\dots N)$, obtained by differentiation of (1.1)-(1.2); then, (2.6) form a system of linear equations relative to $\dot{\alpha}_i, \dot{\beta}_i, \dot{q}_i$ $(i=1,2,\dots N)$, which we will write in the form /19

$$L\dot{\bar{p}} = \bar{S} + \Omega\bar{\omega}, \quad (2.7)$$

where $\bar{p}=(\alpha_1, \beta_1, q_2, \dots q_N)-3N$ is the measured vector of the angles at the leg joints, L is the vector of dimensionality

$3N \times 3N$, the elements of which depend solely on $\bar{p}$; $\bar{S}=(\bar{V}_a-\bar{V}_{c1}, \ldots V_a-V_{cN})$; Ω is the matrix of dimensionality $3 \times 3N$, so that $\Omega\bar{\omega}=(\bar{\omega}\times(\bar{\rho}+\eta_{c1}), \ldots \bar{\omega}\times(\bar{\rho}+\bar{\eta}_{cN}))^T$.

As a result, we obtain the fact that, having determined the angles at the leg joints and constructed the trajectory of motion of the center of mass of the vehicle, one can determine the values of the elements of the matrix L, Ω and the vector S at the moments of lift-off and touchdown. If, in addition, the initial and final value of the angular velocity of the body are known, then, by solving the systems of linear equations (2.7) relative to $\bar{p}$, we will find the boundary values of the velocities at the leg joints at the moments of lift-off and touchdown. The system of equations (2.7) characterizes the dependence of the velocities at the leg joints on the velocity of the center of mass of the vehicle and the angular velocity of the body in that case when the coordinates of the body of the vehicle, the angles at the leg joints, and the absolute velocities of the feet are known. The matrix L in (2.7) deteriorates when the velocity of the foot, as if it were one of the legs, does not determine the velocity of the joints of this leg unequivocally in the relative system of coordinates. This takes place when one of the following conditions is fulfilled: a) the projection of the foot of the $\underline{i}$ leg on the plane coincides with the projection of the point of support of the leg and body, or b) the leg folded up or completely straightened at the knee ($q_1=0$ or $q_1=\pi$). During formation of the initial and final programmed positions of the vehicle, we require that these conditions not be fulfilled. Then, the matrix L is not deteriorated.

With each return to the block of construction of the translational motion of the legs, the value of the initial angular velocity of the body $\bar{\omega}(t_0)=\bar{\omega}^0$ is known. The value of the angular velocity of the body at the moment of touchdown will be determined utilizing the law of conservation of the vector of the kinetic moment. From (1.5) and (1.13), it follows that /20

$$\mathcal{A}^{-1}(I\bar{\omega}+I_L\dot{\bar{p}})\Big|_{t=t_1} = \bar{G}_a^0 \qquad (2.8)$$

where $I_L=(I_1, I_2, \ldots I_N)^T$.

By substituting the values of $\dot{\bar{p}}(t_1)$, obtained by solving the system of linear equations (2.7) and (2.8), we obtain a system of three linear equations relative to $\bar{\omega}(t_1)=\bar{\omega}^1$

$$\mathcal{A}^{-1}(I+I_L L^{-1}\Omega)\bar{\omega}^1 = \bar{G}_a^0-\mathcal{A}^{-1}I_L L^{-1}\bar{S}, \qquad (2.9)$$

by solving which we will find $\bar{\omega}^1$.

As a result, we obtain the values of the coordinates and the velocities at the leg joints at the moments of lift-off and touchdown. Let $\bar{p}=(\alpha_1,\beta_1,\ldots q_N)$. Shifting of the legs in the flight phase will be accomplished so that the obtained boundary conditions will be fulfilled:

$$\bar{p}(t_0)=\bar{p}^0\ ,\quad \dot{\bar{p}}(t_0)=\dot{\bar{p}}^0;\qquad \bar{p}(t_1)=\bar{p}^1\ ,\quad \dot{\bar{p}}(t_1)=\dot{\bar{p}}^1 \qquad (2.10)$$

and the phasic restrictions will not be disrupted:

$$\bar{p}^{min}\leq\bar{p}\leq\bar{p}^{max}, \qquad (2.11)$$

where p^{min}, p^{max} are the given limits of change in the angles at the leg joints in the programmed motion.

Then, for β_i $(i=1,2,\ldots N)$, we have

/21

$$\dot{\beta}_i^0<0\ ,\quad -\dot{\beta}_i^0(t_1-t_0)\gg\beta_i^0-\beta_i^{min},$$
$$\dot{\beta}_i^1>0\ ,\quad \dot{\beta}_i^1(t_1-t_0)\gg\beta_i^1-\beta_i^{min}.$$

Consequently, the shifting of the legs should be accomplished so that $\beta_i(t)$ first slows from the initial position on the phasic plane $(\beta_i^0,\dot{\beta}_i^0)$ to $(\beta_i^{min},0)$, remains for some time at the boundary $(\beta_i^{min},0)$, and then accelerates to a final position $(\beta_i^1,\dot{\beta}_i^1)$. Motion at the knee is accomplished similarly, only $q_i(t)$ $(i=1,2\ldots N)$ reaches the upper limit q_i^{max}. In other words, right after lift-off from the supporting surface, the leg straightens and increases its moment of inertia relative to the axis of the engine, which accomplishes a change in the angle β_i. For more uniform distribution of the forces developed in the leg joints, according to time, we require that the accelerations along the coordinates β_i, q_i, with braking of the leg after lift-off from the supporting surface, be linear functions, which diminish, according to the modulus, to a zero value. In a symmetrical manner, we will accomplish motion along the coordinates β_i, q_i right after the moment of touchdown. We will designate $b=\beta_i, q_i$, $b^{rn}=\beta_i^{min}, q_i^{max}$ $(i=1,2\ldots N)$. We will require that the boundary conditions (2.10) be fulfilled and

$$\ddot{b} = \begin{cases} a_1(\tau_1 - t), & \text{with } t \in [t_0, \tau_1] \\ 0, & \text{with } t \in [\tau_1, \tau_2] \\ a_2(t - \tau_2), & \text{with } t \in [\tau_2, t_1] \end{cases} \qquad (2.12)$$

$$b(t) = b^{rp}, \quad \dot{b}(t) = 0, \quad \text{with } t \in [\tau_1, \tau_2]. \qquad (2.13)$$

Then, it follows from (2.10), (2.12)-(2.13) that /22

$$\begin{aligned} \tau_1 &= t_0 - \frac{3(b^0 - b^{rp})}{\dot{b}^0}, \quad a_1 = -\frac{2\dot{b}^0}{(\tau_1 - t_0)^2}, \\ \tau_2 &= t_1 - \frac{3(b^1 - b^{rp})}{\dot{b}^1}, \quad a_2 = \frac{2\dot{b}^1}{(t_1 - \tau_2)^2}. \end{aligned} \qquad (2.14)$$

From (2.12)-(2.14), we obtain the dependence on time of the coordinates $b = \beta_i, q_i$ $(i=1,2\ldots N)$ with shifting of the legs in the flight phase

$$b(t) = \begin{cases} b^0 + \dot{b}^0(t - t_0) + \frac{1}{6} a_1 (t - t_0)^2 (3\tau_1 - 2t_0 - t), & \text{with } t \in [t_0, \tau_1] \\ b^{rp}, & \text{with } t \in [\tau_1, \tau_2] \\ b^{rp} + \frac{1}{6} a_2 (t - \tau_2)^3, & \text{with } t \in [\tau_2, t_1] \end{cases} \qquad (2.15)$$

At the same time, for the engine, which accomplishes turning of the plane of the leg α_i $(i=1,2\ldots N)$, the moment of inertia of the leg changes slightly. Therefore, according to the coordinates α_i, shifting of the leg will be accomplished in such a manner that will minimize the maximum magnitude of acceleration according to this coordinate, satisfying the boundary conditions (2.10) and the phasic limits (2.11). Construction of motion along the coordinate α_i amounts to the solution of the extremal problem

$$
\begin{aligned}
&\alpha_i(t_0)=\alpha_i^0 , \quad \dot{\alpha}_i(t_0)=\dot{\alpha}_i^0 ,\\
&\alpha_i(t_1)=\alpha_i^1 , \quad \dot{\alpha}_i(t_1)=\dot{\alpha}_i^1 ,\\
&\alpha_i^{max} \leq \alpha_i \leq \alpha_i^{min} ,\\
&\max_t |\ddot{\alpha}_i| \to \inf .
\end{aligned}
\qquad (2.16)
$$

Problem (2.16) was examined in study [3], where its analytic solution was obtained.

As a result, we obtained the dependence of the angles at the leg joints in the flight phase on time and the initial angular velocity of the body.

2.4. Boundary Value Problem. Determination of Initial Angular and Linear Velocity of Body. /23

We will formulate the boundary value problem for determining the initial angular velocity of the body. By substituting the dependence, obtained in paragraph 2.3, of the angles at the leg joints on time and the initial angular velocity of the body $\bar{p}=\bar{p}(t,\bar{\omega}^0)$, where $\bar{p}=(\alpha_1,\beta_1, q_1,\dots,q_N)$, into the law of conservation of the vector of the kinetic moment (1.5), (1.13), we have

$$
I(t,\bar{\omega}^0)\,\bar{\omega} + I_\perp(t,\bar{\omega}^0)\,\dot{\bar{p}}(t,\bar{\omega}^0) = \mathcal{A}\,\bar{G}_0 . \qquad (2.17)
$$

From the kinematic equations of Euler (1.17) and equations (2.17), it follows

$$
\begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\gamma} \end{pmatrix} = \bar{F}(t,\bar{\omega}^0, \psi,\theta,\gamma), \qquad (2.18)
$$

where

$$
\bar{F} = \begin{pmatrix} -\frac{\sin\gamma}{\cos\theta} & 0 & \frac{\cos\gamma}{\cos\theta} \\ \cos\gamma & 0 & \sin\gamma \\ \sin\gamma\,\mathrm{tg}\,\theta & 1 & \cos\gamma\,\mathrm{tg}\,\theta \end{pmatrix} I^{-1}\{\mathcal{A}^{-1}\bar{G}_0 - I_\perp \dot{\bar{p}}\}.
$$

We will write out the boundary conditions (2.1) for the angular coordinates of the body:

$$\psi(t_0)=\psi^0,\ \theta(t_0)=\theta^0,\ \gamma(t_0)=\gamma^0,$$
$$\psi(t_1)=\psi^1,\ \theta(t_1)=\theta^1,\ \gamma(t_1)=\gamma^1. \tag{2.19}$$

We will solve the boundary value problem (2.17)-(2.19) for the system of differential equations, which describe the motion of the vehicle near the center of mass in the flight phase, numerically on the computer. As a result, we obtain the value of the initial angular velocity of the body $\bar{\omega}^0$.

By integrating system (2.18) in the time increment $t\in[t_0,t_1]$ (where t_0 is the beginning and t_1 is the end of the flight phase) by the Runge-Kutta method, we obtain the values of the angular coordinates of the body at the moment of touchdown as functions of $\bar{\omega}^0$ or $\dot{\psi}^0, \dot{\theta}^0, \dot{\gamma}^0$. The boundary value problem (2.17)-(2.19) comes down to the solution of the system of algebraic equations /24

$$\psi(\dot{\psi}^0, \dot{\theta}^0, \dot{\gamma}^0)=\psi^1,$$
$$\theta(\dot{\psi}^0, \dot{\theta}^0, \dot{\gamma}^0)=\theta^1, \tag{2.20}$$
$$\gamma(\dot{\psi}^0, \dot{\theta}^0, \dot{\gamma}^0)=\gamma^1.$$

The system of equations (2.10) was solved numerically on the computer by the modified method of the quickest ascent [4,6].

We would note that translational motion of the legs is created in each step of the iteration process of solution of the system of equations (2.10), and, consequently, the boundary values of the velocities at the leg joints are determined. For this purpose, it is necessary to twice solve the system 3N of linear equations (2.7). The volume of calculations in each step of the iteration process is substantially reduced if, prior to solving the boundary value problems, one solves the corresponding matrix equations by the Gauss method and finds $L^{-1}\bar{S}$ and $L^{-1}\bar{\Omega}$ at the moments of lift-off and touchdown. In this case, in each step of the iteration process of solving the boundary value problem, instead of solving the two systems 3N of linear equations (2.7), it is sufficient to calculate the boundary values of the velocities at the leg joints, according to the formula

$$\dot{\bar{\rho}} = (L^{-1}\dot{\bar{S}}) + (L^{-1}\Omega)\bar{\omega} \Big|_{t=t_0,t_1} . \qquad (2.21)$$

that is, it is sufficient to multiply the matrix $L^{-1}\Omega$ by the value of the vector of the angular velocity of the body, and to add it to the vector $L^{-1}\dot{\bar{S}}$.

Knowing the angular velocity of the body at the moment of lift-off, as a result of solving the boundary value problem (2.17)-(2.19), and the initial velocity of the center of mass of the vehicle from (2.3), we will find the velocity of the center of mass of the body at the moment of lift-off, according to Euler's formula [2] /25

$$\bar{V}^{\circ} = \bar{V}_A^{\circ} + \bar{\omega}^{\circ} \times \bar{\xi}^{\circ} + \dot{\bar{\xi}}^{\circ} . \qquad (2.22)$$

2.5. Simplified Scheme of Construction of Programmed Motion

Let the vehicle have an even number of legs N=2n, suspended symmetrically relative to the plane Oyz, and the right and left leg of each pair of symmetrically suspended legs have identical dynamic and kinematic characteristics.

Let the positions of the vehicle at the moments of lift-off and touchdown satisfy the conditions:

$$\psi(t_0) = \psi(t_1) , \quad \gamma(t_0) = \gamma(t_1) = 0, \qquad (2.23)$$

and the right and left leg of each pair of legs be located symmetrically; then, in the flight phase, the right and left legs will move symmetrically, and the angles of yaw and bank will not change

$$\psi(t) \equiv \psi(t_0) , \quad \gamma(t) \equiv 0. \qquad (2.24)$$

In this case, the problem of constructing the programmed motion can be simplified substantially. Taking into account the symmetry of motion, the law of conservation of the kinetic moment of the vehicle, relative to the center of mass, can be written in the form /26

$$I_{xx}\dot{\theta} + 2\sum_{j=1}^{2}\sum_{i=1}^{N}\{I_{x\alpha_i}\dot{\alpha}_i + I_{x\beta_i}\dot{\beta}_i + I_{x\varphi_i}\dot{\varphi}_i\} = G_{0\xi}^{0}, \quad (2.25)$$

$$G_{0y} \equiv 0, \quad G_{0z} \equiv 0,$$

where

$$I_{xx} = A + M(y_m^2 + z_m^2) + 2\sum_{j=1}^{2}\sum_{i=1}^{n}\{J_{ji}^{11}\sin^2\alpha_i + (J_{ji}^{22}\sin^2\varphi_{ji} + J_{ji}^{33}\cos^2\varphi_{ji})\cos^2\alpha_i +$$
$$+ m_{ji}[(y_m + y_{ji})^2 + (z_m + z_{ji})^2]\},$$
$$I_{x\alpha_i} = \sum_{j=1}^{2}\{(J_{ji}^{33} - J_{ji}^{22})2\sin\varphi_{ji}\cos\varphi_{ji}\cos\alpha_i - m_{ji}(z_m + z_{ji})(x_{ji} - x_{m_i})\},$$
$$I_{x\beta_i} = I_{x\varphi_i} + J_{1i}^{11}\sin\alpha_i + m_{1i}l_{1i}^{c}\{(y_m + y_{1i})\sin\beta_i - (z_m + z_{1i})\cos\beta_i\sin\alpha_i\} +$$
$$+ m_{2i}l_{2i}^{c}\{(y_m + y_{2i})\sin\beta_i - (z_m + z_{2i})\cos\beta_i\sin\alpha_i\},$$
$$I_{x\varphi_i} = J_{2i}^{11}\sin\alpha_i + m_{2i}l_{2i}^{c}\{(y_m + y_{2i})\sin\psi_i - (z_m + z_{2i})\cos\psi_i\sin\alpha_i\}.$$

$$\psi_i = \beta_i + \varphi_i - \pi,$$

$$\varphi_{ji} = \begin{cases}\beta_i, & \text{IF } j=2\\ \varphi_i, & \text{IF } j=1.\end{cases}$$

We will solve the boundary value problem according to the determination of the initial angular velocity of the body, according to the single variable $\dot{\theta}^0$. For the determination of the initial and final velocity at the leg joints, it is sufficient to solve the two systems 3N/2 of linear equations instead of the two systems 3N of linear equations (2.7), that is, lesser dimensionality by twice than with the full scheme of calculations. All the rest of the calculations are also substantially simplified. With regard for the symmetry of motion of the bounding vehicle in the flight phase, planar motion of the vehicle is examined, instead of spatial motion, during construction of the programmed motion according to the simplified scheme.

2.6. Algorithm for Constructing the Programmed Motion Utilizing the Solution of the Boundary Value Problem /27

We will examine the working of the algorithm for constructing the programmed motion of the vehicle in the unsupported phase of the bound as a whole. We will show the sequence of the calculations. The block diagram of the algorithm is given in figure 3; in this case, the number of blocks corresponds to the numbers of points in the description of working of the algorithm given below. The algorithm automatically creates a selection of the simplified or complete scheme of calculations, carried out in blocks 3-6.

1. We will calculate the values of the matrices of transition (1.3), the relative coordinates of the feet and the angles at the leg joints (2.7), and the vector $\bar{\rho}$, directed from the center of mass of the vehicle towards the center of mass of the body (1.1)-(1.2), according to the absolute coordinates of the feet and the coordinates of the body of the vehicle, known from the boundary conditions (2.1), at the moments of lift-off from the supporting surface and touchdown.

2. We will determine the absolute coordinates of the center of mass of the vehicle at the moments of lift-off and touchdown (2.2). We will calculate the duration of the flight phase and the initial velocity of the center of mass of the vehicle (2.3). As a result, the ballistic trajectory of motion of the center of mass of the vehicle is fully determined.

3. We will calculate the elements of the matrix L, Ω and the vector $\bar{S}$, which are part of equations (2.14), at the moments of lift-off and touchdown. By solving the corresponding (2.14) matrix equations, by the Gauss method, we will find the elements of the matrix $L^{-1}\Omega$ and the vector $L^{-1}\bar{S}$ with $t=t_0$ and $t=t_1$ (here and subsequently, t_0 is the moment of the beginning and t_1 is the moment of completion of the flight phase).

4. We will find the initial angular velocity of the body of the vehicle, and the boundary value problem (2.18)-(2.19), which amounts to the solution of the system of equations (2.20). /29
Then, having calculated the initial velocity of the center of mass of the body (2.22), the algorithm completes the work.

Calculation of the left parts of equations (2.20), in the process of the solution of the boundary value problem, takes place in blocks 5,6. We would note that, with each treatment of these blocks, the initial angular velocity of the body of the vehicle is known.

5. Also determined is the angular velocity at the leg joints at the moment of lift-off (2.21), and the vector of the kinetic moment of the vehicle (1.13) is calculated. By solving the system of three linear equations (2.16), we will determine the angular velocity of the body at the moment of touchdown. From relationship (2.21), written out with $t=t_1$, we will find the velocity at the leg joints at the moment of touchdown. We will construct the translational motion of the legs.

6. By integrating the system of differential equations of

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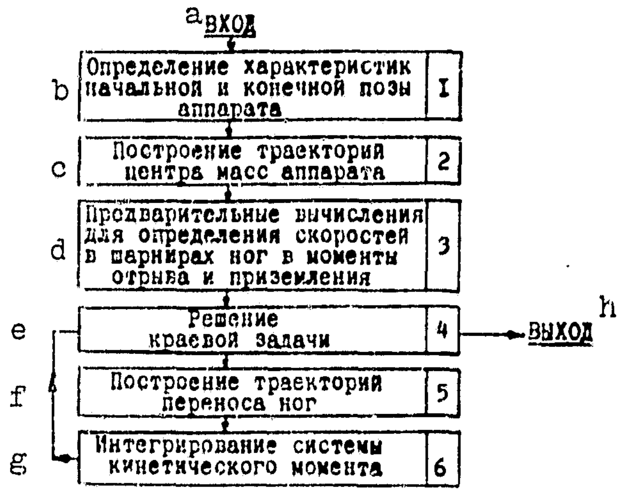

Fig. 3

Key:
a. Input
b. Determination of characteristics of initial and final attitude of vehicle
c. Construction of trajectories of center of mass of vehicle
d. Preliminary calculations for determining the velocities at the leg joints at the moments of lift-off and touchdown
e. Solution of the boundary value problem
f. Construction of the trajectories of shifting of the legs
g. Integration of the system of the kinetic moment
h. Output

the primary integral—the law of conservation of the vector of the kinetic moment (2.18) by the Runge-Kutt method, with given initial conditions and the constructed shifting motion of the legs, we will find the corresponding given initial conditions of the value of the angular coordinates of the body at the moment of touchdown.

Note. In blocks 3-6, all of the references pertain to the formulas which correspond to the complete scheme of calculations. In the case of working of the algorithm according to the simplified scheme (paragraph 2.5), they are simplified, with regard for the symmetry of motion which takes place.

§ 3. Construction of Programmed Motion of Vehicle Utilizing Tables. Standard Motion Conditions. Results of Calculations /30

Calculations were carried out for a four-legged vehicle, having the following dynamic and kinematic characteristics. The mass of the body is 1,000 kg, and the main moments of intertia (in kgm) are: $\tau_x = 530$, $\tau_y = 90$, $\tau_z = 605$. All of the legs are identical, and consist of two sections (thigh and shank). The length of the thigh is 1.2 m, and the length of the shank is 1.4 m. The center of mass of each section is located in the middle. The mass of the thigh is 30 kg, the mass of the shank is 20 kg, the main moments of inertia of the thigh (in kgm) are: $\tau_{x1} = 3.2$, $\tau_{y2} = 0.05$, $\tau_{z2} = 3.2$, and those of the shank are: $\tau_{x1} = 3.6$, $\tau_{y1} = 0.05$, $\tau_{z1} = 3.6$. The coordinates of the points of support of the legs and the body have the values: $x_i = 0.5$ $(i=1,2)$, $x = 0.5$ $(i=3,4)$, $y_i = 1.25$ $(i=1,3)$, $y_i = 1.25$ $(i=2,4)$, $z_i = 0$ $(i=1,2,3,4)$.

The system of units of measurement is the International System of Units. The time is measured in seconds, and the angles in radians. Acceleration of the force of gravity $g = 9.8$ m/sec.

The initial and final programmed positions of the vehicle are determined by the algorithm of formation of the attitude of the vehicle at the moments of lift-off from the supporting surface and touchdown, the working of which is examined in detail in [3]. The index contour in the projection on the horizontal plane $O_1\xi\eta$ is a rectangle, the length of which is equal to the distance between the points of support of the front and rear legs, and the width is equal to twice the lateral extension of the legs from the longitudinal axis of the index contour, which we will designate using Z_b. The input parameters of the algorithm of formation of the attitude, the selection of the values of which is accomplished either by the operator of the bounding vehicle or by higher levels of the motion control system, are: D - the distance of the bound (distance between the centers of the index contours in the projection on the horizontal plane $O_1\xi\eta$), ψ_f - the direction (azimuth) of motion in the flight phase, $\psi^0_{\pi\Lambda}$ - the orientation (azimuth) of the initial index contour, $\psi^1_{\pi\Lambda}$ - the orientation (azimuth) of the final index contour, λ - the tangent of the angle of inclination to the horizontal of the initial velocity of the center of mass of the vehicle, and also information on the supporting surface. At the moments of lift-off from the supporting surface and touchdown, the distance along the vertical from the supporting area to the center of mass of the body is equal to the given value ζ_{nom}, the angles of pitch and bank are equal to zero, $\theta^0 = \theta^1 = \gamma^0 = \gamma^1 = 0$, the angle of yaw is equal to the azimuth of the index contour, $\psi^0 = \psi^0_{\pi\Lambda}$, $\psi^1 = \psi^1_{\pi\Lambda}$. The coordinates ξ and η of the center of mass of the body at the initial and final moments are determined from the condition that, in the projection on the horizontal plane, these points lie on a line connecting the centers of the index contours at a distance $(\zeta_{nom} - \zeta_{min})/\max(1, \lambda^*)$ from the center of the corresponding index contour. Here, ζ_{min} is the minimally permissible /31 distance along the vertical from the supporting surface to the center of mass of the body, and λ^* is the tangent of the angle of inclination of the velocity of the center of mass of the vehicle to the horizontal at the corresponding moment in time.

For the vehicle being examined, we will set ζ_{min}=0.5 m. In study [3], it was shown that the best values, from the point of view of minimization of the maximum magnitudes of the forces developing in the leg joints in the supported phase of motion, are the values ζ_{nom} =1.7 m, ζ_{min}=1 m. Subsequently, we will consider that ζ_{nom} and $l\,b$ take on these values.

The algorithm of construction of the programmed motion of the bounding vehicle was realized in FORTRAN language on the BESI-6 computer. Translation was carried out by the FOREX optimizing translator, developed in the Institute of Applied Mathematics of the Academy of Sciences of the Soviet Union. Construction of the programmed motion of a four-legged vehicle in the flight phase, according to a simplified scheme of calculations for symmetrical motion of the vehicle, takes 0.58 seconds, and for the complete scheme of calculations, 2.9 seconds are required. On the Earth's surface, the duration of the unsupported phase of the bound, a distance of 10 m along /32
the horizontal plane, is roughly 1 second. Therefore, it is necessary either to calculate the programmed motion for the bounding vehicle in advance for the series of bounds, or to have the solution of the problem of construction of the programmed motion for the plight phase in the memory of the computer, which controls the motion of the vehicle. It is advisable to utilize both of these methods for constructing the programmed motion. For a set of standard motion conditions, encompassing a sufficiently broad class of the most frequently encountered motion conditions, it makes sense to retain the solution of the problem of constructing the programmed motion for the flight phase in the memory of the computer. For realization of a series of non-standard bounds, it is necessary to stop the vehicle and make preliminary calculations of the programmed motion.

The basic volume of calculations during construction of the programmed motion of the vehicle for the unsupported phase of the bound is associated with the solution of the boundary value problem on the determination of the initial angular velocity of the body. For purposes of economy of memory of the controlling computer, necessary for remembering the solution of the problem of constructing the programmed motion in the flight phase for standard motion conditions, it is advisable to store only the magnitude of the initial angular velocity of the body. The initial velocity of the center of mass of the body is calculated according to the finite formulas (2.3), (2.22). In this case, with a comparatively small volume of stored information, the volume of calculations for construction of the programmed motion for the unsupported phase of the bound is reduced substantially. Construction of the programmed motion, utilizing the tables, for standard motion conditions makes it possible to reduce the time of solution of this problem up to 0.058 seconds (or roughly by 50 times, as compares with the solution of the problem of construction the programmed motion utilizing the boundary value problem) for the complete scheme of calculations, and for the simplified scheme of calculations—up to 0.037 seconds (roughly by 15 times). /33

Analogous logic is utilized in that case when the vehicle calculates the programmed motion for the series of bounds in advance. Only the initial angular velocity of the body is stored. For realization of motion of the vehicle, the value of the initial velocity of the center of mass of the body is calculated prior to each bound, with regard for corrections which may occur in the process of motion (for example, for the range of the bound).

We will call turning in the flight phase correct if the conditions $\psi_f - \psi^0_{\pi\Lambda} = \psi^1_{\pi\Lambda} - \psi_f$ are fulfilled for the azimuth angle, that is, the direction of motion of the vehicle in the flight phase is the bisector of the angle formed by the longitudinal axes of the index contours at the moments of lift-off from the supporting surface and touchdown. The magnitude $\Delta\psi_{\pi\Lambda} = \psi^1_{\pi\Lambda} - \psi^0_{\pi\Lambda}$ will be called the angle of turning of the vehicle in the flight phase.

We will turn to the description of the standard motion conditions of the vehicle. We require that each of the supporting areas differ little from the section of the horizontal plane. The locale, as a whole, may have a complex form. Under standard motion conditions, the vehicle executes only correct turns within the limits of $|\Delta\psi_{\pi\Lambda}| \leq 0.6$ radians. The range of the bound lies within the range of from 5 to 15 m, or the distance between the front legs of the vehicle at the moment of lift-off from the supporting surface and the rear legs at the moment of touchdown in the projection on the horizontal plane $O_1\xi\eta$ is from one to five times the body of the vehicle. The tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal either takes on one of the standard values

$$\lambda = \{0.6, 0.7, 0.8, 0.9, 1.0, 1.25, 1.5, 1.75, 2.0, 2.5\}, \qquad (3.1)$$

or is calculated in such a manner that the modulus of velocity of the center of mass of the vehicle at the moment of lift-off from the supporting surface for the given bound is minimal. /34

The value of $\lambda<1$ (low-angle trajectory) should be utilized in the "run" condition along a relatively smooth surface [3]. The value of $\lambda>1$ (steep trajectory) should be utilized for bounds on a slippery surface, for bounding across an obstacle of the barrier type, and for overcoming high projections.

The solution of the problem of constructing the programmed motion of the bounding vehicle in the flight phase, for standard motion conditions, depends on the following magnitudes:

D - range of the bound,

$\Delta\psi_{\pi\Lambda}$ - angle of turning of the vehicle in the flight phase,

$\Delta\zeta_{\pi\Lambda}=\zeta^{1}_{\pi\Lambda}-\zeta^{0}_{\pi\Lambda}$ - difference in altitudes of the initial and final supporting areas or the height of the projection being overcome,

λ - tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal.

According to the results of the calculations, we will construct a table of the dependence of the initial angular velocity of the body on the magnitudes of $D, \Delta\zeta_{\pi\Lambda}, \Delta\psi_{\pi\Lambda}$, and λ. We will store the values of the initial angular velocity for the junction points of the table, and for the actual values of D, $\Delta\zeta_{\pi\Lambda}$, and $\Delta\psi_{\pi\Lambda}$, the magnitude of the initial angular velocity will be determined by the method of linear interpolation. The junction points are selected so as to ensure accuracy of interpolation of 0.05 radians/second.

The complete table of the dependence of the initial angular velocity of the body on $D, \Delta\zeta_{\pi\Lambda}, \Delta\psi_{\pi\Lambda}$, and λ for the indicated motion conditions has a sufficiently large volume (about 15,000 elements). For purposes of decreasing the volume of stored information, we will narrow down the class of standard motion conditions. We will exclude the seldom utilized possibilities, placing additional restrictions of the values of the parameters λ and $\Delta\psi_{\pi\Lambda}$, as a function of the height of the projections being surmounted. /35

Let D_0 be the distance between the points of support of the front and rear legs; then, $D_f=D-D_0$ is approximately equal to the distance by which the center of mass of the vehicle is displaced in the flight phase.

According to the height of the projections being surmounted in the given bound, we will divide them into three classes: small projection $|\Delta\zeta_{\pi\Lambda}|\leq 0.05\, D_f$, moderate projection $0.05\, D_f\leq|\Delta\zeta_{\pi\Lambda}|\leq 0.25\, D_f$, high projection $0.25\, D_f\leq|\Delta\zeta_{\pi\Lambda}|\leq 1.5\, D_f$.

We will examine the standard motion conditions which correspond to each class of height of the projection being surmounted.

Small projection $|\Delta\zeta_{\pi\Lambda}| \leqslant 0.05\ D_{f}$. The value of the initial angular velocity of the body, in this case, practically does not depend on $\Delta\zeta_{\pi\Lambda}$.

Given in figure 4 is the dependence of the initial angular velocity, according to the angle of pitch $\dot{\theta}^{0}$, on the distance of the bound D for some values of the tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal λ for bounds without turning $\Delta\psi_{\pi\Lambda}=0$.

In order to ensure the required accuracy of interpolation, it is sufficient, for each λ, to store the value of the initial angular velocity for five junction values of D which are multiples of the length of the body

$$\mathcal{D} = \{5.0, 7.5, 10.0, 12.5, 15.0\}. \qquad (3.2)$$

The tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal takes on one of the standard values (3.1).

Given in figure 5 is the dependence of the initial angular velocity of the body on the angle of turning of the vehicle in azimuth in the flight phase $\Delta\psi_{\pi\Lambda}$ with a bound distance D= 10 m for some values of λ. The values of the initial angular velocity of the body, obtained through calculations, approximate the linear functions well. Analogous results of calculations for other values of D and λ show that it is sufficient to have three junction values of $\Delta\psi_{\pi\Lambda}$

$$\Delta\psi_{\pi\Lambda} = \{0.0, 0.3, 0.6\} \qquad (3.3)$$ /38

With a change in the sign of the angle of turning in azimuth in the flight phase $\Delta\psi_{\pi\Lambda}$, in view of the symmetry of the vehicle, the initial velocity, according to the angle of pitch, is not changed, while the velocities according to the angles of bank and yaw change signs. This makes it possible to determine the initial angular velocities of the body for negative angles of turning $\Delta\psi_{\pi\Lambda}<0$, having in the memory the values of the initial angular velocity of the body which correspond only to positive angles of turning $\Delta\psi_{\pi\Lambda}>0$.

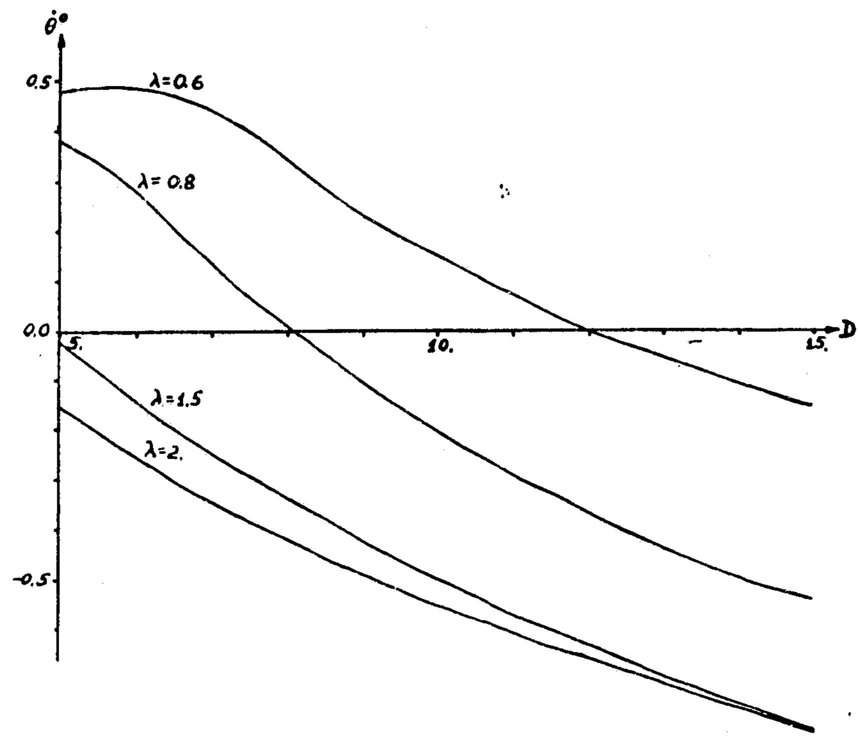
$\dot{\theta}^{\circ}$
0.5
0.0
-0.5
λ=0.6
λ= 0.8
λ=1.5
λ=2.
5.
10.
15.
D

Fig. 4

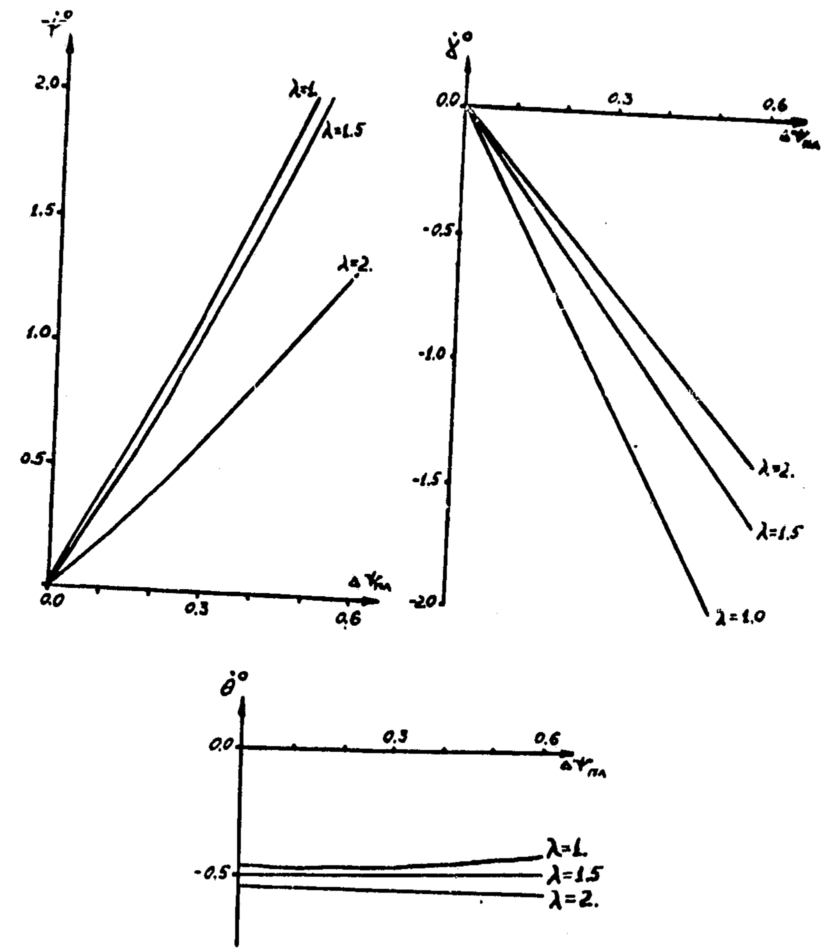
2.0
1.5
1.0
0.5
0.0
0.3
0.6
λ=1.
λ=1.5
λ=2.
0.0
0.3
0.6
-0.5
-1.0
-1.5
-2.0
λ=2.
λ=1.5
λ=1.0
0.0
0.3
0.6
-0.5
λ=1.
λ=1.5
λ=2.

Fig. 5

The stored table of values of the initial angular velocity consists of 350 elements. It is taken into account here that, with $\Delta\psi_{\pi\Lambda}=0$, it is necessary to store only the values of the initial velocity according to the angle of bank $\dot{\theta}^0$ (initial velocities according to the angles of pitch and yaw, in this case, are equal to zero, $\dot{\psi}^0=\dot{\gamma}^0=0$).

High projection $0.25D_f \leq |\Delta\zeta_{\pi\Lambda}| \leq 1.5D_f$. We require that, in surmounting a high projection, the vehicle does not execute a turn in the flight phase, according to the angle of yaw, i.e., $\Delta\psi_{\pi\Lambda}=0$. The tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal will be selected so as to minimize the value of the modulus of velocity of the center of mass of the vehicle at the moment of lift-off from the supporting surface. Taking into account the fact that the center of mass of the vehicle in the flight phase moves along a parabolic trajectory (1.4), we will obtain the optimal value

$$\lambda = \frac{\Delta\zeta_{\pi\Lambda}}{D_a} + \sqrt{\left(\frac{\Delta\zeta_{\pi\Lambda}}{D_a}\right)^2 + 1}, \qquad (3.4)$$

where D_a is the distance traversed by the center of mass of the vehicle in the flight phase in the projection on the horizontal plane $O_1\xi\eta$. In this case, the modulus of velocity of the center of mass of the vehicle at the moment of touchdown will also have a minimum value. Utilized in the calculations was an approximated formula, obtained by the substitution of D_a for its approximated value D_f in (3.4). /40

The dependence of the initial angular velocity of the body of the vehicle on the range of the bound for some values of the relative height of the surmounted projection $\delta=\Delta\zeta_{\pi\Lambda}/D_f$ is given in figure 6.

It follows from the calculations that it is sufficient to have five junction values for the range of the bound (3.2) and twelve junction values of $\Delta\zeta_{\pi\Lambda}$

$$\Delta\zeta_{\pi\Lambda} = \{\pm 0.25D_f, \pm 0.5D_f, \pm 0.75D_f, \pm 1.0D_f, \pm 1.25D_f, \pm 1.5D_f\}. \qquad (3.5)$$

The table of corresponding values of the initial angular velocity of the body contains 60 elements.

Moderate projection $0.05D_f \leq |\Delta\zeta_{\pi\Lambda}| \leq 0.25D_f$. In this case, we will examine two standard motion conditions:

1. The vehicle may execute a turn in the flight phase

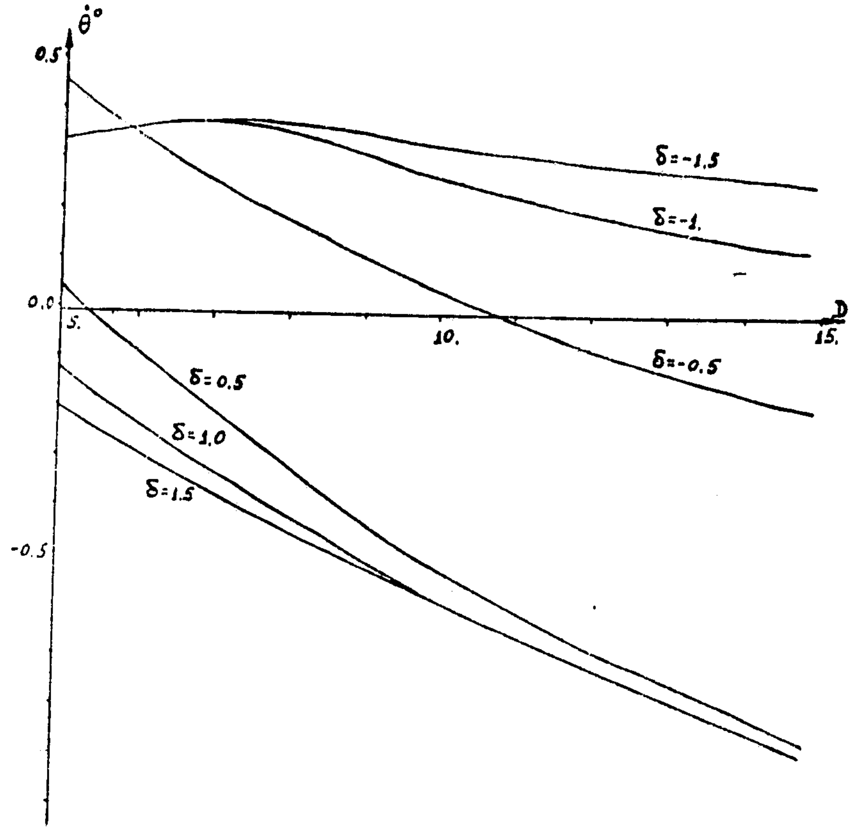

Fig. 6

according to the angle of yaw. The value of the tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle we will calculate as well, just as for surmounting a high projection (3.4).

Given in figure 7 is the dependence of the initial angular velocity of the body of the vehicle on the angle of turning in the flight phase $\Delta\psi_{\pi\Lambda}$ with a range of the bound D=10 m, for some values of the relative height of the surmounted projection $\delta=\Delta\zeta_{\pi\Lambda}/D_f$.

It follows from the results of the calculations that it is sufficient to have five junction values of the range of the bound D (3.2), three junction values of the angle of turning $\Delta\psi_{\pi\Lambda}$ (3.3), and six junction values of the difference in heights of the supporting areas $\Delta\zeta_{\pi\Lambda}$

$$\Delta\zeta_{\pi\Lambda}=\{\pm 0.05 D_f, \pm 0.15 D_f, \pm 0.25 D_f\}. \tag{3.6}$$

The stored table, in this case, contains 210 elements. /42

2. The vehicle surmounts an average projection without turning according to the angle of yaw $\Delta\psi_{\pi\Lambda}=0$. The tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal may take on one of ten standard values (3.1). We require that, at the moment of touchdown, the tangent of the angle of inclination of the velocity of the center of mass of the vehicle to the horizontal is at least 0.5. Then, we obtain the stricter restriction for the maximum value of $\Delta\zeta_{\pi\Lambda}$ with $\Delta\zeta_{\pi\Lambda}>0$

$$\Delta\zeta_{\pi\Lambda} \leq \Delta\zeta_{max}(\lambda, D_f), \tag{3.7}$$

where

$$\Delta\zeta_{max}=\begin{cases} 0.25 D_f & \text{, with } \lambda \geq 1 \\ \dfrac{\lambda-0.5}{2} D_f & \text{, with } \lambda < 1 \end{cases}$$

If a signal with the required values of λ, D, and $\Delta\zeta_{\pi\Lambda}$ enters the input of the algorithm for constructing the programmed motion, so that $\Delta\zeta_{\pi\Lambda}>\Delta\zeta_{max}(\lambda,D_f)$, then the least value of λ^* is selected

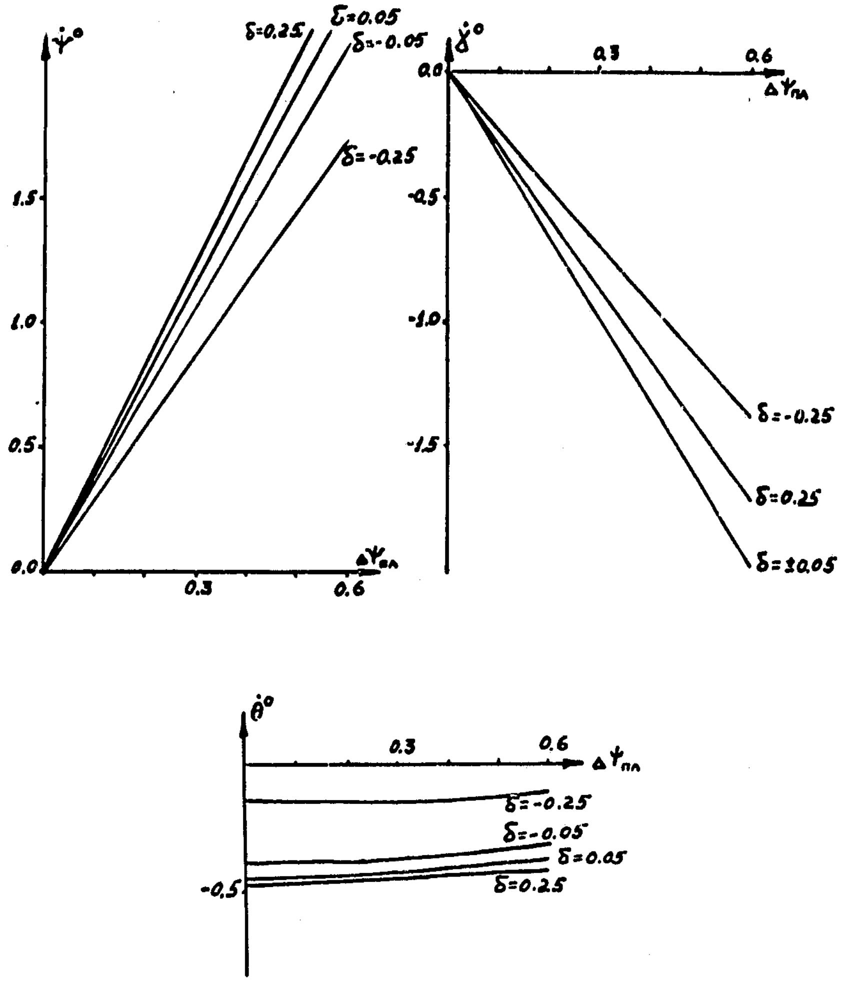
δ=0.25
ε=0.05
δ=-0.05
δ=-0.25
1.5
1.0
0.5
0.0
0.3
0.6
0.0
0.3
0.6
-0.5
-1.0
-1.5
δ=-0.25
δ=0.25
δ=±0.05
0.3
0.6
δ=-0.25
δ=-0.05
δ=0.05
δ=0.25
-0.5

Fig. 7

from (3.1), with which $\Delta\zeta_{\pi\Lambda} \leqslant \Delta\zeta_{max}(\lambda^*, D_f)$. This value of the tangent of the angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal is considered programmed $\lambda = \lambda^*$.

For the required accuracy of interpolation, it is sufficient to have five junction values of D (3.2) and six junction values of $\Delta\zeta_{\pi\Lambda}$

$$\Delta\zeta_{\pi\Lambda} = \{-0.05 D_f, -0.15 D_f, -0.25 D_f, 0.05 D_f, \frac{0.05 D_f + \Delta\zeta_{max}(\lambda, D_f)}{2}, \Delta\zeta_{max}(\lambda, D_f)\}. \quad (3.8)$$

For the given motion conditions, it is necessary to store a table of initial values of the angular velocity of the body consisting of 300 elements.

As a result, we obtain the fact that for storing of the initial angular velocity of the body of the vehicle for all of the examined standard motion conditions, it is necessary to store a table of 920 elements. /43

For standard motion conditions, the values of the initial angular velocity of the body is determined from the table, stored in the memory of the control computer, and the initial velocity of the center of mass of the body is calculated according to finite formulas (2.3), (2.22). Such an approach makes it possible to solve the problem of constructing the programmed motion of the vehicle for the flight phase in the process of motion of a bounding vehicle. With a comparatively small volume of stored information (920 numbers) in the BESI-6 computer, construction of the programmed motion takes 0.058 seconds for the complete scheme of calculations, and 0.037 seconds for the simplified scheme of calculations.

Similar results were obtained for a six-legged vehicle. In this case, construction of the programmed motion, utilizing the solution of the boundary value problem for determining the initial angular velocity of the body, takes 4.6 seconds for the complete scheme of calculations, and 0.86 seconds for the simplified scheme of calculations. For standard motion conditions, construction of the programmed motion for the flight phase, utilizing the tables, takes 0.097 seconds for the complete scheme of calculations, and 0.054 seconds for the simplified scheme of calculations.

Working out of the algorithms of the system of construction of the programmed motion of the vehicle in the unsupported phase of the bound, by the method of mathematical simulation on the computer, showed the effectiveness of the proposed solutions.

We will study the question of the dependence of the solution of the problem of constructing the programmed motion of the vehicle for the unsupported phase of the bound on the relationship of the mass of the body and the total mass of the legs. We will designate this relationship through $\mu = M/\sum_{i=1}^{N}\sum_{j=1}^{3} m_{ji}$. We will establish the mass of the legs, and will change the mass of the body. We will examine bounds without turning of a four-legged vehicle on a horizontal plane at a distance D. /45
The angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal is equal to 45°. In view of the symmetry of motion, the angles of yaw and bank are identically equal to zero. Given in figure 8 is the dependence of the initial angular velocity of the body, according to the angle of pitch $\dot{\theta}^{0}$, on μ for some values of the bound range D. With $\mu \to \infty$, the mass of the body is considerably greater than the total mass of the legs, and, consequently, the translational motion of the legs practically has no effect on the motion of the body $\dot{\theta}^{0} \to 0$. With $\mu \to 0$, the initial angular velocity of the body approaches ∞. The energy of rotational motion of the body $\frac{1}{2}\tau_{x}(\dot{\theta}^{0})^{2}$ is a monotonously diminishing function of μ. That is, $\dot{\theta}^{0}$ diminishes more rapidly than $1/\sqrt{\mu}$. On the whole, for the examined cases of $\mu \geqslant 0.5$, the energy of rotational motion of the body comprises no more than 1% of the kinetic energy of motion of the center of mass of the vehicle at the moment of lift-off from the supporting surface.

The total energy outlays in the supported phase of the bound from place are associated with the transmission of kinetic energy to the center of mass of the vehicle and the kinetic energy of rotational motion to the body and leg sections by the moment of lift-off from the supporting surface, as well as with a change in the potential energy of the vehicle. We will designate the energy outlays in the supported phase of the bound from place using W, and the specific energy outlays per unit of mass of the vehicle using $w = W/(M + \sum_{i=1}^{N}\sum_{j=1}^{3} m_{ji})$. We will study the question of the dependence of the specific energy outlays w on the range of the bound without turning on the horizontal plane D and the relationship of the mass of the body to the total mass of the legs μ. The angle of inclination of the initial velocity of the center of mass of the vehicle to the horizontal is equal to 45°. For the examined cases of $\mu \geqslant 0.5$, the specific energy outlays w practically do not depend on μ. The dependence of the specific energy outlays w (in Joules/kg) on the range of the bound D is given in figure 9. /46
In the supported phase of the bound from place, let the center of mass move with constant acceleration, while the distance traversed by the center of mass of the body, according to the coordinate ζ, is equal to $\zeta_{nom} - \zeta_{min}$, where ζ_{nom} is the given

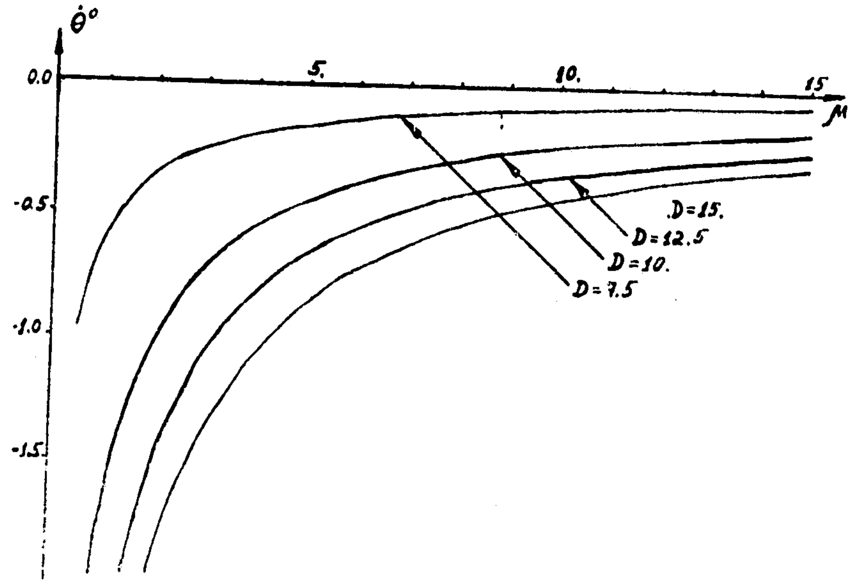

Fig. 8

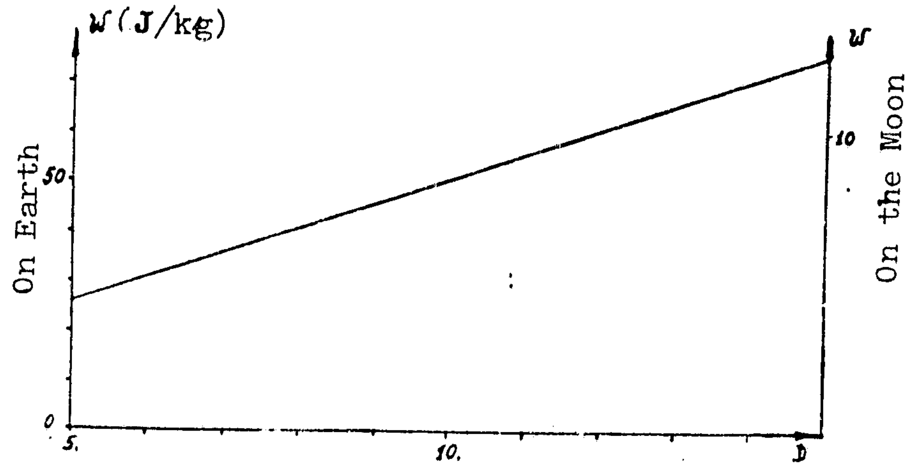
W (J/kg)
W
On Earth
On the Moon
50
10
0
5.
10.
D

Fig. 9

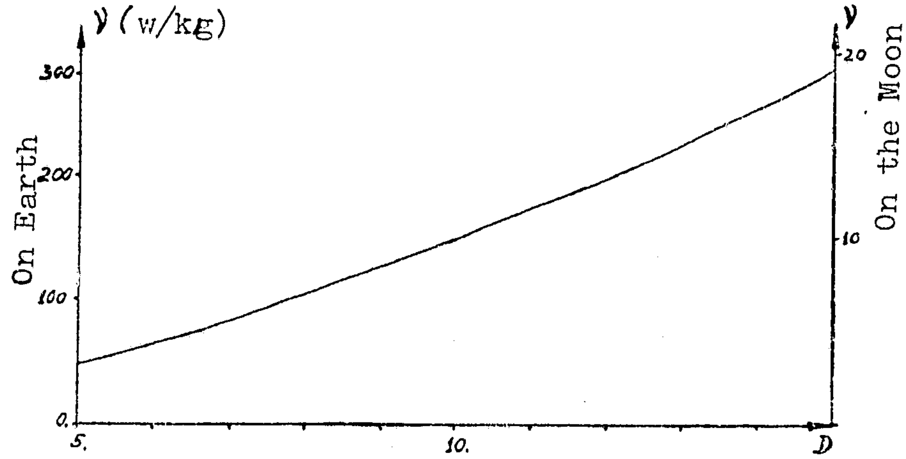
ν (w/kg)
ν
On Earth
On the Moon
300
200
100
0.
20
10
5.
10.
D

Fig. 10

nominal distance from the supporting surface to the center of mass of the body at the moment of lift-off from the supporting surface, and ζ_{min} is the given minimal permissible distance from the supporting surface to the center of mass of the body. For the examined vehicle, we will set $\zeta_{nom}=1.7$ m and $\zeta_{min}=0.5$ m, and we will determine the duration of the supported phase T_{on}. Then, the average specific forces per unit of weight, developed in the supported phase of the bound from place, are equal to $\nu=w/T_{on}$. The dependence of the average specific forces ν (in watts/kg) on the range of the bound is given in figure 10.

We will study the question of the dependence of the specific energy outlays and the average specific outputs from acceleration of the force of gravity. Let g, as before, be the acceleration of the force of gravity on Earth, and g^* be the acceleration of the force of gravity on a planet on which the bounding vehicle is located. In view of the fact that, with a fixed range of the bound the square of the velocity of the center of mass of the vehicle at the moment of lift-off from the supporting surface is proportional to the acceleration of the force of gravity, and the duration of the supported phase of motion is inversely proportional to the initial velocity of the center of mass, we obtain

$$w^* = w\,\frac{g^*}{g}\,, \quad \nu^* = \nu\left(\frac{g^*}{g}\right)^{3/2}.$$

Thus, for example, for a vehicle moving along the surface of the moon, $g^*=1.62$ m/sec. Consequently, on the surface of the moon, the specific energy outlays are roughly 6 times, and the average specific outputs are roughly 15 times, less than on Earth. The obtained results make it possible to draw a conclusion on the possibility of the effective utilization of bounding vehicles under the conditions of planets with a low force of gravity.

The results obtained in the given paragraph can be /48 generalized, utilizing the methods of similarity and dimensionality in mechanics [2]. With a proportional change in all of the linear dimensions by l times, and the mass of the body and the leg sections by m times, the basic characteristics of motion change in the following manner:

duration of unsupported and supported phases of the bound by $\sqrt{l}$ times,

velocity of the center of mass of the vehicle and the body by $\sqrt{l}$ times,

angular velocity of the body and velocities at the leg joints by $1/\sqrt{l}$ times, energy outlays in the supported phase of the bound from place by $m\ell$ times,

specific energy outlays w by ℓ times,

average specific outputs ν by $\sqrt{\ell}$ times.

Conclusion /49

Examined in the study is the problem of constructing the programmed motion of a bounding vehicle in the flight phase. According to the results of the study, one can draw the following conclusions:

1. A mathematical model of the spatial motion of the bounding vehicle in the flight phase is constructed. The primary integrals of the equations of motion are obtained (law of motion of the center of mass of the vehicle and law of conservation of the moment of linear momentum relative to the center of mass).

2. A method is proposed for constructing the translational motion of the legs in a relative system of coordinates, which ensures non-impact of lift-off and the required softness of placement of the legs on the supporting surface.

3. An algorithm is constructed for constructing the programmed motion of the vehicle in the flight phase, which calculates the value of the linear and angular velocity of the body at the moment of lift-off from the supporting surface, necessary for realization of the impending flight phase.

4. The construction of the programmed motion in the flight phase is carried out according to a complete or simplified scheme of calculations, if this permits symmetry of motion of the vehicle. In the case of working of the algorithm according to a simplified scheme, a planar motion is actually examined instead of spatial motion of the vehicle.

5. The basic volume of the calculations for construction of the programmed motion is associated with the solution of the boundary value problem for determining the initial angular velocity of the body. Here, one must numerically integrate the differential equations.

6. For standard motion conditions, which encompass a broad class of the most frequently encountered motion conditions of the vehicle, a table is compiled of the dependence of the initial angular velocity of the body on the parameters of the impending flight phase. The value of the initial angular velocity of the body is determined from this table. The volume of stored information is comparatively small. The value of /50

the initial velocity of the center of mass of the body is computed according to finite formulas. Such an approach sharply reduces the time of calculation, and makes it possible to solve the problem of constructing the programmed motion for the flight phase in the process of motion of the vehicle.

7. The system of construction of the programmed motion of the bounding vehicle in the flight phase, which is part of its motion control system, automatically selects a complete or simplified scheme of calculations both for non-standard and for standard motion conditions.

8. The algorithms for the system of construction of the programmed motion of the vehicle were worked out for the unsupported phase of motion by means of mathematical simulation on a computer.

Appendix

Coefficients of the Integral of the Kinetic Moment

$$I_{xx} = J_x + M(y_m^2 + z_m^2) + \sum_{i=1}^{N} \sum_{j=1}^{2} \{ J_{ji}^{11} \sin^2\alpha_i + (J_{ji}^{22} \sin^2\varphi_{ji} + J_{ji}^{33} \cos^2\varphi_{ji}) \cos^2\alpha_i +$$
$$+ (J_{ji}^{12} \sin\varphi_{ji} + J_{ji}^{13} \cos\varphi_{ji}) \sin 2\alpha_i + J_{ji}^{23} \cos\alpha_i \sin 2\varphi_{ji} + m_{ji} [(y_m + y_{ji})^2 +$$
$$+ (z_m + z_{ji})^2] \}$$

$$I_{yy} = J_y + M(x_m^2 + z_m^2) + \sum_{i=1}^{N} \sum_{j=1}^{2} \{ J_{ji}^{11} \cos^2\alpha_i + (J_{ji}^{22} \sin^2\varphi_{ji} + J_{ji}^{33} \cos^2\varphi_{ji}) \sin^2\alpha_i -$$
$$- (J_{ji}^{12} \sin\varphi_{ji} + J_{ji}^{13} \cos\varphi_{ji}) \sin 2\alpha_i + J_{ji}^{23} \sin\alpha_i \sin 2\varphi_{ji} + m_{ji} [(x_m + x_{ji})^2 +$$
$$+ (z_m + z_{ji})^2] \}$$

$$I_{zz} = J_z + M(x_m^2 + y_m^2) + \sum_{i=1}^{N} \sum_{j=1}^{2} \{ J_{ji}^{22} \cos^2\varphi_{ji} + J_{ji}^{33} \sin^2\varphi_{ji} - J_{ji}^{23} \sin 2\varphi_{ji} +$$
$$+ m_{ji} [(x_m + x_{ji})^2 + (y_m + y_{ji})^2] \}$$

$$I_{xy} = -M x_m y_m - \sum_{i=1}^{N} \sum_{j=1}^{2} \{ (J_{ji}^{11} - J_{ji}^{22} \sin^2\varphi_{ji} - J_{ji}^{33} \cos^2\varphi_{ji} - J_{ji}^{23} \sin 2\varphi_{ji}) \sin\alpha_i \cos\alpha_i +$$
$$+ m_{ji} (x_m + x_{ji})(y_m + y_{ji}) \}$$

$$I_{xz} = -M x_m z_m - \sum_{i=1}^{N} \sum_{j=1}^{2} \{ [(J_{ji}^{22} - J_{ji}^{33}) \frac{\sin 2\varphi_{ji}}{2} + J_{ji}^{23} \cos 2\varphi_{ji}] \cos\alpha_i +$$
$$+ (J_{ji}^{12} \cos\varphi_{ji} + J_{ji}^{13} \sin\varphi_{ji}) \sin\alpha_i + m_{ji} (x_m + x_{ji})(z_m + z_{ji}) \}$$

$$I_{yz} = -M y_m z_m - \sum_{i=1}^{N} \sum_{j=1}^{2} \{ [(J_{ji}^{22} - J_{ji}^{33}) \frac{\sin 2\varphi_{ji}}{2} + J_{ji}^{23} \cos 2\varphi_{ji}] \sin\alpha_i -$$
$$- (J_{ji}^{12} \cos\varphi_{ji} + J_{ji}^{13} \sin\varphi_{ji}) \cos\alpha_i + m_{ji} (y_m + y_{ji})(z_m + z_{ji}) \}$$

$$I_{x\alpha_i} = \sum_{j=1}^{2} \{ (-J_{ji}^{12} \cos\varphi_{ji} + J_{ji}^{13} \sin\varphi_{ji}) \sin\alpha_i - J_{ji}^{23} \cos\alpha_i \cos 2\varphi_{ji} +$$
$$+ (J_{ji}^{33} - J_{ji}^{22}) \frac{\sin 2\varphi_{ji}}{2} \cos\alpha_i - m_{ji} (z_m + z_{ji})(x_{ji} - x_{ai}) \}$$

$$I_{x\beta_i} = I_{x\varphi_i} + J_{2i}^{11} \sin\alpha_i + (J_{2i}^{12} \sin\beta_i + J_{2i}^{13} \cos\beta_i) \cos\alpha_i + m_{2i} l_{2i} \{ (y_m + y_{2i}) \sin\beta_i -$$
$$- (z_m + z_{2i}) \cos\beta_i \sin\alpha_i \} + m_{3i} l_{3i}^{*} \{ (y_m + y_{3i}) \sin\beta_i - (z_m + z_{3i}) \cos\beta_i \sin\alpha_i \}$$

$$I_{x\varphi_i} = J_{1i}^{11} \sin\alpha_i + (J_{1i}^{12} \sin\varphi_i + J_{1i}^{12} \cos\varphi_i) \cos\alpha_i + m_{1i} l_{1i}^{*} \{ (y_m + y_{1i}) \sin\varphi_i -$$
$$- (z_m + z_{1i}) \cos\varphi_i \sin\alpha_i \}$$

$$I_{y\alpha_i} = \sum_{j=1}^{2} \{ (J_{ji}^{33} - J_{ji}^{22}) \frac{\sin 2\varphi_{ji}}{2} \sin\alpha_i - J_{ji}^{23} \cos 2\varphi_{ji} \sin\alpha_i + (J_{ji}^{12} \cos\varphi_{ji} -$$
$$- J_{ji}^{13} \sin\varphi_{ji}) \cos\alpha_i - m_{ji} (z_m + z_{ji})(y_{ji} - y_{ai}) \}$$

$$I_{y\beta_i} = I_{yq_i} - J_{2i}^{11}\cos\alpha_i + (J_{2i}^{12}\sin\beta_i + J_{2i}^{13}\cos\beta_i)\sin\alpha_i - m_{1i}\,l_{2i}\{(x_m + x_{1i})\sin\beta_i -$$
$$-(z_m + z_{1i})\cos\beta_i\cos\alpha_i - m_{2i}\,l_{2i}^{43}\{(x_m + x_{2i})\sin\beta_i - (z_m + z_{2i})\cos\beta_i\cos\alpha_i\}$$

$$I_{yq_i} = -J_{1i}^{11}\cos\alpha_i + (J_{1i}^{12}\sin\varphi_i + J_{1i}^{13}\cos\varphi_i)\sin\alpha_i - m_{1i}\,l_{1i}^{43}\{(x_m + x_{1i})\sin\varphi_i -$$
$$-(z_m + z_{1i})\cos\varphi_i\cos\alpha_i\}$$

$$I_{\alpha\alpha_i} = \sum_{j=1}^{2}\{J_{ji}^{22}\cos^2\varphi_{ji} + J_{ji}^{33}\sin^2\varphi_{ji} - J_{ji}^{23}\sin 2\varphi_{ji} + m_{ji}[(x_m + x_{ji})(x_{ji} - x_{ni}) +$$
$$+(y_m + y_{ji})(y_{ji} - y_{ni})]\}$$

$$I_{z\beta_i} = I_{zq_i} + J_{2i}^{13}\sin\beta_i - J_{2i}^{12}\cos\beta_i + \{m_{1i}\,l_{2i}[(x_m + x_{1i})\sin\alpha_i - (y_m + y_{1i})\cos\alpha_i] +$$
$$+ m_{2i}\,l_{2i}^{43}[(x_m + x_{2i})\sin\alpha_i - (y_m + y_{2i})\cos\alpha_i]\}\cos\beta_i$$

$$I_{zq_i} = J_{1i}^{13}\sin\varphi_i - J_{1i}^{12}\cos\varphi_i + m_{1i}\,l_{1i}^{43}\cos\varphi_i\{(x_m + x_{1i})\sin\alpha_i - (y_m + y_{1i})\cos\alpha_i\},$$

where

$$\varphi_i = \beta_i + q_i - \pi,$$

$$\varphi_{ji} = \begin{cases}\beta_i, & \text{IF} \quad j=2\\ \varphi_i, & \text{IF} \quad j=1\,.\end{cases}$$

REFERENCES

1. Alekseeva, L. A., Golubev, Yu. F., "Model' dinamiki shagayushchego apparata" Model of Dynamics of a Walking Vehicle , Izvestiya AN SSSR, Tekhnicheskaya kibernetika, 3 (1975).

2. Appel', P., Teoreticheskaya mekhanika 1 & 2, Moscow, Fizmatgiz, 1960.

3. Lapshin, V. V., "Opornaya faza dvizheniya mnogonogogo prygayushchego apparata" Supported Phase of Motion of a Multi-Legged Bounding Vehicle , Preprint IMP AN SSSR, 126 (1978).

4. Lebedev, V. I., "Raschet dvizheniya kosmicheskogo apparata s maloy tyagoy" Calculation of Motion of Space Vehicle with Small Thrust , Series Matematicheskie metody v dinamike kosmicheskikh apparatov, 4, VTs AN SSSR, 1963.

5. Okhotsimskiy, D. Ye., Golubev, Yu. F., Alekseeva, L. A., "Upravleniye dinamicheskoy model'yu shagayushchego apparata" Control of a Dynamic Model of a Walking Vehicle , Preprint IPM AN SSSR, 20 (1974).

6. Eneyev, T. M., "Nekotorye voprosy primeneniya metoda naiskoreyshego spuska" Some Questions of the Use of the Method of Most Rapid Launch , Preprint IPM AN SSSR, 17 (1970).