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STATISTICAL ENERGY ANALYSIS RESPONSE PREDICTION METHODS FOR STRUCTURAL SYSTEMS

FINAL REPORT

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PREFACE

This document is submitted by the McDonnell Douglas Astronautics Company to the National Aeronautics and Space Administration and was prepared under Contract NAS8-33191, "Statistical Energy Analysis of Complex Structures." The study was directed by R. F. Davis. J. B. Herring of the Vibration Analysis Branch of the Systems Dynamics Laboratory of Marshall Space Flight Center administered and directed the contract.

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SUMMARY

The aerodynamic and external acoustic environments of current aerospace vehicles generate very significant high-frequency random vibration structural response. This vibration response has proven to be a primary design consideration for the short life requirements (one flight) of past space vehicles. The effects of this adverse vibration environment will increase for the reusable vehicles that are currently being developed for space exploration.

The most efficient method to achieve optimum vehicle design for this high-frequency vibration environment is to generate meaningful design and test criteria early in the design phase of the system and to periodically update these criteria throughout the various phases of vehicle development. The methods included under the term "statistical energy analysis," or "SEA," provide a means of predicting such high-frequency vibration criteria in systems that do not conform well to other analysis methods. These energy analysis techniques are denoted "statistical" because they involve averaging structural responses over portions of the structure. This averaging is performed over time and space and in frequency bands.

Response predictions for a structure are made by modeling the structure as a number of elements, deriving power flow equations for each of these elements (including acoustic or mechanical energy sources), and simultaneously solving the resultant system of equations for the element response levels.

This report presents the results of an effort to document methods for accomplishing such response predictions for commonly encountered aerospace structural configurations. The effort included application of these methods to specified aerospace structure to provide sample analyses. The report has been arranged in the form of an applications manual, with the structural analyses appended as example problems. Comparisons of the response

predictions with measured data are provided for three of the example problems. Other appendices provide a derivation of statistical energy analysis response equations, application guidelines, and response solution programs for programmable calculators.

SYMBOLS

A	area
a	acceleration
C/C_c	fraction of critical damping
C_g	group velocity ($= 1.07\,[\omega\, C_\ell\, t]^{1/2}$)
C_ℓ	longitudinal wave velocity
c_o	speed of sound in air
D	dissipation of damping; bending stiffness
E	total energy of an element, modulus of elasticity
f	frequency
G, g	gravitational acceleration
h	thickness
I	moment of inertia
L, ℓ	length
m	mass
N	number of modes
$n(\omega)$	modal density
P	pressure
r	radius of curvature
S	power input
t	thickness
V	volume; velocity
v	velocity
W, w	width, weight
$\overline{w}$	weight density
Δ	incremental value
η_a	damping loss factor $\left(= \frac{D}{2\pi E} = 2C/C_c\right)$
$\eta_{a,b}$	coupling loss factor $\left(= \frac{\phi_{a,b} N_b}{\omega}\right)$
κ	radius of gyration
ν	Poisson's ratio
π	3.14159

ρ	density
σ	radiation efficiency
$\phi_{a,b}$	average mode-to-mode coupling between Elements a and b
ω	angular frequency, normally center frequency of a frequency band

NOTATION

Element	a set of modes modeled as one unit of a system, all modes in a frequency band having identical energy (on the average)
System	the total structure and associated energy sources under consideration (may be only a portion of an actual structure)
$\langle\overline{\quad}\rangle$	indicates averaging over both time and space

INTRODUCTION

Three methods are in common usage for predicting the vibration response of aerospace structural systems: classical modal techniques, comparative scaling using available data banks, and empirical formulas. Each of these methods is limited in application by inherent characteristics of the method. Although classical dynamic analysis techniques for predicting dynamic response work well in the frequency range of the lower structural resonances, their application to high-frequency regimes is limited by model complexity, computer size capability, and cost, and is not amenable to rapid estimates. Use of data banks is limited to structural configurations which are very similar to the previous designs on which the data bank is based. Likewise, empirical formulas can, in general, be applied validly only to structures of the specific configuration for which they were derived and which match previous designs.

The advent of reusable space vehicles featuring new configurations with unique forcing fields requires extending these techniques beyond the configurations from which they were developed. An alternative and relatively simple approach to high-frequency vibration analysis has been developed which is known as statistical energy analysis (SEA).

Statistical energy methods have been developed to consider the distribution and transfer of energy among the modes of a vibrating system. These methods assume that the modes of a system being analyzed contain all the vibratory energy of that system. Therefore, for SEA to have valid application, all significant energy of a system must be "resonant" as opposed to "nonresonant." A parameter for evaluating this condition is examined in Reference 1.

The SEA methods separate the frequency range of interest into frequency bands, which are analyzed independently. The methods assume that the energy in the modes of one frequency band is not transmitted (through coupling) to modes in other frequency bands, either within an element or among the elements of a system.

An important factor in validating the space and frequency averaging inherent in SEA is the number of modes included in each frequency band. With many modes excited in one frequency band of an element, the vibratory energy may be expected to be well distributed throughout the element and among the various modes, and averaging will furnish a valid approximation of actual values. This effect is demonstrated in Figure 1 which shows, for a particular structural system, that predictions made with fewer than 20 modes per element per frequency band exhibit considerably more scatter than predictions with more contributing modes. Since constant percentage bandwidths (such as octave or one-third-octave) are generally utilized to obtain predictions, the narrower bandwidths in the lower frequencies result in fewer contributing modes and therefore less accuracy at these lower frequencies. Accordingly, SEA is generally applied to frequencies of 100 Hz or greater for typical aerospace structure. Also, model elements are chosen as generally gross portions of structure rather than representing fine details in order to maintain a high number of contributing modes per element. An example of such modeling for a section of skirt structure on a launch vehicle would utilize three elements, one element representing the skin/stringer external shell, another representing an equipment-mounting panel, and the third representing the components on the panel (which are considered to be "smeared" over the panel in an average sense).

The assumptions, then, upon which statistical energy analysis is based are:

A. The modes of the elements of a system contain all the vibratory energy of the system.

B. Only modes occurring within the same frequency band are coupled.

C. The energy in one frequency band of a system element is equally distributed among the modes of that element occurring in the frequency band.

D. For two coupled elements, all of the modes occurring in one of the elements in one frequency band are equally coupled to each mode occurring in the same frequency band in the other element.

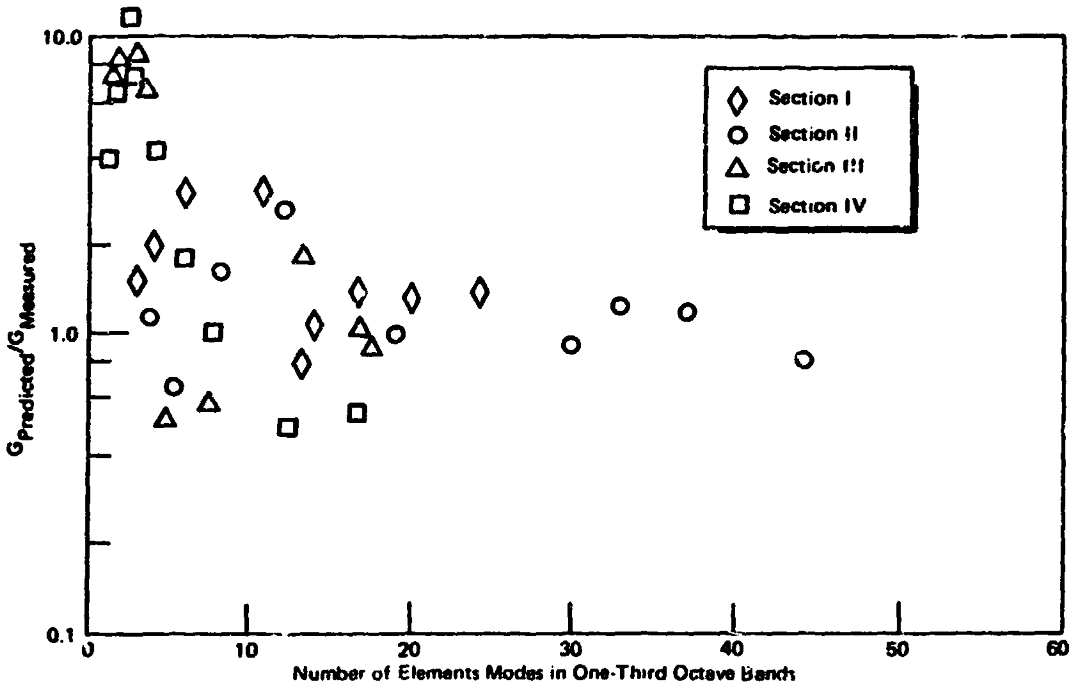


Figure 1. Prediction Accuracy of SEA as a Function of Number of Element Modes Participating (from Reference 2)

DESCRIPTION

The following section will demonstrate the application of SEA methods to a common structural system of interest. While particular systems will require unique adaptations of the methods, the analytical procedure should be fairly general.

Consider a section of airframe of an aerospace vehicle, such as a missile, reentry vehicle, or airplane, consisting of the external shell, an internally mounted equipment panel, and a component mounted on the panel.

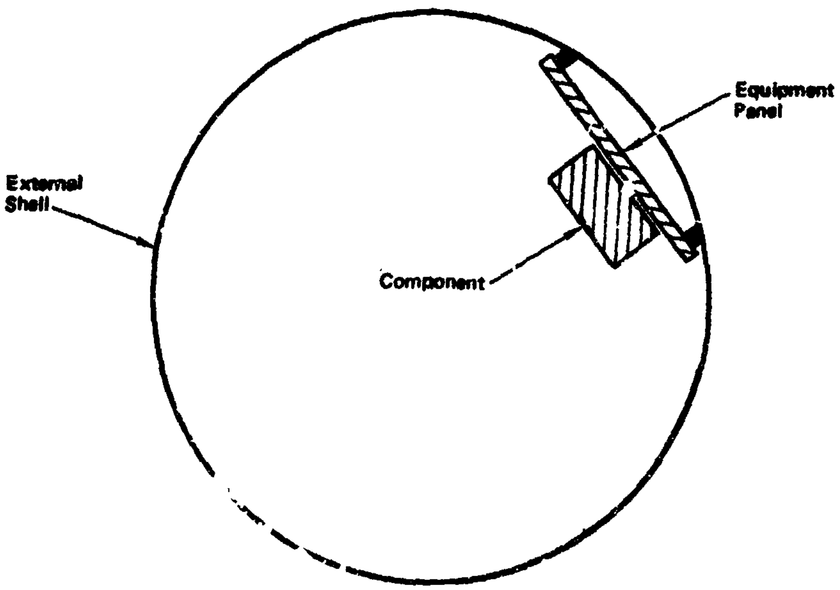
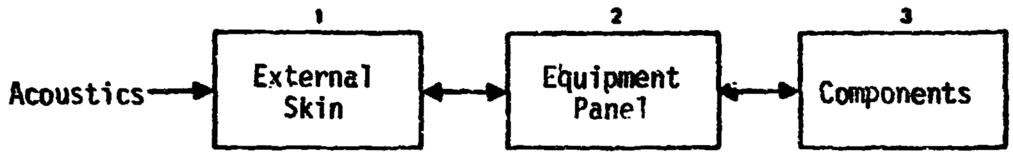


Figure 2. Typical Aerospace Equipment Panel Installation

Several reasons may exist for analyzing the noted section of interest without performing an analysis that encompasses the entire vehicle: a local structural change in the section for an operational vehicle, a design evaluation examining several locations and configurations for the panel, or evaluation of an alternate location for the component.

This segment of structure can be represented with an SEA model of three elements as shown below.



Note that this model will reduce to two elements for the analysis of a component mounted directly to the shell.

Another common structural configuration of aerospace interest consists of a airframe section containing an internal bulkhead, with a component mounted on the bulkhead.

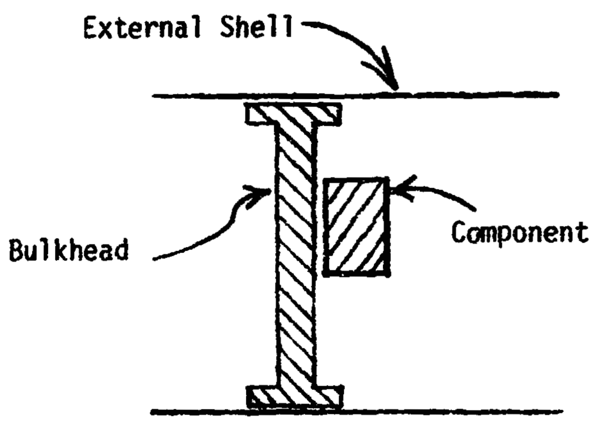
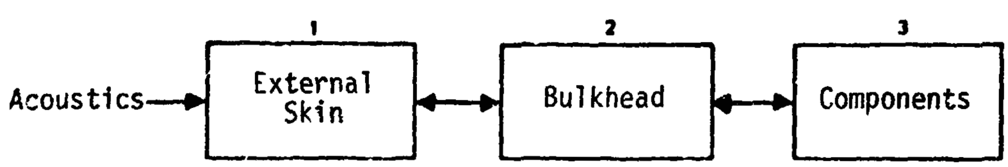


Figure 3. Typical Aerospace Bulkhead Installation

The corresponding SEA model for this structural system requires three elements and is of identical form to the shell/equipment panel/component model.



The analytical differences between the two structures will appear in the parameter values selected to represent the dynamic properties of the various elements. There is no difference in the configuration of the two models.

The elements of these models are a very straightforward representation of the structures. The only significant decision required is in definition of the amount of external skin to include with the model. Selection of the correct skin area yields a balanced system, with the energy flowing into the model subsystem mechanically from the remainder of the structure equal to the energy transported mechanically out of the subsystem. This situation therefore exhibits a zero net flow of energy across the boundaries of the model. In many cases this ideal condition can be approximately achieved by establishing the model boundaries at points halfway between major structural loading points, i.e., halfway between attach points of two adjacent equipment panels, halfway between the panel attach point and a fuel tank bulkhead, halfway between the panel attach point and a large component, etc. The effect on response predictions of incorrect estimation of the skin area will be in essentially direct proportion to the error: selection of an area too large by 10% will result in predicted levels (g^2/Hz) that are too high by approximately 10%.

Once the structural system has been modeled, the next step is to select the applicable equations for the model. These equations, listed below, were determined by examination (recognizing that the coupling between the skin and component elements is zero) of the SEA system equations listed in the Conclusions section of this report.

$$(\omega\eta_1 + N_2\phi_{12})\,E_1 - N_1\phi_{12}E_2 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12} + N_3\phi_{23})\,E_2 - N_2\phi_{23}E_3 = 0$$

$$-N_3\phi_{23}E_2 + (\omega\eta_3 + N_2\phi_{23})\,E_3 = 0$$

where

ω = angular frequency (average) of system

η_a = Element a loss factor $\left(\frac{1}{\text{critical damping ratio}}\right)$

N_a = number of modes resonant in Element a

$\phi_{a,b}$ = power transfer coefficient for coupling between modes through the structural joint ($\phi_{a,b} = \phi_{b,a}$)

E_a = total energy of Element a

S_a = power introduced into Element a from an external source

Each of these equations represents a power flow equation for one of the elements of the model. Together, the equations form an algebraic system for the solution of the E_i, provided the other terms can be evaluated for a structural system. The input term, S_i, will generally represent an acoustic excitation of the system. A derivation of the SEA response equations is provided in Appendix I.

The next step is to evaluate modal density, damping, and coupling parameters for the system.

Modal Density

The shell element is composed of the cylindrical skin, stringers and ring frames. The modal density of these subelements can be determined with the appropriate equations of Table 1 in Appendix II.

Skin: $$n_{skin}(\omega) = \frac{A_s}{4\pi\kappa_p c_\ell}\left(\frac{\omega}{\omega_r}\right)^{2/3}$$

Stringers (assuming plate-type response for high frequencies):

$$n_{str}(\omega) = \frac{A_s}{4\pi\kappa_p c_\ell}$$

Ring frames (assuming beam-type response):

$$n_{rf}(\omega) = \frac{L}{2\pi}\frac{1}{\sqrt{\omega\kappa_b c_\ell}}$$

where

n_a = modal density of Element a = $\frac{N_a}{\Delta\omega}$

$\Delta\omega$ = frequency bandwidth selected for analysis

A_s = surface area

κ_p = radius of gyration of plate cross section

c_ℓ = longitudinal wave velocity

ω_r = structure ring frequency = $\frac{c_\ell}{r}$

r = radius of curvature

L = length

κ_b = radius of gyration of beam

Summing these contributions.

$$n_1 = n_{skin} + n_{str} + n_{rf}$$

and

$$N_1 = n_1(\Delta\omega)$$

The equipment panel (or alternately, the bulkhead) is a ribbed plate which can be subdivided into plates and beams:

$$n_{bulkhead} = n_{plate} + n_{ribs}$$

The component can generally also be subdivided into plate-, beam-, or shaft-type elements, depending upon specific design. If, however, experimental modal response data are available on similar components, an estimate of the component modal density can be obtained from a plot of mode number versus frequency. The slope of the approximate line joining the points, as indicated in Figure 4, is the modal density.

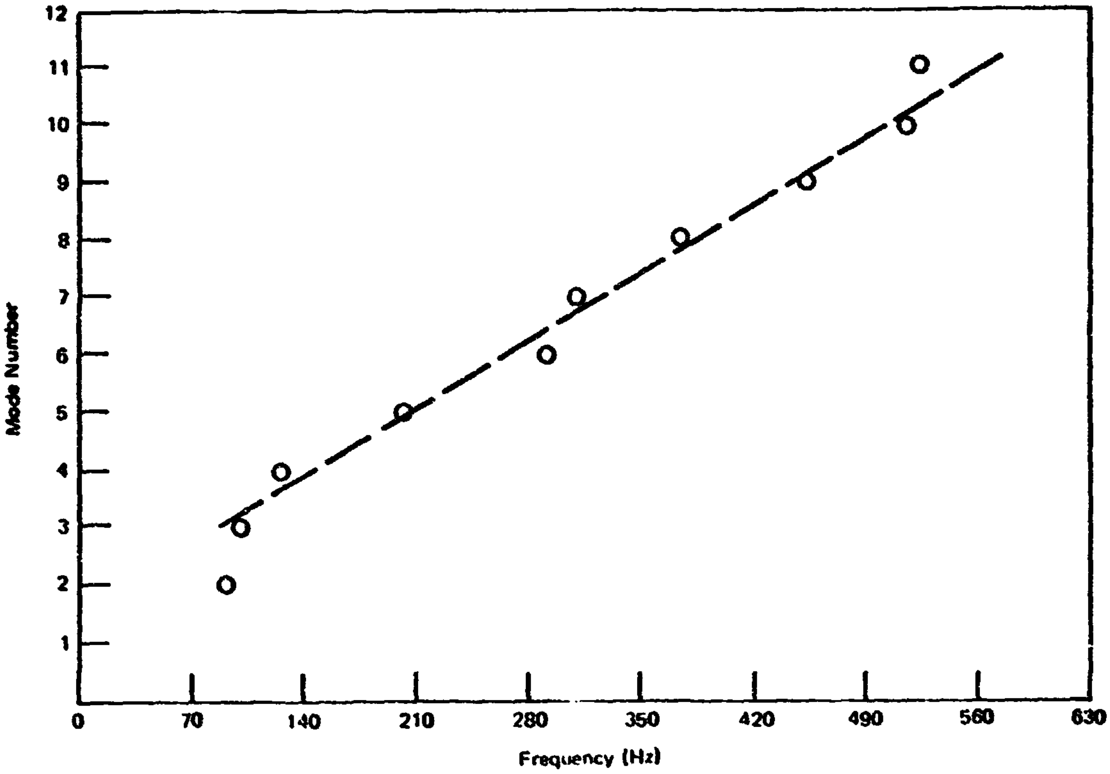


Figure 4. Graphical Approximation of Modal Density

Damping

The damping parameter values for the elements should be estimated based on experience with similar structures. References are provided in Appendix II which will assist the user in selection of values.

Coupling

Appendix II provides coupling factors that apply for a number of structural joints. For the typical structure under consideration, coupling factors are provided for specific skin/equipment panel joint configurations (also for skin/bulkhead joint configurations of the alternate structural system) that are similar to many aerospace installations.

A wide variety of configurations may be encountered for the component/equipment panel joint. The first approach to evaluating coupling factors for this joint is to check Appendix II and other available sources for a similar joint. When coupling factors for a similar joint cannot be found, this parameter may be determined through an SEA evaluation of response data with a similar joint. The SEA parameter for the similar joint may be evaluated by the following procedure.

The response relationship between a component and mounting panel is, in general, provided by an equation in the form of the third of the set of SEA equations:

$$-N_c \phi_{pc} E_p + (\omega \eta_c + N_p \phi_{pc}) E_c = 0$$

where the subscript notation is: c = component
p = equipment panel

This equation provides a means of solving for the coupling factor if the other system parameters can be evaluated and response data are available.

$$\phi_{pc} = \frac{\omega \eta_c}{N_c \frac{E_p}{E_c} - N_p}$$

The total energy term for an element is provided by

$$E_i = m_i \langle \overline{V_i^2} \rangle = m_i \frac{\langle \overline{a_i^2} \rangle}{\omega^2}$$

m_i = element i mass

v_i = element i velocity

a_i = element i acceleration

$\langle \overline{\ } \rangle$ indicates averaging over time and over area

which yields

$$\phi_{pc} = \frac{\omega \eta_c}{N_c \frac{m_p}{m_c} \frac{\langle a_p^2 \rangle}{\langle a_c^2 \rangle} - N_p}$$

This equation defines the coupling factor based on the relative response level, $\frac{\langle a^2_p \rangle}{\langle a^2_c \rangle}$, of the equipment panel to the component for the similar joint configuration. (Similarity to the primary structural system under analysis assists in defining the SEA parameters in the equation, generally reducing the effort required to determine the coupling parameter using response data.)

The final step before obtaining the response solution for the system is to define S_i, the input term. For the sample structure, excited by a reverberant acoustic field, this term is

$$S_i = \frac{2\pi^2 c_0^2 A_i \langle \overline{P^2} \rangle \sigma N_i \,(\text{surface})}{\omega_0^2 (\Delta\omega)\, m_i}$$

c_0 = speed of sound in fluid medium

A_i = surface area of Element i

P = acoustic pressure

σ = radiation efficiency

N_i = number of surface modes of Element i (excludes modes of ring frames, stiffeners, etc.)

m_i = mass of Element i

Appendix II provides radiation efficiency values for both flat panels and circular cylinders.

The parameter values may now be substituted into the system of SEA equations (for each bandwidth of interest) and the response solution for the E_i obtained. The previously noted relation,

$$E_i = m_i \frac{\langle \overline{a^2_i} \rangle}{\omega^2}$$

can then be used to present the element responses in the form of $\frac{\sqrt{\langle a_i^2 \rangle}}{g}$ (for response in g's) or $\frac{\langle a_i^2 \rangle}{(\Delta\omega)g^2}$ (in g^2/Hz), where g is the gravitational acceleration constant.

SUMMARY OF STATISTICAL ENERGY ANALYSIS LIMITATIONS AND EQUATIONS

The methods presented in this document will provide estimates of the high-frequency vibration environment for structural systems. The user should be aware of the requirements and limitations for the application of these methods. A summary of the principal limitations is provided below.

1. Application is more valid in frequency ranges where many modes are excited. Care must be taken in evaluating the lower frequency limit of applicability for a particular structural system.

2. Application is valid only for systems containing all their energy in modal resonances, therefore SEA does not apply to heavily damped systems. Reference 1 provides a means of evaluating this requirement for specific systems.

3. Response predictions determined with these methods represent averages over generally gross portions of structure. Therefore caution should be taken in applying these average values with nonuniformly configured structural elements such as panels with relatively massive integral stiffeners, where response amplitude of the panel segments may be expected to differ considerably from that on the stiffeners.

A summary of the SEA response prediction equations is provided below. Inspection of the system equations will indicate the terms required to expand the set of equations to accommodate a system with any number of elements. Likewise, the simplification possible for systems which do not have each element connected to every one of the other elements can be determined by setting the appropriate ϕ_{ij} terms to zero.

Guidelines for structural modeling and parameter evaluation are provided in Appendix II.

Statistical Energy Analysis Equations for a Four-Element System

$$(\omega\eta_1 + N_2\phi_{12} + N_3\phi_{13} + N_4\phi_{14})\,E_1 - N_1\phi_{12}E_2 - N_1\phi_{13}E_3 - N_1\phi_{14}E_4 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12} + N_3\phi_{23} + N_4\phi_{24})\,E_2 - N_2\phi_{23}E_3 - N_2\phi_{24}E_4 = S_2$$

$$-N_3\phi_{13}E_1 - N_3\phi_{23}E_2 + (\omega\eta_3 + N_1\phi_{13} + N_2\phi_{23} + N_4\phi_{34})\,E_3 - N_3\phi_{34}E_4 = S_3$$

$$-N_4\phi_{14}E_1 - N_4\phi_{24}E_2 - N_4\phi_{34}E_3 + (\omega\eta_4 + N_1\phi_{14} + N_2\phi_{24} + N_3\phi_{34})\,E_4 = S_4$$

ω_i = angular frequency (center frequency of analysis band)

η_i = element damping factor

N_i = number of modes excited in the element in frequency band of analysis

ϕ_{ij} = element coupling value (symmetric: $\phi_{ij} = \phi_{ji}$)

$E_i = m_i \dfrac{\overline{\langle a_i^2 \rangle}}{\omega^2}$

a_i = angular acceleration

S_i = energy input term = $\dfrac{2\pi^2 C_o^2 A_j \, \overline{\langle P_j^2 \rangle}\, \sigma_j}{\omega_o^2\,(\Delta\omega)\, m_j} N_j$ (for reverberant acoustic excitation only)

C_o = local speed of sound in surrounding medium

A_j = acoustically excited surface area of element

P_j = acoustic pressure

σ_j = radiation efficiency

m_j = mass of element

These equations are frequently expressed in an alternate format by combining terms into a coupling loss factor:

$\eta_{ij} = \dfrac{\phi_{ij}N_j}{\omega}$ = coupling loss factor (nonsymmetric: $\eta_{ij} \neq \eta_{ji}$)

REFERENCES

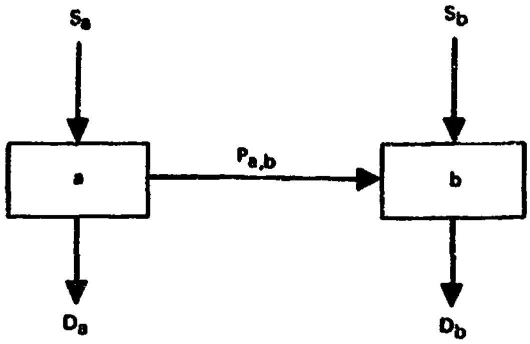
1. L. D. Pope. On the Transmission of Sound through Finite, Closed Shells: Statistical Energy Analysis, Modal Coupling, and Nonresonant Transmission. Report 21, University of Houston, August 1970.

2. R. F. Davis and D. E. Hines. Final Report - Performance of Statistical Energy Analysis. McDonnell Douglas Astronautics Company report MDC G4741, June 1973; also NASA CR-124322, June 1973.

Appendix I

DERIVATION OF STATISTICAL ENERGY ANALYSIS RESPONSE EQUATIONS

Consider a simple structure modeled as the two-element system in the following schematic.



In this schematic of the two elements, a and b, the following nomenclature is used:

S_a = power introduced into Element a from an external source.

D_a = power dissipated within Element a.

$P_{a,b}$ = net power transmitted from Element a to Element b (= $-P_{b,a}$).

These values and the following derivations are for only a single frequency band; solution for the complete spectrum of interest is accomplished by summing the predictions for the contributing frequency bands.

Power flow equations for all of the energy passing through the two elements may be expressed as

$$D_a + P_{a,b} = S_a$$

$$D_b - P_{a,b} = S_b$$

The energy dissipated per unit time is defined in terms of the element loss factor as

$$D_a = \omega \eta_a E_a$$

where

ω = angular frequency (average) of system

η_a = Element a loss factor $\left(\frac{1}{\text{critical damping ratio}}\right)$

E_a = total energy of Element a

The net power transmitted from the resonant modes of Element a ... the resonant modes of Element b is

$$P_{a,b} = N_b \phi_{a,b} E_a - N_a \phi_{a,b} E_b$$

$$= \text{(power transmitted from b to a)} - \text{(power transmitted from a to b)}$$

where

N_a = number of modes resonant in Element a

$\phi_{a,b}$ = power transfer coefficient for coupling between modes through the structural joint ($\phi_{a,b} = \phi_{b,a}$)

Performing the indicated substitutions, the power flow equations become

$$\omega \eta_a E_a + N_b \phi_{a,b} E_a - N_a \phi_{a,b} E_b = S_a$$

$$\omega \eta_b E_b + N_a \phi_{a,b} E_b - N_b \phi_{a,b} E_a = S_b$$

For systems with the parameters η, N, ϕ defined and the inputs, S, known, the power flow equations form a set of linear, simultaneous equations for the unknowns E_a and E_b at the average frequency ω.

Appendix II

GUIDELINES FOR APPLICATION OF STATISTICAL ENERGY ANALYSIS

Definition of Structural Models

One of the initial steps in the SEA applications procedure will be selection and definition of suitable models. The basic considerations of modeling are to (1) determine the structural definition and detail required, (2) evaluate energy sources, and (3) partition the significant portion of the structure into the actual model elements. The first consideration, requirements for structural definition, is to insure that the model will both provide the information desired and omit useless details. Structural assemblies may be lumped together as a single element if finer definition is not required since SEA uses averaged quantities, and averaging is equally valid for multiple portions of a structure as for a single part. Such lumping of elements also reduces the bookkeeping associated with the analysis.

The second consideration in SEA modeling is evaluation of the energy sources. This consideration assists in limiting the size of a model. Basically, any structural boundary across which the net energy flow is zero represents a limit to the need for modeling.

The final step in modeling is to actually partition the significant structure into elements in line with the previously stated principles. The elements represent generally gross, continuous portions of the structure.

Damping

The structural damping must be defined for each element of the models as one of the procedural steps. This parameter is not unique to SEA and must appear in some form in every response analysis. While much investigation of structural damping has been accomplished, and over a long period of time, selection of appropriate values remains very much a matter of engineering judgement based on past experience. References 1 and 2 provide information useful in defining the damping of structures.

Modal Density

Modal density is the parameter which is used to evaluate the number of resonant modes present within a particular frequency band of a given structural subset. Approximation equations are presented in Table II-1 (from Reference 4) which define this parameter for specific structural shapes.

Alternate methods of evaluating the modal densities of specific structural configurations are available. One alternate method makes use of computer programs generally available for analyzing the response of commonly encountered structural shapes such as pinned-end cylinders, liquid-filled cylinders, etc. These programs are utilized to determine the frequency response of the elements, thus yielding directly the number of resonant modes within the frequency band.

A second alternate method has utilized classical low-frequency modal analyses of the structure to evaluate the modal density for model elements. This method, of course, is only practical for structures which have previously received modal analyses, or if a low-frequency modal analysis is being performed in conjunction with the high-frequency SEA prediction. This method, indicated in Figure II-1, involves graphically plotting the response frequencies from the modal analysis versus mode number for the structure which is being represented by an SEA model element. The slope of the plotted points can then be determined which, assuming extension of the curve into the high frequencies to be valid, yields the value for modal density of the element.

Structural Coupling

The SEA parameter for the structural coupling between elements is unique to this form of analysis, and its definition represents one of the most significant steps in the procedure. As a consequence of the relative newness and lack of previous application of the SEA methods, very little information has been available concerning appropriate values of this parameter for structural joints in general. The coupling values for two typical joints are shown in Figures II-2 and II-3.

Table II-1

MODAL DENSITIES OF SOME UNIFORM SYSTEMS*

System	Motion	Modal Density, $n(\omega)$**	Auxiliary Expressions
String	Lateral	$L/\pi c_S$	$c_s = \sqrt{T/\rho A}$
Shaft, Beam	Torsion	$L/\pi c_T$	$c_T = \sqrt{Gk/\rho J}$
Shaft, Beam	Longitudinal	$L/\pi c_\ell$	$c_\ell = \sqrt{E/\rho}$
Beam	Flexure	$\frac{L}{2\pi}\left(\omega\kappa_b c_\ell\right)^{-1/2}$	$\kappa_b c_\ell = \sqrt{EI/\rho A}$
Membrane	Lateral	$A_s\ \omega/2\pi c_m^{\ 2}$	$c_m = \sqrt{S/\rho h}$
Plate	Flexure	$A_s/4\pi\kappa_p c_\ell$	$\kappa_p c_\ell = \sqrt{D/\rho h}$ $= \sqrt{Eh^2/12\rho(1-\nu^2)}$
Room, (Acoustic Volume)	Sound (Compression)	$V_o\omega^2/2\pi^2 c_a^{\ 3}$	
Cylindrical Shells (Ref. 3)		$\begin{cases} \sim n_p & \text{for } \omega/\omega_r > 1 \\ \simeq n_p\left(\frac{\omega}{\omega_r}\right)^{2/3} & \text{for } \omega/\omega_r < 1 \end{cases}$	$\omega_r = c_\ell/a$ $np = A_s/4\pi\kappa_p c_\ell$
Doubly Curved Shells	Flexure	Expressions are complex, given in Reference 4	

*See next page for definitions of symbols.

**$N = \Delta\omega \cdot \eta(\omega)$

Symbol Definitions for Table II-1

A	cross-section area
A_s	surface area
c_a	acoustic wave velocity
c_ℓ	Longitudinal wave velocity
c_m	membrane wave velocity
c_s	string wave velocity
c_T	torsional wave velocity
D	plate rigidity
E	Young's modulus
G	shear modulus
h	thickness
I	centroidal moment of inertia of A
J	polar moment of inertia of A
K	torsional constant of A
I	length
S	membrane tension force/unit edge length
T	string tension force
V_o	volume
κ_b	radius of gyration of A
κ_p	radius of gyration of plate cross section
ν	Poisson's ratio
ω	frequency (radians/time)
ρ	material density

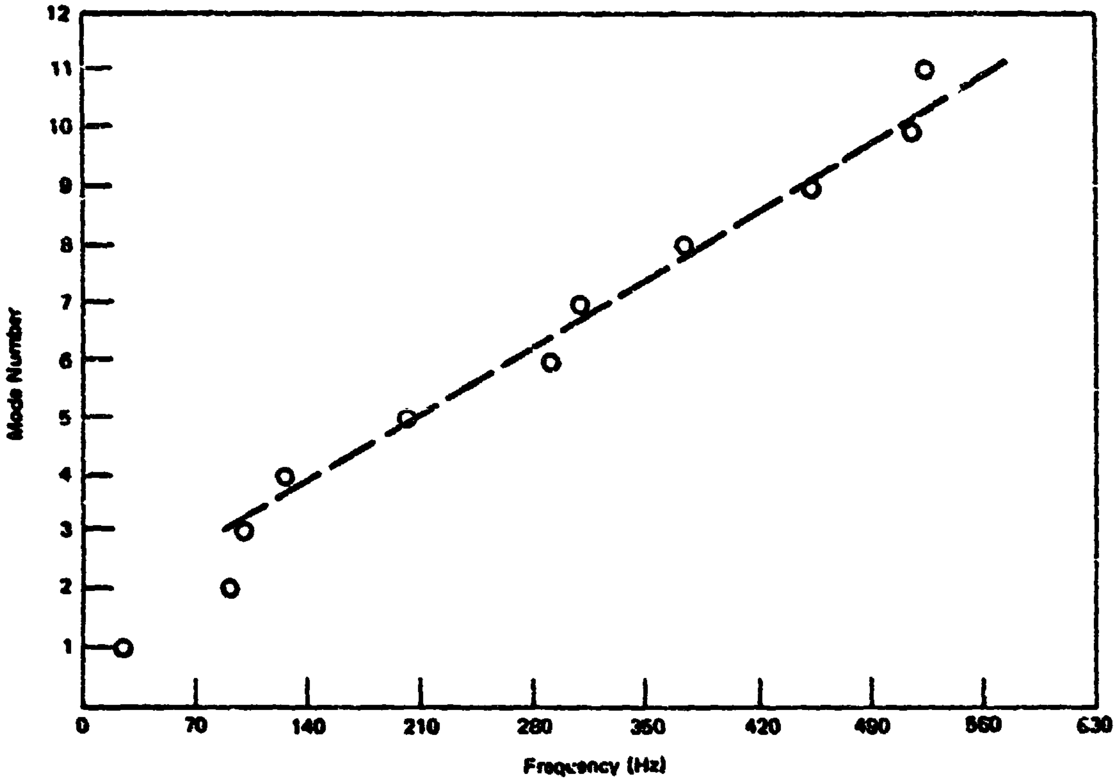


Figure II-1. Approximation of Modal Density by Graphical Method (Delta Fairing Modes)

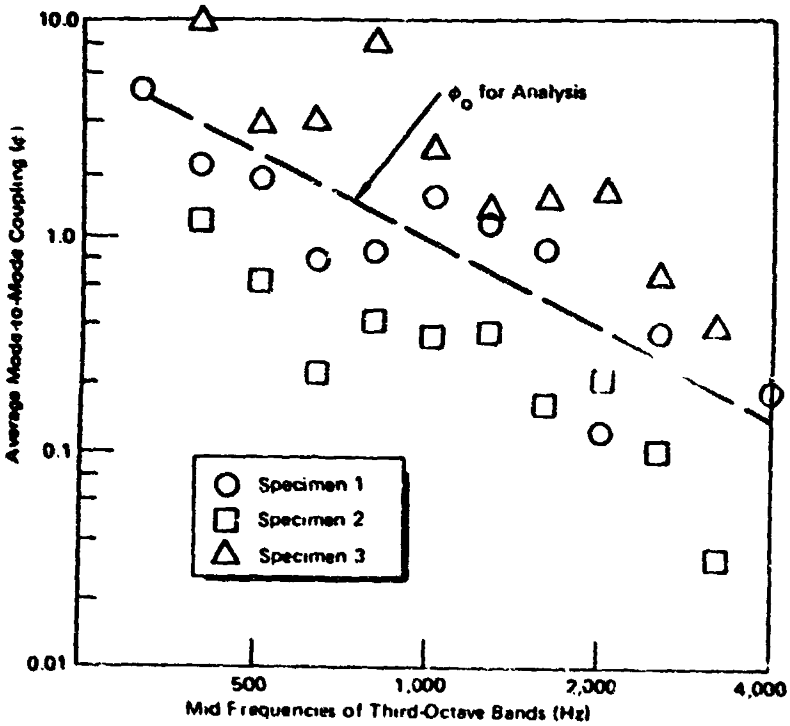


Figure II-2. SEA Coupling Parameter for Small Reentry and Intercept Vehicle Field Joints (Typical, from UpSTAGE Ground Test Data)

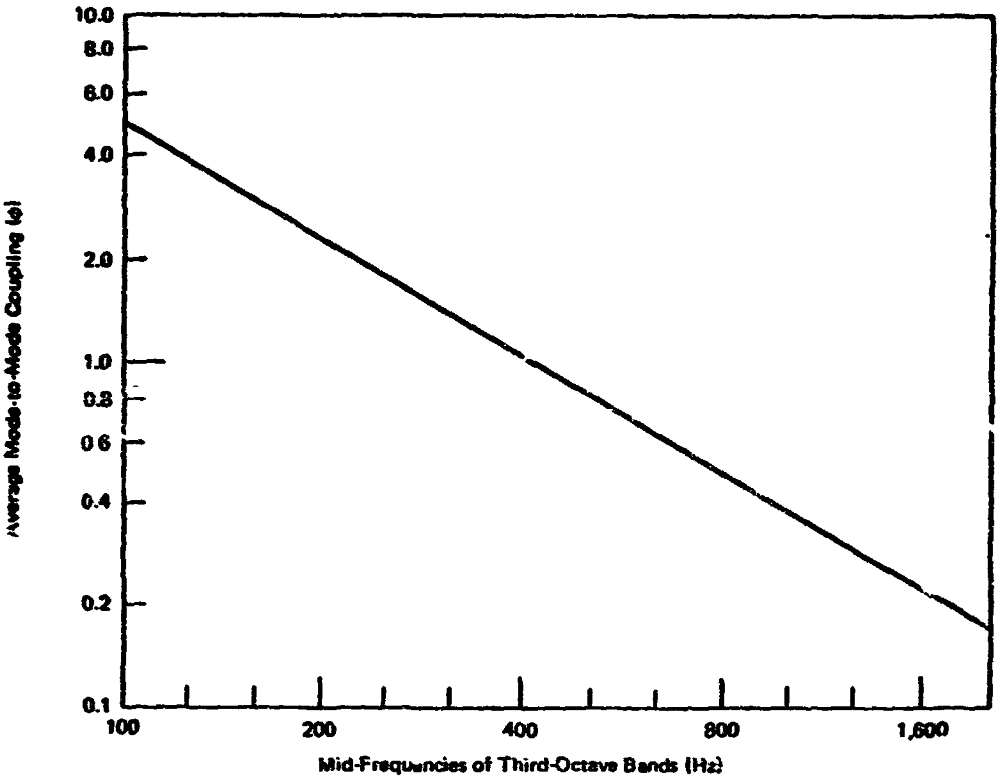


Figure II-3. SEA Coupling Parameter for Launch Vehicle Tank/Skirt Joint (Typical, Based on S-II Data)

Reference 6 provides equations for evaluating the coupling values of various beam and plate joints. Two of the most useful of these relations are presented below.

Two plates of approximately equal stiffness joined at right angles:

$$\phi_{12} = \frac{C_g L}{\pi A_1 N_2} \frac{8}{27}$$

$$C_g = 1.07(\omega C_\ell t)^{1/2}$$

$$C_\ell = \sqrt{\frac{E}{\rho(1 - \nu^2)}}$$

L = joint length

t = plate thickness

Beam cantilevered to a plate of equal thickness:

$$\phi_{12} = \frac{2\pi f}{N_p} \frac{W}{4\ell}$$

N_p = number of modes in plate

W = width of beam

ℓ = length of beam

Acoustic Coupling

The input term will generally involve a transfer function to couple a fluctuating pressure field to the structural system. A reverberant acoustic field may be coupled to a structure with the relation presented in the Conclusions section of this report. Use of this expression for predicting response from other acoustic fields requires the definition of an "equivalent" reverberant field, or the coupling terms must be modified. The development of the reverberant coupling terms will indicate an approach that could be used in defining coupling terms for other pressure fields.

The radiation efficiency term, σ, appearing in the input relation for reverberant acoustic fields of the Conclusions section may be determined from Figures II-4 and II-5, which are taken from Reference 5.

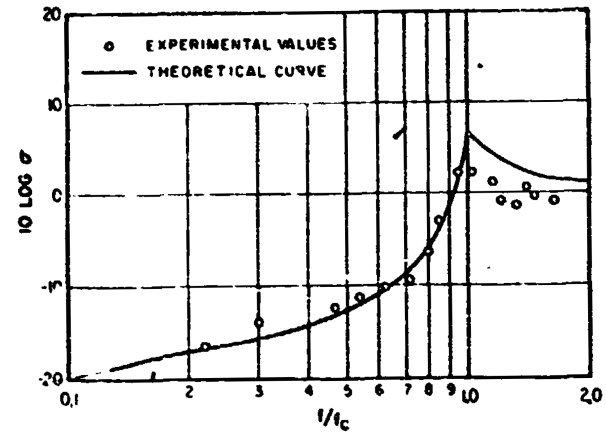


Figure II-4. Radiation Efficiency σ of a Baffled Panel (from Reference 5)

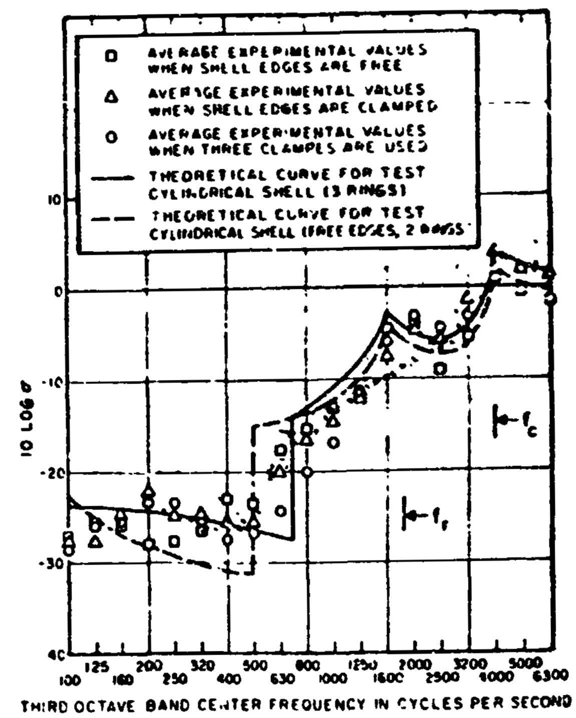


Figure II-5. Radiation Efficiency σ of a Cylindrical Shell (The peak in the radiation efficiency about the ring frequency f_r is associated with increase in wavespeed due to curvature.) (From Reference 5)

REFERENCES TO APPENDIX II

1. B. J. Lazan. Damping of Materials and Members in Structural Mechanics. Pergamon Press Inc., 1968.

2. L. T. Lee. A Graphical Compilation of Damping Properties of Both Metallic and Non-Metallic Materials. Technical Report No. AFML-TR-66-169, Air Force Materials Laboratory, May 1966.

3. M. Heckl. Vibrations of Point Driven Cylindrical Shells. JASA V. 34, pp. 1553-57, October 1962.

4. V. V. Bolotin. On the Density of the Distribution of Natural Frequencies of Thin Elastic Shells. J. Appl. Math. and Mech., Vol. 27, No. 2, 1963, pp 538-543.

5. R. H. Lyon and G. Maidanik. Statistical Methods in Vibration Analysis. AIAA Journal, Vol. 2, No. 6, June 1964, pp 1015-1024.

6. R. H. Lyon. Statistical Energy Analysis of Dynamical Systems: Theory and Applications. The MIT Press, 1975.

Appendix III

EXAMPLE PROBLEMS

Example Problem Number 1

SPACE SHUTTLE EXTERNAL TANK - UNLOADED STRUCTURE

The structure to be analyzed is located in the Space Shuttle External Tank intertank area at 270° (-Y) on the station 1034.2 frame. This location corresponds to a vibration measurement location in use during Main Propulsion Test Article (MPTA) testing for the Space Shuttle. The measurement location is indicated in Figure III-1. This location is on a ring frame which is surrounded by skin panels with external stringers. This portion of the structure is not loaded by component installations and can therefore be considered typical of unloaded aerospace shell structures.

Model

The unloaded structure can be represented by the simplest of SEA models, consisting of a single element excited by the external acoustic field. The model element will include the area 45° to either side of the measurement location (225° to 315°) and halfway to the adjacent frames at stations 985 and 1082. This element is indicated in Figures III-2 and III-3.

Response Equation

The SEA response equation for the one-element system is

$$\omega \eta_1 E_1 = S_1$$

This equation simply equates the energy dissipated by damping within the element to the energy transmitted from the external acoustic field.

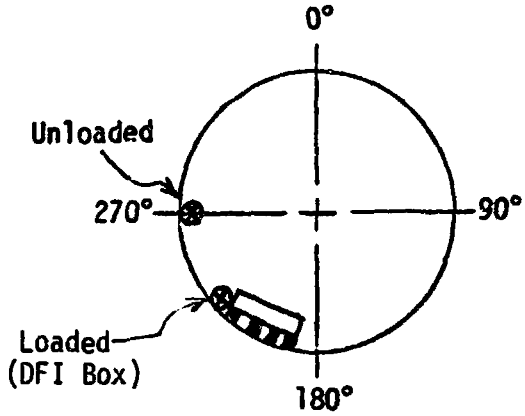


Figure III-1. MPTA Vibration Measurement Locations (Station 1034)

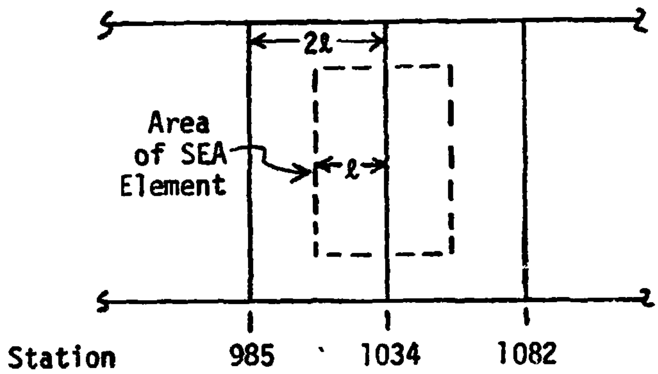


Figure III-2. SEA Model Element for Unloaded Structure

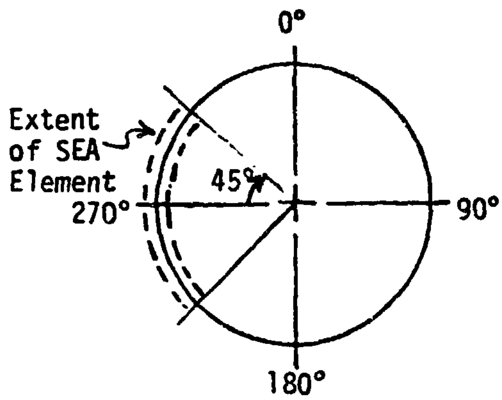


Figure III-3. SEA Model Element for Unloaded Structure (Cross Section)

Damping

The damping parameter was evaluated with response data measured during an acoustic fatigue test of Saturn S-IVB/V interstage panels (Reference III-1). These panels feature a skin/stringer construction similar to the current structure and are of similar size. The approximate relation for damping of $\eta = \frac{(\Delta f)^{\circ}}{f}$, where $(\Delta f)^{\circ}$ represents the bandwidth to the half-power points, was evaluated for a number of response measurements on the test. The resulting values have been plotted in Figure III-4. Straight lines have been faired through these data for a simple graphical approximation to the damping which is used for this analysis. The approximation lines have been positioned on the low side of the obvious mean of the data since low values for damping result in high predicted responses that are conservative for design purposes.

Element Energy

The energy of the model element is represented by

$$E_1 = m_1 \langle \overline{V_1^2} \rangle = m_1 \frac{\langle \overline{a_1^2} \rangle}{\omega^2}$$

The element mass was estimated from available detail drawings by summing the volume of the element subsections (skin panels, stringers, frame sections, etc.) and multiplying by the material density.

$$m_1 \simeq \frac{310 \text{ lbs}}{g}$$

Acoustic Power Input

The external acoustic field is assumed to be reverberant, therefore the input term can be represented by

$$S_1 = \frac{2\pi^2 c_0^2 A_1 \langle \overline{P^2} \rangle \, \sigma \, N_1 \, \underline{(\text{surface})}}{\omega_0^2 \, (\Delta\omega) \, m_1}$$

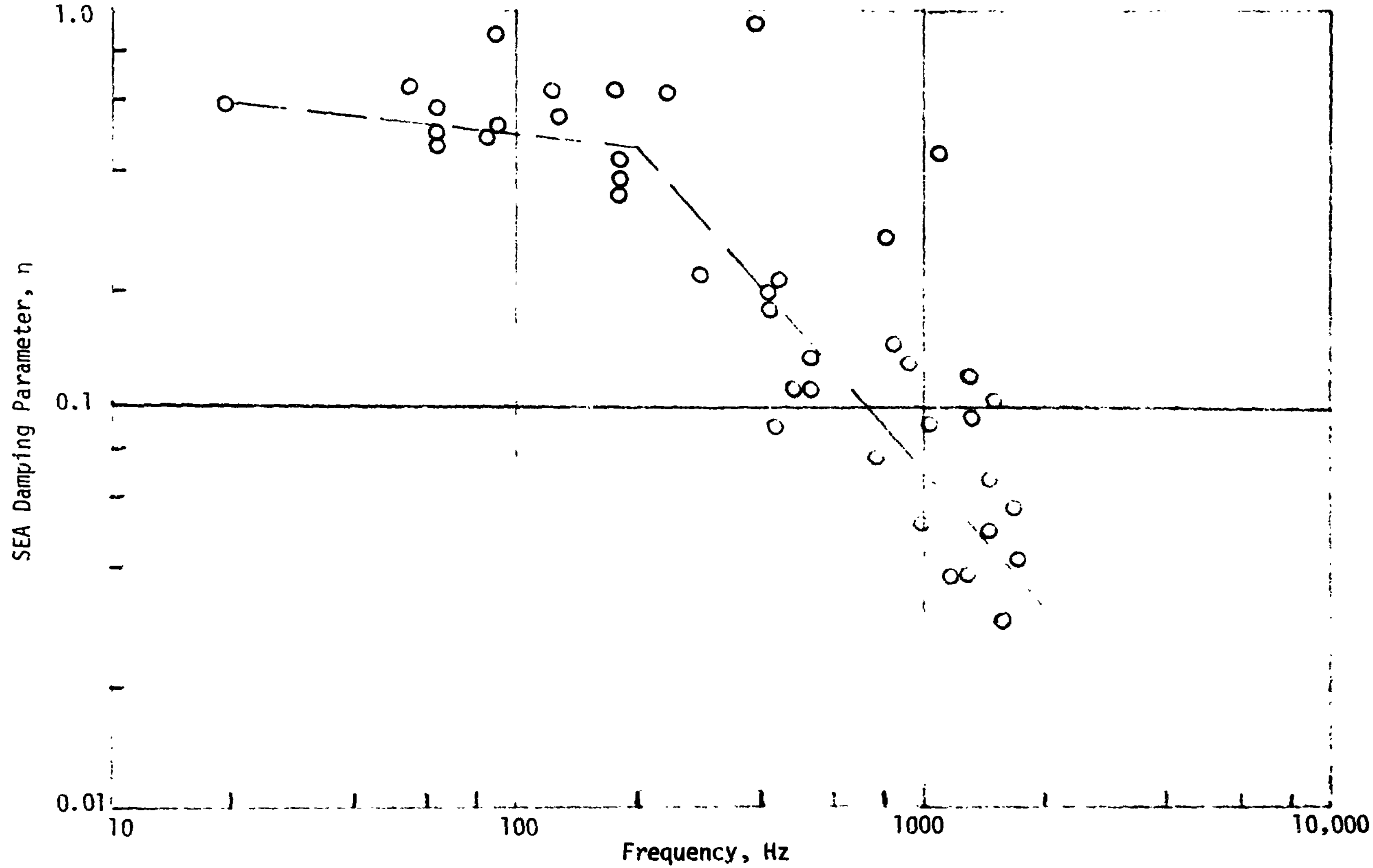


Figure III-4. Damping Values for Saturn S-IVB/V Interstage Panels (Dashed line indicates values selected for current analysis)

The surface of the subject structure is composed partly of skin panels and partly of stringers. The correct representation of the term $\frac{A_1 N_1}{m_1}$ is thus

$$\frac{A_{1p}\, N_{1p}}{m_{1p}} + \frac{A_{1s}\, N_{1s}}{m_{1s}}$$

where the subscripts p and s indicate panel and stringer values, respectively. Since the stringers tend to bound the surface into plate areas of relatively small curvature, the approximate relation for high frequency modal density of plates,

$$n(\omega) = \frac{A}{4\pi \kappa_p C_\ell}$$

was utilized for both skin and stringer areas.

$$N = n(\Delta\omega)$$

also $$m = \frac{\overline{w}}{g} V = \frac{\overline{w}}{g} At$$

$\overline{w}$ = weight density (7075 Al = .101 lb/in^3)

V = volume

t = thickness

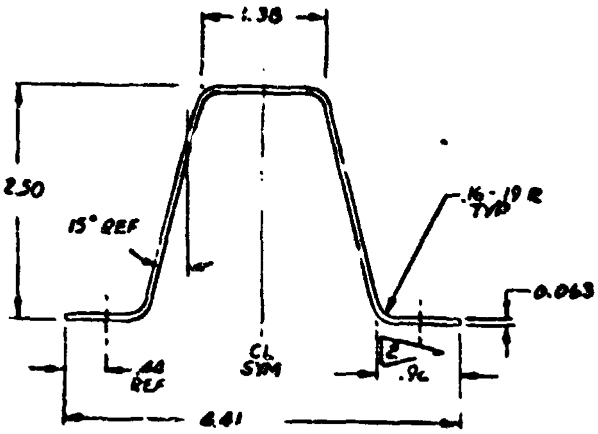
yielding

$$\frac{AN}{m} = \frac{A g (\Delta\omega)}{4\pi \overline{w}\, t\, \kappa_p C_\ell}$$

also: $$\Delta\omega = 2\pi(\Delta f), \quad \kappa_p C_\ell = \sqrt{\frac{E t^2 g}{12 \overline{w} (1 - \nu^2)}}$$

so $$\frac{AN}{m} = \frac{A(\Delta f)}{2t^2} \sqrt{\frac{12 g (1 - \nu)^2}{\overline{w} E g}}$$

The surface area of the element is approximately 48.5 in x 259 in, with 36 stringers of the cross-sectional dimensions indicated below.



The effective width of the stringer is approximately 8.5 inches.

$$A_s = 36(8.5 \times 48.5) = 14{,}841 \text{ in}^2$$

The skin panels are 0.071 inch thick and have an area of

$$A_p = 48.5 \times [259 - 36(4.41)] = 4862 \text{ in}^2$$

Then

$$\frac{A N}{m} = \frac{\Delta f}{2}\sqrt{\frac{12g(1-\nu^2)}{WE}}\left[\frac{A_p}{t_p^2} + \frac{A_s}{t_s^2}\right]$$

$$= 151{,}934\ \Delta f$$

Acoustic levels measured during the MPTA test are presented in Figure III-5. The three measurements were averaged for each one-third octave band center frequency and the average value used as the required acoustic pressure input:

$$\langle \overline{P^2} \rangle = 10 \exp\left[\frac{(SPL)_{avg}}{10}\right] \times 8.41 \times 10^{-18}$$

The values for $\langle \overline{P^2} \rangle$ are listed in Table III-1.

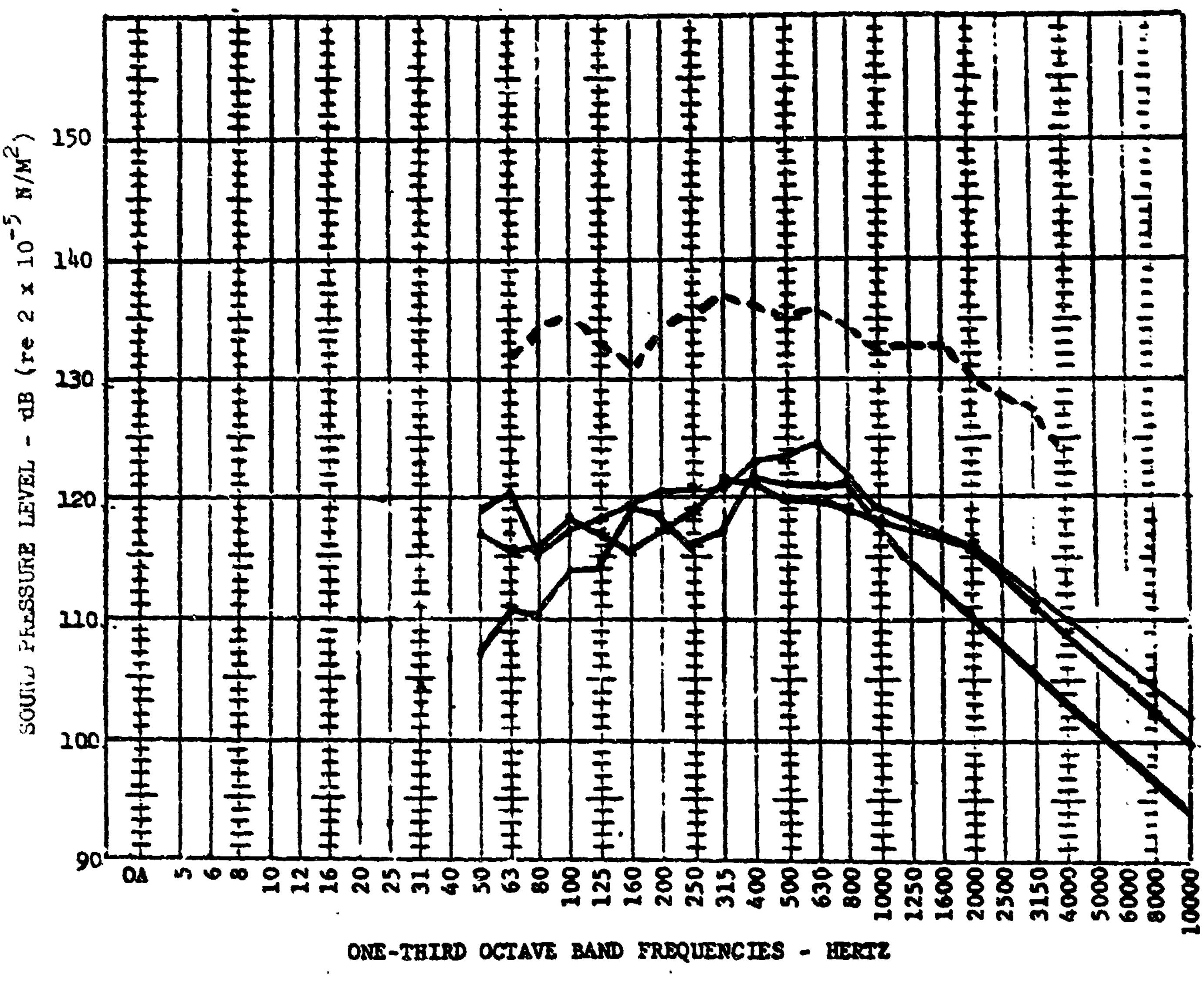


Figure III-5. Space Shuttle External Tank Main Propulsion Test Acoustic Data. Three measurements external to intertank area (solid lines); upper, dashed line represents measurement subsequent to analysis (see text).

Table III-1

f	η_1	$\langle \overline{p^2} \rangle$ (dB)	σ
50	0.52	116.5	.000794
63	0.50	117.2	.00100
80	0.48	114.4	.00126
100	0.47	116.6	.00158
125	0.46	116.6	.00200
160	0.45	118.1	.00251
200	0.44	118.9	.00316
250	0.36	118.9	.00398
315	0.25	120.0	.00631
400	0.20	122.2	.00316
500	0.15	121.7	.00251
630	0.12	121.9	.00251
800	0.090	120.8	.00251
1000	0.070	118.3	.0100
1250	0.054	116.8	.0200
1600	0.040	115.7	.0316
2000	0.032	114.7	.0631

Radiation efficiency values are taken from Appendix II, based on a coincidence frequency of f_c = 7683 Hz (for the 0.063-inch thickness of the stringers since they furnish most of the surface area) and a ring frequency of f_r = 198 Hz. These values are also listed in Table III-1.

Response Solution

The response predictions for the element were determined in each one-third octave band from 50 to 2000 Hz. A sample prediction for the 50 Hz octave band is presented below as an example.

$$\omega \eta_1 E_1 = S_1$$

$$[2\pi \times 50][0.52]\left[\frac{310}{g}\,\frac{\overline{\langle a_1^2 \rangle}}{(2\pi \times 50)}\right] =$$

$$\frac{2\pi^2[(1116 \times 12)^2]\left[(151{,}934)\,\frac{50}{4.33}\right]\left[10^{\frac{116.5}{10}} \times 8.41 \times 10^{-18}\right][.000794]}{\left[(2\pi \times 50)^2\right]\left[2\pi \times \frac{50}{4.33}\right]}$$

where $\Delta f = \frac{f}{4.33}$ for one-third octave bands.

Dividing by g and solving for the mean squared acceleration:

$$\frac{\overline{\langle a_1^2 \rangle}}{g^2} = 0.0130$$

The acceleration spectral density level is

$$\frac{\overline{\langle a_1^2 \rangle}}{g^2(\Delta f)} = \frac{.0130}{\left(\frac{50}{4.33}\right)} = 0.00113$$

and the root-mean-squared acceleration in this one-third octave band is

$$g_{rms} = \sqrt{\frac{\overline{\langle a_1^2 \rangle}}{g^2}} = \sqrt{0.0130} = 0.114$$

The predicted response levels are plotted in Figures III-6 and III-7 for the respective acceleration spectral density and g_{rms} in one-third octave bands.

The increasing levels above 1000 Hz which are most noticeable in Figure III-7 are an unexpected result. Inspection of the input parameter values for the response solution shows this result can be attributed to the radiation efficiency. The value for this parameter is controlled by the coincidence frequency of f_c = 7683 Hz for the 0.063-inch-thick stringers. This frequency corresponds to the peak radiation efficiency values and is higher than typically encountered with aerospace structures, thus causing the radiation efficiency to be still increasing at 2000 Hz with the resulting high predicted levels.

Reference III-4 demonstrates that ±3 dB accuracy may be expected for an SEA response prediction when 20 or modes per analysis band are excited in each element or above the ring frequency for cylindrical structure. That result was based on a relatively small, stiff vehicle with an elliptical shape. These requirements correspond to 20 Hz (20 modes) and 200 Hz (ring frequency), respectively. for the current structure. Therefore, the predictions may be expected to demonstrate ±3 dB accuracy at frequencies above 200 Hz, and probably above 20 Hz.

<u>Comparison of Prediction with Measured Test Data</u>

Subsequent to completion of this prediction, MPTA response data were made available for comparison. However, one of the acoustic measurements provided with the response data showed an increase of more than 10 dB throughout the spectrum (see Figure III-5), while another measurement agreed with the previous acoustic data. Further investigation revealed that the initial acoustic data were for a 70% thrust level, and that the high measured levels were probably valid for one side (adjacent to Orbiter engines) of the external tank at the 100% thrust level with a reduction in acoustic levels around the tank to the opposite side. Because of the resulting

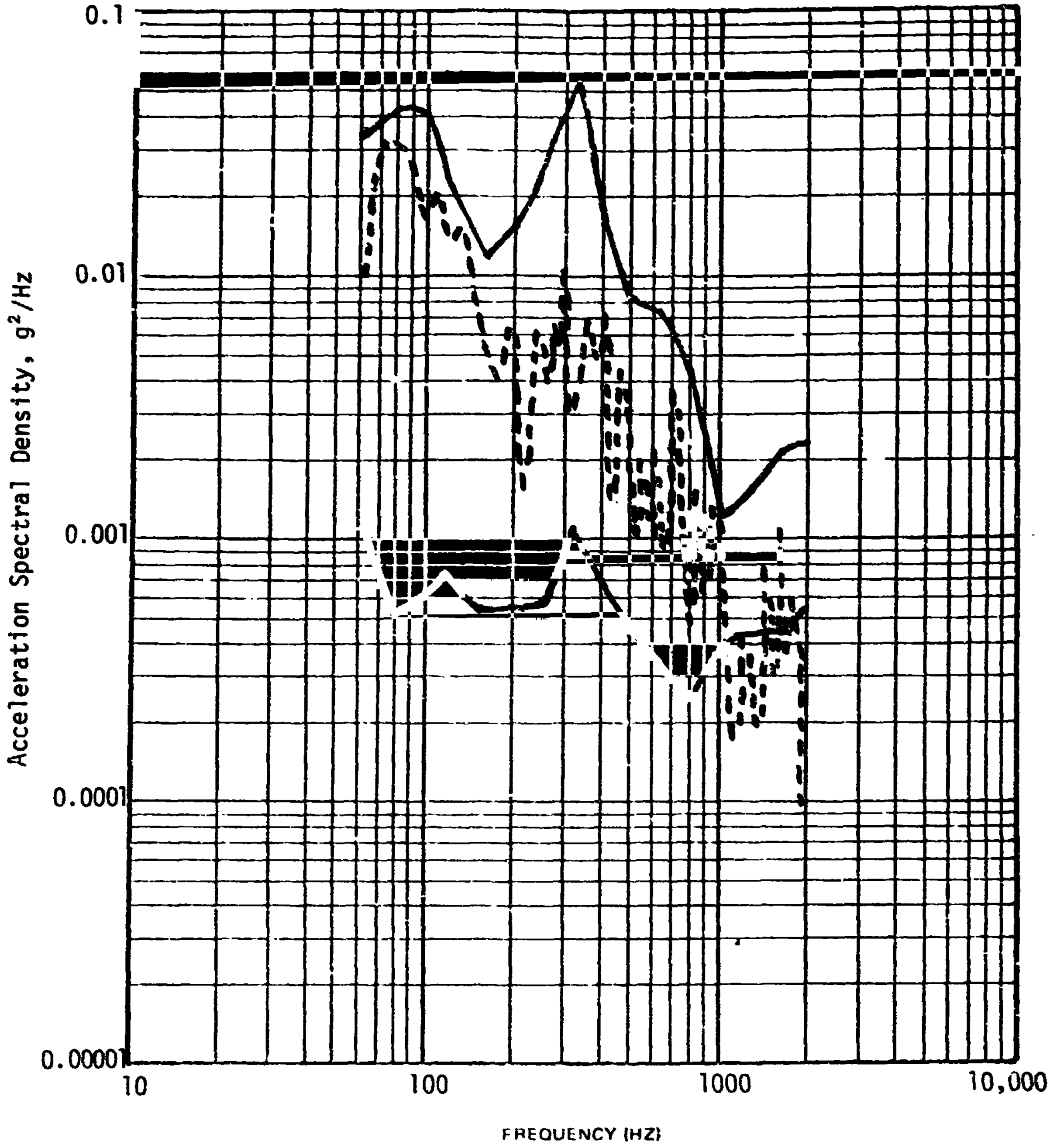


Figure III-6. Measured Test Data (dashed line) vs. Predicted Response for Two Acoustic Input Levels (solid lines) on External Tank Interstage Area - Unloaded Shell Structure

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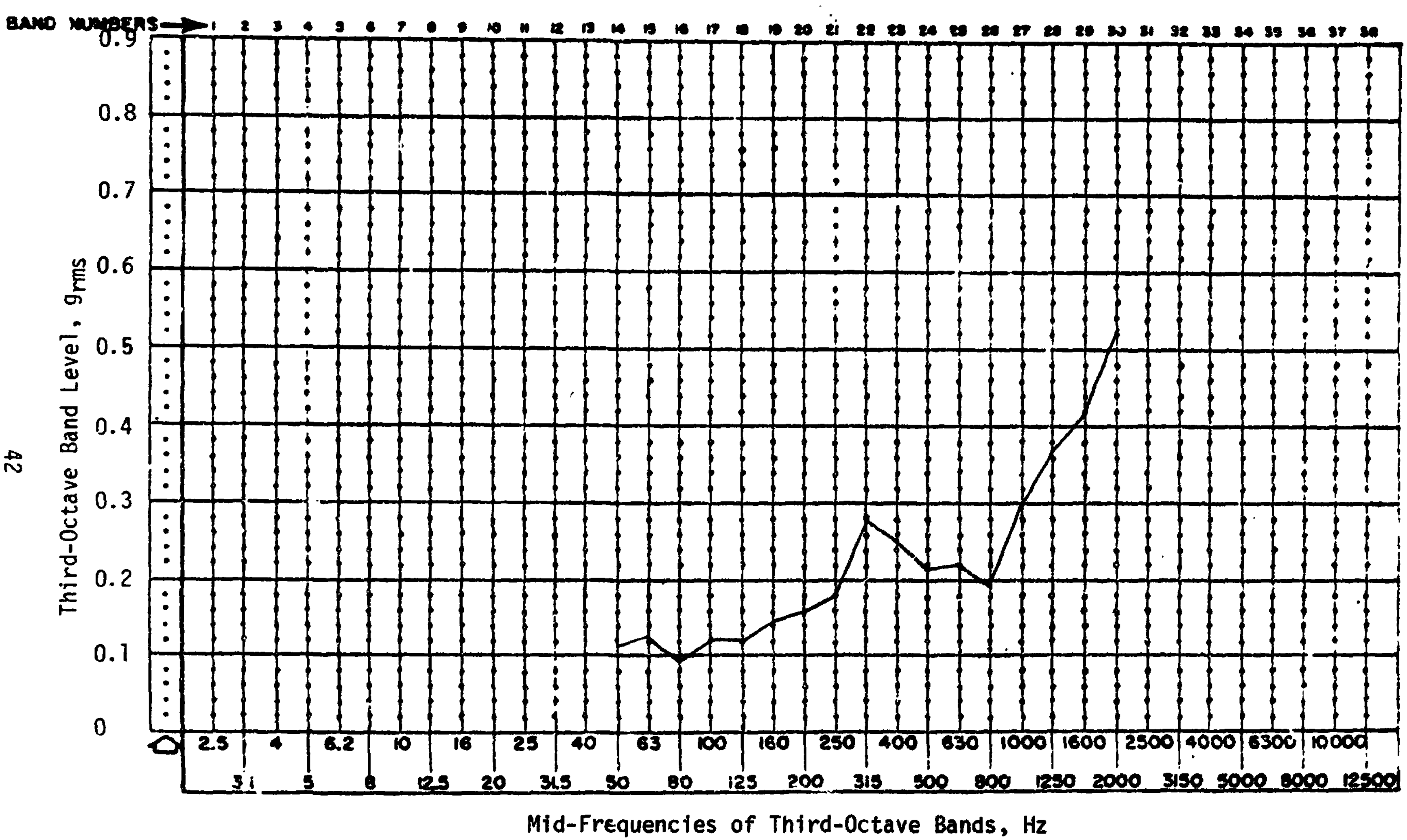


Figure III-7. Predicted Response (g_{rms}) for External Tank Interstage Area - Unloaded Shell Structure uncertainty in the exact input acoustic levels, the response predictions for the data comparison are presented in Figure III-6 for two input levels - the original, averaged value and the high measurement level - with the expectation that the correct level actually lies between the two. The vibration response data correspond with this expectation and lie chiefly between the two predictions (SEA response predictions should be compared to <u>average</u> values of the response over one-third octave bands rather than peak response values). At 1000 Hz, the predicted response exhibits a change in slope and begins to increase at higher frequencies, a result determined to be due to an increase in radiation efficiency values near the coincidence frequency of the external stringers. This discrepancy in response with the test data is most likely due to improper definition of damping values about the coincidence frequency. This overprediction of response relative to the measured levels would lead to a conservative result when used for design purposes.

Example Problem Number 2
SPACE SHUTTLE EXTERNAL TANK - LOADED STRUCTURE

The structure to be analyzed is located in the Space Shuttle External Tank intertank area at about 200° on the station 1034.2 frame (refer to Figure III-1). This location corresponds to a vibration measurement location used during Main Propulsion Test Article (MPTA) testing for the Space Shuttle, as for Example Problem 1. The structure is identical to the skin/stringer/ring frame structure of Example Problem Number 1, but is loaded by the 260 pound DFI box installation and can therefore be considered as typical for aerospace shell structure loaded by heavy components.

Model

The configuration to be analyzed represents structure which is loaded by the DFI package. This package mounts on supports between frames. The model for this structure will have two elements, consisting of the external shell and the DFI package with support intercostals. The shell structure to be included will extend halfway to the frames adjacent to those carrying the support structure and a circumferential width identical to twice the support frame width, as indicated by Figures III-8 and III-9.

Response Equations

The SEA response equations for the two-element system with external acoustic excitation are

$$(\omega\eta_1 + N_2\phi_{12})\,E_1 - N_1\phi_{12}E_2 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12})\,E_2 = 0$$

where the subscript 1 denotes external shell values, and 2 denotes DFI box and intercostal values.

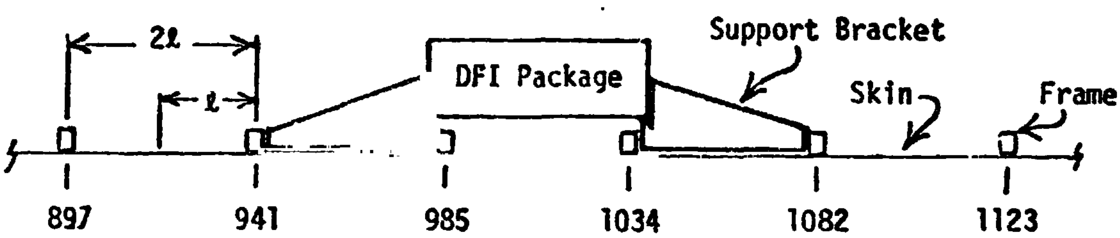


Figure III-8. Structural Configuration for SEA Model of Loaded Structure

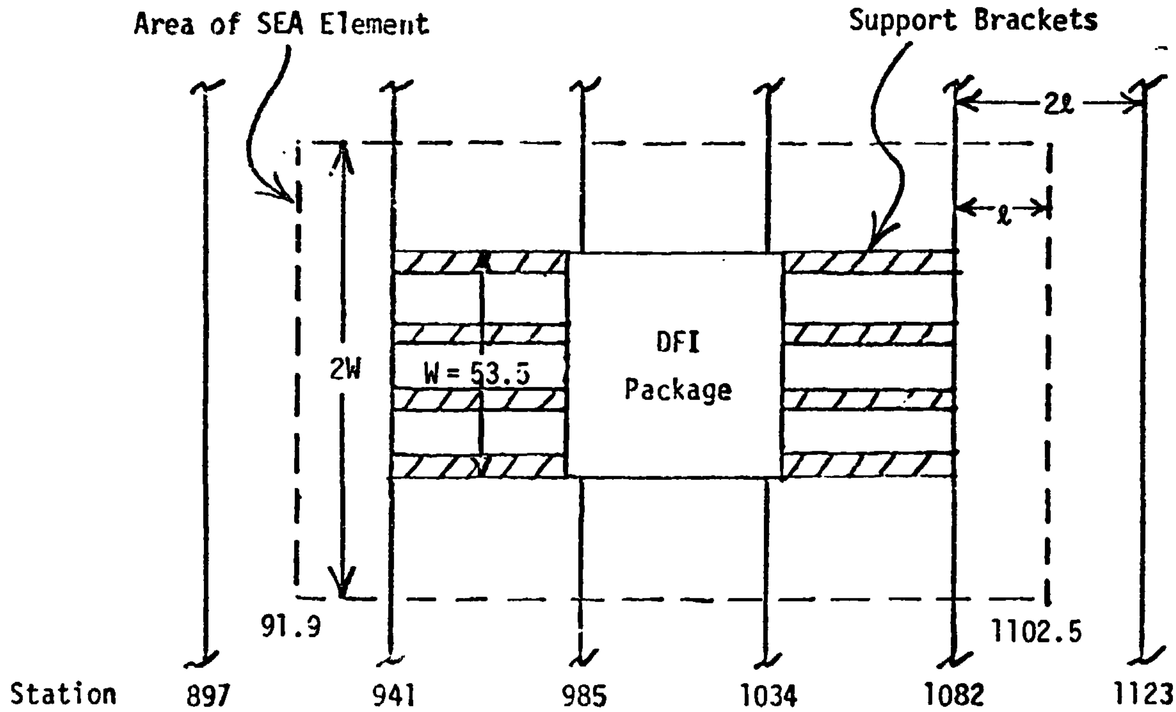


Figure III-9. Extent of SEA Model for Loaded Structure

Damping

The damping parameter for the external shell is identical to that of Example Problem 1 in Figure III-4.

Test experience with smaller electronic packages during the Delta program indicates response amplifications of $Q = 6$ to 10 should be expected for the DFI package. Therefore, a value for the loss factor of $\eta = 0.1$ ($Q = 10$) was adopted for this analysis.

Modal Density

The expression for number of modes, N_i, required in the response equation is determined by

$$N_i = n_i \; (\Delta f)$$

where

n_i = modal density

Δf = bandwidth of analysis

The portion of intertank structure which has been designated as the external shell element is actually composed of several hundred individual parts (skin panels, stringers, stiffeners, ring frame segments, fittings, brackets, etc.). The modeling of all these parts by one element is a feature of the averaging assumptions of the SEA approach and is one of the most attractive aspects of SEA. Almost without exception, the individual parts are plates or are formed with multiple plate sections. Therefore, the shell element modal density was calculated by summing the modal density of the individual plate sections determined by the approximate relation for high frequency modal density of plates,

$$n_1(f) = \sum \frac{A}{2 \kappa_p C_\ell}$$

$$= \frac{1}{2\sqrt{\dfrac{Eg}{12 \overline{w} (1-\nu^2)}}} \sum \frac{A}{t}$$

$$= 6.52 \text{ modes/Hz}$$

where $\sum \frac{A}{t}$ represents the summation of values for all parts and their subsections.

The second element is made up of the DFI package plus the supporting intercostal structure. The DFI package consists of a box, whose plate element modal densities can be determined as above, plus a panel loaded with electronic components. Reference 2 indicates that the loaded panel will exhibit a greater stiffness (and resultant lower modal density) than an identical unloaded panel. An increase in stiffness by a factor of 2 was assumed for the loaded panel. The resulting modal density for the box element is

$$n_{DFI}(f) = (n_2)_{box} + (n_2)_{panel}$$

$$= \frac{1}{2\sqrt{\frac{Eg}{12\bar{w}(1-\nu^2)}}} \left[\left(\frac{A}{t}\right)_{box} + \frac{1}{\sqrt{2}}\left(\frac{A}{t}\right)_{panel}\right]$$

$$\approx 0.27 + 0.12$$

$$= 0.39 \text{ modes/Hz}$$

The modal density for the supporting intercostals is also determined with the plate equation:

$$n_I(f) = \frac{1}{2\sqrt{\frac{Eg}{12\bar{w}(1-\nu^2)}}} \sum \frac{A}{t} = 0.32$$

Therefore the total modal density for this element is

$$n_2(f) = n_{DFI}(f) + n_I(f) = 0.71$$

Modal Coupling

The model elements are coupled by the joint between the vehicle shell and the DFI support intercostals. This joint is essentially two plates joined at right angles. For the case of equal plate stiffnesses which is approximately satisfied (.071 inch intercostals and .071 inch skin with supplementary stiffeners), Reference 3 gives the relation for coupling loss factor of

$$\eta_{12} = \frac{C_g L}{2\pi^2 f A_1}\left(\frac{8}{27}\right)$$

$$C_g = 1.07(\omega C_\ell t)^{1/2}$$

$$C_\ell = \sqrt{\frac{Eg}{w(1-\nu^2)}}$$

L = Joint length

From the basic definition,

$$\eta_{12} = \frac{\phi_{12} N_2}{\omega}$$

so that

$$\phi_{12} = \frac{\omega}{N_2}\frac{C_g L}{2\pi^2 f A_1}\left(\frac{8}{27}\right) = \frac{1.20}{\sqrt{f}}$$

which makes use of $\frac{f}{\Delta f}$ = 4.33 for the one-third octave bandwidths to be used for analysis.

Element Energy

The element energy was handled as in Example Problem 1.

$$E_i = m_i \frac{\langle \overline{a_i^2} \rangle}{\omega^2}$$

$$m_1 = \frac{484 \text{ lbs}}{g}$$

$$m_2 = \frac{280 \text{ lbs}}{g}$$

Acoustic Power Input

This term was also handled as in Example Problem 1. The applicable term for $\frac{AN}{m}$ is

$$\left(\frac{A_1 N_1}{m_1}\right)_{surface} = 237{,}473 \; \Delta f$$

Response Solution

The response predictions for each element were determined in each one-third octave band from 50 to 2000 Hz. A sample prediction for the 50 Hz band is presented below as an example:

$$(\omega\eta_1 + N_2\phi_{12})\, E_1 - N_1\phi_{12} E_2 = S_1$$

$$-N_2\phi_{12} E_1 + (\omega\eta_2 + N_1\phi_{12})\, E_2 = 0$$

Substituting for the parameter values, the expressions become

$$\left\{[2\pi \times 50][0.52] + \left[0.71\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right]\right\}\left[\frac{484}{g}\,\frac{\langle \overline{a_1^2} \rangle}{(2\pi \times 50)}\right]$$

$$- \left[6.52\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right]\left[\frac{290}{g}\,\frac{\langle \overline{a_2^2} \rangle}{(2\pi \times 50)}\right]$$

$$= \frac{2\pi^2\left[(1116 \times 12)^2\right]\left[(237{,}473)\,\frac{50}{4.33}\right]\left[10^{\frac{116.5}{10}} \times 8.41 \times 10^{-18}\right][.000794]}{\left[(2\pi \times 50)^2\right]\left[2\pi \times \frac{50}{4.33}\right]}$$

and

$$-\left[0.71\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right]\left[\frac{484}{g}\ \frac{\langle\overline{a_1^2}\rangle}{(2\pi\ \text{x}\ 50)^2}\right]$$

$$+\left\{[2\pi\ \text{x}\ 50][0.1] + \left[6.52\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right]\right\}\left[\frac{290}{g}\ \frac{\langle\overline{a_2^2}\rangle}{(2\pi\ \text{x}\ 50)^2}\right] = 0$$

where $\Delta f = \frac{f}{4.33}$ for one-third octave bands.

Dividing by g and solving for the mean squared accelerations:

$$\frac{\langle\overline{a_1^2}\rangle}{g^2} = 0.0130 \qquad \frac{\overline{a_2^2}}{g^2} = 0.000662$$

The acceleration spectral density levels are

$$\frac{\langle\overline{a_1^2}\rangle}{g^2(\Delta f)} = \frac{0.0130}{\left(\frac{50}{4.33}\right)} = 0.00113, \qquad \frac{\overline{a_2^2}}{g^2(\Delta f)} = \frac{0.000662}{\left(\frac{50}{4.33}\right)} = 0.0000572$$

and the root-mean-squared acceleration in this one-third octave band is

$$(g_{rms})_1 = \sqrt{\frac{\langle\overline{a_1^2}\rangle}{g^2}} = \sqrt{0.0130} = 0.114$$

$$(g_{rms})_2 = \sqrt{0.000662} = 0.0257$$

The predicted response levels are plotted in Figures III-10 through III-12 in acceleration spectral density and g_{rms} in one-third octave band formats.

The criterion of 20 modes per analysis band in each element as a requirement for SEA prediction accuracy is satisfied for the model at 125 Hz, indicating predictions with ±3 dB accuracy above this frequency.

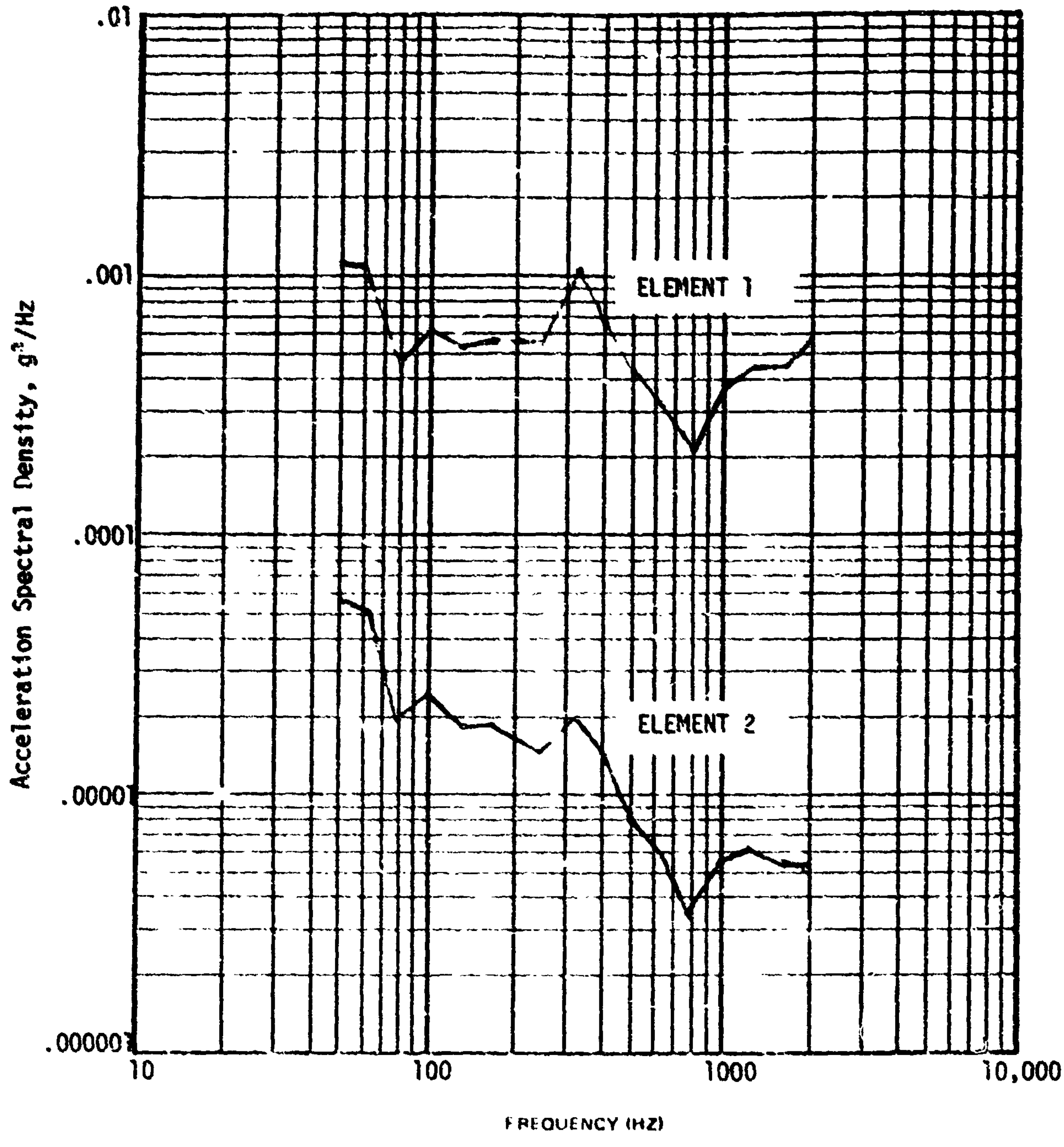


Figure III-10. Predicted Response (g^2/Hz) for External Tank Interstage Area

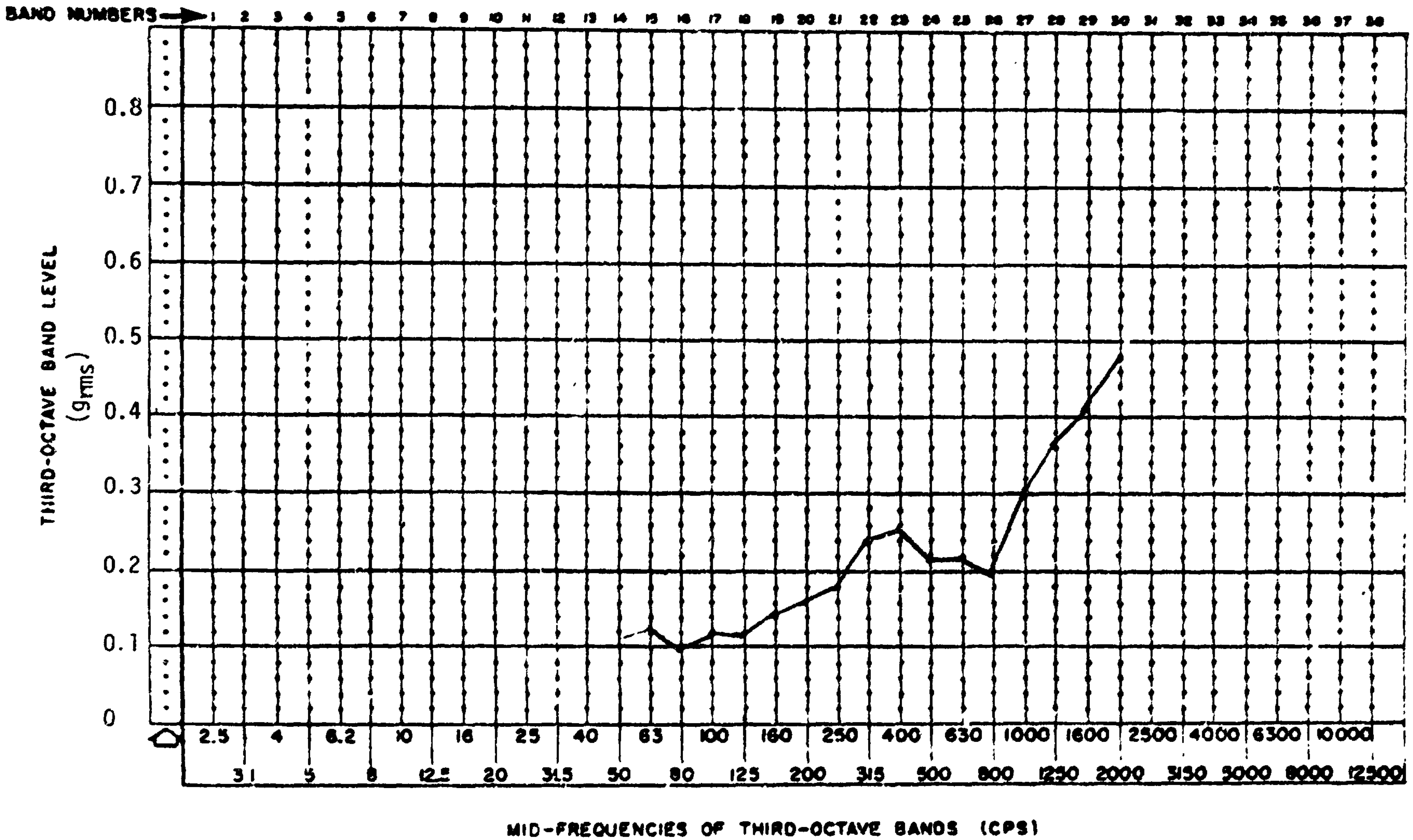


Figure III-11. Predicted Response (g_{rms}) for External Tank Interstage Area - Loaded Shell Structure (Element 1)

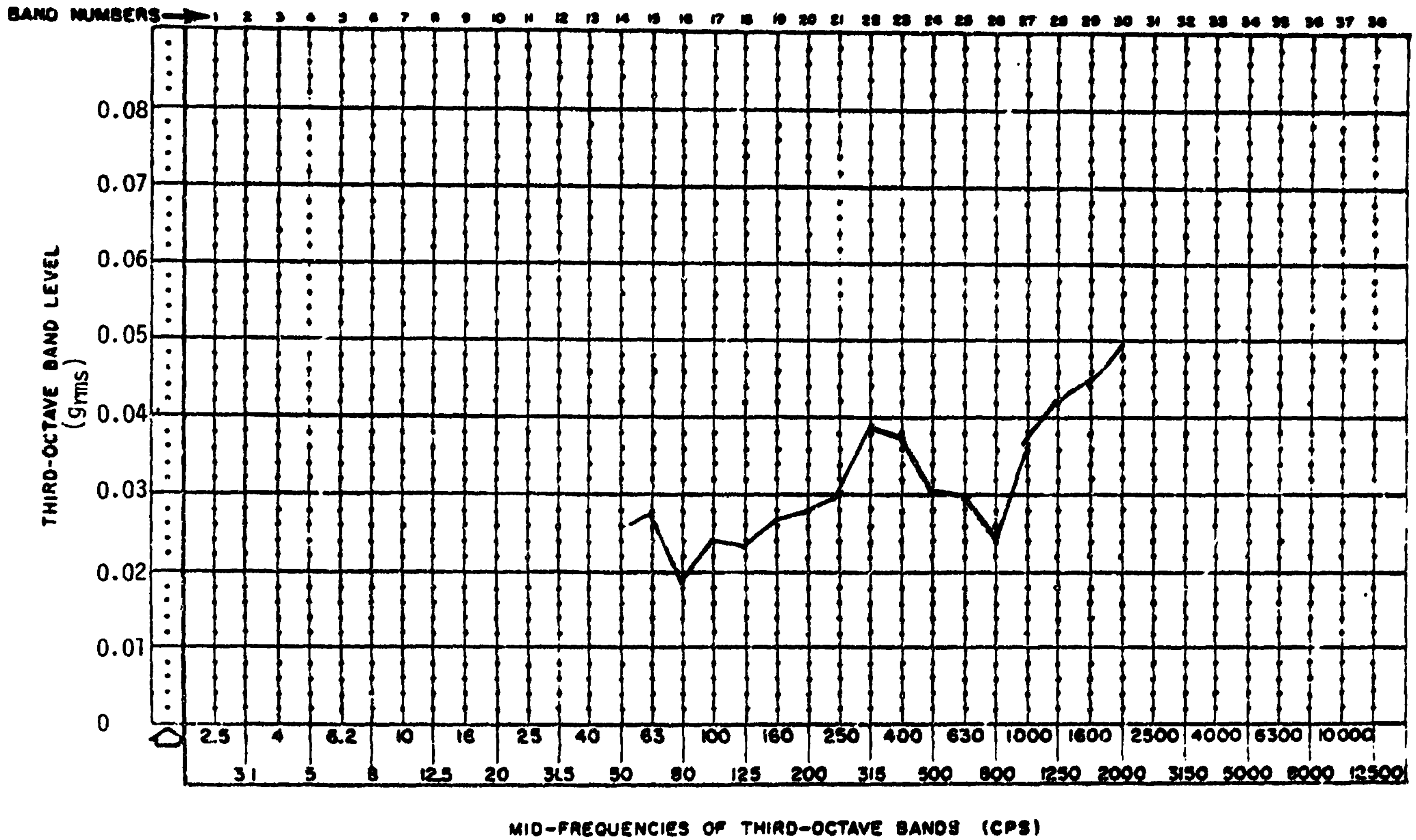


Figure III-12. Predicted Response (g_{rms}) for External Tank Interstage Area - DFI Box and Supports (Element 2)

Comparison of Prediction with Measured Test Data

The structures analyzed in Example Problems Numbers 1 and 2 were selected to provide an evaluation of SEA methods in application to loaded and unloaded structure. To circumvent t . effect of the uncertainty in the acoustic input levels as noted in Example Problem Number 1, the available test data have been compared to the predicted values in a relative sense between the loaded structure of this example problem and the unloaded structure of Example Problem Number 1. A comparison of the element 1 response in Figure III-10 with the lower predicted curve of Figure III-6 shows the structural loading to have no effect on the predicted values which are essentially identical. Figure III-13 shows the approximate average spectrum values (faired graphically through the data) for the test measurements. These data show the loaded structure to have reduced response relative to the unloaded structure below approximately 800 Hz, and essentially the same average spectrum levels above that frequency. For this case and based on these specific measurement locations, the SEA method furnishes accuracy within 3 dB only above 500 Hz, which corresponds to 80 modes per analysis band in each element.

Prediction of the relative response of the loaded structure through traditional mass scaling would result in a reduction of response by the ratio

$$\frac{484 \text{ lbs}}{484 \text{ lbs} + 280 \text{ lbs}} = 0.630$$

or approximately 2 dB. The SEA method obviously yields more accurate results in the higher frequencies.

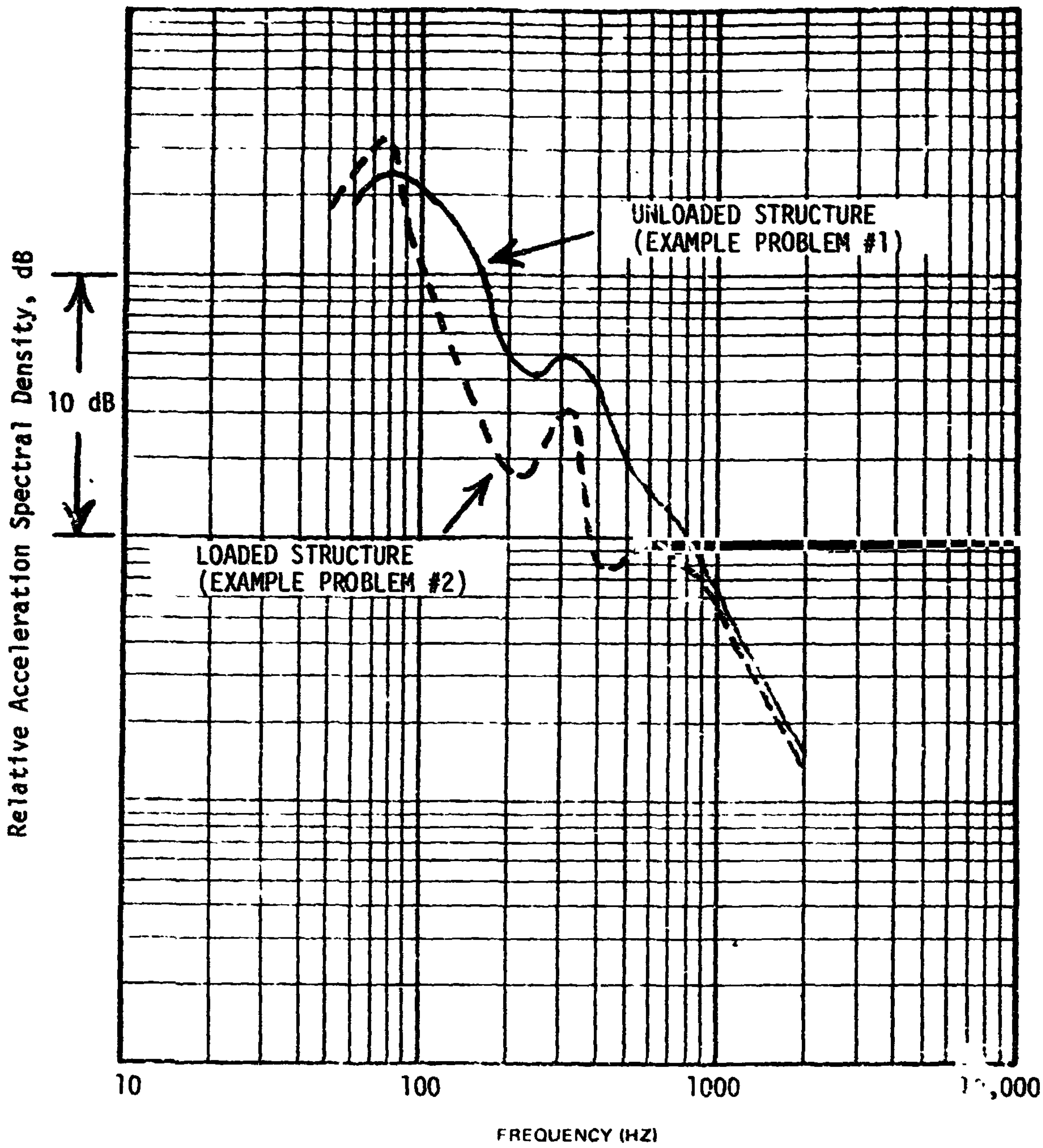


Figure III-13. Comparison of Relative Response for Test Measurements on External Tank Interstage Area Unloaded and Loaded Structure - Approximate Average Values

Example Problem Number 3

SPACE SHUTTLE EXTERNAL TANK - DETAILED ANALYSIS OF LOADED STRUCTURE

The structure to be analyzed is identical to that of Example Problem Number 2 and is located in the Space Shuttle External Tank intertank area at about 200° on the station 1034.2 frame (refer to Figure III-1). This location corresponds to vibration measurement locations used during Main Propulsion Test Article (MPTA) testing for the Space Shuttle, as for Example Problems 1 and 2. The structure is identical to the skin/stringer/ring frame structure with DFI box installation of Example Problem Number 2, but will be modeled in more detail, including internal acoustic excitation, to provide a response comparison for a measurement location on an equipment panel inside the DFI box.

Model

The configuration to be analyzed represents structure which is loaded by the DFI package. This package mounts on supports between frames. The model for this structure will have four elements, consisting of external shell, DFI support intercostals, DFI box equipment panel, and DFI box cover. The shell structure to be included will extend halfway to the frames adjacent to those carrying the support structure and a circumferential width identical to twice the support frame width, as indicated by Figures III-14 and III-15.

Response Equations

The SEA response equations for the four-element system with external acoustic excitation are:

$$(\omega\eta_1 + N_2\phi_{12})\ E_1 - N_1\phi_{12}E_2 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12} + N_3\phi_{23})\ E_2 - N_2\phi_{23}E_3 = 0$$

$$-N_3\phi_{23}E_2 + (\omega\eta_3 + N_2\phi_{23} + N_4\phi_{34})\ E_3 = N_3\phi_{34}E_4 = 0$$

$$-N_4\phi_{34}E_3 + (\omega\eta_4 + N_3\phi_{34})\ E_4 = S_4$$

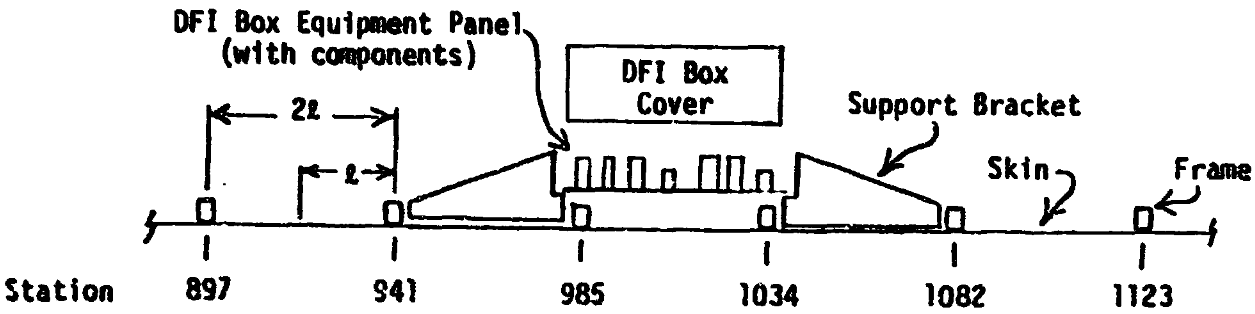


Figure III-14. Structural Configuration for SEA Model

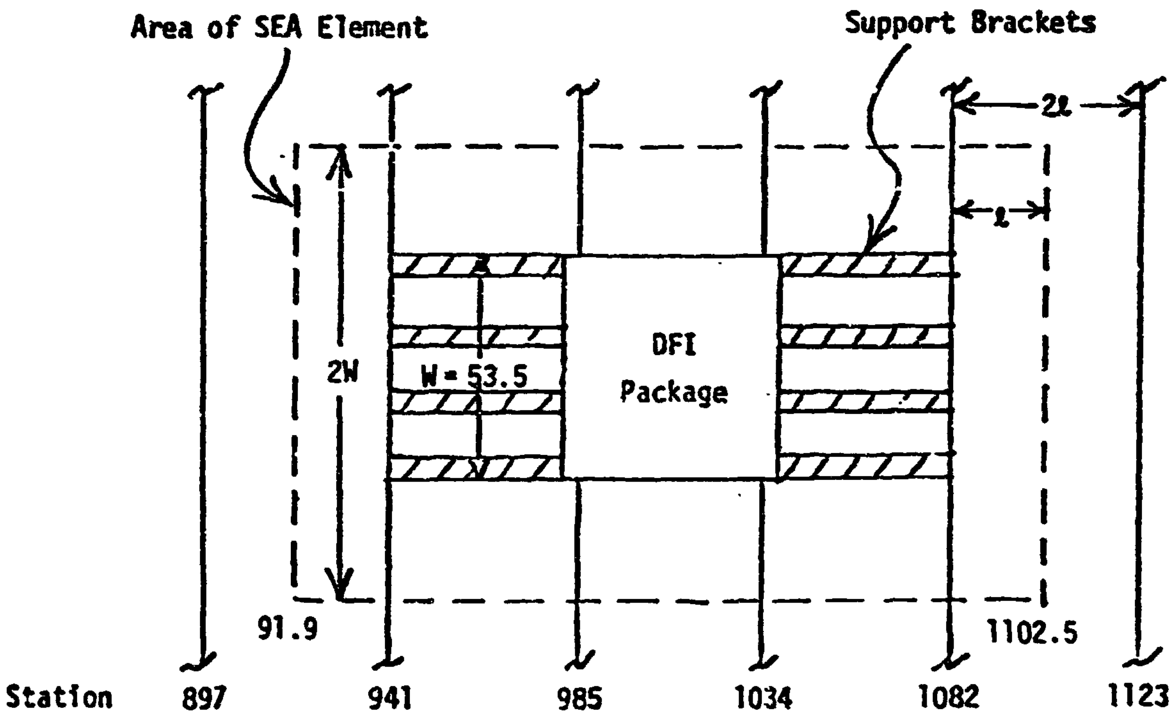


Figure III-15. Extent of SEA Model for Loaded Structure where subscript 1 denotes external shell values, 2 denotes intercostal values, 3 denotes DFI box equipment panel values, and 4 denotes DFI box cover values.

Damping

The damping parameter for the external shell is identical to that of Example Problem 1 in Figure III-4.

The damping parameter for the DFI box/intercostal element of Example Problem 2 was assigned a loss factor value of $\eta = 0.1$. This value was also adopted in the current example problem for each of the three elements (panel, cover, intercostals) resulting from subdivision of the previous DFI box/intercostal element.

Modal Density

The modal density parameters were calculated during Example Problem 2 as

$$n_1(f) = 6.52 \text{ modes/Hz}$$
$$n_2(f) = 0.32 \text{ modes/Hz}$$
$$n_3(f) = 0.12 \text{ modes/Hz}$$
$$n_4(f) = 0.27 \text{ modes/Hz}$$

Modal Coupling

Three joints are included with the model. The shell/intercostal joint was evaluated during Example Problem 2 and assigned the value

$$\phi_{12} = \frac{1.20}{\sqrt{f}}$$

The DFI box is connected to the intercostals by eight flanges which connect to right-angle intercostal brackets. Therefore the joints were considered as two plates jointed at right angles and evaluated following the approach used for the shell/intercostal joint.

$$\phi_{23} = \frac{\omega}{N_3} \frac{C_g L}{2\pi^2 f A_2} \left(\frac{8}{27}\right) = \frac{24.6}{\sqrt{f}}$$

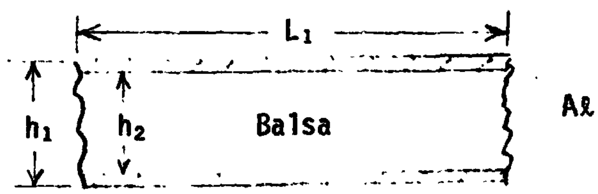
The joint between the DFI box cover and equipment panel is also a right angle connection between two plates and was evaluated as in Example Problem 2.

$$\phi_{34} = \frac{\omega}{N_4} \frac{C_g L}{2\pi^2 f A_3} \left(\frac{8}{27}\right) = \frac{255}{\sqrt{f}}$$

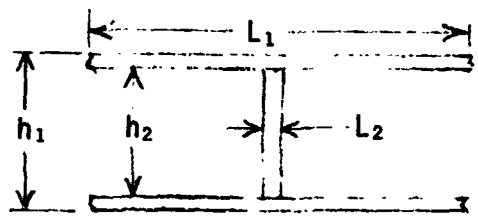
The effective thickness of the laminated aluminum and balsa plates required to evaluate the term

$$C_g = 1.07(\omega C_\ell t)^{1/2}$$

was determined through evaluation of the bending rigidity, $D = m\kappa^2 C_\ell^2$, for an equivalent single-layer plate of aluminum

Aluminum/Balsa Plate: 

The equivalent laminated all-aluminum plate with the same rigidity has the dimensions



where $L_2 = \frac{E_{balsa}}{E_{Al}} \cdot L_1 = 0.04\ L_1$, since in the center layer

$$D_{balsa} = m_B \kappa^2 C_{\ell B}^2 = (\rho_B V_B) \frac{h_2^2}{12} \left(\frac{E_B}{\rho_B}\right) = V_B \frac{h_2^2}{12} E_B = (h_2 \cdot L_1 \cdot 1) \frac{h_2^2}{12} E_B$$

where the volume term, V, is evaluated per unit depth.

For the equivalent center section of aluminum:

$$D_{A\ell} = m_{A\ell}\,\kappa^2\,C_{\ell A\ell}^2 = (\rho_{A\ell} V_{A\ell})\,\frac{h_2^2}{12}\left(\frac{E_{A\ell}}{\rho_{A\ell}}\right) = V_{A\ell}\,\frac{h_2^2}{12}\,E_{A\ell} = (h_2 \cdot L_2 \cdot 1)\,\frac{h_2^2}{12}\,E_{A\ell}$$

Equating the terms

$$D_{balsa} = D_{A\ell} = (h_2 \cdot L_1 \cdot 1)\,\frac{h_2^2}{12}\,E_B = (h_2 \cdot L_2 \cdot 1)\,\frac{h_2^2}{12}\,E_{A\ell}$$

or

$$L_2 = \frac{E_B}{E_{A\ell}} \cdot L_1$$

The effective thickness of a single-layer plate of aluminum with the same rigidity is defined by

$$D_{A\ell} = D'_{A\ell}$$

or

$$m_p\,\kappa^2\,C_\ell^2 = m_p(\kappa')^2\,C_\ell^2$$

$$\kappa^2 = (\kappa')^2$$

For the equivalent laminated aluminum plate,

$$\kappa^2 = \frac{h_1^2 - \left(\frac{L_1 - L_2}{L_1}\right) h_2^2}{12} = \frac{h_1^2 - .96\ h_2^2}{12}$$

and for a single-layer plate,

$$(\kappa')^2 = \frac{t^2}{12}$$

yielding $t^2 = h_1^2 - .96\ h_2^2$

For the DFI box equipment panel, h_1 = 0.96 in, h_2 = 0.92 in, so

$$t = 0.33 \text{ in}$$

is the required effective thickness.

Element Energy

The element energy was handled as in Example Problems 1 and 2:

$$E_i = m_i \frac{\overline{\langle a_i^2 \rangle}}{\omega^2}$$

$$m_1 = \frac{40.1 \text{ lbs}}{g}$$

$$m_2 = \frac{19.7 \text{ lbs}}{g}$$

$$m_3 = \frac{215.3 \text{ lbs}}{g}$$

$$m_4 = \frac{44.7 \text{ lbs}}{g}$$

Acoustic Power Input

This term for the external acoustic power input is handled as in Example Problems 1 and 2. The applicable term for $\frac{An}{m}$ is

$$\left(\frac{A_1 N_1}{m_1}\right)_{\text{surface}} = 237{,}473 \; \Delta f$$

The internal acoustic field was assumed to be reverberant. Therefore, the internal acoustic excitation was handled in the same manner as the external acoustic excitation. The input term is

$$S_4 = \frac{2\pi^2 c_0^2 \Lambda_4 \overline{\langle P_I^2 \rangle} \, \sigma \, N_4(\text{surface})}{\omega_0^2 \, (\Delta\omega) \, m_4}$$

All of the modes of this element are surface modes, so

$$n_{4(\text{surface})} = n_4 = .27 \text{ modes/Hz}$$

The surface weight density for the cover is 0.00603 lb/in². Therefore the $\frac{AN}{m}$ term is

$$\frac{A_4 N_4}{m_4} = \frac{386}{.00603} (.27\ \Delta f) = 17,284\ \Delta f$$

The internal sound pressure levels were measured during MPTA testing and are listed in Table III-2.

The radiation efficiency values for the DFI box cover are from Appendix II, based on a coincidence frequency of 1434 Hz. This coincidence frequency for the laminated cover is given by

$$f_c = \frac{C_o^2}{2\pi} \left\{ \frac{\left(\frac{W_4}{g A_4}\right)}{\frac{E t^3}{12(1-\nu^2)}} \right\}^{1/2} = 1434 \text{ Hz}$$

where the equivalent thickness for the cover was determined, as in the Modal Coupling section of this example problem, to be

$$t = 0.189 \text{ in.}$$

The radiation efficiency values are also listed in Table III-2.

Table III-2

Frequency (Hz)	$\langle \overline{p^2} \rangle$ (dB re 2×10^{-5} N/m^2)	σ
50	122.9	0.00316
63	124.4	0.00398
80	125.6	0.00501
100	126.2	0.00631
125	126.5	0.00794
160	126.5	0.0126
200	126.3	0.0158
250	126.0	0.0200
315	125.5	0.0251
400	124.7	0.0316
500	123.6	0.0398
630	122.5	0.0501
800	120.8	0.0631
1000	119.5	0.126
1250	117.5	0.501
1600	116.5	3.98
2000	115.5	2.00

Response Solution

The response solutions for each element were determined in each 1/3 octave band from 50 to 2000 Hz. A sample prediction for the 50 Hz band is presented below as an example.

$$(\omega\eta_1 + N_2\phi_{12})\, E_1 - N_1\phi_{12}E_2 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12} + N_3\phi_{23})\, E_2 - N_2\phi_{23}E_3 = 0$$

$$-N_3\phi_{23}E_2 + (\omega\eta_3 + N_2\phi_{23} + N_4\phi_{34})\, E_3 - N_3\phi_{34}E_4 = 0$$

$$-N_4\phi_{34}E_3 + (\omega\eta_4 + N_3\phi_{34})\, E_4 = S_4$$

Substituting for the parameter values, the expressions become

$$\left\{[2\pi \times 50][0.52] + \left[.32\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right]\right\}\left[\frac{484}{g}\,\frac{\langle \overline{a_1^2}\rangle}{(2\pi \times 50)^2}\right]$$

$$-\left[6.52\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right]\left[\frac{19.7}{g}\,\frac{\langle \overline{a_2^2}\rangle}{(2\pi \times 50)^2}\right]$$

$$= \frac{2\pi^2[(1116 \times 12)^2]\left[(237{,}473)\,\frac{50}{4.33}\right]\left[10^{\frac{116.5}{10}} \times 8.41 \times 10^{-18}\right][.000794]}{[(2\pi \times 50)^2]\left[2\pi \times \frac{50}{4.33}\right]}\ ;$$

$$-\left[0.32\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right]\left[\frac{484}{g}\,\frac{\langle \overline{a_1^2}\rangle}{(2\pi \times 50)^2}\right]$$

$$+\left\{[2\pi \times 50][0.1] + \left[6.52\left(\frac{50}{4.33}\right)\right]\left[\frac{1.2}{\sqrt{50}}\right] + \left[0.12\left(\frac{50}{4.33}\right)\right]\left[\frac{24.6}{\sqrt{50}}\right]\right\}$$

$$\left[\frac{19.7}{g}\,\frac{\langle \overline{a_2^2}\rangle}{(2\pi \times 50)^2}\right] - \left[0.32\left(\frac{50}{4.33}\right)\right]\left[\frac{24.6}{\sqrt{50}}\right]\left[\frac{215.3}{g}\,\frac{\langle \overline{a_3^2}\rangle}{(2\pi \times 50)^2}\right] = 0\ ;$$

$$- \left[0.12\left(\frac{50}{4.33}\right)\right]\left[\frac{24.6}{\sqrt{50}}\right]\left[\frac{19.7}{g}\,\frac{\langle \overline{a_2^2} \rangle}{(2\pi \times 50)^2}\right]$$

$$+ \left\{[2\pi \times 50][0.1] + \left[0.32\left(\frac{50}{4.33}\right)\right]\left[\frac{24.6}{\sqrt{50}}\right] + \left[0.27\left(\frac{50}{4.33}\right)\right]\left[\frac{255}{\sqrt{50}}\right]\right\}$$

$$\cdot \left[\frac{215.3}{g}\,\frac{\langle \overline{a_3^2} \rangle}{(2\pi \times 50)^2}\right] - \left[0.12\left(\frac{50}{4.33}\right)\right]\left[\frac{255}{\sqrt{50}}\right]\left[\frac{44.7}{g}\,\frac{\langle \overline{a_4^2} \rangle}{(2\pi \times 50)^2}\right] = 0 \ ;$$

and

$$- \left[0.27\left(\frac{50}{4.33}\right)\right]\left[\frac{255}{\sqrt{50}}\right]\left[\frac{215.3}{g}\,\frac{\langle \overline{a_3^2} \rangle}{(2\pi \times 50)^2}\right]$$

$$+ \left\{[2\pi \times 50][0.1] + \left[0.12\left(\frac{50}{4.33}\right)\right]\left[\frac{255}{\sqrt{50}}\right]\right\}\left[\frac{44.7}{g}\,\frac{\langle \overline{a_4^2} \rangle}{(2\pi \times 50)^2}\right]$$

$$= \frac{2\pi^2[(1116 \times 12)^2]\left[(17{,}284)\,\frac{50}{4.33}\right]\left[10^{\frac{122.9}{10}} \times 8.41 \times 10^{-18}\right][.00316]}{[(2\pi \times 50)^2]\left[2\pi \times \frac{50}{4.33}\right]}$$

where $\Delta f = \frac{f}{4.33}$ for 1/3 octave bands.

Dividing by g and solving for the mean squared accelerations:

$$\frac{\langle \overline{a_1^2} \rangle}{g^2} = .0135$$

$$\frac{\langle \overline{a_2^2} \rangle}{g^2} = .130$$

$$\frac{\langle \overline{a_3^2} \rangle}{g^2} = .0433$$

$$\frac{\langle \overline{a_4^2} \rangle}{g^2} = .646$$

The acceleration spectral density levels are

$$\frac{\langle \overline{a_1^2} \rangle}{g^2\ (\Delta f)} = \frac{.0135}{\left(\frac{50}{4.33}\right)} = .00117$$

$$\frac{\langle \overline{a_2^2} \rangle}{g^2\ (\Delta f)} = \frac{.130}{\left(\frac{50}{4.33}\right)} = .0112$$

$$\frac{\langle \overline{a_3^2} \rangle}{g^2\ (\Delta f)} = \frac{.0433}{\frac{50}{4.33}} = .00375$$

$$\frac{\langle \overline{a_4^2} \rangle}{g^2\ (\Delta f)} = \frac{.646}{\frac{50}{4.33}} = .0560$$

and the root-mean-squared acceleration in this 1/3 octave band is

$$(g_{rms})_1 = \sqrt{\frac{\langle \overline{a_1^2} \rangle}{g^2}} = \sqrt{.0135} = .116$$

$$(g_{rms})_2 = \sqrt{.130} = .360$$

$$(g_{rms})_3 = \sqrt{.0433} = .208$$

$$(g_{rms})_4 = \sqrt{.646} = .804$$

The predicted response levels are plotted in Figures III-16 through III-21 in acceleration spectral density and g_{rms} in 1/3 octave band formats.

The criterion of 20 modes per analysis band in each element as a requirement for SEA prediction accuracy is satisfied for the model at 800 Hz, indicating predictions with ± 3 dB accuracy above this frequency.

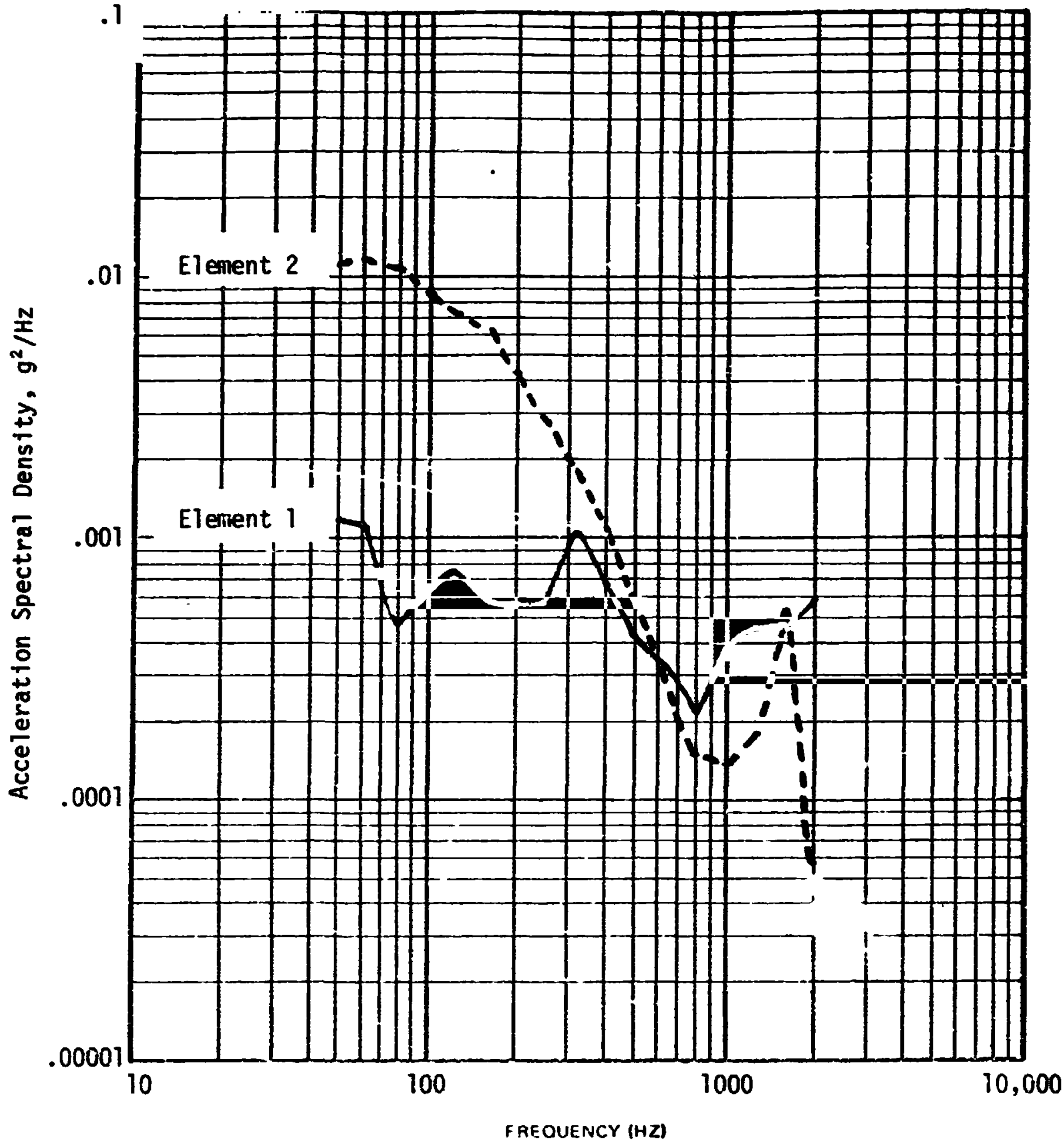


Figure III-16. Predicted Response (g^2/Hz) for External Tank Interstage Area (Elements 1 and 2)

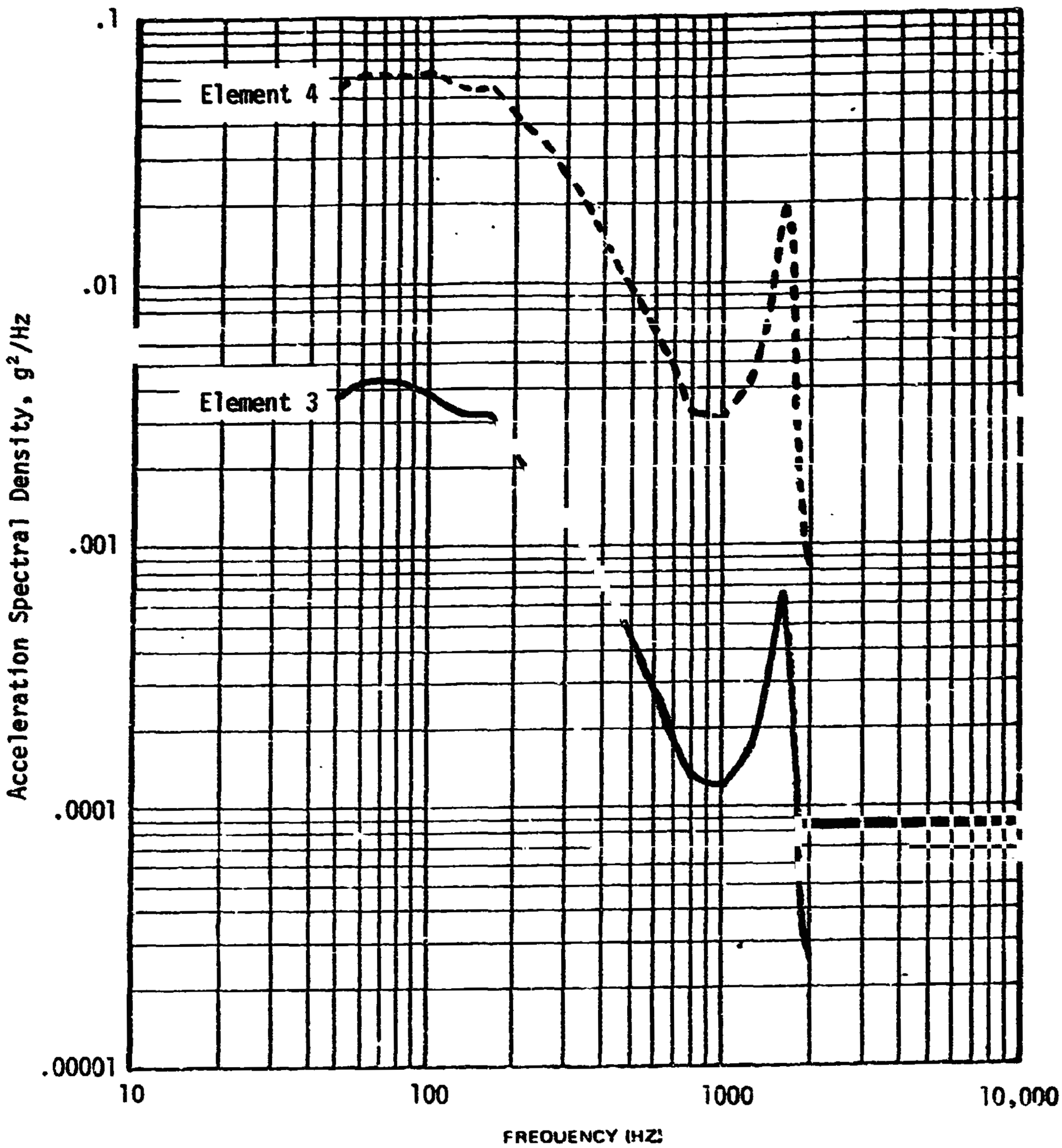


Figure III-17. Predicted Response (g^2/Hz) for External Tank Interstage Area (Elements 3 and 4)

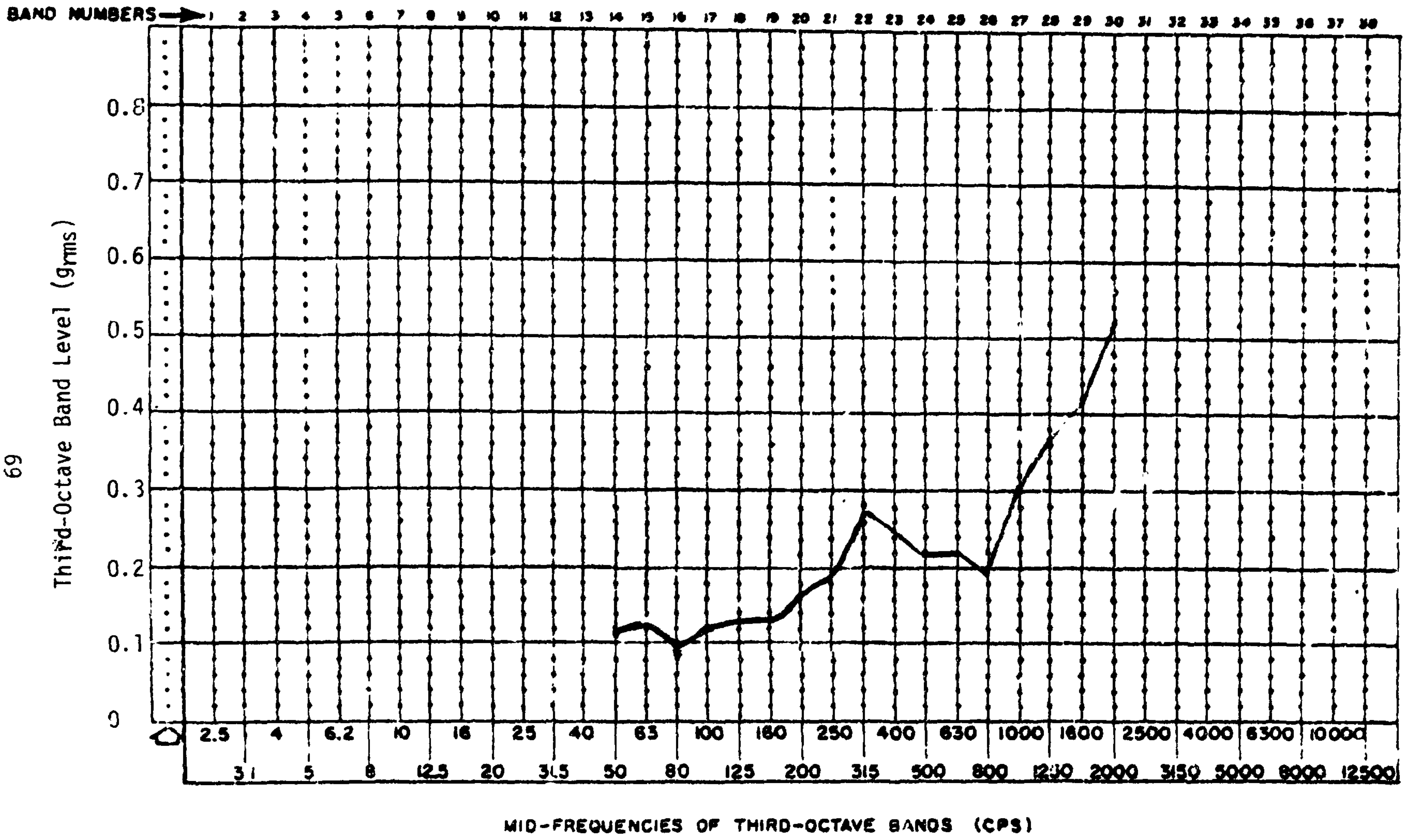


Figure III-18. Predicted Response for External Tank Interstage Area - Shell Structure (Element 1)

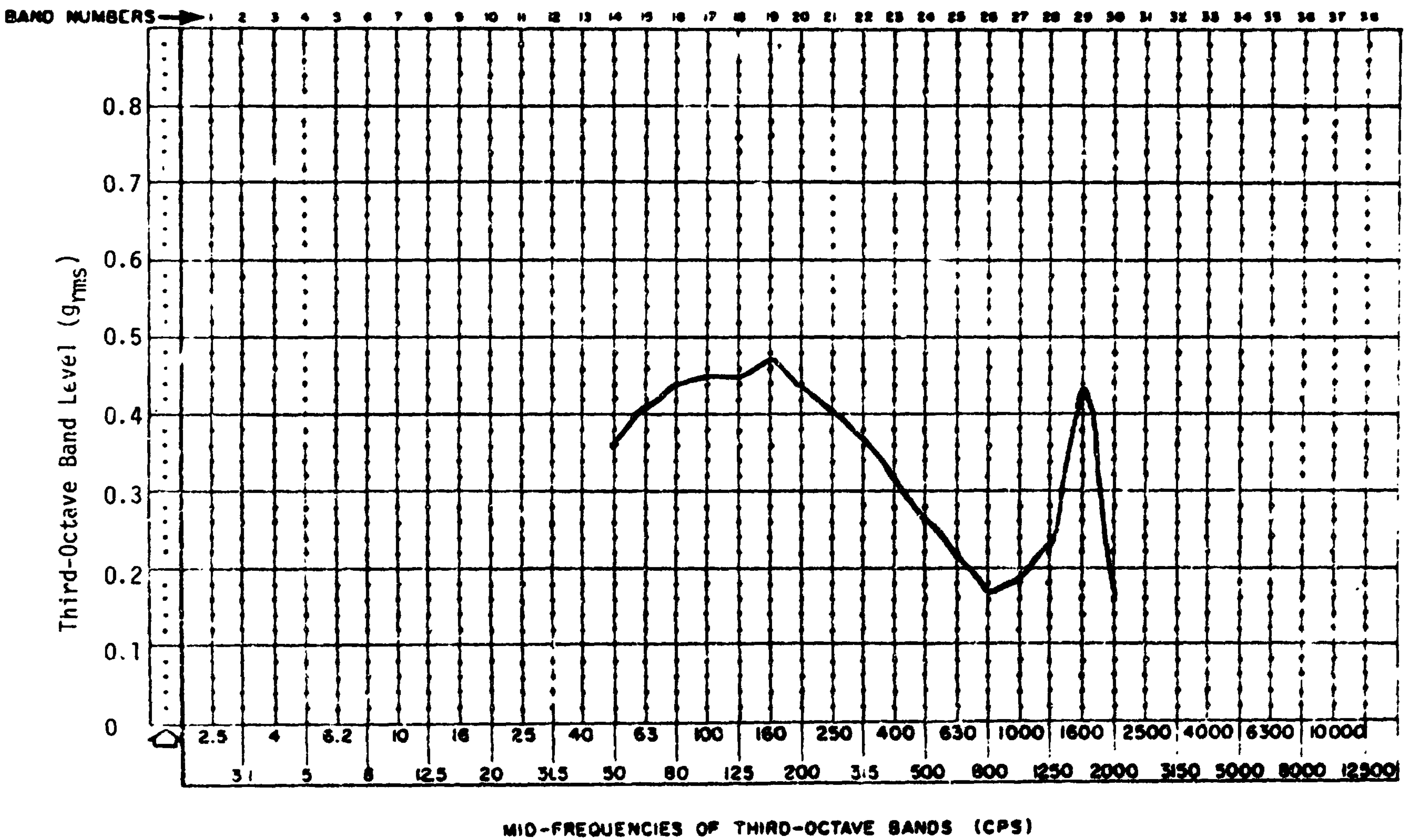


Figure III-19. Predicted Response (g_{rms}) for External Tank Interstage Area - DFI Support Intercostals (Element 2)

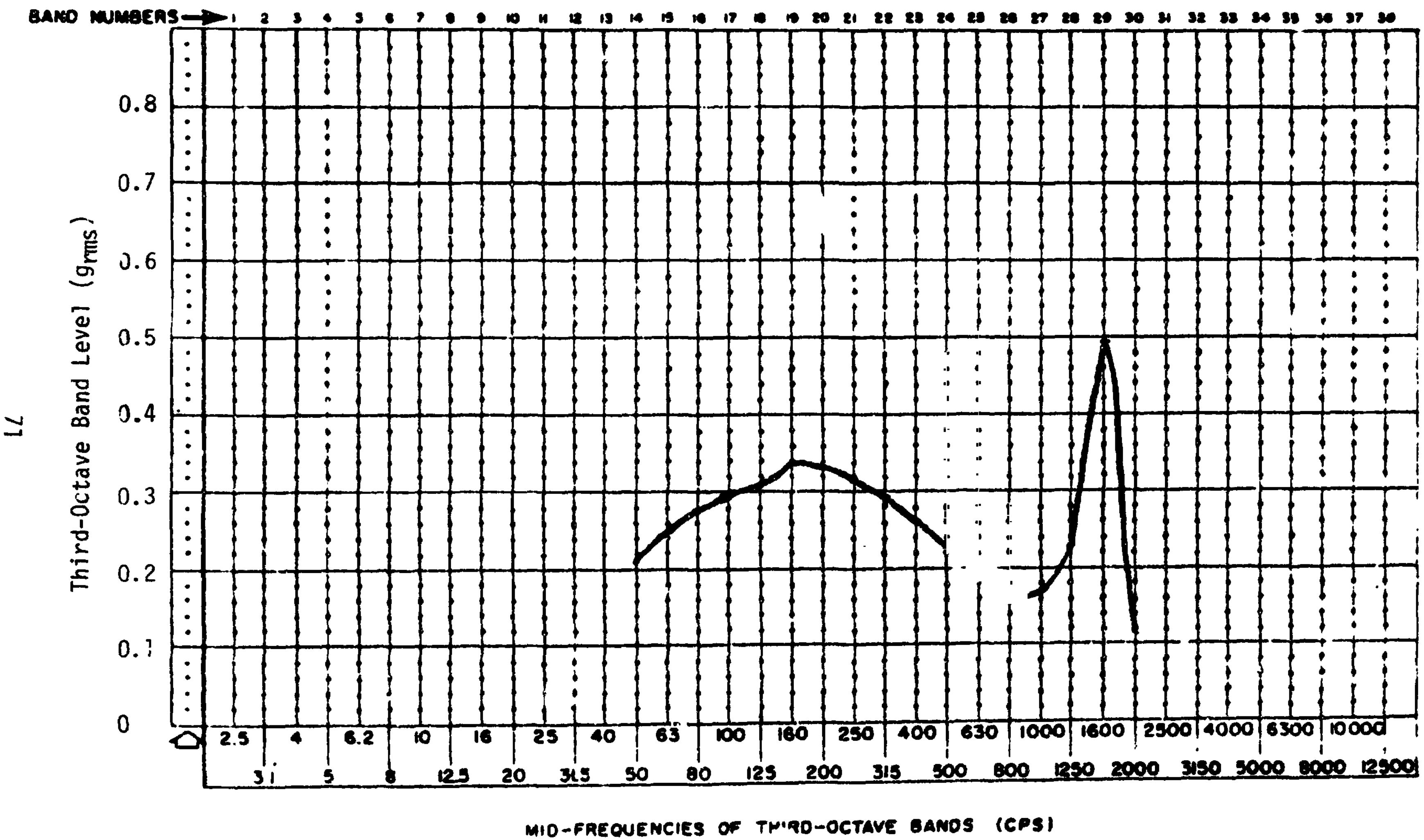


Figure III-20. Predicted Response (g_{rms}) for External Tank Interstage Area - DFI Equipment Panel (Element 3)

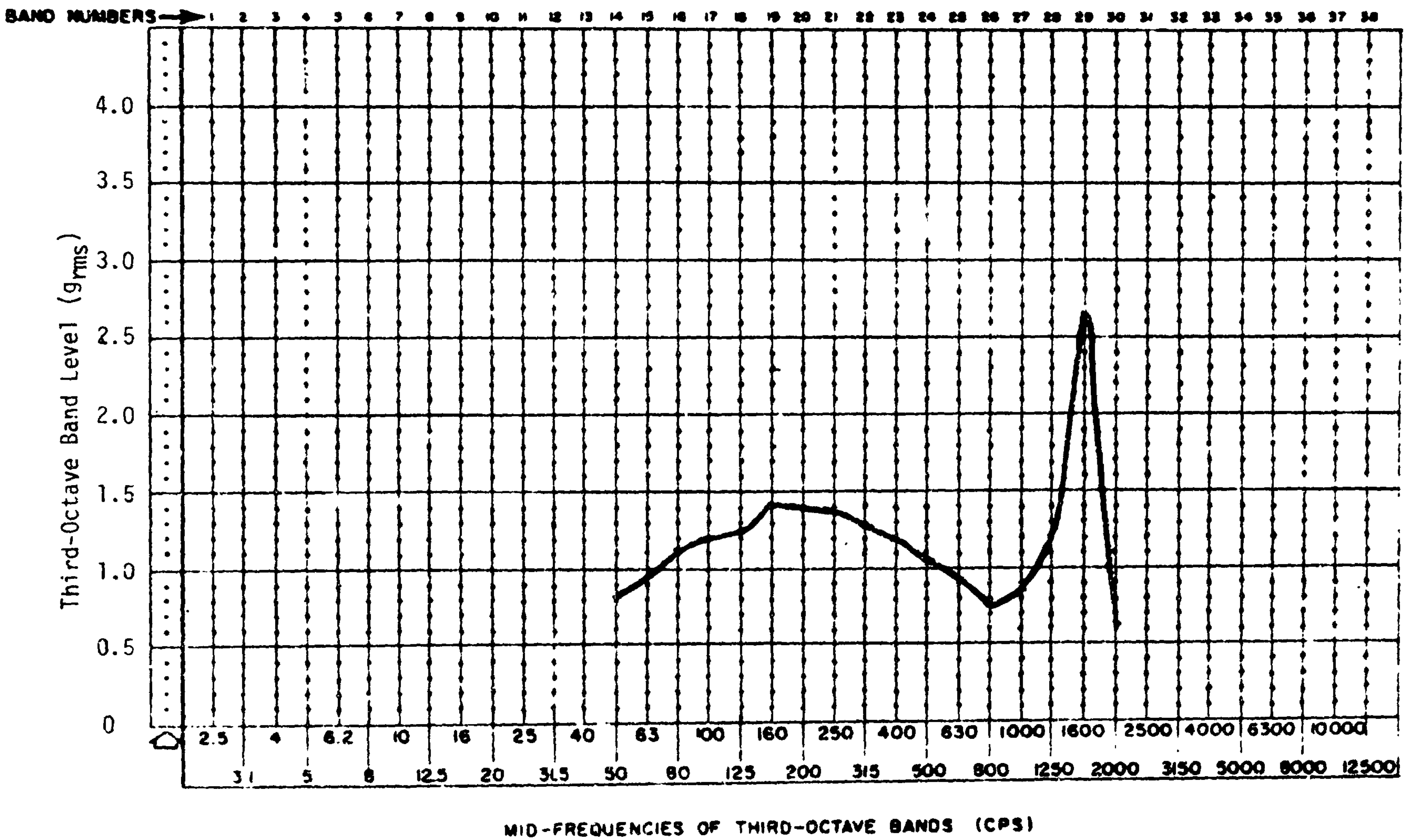


Figure III-21. Predicted Response (g_{rms}) for External Tank Interstage Area - DFI Box Cover (Element 4)

Comparison of Prediction with Measured Test Data

Subsequent to completion of this analysis, MPTA response data were made available for comparison with the predicted responses of elements 1 and 3. The external shell (element 1) prediction is essentially the same as for Example Problem Number 1 and has been previously discussed. The data comparison for the DFI equipment panel (element 3) is presented in Figure III-22. The comparison shows the prediction to furnish a good approximation to the average response at frequencies above 100 Hz. The peak in predicted response at 1600 Hz is due to a coincidence frequency effect of the DFI box cover (element 4), similar to the external shell response peaking discussed in Example Problem Number 1, which was attributed to improper damping definition near the coincidence frequency.

A supplementary check case was performed to evaluate sensitivity of the panel response prediction to variation in the external acoustic levels. The external acoustic levels were increased by 6 dB for the check case and resulted in essentially no change in the predicted DFI panel response, showing it to be mainly driven by the internal acoustics.

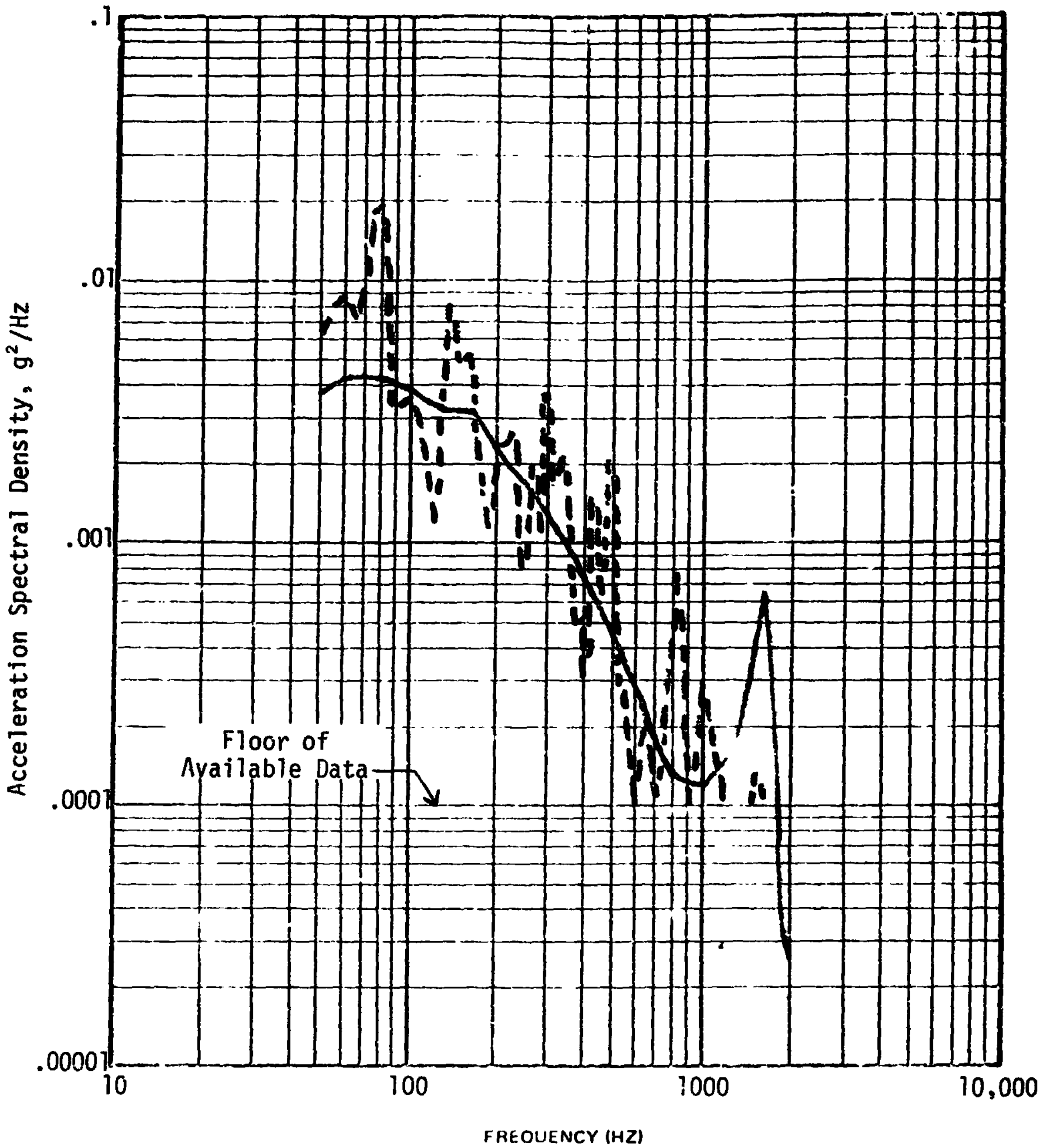


Figure III-22. Comparison of Measured Test Data (dashed line) with Predic'ed Response (solid line) for DFI Equipment Panel (Element 3)

Example Problem Number 4
SPACE SHUTTLE RETRIEVABLE SPACECRAFT

The structure to be analyzed consists of the fairing for a Delta launch vehicle and a payload spacecraft. The payload attaches directly to the fairing rather than to a lower stage of the Delta launch vehicle. This analysis makes use of the averaging abilities of SEA to provide a gross estimate of the payload response for evaluation of the attachment configuration. The analysis utilizes information and data obtained during a number of previous modal analyses of the Delta launch vehicle and payload spacecraft. Since the fairing response to the flight acoustic environment had been previously measured, the system was treated as having a mechanical input from the fairing to the spacecraft. This treatment assumes no change in fairing average response with the spacecraft connected directly to the fairing. Since this configuration may be expected to attenuate fairing response somewhat due to mass loading, the spacecraft response predictions presented herein are expected to be a conservative estimation.

Model

The model for the structure will have two elements, one for the fairing and one for the payload. These elements are illustrated in Figure III-23.

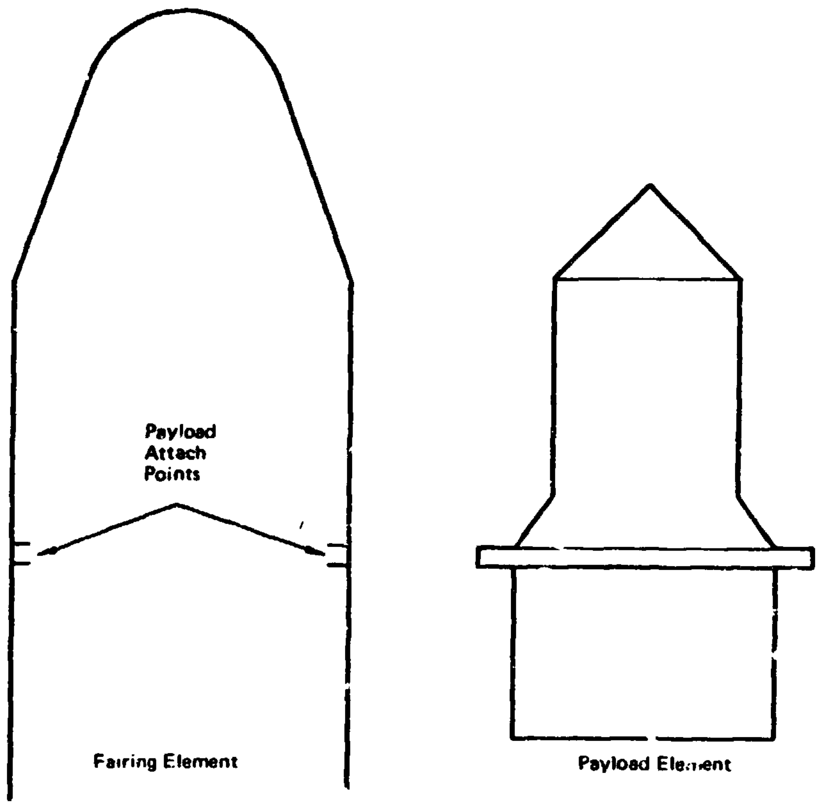


Figure III-23. SEA Model of Fairing and Payload Spacecraft

Response Equations

The SEA response equations for the two-element system with external acoustic excitation are

$$(\omega\eta_1 + N_2\phi_{12})\, E_1 - N_1\phi_{12}E_2 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12})\, E_2 = 0$$

where the subscript 1 denotes external shell values, and 2 denotes DFI box and intercostal values.

Since the response of element 1 is known, the response of element 2 can be determined using only the second equation:

$$E_2 = \frac{N_2\phi_{12} E_1}{(\omega\eta_2+N_1\phi_{12})}$$

Damping

A damping value of 1-1/2% (η=0.03) was used for both of the model elements. This damping value had previously been selected for use with modal analyses of the Delta fairing and payloads.

Modal Density

The modal density of the fairing was calculated by assuming the isogrid structure to be composed of beam and plate elements. Modal densities were then calculated for each of the beam and plate sub-elements, and these sub-element values were summed to give the total modal density of the fairing.

The modal density of a single plate sub-element is:

$$n_p(\omega) = \frac{A}{4\pi h \sqrt{\dfrac{E}{12\rho(1-\nu^2)}}}$$

and for a beam:

$$n_b(\omega) = \frac{L}{2\pi(\omega)^{1/2}(EI/\rho A)^{1/4}}$$

Totalling the modal densities:

$$n(\omega) = n_b + n_p = \frac{1}{2\pi(\omega)^{1/2}(E/\rho)^{1/4}}\left[\sum \frac{L}{(I/A)^{1/4}}\right] + \frac{1}{4\pi\sqrt{\frac{E}{12\rho(1-\nu^2)}}}\left[\sum \frac{A}{h}\right]$$

The summations account for all the modal properties of the stiffeners and skin. The values for these sums used for the fairing were:

$$\sum \frac{L}{(I/A)^{1/4}} = 133,\text{[illegible]}\ \text{in}^{1/2}; \quad \sum \frac{A}{h} = 2,181,000\ \text{in}$$

These modal densities were corrected for curvature effects based on a ring frequency of 650 Hz. Correction factors were obtained from Reference 4. The resulting number of modes for each frequency band of analysis is listed in Table III-3. The plate modal density equation presented is actually for free-free boundary conditions, but yields a valid approximation for all boundary conditions at high frequencies. However, this technique is expected to overestimate the modal density in the lower frequencies of analysis.

The modal density of a "typical" payload is estimated to be approximately the same as for the fairing. This estimate is based on comparisons of the results of modal analyses for the fairing and for 2000-pound and 4000-pound Delta payloads. The results of the analyses, presented in Figures III-24 and III-25, yield essentially identical estimates for the modal densities of the two elements. Although simple, beam-type models were used for these analyses, the relative equality of the modal densities may be expected to remain when the higher frequency modes are considered.

Table III-3

f	N	PSD (g^2/Hz)
250	633	3.0
315	990	4.5
400	1500	15.0
500	2320	18.0
630	3960	10.0
800	4780	3.5
1000	5280	0.8
1250	6230	0.5
1600	7410	0.45
2000	9120	0.45

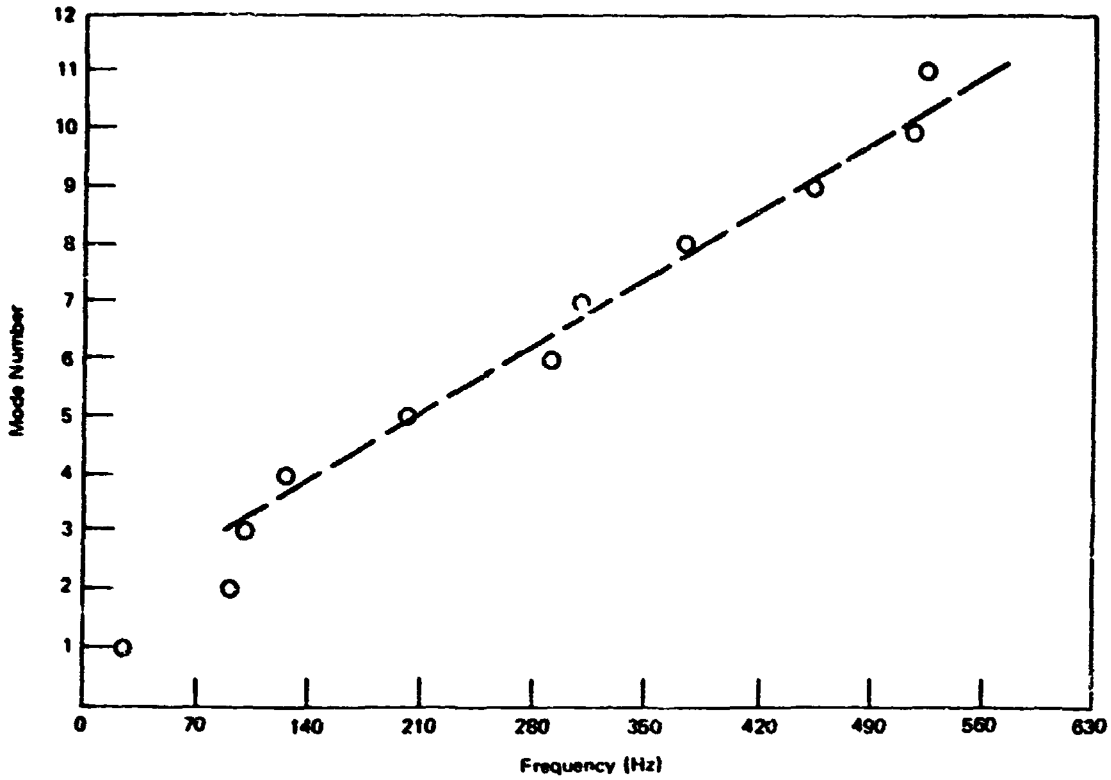


Figure III-24. Delta Fairing Modes

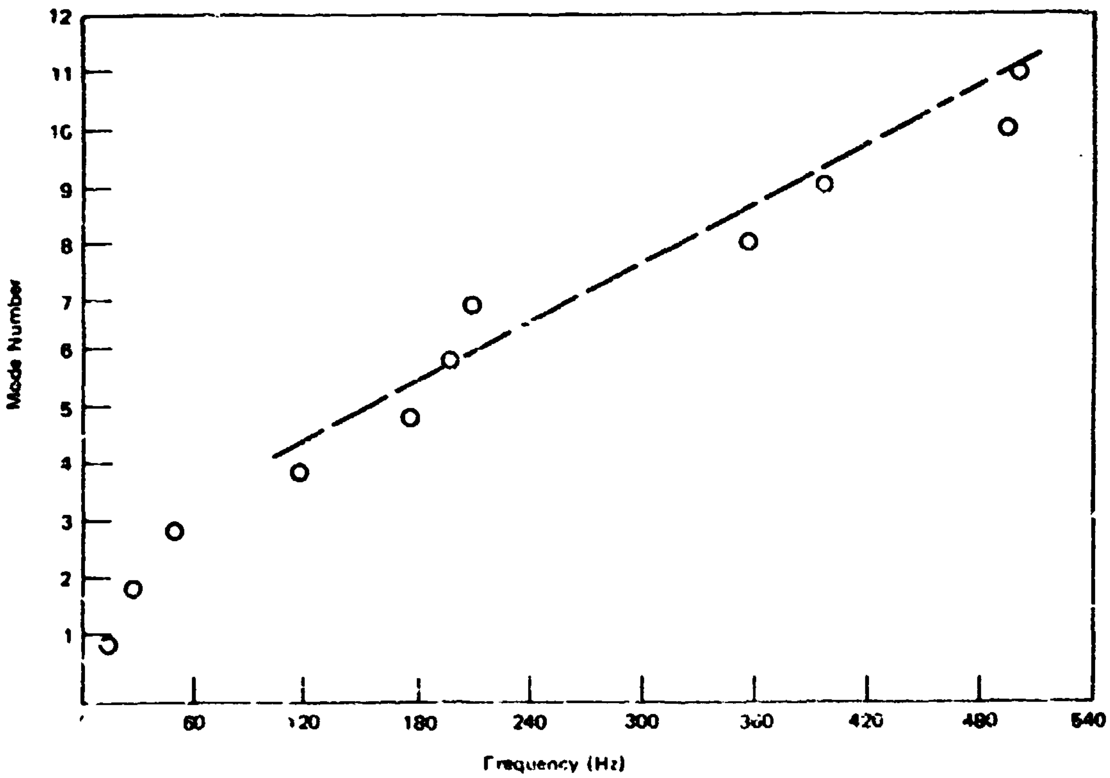


Figure III-25. Delta Payload Modes (2,0.0 lb or 4,000 lb)

Modal Coupling

In order to estimate the coupling parameter, the structural joint between the Saturn S-II aft skirt and thrust structure assemblies was selected as typical of the fairing/payload joint. The S-II aft skirt/thrust structure was modeled as an SEA system of two elements with only the aft skirt externally excited. This model permitted solving for the coupling parameter of the system using the explicit response solution

$$\frac{E_T}{E_A} = \frac{\overline{\langle a_T^2 \rangle}}{\overline{\langle a_A^2 \rangle}} \frac{M_T}{M_A} = \frac{N_T \phi}{\omega \eta_T + N_A \phi}$$

where "T" subscripts indicate the thrust structure and "A" subscripts indicate the aft skirt.

Assuming thrust structure damping to be a maximum of 1 percent and that the coupling decreases with frequency at the same rate which MDAC experienced with the UpSTAGE evaluation, the S-II aft skirt/thrust structure SEA coupling parameter was estimated to have the values shown in Figure III-26. These values were used for the coupling parameter of the fairing/payload model.

Element Energy

The element energy was handled as in Example Problem 1.

$$E_i = m_i \frac{\overline{\langle a_i^2 \rangle}}{\omega^2}$$

$$m_1 = \frac{1348 \text{ lbs}}{g}$$

$$m^2 = \frac{3900 \text{ lbs}}{g}$$

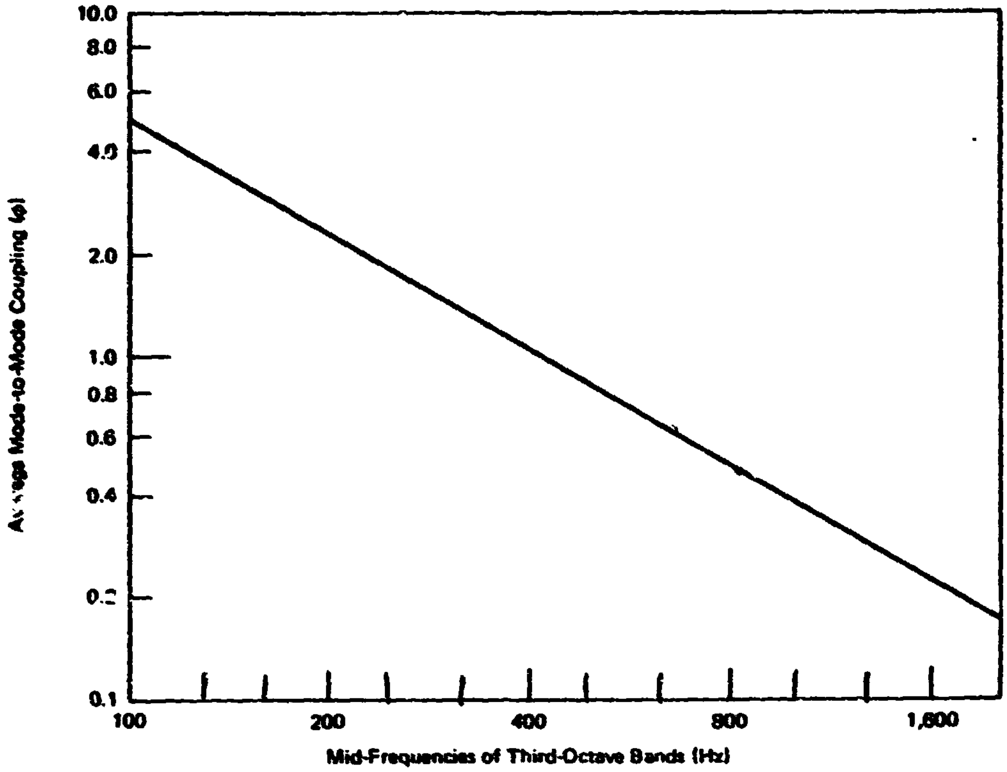


Figure III-26. Field-Joint Coupling (ϕ_0) for Delta/Payload Analysis (Based on S-II Data)

The measured response of element 1 is shown in Figure III-27 in an acceleration spectral density format with units of

$$PSD = \frac{G^2}{Hz} = \frac{<a_1^2>}{g^2\ (\Delta f)}$$

Therefore

$$<a_1^2> = g^2(\Delta f)\cdot PSD$$

can be evaluated from Figure III-27 by using the PSD value for the center frequency as representative over the bandwidth, Δf. The PSD values used for analysis are listed in Table III-3.

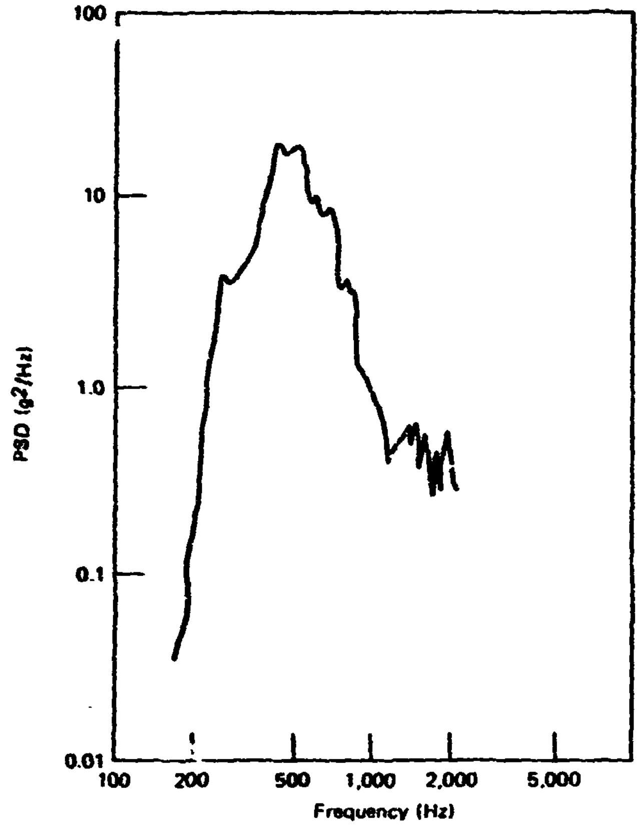


Figure III-27. Fairing Skin Response (Radial)

Response Solution

The response predictions for each element were determined in each one-third octave band from 250 to 2000 Hz. A sample prediction for the 250 Hz band is presented below as an example:

$$E_2 = \frac{N_2 \ \phi_{12}}{(\omega \eta_2 + N_1 \ \phi_{12})} \ E_1$$

Substituting for the parameter values, the expression becomes

$$\frac{3900}{g} \ \frac{\langle \bar{a}_2^2 \rangle}{(2\pi X250)^2} = \frac{(6.33)(1.75)}{\left|[2\pi X250][0.03]+[(633)(1.75)]\right|} \left\{ \frac{1348}{g} \ \frac{g^2 \left(\frac{250}{4.33}\right)(3.0)}{(2\pi X250)^2} \right\}$$

where $\Delta f = \frac{f}{4.33}$ for one-third octave bands.

The acceleration spectral density level is

$$\frac{\overline{\langle a_2^2 \rangle}}{g^2(\Delta f)} = 0.43$$

and the root-mean-squared acceleration in this one-third octave band is

$$(g_{rms})_2 = \sqrt{\frac{\overline{\langle a_2^2 \rangle}}{g^2}} = \sqrt{24.8} = 4.98$$

The predicted response levels are plotted in Figures III-28 and III-29 in acceleration spectral density and g_{rms} in one-third octave band formats. As previously noted, these levels can be expected to represent a conservative overestimate of the actual response due to an expected attenuation of the fair response in this configuration.

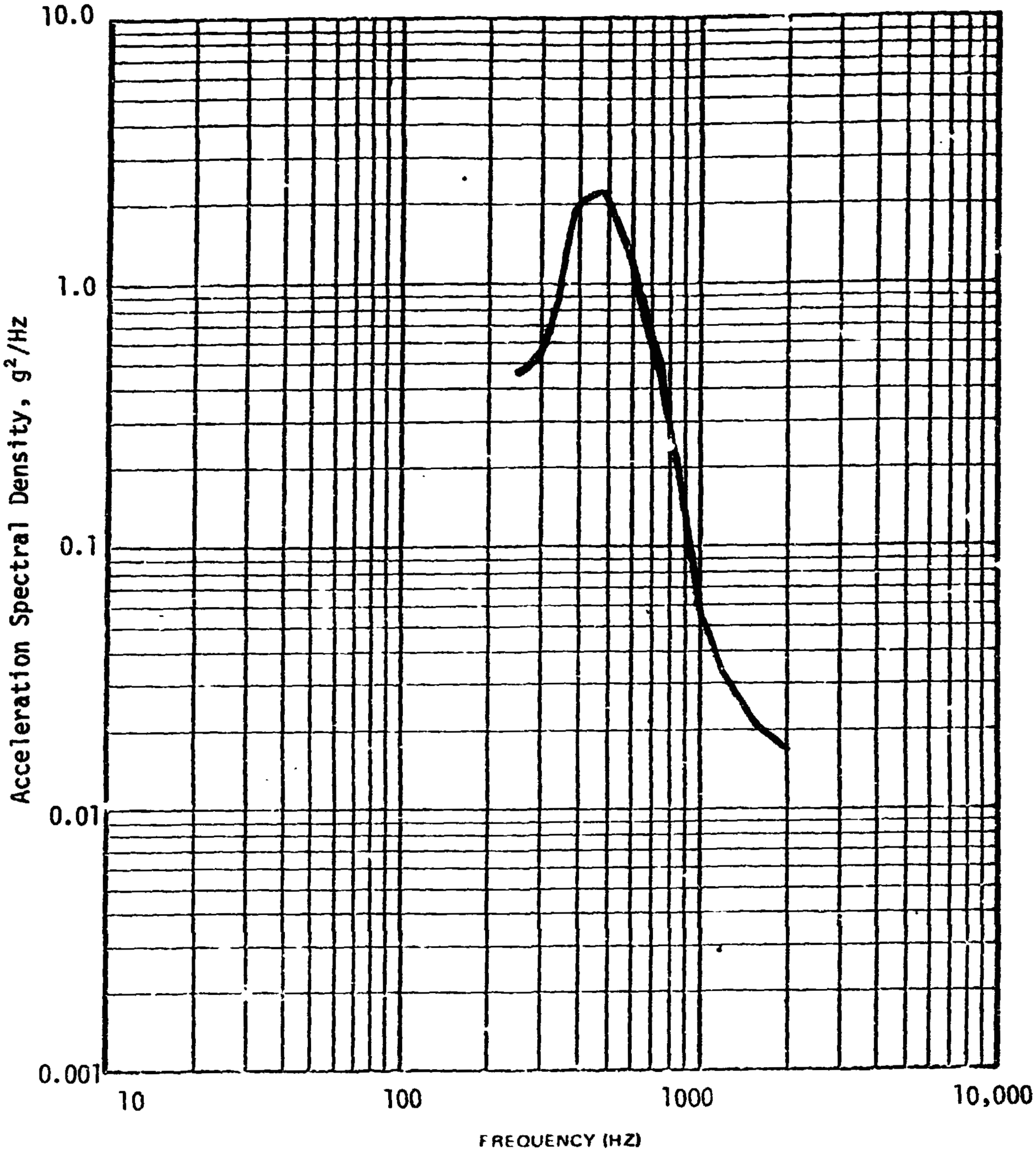


Figure III-28. Predicted Response (g^2/Hz) for Payload Element

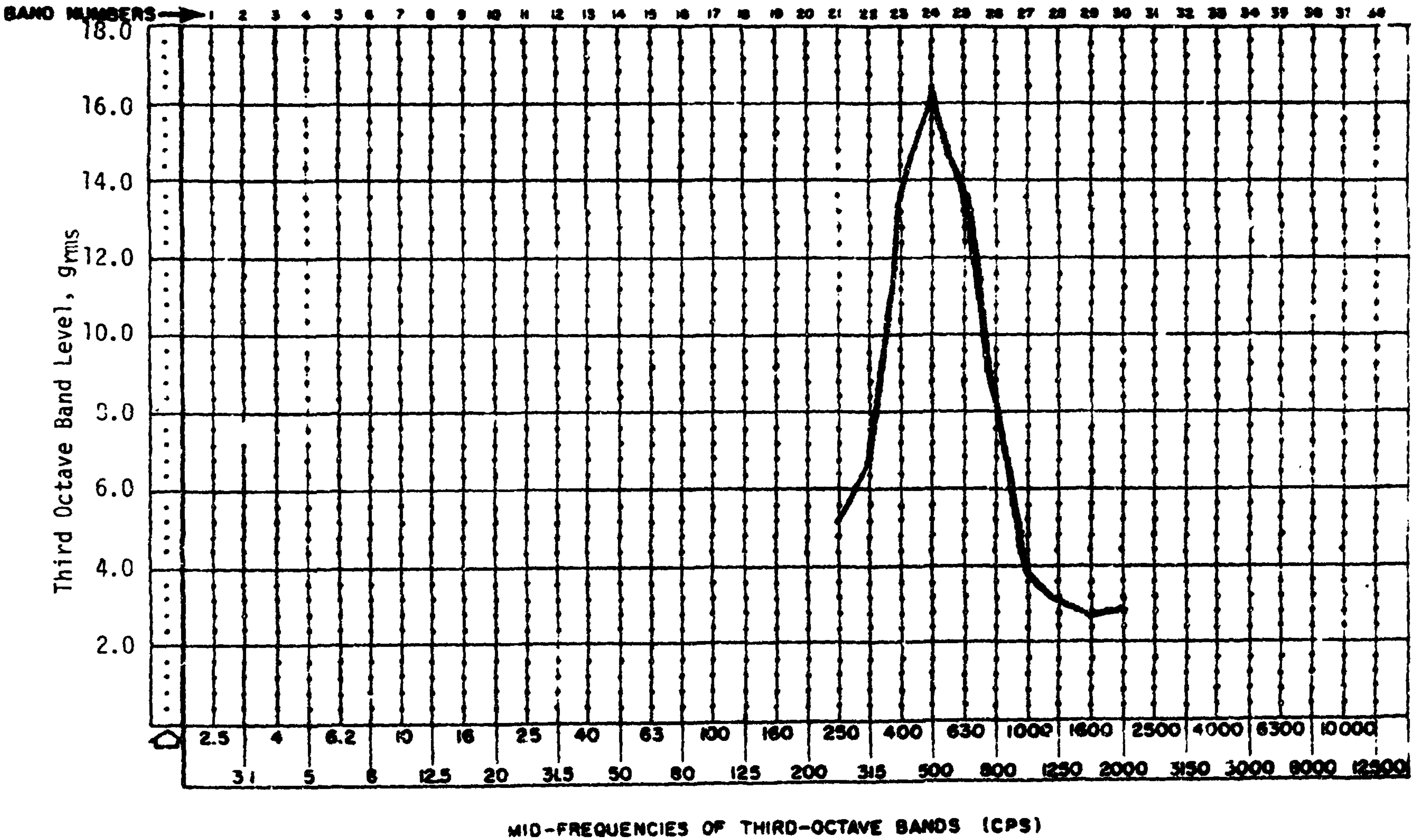


Figure III-29. Predicted Response (g_{rms}) for Payload Element

REFERENCES TO APPENDIX III

1. D. N. Roudebush. Design Development Acoustic Fatigue Test of S-IVB/V Structural AFT Interstage Panels. Douglas Report SM-47320, Douglas Missile and Space Systems Division, 15 April 1966.

2. R. W. Sevy and D. A. Earls. The Prediction of Internal Vibration Levels of Flight Vehicle Equipments Using Statistical Energy Methods. Technical Report AFFDL-TR-69-54, Air Force Flight Dynamics Laboratory, January 1970.

3. R. H. Lyon. Statistical Energy Analysis of Dynamical Systems: Theory and Applications. The MIT Press, 1975.

4. R. F. Davis and D. E. Hines. "Final Report - Performance of Statistical Energy Analysis." McDonnell Douglas Astronautics Company Report MDC G4741, June 1973; also NASA CR-124322, June 1973.

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Appendix IV

SEA RESPONSE SOLUTION PROGRAMS FOR HEWLETT-PACKARD AND TEXAS INSTRUMENTS CALCULATORS

SOLUTION FOR MODEL RESPONSES

One of the final steps of the SEA applications procedure will be the mathematical solution for the system response. Because of the repetitive nature of the response calculations for the various frequency bands of an analysis (often the 14 one-third octave bands from 100 to 2000 Hz will be included in an analysis), response calculations can be expediently accomplished through the use of preprogrammed solutions. Such programs also eliminate the errors which occur with monotonous, repetitive, hand calculations. The capability of many programmable hand calculators is suitable for solution of the smaller common model sizes. Programs for both Hewlett-Packard and Texas Instruments programmable calculators are provided, thus encompassing the more popular equipment in current use, as well as providing examples in both reverse polish and algebraic notation for modification to additional calculator systems.

HEWLETT-PACKARD (HP-67, HP-97) SEA RESPONSE PROGRAM - TWO ELEMENTS

<u>Description</u>

This program provides the SEA response solution for a system of two elements. The response equations are

$$(\omega\eta_1 + N_2\phi_{12})\,E_1 - N_1\phi_{12}E_2 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12})\,E_2 = S_2$$

The energy input terms may be input directly to the program or can be calculated if a reverberant acoustic field provides the structural excitation. The required inputs to the program are listed below:

f - center frequency of analysis band (Hz)

Δf - bandwidth (Hz)

η_1 - element 1 damping factor (dimensionless)

η_2 - element 2 damping factor (dimensionless)

ϕ_{12} - coupling value for elements 1-2 $\left(\frac{1}{\text{mode} \cdot \text{sec}}\right)$

n_1 - element 1 modal density (modes/Hz)

n_2 - element 2 modal density (modes/Hz)

W_1 - weight of element 1 (lbs)

W_2 - weight of element 2 (lbs)

S_1 - element 1 energy input $\left(\frac{\text{in-lb}}{\text{sec}}\right)$

S_2 - element 2 energy input $\left(\frac{\text{in-lb}}{\text{sec}}\right)$

Additional requirements if the S_i are to be calculated for a reverberant acoustic field:

C_o - speed of sound in surrounding medium (in/sec)

$\left(\frac{An}{m}\right)_1$ - element 1 surface mass parameter $\left(\frac{\text{in}^3\text{-modes}}{\text{lb-sec}^2\text{-Hz}}\right)$

$\left(\frac{An}{m}\right)_2$ - element 2 surface mass parameter $\left(\frac{\text{in}^3\text{-modes}}{\text{lb-sec}^2\text{-Hz}}\right)$

$(SPL)_1$ - element 1 sound pressure level (dB)

$(SPL)_2$ - element 2 sound pressure level (dB)

σ_2 - element 1 radiation efficiency (dimensionless)

σ_2 - element 2 radiation efficiency (dimensionless)

Output from the program is the root-mean-squared acceleration of the elements over the input frequency interval:

$$\sqrt{\frac{\langle a_1^2 \rangle}{g^2}} \quad \text{and} \quad \sqrt{\frac{\langle a_2^2 \rangle}{g^2}}$$

USER INSTRUCTIONS

Step		Enter	Press	Display
0		--	RTN	--
1	Input number of elements (N = 2)	2.0	R/S	2.0
2	Input center frequency	f	R/S	$2\pi f$
3	Input bandwidth	Δf	R/S	Δf
4	Input damping of element 1	η_1	R/S	η_1
5	Input damping of element 2	η_2	R/S	η_2
6	Input element coupling	ϕ_{12}	R/S	ϕ_{12}
7	Input modal density of element 1	n_1	R/S	N_1
8	Input modal density of element 2	n_2	R/S	N_2
9	Input weight of element 1	W_1	R/S	W_1
10	Input weight of element 2	W_2	R/S	W_2
11a	Input "0" if element energy input terms (S_i) are to be input to program; input "1" if reverberant acoustic input is to be calculated by program and see below.	0	R/S	0
12a	Input S_1	S_1	R/S	S_1
13a	Input S_2, obtain solution	S_1	R/S	$\sqrt{\frac{\langle a_1^2\rangle}{g^2}}$
14a	Obtain solution for element 2	$\sqrt{\frac{\langle a_1^2\rangle}{g^2}}$	R/S	$\sqrt{\frac{\langle a_2^2\rangle}{g^2}}$
15a	Check for end of program	--	R/S	0
11b	Alternate solution with calculations for reverberant acoustics	1	R/S	1
12b	Input speed of sound (in/sec)	C_0	R/S	C_0
13b	Input $\left(\frac{An}{m}\right)$ for element 1	$\left(\frac{An}{m}\right)_1$	R/S	$\left(\frac{An}{m}\right)_1$
14b	Input $\left(\frac{An}{m}\right)$ for element 2	$\left(\frac{An}{m}\right)_2$	R/S	$\left(\frac{An}{m}\right)_2$
15b	Input SPL for element 1	$(SPL)_1$	R/S	$(SPL)_1$
16b	Input SPL for element 2	$(SPL)_2$	R/S	$(SPL)_2$
17b	Input radiation efficiency for element 1	σ_1	R/S	σ_1
18b	Input radiation efficiency for element 2, obtain solution	σ_2	R/S	$\sqrt{\frac{\langle a_1^2\rangle}{g^2}}$

Step		Enter	Press	Display
19b	Obtain solution for element 2	$\sqrt{\dfrac{\langle a_1^2 \rangle}{g^2}}$	R/S	$\sqrt{\dfrac{\langle a_1^2 \rangle}{g^2}}$
20b	Check for end of program	--	R/S	0

Data Registers

A	η_1	7	a_{12}
B	η_2	8	a_{21}
C	W_1	9	a_{22}
D	W_2	P0	--
E	n_1	P1	C_0
I	$(2\pi f)^2/386$	P2	σ_1
0	n_2	P3	σ_2
1	ϕ_{12}	P4	$(SPL)_1$
2	S_1	P5	$(SPL)_2$
3	S_2	P6	$\left(\frac{An}{m}\right)_1$
4	N	P7	$\left(\frac{An}{m}\right)_2$
5	$2\pi f$	P8	Δf
6	a_{11}	P9	--

where the a_{ij} represent program-generated matrix elements.

PROGRAM LISTING

Program Step	Key	Program Step	Key	Program Step	Key
001	*LBLA	058	CHS	115	RCL1
002	STO4	059	x	116	÷
003	R/S	060	P⇌S	117	STO5
004	Pi	061	RCL3	118	RCLE
005	x	062	X²	119	RCL1
006	2	063	+	120	x
007	x	064	P⇌S	121	RCLD
008	STO5	065	STO1	122	x
009	R/S	066	RCL6	123	RCLI
010	STO8	067	x	124	÷
011	R/S	068	RCL2	125	CHS
012	STO4	069	x	126	STO7
013	R/S	070	RCL4	127	RCL0
014	STO5	071	1	128	RCL1
015	R/S	072	0	129	x
016	STO1	073	÷	130	RCLC
017	R/S	074	10ˣ	131	x
018	RCL8	075	x	132	RCLI
019	x	076	P⇌S	133	÷
020	STOE	077	STO2	134	CHS
021	R/S	078	P⇌S	135	STO8
022	RCL8	079	RCL1	136	RCL5
023	[illegible]	080	RCL7	137	RCL6
024	STO8	081	x	138	x
025	R/S	082	RCL3	139	RCLE
026	STOC	083	x	140	RCL1
027	R/S	084	RCL5	141	x
028	STOD	085	1	142	+
029	R/S	086	0	143	RCLD
030	X=0?	087	÷	144	x
031	GTO1	088	10ˣ	145	RCLI
032	P⇌S	089	x	146	÷
033	R/S	090	P⇌S	147	STO9
034	STO1	091	STO3	148	1/X
035	R/S	092	STO2	149	RCL3
036	STO6	093	*LBL1	150	x
037	R/S	094	R/S	151	RCL7
038	STO7	095	STO2	152	x
039	R/S	096	R/S	153	CHS
040	STO4	097	STO3	154	RCL2
041	R/S	098	*LBL2	155	+
042	STO5	099	RCL5	156	RCL8
043	R/S	100	X²	157	RCL9
044	STO2	101	3	158	+
045	R/S	102	8	159	RCL7
046	STO3	103	6	160	x
047	Pi	104	+	161	CHS
048	RCL1	105	STO1	162	RCL6
049	X²	106	RCL5	163	+
050	x	107	RCLA	164	÷
051	8	108	x	165	SF1
052	.	109	RCL0	166	SF2
053	÷	110	RCL1	167	√X
054	1	111	x	168	PRTX
055	EEX	112	+	169	R/S
056	1	113	RCL2	170	[illegible]
057	8	114	x	171	RCL8

C-2

Program Step	Key
172	×
173	CHS
174	RCL3
175	+
176	RCL9
177	÷
178	√x
179	PRT
180	R/S
181	e
182	RTN
183	R/S

TEXAS INSTRUMENTS (TI PROGRAMMABLE 59) SEA RESPONSE PROGRAM - TWO OR THREE ELEMENTS

Description

This program provides the SEA response solution for a system of two or three elements. The response equations are:

$$(\omega\eta_1 + N_2\phi_{12} + N_3\phi_{13})\,E_1 - N_1\phi_{12}E_2 - N_1\phi_{13}E_3 = S_1$$

$$-N_2\phi_{12}E_1 + (\omega\eta_2 + N_1\phi_{12} + N_3\phi_{23})\,E_2 - N_2\phi_{23}E_3 = S_2$$

$$-N_3\phi_{13}E_1 - N_3\phi_{23}E_2 + (\omega\eta_3 + N_1\phi_{13} + N_2\phi_{23})\,E_3 = S_3$$

The energy input terms, S_i, may be input directly to the program or can be calculated if a reverberant acoustic field provides the structural excitation. The required inputs to the program are listed below:

- f - center frequency of analysis band (Hz)
- Δf - bandwidth (Hz)
- η_1 - element 1 damping factor (dimensionless)
- η_2 - element 2 damping factor (dimensionless)
- η_3 - element 3 damping factor (dimensionless)

ϕ_{12} - coupling value for elements 1-2 $\left(\frac{1}{\text{mode-sec}}\right)$

ϕ_{13} - coupling value for elements 1-3 $\left(\frac{1}{\text{mode-sec}}\right)$

ϕ_{23} - coupling value for elements 2-3 $\left(\frac{1}{\text{mode-sec}}\right)$

n_1 - element 1 modal density (modes/Hz)

n_2 - element 2 modal density (modes/Hz)

n_3 - element 3 modal density (modes/Hz)

W_1 - weight of element 1 (lbs)

W_2 - weight of element 2 (lbs)

W_3 - weight of element 3 (lbs)

S_1 - element 1 energy input $\left(\frac{\text{in-lb}}{\text{sec}}\right)$

S_2 - element 2 energy input $\left(\frac{\text{in-lb}}{\text{sec}}\right)$

S_3 - element 3 energy input $\left(\frac{\text{in-lb}}{\text{sec}}\right)$

Additional requirements if the S_i are to be calculated for a reverberant acoustic field:

C_0 - speed of sound in surrounding medium (in/sec)

$\left(\frac{An}{m}\right)_1$ - element 1 surface mass parameter $\left(\frac{\text{in}^3\text{-modes}}{\text{lb-sec}^2\text{-Hz}}\right)$

$\left(\frac{An}{m}\right)_2$ - element 2 surface mass parameter $\left(\frac{\text{in}^3\text{-modes}}{\text{lb-sec}^2\text{-Hz}}\right)$

$\left(\frac{An}{m}\right)_3$ - element 3 surface mass parameter $\left(\frac{\text{in}^3\text{-modes}}{\text{lb-sec}^2\text{-Hz}}\right)$

$(SPL)_1$ - element 1 sound pressure level (dB)

$(SPL)_2$ - element 2 sound pressure level (dB)

$(SPL)_3$ - element 3 sound pressure level (dB)

σ_1 - element 1 radiation efficiency (dimensionless)

σ_2 - element 2 radiation efficiency (dimensionless)

σ_3 - element 3 radiation efficiency (dimensionless)

USER INSTRUCTIONS

Procedure - N = 2 (see following procedure for N = 3)

Step		Enter	Press	Display
0a	Partition memory	4	Op 17	639.39
0b	Insert program	--	--	--
0c	Initialize program		RST	
1	Input number of elements, N	2	R/S	Previous t-register
2	Input center frequency	f	R/S	$2\pi f$
3	Input bandwidth	Δf	R/S	Δf
4	Input damping of element 1	η_1	R/S	η_1
5	Input damping of element 2	η_2	R/S	2
6	Input element coupling	ϕ_{12}	R/S	2
7	Input modal density of element 1	n_1	R/S	n_1
8	Input modal density of element 2	n_2	R/S	2
9	Input weight of element 1	W_1	R/S	W_1
10	Input weight of element 2	W_2	R/S	386
11a	Input "0" if element energy input terms are to be input to program; input "1" if reverberant acoustic input is to be calculated by program, and see below	0	R/S	0
12a	Input S_1	S_1	R/S	S_1
13a	Input S_2, obtain solution	S_2	R/S	$\sqrt{\frac{\langle a_1^2\rangle}{g^2}}$
14a	Obtain solution for element 2	--	R/S	$\sqrt{\frac{\langle a_2^2\rangle}{g^2}}$
15a	Check for end of program	--	R/S	0
11b	Alternate solution with calculations for reverberant acoustics	1	R/S	0
12b	Input speed of sound (in/sec)	C_0	R/S	C_0
13b	Input $\left(\frac{An}{m}\right)$ for element 1	$\left(\frac{An}{m}\right)_1$	R/S	$\left(\frac{An}{m}\right)_1$
14b	Input $\left(\frac{An}{m}\right)$ for element 2	$\left(\frac{An}{m}\right)_2$	R/S	2
15b	Input SPL for element 1	$(SPL)_1$	R/S	$(SPL)_1$
16b	Input SPL for element 2	$(SPL)_2$	R/S	2

Step		Enter	Press	Display
17b	Input radiation efficiency for element 1	σ_1	R/S	σ_1
18b	Input radiation efficiency for element 2, obtain solution	σ_2	R/S	$\sqrt{\frac{\langle a_1^2 \rangle}{g^2}}$
19b	Obtain solution for element 2	--	R/S	$\sqrt{\frac{\langle a_2^2 \rangle}{g^2}}$
20b	Check for end of program	--	R/S	0

Procedure - N = 3

0a	Partition memory	4	Op 17	639.39
0b	Insert program	--	--	--
0c	Initialize program		RST	
1	Input number of elements	3	R/S	Previous t-register
2	Input center frequency	f	R/S	$2\pi f$
3	Input bandwidth	Δf	R/S	Δf
4	Input damping of element 1	η_1	R/S	η_1
5	Input damping of element 2	η_2	R/S	2
6	Input damping of element 3	η_3	R/S	η_3
7	Input coupling for elements 1-2	ϕ_{12}	R/S	2
8	Input coupling for elements 1-3	ϕ_{13}	R/S	ϕ_{13}
9	Input coupling for elements 2-3	ϕ_{23}	R/S	ϕ_{23}
10	Input modal density of element 1	n_1	R/S	n_1
11	Input modal density of element 2	n_2	R/S	2
12	Input modal density of element 3	n_3	R/S	n_3
13	Input weight of element 1	W_1	R/S	W_1
14	Input weight of element 2	W_2	R/S	2
15	Input weight of element 3	W_3	R/S	386
16a	Input "0" if element energy input terms are to be input to program; input "1" if reverberant acoustic input is to be calculated by program and see below	0	R/S	0

Step		Enter	Press	Display
17a	Input S_1	S_1	R/S	S_1
18a	Input S_2	S_2	R/S	2
19a	Input S_3, obtain solution	S_3	R/S	$\sqrt{\frac{\langle a_1^2 \rangle}{g^2}}$
20a	Obtain solution for element 2	--	R/S	$\sqrt{\frac{\langle a_2^2 \rangle}{g^2}}$
21a	Obtain solution for element 3	--	R/S	$\sqrt{\frac{\langle a_3^2 \rangle}{g^2}}$
22a	Check for end of program	--	R/S	0
16b	Alternate solution with calculations for reverberant acoustics	1	R/S	0
17b	Input speed of sound (in/sec)	C_0	R/S	C_0
18b	Input $\left(\frac{An}{m}\right)$ for element 1	$\left(\frac{An}{m}\right)_1$	R/S	$\left(\frac{An}{m}\right)_1$
19b	Input $\left(\frac{An}{m}\right)$ for element 2	$\left(\frac{An}{m}\right)_2$	R/S	2
20b	Input $\left(\frac{An}{m}\right)$ for element 3	$\left(\frac{An}{m}\right)_3$	R/S	$\left(\frac{An}{m}\right)_3$
21b	Input SPL for element 1	$(SPL)_1$	R/S	$(SPL)_1$
22b	Input SPL for element 2	$(SPL)_2$	R/S	2
23b	Input SPL for element 3	$(SPL)_3$	R/S	$(SPL)_3$
24b	Input radiation efficiency for element 1	σ_1	R/S	σ_1
25b	Input radiation efficiency for element 2	σ_2	R/S	2
26b	Input radiation efficiency for element 3, obtain solution	σ_3	R/S	$\sqrt{\frac{\langle a_1^2 \rangle}{g^2}}$
27b	Obtain solution for element 2	--	R/S	$\sqrt{\frac{\langle a_2^2 \rangle}{g^2}}$
28b	Obtain solution for element 3	--	R/S	$\sqrt{\frac{\langle a_3^2 \rangle}{g^2}}$
29b	Check for end of program	--	R/S	0

Data Registers

00	--
01	--
02	C_0
03	σ_1
04	σ_2
05	σ_3
06	$(SPL)_1$
07	$(SPL)_2$
08	$(SPL)_3$
09	$\left(\frac{An}{m}\right)_1$
10	$\left(\frac{An}{m}\right)_2$
11	$\left(\frac{An}{m}\right)_3$
12	η_1
13	η_2
14	η_3
15	W_1
16	W_2
17	W_3
18	n_1
19	n_2
20	n_3
21	ϕ_{12}
22	ϕ_{13}
23	ϕ_{23}
24	S_1
25	S_2
26	S_3
27	$(2\pi f)^2/386$
28	N
29	Δf
30	$2\pi f$
31	a_{11}
32	a_{12}
33	a_{13}
34	a_{21}
35	a_{22}
36	a_{23}
37	a_{31}
38	a_{32}
39	a_{33}

where the a_{ij} represent program-generated matrix elements.

PROGRAM LISTING

LOC	CODE	KEY
000	42	STO
001	28	28
002	32	X:T
003	91	R/S
004	65	×
005	02	2
006	65	×
007	70	RAD
008	01	1
009	94	+/-
010	22	INV
011	39	COS
012	95	=
013	42	STO
014	30	30
015	91	R/S
016	42	STO
017	29	29
018	91	R/S
019	42	STO
020	12	12
021	91	R/S
022	42	STO
023	13	13
024	02	2
025	67	EQ
026	22	INV
027	91	R/S
028	42	STO
029	14	14
030	76	LBL
031	22	INV
032	91	R/S
033	42	STO
034	21	21
035	02	2
036	67	EQ
037	23	LNX
038	91	R/S
039	42	STO
040	22	22
041	91	R/S
042	42	STO
043	23	23
044	76	LBL
045	23	LNX
046	91	R/S
047	42	STO
048	18	18
049	91	R/S
050	42	STO
051	19	19
052	02	2
053	67	EQ
054	24	CE

LOC	CODE	KEY
055	91	R/S
056	42	STO
057	20	20
058	76	LBL
059	24	CE
060	91	R/S
061	42	STO
062	15	15
063	91	R/S
064	42	STO
065	16	16
066	02	2
067	67	EQ
068	25	CLR
069	91	R/S
070	42	STO
071	17	17
072	76	LBL
073	25	CLR
074	03	3
075	08	8
076	06	6
077	42	STO
078	27	27
079	91	R/S
080	32	X:T
081	00	0
082	67	EQ
083	32	X:T
084	91	R/S
085	42	STO
086	02	02
087	91	R/S
088	42	STO
089	09	09
090	91	R/S
091	42	STO
092	10	10
093	43	RCL
094	28	28
095	32	X:T
096	02	2
097	67	EQ
098	33	X²
099	91	R/S
100	42	STO
101	11	11
102	76	LBL
103	33	X²
104	91	R/S
105	42	STO
106	06	06
107	91	R/S
108	42	STO
109	07	07

LOC	CODE	KEY
110	02	2
111	67	EQ
112	34	√X
113	91	R/S
114	42	STO
115	08	08
116	76	LBL
117	34	√X
118	91	R/S
119	42	STO
120	03	03
121	91	R/S
122	42	STO
123	04	04
124	02	2
125	67	EQ
126	35	1/X
127	91	R/S
128	42	STO
129	05	05
130	76	LBL
131	35	1/X
132	01	1
133	94	+/-
134	22	INV
135	39	COS
136	65	×
137	43	RCL
138	02	02
139	33	X²
140	65	×
141	08	8
142	93	.
143	04	4
144	01	1
145	52	EE
146	01	1
147	08	8
148	94	+/-
149	55	÷
150	43	RCL
151	30	30
152	27	INV
153	52	EE
154	33	X²
155	95	=
156	42	STO
157	01	01
158	65	×
159	01	1
160	00	0
161	45	Y^X
162	53	(
163	43	RCL
164	06	06

LOC	CODE	KEY
165	55	÷
166	01	1
167	00	0
168	54	)
169	65	×
170	43	RCL
171	09	09
172	65	×
173	43	RCL
174	03	03
175	95	=
176	42	STO
177	24	24
178	43	RCL
179	01	01
180	65	×
181	01	1
182	00	0
183	45	Y^x
184	53	(
185	43	RCL
186	07	07
187	55	÷
188	01	1
189	00	0
190	54	)
191	65	×
192	43	RCL
193	10	10
194	65	×
195	43	RCL
196	04	04
197	95	=
198	42	STO
199	25	25
200	02	2
201	67	EQ
202	42	STO
203	43	RCL
204	01	01
205	65	×
206	01	1
207	00	0
208	45	Y^x
209	53	(
210	43	RCL
211	08	08
212	55	÷
213	01	1
214	00	0
215	54	)
216	65	×
217	43	RCL
218	11	11
219	65	×

LOC	CODE	KEY
220	43	RCL
221	05	05
222	95	=
223	42	STO
224	26	26
225	61	GTO
226	42	STO
227	76	LBL
228	32	X:T
229	91	R/S
230	42	STO
231	24	24
232	91	R/S
233	42	STO
234	25	25
235	43	RCL
236	28	28
237	32	X:T
238	02	2
239	67	EQ
240	42	STO
241	91	R/S
242	42	STO
243	26	26
244	76	LBL
245	42	STO
246	00	0
247	42	STO
248	31	31
249	42	STO
250	35	35
251	43	RCL
252	30	30
253	33	X^2
254	55	÷
255	43	RCL
256	27	27
257	95	=
258	42	STO
259	27	27
260	43	RCL
261	28	28
262	32	X:T
263	02	2
264	67	EQ
265	43	RCL
266	43	RCL
267	29	29
268	65	×
269	43	RCL
270	18	18
271	65	×
272	43	RCL
273	22	22
274	65	×

LOC	CODE	KEY
275	43	RCL
276	17	17
277	55	÷
278	43	RCL
279	27	27
280	95	=
281	94	+/-
282	42	STO
283	33	33
284	43	RCL
285	29	29
286	65	×
287	43	RCL
288	19	19
289	65	×
290	43	RCL
291	23	23
292	65	×
293	43	RCL
294	17	17
295	55	÷
296	43	RCL
297	27	27
298	95	=
299	94	+/-
300	42	STO
301	36	36
302	43	RCL
303	29	29
304	65	×
305	43	RCL
306	20	20
307	65	×
308	43	RCL
309	22	22
310	65	×
311	43	RCL
312	15	15
313	55	÷
314	43	RCL
315	27	27
316	95	=
317	94	+/-
318	42	STO
319	37	37
320	43	RCL
321	29	29
322	65	×
323	43	RCL
324	20	20
325	65	×
326	43	RCL
327	23	23
328	65	×
329	43	RCL

LOC	CODE	KEY
330	16	16
331	55	÷
332	43	RCL
333	27	27
334	95	=
335	94	+/-
336	42	STO
337	38	38
338	43	RCL
339	30	30
340	65	×
341	43	RCL
342	20	20
343	85	+
344	43	RCL
345	39	39
346	65	×
347	53	(
348	43	RCL
349	18	18
350	65	×
351	43	RCL
352	22	22
353	85	+
354	43	RCL
355	19	19
356	65	×
357	43	RCL
358	23	23
359	54	)
360	95	=
361	65	×
362	43	RCL
363	17	17
364	55	÷
365	43	RCL
366	27	27
367	95	=
368	42	STO
369	39	39
370	43	RCL
371	20	20
372	65	×
373	43	RCL
374	22	22
375	95	=
376	42	STO
377	31	31
378	43	RCL
379	20	20
380	65	×
381	43	RCL
382	23	23
383	95	=
384	42	STO

LOC	CODE	KEY
385	35	35
386	76	LBL
387	43	RCL
388	43	RCL
389	29	29
390	65	×
391	43	RCL
392	18	18
393	65	×
394	43	RCL
395	21	21
396	65	×
397	43	RCL
398	16	16
399	55	÷
400	43	RCL
401	27	27
402	95	=
403	94	+/-
404	42	STO
405	32	32
406	43	RCL
407	29	29
408	65	×
409	43	RCL
410	19	19
411	65	×
412	43	RCL
413	21	21
414	65	×
415	43	RCL
416	15	15
417	55	÷
418	43	RCL
419	27	27
420	95	=
421	94	+/-
422	42	STO
423	34	34
424	43	RCL
425	30	30
426	65	×
427	43	RCL
428	12	12
429	85	+
430	43	RCL
431	29	29
432	65	×
433	53	(
434	43	RCL
435	19	19
436	65	×
437	43	RCL
438	21	21
439	85	+

LOC	CODE	KEY
440	43	RCL
441	31	31
442	54	)
443	95	=
444	65	×
445	43	RCL
446	15	15
447	55	÷
448	43	RCL
449	27	27
450	95	=
451	42	STO
452	31	31
453	43	RCL
454	30	30
455	65	×
456	43	RCL
457	13	13
458	85	+
459	43	RCL
460	29	29
461	65	×
462	53	(
463	43	RCL
464	18	18
465	65	×
466	43	RCL
467	21	21
468	85	+
469	43	RCL
470	35	35
471	54	)
472	95	=
473	65	×
474	43	RCL
475	16	16
476	55	÷
477	43	RCL
478	27	27
479	95	=
480	42	STO
481	35	35
482	43	RCL
483	28	28
484	36	PGM
485	02	02
486	11	A
487	01	1
488	36	PGM
489	02	02
490	12	B
491	43	RCL
492	31	31
493	36	PGM
494	02	02

LOC	CODE	KEY
495	91	R/S
496	43	RCL
497	34	34
498	36	PGM
499	02	02
500	91	R/S
501	43	RCL
502	28	28
503	32	X:T
504	02	2
505	67	EQ
506	44	SUM
507	43	RCL
508	37	37
509	36	PGM
510	02	02
511	91	R/S
512	76	LBL
513	44	SUM
514	43	RCL
515	32	32
516	36	PGM
517	02	02
518	91	R/S
519	43	RCL
520	35	35
521	36	PGM
522	02	02
523	91	R/S
524	43	RCL
525	28	28
526	32	X:T
527	02	2
528	67	EQ
529	45	Y^x
530	43	RCL
531	38	38
532	36	PGM
533	02	02
534	91	R/S
535	43	RCL
536	33	33
537	36	PGM
538	02	02
539	91	R/S
540	43	RCL
541	36	36
542	36	PGM
543	02	02
544	91	R/S
545	43	RCL
546	39	39
547	36	PGM
548	02	02
549	91	R/S
550	76	LBL
551	45	Y^x
552	36	PGM
553	02	02
554	13	C
555	01	1
556	36	PGM
557	02	02
558	14	D
559	43	RCL
560	24	24
561	36	PGM
562	02	02
563	91	R/S
564	43	RCL
565	25	25
566	36	PGM
567	02	02
568	91	R/S
569	43	RCL
570	28	28
571	32	X:T
572	02	2
573	67	EQ
574	52	EE
575	43	RCL
576	26	26
577	36	PGM
578	02	02
579	91	R/S
580	76	LBL
581	52	EE
582	36	PGM
583	02	02
584	25	CLR
585	36	PGM
586	02	02
587	15	E
588	01	1
589	36	PGM
590	02	02
591	16	A'
592	36	PGM
593	02	02
594	91	R/S
595	34	ΓX
596	91	R/S
597	36	PGM
598	02	02
599	91	R/S
600	34	ΓX
601	91	R/S
602	43	RCL
603	[illegible]	28
604	3.	X:T
605	02	2
606	67	EQ
607	53	(
608	36	PGM
609	02	02
610	91	R/S
611	34	ΓX
612	91	R/S
613	76	LBL
614	53	(
615	00	0
616	91	R/S