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RESONANT FREQUENCY OF MICROSTRIP ANTENNAS CALCULATED FROM TE-EXCITATION OF AN INFINITE STRIP EMBEDDED IN A GROUNDED DIELECTRIC SLAB

FOR REFERENCE

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RESONANT FREQUENCY OF MICROSTRIP ANTENNAS

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SUMMARY

The calculation of currents induced by a plane wave normally incident
upon an infinite strip embedded in a grounded dielectric slab is used to
infer the resonant width (or frequency) of rectangular microstrip antennas.
By placing the strip inside the dielectric, the effect of a dielectric
cover of the same material as the substrate can be included in the calculation
of resonant frequency. A comparison with measured results indicated agreement
of 1 percent or better for rectangular microstrip antennas constructed on
teflon-fiberglass substrate.

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INTRODUCTION

Microstrip antennas are being used extensively in applications where low-profile, inexpensive, rugged, high efficient antennas are desirable. Due to the thin substrate on which the antenna is constructed, the microstrip antenna is inherently very narrow band (usually less than 5 percent); therefore, an accurate method of determining the resonant frequency is needed in order to adequately design microstrip antennas to meet specified application requirements.

It has been demonstrated experimentally (Ref. 1) that the resonant frequency of rectangular microstrip antennas depends primarily upon the dielectric constant and thickness of the substrate material (and also the cover if present) and upon the E-plane dimension of the antenna. Secondary effects would be due to the H-plane dimension and the detailed feed design. The purpose of this paper is to show that the primary effects upon the resonant frequency can be calculated by considering an infinite strip embedded in a grounded dielectric slab and excited by a plane wave polarized transverse to the strip. The theoretical development of the analysis is quite lengthy and will be fully documented in a later report along with a computer code. Only a brief synopsis of the analytical and numerical methods will be presented here.

d	thickness of dielectric
E_y^i	electric field intensity incident on strip
G_y	Green's function
j	$\sqrt{-1}$
J_n	complex amplitude of nth current pulse
J_y	electric current density
k	wave number in dielectric $(2\pi/\lambda_\varepsilon)$
k_o	wave number in free space
k_y	Fourier transform variable
m	indicates m^{th} current pulse
n	indicates n^{th} current pulse
N	total number of current pulses
w	width of strip
x, y, z	Cartesian coordinates
y'	variable of integration
y_m	location of center of mth pulse
y_n	location of center of nth pulse
z'	height of strip above ground plane
ε	permittivity of dielectric
ε_o	permittivity of free space
ε_r	dielectric constant $(\varepsilon/\varepsilon_o)$

η_o	intrinsic impedance of free space
λ_ε	wavelength in dielectric
μ	permeability of dielectric
μ_o	permeability of free space
μ_r	relative permeability (μ/μ_o)
ω	angular frequency

SYNOPSIS OF THEORY

The geometry for the analytical model is illustrated in Fig. 1. The conducting strip is infinite in the x-direction and parallel to the xy-plane. All electromagnetic quantities are assumed to be invarient in the x-direction. The problem then becomes a 2-dimensional one in the yz-plane.

The problem is analyzed by first deriving a Green's function which satisfies the boundary conditions for a line source parallel to the x-axis, embedded in the grounded dielectric layer, and polarized in the y-direction. Since the problem is 2-dimensional and only a y-component of electric current is assumed, a scalar Green's function is sufficient to completely characterize the electromagnetic fields. The Green's function for the region inside the dielectric is:

$$G_y(y,y',z) = \int_{-\infty}^{\infty} \frac{j\mu}{2\,k_z'}\Bigg\{\left[e^{j\,k_z'\,(z-z')} - e^{-j\,k_z'\,|z-z'|}\right]$$

$$-2j\,\sin(k_z'z)\left[\frac{\epsilon k_z\,\cos(k_z'\,(d-z')) + j\epsilon_o\,k_z'\,\sin(k_z'\,(d-z'))}{\epsilon k_z\,\cos(k_z'\,d) + j\epsilon_o\,k_z'\,\sin(k_z'\,d)}\right]\Bigg\}e^{jk_y\,(y-y')}\,dk_y$$

$$(1)$$

where

$$k_z = \begin{cases} \sqrt{k_o^2 - k_y^2} & k_y \leq k_o \\[2ex] -j\sqrt{k_y^2 - k_o^2} & k_y > k_o \end{cases}$$

$$k_z' = \begin{cases} \sqrt{k^2 - k_y^2} & k_y \leq k \\[2ex] -j\sqrt{k_y^2 - k^2} & k_y > k \end{cases}$$

By weighting the Green's function with the electric current density and integrating across the strip, the radiated electromagnetic fields can be calculated at any point inside the dielectric layer. Since the current distribution on the strip is not known, it must first be determined. It is in this calculation of the current density on the strip which allows one to predict the resonant frequency of microstrip antennas. The determination of the strip current is accomplished by imposing the restraint of zero tangential electric field on the surface of the perfectly conducting strip to arrive at the following integro-differential equation for the unknown current density:

$$E_y^i (y,z') = \frac{j\omega}{k^2} \left[k^2 + \frac{\partial^2}{\partial y^2} \right] \int_{-w/2}^{w/2} J_y (y') \, G_y (y,y',z') \, dy' \qquad (2)$$

where the Green's function is evaluated at the strip (z=z'). The electric field incident on the strip is a known quantity since plane wave excitation is assumed.

The integro-differential equation is solved for the unknown current density by employing the method of moments (Ref. 2) using piecewise linear expansion of the current and triangular pulse testing to arrive at a set of N simultaneous equations.

$$E_y^i (y_m, z') = \left(\frac{j k_o \eta_o z' \Delta}{\pi \epsilon_r} \right) \sum_{n=1}^{N} J_n \int_0^{\infty} \left\{ \cos \left[k_y (y_m - y_n) \right] \left[\frac{\sin \left(\frac{k_y \Delta}{2} \right)}{\frac{k_y \Delta}{2}} \right]^2 \right.$$

$$\left[\mu_r \epsilon_r - \left(\frac{2}{k_o \Delta} \right)^2 \sin^2 \left(\frac{k_y \Delta}{2} \right) \right] \left[\frac{\sin (k_z' z')}{k_z' z'} \right]$$

$$\left. \left[\frac{\epsilon k_z \cos \left(k_z' (d-z') \right) + j \epsilon_o k_z' \sin k_z' \left((d-z') \right)}{\epsilon k_z \cos (k_z' d) + j \epsilon_o k_z' \sin (k_z' d)} \right] \right\} dk_y$$

$$m = 1, 2, 3, \ldots, N \qquad (3)$$

Using numerical integration and matrix inversion, the unknown complex amplitudes of the current pulses can be calculated from Eq. (3).

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RESULTS

All calculations presented here are for a lossless dielectric with a dielectric constant of 2.5, which corresponds closely to the properties of teflon-fiberglass.

The distribution of current density across a strip located at the surface (z'=d) of a 0.02 λ_ϵ grounded dielectric is plotted in Fig. 2 for strip widths of 1/4, 1/2, and 3/4 wavelength. One readily notices that the current on the strip is excited more strongly when the strip width is near a half-wavelength. Figure 3 illustrates how the amplitude of the current varys near a strip width of 0.5 λ_ϵ reaching a peak when the strip width is slightly less than a half-wavelength. If one plots the real and imaginary parts of the strip current as a function of the strip width, as is done in Fig. 4, the plot has the characteristic resonant behavior of a dipole antenna.

Figure 5 shows the resonant width of the strip (width for zero imaginary current) as a function of the thickness of the dielectric. The width of the strip for resonance decreases in a monotonic fashion as the dielectric thickness increases. This behavior is also generally characteristic of microstrip antennas (Refs. 1 and 3). The measured data for a rectangular microstrip antenna (Ref. 1) is plotted in Fig. 5 for comparison. A comparison is also given in Fig. 6 between the calculated resonant width of a strip embedded in the dielectric and the measured resonant width (Ref. 1) for a rectangular microstrip antenna with a teflon-fiberglass cover.

CONCLUSION

It is demonstrated that a plane-wave excited infinite strip on, or embedded in, a grounded dielectric slab can be used as an analytical model to accurately calculate the resonant frequency of rectangular microstrip antennas with or without a dielectric cover. A comparison with measurements indicates an accuracy of 1 percent or better for teflon-fiberglass material.

REFERENCES

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3. M. C. Bailey and F. G. Parks, "Design of Microstrip Disk Antenna Arrays," NASA TM-78631, NASA Langley Research Center, February 1978.
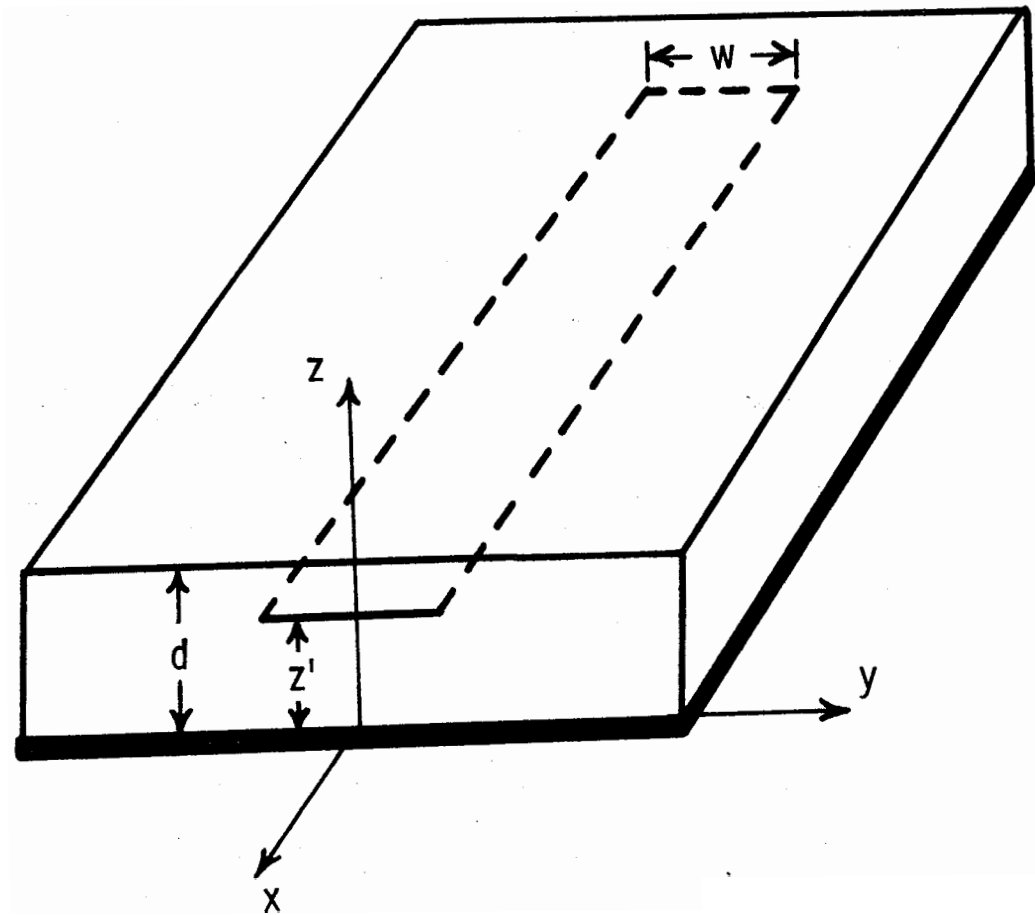
Figure 1: Infinite strip embedded in grounded dielectric slab of infinite extent.
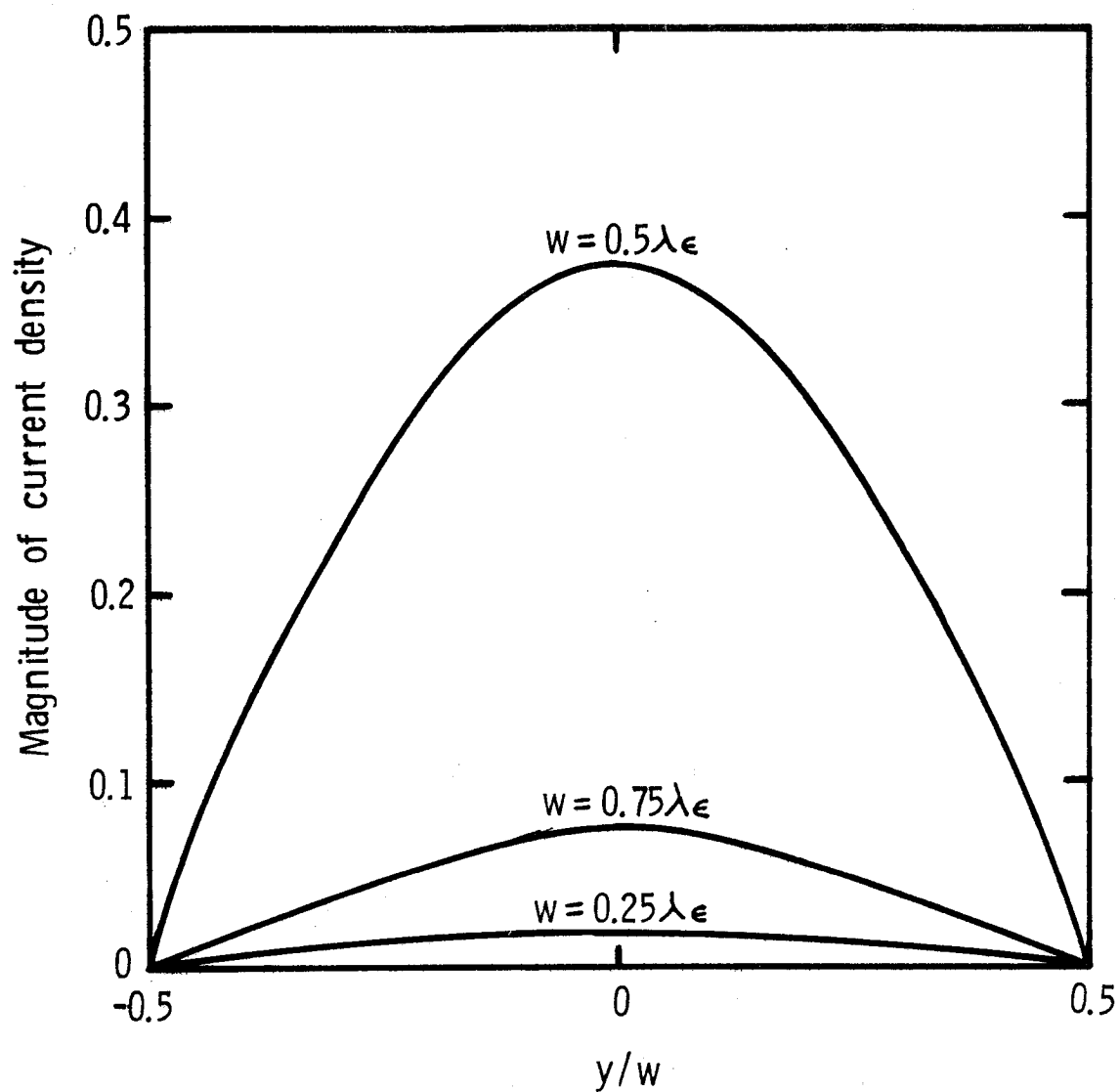
Figure 2: Magnitude of current density across a TE plane wave excited strip at surface of a grounded dielectric slab ($z' = d = 0.02 \lambda_\epsilon$, $\epsilon_r = 2.5$).
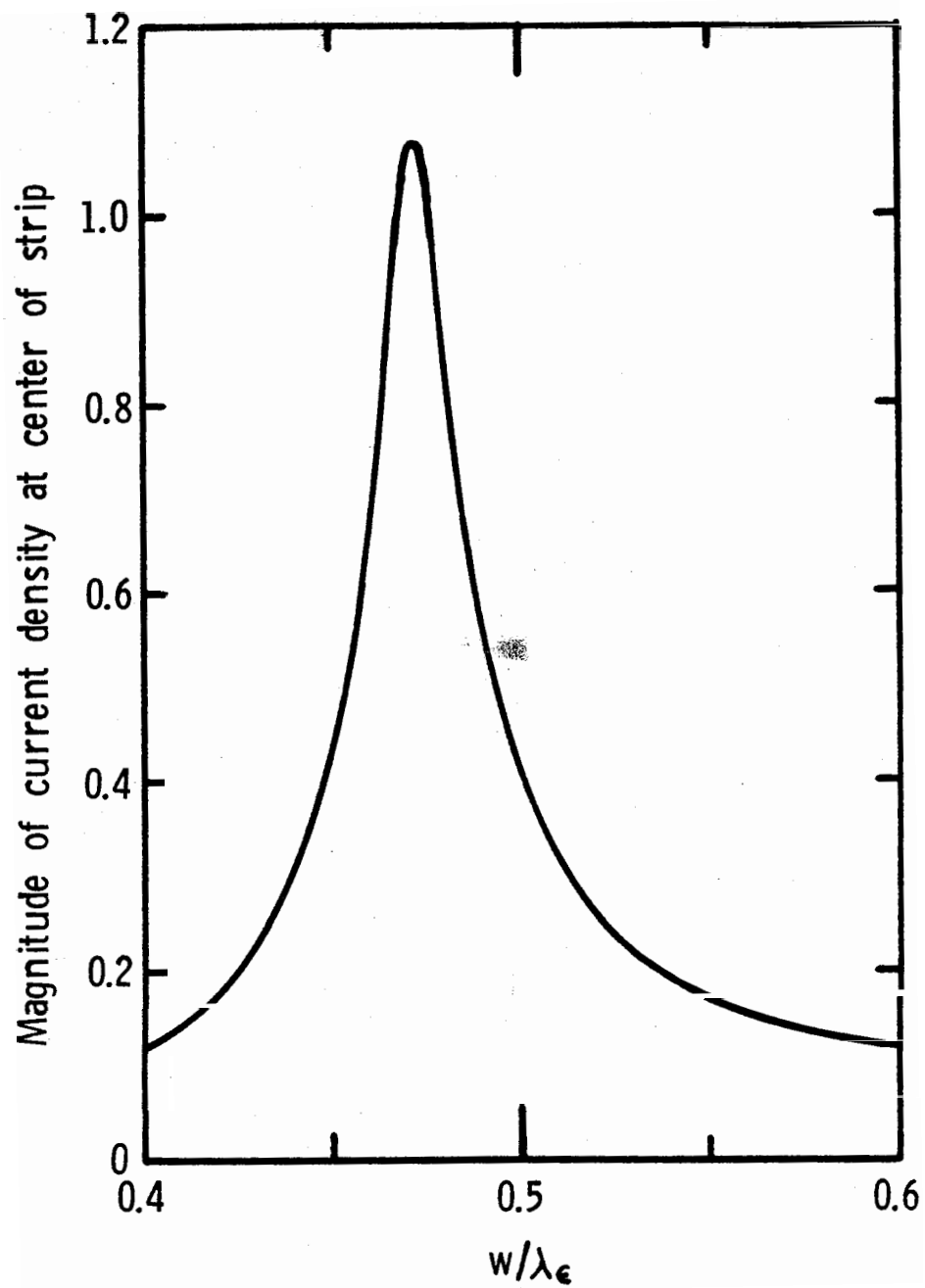
Figure 3: Magnitude of current at center of strip versus strip width $(z' = d = 0.02\lambda_\epsilon, \epsilon_r = 2.5)$.
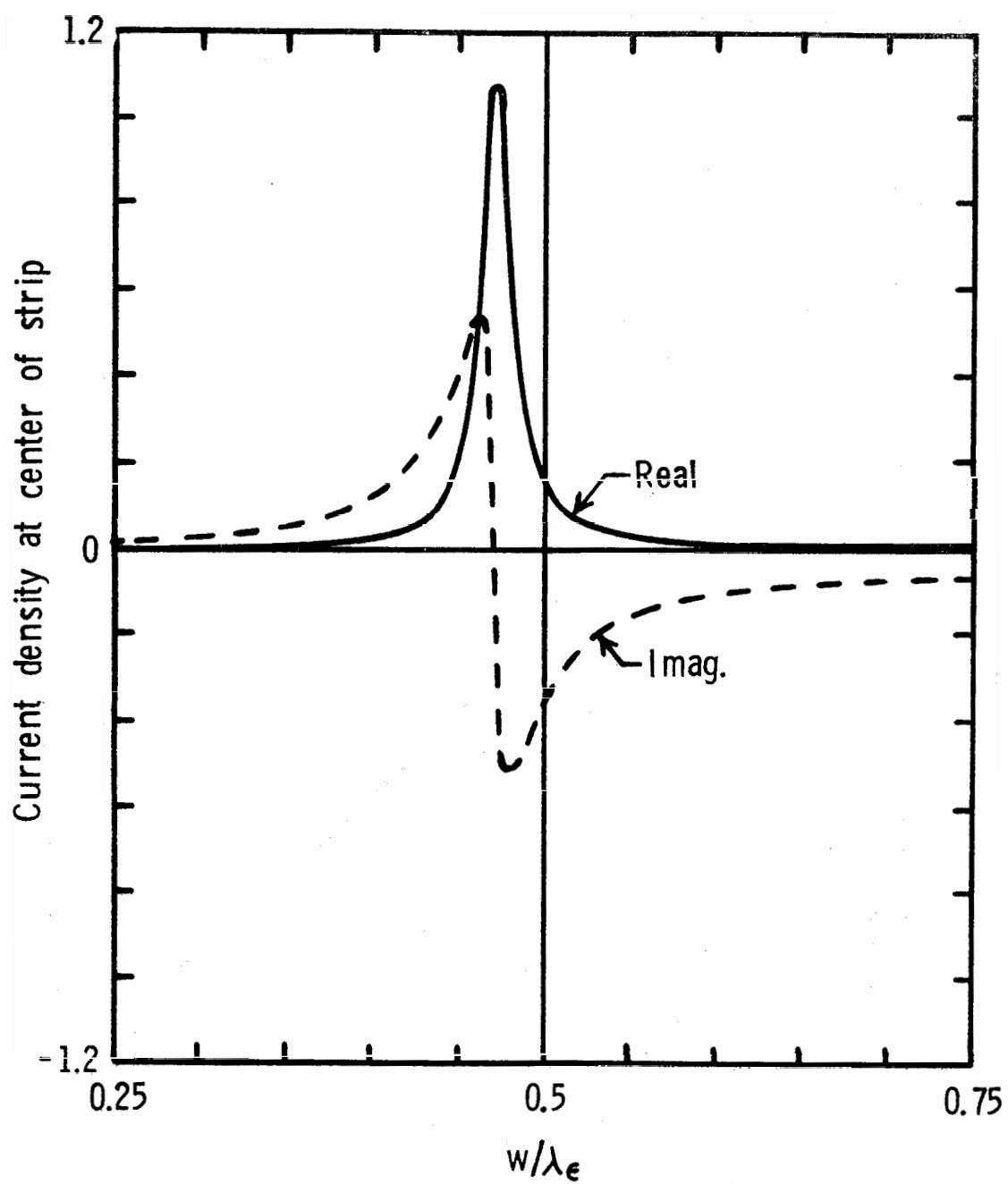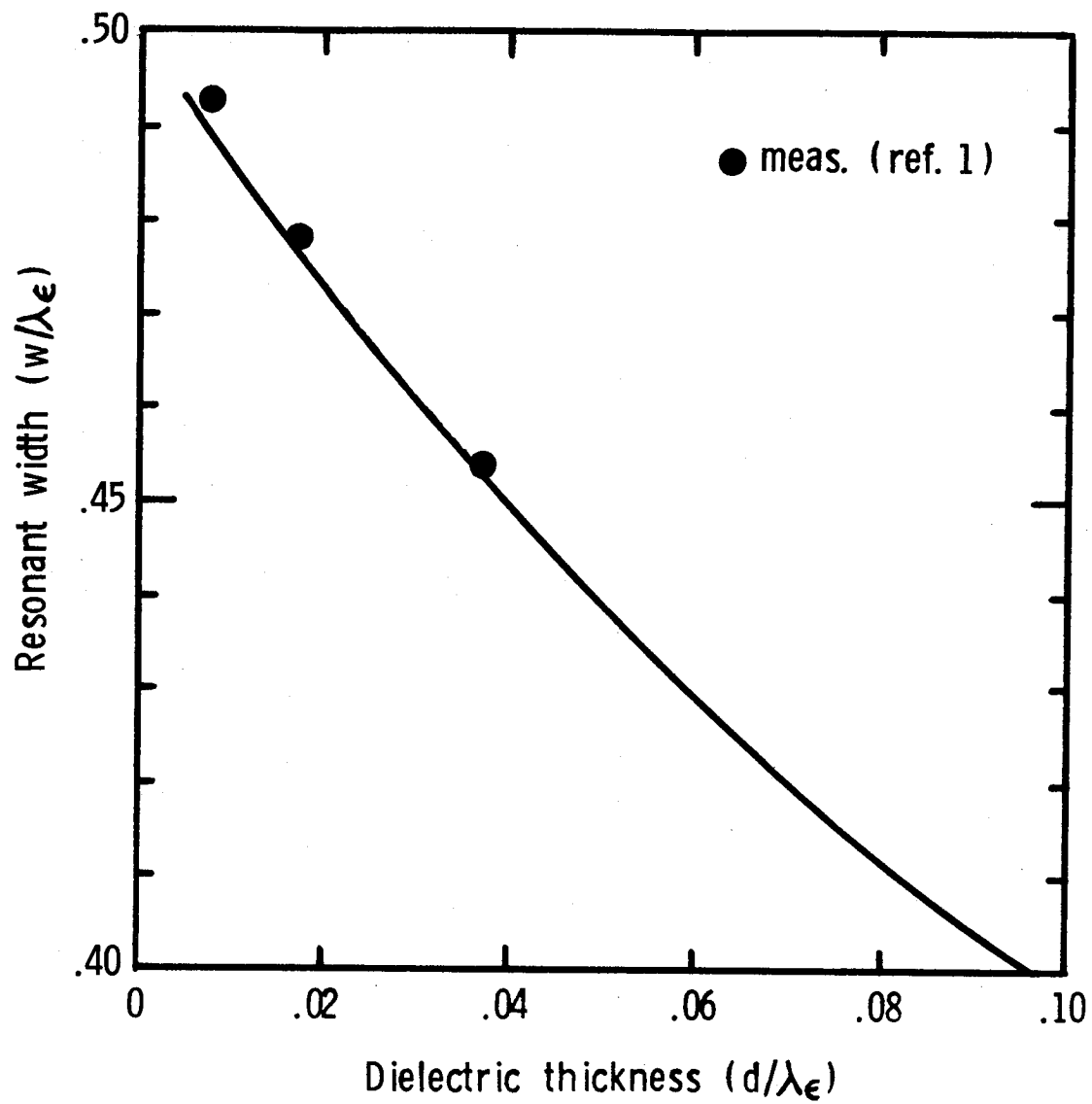
Figure **4**: Complex current density at center of strip versus strip width ($z'=d=0.02\lambda_\epsilon, \epsilon_r=2.5$).

Figure 5: Comparison between calculated resonant width of strip and measured resonant width of microstrip antenna ($z'=d$, $\epsilon_r=2.5$).
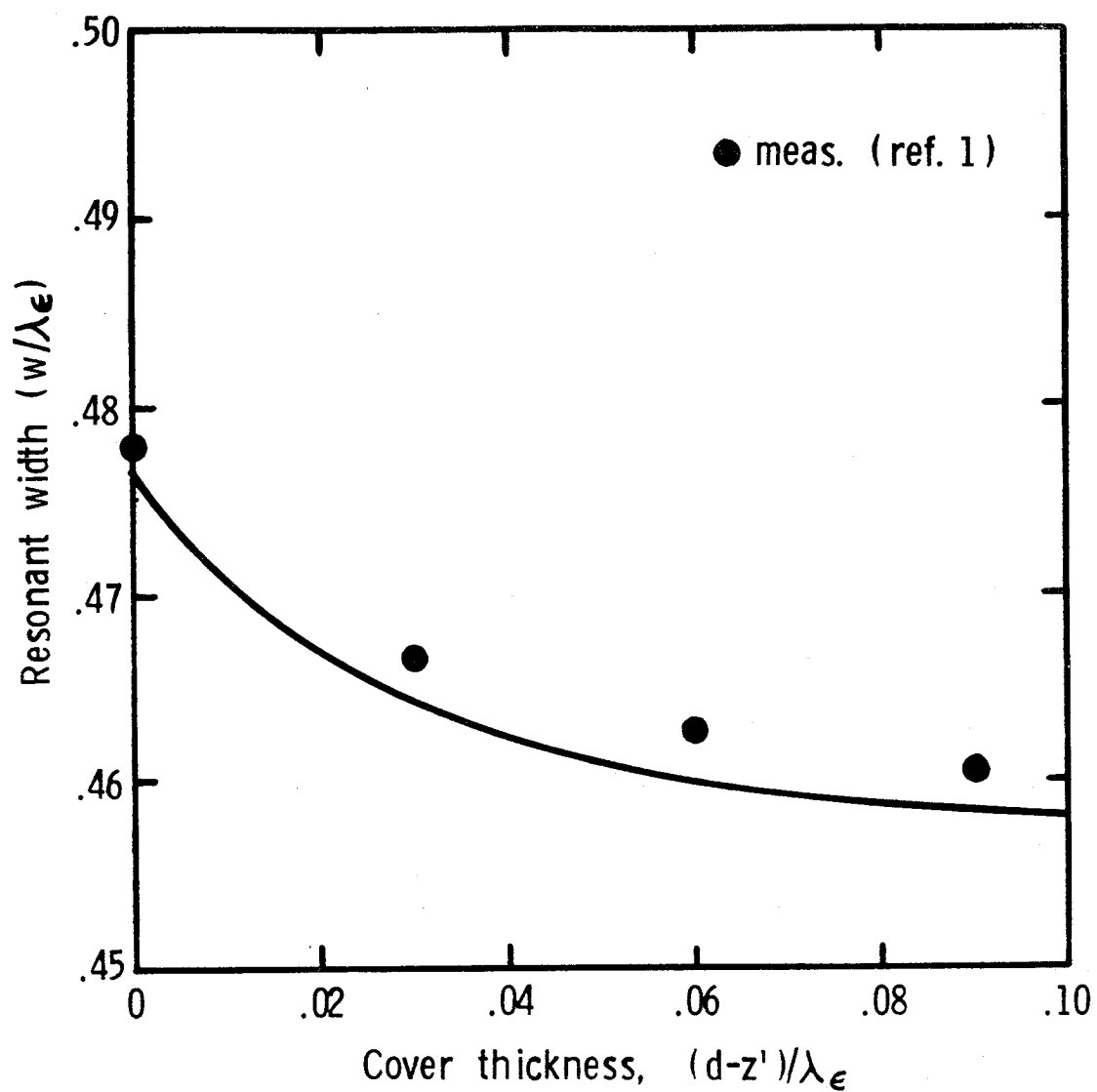
Figure 6: Comparison between calculated resonant width of strip and measured resonant width of microstrip antenna with cover ($z'=0.0175\lambda_\epsilon, \epsilon_r=2.5$).

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15. Supplementary Notes

16. Abstract

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