NASA-TP-1611 19800005789

NASA Technical Paper 1611

Sonic-Boom Wave-Front Shapes and Curvatures Associated With Maneuvering Flight

Raymond L. Barger

DECEMBER 1979

NASA

NASA Technical Paper 1611

Sonic-Boom Wave-Front Shapes and Curvatures Associated With Maneuvering Flight

Raymond L. Barger
Langley Research Center
Hampton, Virginia

National Aeronautics
and Space Administration

Scientific and Technical
Information Branch

1979

SUMMARY

Sonic-boom wave shapes and caustic lines generated by an airplane performing a general maneuver are studied. The equations are programmed for graphical output as a perspective view of the wave shape. This quasi-three-dimensional presentation provides a qualitative insight into the effects of the maneuver on the wave shape and the caustic locations.

For the special case of planar maneuvers, the principal curvatures of the wave front are derived. These curvatures are needed to calculate the sound field in the vicinity of a caustic.

The results of the analysis are applicable not only to sonic-boom studies but also to the calculation of noise generated by a supersonic rotor or propeller blade tip.

INTRODUCTION

When a supersonic aircraft accelerates or turns, the wave front generated by the flight trajectory develops folds or caustic lines. Special interest is attached to the regions near caustics because the sonic boom overpressures are enhanced in such regions. The sonic-boom literature contains a number of analyses which treat the local conditions near a caustic. (See, for example, refs. 1 to 3.) Heretofore, these analyses have assumed expressions for the local wave-front shape and curvature which are required to perform the calculation in the neighborhood of the caustic. The problem that remains is to relate the wave-front shape and curvature to the actual maneuver that is being performed by the aircraft generating the wave. The solution to this problem is the primary subject of this paper.

Because of the complexity of the analysis, the expressions derived from the wave-front curvatures are limited to planar maneuvers. This means simply that the torsion of the flight trajectory is zero. The maneuver plane may be horizontal, vertical, or skewed.

On the other hand, the equation for the rays and the wave-front shape is not limited to planar maneuvers. Some types of sonic-boom problems, especially those that are strongly dependent on refractive effects, are more advantageously formulated in terms of the rays (refs. 4 and 5). However, when the flight program involves turns and acceleration, the wave shape is no longer obvious, and it is desirable to know the location of the entire wave front at any given time. Other advantages of the wave-front formulation are discussed in reference 1. However, from an engineering standpoint, probably the greatest advantage of the wave-front formulation is the intuitive understanding of the effects of maneuvers that it can provide. Toward this end, the wave-front equation has been programmed with graphical output in the form of a perspective view of the wave. This type of quasi-three-dimensional picture provides a rapid qualitative insight into the relationship of the wave-front shape and caustic locations to the aircraft maneuvers.

SYMBOLS

In numeric examples, distances are nondimensionalized in terms of the distance that sound travels in 1 second.

a	speed of sound
$\bar{b}$	$= \bar{T} \times \bar{n}$
A,B,C,D	labels for groups of expressions defined by equations (A4a) to (A4d), respectively
E,F,G	coefficients of first fundamental form for a surface, equations (A1b) to (A1d), respectively
e,f,g	coefficients of second fundamental form for a surface, equations (A2a) to (A2c), respectively
$\bar{i},\bar{j},\bar{k}$	orthonormal base vectors, fixed in space
M	Mach number
$\bar{n}$	unit vector perpendicular to flight direction, in plane of maneuver
P	curvature parameter (see fig. 8)
R	$= a(t - \tau)$
$\bar{r}$	generic vector of point on shock wave
$\bar{r}_t$	position vector of aircraft
s	distance along flight path
$\bar{T}$	unit base vector in direction of local tangent to flight path
t	time, sec
V	local flight speed
β	$= \sqrt{M^2 - 1}$
Δ	denotes an increment in a quantity
θ	angle variable for curved flight path
K	total surface curvature

κ curvature

ρ $1/\kappa$ radius of curvature of flight path

ρ_1, ρ_2 principal curvatures of wave front

τ time at which aircraft is at position $\bar{r}$

ϕ angle variable in plane of characteristic, measured from horizontal

Subscripts:

t flight path or trajectory

τ derivative with respect to τ

ϕ derivative with respect to ϕ

1 first

2 second

A prime denotes a derivative with respect to time.

ANALYSIS

Wave-Front Shape

It is convenient to use two systems of unit base vectors. One is the usual system of unit vectors $(\bar{i}, \bar{j}, \bar{k})$ fixed in space, with $\bar{k}$ taken normal to the plane of the maneuver for planar trajectories. The other system of unit base vectors $(\bar{T}, \bar{n}, \bar{b})$ moves with the aircraft. (See fig. 1.) The first vector, $\bar{T}$, is in the direction of flight and is defined by

$$\bar{T} = \frac{d\bar{r}_t}{ds} \tag{1a}$$

or

$$\frac{d\bar{r}_t}{d\tau} = V\bar{T} \tag{1b}$$

The second vector, $\bar{n}$, is defined by

$$\frac{d\bar{T}}{ds} = \kappa_t \bar{n} \tag{2a}$$

or

$$\frac{d\bar{T}}{d\tau} = V\kappa_t \bar{n} \tag{2b}$$

where κ_t is the curvature of the flight path. (See ref. 6, p. 18.) Since $\bar{T}$ is a unit vector, $\bar{n}$ is normal to $\bar{T}$ in the plane of the trajectory. For planar maneuvers, the third vector $\bar{b}$ is parallel to $\bar{k}$ but may be oppositely directed.

The flight program may be defined in several different forms. It may be given as a vector position of time $\bar{r}_t(\tau)$, as a trajectory $\bar{r}_t(s)$ and velocity function of arc length $V(s)$, or as a trajectory $\bar{r}_t(s)$ and velocity function of time $V(\tau)$. In any case, the time and arc length parameters can be related by

$$\tau = \int_0^s \frac{ds}{V}$$

or

$$s = \int_0^\tau V\, d\tau$$

At time t the pressure disturbance created by the moving aircraft at time τ has spread a distance $R = a(t - \tau)$. If $\bar{r}(t,\tau)$ is the generic point of this disturbance, and $\bar{r}_t(\tau)$ is the source location at time τ, then the equation for the disturbance wave front is

$$(\bar{r} - \bar{r}_t) \cdot (\bar{r} - \bar{r}_t) = a^2(t - \tau)^2 = R^2 \tag{3}$$

The shock surface is the envelope of this one-parameter (τ) family of spheres. The characteristic lines that comprise this envelope are found by differentiating equation (3) with respect to τ and solving the resulting equation simultaneously with equation (3). Thus,

$$(\bar{r} - \bar{r}_t) \cdot V\bar{T} = aR \tag{4a}$$

or

$$(\bar{r} - \bar{r}_t) \cdot \bar{T} = R/M \tag{4b}$$

Thus the component in the flight direction of a ray to the shock envelope is R/M, which means that the characteristic line is a circle with center at $\bar{r}_t + (R/M)\bar{T}$ and with radius $\beta R/M$. In the plane of this circle let ϕ denote the angle with the horizontal. (See fig. 2.) Then the vector equation for the shock surface at time t can be written explicitly in terms of the moving trihedral base vectors and the two parameters τ and ϕ as follows:

$$\bar{r} = \frac{a(t - \tau)}{M(\tau)} \bar{T}(\tau) + \frac{a\beta(\tau)(t - \tau) \cos \phi}{M(\tau)} \bar{n}(\tau) + \frac{a\beta(\tau)(t - \tau) \sin \phi}{M(\tau)} \bar{b}(\tau) + \bar{r}_t \tag{5}$$

If, on the other hand, both τ and ϕ are held constant and t is varied, then equation (5) describes the trajectory of an element of the wave front, that is, a ray.

The equation for a caustic line in the wave front at time t is found by differentiating equation (4) with respect to τ and solving the resulting equation simultaneously with equations (3) and (4). This is the mathematical equivalent of the geometric condition that two consecutive characteristic circles be tangent to each other. The derivative of equation (4) with respect to τ is

$$-\bar{r}_t' \cdot V\bar{T} + (\bar{r} - \bar{r}_t) \cdot \left(V^2 \frac{d\bar{T}}{ds} + V'\bar{T}\right) = -a^2$$

Substituting from equations (1) and (2) yields

$$(\bar{r} - \bar{r}_t) \cdot (V^2\kappa_t\bar{n} + V'\bar{T}) = \beta^2a^2 \tag{6}$$

Combined with equation (4), this equation becomes

$$(\bar{r} - \bar{r}_t) \cdot \bar{n} = \rho_t\left(\frac{\beta^2}{M^2} - \frac{V'R}{V^2M}\right) \tag{7a}$$

Equations (4) and (7a), respectively, give the components in the $\bar{T}$ and $\bar{n}$ directions of the ray to a point on the caustic. The component in the $\bar{b}$ direction follows from equation (1) as follows:

$$(\bar{r} - \bar{r}_t) \cdot \bar{b} = \pm\sqrt{\frac{\beta^2R^2}{M^2} - \rho_t^{\,2}\left(\frac{\beta^2}{M^2} - \frac{V'R}{V^2M}\right)} \tag{7b}$$

Equations (4) and (7) were given in reference 7 in a slightly different form. In order to calculate the wave shape from equation (5) or the caustic line from equations (4) and (7), the moving vectors $(\bar{T},\bar{n},\bar{b})$ must be expressed in terms of the fixed base vectors $(\bar{i},\bar{j},\bar{k})$ by using equations (1) and (2).

In the case of a straight accelerating flight, κ_t is 0 and the coefficient of $\bar{n}$ in equation (6) becomes zero. For this axisymmetric case, the caustic line at time t is the circle determined by the value of τ that satisfies the equation

$$\tau + \frac{\beta^2(\tau)M(\tau)}{M'(\tau)} = t$$

which is obtained from equations (4) and (6).

Wave-Shape Examples

Even relatively simple maneuvers present some interesting examples of wave shapes. Figure 3 shows in perspective the caustic line and a number of the characteristic circles comprising the wave front that results from a simple turn at constant speed. Figure 4 shows how the wave folds on itself to form a caustic when the flight trajectory changes from a straight flight to a turn. Only the half of the wave above the flight plane is shown in order to avoid the confusion of too many overlapping lines. Figure 5 shows a different perspective of a similar maneuver, but with the airplane decelerating at a constant rate from an initial Mach number of 2.4. The principal effect of the deceleration is to move the caustic line farther from the flight trajectory than it would be for constant flight speed. Figure 6 shows a similar maneuver but with the airplane accelerating from an initial Mach number of 1.1. Two caustic lines appear in this case. The first occurs near the beginning of the acceleration region and is due to the acceleration, while the other is associated with the turning maneuver.

The nature of the acceleration caustic is better seen in a detailed plot of the wave cross section (fig. 7). At some distance from the flight path a single caustic line forms initially, but two caustic lines appear as the wave front crosses itself. These lines appear in the cross-sectional plot as cusp-like points of reversal of direction. Note, however, that they do not represent cusp or arête points in a caustic line. Some experimental data on sonic-boom ground patterns and signature shapes are presented in reference 8.

Determination of Wave-Front Curvature

In order to calculate the flow field in the vicinity of a caustic, it is necessary to know the curvature of the wave front approaching the caustic line (refs. 1 to 3). In addition, knowledge of the curvature distribution over the wave front provides an insight into the relative expansion or compression of the wave.

The shock wave envelope is, except for straight, constant-speed flight, a surface of double curvature. At a fixed time t, the equation of the surface (eq. (5)) is a function of the two variables τ and ϕ. The theory for calculating the curvatures with the equation in this form is described in chapter 2 of reference 7. The procedure is to calculate the total radius of curvature, which is the reciprocal of the total curvature. The details of the calculation, which are somewhat tedious, are given in the appendix. The result is as follows:

$$\frac{1}{K} = \rho_1 \rho_2 = -a(t - \tau)\left[\frac{\beta a}{\dfrac{M'}{\beta M} + Ma\kappa_t \cos\phi} - a(t - \tau)\right] \tag{8}$$

Some insight into the nature of the principal curvatures can be gained by examining the example of straight, accelerating flight. Since for this case $\kappa_t = 0$, then D, f, and F are all zero (see appendix). Consequently, the curvature directions are orthogonal and the principal curvatures can be computed by the following equations (see ref. 6, p. 81):

$$\kappa_1 = \frac{e}{E} = -\frac{M'}{\beta^2}\frac{A}{A^2 + B^2} = -\frac{M'}{\beta^2}\frac{-\beta B}{M^2 B^2} = \frac{M'}{\beta M^2 B}$$

Therefore,

$$\rho_1 = \frac{\beta M^2 B}{M'} \tag{9a}$$

Substituting for B gives

$$\rho_1 = a(t - \tau) - \frac{\beta^2 Ma}{M'} \tag{9b}$$

and

$$\rho_2 = \frac{G}{g} = \frac{MC}{\beta} = a(t - \tau) \tag{10}$$

Thus, κ_1 is the curvature of the meridians along the shock envelope. This curvature results from the acceleration maneuver. On the other hand, κ_2 is the curvature associated with the normal acoustic spreading of the wave front.

Equation (10) could therefore have been obtained by the following argument. Since, according to equation (4), the wave surface consists of circles of radius $\frac{\beta a(t - \tau)}{M}$, the radius of curvature normal to the surface is this radius divided by the cosine of the angle between the normal to the surface and the radius vector of the circle. This result is in accordance with Meusnier's theorem (ref. 6, p. 76). Since this cosine is β/M, the normal radius of curvature is simply $a(t - \tau)$, which is the same as equation (10).

This latter argument is applicable locally not only for the case of straight flight but also for a general maneuver. At any point on the shock surface one of the principal radii of curvature is $\rho_2 = a(t - \tau)$. Consequently, according to equation (8), the other radius of curvature is

$$\rho_1 = a(t - \tau) - \frac{\beta^2 Ma}{M' + \beta M^2 a\kappa_t \cos \phi} \tag{11}$$

The curvature κ_1 results from the maneuver. The curvature direction associated with κ_1 is locally normal to the characteristic circle. Consequently, near a caustic, where $\rho_1 = 0$, the curvature direction is nearly normal to the caustic, since the caustic is locally tangent to a characteristic line.

Thus, in the vicinity of a caustic line whose curvature is small compared with κ_1, the wave is approximately two-dimensional in character with its radius of curvature given by equation (11).

The essential expression in the curvature calculation is the last term in equation (11). This term (denoted by P) represents the distance of a point on the caustic line from the trajectory point at which it was emitted. It is readily seen that a positive acceleration decreases this distance and the radius of curvature at a given distance along the ray, whereas a deceleration has the opposite effect. These results are shown in the plots of figures 8(a) and 8(b). The Mach numbers (1.2 and 1.6) for which these examples were calculated are the instantaneous Mach numbers at the time the caustic ray was emitted. From the parameter P plotted in figure 8, the curvature ρ_1 can be calculated as R - P, where R and P are the first and last terms on the right side of equation (11). The total curvature is then $1/R\rho_1$. Equation (11) could be obtained in a brief but intuitive manner as follows:

Substitute

$$(\bar{r} - \bar{r}_t) \cdot \bar{T} = \frac{a(t - \tau)}{M}$$

and

$$(\bar{r} - \bar{r}_t) \cdot \bar{n} = \frac{\beta a(t - \tau) \cos \phi}{M}$$

into equation (6). This yields

$$a(t - \tau)(V^2\kappa_t\beta \cos \phi + V') = \beta^2 Ma \tag{12a}$$

or

$$a(t - \tau) - \frac{\beta^2 Ma}{M' + \beta M^2 a\kappa_t \cos \phi} = 0 \tag{12b}$$

This is the condition that exists at a caustic point. It can also be obtained by setting $\rho_1 = 0$ in equation (11). Therefore, by using dimensional considerations and the fact that the rays are straight, one can argue that equation (11) follows from equation (12) without making the extensive curvature calculation.

Ray-Tube Area

An incidental result of the preceding calculation for the curvature is a formula for the ray-tube area. As given on page 63 of reference 6, an elemental area of the wave front is determined by the square root of the discriminant of the first fundamental form:

$$\Delta A = \sqrt{EG - F^2}\, d\tau\, d\phi$$

Thus, the variation of the cross-sectional area of a ray tube is determined by allowing t to vary in the expression $\sqrt{EG - F^2}$, while holding τ and ϕ constant. Equations (A4a), (A4c), and (A8) yield $R = a(t - \tau)$:

$$\Delta A = \frac{\beta^2 a}{M} R \left[1 - \left(\frac{M'}{\beta^2 Ma} + \frac{M\kappa_t \cos\theta}{\beta}\right) R\right] d\tau\, d\phi \tag{13}$$

Equation (13) is the same result that was obtained by Rao (refs. 9 and 10) by means of a somewhat more involved development. It may also be noted that equation (5), for the wave shape, and equation (13), for the ray-tube area, are useful not only for sonic-boom calculations associated with maneuvering supersonic airplanes but also for computing the noise field of supersonic propeller or rotor tips.

Higher-Order Caustic Locations

A caustic point represents a point of tangency of two successive characteristic circles of the wave front. Under certain conditions, caustic points may also exist at which a higher-order contact occurs among successive characteristics. Additional compression, beyond that normally experienced at a caustic point, would be realized at these points.

The location of such a point is found by differentiating equation (6) with respect to τ and solving the resulting equation simultaneously with equations (3), (4), and (6). The derivative of equation (6) is

$$-V\bar{T} \cdot (V^2\kappa_t\bar{n} + V'\bar{T}) + (VV'\kappa_t\bar{n} + V''\bar{T} - V^3\kappa_t{}^2\bar{T} + 2VV'\kappa_t\bar{n}$$

$$+ V^2\kappa_t'\bar{n}) \cdot (\bar{r} - \bar{r}_t) = 2VV'$$

or, with terms collected,

$$(\bar{r} - \bar{r}_t) \cdot \left[\left(3VV'\kappa_t + V^2\kappa_t'\right)\bar{n} + \left(V'' - V^3\kappa_t^2\right)\bar{T}\right] = 3VV' \tag{14}$$

For some maneuvers, equation (14) does not have a solution. One example is a straight linear acceleration, for which $\kappa_t = 0$ and $V'' = 0$. Another example is a constant speed, constant radius turn, for which $V' = 0$ and $\kappa_t' = 0$.

It should also be noted that, even if it is possible to solve equation (14), the caustic line does not necessarily have a cusped shape. A simple example is straight nonlinear acceleration ($V'' \neq 0$). If a value of $t - \tau$ exists for which both equations (6) and (14) are satisifed, then there is a higher-order folding of the wave along the corresponding characteristic circle, but no cusp is formed.

CONCLUDING REMARKS

The relationship of sonic-boom wave shapes and caustic lines to the maneuver performed by the generating aircraft has been studied. The analysis treated general turning and accelerating maneuvers in an isothermal atmosphere. The equations were programmed for graphical output in such a way that the wave shape could be observed in a quasi-three-dimensional perspective view in order to provide a qualitative insight into the effects of the maneuver on the wave shape.

For the special case of planar maneuvers, the principal curvatures of the wave front were calculated. These curvatures are of interest in the calculation of the sound field in the immediate vicinity of a caustic line.

Although the analysis is based on equations originally derived for a maneuvering supersonic airplane, the results are also applicable to the propagation of sound from a supersonic rotor or propeller tip.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
December 7, 1979

APPENDIX

METHOD OF CALCULATION OF TOTAL RADIUS OF CURVATURE

This appendix provides the details of the calculation of the wave-front curvature K, which is the product of the two principal curvatures, κ_1 and κ_2. The basic formula, which is given on page 83 of reference 6, is

$$K = \frac{eg - f^2}{EG - F^2} \tag{A1a}$$

In this expression, the numerator is the discriminant of the second fundamental form of the equation of the surface, and the denominator is the discriminant of the first fundamental form. The individual quantities are the coefficients which are defined by

$$E = \bar{r}_\tau \cdot \bar{r}_\tau \tag{A1b}$$

$$F = \bar{r}_\tau \cdot \bar{r}_\phi \tag{A1c}$$

$$G = \bar{r}_\phi \cdot \bar{r}_\phi \tag{A1d}$$

Also required are the coefficients of the second fundamental form, defined by

$$e = (\bar{r}_{\tau\tau} \cdot \bar{r}_\tau \times \bar{r}_\phi)\Big/\sqrt{EG - F^2} \tag{A2a}$$

$$f = (\bar{r}_{\tau\phi} \cdot \bar{r}_\tau \times \bar{r}_\phi)\Big/\sqrt{EG - F^2} \tag{A2b}$$

$$g = (\bar{r}_{\phi\phi} \cdot \bar{r}_\tau \times \bar{r}_\phi)\Big/\sqrt{EG - F^2} \tag{A2c}$$

The derivatives of $\bar{r}$ are obtained by differentiating equation (5):

$$\bar{r}_\tau = \left[\frac{\beta^2 a}{M} - \frac{M'a(t - \tau)}{M^2} - \beta\kappa_t a^2(t - \tau)\cos\phi\right]\bar{T} + \left[a^2\kappa_t(t - \tau) + \frac{M'a(t - \tau)\cos\phi}{\beta M^2} - \frac{\beta a\cos\phi}{M}\right]\bar{n} + \left[\frac{M'a(t - \tau)}{\beta M^2} - \frac{\beta a}{M}\right]\sin\phi\,\bar{b} \tag{A3}$$

In order to simplify the calculations, the mathematical expressions may be grouped as follows:

$$A \equiv \frac{\beta^2 a}{M} - \frac{M' a(t - \tau)}{M^2} - \beta \kappa_t a^2 (t - \tau) \cos \phi \tag{A4a}$$

$$B \equiv \frac{M' a(t - \tau)}{\beta M^2} - \frac{\beta a}{M} \tag{A4b}$$

$$C \equiv \frac{\beta a(t - \tau)}{M} \tag{A4c}$$

$$D \equiv \kappa_t a^2 (t - \tau) \tag{A4d}$$

Thus,

$$A = -\beta(B + D \cos \phi) \tag{A4e}$$

In terms of these groupings, equation (A3) becomes

$$\bar{r}_\tau = A\bar{T} + (D + B \cos \phi)\bar{n} + B \sin \phi \, \bar{b} \tag{A5}$$

The derivative of equation (5) with respect to ϕ is

$$\bar{r}_\phi = -C \sin \phi \, \bar{n} + C \cos \phi \, \bar{b} \tag{A6}$$

The coefficients of the first fundamental form can now be obtained from equations (A1), (A5), and (A6):

$$E = A^2 + D^2 + 2BD \cos \phi + B^2 \tag{A7a}$$

$$F = -CD \sin \phi \tag{A7b}$$

$$G = C^2 \tag{A7c}$$

The discriminant of the first fundamental form is found by substituting equations (A7) into the expression $EG - F^2$ and simplifying by using equations (A4). This yields

$$EG - F^2 = \frac{M^2C^2A^2}{\beta^2} \tag{A8}$$

The second derivatives are required in the calculation of the coefficients of the second fundamental form:

$$\bar{r}_{\tau\tau} = \left[A' - V\kappa_t(B \cos \phi + D)\right]\bar{T} + (V\kappa_t A + B' \cos \phi + D')\bar{n} + B' \sin \phi \, \bar{b} \tag{A9a}$$

$$\bar{r}_{\tau\phi} = \beta D \sin \phi \, \bar{T} - B \sin \phi \, \bar{n} + B \cos \phi \, \bar{n} \tag{A9b}$$

$$\bar{r}_{\phi\phi} = -C(\cos \phi \, \bar{n} + \sin \phi \, \bar{b}) \tag{A9c}$$

From equations (A5) and (A6),

$$\bar{r}_\tau \times \bar{r}_\phi = C(B + D \cos \phi)\bar{T} - AC \cos \phi \, \bar{n} - AC \sin \phi \, \bar{b}$$

or, from equation (A4e),

$$\bar{r}_\tau \times \bar{r}_\phi = -AC\left(\frac{1}{\beta}\,\bar{T} + \cos \phi \, \bar{n} + \sin \phi \, \bar{b}\right) \tag{A10}$$

The first coefficient, e, is obtained from equations (A2a), (A9a), (A8), and (A10):

$$\frac{M}{\beta} e = -\frac{1}{\beta}\left[A' - V\kappa_t(B \cos \phi + D)\right] - \cos \phi(V\kappa_t A + B' \cos \phi + D') - B' \sin^2 \phi \tag{A11}$$

From equation (A4e),

$$A' = -\beta(B' + D' \cos \phi) - \frac{MM'}{\beta}(B + D \cos \phi)$$

$$= -\beta(B' + D' \cos \phi) + \frac{MM'}{\beta^2} A$$

Consequently, the first term and the last three terms on the right side of equation (A11) combine to yield $MM'A/\beta^3$. Thus,

$$e = -\frac{M'A}{\beta^2} + \frac{V\kappa_t}{M}\left[(B - \beta A) \cos \phi + D\right]$$

However,

$$B \cos \phi + D = (B + D \cos \phi) \cos \phi + D \sin^2 \phi$$

$$= -\frac{A \cos \phi}{\beta} + D \sin^2 \phi$$

Therefore,

$$e = -\left(\frac{M'}{\beta^2} + \frac{V\kappa_t}{\beta M} \cos \phi + \frac{V\kappa_t\beta \cos \phi}{M}\right)A + a\kappa_t D \sin^2 \phi$$

or

$$e = -\left(\frac{M'}{\beta^2} + \frac{M^2 a\kappa_t \cos \phi}{\beta}\right)A + a\kappa_t D \sin^2 \phi \qquad \text{(A12a)}$$

The second coefficient, f, is obtained from equations (A9b) and A10):

$$f = -ACD \sin \phi\left(\frac{\beta}{MCA}\right)$$

$$f = -\frac{\beta D \sin \phi}{M} \tag{A12b}$$

Similarly, from equations (A9c) and (A10),

$$g = AC^2\left(\frac{\beta}{MCA}\right)$$

$$g = \frac{\beta C}{M} \tag{A12c}$$

The discriminant of the second fundamental form can be obtained from equations (A12):

$$eg - f^2 = -\left(\frac{M'}{\beta M} + Ma\kappa_t \cos \phi\right) CA + \frac{\beta a \kappa_t CD \sin^2 \phi}{M} - \frac{\beta^2 D^2 \sin^2 \phi}{M^2}$$

Since $Ca\kappa_t = \beta D/M$, the last two terms cancel:

$$eg - f^2 = -\left(\frac{M'}{\beta M} + Ma\kappa_t \cos \phi\right) CA \tag{A13}$$

The total curvature can then be calculated by using equations (A8) and (A13):

$$K = \kappa_1 \kappa_2 = \frac{eg - f^2}{EG - F^2} = -\frac{\beta^2}{M^2 AC}\left(\frac{M'}{\beta M} + Ma\kappa_t \cos \phi\right)$$

It is more convenient to work with the reciprocal, which after substituting from equation (11), is

$$\frac{1}{K} = \rho_1 \rho_2 = -a(t - \tau)\left[\frac{\beta a}{\dfrac{M'}{\beta M} + Ma\kappa_t \cos\phi} - a(t - \tau)\right] \tag{A14}$$

REFERENCES

1. Hayes, Wallace D.: Similarity Rules for Nonlinear Acoustic Propagation Through a Caustic. Second Conference on Sonic Boom Research, NASA SP-180, 1968, pp. 165-171.

2. Cramer, M. S.; and Seebass, A. R.: Focusing of Weak Shock Waves at an Arête. J. Fluid Mech., vol. 88, pt. 2, Sept. 27, 1978, pp. 209-222.

3. Pierce, Allan D.: Maximum Overpressures of Sonic Booms Near the Cusps of Caustics. Noise and Vibration Control Engineering, Malcolm J. Crocker, ed., Purdue Univ., July 1971, pp. 544-553.

4. Lansing, Donald L.: Application of Acoustic Theory to Prediction of Sonic-Boom Ground Patterns From Maneuvering Aircraft. NASA TN D-1860, 1964.

5. Hayes, Wallace D.; Haefeli, Rudolph C.; and Kulsrud, H. E.: Sonic Boom Propagation in a Stratified Atmosphere, With Computer Program. NASA CR-1299, 1969.

6. Struik, Dirk J.: Differential Geometry. Addison-Wesley Pub. Co., Inc., c.1950.

7. Barger, Raymond L.: Some Effects of Flight Path and Atmospheric Variations on the Boom Propagated From a Supersonic Aircraft. NASA TR R-191, 1964.

8. Wanner, Jean-Claude L.; Vallee, Jacques; Vivier, Claude; and Thery, Claude: Theoretical and Experimental Studies of the Focus of Sonic Booms. J. Acoust. Soc. of America, vol. 52, no. 1, pt. 1, July 1972, pp. 13-32.

9. Rao, P. Sambasiva: Supersonic Bangs - Part 1. Aeron. Quart., vol. VII, pt. I, Feb. 1956, pp. 21-44.

10. Rao, P. Sambasiva: Supersonic Bangs - Part 2. Aeron. Quart., vol. VII, pt. II, May 1956, pp. 135-155.

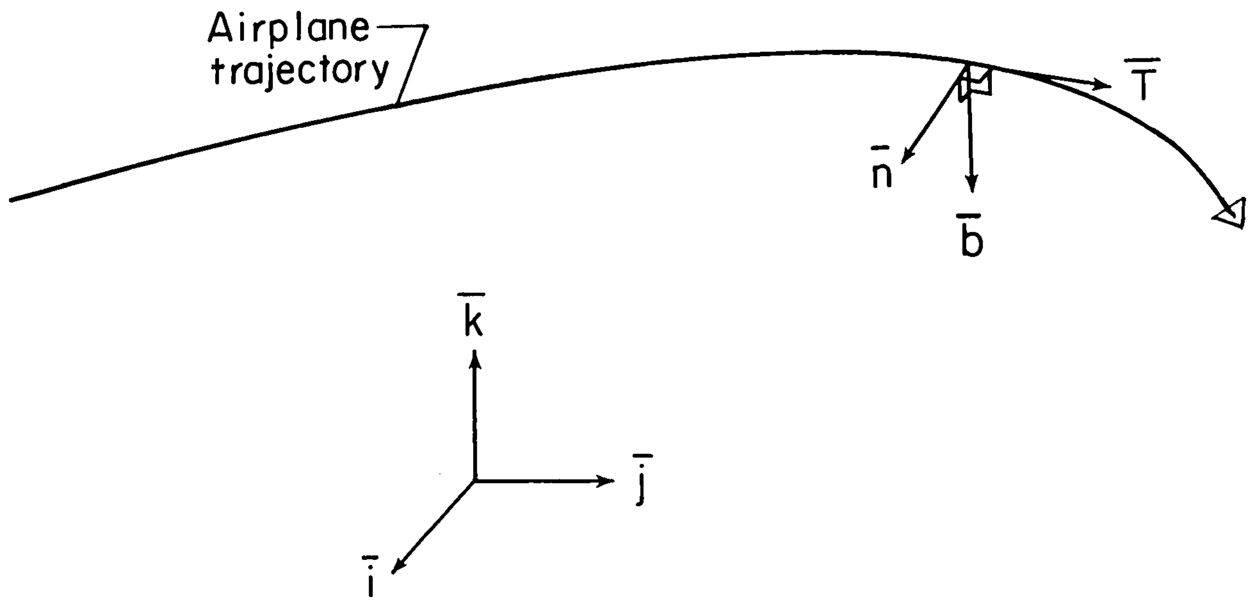


Figure 1.- Fixed and moving systems of unit base vectors.

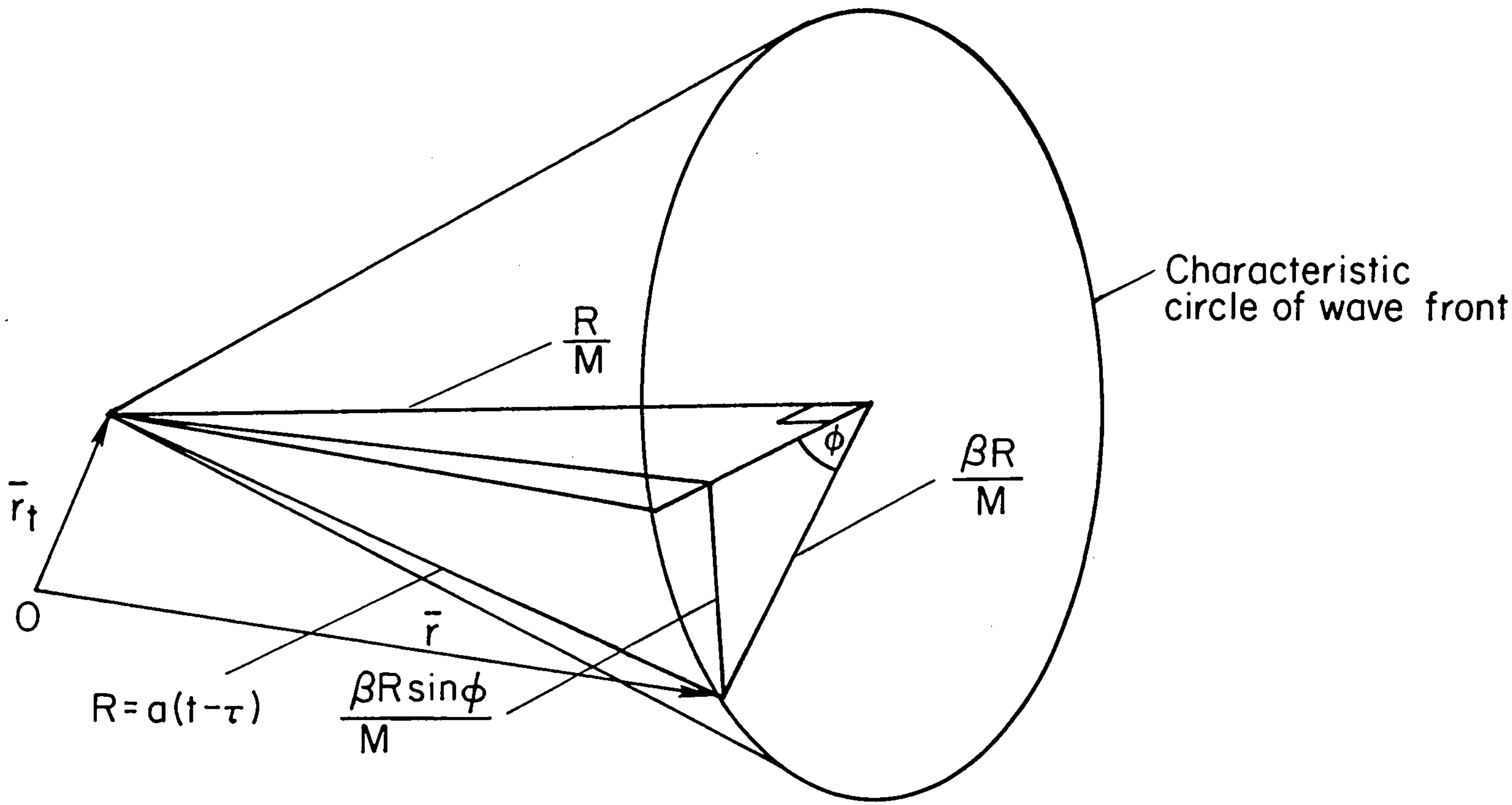


Figure 2.- Ray and wave-front geometry.

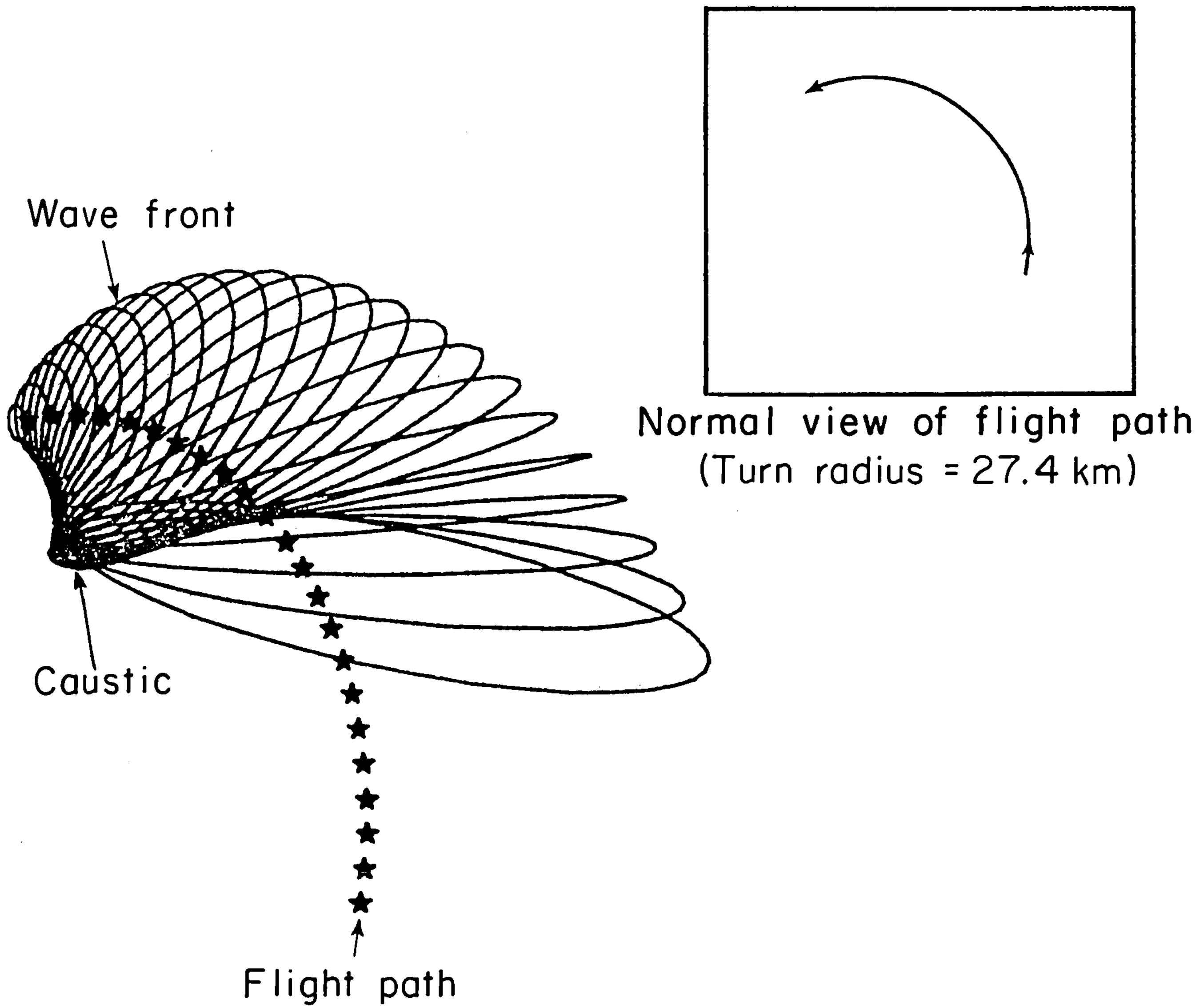


Figure 3.- Perspective view of wave front with caustic resulting from a constant-curvature turn at $M = 1.16$.

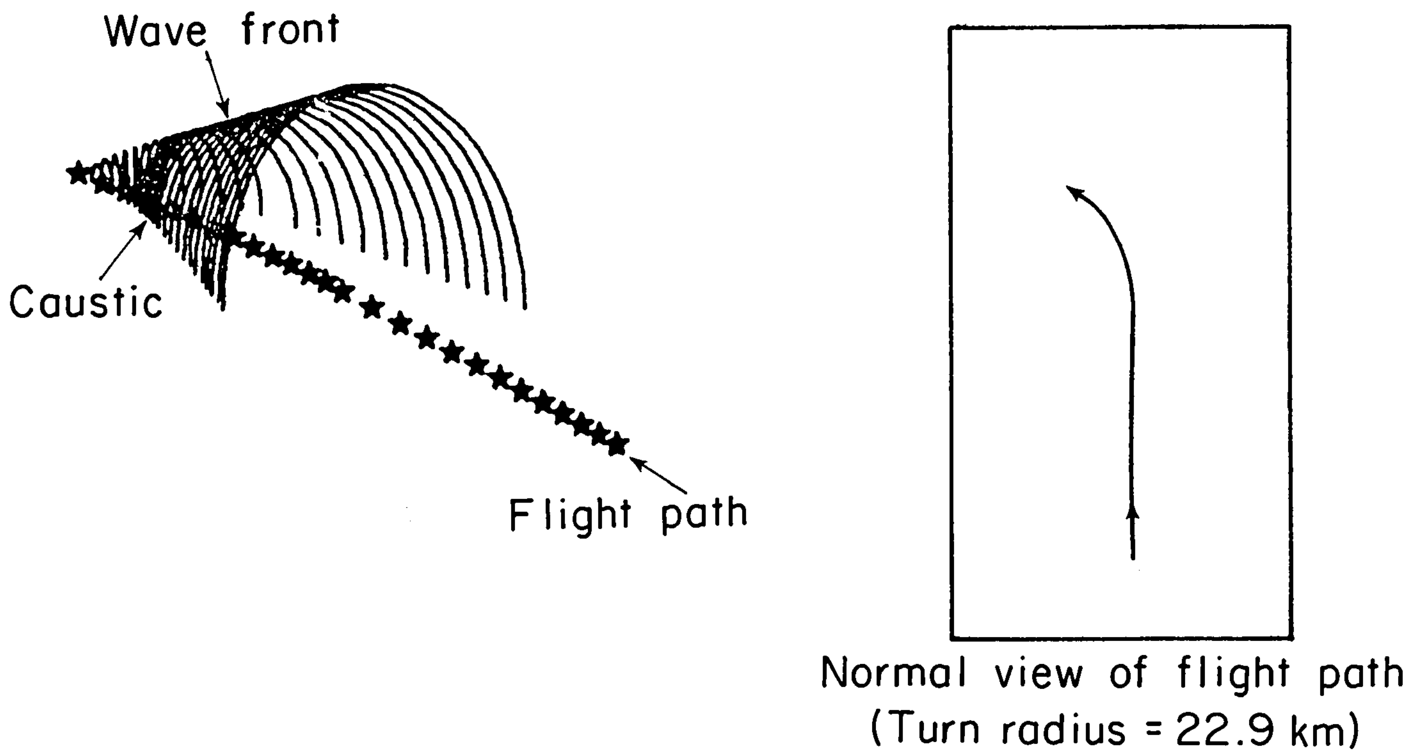


Figure 4.- Portion of wave front above flight plane for a straight flight path succeeded by a constant-radius turn at $M = 1.3$.

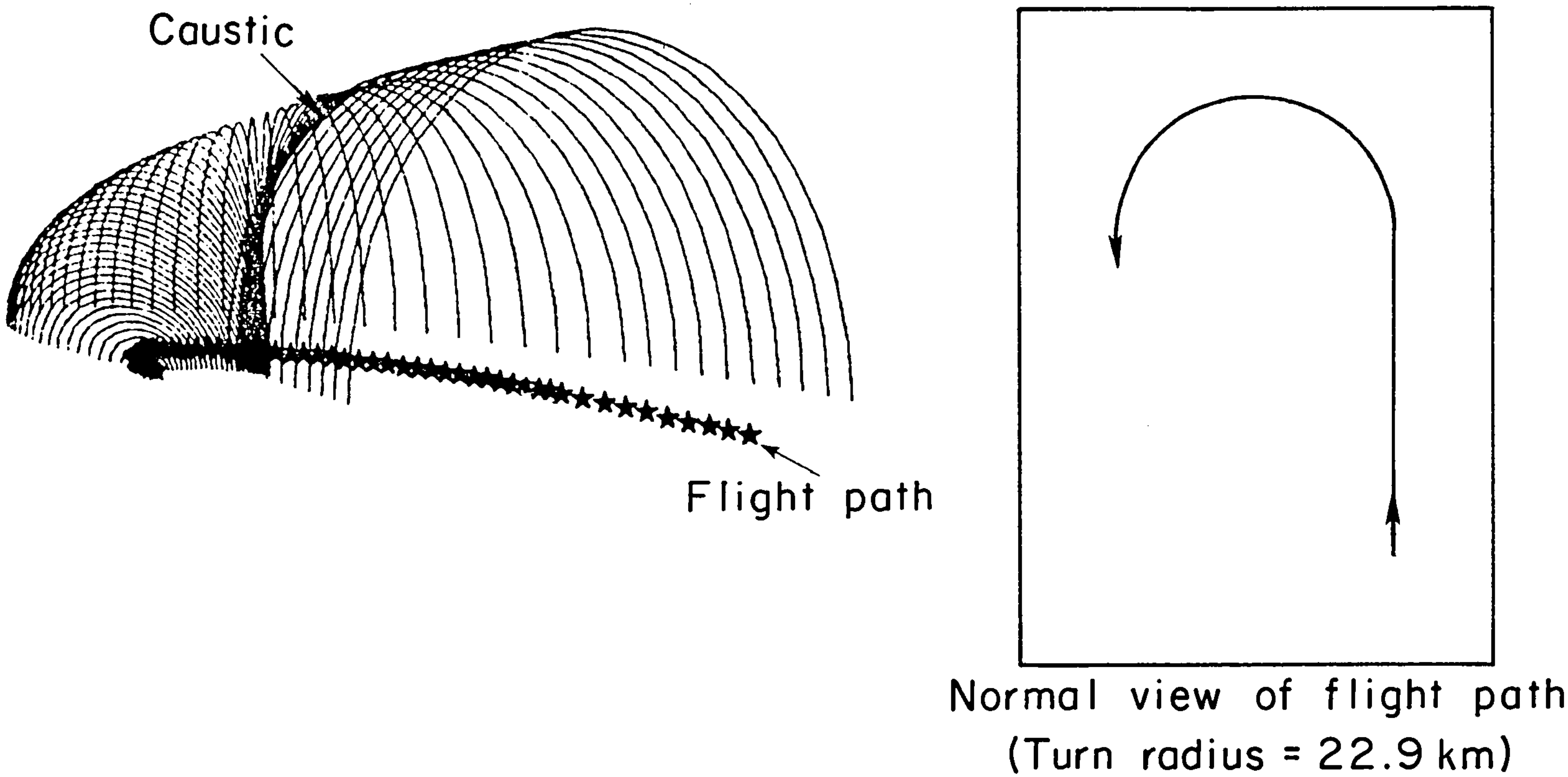


Figure 5.- Portion of wave front above flight plane for a straight flight path succeeded by a constant-radius turn with aircraft decelerating from $M = 2.4$ to $M = 1.52$ at $dM/d\tau = -0.004$/sec.

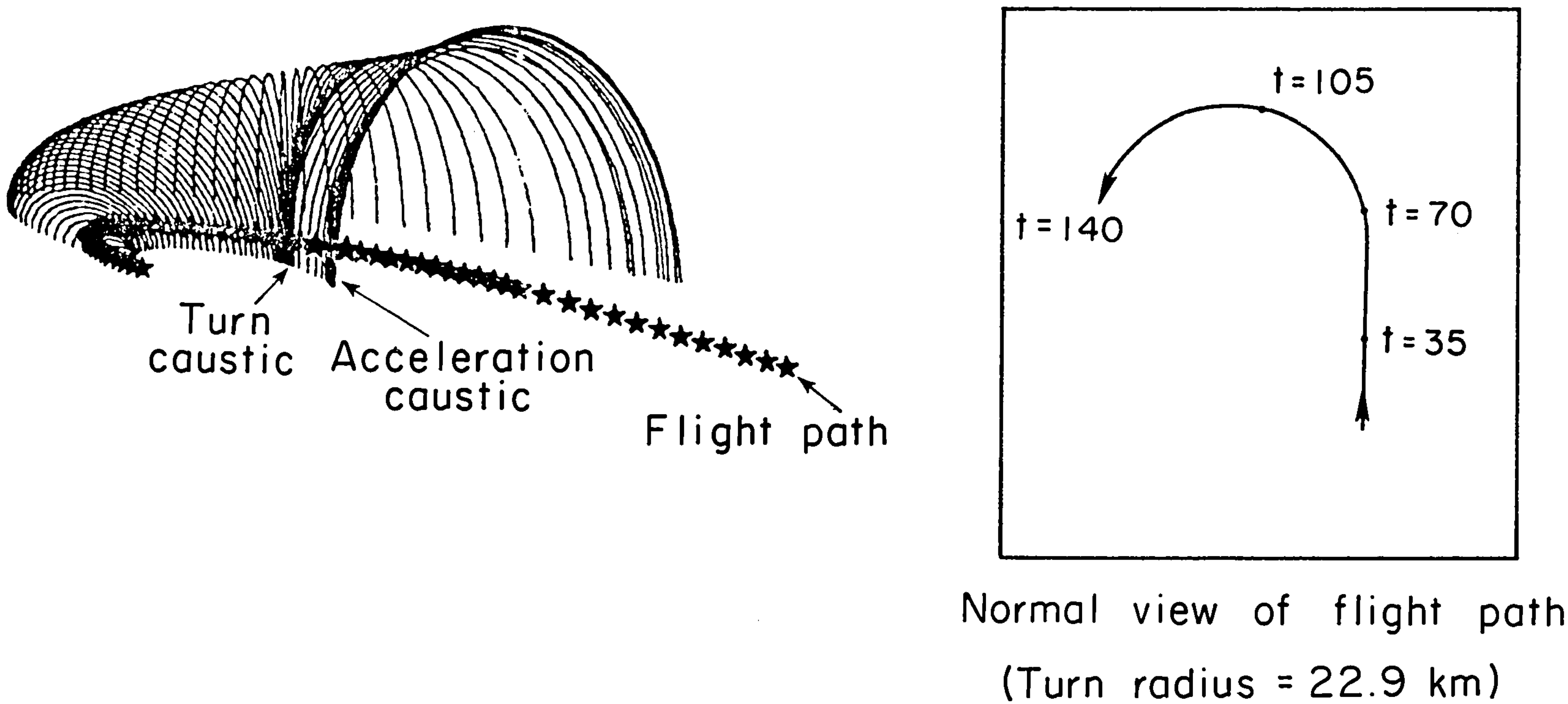


Figure 6.- Portion of wave front above flight plane for a straight flight path succeeded by a constant radius with aircraft accelerating from M = 1.1 to M = 3.1 at dM/dτ = 0.016/sec.

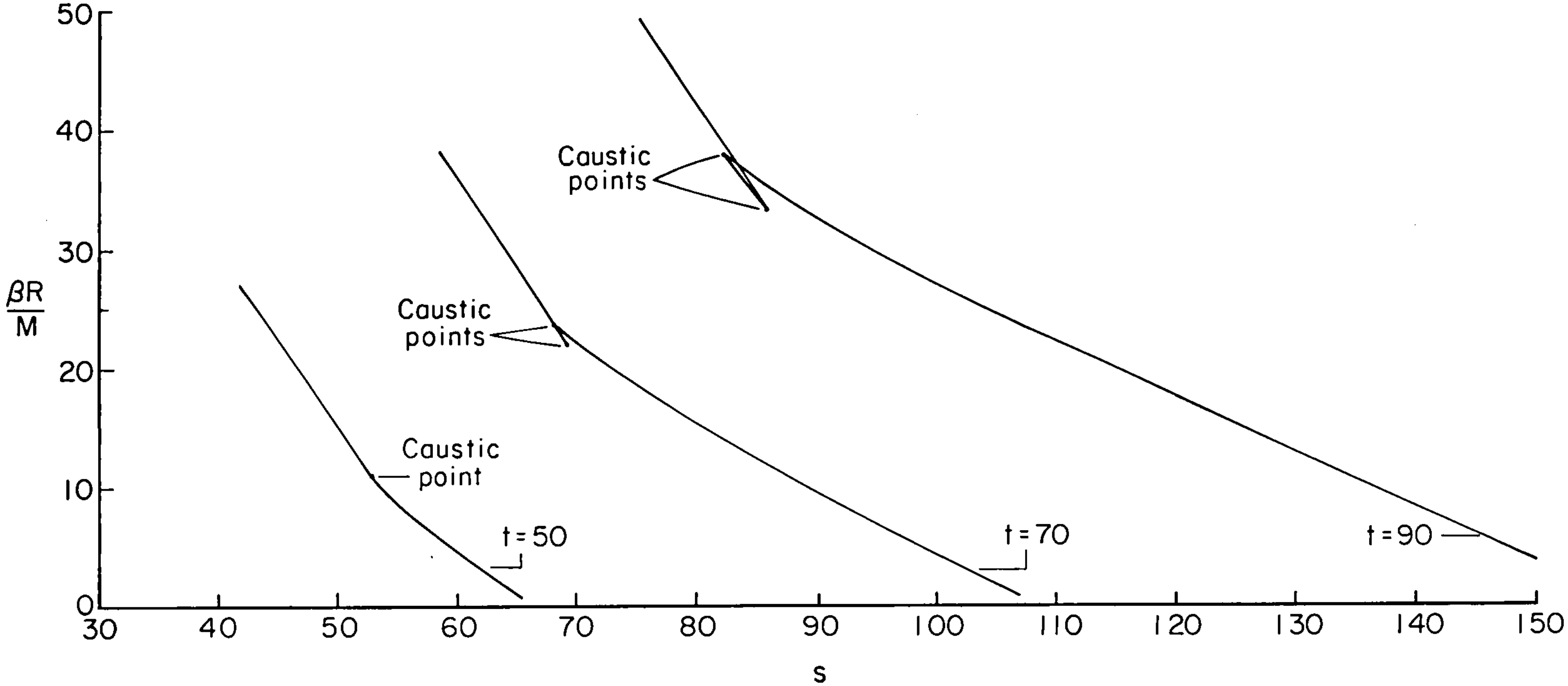
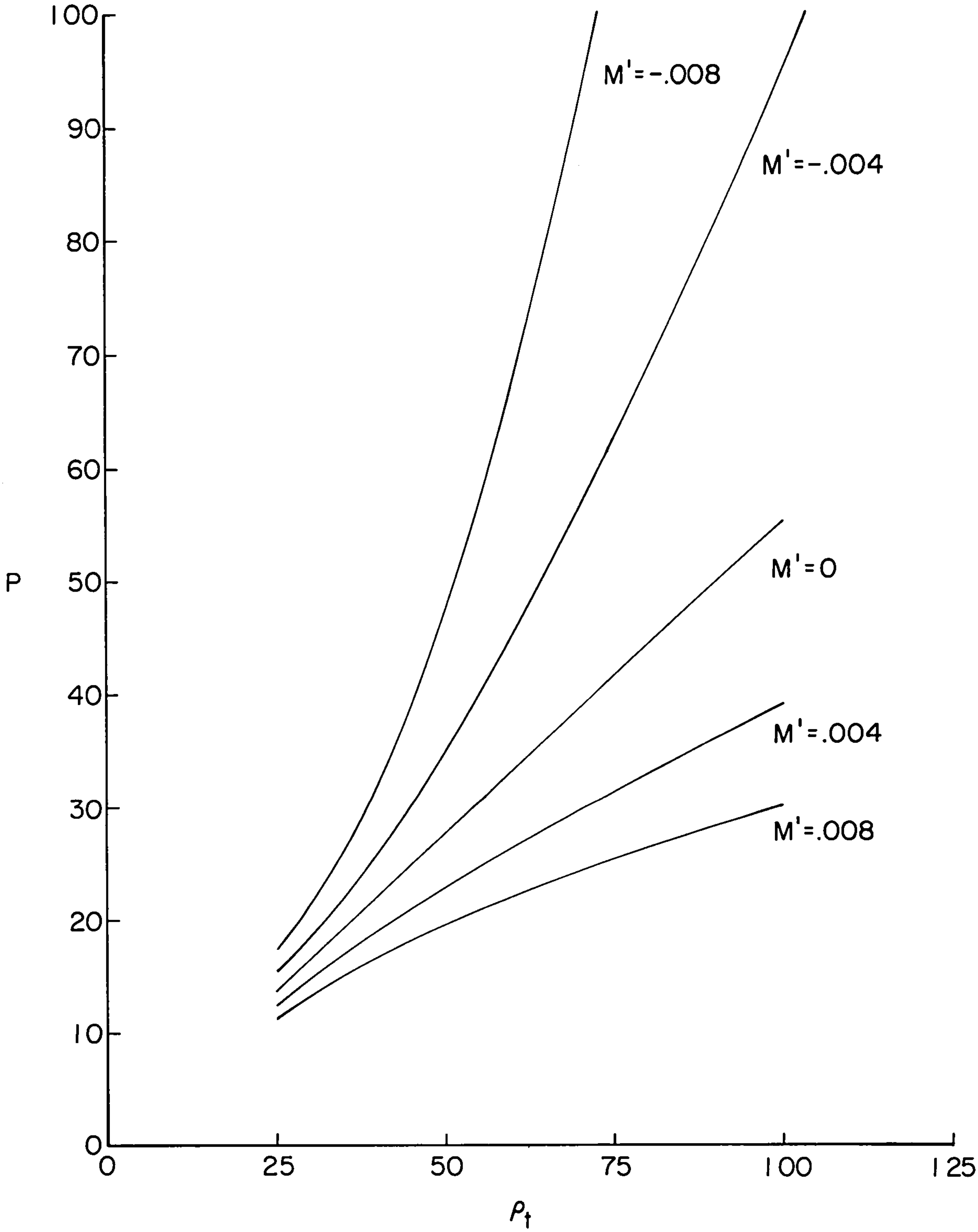
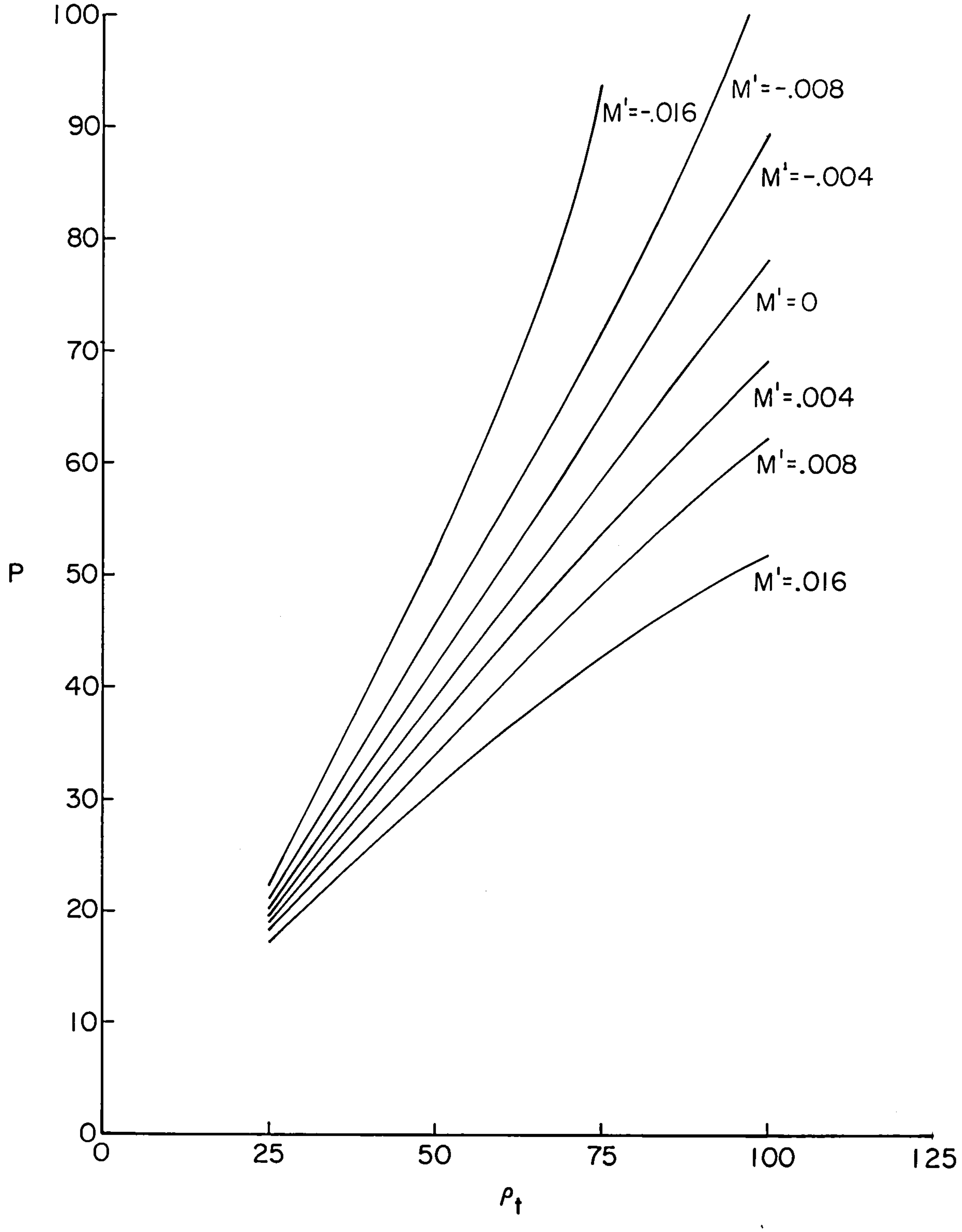


Figure 7.- Cross section of developing wave resulting from straight linear acceleration for $\tau < 30$ sec, $M = 1.2$; for $30 < \tau < 60$, $V' = 1g$; and for $\tau > 60$, $V' = 0$.



(a) M = 1.2.

Figure 8.- Wave-front curvature parameter $P = \dfrac{\beta^2 Ma}{M' + \beta M^2 a\kappa_t \cos\phi}$ as a function of trajectory radius of curvature for various values of acceleration.



(b) M = 1.6.

Figure 8.- Concluded.

1. Report No. NASA TP-1611	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle SONIC-BOOM WAVE-FRONT SHAPES AND CURVATURES ASSOCIATED WITH MANEUVERING FLIGHT		5. Report Date December 1979
		6. Performing Organization Code
7. Author(s) Raymond L. Barger		8. Performing Organization Report No. L-13339
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665		10. Work Unit No. 533-01-43-04
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		13. Type of Report and Period Covered Technical Paper
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

Sonic-boom wave shapes and caustic lines generated by an airplane performing a general maneuver are studied. The equations are programmed for graphical output as a perspective view of the wave shape. This quasi-three-dimensional presentation provides a qualitative insight into the effects of the maneuver on the wave shape and the caustic locations. For the special case of planar maneuvers, the principal curvatures of the wave front are derived. These curvatures are needed to calculate the sound field in the vicinity of a caustic. The results of the analysis are applicable not only to sonic-boom studies but also to the calculation of noise generated by a supersonic rotor or propeller blade tip.

17. Key Words (Suggested by Author(s))		18. Distribution Statement
Sonic boom	Wave-front curvature	Unclassified - Unlimited
Wave shape	Maneuver	
Caustic	Performance	
		Subject Category 02

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	27	$4.50

* For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1979

National Aeronautics and
Space Administration

Washington, D.C.
20546

Official Business

Penalty for Private Use, $300

THIRD-CLASS BULK RATE

Postage and Fees Paid
National Aeronautics and
Space Administration
NASA-451

U.S.MAIL

NASA

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return