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DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
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CONTROL SYSTEMS DESIGN FOR LARGE FLEXIBLE SPACE STRUCTURES

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Prepared for the
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Under
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BY

S. M. Joshi[1] and A. S. Roberts, Jr.[2]

SUMMARY

Large, lightweight space structures represent the basic requirement of many potentially important, new, space initiatives. This report contains a description of the research performed under grant NSG 1473, in the area of control systems design for large space structures (LSS). Several approaches for the design of reduced-order LQG-type controllers for LSS were proposed and evaluated using a continuous model of a long free-free beam. Sufficient conditions were derived for the asymptotic stability with this type of controller. A finite-element model of a free-free-free-free square plate was obtained for use in control systems studies. A method was developed for optimal damping enhancement in LSS.

INTRODUCTION

Many of the potentially important new space initiatives and missions which have been identified in reference 1 require large space structures with dimensions which range from one hundred to several thousands of meters. Example structures include very large microwave reflectors, microwave antennas, antenna platforms, solar energy collectors, radiators, solar sails, and telescopes. When the Space Shuttle becomes operational, development of large space structures such as these will become feasible.

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Control systems design for structures of the size being contemplated is a complex and challenging problem because weight and volume constraints on the structural members will result in extremely low frequency bending modes which are closely spaced in the frequency domain. Because of stability and pointing requirements, a number of lower frequency modes will probably fall within the bandwidth of the controller. Thus, the active control of some of the structural modes appears unavoidable. The tools of modern control theory, such as Linear-Quadratic-Gaussian (LQG) control theory, can be effectively applied to the solution of the problem, but not using the standard techniques. This is because the order of the model required to accurately describe the structure, or plant, will be too high to permit a practical solution using standard LQG theory with full-order estimators and regulators. The feasible solution, therefore, is almost certain to consist of regulators and estimators of order much lower than the plant.

Unfortunately, stability of the system with lower order regulators and estimators is not guaranteed because of the possible excitation of uncontrolled or "residual" modes. In reference 2, the very descriptive terms "controller spillover" and "observation spillover" were introduced to describe the unwanted forcing of the residual modes by the control inputs and the unwanted contribution of the residual modes to the observations respectively. If the residual modes have some natural damping, the closed-loop system would be asymptotically stable in the absence of either or both spillovers. However, spillover terms are present in practice and must be considered in overall system performance.

This report consists of a summary of the research performed during the period of the grant. For further information, readers are referred to the papers and reports written by S. M. Joshi during the course of the study.

REDUCED-ORDER LQG-CONTROLLERS

The objective of this research was to develop a controller design methodology for large space structures (LSS). The first step towards this objective was the selection of an appropriate model representing the important dynamic characteristics of LSS while being sufficiently simple to be mathematically tractable. These considerations led to the choice of a uniform free-free beam as the basic model for developing controller design methodology. Reference 3 consists of the development of a model of a free-free beam. Some interesting properties of the mode shapes were also derived in that report.

In reference 4, several approaches to the design of reduced order controllers for large space structures were presented and discussed. These approaches were based on LQG control theory and included truncation, modified truncation regulators and estimators, use of higher order estimators, and selective modal suppression. Also, the use of direct sensor feedback, instead of a state estimator, was investigated for some of the approaches. Numerical results were obtained for a long free-free beam. In addition, sufficient conditions for asymptotic stability were obtained in references 4 and 5. Reference 6 considered a number of approaches to the LQG controller design for LSS, and also proposed the use of "polynomial estimators" for explicit estimation of the observation spillover. Results of reference 6 indicated that the modified truncation regulator and estimator (MTR and MTE) would be satisfactory design approaches. The direct sensor feedback (DSF) implementation was also found to be superior under certain conditions. The use of a higher order estimator was also found to be a potentially useful method. However, the "polymonial estimator" method was not found to be satisfactory.

FINITE ELEMENT MODEL OF A THIN PLATE

The research described above used a planar model of a free-free beam. However, a more realistic three-axis model of a large space structure is more desirable for controller design studies. Since no such model was available, the task of developing a finite-element model was undertaken. A 304.8 m × 304.8 m × 0.254 cm (100 ft × 100 ft × 0.1 in. thick), square aluminum plate was selected for this purpose. A finite-element model was developed using a 25 × 25 mesh, by applying the SPAR program for structural analysis. Modal frequency and mode shape data for 44 structural modes (which represent about 80 kilowords) were stored on a magnetic tape, and were also printed out in a book form. This model should be useful to the NASA researchers working in the LSS control area. Figures 1 to 6 show some of the typical mode shapes, and table 1 shows the modal frequencies.

MODAL DAMPING ENHANCEMENT IN LSS

It was found in the above described work that the closed-loop stability of a LSS is heavily dependent on natural damping of the residual modes; therefore, it is highly desirable to increase the damping of the residual modes where possible. One method of achieving this is to use "member dampers" (ref. 7). However, a systematic method is needed for the selection of member damper gains. Therefore, a method for obtaining optimal member damper gains was developed (see Appendix). This method has significant potential when used in conjunction with the methods discussed above for the design of the primary controller.

CONCLUDING REMARKS

The objective of this project was to develop controller design methodologies for LSS. To that effect, several approaches to the controller design were proposed and discussed. It appears that the LQG controller theory is very well suited as the basic design tool. Of the approaches considered, the Modified Truncation Regulator and Estimator (MTR and MTE) design was found to be potentially the best, especially when used with Direct Sensor Feedback (DSF) implementation. The use of higher order estimators has also shown promise. The importance of the natural damping of the LSS to the closed-loop stability cannot be overemphasized. The natural damping should be increased wherever possible. To that effect, a method was developed for the design of optimal member damper gains. Further investigation is needed in this area, and also in the area of control actuator and sensor analysis and design.

APPENDIX: OPTIMAL MEMBER DAMPER CONTROLLER DESIGN FOR LARGE SPACE STUCTURES

INTRODUCTION

Control systems design for large flexible space structures is a complex and challenging problem because of their special dynamic characteristics. Large space structures tend to have extremely low-frequency, lightly damped, bending modes which are closely spaced in the frequency domain. Because of pointing requirements, a number of lower-frequency modes will probably fall within the bandwidth of the primary controller, thus making active control of some of the modes unavoidable. Since control of all the modes is impractical, the primary controller will be reduced order. This introduces stability problems because of observation and control spillover (refs. 4, 5, 8). The stability of a system with a reduced-order controller is heavily dependent on the natural damping of the residual (uncontrolled) modes. Therefore, it is desirable to increase the damping of the residual modes where possible.

The member damper approach and the application of multiple member dampers in an output velocity feedback configuration were discussed in reference 7. The member damper approach includes local damping elements which could consist of collocated actuators and velocity sensors. Each actuator/sensor pair is configured as a single-loop control system and the member dampers work independently of each other. In the output velocity feedback configuration, all the sensor signals are distributed by a gain matrix to interconnect all the actuators and sensors. This concept was further investigated in references 9 and 10. It has been proved in these references that direct velocity feedback (DVFB) cannot destabilize the system. Such controllers may be used in conjunction with a conventional (modern) active controller, and have the potential to effect significant improvement in the overall performance.

Selection of velocity feedback gains for individual member dampers is an important part of the design. The root locus technique may be used for this purpose; however, this could be a complex task, especially if a large number of actuators are used. In this note, the problem of selecting velocity feedback gains is formulated as an optimal output feedback regulator problem, and necessary conditions are derived for minimizing a quadratic performance function. The special structure of the gain matrix (i.e., diagonal) is taken into account, and the knowledge of process noise and sensor noise is used to advantage.

EQUATIONS OF MEMBER DAMPER CONTROLLERS

The structural model of a large space structure can be (approximately) described by the equations:

$$\ddot{q}_o + D_o\dot{q}_o + \Lambda_o q_o = \Phi_o^T f \qquad (1)$$

$$y_o = \Phi_o q_o \qquad (2)$$

where q_o is the n_o-dimensional vector of modal amplitudes; Φ_o is the $m \times n_o$ "mode shape" matrix; f is the $m \times 1$ generalized force vector (components of f represent applied forces or torques); y_o is the $m \times 1$ vector of generalized displacements (linear and angular) at the m points of application $(\ell_1, \ell_2, \ldots \ell_m)$ of the generalized forces; D_o is the inherent damping matrix, and Λ_o is the diagonal matrix of squared natural frequencies. These equations describe truncated normal-coordinate continuous models, or finite-element models.

If a member damper is connected between two points, equal and opposite forces (torques), proportional to the sensed relative velocity (relative angular velocity) between the points, are applied at the points. Thus, if a single-member damper is connected between points ℓ_1 and ℓ_2, the equations are

$$\ddot{q}_o + D_o\dot{q}_o + \Lambda_o q_o = [\phi_1 \ \phi_2] \begin{bmatrix} f \\ -f \end{bmatrix} = [\phi_1 - \phi_2] \ f \tag{3}$$

where the n_o -vector ϕ_i is the ith column of the Φ_o^T matrix, and f is the scalar force.

$$f = g_1 \ \dot{y}_1 \tag{4}$$

$$y_1 = (\phi_1^T - \phi_2^T) \ q_o \tag{5}$$

where g_1 is the DVFB gain (g_1 and y_1 are scalars).

Substitution of (4) and (5) into (3) yields

$$\ddot{q}_o + (D_o - g_1 \ \psi_1 \ \psi_1{}^T) \ \dot{q}_o + \Lambda_o q_o = 0 \tag{6}$$

where

$$\psi_1 = \phi_1 - \phi_2 \tag{7}$$

If p member dampers are used, the closed-loop equation becomes

$$\ddot{q}_o + (D_o - \sum_{i=1}^{p} \ g_i \ \psi_i \ \psi_i{}^T) \ \dot{q}_o + \Lambda_o q_o = 0 \tag{8}$$

where g_i is the feedback gain and ψ_i the effective input matrix [similar to eq. (7)] for the ith damper. If $g_i \leq 0$ ($i = 1, 2, \ldots, p$), and $D_o \geq 0$, then the effective damping matrix (coefficient matrix of $\dot{q}_o$ in eq. 8) is positive semidefinite, and the system is stable in the sense of Lyapunov; if the effective damping matrix is positive definite, the system is asymptotically stable (ref. 9). It should be noted that this is only a sufficient condition, and the system can be asymptotically stable even though the effective damping matrix is only positive semidefinite.

OPTIMAL OUTPUT FEEDBACK FORMULATION

For the purpose of controller design, n of the n_o modes of the structure are considered. Thus, the "design model" is of order $2n$. Although the member-damper control system is based on the lower order "design model," it cannot destabilize the higher order model. Therefore, this design does not suffer from the problems associated with the use of reduced-order models in conventional optimal regulator and estimator design. Let q denote the modal amplitude vector for the modes in the "design." The state equations for the system under consideration, including process noise and sensor noise, may be written as:

$$\dot{x} = Ax + Bu + v \tag{9}$$

$$z = \psi^T \dot{q} + w = Cx + w \tag{10}$$

where

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}_{2nX1} , \quad A = \begin{bmatrix} 0 & I \\ -\Lambda & -D \end{bmatrix}_{2nX2n} , \quad B = \begin{bmatrix} 0 \\ \psi \end{bmatrix}_{2nXp} \tag{11}$$

$$C = [0 \quad \psi^T]_{pX2n} , \quad \psi = [\psi_1 \ \psi_2 \ . \ . \ . \ \psi_n]_{nXp} \tag{12}$$

and where x is the $2nX1$ state vector, u is the $pX1$ vector of effective inputs, and v and w are zero-mean, white process noise and measurement noise vectors with covariance intensities V and W. It should be noted that $C = B^T$. It is necessary to obtain an input of the type

$$u = Gz = G\ (Cx + w) \tag{13}$$

where

$$
G = \begin{bmatrix} g_1 & 0 & . & . & . & 0 \\ 0 & g_2 & . & . & . & 0 \\ 0 & 0 & . & . & . & g_p \end{bmatrix} \tag{14}
$$

which will minimize

$$
J = \lim_{t_f \to \infty} \frac{1}{t_f} \; \varepsilon \int_0^{t_f} \left\{ x^TQx + (GCx)^TR\,(GCx) \right\} dt \tag{15}
$$

where $Q = Q^T \geq 0$, and $R = R^T > 0$ are the state and control weighting matrices, and ε denotes expected value. The noise-dependent part of u is excluded from the performance function J since it makes J unbounded (ref. 11). The optimal output feedback problem for a general nondiagonal rectangular G matrix was solved in reference 11. However, in this case, since G is diagonal, the minimization has to be performed with respect to the p variable $g_1, g_2, \ldots, g_p$. (If cross-feedbacks are allowed, the problem becomes the same as the general optimal output feedback problem, with the constraint that the closed-loop system is stable.) Let g denote the vector $(g_1, g_2, \ldots, g_p)^T$.

Let the symbol $\alpha * \beta$ denote the element by element product (matrix) of matrices α and β. That is,

$$
\left\{\alpha * \beta\right\}_{ij} = \alpha_{ij}\, \beta_{ij} \tag{16}
$$

Define the $\ell \times 1$ vector-function Δ of a $\ell \times \ell$ matrix α as

$$
\Delta(\alpha) = \begin{bmatrix} \alpha_{11} \\ \alpha_{22} \\ \vdots \\ \alpha_{\ell\ell} \end{bmatrix} \tag{17}
$$

Theorem.- The necessary conditions for the minimization of J in equation (15), with constraints of equations (9), (10), and (13), are given by

$$g = - [R * (B^T \Sigma B) + W * (B^T PB)]^{-1} \Delta(B^T \Sigma PB) \tag{18}$$

$$(A + BGB^T)^T P + P(A + BGB^T) + Q + BGRGB^T = 0 \tag{19}$$

$$(A + BGB^T)\Sigma + \Sigma(A + BGB^T)^T + V + BGWGB^T = 0 \tag{20}$$

where P and Σ are 2nX2n symmetric matrices.

Proof.- The structure of the proof is very similar to that used for the general optimal output feedback problem (ref. 11). (It should be noted that the proof given in ref. 11 needs slight modification in light of ref. 12, although the end result is the same). The only difference is that the derivative of the Hamiltonian with respect to the vector g (rather than the matrix G) is equated to zero. The following easily proved properties of a matrix trace are used (G is a diagonal matrix, α and β are square matrices of compatible dimension):

$$\frac{\partial}{\partial g} \text{Tr } [G\alpha G\beta] = \frac{\partial}{\partial g} [g^T (\alpha * \beta)g] = \left\{\alpha * \beta + (\alpha * \beta)^T\right\} g \tag{21}$$

$$\frac{\partial}{\partial g} \text{Tr } [G\alpha] = \Delta(\alpha) \tag{22}$$

The fact that $C = B^T$ is also used.

It should be noted that, as in the case of the general optimal output feedback problem, the theorem does not guarantee the existence of a g that will make the system asymtotically stable, although the necessary conditions assume the existence. Indeed, the performance function of equation (15) will be meaningful only if such a g exists.

The optimal gain vector g may be computed using the algorithm given in reference 11, using equation (18) or using a numerical minimization method (such as Davidon-Fletcher-Powell).

CONCLUDING REMARKS

Necessary conditions were obtained for minimizing a quadratic performance function under the framework of the member damper concept. Knowledge of noise covariances is used in the design. The method presented offers a systematic approach to the design of a class of controllers for enhancing structural damping in large space structures. This type of controller has significant potential if used in conjunction with a reduced-order optimal controller that is designed to control rigid-body modes and some selected structural modes.

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Table 1. Modal frequencies of 304.8 m × 304.8 m × 0.254 cm (100 ft × 100 ft × 0.1 in. thick) free-free-free-free square aluminum plate.

MODE	FREQ(RAD/SEC)	FREQ(HZ)
1	.54999E-01	.87534E-02
2	.80024E-01	.12736E-01
3	.99111E-01	.15774E-01
4	.14211E+00	.22618E-01
5	.14211E+00	.22618E-01
6	.24948E+00	.39707E-01
7	.24948E+00	.39707E-01
8	.26008E+00	.41392E-01
9	.28286E+00	.45018E-01
10	.31515E+00	.50157E-01
11	.43068E+00	.68545E-01
12	.43068E+00	.68545E-01
13	.47824E+00	.76114E-01
14	.50003E+00	.79583E-01
15	.53689E+00	.85449E-01
16	.53689E+00	.85449E-01
17	.62422E+00	.99347E-01
18	.65958E+00	.10497E+00
19	.68808E+00	.10951E+00
20	.80973E+00	.12887E+00
21	.80973E+00	.12887E+00
22	.83371E+00	.13269E+00
23	.87377E+00	.13906E+00
24	.87982E+00	.14003E+00
25	.87982E+00	.14003E+00
26	.99216E+00	.15791E+00
27	.99216E+00	.15791E+00
28	.11483E+01	.18275E+00
29	.11922E+01	.18974E+00
30	.11996E+01	.19093E+00
31	.12194E+01	.19407E+00
32	.12250E+01	.19497E+00
33	.12532E+01	.19946E+00
34	.12532E+01	.19946E+00
35	.13742E+01	.21872E+00
36	.14082E+01	.22412E+00
37	.14871E+01	.23668E+00
38	.14871E+01	.23668E+00
39	.16054E+01	.25558E+00
40	.16059E+01	.25558E+00
41	.16972E+01	.27012E+00
42	.16972E+01	.27012E+00
43	.17111E+01	.27233E+00
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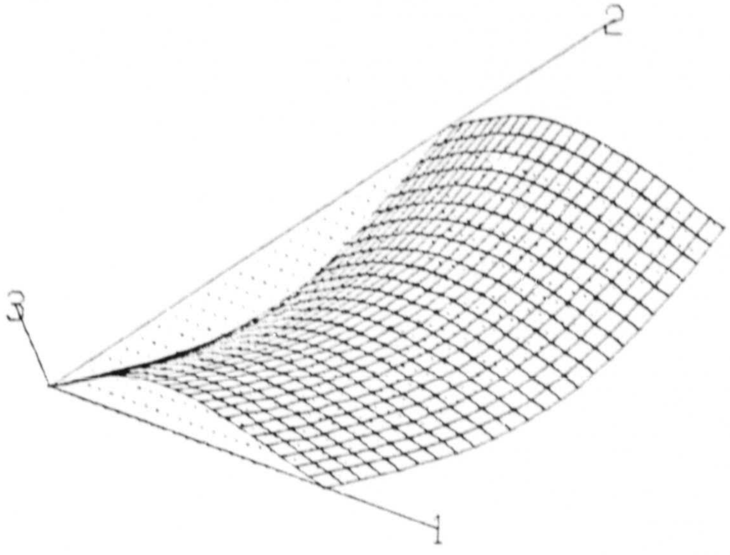


Figure 1. Mode No. 2.

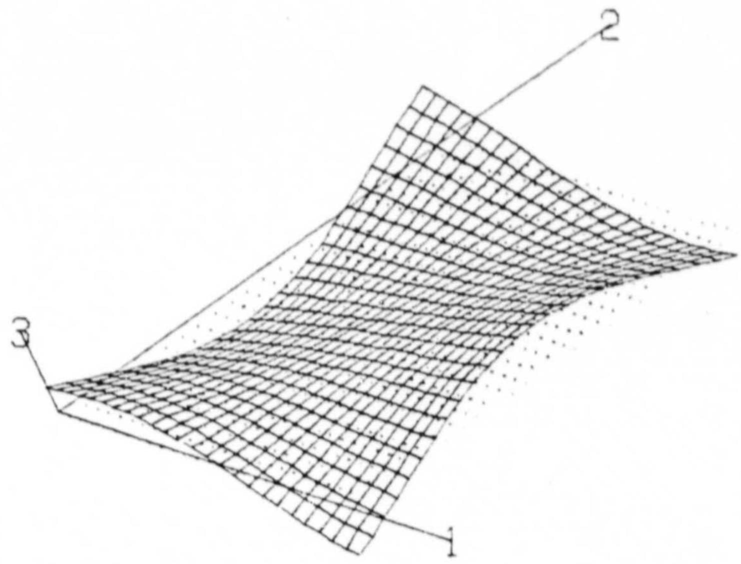
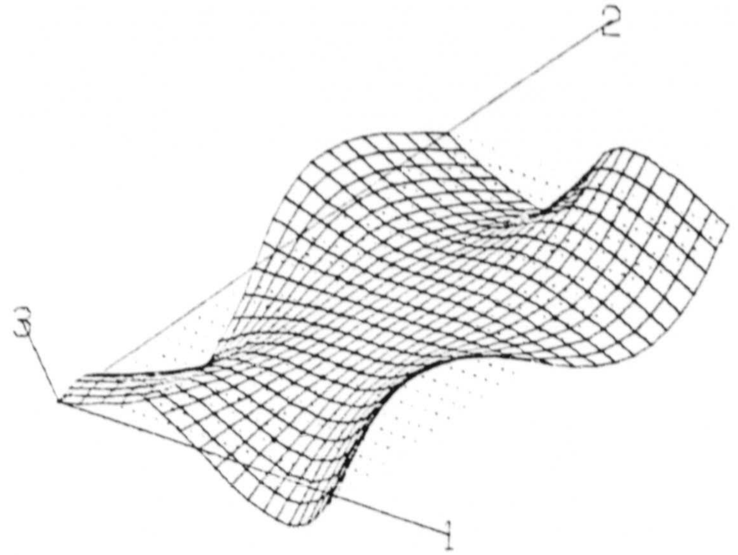


Figure 2. Mode No. 5.



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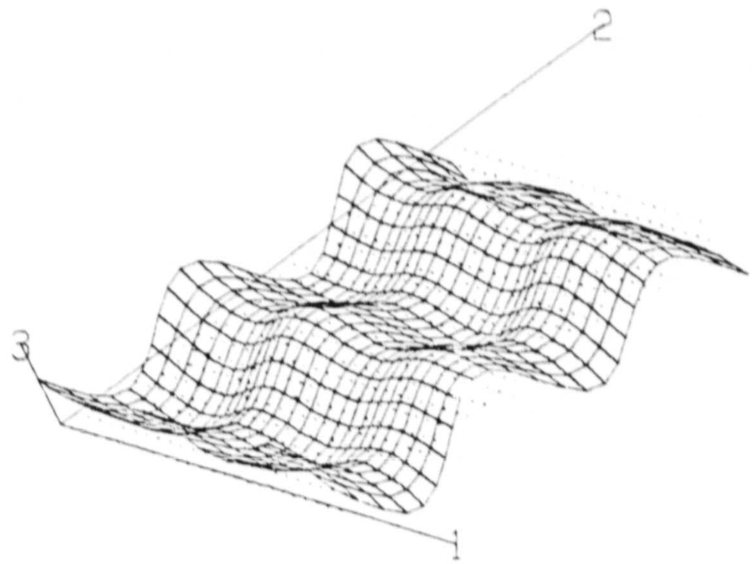

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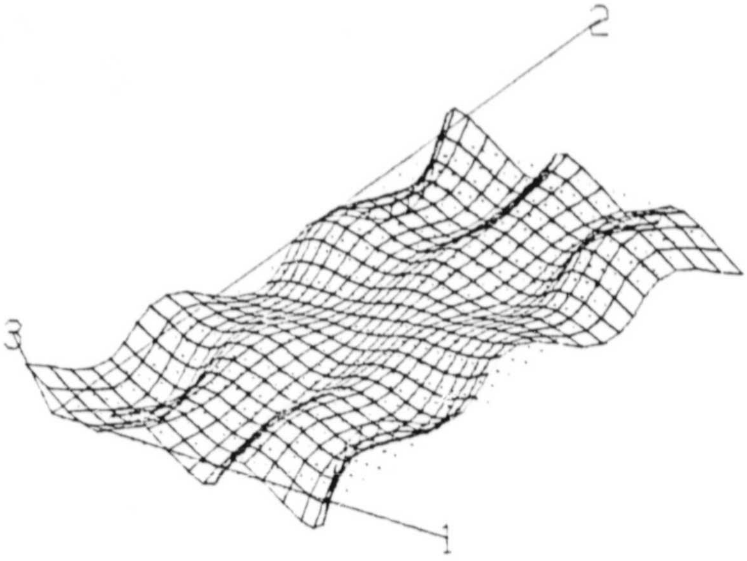

Figure 5. Mode No. 33.

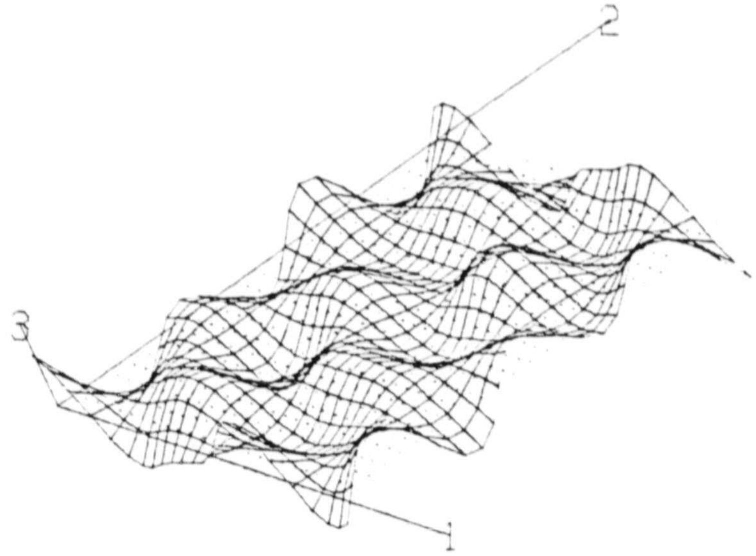

Figure 6. Mode No. 40.