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### COMBUSTION NOISE

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Stanford, California.

## SUMMARY

A review of the subject of combustion generated noise is presented. Combustion noise is an important noise source in industrial furnances and process heaters, turbopropulsion and gas turbine systems, flaring operations, Diesel engines and rocket engines. The state-of-the-art in combustion noise importance, understanding, prediction and scaling is presented for these systems. The fundamentals and currently available theories of combustion noise are presented. Controversies in the field are discussed and recommendations for future research are made.

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## NOMENCLATURE

A	Area bounding reaction region
a	Constant in Eq. (10) or pressure admittance coefficient
b	Constant in Eq. (10) or entropy admittance coefficient
c	Constant in Eq. (10) or speed of sound
$c_p$	Specific heat at constant pressure
D	Burner diameter
$Da_i$	Damkohler's first similarity group for the $i^{\text{th}}$ reaction; ratio of flow time to chemical time
d	Constant in Eq. (10)
F	Fuel mass fraction
f	frequency
H	Fuel heating value
I	Emission intensity, local
	Emission intensity, total
i	$(-1)^{1/2}$
$l_e$	Integral length scale of turbulence
M	Mach number
$m_f$	Fuel mass flow rate
$\underline{n}, \underline{n}_0$	Outward unit normal vectors
P	Sound power
Pr	Prandtl number
p	Pressure
$\dot{Q}$	Volumetric heat release rate
$R_{xy}$	Cross correlation junction
Re	Reynolds number
$\underline{r}, \underline{r}_0$	Position vectors
Sc	Schmidt number
St	Strouhal number
$S_{xy}$	Cross spectral density
$S_L$	Laminar Flame speed
$S_T$	Turbulent flame speed; cold fluid velocity normal to the mean flame surface



## NOMENCLATURE (CON'T)

T	Temperature or sample time in random data analysis
t	Time
U	Mass weighted average axial fluid velocity
u	Axial velocity component
V	Volume enclosing reaction
$\vec{v}, v_i$	Velocity vector
$x_i$	Cartesian coordinate in $i^{\text{th}}$ direction
x,y	Two functions of space and time
$\alpha$	Thermal diffusivity
$\gamma$	Ratio of specific heats
$\gamma^2$	Coherence function
$\eta_{ta}$	Ratio of sound power to thermal input, thermoacoustic efficiency
$\mu$	Mean value
$\rho$	Density
$\omega$	Angular frequency
$\sigma$	Dimensionless entropy fluctuation
$\tau$	Time delay in cross correlation

### Superscripts

$\hat{\quad}$	Fluctuation about a mean value
-	Steady state or mean quantity, usually referring to an inlet or cold quantity
$\overline{\quad}^2$	Mean square value

### Subscripts

b	Burner
e	Combustor exit plane
f	Adiabatic flame
$\omega$	Fourier transform

## 1. INTRODUCTION

Noise is a pollutant emitted by the combustion process in several energy - producing systems. Unlike many chemical pollutants, noise has an immediate effect upon observers. At the time of emission, noise causes annoyance and physiological change and also impedes efficiency of observers. Noise may also cause immediate physiological damage. Long range effects of noise are structural and physiological impairment and property value decrease. Generally, a different scientific community deals with noise problems than deals with chemical pollutants. An area where the disciplines begin to overlap, however, is the field of combustion generated noise.

Noise arises in combustion systems for two reasons: (a) All practical burners, except for some small scale devices, operate with the working fluid in turbulent motion, and (b) heat is being liberated. Turbulence is a motion of the fluid which is random in space and time. This will cause noise of its own accord, since a pressure field which is unsteady with respect to a fixed observer is generated. Combustion noise is a turbulence-combustion interaction noise.

Practical combustors are usually turbulent, because it is possible to release more heat per unit volume per unit time in such a system than in a laminar one. Turbulence changes dominant transport mechanisms from the molecular level to the macroscopic level and increases the rapidity of transport processes. While combustion still requires interactions at the molecular level, the fact that combustion is imbedded in a turbulent field alters the noise production mechanism as compared with non-reacting turbulent fields.

Although somewhat arbitrary, it has been stated<sup>1</sup> that a noise abatement problem exists in combustors firing above  $2 \times 10^6$  Btu/hr (600 kw, .03 lb/sec or 14 gm/sec or 108 lb/hr of a 18,500 Btu/lb fuel, whichever units the reader prefers). Whether a problem exists, of course, depends upon the proximity of an observer, and this depends upon the application. However, the number quoted is quite small compared with those of practical interest. For example, a

single Pratt and Whitney JT8D turbofan aircraft engine has a firing rate of 40,000 kw. Combustion noise has been identified as an important noise source in industrial furnaces and process heaters, gas flaring operation, aircraft turbopropulsion systems, gas turbine units and Diesel engines. The fundamentals of the problem are the same in all systems, however.

Combustion noise occurs in two forms - direct and indirect noise<sup>2</sup>. The first is noise generated in and radiated from a region undergoing turbulent combustion. It is caused by a temporal fluctuation in the aggregate heat release of the reacting region. This overall fluctuation, while small, exists and generates pressure waves. The second type of noise is generated downstream of the combustion region. In any turbulent combustion situation, all streamlines are not equally heated by the combustion process; hot spots (or cold spots) are generated in the flow. These regions of fluid nonuniformity cause noise when they interact with downstream components of the device, as will be explained below. Depending on the nature of the device, either direct or indirect noise may be dominant. Most of the problems arise from direct noise, however, so that greater attention will be focused upon it.

It is important to recognize that the discussion here is not concerned with combustion instability, although there is a weak relation between combustion noise and stability. Combustion noise is a random output of sound, containing all frequencies with random amplitude. Combustion instability, on the other hand, is a phase-coherent, fixed frequency feedback oscillation. While there may be feedback between acoustic waves and the combustion process in a device making combustion noise, it is not of such a strength to "lock in" to a phase-coherent oscillation. Typically, a well-behaved device will radiate  $10^{-6}$  to  $10^{-5}$  of its thermal input as noise sound power. If the system is in a true instability, the number will rise to about  $10^{-4}$ . The figure of  $10^{-6}$  -  $10^{-5}$  is, of course, of no consequence concerning energy loss to the system. However, the noise is quite sufficient to cause concern in many instances.

## 2. PHYSICAL DESCRIPTION OF DIRECT COMBUSTION NOISE

A very neat description of the physics of combustion noise is given by the experiment and analysis of Ref. 3. The experiment consisted of a soap bubble containing a premixed fuel-air mixture which was centrally spark ignited. The situation, shown in Fig. 1, produces, upon ignition, a transient burning situation accompanied by a transient volume increase in the bubble gases. This spherically symmetric situation will emit a spherically symmetric sound wave with a waveform as shown in Fig. 1. This is an example of a monopole sound radiator, and the experimental waveforms were accurately reproduced by theory. It was concluded from this experiment that the ratio of sound energy produced to the thermal input should not exceed about  $10^{-6}$  for typical hydrocarbon-air flames. Interestingly, this conclusion has in fact proved to be valid in much more complex turbulent situations, as long as the sound is radiated to a free field (no reflecting surfaces). The reason for the success is that this simple experiment contains the fundamentals of combustion noise generation - an expansion by combustion of pockets of gas forming a time-unsteady process with respect to an observer.

Preceding these experiments was a theory by Bragg<sup>4</sup>, essentially based on the above arguments but applied to a turbulent flame. The turbulent flame was envisioned as a collection of statistically uncorrelated "eddies", "turbules", or whatever term the reader prefers, which are undergoing various stages of reaction. These eddies have their own heat release rate which is statistically independent of those of neighboring eddies. The rate at which the entire combustion volume is contracting or expanding is the sum of the contraction or expansion of the individual eddies. Assuming this results in a net monopole radiator, simple source theory says that<sup>3,5</sup>

$$P = \overline{\left(\frac{d^2V}{dt^2}\right)^2} \frac{\bar{V}}{4\pi\bar{c}} \quad (1)$$

is the total radiated power, where the time derivatives of  $V$  are obtained by summing over the contributions of the individual eddies.\*

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\*Some liberties have been taken in this exposition, as compared with Bragg's original work, to facilitate comparison with more modern work.

The estimation of the quantities in Eq. (1), in terms of combustion variables, was the next step in Bragg's theory. However, the results are at variance with more modern work and will not be pursued here. The major point, however, which has stood the test of time, is that direct combustion noise is a dilatation of the flow (volumetric expansion and contraction). It is caused by a fluctuation in the aggregate heat release rate in the flame. Moreover, under suitable restrictions, a monopole radiation pattern (spherically symmetric) is obtained.

To proceed with fundamental understanding of this noise it is recognized that Bragg's theory: (a) Does not follow from the conservation principles of fluid mechanics so that it is unclear what approximations are truly involved (b) Postulates a net monopole radiation that may or may not follow from a rigorous treatment (and, indeed, does not under some circumstances) and (c) Often produces incorrect answers if pursued along original lines from Eq. (1) to combustor variables. Nevertheless, it was an ingenious exposition of the underlying cause of direct combustion noise.

### 3. DESCRIPTORS OF NOISE

The ear is sensitive to fluctuations of air pressure,  $p'$ , about the mean atmospheric pressure,  $\bar{p}$ . In general, the fluctuating pressure is a function of time and space variables,  $p'(\underline{r}, t)$ . Although the details will not be covered here, it will be demanded that all fluctuating variables will be weakly stationary in time so that the cross correlation function of two variables  $x(\underline{r}_1, t)$ ,  $y(\underline{r}_2, t)$

$$R_{xy}(\underline{r}_1, \underline{r}_2, t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \left[ x(\underline{r}_1, t) y(\underline{r}_2, t+\tau) \right] \quad (2)$$

exists, independent of the time origin. Without loss of generality,  $x$  and  $y$  are fluctuating variables with a zero mean value

$$\mu_{x,y} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x \, dt = 0$$

Pressure fluctuations about atmospheric pressure, for example, conform to this last statement for the noise under consideration.

Using the treatment of Ref. 6, the "two-sided" cross spectral density of  $x$  and  $y$  is given by

$$S_{xy}(f, \underline{r}_1, \underline{r}_2) = \int_{-\infty}^{\infty} R_{xy}(t, \underline{r}_1, \underline{r}_2) e^{-2\pi i f t} dt \quad (3)$$

Equation (3) is a Fourier transform operation on  $R_{xy}$ , and it follows by inversion that

$$R_{xy} = \int_{-\infty}^{\infty} S_{xy} e^{2\pi i f t} df \quad (4)$$

The term "two sided" for  $S_{xy}$  is used to denote the fact that  $S_{xy}$  is defined for negative as well as positive frequencies.

The cross spectral density and the cross correlation function, being derivable from each other, contain the same information. Which one is used is purely a matter of convenience. For computation, it is known that  $S_{xy}$  may

be estimated by a Fourier transform operation on  $x$  and  $y$

$$S_{xy} = \frac{x_{\omega} y_{\omega}}{T} \quad (5)$$

with  $T$  a finite sample time which is large compared with the minimum period (inverse of a frequency) to be resolved. Here

$$x_{\omega} = \int_0^T x e^{-2\pi i f t} dt = \int_0^T x e^{-i\omega t} dt \quad (6)$$

is the convention used.

Special cases of Eqs. (2) and (3) immediately follow. If  $x$  and  $y$  are the same variable at the same location, the autocorrelation function and the spectral density are

$$R_{xx}(\tau) = \frac{1}{T} \int_0^T dt x(t) x(t+\tau) = R_x$$

$$S_{xx} = \int_{-\infty}^{\infty} R_x e^{-i\omega\tau} d\tau = S_x \quad (7)$$

Moreover, the mean square value of  $x$  follows from Eqs. (7) as

$$\overline{x^2} = R_{xx}(0) = \frac{1}{T} \int_0^T x(t)^2 dt \quad (8)$$

and is calculable from Eq. (4) via the spectral density by

$$\overline{x^2} = \int_{-\infty}^{\infty} S_{xx} df \quad (9)$$

The spectral density of  $x$  therefore gives the frequency distribution of the mean square fluctuation.

Often, in what follows,  $x$  will be the fluctuating pressure,  $p'$ . The noise magnitude is usually described in terms of  $\overline{p'^2}$  and its frequency distribution in terms of  $S_{p'p'} = S_{p'}$ . From experimental data Eqs. (5) and (6)

may be used and all quantities of interest will follow.

Another useful function, constructed from the above, is the coherence function defined by

$$\gamma^2 = \frac{S_{xy} S_{xy}^*}{S_x S_y}$$

If a linear relation exists between  $x$  and  $y$  the transforms are related through a transfer function  $H_\omega$  such that

$$x_\omega = H_\omega y_\omega$$

Use of Eq. (5) then shows that the coherence function is unity.  $\gamma^2$  consequently shows, as a function of frequency, the existence (or non-existence) of a linear, causal relation between  $x$  and  $y$ .

Somewhat similar to the coherence function, but not quite as powerful, is the cross correlation coefficient, constructed by

$$R_{xy} = \frac{R_{xy}}{R_{xx}^{1/2} R_{yy}^{1/2}}$$

It may readily be shown that if  $x = y$  and  $x$  and  $y$  are shifted in phase by a time  $\bar{\tau}$ ,  $R_{xy}(\bar{\tau}) = 1$ . These requirements are more stringent than the existence of  $H_\omega$  in the case of the coherence function, since  $H_\omega$  is an arbitrary transfer function. Nevertheless, the cross correlation coefficient is useful for investigation of causal relations involving direct proportionalities and simple phase delays.

All of the above descriptors of noise are useful in a discussion of combustion noise. Overall sound strength is measured in terms of  $\overline{p^2}$  and its frequency distribution in terms of  $S_p$ . The cross correlation and coherence functions are useful to show causal relations between the combustion process and the emitted sound. The cross correlation and cross power spectrum at



spatially separated locations are useful in characterizing indirect combustion noise. One further relation is the connection between sound power and mean square pressure. In the absence of a mean flow the sound power crossing an area A is given by

$$P = \int_A \overline{p' \vec{v}' \cdot \vec{n}} \, dA$$

often,  $p' \propto \vec{v}' \cdot \vec{n}$  so that

$$P \propto \int_A \overline{p'^2} \, dA$$

Consequently, discussions of the mean square pressure and sound power are often synonymous. This was in fact the case with Eq. (1) for a flame radiating sound to a free field.

Finally, sound units are most often quoted in decibels, where the overall sound pressure level (OASPL) is defined by

$$\text{OASPL} \equiv 10 \log_{10} \frac{\overline{p'^2}}{p_0^2}$$

with  $p_0 = 2 \times 10^{-5}$  Pa, by convention. When the sound pressure level (SPL) is discussed one is usually speaking of the spectral distribution of the OASPL or  $\text{SPL} = 10 \log_{10} S_{p'}/p_0^2$ . However, one must be careful here because  $S_{p'}$  is strictly in units of pressure<sup>2</sup>/Hz, through Eq. (9). More often, a different bandwidth than one Hz is used to express  $S_{p'}$ , depending upon the analysis equipment. The convention here is to state SPL in dB/(ΔfHz) where Δf is the bandwidth used to construct  $S_{p'}$ .

#### 4. EXPERIMENT AND THEORY FOR SIMPLE PREMIXED TURBULENT JET FLAMES RADIATING TO A FREE FIELD

##### Experimental

One of the simplest possible flame configurations is the turbulent Bunsen burner flame of premixed gases. Here, the flame burns in the open, anchored at the end of a tube. Numerous studies of the noise characteristics of such flames have been reported.<sup>7-12</sup> Of these studies, Refs. 7 and 9-11 have the simplest turbulence field upstream of the flame zone and also more correlation work associated with the results. Consequently, they will be concentrated upon here.

When theory is deficient a common practice is to attempt a correlation of some interesting quantity, such as sound power, with dimensionless variables which characterize the problem. For example, the sound power, made dimensionless by the thermal input rate, should be expressible in the form

$$\frac{P}{\dot{m}_f H} \eta_{ta} = f \left( M, F, Re, Pr, Sc, \gamma, Da_1, \dots, H/c_p \bar{T} \right)$$

where the  $Da_i$  are the ratios of flow to reaction time for all relevant reactions in the process of conversion from reactants to products. Often, the details of the individual reactions are not known, and it is common to replace the aggregate of reactions with an effective single (global) reaction. In this case a measure of the single effective  $Da$  is given by the laminar flame speed through the formula

$$Da = S_L^2 D / \nu^2$$

The laminar flame speed for various mixtures is measurable and tabulated.

The experiments cited were sufficient to vary the four dimensionless groupings of  $F, M, Re$  and  $Da$ . Moreover, theory suggested that the results should behave according to a power law<sup>10</sup> with

$$\eta_{ta} \sim M^a Re^b Da^c F^d \quad (10)$$

In fact, it was found that

$$\begin{array}{ll} a = 2.68 & b = -0.09 \\ c = 0.92 & d = -1.26 \end{array}$$

The standard deviation of this result is  $\pm 1.5$  dB, which is quite good for noise measurements. The acoustic power involved was varied by a factor of  $10^4$ , or 40 dB.

From the physical viewpoint the most important fact of Eq. (10) is the absence of a strong dependence on Reynolds number, indicating that transport processes play very little role in the noise generation process. From the turbulence viewpoint this indicates that the large scale or energy containing eddies may dominate the noise process.

A typical frequency spectrum from the cited results is shown in Fig. 2. This is a plot of the SPL relative to some arbitrary datum. Combustion noise from open premixed flames is typically a broad band noise with a single spectral maximum. The frequency at which the noise maximizes will be called the peak frequency. A Strouhal number,  $St \equiv f_p D/U$ , can also be correlated as in Eq. (10) and the result is

$$St \propto M^{-0.40} Pe^{0.02} Da^{0.39} F^{-1.1} \quad (11)$$

Again, the independence with regard to Reynolds number is striking. In both Eqs. (10) and (11) the chemistry (through  $Da$ ) and/or the flame temperature (through  $F$ ) clearly play a strong role in the noise generation process. The dependence upon Mach number should not be taken to imply strong compressibility effects, but, as will be seen, it naturally enters acoustic problems because the power radiated depends directly on the speed of sound of the medium.

## Theory

It is now of interest to see whether or not the results of Eqs. (10) and (11) may be explained theoretically. A simple theory has been given in Ref. 10. Consider Fig. 3 as the flame schematic and consider the limit that the flame shrinks to zero size with the surface A remaining fixed. Also consider that wavelengths emitted by the flame are long compared with a maximum dimension of the volume V which is surrounded by A (this is known as a compactness assumption). In this limit of zero flame size nearly complete mixing of the post-flame gases has occurred with the ambient air so that the speed of sound outside of A is nearly constant and the flow speed is nearly zero. Consequently, the simple wave equation holds outside of A, the Fourier transform of which is the Helmholtz equation given by

$$\nabla^2 p_\omega + k^2 p_\omega = 0$$

The solution to this equation, subject to the condition of only outgoing radiation of sound (free field), is<sup>12</sup>

$$p_\omega = \int_A \left( p_\omega \frac{\partial g_\omega}{\partial n_o} - g_\omega \frac{\partial p_\omega}{\partial n_o} \right) dA(\underline{r}_o) \quad (12)$$

with

$$g_\omega = \frac{e^{-ikR}}{4\pi R} \quad R = |\underline{r}_o - \underline{r}| \quad (13)$$

In the above  $\underline{r}_o$  is the source position,  $\underline{r}$  is the observation point, R is the distance between the two and  $g_\omega$  is termed the free space Green's function. Without working out the details, it may be shown that if observation is made in the far field ( $R \gg |\underline{r}|$ ) the first term of Eq. (12) is negligible compared to the second. Moreover, if the surface A is compact so that  $k|\underline{r}_o| \ll 1$ ,  $g_\omega$  may be brought outside of the integral sign and Eq. (12) simplifies to

$$p_{\omega} = \frac{e^{-ik|\underline{r}|}}{4\pi|\underline{r}|} \int_A \frac{\partial p_{\omega}}{\partial n_o} dA \quad (14)$$

The unsteady momentum equation in the absence of a mean flow is  $\bar{\rho} \partial \underline{v}' / \partial t = -\nabla p'$ . Upon taking the Fourier transform and using the component in the  $\underline{n}_o$  direction Eq. (14) becomes

$$p_{\omega} = \frac{e^{-ik|\underline{r}|}}{4\pi|\underline{r}|} \bar{\rho} \int_A \left( \frac{\partial v'_{n_o}}{\partial t} \right)_{\omega} dA \quad (15)$$

Therefore, the flame induces noise by causing a fluctuation in the outward normal component of velocity on A.

Although not too obvious, Eq. (16) is actually combustion noise, as will be seen. Notice that  $p_{\omega}$  only depends upon  $|\underline{r}|$  so that it is spherically symmetric, monopole radiation. This has been amply demonstrated experimentally. Equation (15) may also be shown to reproduce exactly Bragg's result of Eq. (1). For note that the integral in Eq. (15) is precisely  $(d^2v/dt^2)_{\omega}$ . Multiplying  $p_{\omega}$  by  $p_{\omega}^*$  the exponential terms of Eq. (15) disappear. Then using Eqs. (15) and (9)

$$\overline{p'^2} = \frac{\bar{\rho}^2}{(4\pi|\underline{r}|)^2} \left( \overline{\frac{d^2v}{dt^2}} \right)^2 \quad (16)$$

Although the details will not be covered here, the suggestions of Section 3 hold here with regard to acoustic power, and, in fact,

$$P = \overline{p'^2} \frac{4\pi|\underline{r}|^2}{\bar{\rho} \bar{c}} \quad (17)$$

from which Eq. (1) directly follows.

Summarizing, in this simple flame limit which also holds for diffusion flames, combustion noise is a simple monopole noise field caused by dilatation of the field enclosing the flame.

### Comparison of Theory and Experiment

The next step is to see whether or not Eqs. (15) - (17) can recover the experimental results of Eqs. (10) and (11). Equations (15) - (17) may be written as

$$\eta_{ta} = \frac{P}{\dot{m}_f H} = \frac{\bar{p}}{4\pi\bar{c}} \left( \int \frac{\partial v'_{n_0}}{\partial t} dA \right)^2 \frac{1}{\dot{m}_f H} \quad (18)$$

and the primary difficulty is in evaluation of the area integral in Eq. (18). The time derivative inside the integral sign emphasises the dominant frequency of combustion noise and it is assumed  $\frac{\partial}{\partial t} \propto f_p$ . The fundamental assumption is made that  $v'_{n_0} \propto S_T T_f / \bar{T}$ . This would be the velocity downstream of an isobaric adiabatic one-dimensional flame propagating at the turbulent flame speed. This assumption is equivalent to saying that the impetus for causing fluctuations is proportional to the impetus for downstream velocity (the heat release) in the steady state. This is an argument which is motivated by the experimental absence of an Re effect on sound and the knowledge that the incoming turbulence was fully developed pipe flow, whose turbulence properties are independent of Re. Therefore, only large scale properties of the turbulence are assumed to be involved in the noise generation process. A is taken as proportional to the observed flame area<sup>14,15,10</sup> and the turbulent flame speed is measured. For the moment, the observed frequency law will be used, Eq. (11), and the necessary relations are

$$\eta_{ta} \propto \frac{\bar{p}}{\bar{c}} A^2 St^2 \left(\frac{U}{D}\right)^2 \left(S_T \frac{T_f}{\bar{T}}\right)^2 \frac{1}{\dot{m}_f H}$$

$$c_p(T_f - \bar{T}) \approx c_p T_f = FH$$

$$S_T \propto U^{0.24} S_L F^0 D^{0.5}$$

$$A \propto U D^2 / S_T \propto U^{0.76} D^{1.5} S_L^{-1} F^0$$

$$\dot{m}_f \propto M^{2.2} Re^{0.04} Da^{0.78} F^{-1.2} \left(\frac{H}{c_p \bar{T}}\right)$$

The result is

$$\eta_{ta} \propto M^{2.2} Re^{0.04} Da^{0.78} F^{-1.2} \left(\frac{H}{c_p \bar{T}}\right) \quad (19)$$

which recovers Eq. (10) exceedingly closely and, in addition, provides a prediction concerning the effect of the fuel heating value. Because of the introduction of observed flame shape and turbulent flame speed, Eq. (19) is not valid for diffusion flames whereas Eq. (18) is valid for such flames. A different line of argument would be necessary to develop scaling laws for diffusion flames from Eq. (18). The major point is that theory and experiment appear to be consistent for premixed turbulent flames.

A better a priori prediction could be made if a theoretical estimate for frequency were available, rather than the empirical one used to generate Eq. (19). This was attempted in Ref. 10, using the argument that the large scale eddy characteristic times,  $l_e/u'$ , were responsible for the frequency content of the noise. The inability to predict the effect of  $F$  in Eq. (11) is a drawback to the theory, however. Moreover, the strong effect of  $Da$  in Eq. (11) leads one to believe that the chemical rates play a role in the frequency content. Consequently, at this time, the writer believes that there is no adequate theory for the frequency content and experimental results provide the only guidance for predictive purposes.

The connection between Eq. (15) and combustion fluctuations is one of the connection between flow dilatation and heat release. For the limit considered here, that the flame is essentially a point source, it is known from theoretical acoustics that  $\lim v'/p' \rightarrow \infty$  as the origin is approached. As will be shown later in detail, the energy equation becomes

$$\gamma \bar{p} \nabla \cdot v' = (\gamma - 1) \dot{Q}'$$

in this limit. Integrating over the reacting volume and applying the divergence theorem shows that

$$\int_A v'_{n_o} dA = \frac{\gamma-1}{\gamma \bar{p}} \int_V \dot{Q}' dV \quad (20)$$

in the limit that the origin is approached. This shows that a volume integration of heat release fluctuations is synonymous with dilatation fluctuations and that Eq. (15) indeed represents combustion noise.

### The Existence of Combustion Noise

So far only indirect proof has been given that turbulent combustion indeed generates noise. It would be nice to show that actual events inside the reacting region are responsible for the noise generation. With this view in mind Price et. al.<sup>17</sup> used an optical technique to relate combustion processes to noise generation. The method, based on a proven link<sup>18</sup> between the rate of combustion and the emission intensity of radicals like C<sub>2</sub> or CH, was applied to ethylene/air premixed flames, hydrogen/methane diffusion flames and acetone/air spray type diffusion flames. the method presumes a relation exists of the form

$$\frac{\partial \dot{Q}}{\partial t} = \alpha \frac{\partial I}{\partial t}$$

at any point in the flame. Then viewing the entire flame volume and using Eqs. (15) and (20)

$$p_{\omega} = \frac{e^{ik|\underline{r}|}}{4\pi|\underline{r}|} \frac{1}{\rho} \frac{\alpha(\gamma-1)}{\gamma\bar{p}} \int_V \left( \frac{\partial I}{\partial t} \right)_{\omega} dt \quad (21)$$

which may be inverted to yield

$$P'(|\underline{r}|, t) \propto \left. \frac{d\dot{Q}}{dt} \right]_{t-\bar{r}}$$

where  $\dot{Q}$  is the total emission intensity. Therefore, if one compares a time-delayed, time-differentiated emission trace with a far field pressure fluctuation trace, there should be an overlap of the two. Also, the analytical form of the pressure-emission relation suggests it is highly suitable for cross-correlation coefficient analysis (see Section 3). Direct comparisons of these traces and their spectra have been made in Refs. 18 and 19. Cross-correlation information has been investigated in Refs. 19 and 20.

A typical set of traces and cross-correlations are taken from Ref. 19 and shown as Figs. 4 and 5. These results, taken from C<sub>2</sub> radiation (5165 Å), are typical of those in Refs. 17 and 19-20. Differentiation of the emission in-



tensity in time tends to introduce high frequency noise; consequently, there is usually some upper frequency limit beyond which filtration of the emission signal is required. The filtration range is shown on Fig. 4 and 5. First viewing Fig. 4, the direct traces are compared. In some cases good visual correspondence is obtained and in some cases it is difficult to see a similarity between traces. For this reason the cross-correlation coefficient is valuable. In Fig. 5 the existence of the cross correlation peak at the time delay required for the acoustic signal to reach the microphone is quite impressive. Unfortunately the values of the peak cross correlation coefficients were not reported in Ref. 19. In Ref. 20 point measurements of the emission intensity were carried out, and the result was a high correlation between the noise and light emission in the downstream portion of the flame where most of the noise is known to be generated.<sup>7,9</sup> These experiments prove combustion noise exists - it is noise generated in and radiated from a region undergoing turbulent combustion. Moreover, these experiments have a direct bearing on a theoretical controversy in the field which will be returned to later.

#### Generalization From Simple Flame Results

The use of Eq. (18) for arbitrary flames is not in general possible. It was generated for compact flames radiating to a free field. The effects of enclosures or large flames will destroy the validity of Eq. (18). In going from Eq. (18) to Eq. (19) more difficulties arise in different situations. In each flame configuration one needs to know geometric scaling laws for A and the details of the fluid mechanics - combustion interaction which makes up the frequency content and the velocity (or heat release) fluctuation magnitude. It is no surprise, therefore, that in a perusal of many laboratory flame noise results a hodgepodge of scaling laws is apparent.<sup>7-12,21-27</sup> Only when all of the parameters in Eq. (18) can be estimated does a quantitative comparison with theory emerge; another example where theory and experiment have been compared is in Ref. 27. Nevertheless Eqs. (18) and (20) do give some guides. For example, clearly the larger the physical size of the burner, the larger should be  $\eta_{ta}$  (through A). By the connection between heat release and  $v'_n$ , one would expect  $\eta_{ta}$  to be larger the higher the heat release. If  $v'_n$  is connected to the mean

velocity and the mean velocity also plays a role in the frequency content, one can expect  $\eta_{ta}$  to be a quite strong increasing function of flow velocity. These statements usually turn out to be correct, but quantitative principles are lacking.

## 5. DUCTING EFFECTS ON COMBUSTION NOISE

When an acoustic source is placed within an enclosure its sound radiation characteristics change as compared with radiation to a free field. For example, if a monopole (contracting - expanding sphere) oscillating at frequency  $\omega$  radiates to free space, the acoustic power is proportional to  $\omega^2$  for a fixed velocity amplitude. However, if this monopole were placed in a infinite circular duct the acoustic power would remain finite as  $\omega \rightarrow 0$ . The difference is the relief effect of three dimensional space, or, equivalently, the effect of wall reflections on the source. This phenomenon is called a change in the radiation impedance of the source.

Other phenomena caused by enclosures are the excitation of resonant modes and possible feedback between the reflected waves and the source. The latter problem is especially acute in the case of combustion noise because of the well known feedback problem of combustion instability.<sup>29</sup>

There have been two works specifically addressing the effects of enclosures or ducting upon the combustion noise problem,<sup>30,31</sup> although the effects of enclosures have been clearly recognized for some time.<sup>32</sup> In Ref. 30 a theory was constructed to show that heat release fluctuations can indeed drive resonances within a duct even without feedback. Experiments with a can type combustor terminated by a duct verified that a) such interior resonances occur and b) they are transmitted to the far field, out the duct tailpipe. No comparison between theory and experiment was attempted because the details of the heat release fluctuation magnitude and spectrum were unknown. Moreover, it was recognized that feedback effects between reflected pressure waves and the combustion process might be important but could not be quantified.

An attempt to compare theory and experiment was given in Ref. 31. Here, a burner from the work of Ref. 9 was placed in a cylindrical tube as shown in Fig. 6. The rather strange looking ends of the tube are "mufflers" to catch all incident waves and allow no longitudinal reflections. The idea here is that an infinite tube is the simplest enclosure from the standpoint of theory and allows the fewest reflections to penetrate the flame zone (assuming the mufflers work properly) because longitudinal reflections are suppressed. The

theory presumed that the flame was a point monopole source, which is precisely that used in Sect. 4 for the simple free flame, but that it was enclosed by a tube. This acoustic problem is solved in Ref. 33. The source characteristics are deduced by free field experiments, and, assuming no change when placed in the tube, the duct radiation characteristics should be calculable. Of course, frequencies were restricted to those below the first transverse mode of the duct gases in order to keep the theory simple. In such a case, only plane waves with no resonances are expected.

The results are shown in Fig. 7. Shown are the free field spectrum, from which Eqs. (15) and (5) determine the source spectrum, and the measured spectrum in the tube, which should be calculable knowing the source spectrum. The spectrum in the tube shows the proper behavior, with a rotation of the spectrum according to a multiplication of  $1/\omega^2$ . Some imperfect behavior of the mufflers at low frequency is seen in the appearance of the first two longitudinal modes. The OASPL prediction in these experiments was  $\pm 2$  dB within the experimental values. The conclusions were that for highly damped configurations (few wall reflections) a) the feedback effect was negligible and b) if the free field source characteristics of the flame were known, the ducted characteristics of the noise could be predicted. Of course, it has been known for some time that if the damping far outweighs the feedback the feedback should be unimportant for stability considerations.<sup>29</sup> However, it was not certain just how well the flame-in-duct performance could be predicted from a free field measurement.

These results demonstrate that if one is well removed from a stability problem the feedback problem may not influence the noise to a great degree. However, it is not usually possible to obtain necessary data for an in-duct noise prediction from free field data because many combustors have the flame structure and duct configuration interdependent. Moreover, many combustors operate at elevated pressure and it is difficult to obtain elevated pressure free field data. In recent years the duct combustion noise problem has been treated from the outset as a combined problem, rather than trying to predict from model free field data, for the reasons suggested.

For ducted systems the major problem is that the entire duct acoustics problem must be solved along with the flame structure problem in order to

predict noise output from the device. In real burners, the ducts are often complex in shape and construction so that their acoustical behavior is not well known. As an example, to calculate from first principles the direct combustion noise generated by a gas turbine unit with a can type combustor one would have to know a) the complete turbulence structure of the flame and b) the combined acoustical behavior of the can liner, turbine and nozzle assembly. At the current time such a treatment is not possible, and other approaches are in use, as seen below.

## 6. DIRECT COMBUSTION NOISE IN VARIOUS SYSTEMS

### Turbopropulsion and Gas Turbine Systems

During the past decade there has been rather intense research on the question of "core engine", "core" or "excess" noise from gas turbine units for stationary and aircraft power plants. This is noise which emanates from the core (working part of the airflow which passes through the turbine) of the engine. All noise due to the fan on a turbofan engine or the jet exhaust is excluded; moreover, turbine noise is also excluded from the definition for reasons to be detailed later. It is now well documented that jet exhaust noise sound power scales as  $U^8$  at subsonic speeds, the physics being first given by Lighthill.<sup>34</sup> At sufficiently low exhaust speeds, however, actual gas turbine units break away from the  $U^8$  law, more towards a  $U^3$  or  $U^4$  law. This fact seems to have first been documented by Mawardi and Dyer.<sup>35</sup> Low exhaust velocity is produced by a) using a modern high bypass ratio turbofan for aircraft application b) taking out high shaft power in the case of gas turbine powerplants or c) cutting back power in aircraft applications.

That core noise may be due to mass flow fluctuations coming out the jet exhaust was first suggested in Ref. 36. Quantitative estimates of this effect, using "reasonable" numbers from jet engine experience, were given in Ref. 37. Indeed, it appears that exit plane mass flow fluctuations are strong enough to account for the exhaust speed effect on noise. Such mass flow fluctuations can arise from highly correlated intense turbulence at the exit plane or acoustical (dilatation) waves produced upstream of the exit by the combustion process, for example.

The importance of core noise, relative to other turbopropulsion or gas turbine noise sources, is a subject of controversy.<sup>38-40</sup> In aircraft applications it is clear, however, that it currently is a noise floor which must be breached for further extensive noise reduction. In stationary gas turbines, core noise is a major noise source.<sup>41</sup>

That the turbulent combustion process may be the root cause of core noise has been suggested by many workers.<sup>37-49</sup> Whether it is direct or indirect

combustion noise is still an open issue, however. For now, it will be assumed that direct combustion noise controls core noise, and the question of indirect noise will be returned to later. The most extensive investigation, based upon the hypothesis that direct combustion noise controls core noise is given by Mathews et. al.<sup>40</sup>

Kushida and Rupe<sup>43</sup> appear to be the first investigators to show a relation between the pressure fluctuation level in a rocket motor combustion chamber and a far field noise level. Abdelhamid et. al.<sup>44</sup> have shown a strong cross correlation of far field noise with interior chamber pressure fluctuations in a can type combustor terminated by a high speed nozzle. In actual engine firings Mathews et. al.<sup>40</sup> have shown a strong cross correlation between combustor fluctuating pressure and far field noise. A typical plot of the cross correlation coefficient from Ref. 40 is shown in Fig. 8. The time delay in this figure at the cross correlation peak corresponds to a wave travel time from the combustor to the far field location. In the frequency range of 0-250 Hz Karchmer and Reshotko<sup>50</sup> have found high coherence between combustion and far field fluctuating pressures. They also showed a strong cross-correlation, but the result has a subtle difference as compared with the work of Ref. 40. In Ref. 50 the peak cross correlation is negative whereas Fig. 8 shows a positive peak. This will be discussed later. Again, these results do not prove that direct combustion noise is the culprit in core engine noise, but Ref. 40 is able to correlate Pratt and Whitney core noise results on the basis of a direct combustion noise theory motivated by Ref. 42. Unfortunately, the correlation does not hold true for engines of another manufacturer.

Several groups have obtained combustor rig data, in which a combustor is fired at essentially atmospheric pressure and exhausted directly to the atmosphere.<sup>40, 51-53</sup> In such configurations, when care is taken to keep the exhaust noise low, virtually all the noise propagated outside is direct combustion noise. Indirect noise cannot be produced because there is no significant pressure gradient imposed on the exhaust. All flow noise have been proven to be significantly lower than the combustion-on noise and the noise has all of the general characteristics of free flame noise with one exception - the

modifying effect of the duct. A typical spectrum for a combustor can test using JP-4 fuel is shown in Fig. 9. The peak at 400 Hz is the quarter wave resonance, whereas the basic broadband spectrum peaks around 200 Hz, in this configuration. Coherence between an internal microphone and external microphone is typically as shown in Fig. 10. The low coherence at very low frequencies is attributed to non-propogational hydrodynamic (scrubbing noise at the internal microphone location) noise contaminating the interior microphone signal. An upper frequency limit of high coherence also exists for reasons explained in Ref. 53. However, there is a wide range of high coherence, indicating a linear, causal relation between interior and exterior acoustic events. This is to be theoretically expected.<sup>53</sup>

Some interesting generalizations may be made from the aggregate of combustor rig testing. These are a) the ducting effect raises the thermoacoustic efficiency about a factor of 10 above the usual free flame value of about  $10^{-6}$  and introduces resonances, the strength of which depends upon the individual combustor design, b) the frequency content is low, say  $f_p = 200 - 500\text{Hz}$ , depending upon the combustor, and is notoriously insensitive to flow and design variables or fuel type. This latter observation is one of the most puzzling to workers in the field. It will be recalled that in the case of the free flame the frequency content was rather insensitive to flow and fuel variables but could be interpreted as a combination of turbulence and reaction times. However, in Refs. 51 and 54 changes in a) fuels with different reactivities, b) flow rate and fuel/air ratio and c) can liner hole size have virtually no effect on the frequency content. In Ref. 40, using different overall combustor designs,  $f_p \propto v_b^{-1}$  was obtained. This was interpreted as an effect of reaction time; such an interpretation presumes that the smaller the burner, to achieve complete reaction, the faster must have been the reaction time (at fixed throughput rate). Since the fuel was the same in all tests, however, it is really a statement on mixing time, rather than reaction time.

After some rewriting and approximations the correlation for thermoacoustic efficiency obtained in Ref. 40 may be expressed as

$$\eta_{ta} \propto M^3 \left( \frac{F_{fl}}{c_p T} \right) \quad (22)$$



A comparison of Eq. (22) with other empirical and semi-theoretical approaches<sup>51,52,47</sup> reveals differences in detail, but the general result that the efficiency rises with flow speed (through  $M$ ) and temperature rise across the combustor (the second parameter of Eq. (22)) is now generally established. Independent checks on the effect of the heat of combustion<sup>51,55</sup> have also shown the general trend of Eq. (22). Equation (22) should also be compared with Eq. (19) to note the similarities.

In moving from combustor rig tests to installed configurations the measurements must distinguish between combustor-related noise and all other noise sources. This is done by a variety of coherence, cross correlation, and fan and jet noise suppression techniques. Moreover, as indicated above, combustion noise is recognized as low frequency noise, whereas turbine noise is generally high frequency noise.<sup>56</sup> Consequently, turbine noise is easily separable from combustion noise, and, in fact, is usually carried along with its own name and not included in the term "core noise". The existence of the turbine and nozzle assemblies is expected to drop the overall sound power over that which would be attained in a combustor rig test. In fact, it appears to drop it by about a factor of ten.<sup>40</sup> Consequently, the effects of ducting and then terminating by complex hardware are compensating and the net result is still a thermoacoustic efficiency of the order of  $10^{-6}$ , as predicted in Ref. 3. There are great variances to this number, however, depending on design and operating variables. One particular design variable which has a great influence is the end-use of the gas turbine unit. As might be expected, the greater the turbine work output, the less the combustion noise<sup>47</sup>, since the turbine becomes a greater barrier to noise propagation.

Many of the above comments and cited results have been related to gas turbines for aircraft use. Most of the combustion noise research has taken place using such units. However, in large stationary gas turbine power plants the same principles apply. A problem unique to large units is the availability of resonances at extremely low frequency, because of the large gas path lengths. Resonances at 10's of Hz are not uncommon, which border on infrasound. At least these are generally out of the audible range, but they may cause serious structural vibration problems.

Other results on ducted combustors and further discussion of some points raised above is delayed until the section on indirect combustion noise.

### Industrial Furnaces and Process Heaters

Much of the literature in industrial furnace combustion noise is due to Putnam<sup>22,23,26,32,57,58</sup>, who has given the term "combustion roar" to direct combustion noise. The fundamentals of the problem are no different in industrial systems than in aircraft gas turbine units, but some of the parameters at the control of the designer or noise control engineer are different. One nice thing about ground based equipment is the possibility of the use of sound suppression devices. Whereas in volume and weight limited aircraft systems the use of silencing techniques carries a large penalty, this is often not a severe constraint in ground based equipment. In industrial burners using standard hydrocarbon fuels the frequency content of combustion roar is no different than described above - it is low frequency broad band noise, typically shown in Fig. 2. The spectrum is, of course, modified by enclosure effects, as discussed above. The complications that enclosures give in industrial burners are discussed in Ref. 32.

Design variables include burner fuel pressure, number of burner units/firebox, firebox size, heat release per burner or fuel/air ratio, turbulence level, and perhaps, fuel quality. In discussing potential noise reduction techniques Putman uses a variant of Bragg's theory for interpretation.<sup>57</sup> One of the consequences of this theory is that

$$\eta_{ta} \propto \left(\frac{u'}{U}\right)^2 M^2$$

where the turbulence intensity explicitly appears. The law that  $\eta_{ta}$  is proportional to the square of the heat release rate (through the throughput in the velocity in  $M$ ) seems to have been reasonably well established.<sup>58</sup> A caution here, however, which applies to all other correlations above, is that this kind of correlation can not hold over too large a range of throughput. If it did, the thermoacoustic efficiency could be made greater than unity at sufficiently large throughput. Such correlations are based upon flow speeds which are in the low subsonic range, and this is probably the restriction on their use.

Putnam et. al.<sup>59</sup> have coined the term "superturbulent" combustion noise, which is a dramatic noise increase at operating conditions near a blowoff limit. It is accompanied by erratic flame behavior and/or a change in flame configuration. It is not a desirable operating condition and generally limits the design firing rate.

Industrial units operate with either pure diffusion flames or premixing just before burning (to prevent long flashback paths) and are either gas or oil fired. There is little difference in the fundamentals or characteristics of the noise with any of these different types of burners. Combustion noise in industrial furnaces is a gas phase combustion-turbulence interaction, as it is in laboratory premixed flames or aircraft gas turbine spray diffusion flames. Although scaling rules can be successfully generated for sound power, on the basis of rather simple theory, the scaling rules for frequency still defy interpretation. The frequency scaling rule generated in Ref. 57 is not consistent with the results of Eq. (11), and it appears that until a detailed turbulent combustion model becomes available the frequency content of combustion noise will remain mysterious.

Finally, indirect combustion noise does not generally arise in industrial applications since a high velocity stream of heated gas is not desired. There is no strong pressure gradient imposed on the combustor exhaust gases.

#### Flaring Operations

Although there appears to be a difference of opinion concerning the importance of combustion noise in hydrocarbon flaring operations, relative to other noise sources,<sup>60,61</sup> combustion noise does exist. Flaring is a disposal process of waste gases from refineries, fields and hydrocarbon plants where the gases are burned in open air. From the noise standpoint there is nothing basically different in flames of this type than in those discussed above. To drop combustion noise in these burners one has an option of lowering the turbulence level. Such a fix does not work well in systems where high volumetric heat release rates are desired. For flaring, such a constraint is not imposed. As in industrial burners, indirect noise does not arise in flaring.

## Diesel Engines

It has recently been found that combustion roar may play a role in the noise radiated from Diesel engines<sup>62,63</sup>. It has known for some time that the form of the pressure-time (p-t) diagram in the cylinder is important to the noise problem;<sup>64</sup> however, that the turbulence-combustion interaction may be important is a new revelation. The Diesel engine is a different type of combustor, as compared with those discussed above. It operates as a nearly constant volume, intermittent device, rather than a steady flow, nearly constant pressure device. Nevertheless, the gases are in turbulent motion during combustion, and from cycle to cycle it is impossible to reproduce in space and in time (crank angle) the exact same event. That is, superimposed upon a mean, periodic p-t trace there will be random fluctuations due to the turbulence.

This is illustrated by a spectral analysis of p-t traces on Fig. 11. The lower spectrum is the spectrum of the p-t trace resulting from an average of 100 p-t traces. That is, at the same crank angle 100 cycles are averaged to obtain the average pressure at that crank angle. This operation smooths out the random fluctuations and produces the mean, periodic p-t trace. The upper spectrum is the average spectral density of each of the 100 firings. That is, the spectral density of each firing is calculated and the results are averaged. This latter spectral density includes the effects of randomness as well as the mean trace. The difference between the two spectra is the spectrum of the random part of the p-t trace and is the combustion noise. In this particular case the noise dominates the spectral density of the pressure at frequencies above 1500 Hz.

The connection between this result and radiated noise is shown in Fig. 12 where the coherence between the cylinder pressure and exterior noise-measuring microphone is shown. Also shown is the spectrum of the noise. It is seen that a) the noise is highly coherent with the cylinder pressure and b) the cylinder pressure is dominated by randomness in a large frequency range where the noise occurs. Consequently, the noise is caused by the combustion randomness, to a large degree. For this run about one-half the total OASPL is due to combustion roar.

If one restricts consideration to frequencies below the first transverse wave mode frequency, a theory of the random fluctuation in cylinder pressure yields<sup>62</sup>

$$p_{\omega} = \frac{(\gamma-1) i}{\omega} \frac{1}{V} \int_V \dot{Q}_{\omega} dV \quad (23)$$

where  $V$  is now time variable because of the piston motion. There is an interesting difference between Eq. (23) and Eq. (15). In the former the  $1/\omega$  factor is replaced by a factor of  $\omega$  to essentially recover the behavior of Eq. (15). Notice that time differentiation in the time domain is like frequency multiplication in the frequency domain. In a nearly constant volume system the time derivative of the pressure is proportional to the heat release, whereas in a nearly constant pressure situation the fluctuating pressure is determined by a time derivative of the heat release. Therefore, if heat release fluctuation spectra are similar in the two systems the spectrum of  $p'$  will fall off with frequency much faster than in most steady flow combustors. This is indeed the case, as may be seen in comparison of Figs. 2 and 11.

In any event, Eq. (23) has been used to estimate  $p'$ , using what is known about  $\dot{Q}'$  from steady flow combustor data.<sup>62</sup> The numbers check in order of magnitude, lending credence to the interpretation of Fig. 11 as combustion noise. Further documentation of this interpretation is given in Ref. 63.

### Rocket Engines

Because of the intense noise generated by a supersonic exhaust jet, combustion noise generated inside a rocket motor chamber is not an audible noise problem; it is masked by the jet noise. However, the pressure fluctuations generated inside the motor cavity do give rise to motor vibrations which may trouble a guidance system, in some instances.

In liquid propellant rockets the mechanism of noise generation is probably close to that already discussed in steady flow combustors. If turbulence of the flow penetrates the flame zone of a homogeneous solid propellant rocket, the mechanism is probably also the same here. In a heterogeneous (composite) solid propellant rocket, however, there is a mechanism for noise production purely

due to the mass flow fluctuations provided by the heterogeneity. From the schematic of Fig. 13 one can see that as the propellant regresses an unsteady velocity field will be seen by a fixed observer. This has an associated unsteady pressure field and will give rise to acoustic waves in the motor cavity. At the current time, however, the strength of this source has not been found sufficient<sup>65</sup> to produce pressure fluctuation levels that are commonly observed (1-2% of the mean pressure).

Rocket engines are somewhat unique in that they often operate very close to a limit of combustion instability. Consequently, there can be expected a strong interaction between the forced oscillation (noise generation process) and the feedback mechanism. Investigations are currently underway in this area of rocket motor vibration. It must also be borne in mind that indirect noise may be a strong contributor here.

## 7. INDIRECT COMBUSTION NOISE

In a turbulent combustor, such as in a gas turbine, which burns by a diffusion flame, each parcel of fluid does not burn at the same mixture ratio. Moreover, a large distance down stream of the flame zone would be required for the turbulent and molecular mixing process to smooth out the flow and make a flow with uniform thermodynamic properties. The major consequence of combustion at spatially and temporally varying mixture ratio is the creation of hot spots (or cold spots) in the flow which persist to the next engine component - the turbine in the case of the gas turbine engine. In turbopropulsion systems, at least, the turbine nozzle guide vanes operate at a choked or nearly choked condition and therefore impose a strong pressure gradient on the entering flow. A quick look at the inviscid momentum equation  $Dy/Dt = - \nabla p/\rho$  shows that hot spots cause a problem. For, if an element having a different  $\rho$  from a neighboring element transverses the same pressure gradient, its acceleration will differ from that of the neighboring element, and, by continuity, a steady flow is impossible. What happens is that an unsteady acoustic field is set up to accommodate this situation, and this is called indirect combustion noise.

Indirect noise is also called entropy noise, because a hot spot has an entropy different from that of its surroundings. In this regard, molecular weight differences cause precisely the same effect on  $\rho$  in the momentum equation. Also, specific heat variations can also be shown to cause a similar but not exactly analogous effect.<sup>66</sup> In any event, molecular weight and specific heat variations are usually small compared to the temperature effect because of the preponderance of  $N_2$  in a gas turbine flow. Consequently, virtually all work in this field concentrates upon the temperature variations in a field of otherwise homogeneous composition.

The order of magnitude of the effect under consideration is easily shown by the impedance relation for a choked nozzle.<sup>67</sup> If a choked nozzle terminates a constant area duct, the condition at the nozzle entrance plane for plane acoustic waves is

$$\frac{u'}{c} + a \frac{p'}{p} + b \sigma' = 0 \quad (24)$$

with  $\sigma'$  as the dimensionless entropy fluctuation. For perfect gases,  $\sigma' = p'/\gamma\bar{p} - \rho'/\bar{\rho}$ ;  $\sigma' = -T/\bar{T}$  in the case of pure incident entropy waves, since  $\sigma' = 0$  for isentropic acoustic waves. For short nozzles (wavelength long compared with the nozzle length) the condition of constant entrance plane Mach number must prevail. In such a case

$$a = - \frac{M_e}{2} \frac{\gamma-1}{\gamma} \quad b = + \frac{M_e}{2}$$

Consequently, if there is a pure entropy wave incident on the nozzle entrance a pressure fluctuation of the same order of magnitude as  $\sigma'$  is set up. This pressure wave consists of reflected and transmitted components. For plane waves incident upon any nozzle a relation of the form of Eq. (24) also holds, but the a and b numbers, in general, depend upon the frequency content of the waves and the physical form of the universe downstream of the nozzle. Only in the case of a short, choked nozzle are the numbers a and b easy to compute.

The general problem of indirect noise requires that the impedance condition at the combustor termination be known and that the transmission properties of the termination be known. Incident entropy waves cause both reflection and transmission; but the reflected waves come back to the termination by subsequent reflection from the head end of the combustor. Consequently, the problem becomes a bit complicated; there is a direct generation of transmitted waves by the incident entropy waves but there is a buildup of isentropic sound in the combustor which has its own propagational characteristics through the combustor termination. A more complete picture than presented heretofore of combustion noise is this: Aggregate heat release fluctuations produce an isentropic sound buildup in a combustor, the transmission of which depends upon the impedance of the combustor termination; hot spots, convected toward the termination at the fluid speed, contribute to the isentropic sound buildup and transmission, but, in addition, create transmitted sound during their passage through the termination.

Analytical studies of sound generation by entropy spots are reported by Marble,<sup>68</sup> Candel<sup>69</sup> and Ffowcs Williams and Howe.<sup>70</sup> Experimental verification of Refs. 68 and 69 is reported in Ref. 71. Further theoretical development and



application to jet engines is given by Pickett<sup>72</sup> and Cumpsty.<sup>73</sup> In both Refs. 72 and 73 core noise of particular engines is explained in terms of indirect noise. However, there was apparently a numerical error in Ref. 72, which, when corrected and applied in the work of Ref. 40, results in a conclusion reversal - direct combustion noise dominates indirect combustion noise by about a factor of 10 in overall sound power.

The controversy concerning the relative dominance of direct or indirect noise has been recently given some fuel by the results of Ref. 54. In Ref. 54 theory and experiments are reported for a can type combustor terminated by a converging nozzle. Some results are shown in Figs. 14 and 15. Shown in Fig. 14 are exterior radiated noise spectra as mass flow is changed at fixed fuel/air ratio and contraction ratio, and as contraction ratio is changed at fixed mass flow. There are clear spectral changes involved which follow theoretical predictions<sup>54</sup> and which are caused by a takeover of entropy noise from direct noise as the pressure gradient imposed on the exhaust gases is increased. Moreover, the coherence between the effective one dimensional  $\sigma'$  and the radiated sound is shown to take over from the coherence between the interior microphone and radiated sound in Fig. 15, showing that at large exhaust pressure gradient the hot spots dominate the noise generation process. Consequently, the conclusions between Refs. 40 and 54 are at odds with each other. Moreover, the results of Ref. 73 must be considered.

There is another complication which enters the picture. As pointed out in Ref. 72 and hinted at by the experimental results of Ref. 54 another noise source may be clouding the picture. Reference to Eq. (24) shows that axial velocity fluctuations entering the nozzle will cause a pressure reaction; such velocity fluctuations are present, independently of combustion, in the turbulence. Work in this direction is required.

One reason for the apparent takeover of indirect from direct noise in the work of Ref. 54 may be given by the analysis of Ref. 70. In Ref. 70 it is shown that hot spots will radiate sound when passing through a nozzle exhausting to a free field as  $P \propto U^6$ , whereas when ducted  $P \propto U^4$ . The experiments of Ref. 54

used a nozzle exhausting to a free field, whereas the engine experiments of Ref. 40 were effectively a ducted configuration. On the other hand, the theory of Ref. 70 is restricted to low Mach number flows which are not the case of interest. Detailed theoretical calculations were not carried out in Ref. 54 to show the effective velocity scaling laws. Consequently, further work is required in the area of core noise to resolve the outstanding question of relative dominance of indirect and direct noise.

This issue clouds the results of all combustor work, when the combustor is terminated by a high speed nozzle,<sup>30,44,54,55</sup> and engine work.<sup>40,41,43,50</sup> It may contribute to the inconsistency, mentioned in Sect. 7, of the cross correlation results between Refs. 40 and 50. While Ref. 50 presents a theory to explain the result, the theory is strictly inapplicable to a ducted flow. Consequently, there may have been a difference in dominant noise source, which led to differences in the behavior of cross-correlation of interior and exterior pressures, as compared with the results of Ref. 40.

It is reiterated that the issue of indirect combustion noise only exists in systems which terminate the combustor after a short running length with an extreme pressure gradient. Consequently, jet and rocket engines are the only systems of primary interest in this regard. Large stationary gas turbines are probably free of the problem because of long gas paths (removal of hot spots by mixing) and the fact that the turbines in such systems operate away from a choking condition.

## 8. THEORY OF COMBUSTION NOISE

Historically, the theories of combustion noise follow in the sequence of Refs. 4, 21, 42, 74, 10, 30, 75, 76 and 53. Several different approaches have been taken in these theories and several different answers have been obtained. The fundamental reason for such divergence is that the mathematical formalism used, which belongs to the subject of aeroacoustics,<sup>76</sup> are all inexact. Approximations are necessary for analytical tractability. The two fundamental approaches which have been most widely tested against experiment are those of Refs. 42 and 74, and they will be looked at here for the simple case of a free flame radiating to a uniform free field.

Both theories are known as acoustic analogy theories,<sup>77</sup> one based upon Lighthill's equation and one on Phillips' equation.<sup>76</sup> A slightly improved form of the theory of Ref. 42 is described here. The starting point for both theories is in the statement of the conservation principles for an inviscid, non-heat conducting fluid.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial v_i} (\rho v_i) = 0 \quad (25)$$

$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} = \rho \frac{Dv_i}{Dt} \quad (26)$$

$$\frac{Ds}{Dt} = \frac{\dot{Q}}{\rho T} \quad (27)$$

The effect of combustion is felt through the heat source in Eq. (27). No molecular transport phenomena are included in Eqs. (25) - (27) since they are unimportant to the noise problem.<sup>34</sup> While the processes of turbulence generation, spectral transfer and decay demand consideration of molecular transport, these processes are slow on the time scale of the "period" of noise generation. It is the large scale motions of the turbulence, which operate relatively independent of molecular transport, which are responsible for noise.

It is assumed that the fluid is a perfect gas so that

$$p = \rho RT, \quad c^2 = \gamma p / \rho, \quad \frac{s}{c_p} = \frac{1}{\gamma} \ln p - \ln \rho \quad (28)$$

A more useful form of the energy equation is obtained through use of Eqs. (28) and (25); there results

$$\frac{D \ln p}{Dt} + \gamma \frac{\partial v_i}{\partial x_i} = (\gamma - 1) \frac{\dot{Q}}{p} \quad (29)$$

The approach of Ref. 42 is to construct a Lighthill equation by time differentiating Eq. (25), taking the divergence of Eq. (26), subtracting the two and subtracting  $\partial[\bar{c}^2 \partial \rho / \partial x_i] / \partial x_i$  from both sides of the resulting equation. The result is

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\partial}{\partial x_i} \left( \bar{c}^2 \frac{\partial \rho}{\partial x_i} \right) = - \frac{\partial^2}{\partial x_i \partial x_j} (\rho v_i v_j) + \frac{\partial}{\partial x_i} \left( - \frac{\partial p}{\partial x_i} - \bar{c}^2 \frac{\partial p}{\partial x_i} \right) \quad (30)$$

What one has accomplished with Eq. (30) is to create a wave operator for a stationary medium, but variable speed of sound, on the left hand side and an apparent inhomogeneity on the right hand side.

On the other hand, the approach of Ref. 74 is to construct a Phillips equation by using the variable  $\eta \equiv \ln p$ , taking the substantial time derivative of Eq. (29), taking the divergence of Eq. (26) and subtracting the two. The result is

$$\frac{D^2 \eta}{Dt^2} + \frac{\partial}{\partial x_i} \left( \bar{c}^2 \frac{\partial \eta}{\partial x_i} \right) = (\gamma - 1) \frac{D}{Dt} \left( \frac{\dot{Q}}{p} \right) + \gamma \frac{\partial v_j}{\partial x_i} \frac{\partial v_i}{\partial x_j} \quad (31)$$

This produces a convective, variable speed of sound wave operator on the left hand side with an apparent inhomogeneity on the right hand side.

Through arguments developed in Refs. 42 and 74 the last term on the right hand side of Eq. (30) and the first term on the right hand side of Eq. (31) are selected as the culprits in combustion noise generation. They are presumed known so that it is a question of solving an inhomogeneous wave equation. Both approaches are inexact and, indeed, incorrect. The difficulty is this: the velocity terms on the right hand sides of Eqs. (30) and (31) contain fluid

dilatation terms, but  $\dot{Q}$  and  $\partial v_i / \partial x_i$  are actually linked by Eq. (29); to throw away these terms is to throw out parts of the problem which may be important. A disadvantage to Eq. (31) as compared with Eq. (30) is that a linearization must be performed before proceeding, whereas Eq. (30) is linear to start with by construction (for the terms which are retained in the combustion noise analysis). Consequently, no smallness assumption on the allowable fluctuations of  $\rho$  are required in Eq. (30) as they are on  $\dot{Q}$  and  $p$  in Eq. (31). On the other hand, since Eq. (31) is a convected wave equation one has the gut feeling that it will produce better results in a flow situation. However, Phillips' wave operator is not even correct for a unidirectional transversely sheared mean flow acoustics problem.<sup>77</sup> Moreover, if one linearizes Eqs. (29) and (26) for small disturbances about a steady state and then takes the limit of a low Mach number flow<sup>53</sup> one recovers the result obtained by the process of using Eq. (30); this is not recovered by making a low Mach number approximation in the result obtained after solution of Eq. (31). Resort to experiment is required to validate either of the approaches.

Without going into the details, the solutions obtained by both approaches are given for the compact flame of Sect. 3 viewed from the far field

Eq. (30)

$$p' \propto \frac{1}{|r|} \int_V \frac{1}{\bar{c}^2} \left. \frac{\partial \dot{Q}}{\partial t} \right|_{\xi_0, t - \frac{|r|}{c_0}} dv(\xi_0) \quad (32)$$

Eq. (31)

$$p' \propto \frac{1}{|r|} \int_V \frac{1}{\bar{c}^2} \left[ \frac{\partial \dot{Q}}{\partial t} + \bar{v}_i \frac{\partial \dot{Q}}{\partial x_i} + \left( v_i' - \frac{T'}{T} \bar{v}_i \right) \frac{\partial \bar{Q}}{\partial x_i} \right] \left. dv(\xi_0) \right|_{\xi_0, t - \frac{|r|}{c_0}} \quad (33)$$

Equation (32) is an improved version of the result of Ref. 42, because Eq. (30) has used a variable speed of sound formulation, whereas the original work assumed  $\bar{c}$  as a constant. The solution procedure in this case has used the "parametrix" procedure used by Chiu and Summerfield.<sup>74</sup> By comparison with

Eq. (15), Eq. (32) does the best job of recovering the simple flame theory result, is simpler, conforms to the light emission results mentioned in Sect. 3, and has been tested against the most flame results. It is consequently believed that it comes from a superior aeroacoustics approach to the problem. All of the evidence is not yet in, however, since work is currently in progress to measure the effects of the additional terms of Eq. (33). Based upon the results of Ref. (52), however, it is at least confirmed that the first term of Eq. (33) is an extremely strong contributor to combustion noise.

Concerning indirect noise, Eq. (27) shows exactly what is involved. Following the fluid, the history of heat release determines the entropy at any downstream location. If this fluid then interacts with a boundary, such as a nozzle entrance plane, which is sensitive to entropy fluctuations, such as through Eq. (25), noise may be generated. This is different from an aggregate fluctuation in heat release rate, such as given in Eq. (32). Both types of noise have their origin in heat release fluctuations, and it is shown in Ref. 54 that there are some frequency regimes (low frequency) where both types of noise may be highly coherent with each other.

The ability to make a calculation of the sound output and spectra from equations like (27), (32) or (33) requires that the turbulence details be known. While certain physical suppositions have been successful in using these equations to develop scaling rules for sound power, absolute prediction and spectral information has not been possible. The sources of this difficulty are the lack of detailed turbulence theory for the combustion process and experimental information on the statistics of the process. In this regard, it may be instructive to point out that turbulence information may be obtained by the acoustic output, if the theoretical link between the two is believable. The acoustic measurement is non-intrusive to the combustion process, as are many diagnostics based upon a form of radiation, acoustic or electromagnetic. Work in this direction may prove fruitful.

## 9. CONCLUDING REMARKS

Combustion noise of the indirect or direct type can be an important noise source in various systems. The physics of the problem are well enough understood to predict the effects of various design variables on the noise output, but it is as yet impossible to make an a priori prediction of noise output and spectral content, because of the lack of knowledge of turbulence details in a turbulent flame. Combustion noise, when usual hydrocarbons are used, is low frequency noise, which is extremely difficult to silence by conventional absorbers. Consequently, it is desirable to be in a better theoretical position, than is currently available, to make absolute predictions of the noise output. This will require continued testing of the available theories, perhaps construction of new theories, continued monitoring of advances in turbulent flame theory and experiment, and, perhaps, the use of acoustics as a turbulence diagnostic.

Care has been taken in this review to not delve too heavily into organized combustion oscillations or feedback interaction between the flame and acoustic fields. This would require a review in itself. In some systems the noise problem may be strongly linked to a feedback problem, and continued research in this area is warranted.

Careful perusal of some of the correlations presented above may indicate what the future combustion noise problems may be. For example, the use of hydrogen as a fuel should increase the thermoacoustic efficiency and raise the frequency content (assuming the turbulence field is the same). On the other hand, with hydrogen it may be possible to use lower turbulence levels in the combustor, which would reduce the noise output. The use of lower grade fuels (in heating value or reactivity) would have the opposite effect. A rise in the frequency is good in the sense that sound absorption devices become more effective for a given size constraint. A lowering of the frequency can also be nice if it takes the noise sufficiently close to the audible limit; on the other hand, vibration problems are often encountered if the excitation frequency is low. Whatever the fuels of the future may be, it is unlikely that combustors will become laminar flow devices; consequently, the combustion-turbulence interaction which produces noise will still be present.

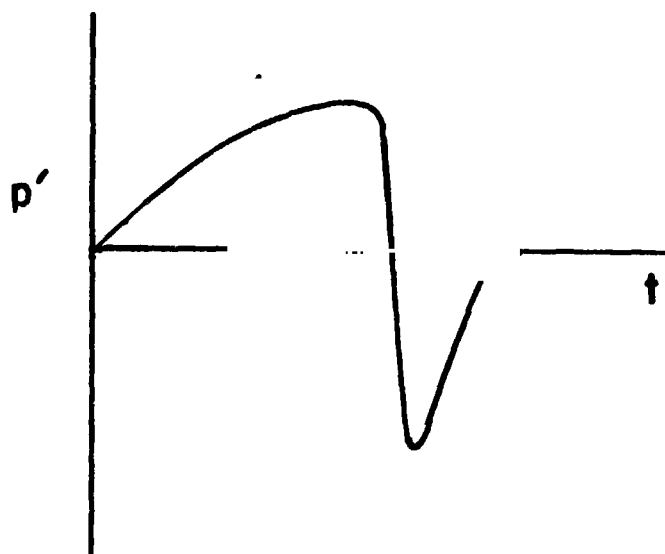
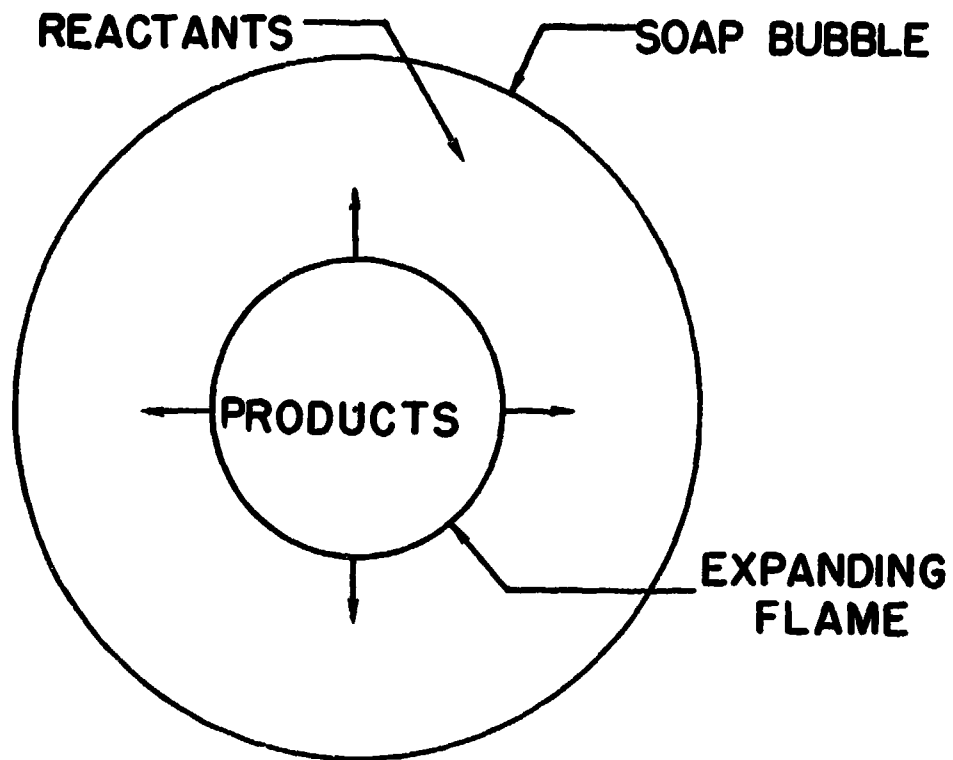


Figure 1. Schematic of an expanding spherical flame front inside a soap bubble and the associated pressure-time trace seen by a microphone outside the soap bubble (after Thomas and Williams, Ref. 3).



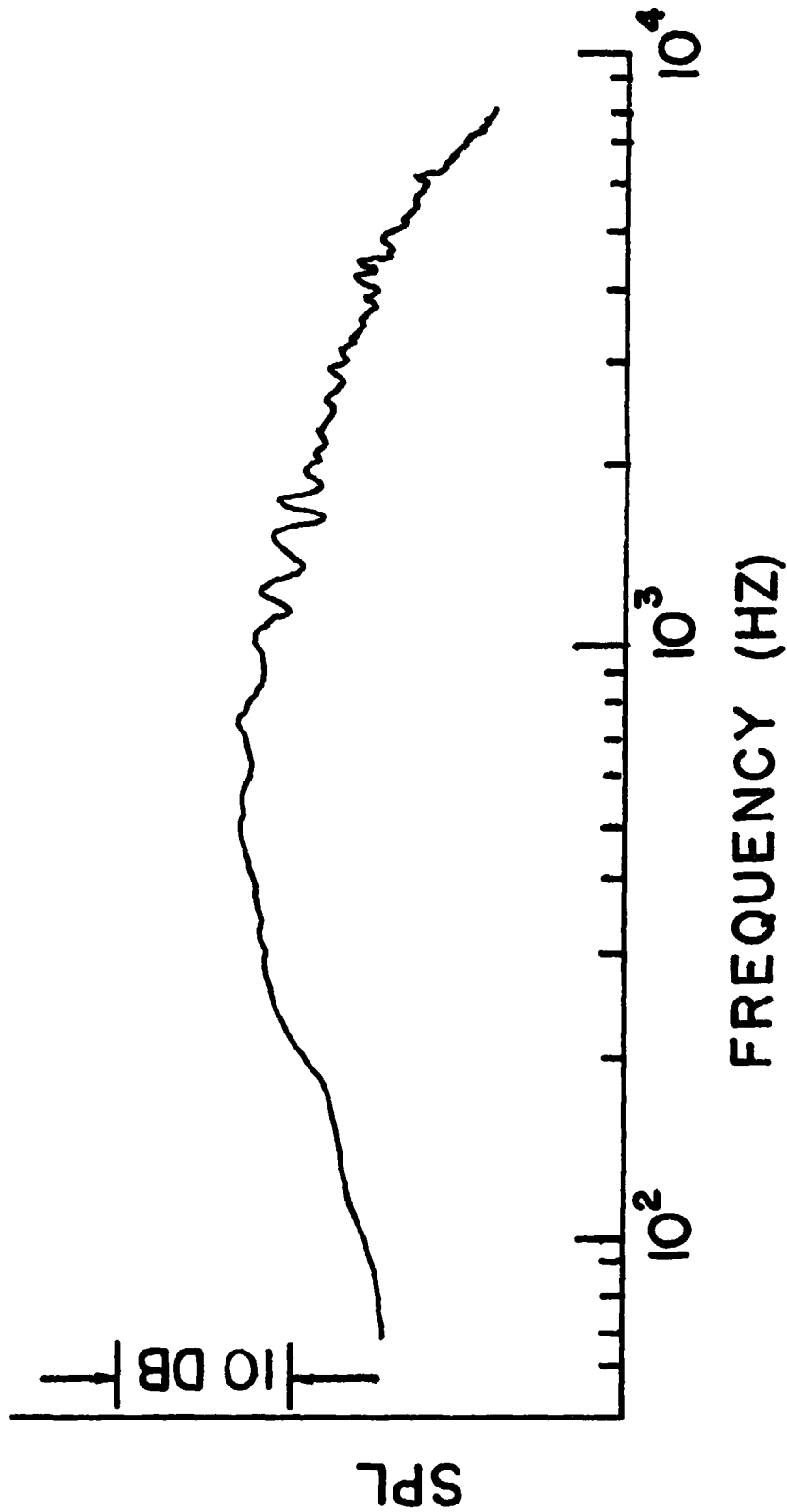


Figure 2. Spectral shape of combustion noise for a premixed free jet flame burning in anechoic surroundings. The flame is of ethylene-air flowing at 100 ft/sec at an equivalence ratio of unity from a burner of 0.4 in diameter. The spectral analysis bandwidth is 15.6 Hz. The relatively high frequency for the broad band peak, compared with more common hydrocarbons, is due to the high reactivity of ethylene compared with, say, propane; propane-air at these conditions would peak at about 250 Hz.

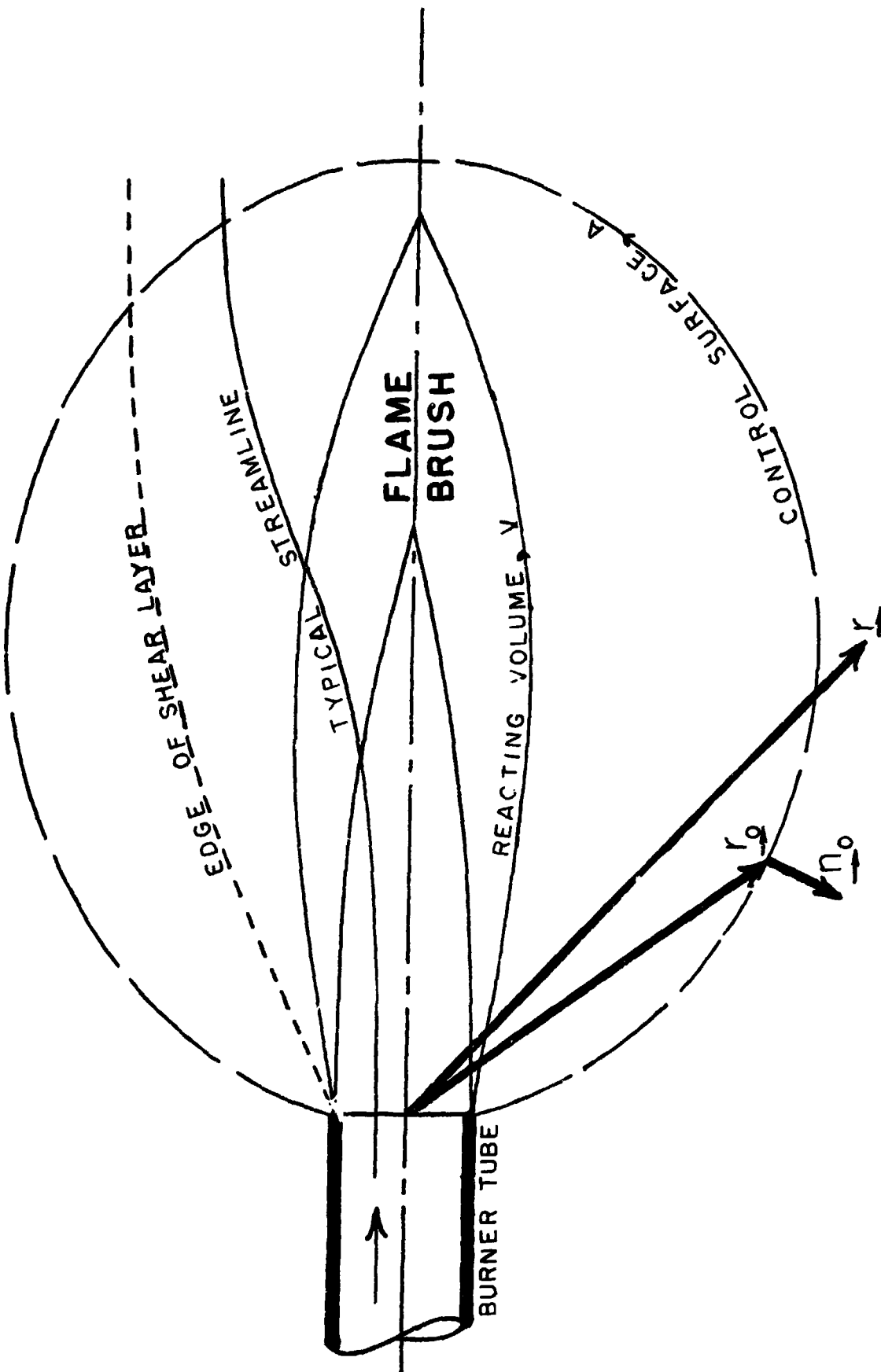


Figure 3. Schematic for the analysis of noise emitted by a turbulent premixed open jet flame.

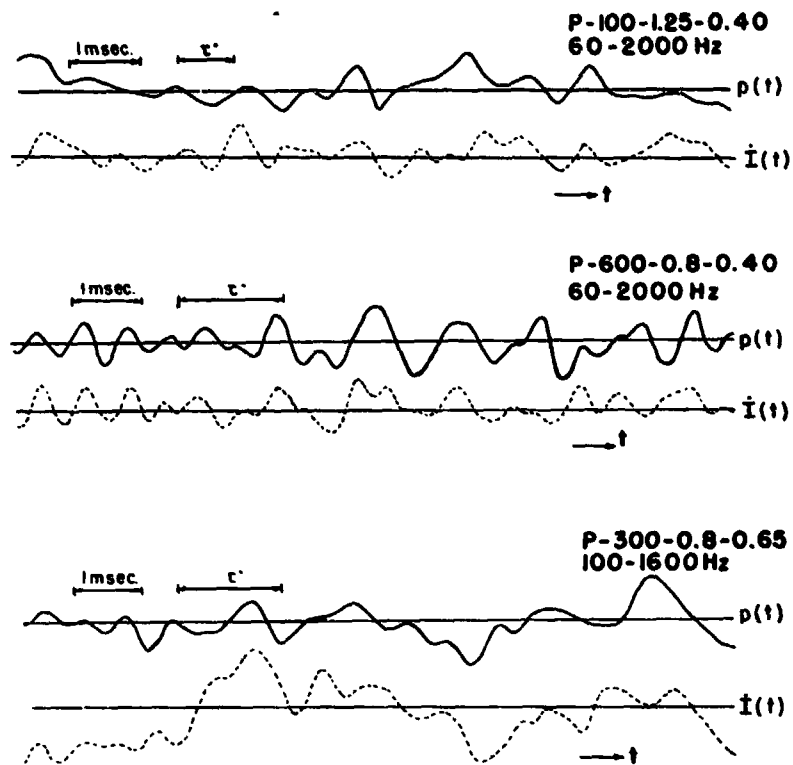


Figure 4. Direct comparison of the fluctuating pressure and time derivative of the overall  $C_2$  emission traces after shifting by the acoustic wave travel delay time,  $\tau^*$ . P stands for propane-air, the second number is the flow velocity in ft/sec, the third number is the equivalence ratio and the fourth number is the burner diameter in inches. The filter bandwidth is also shown.

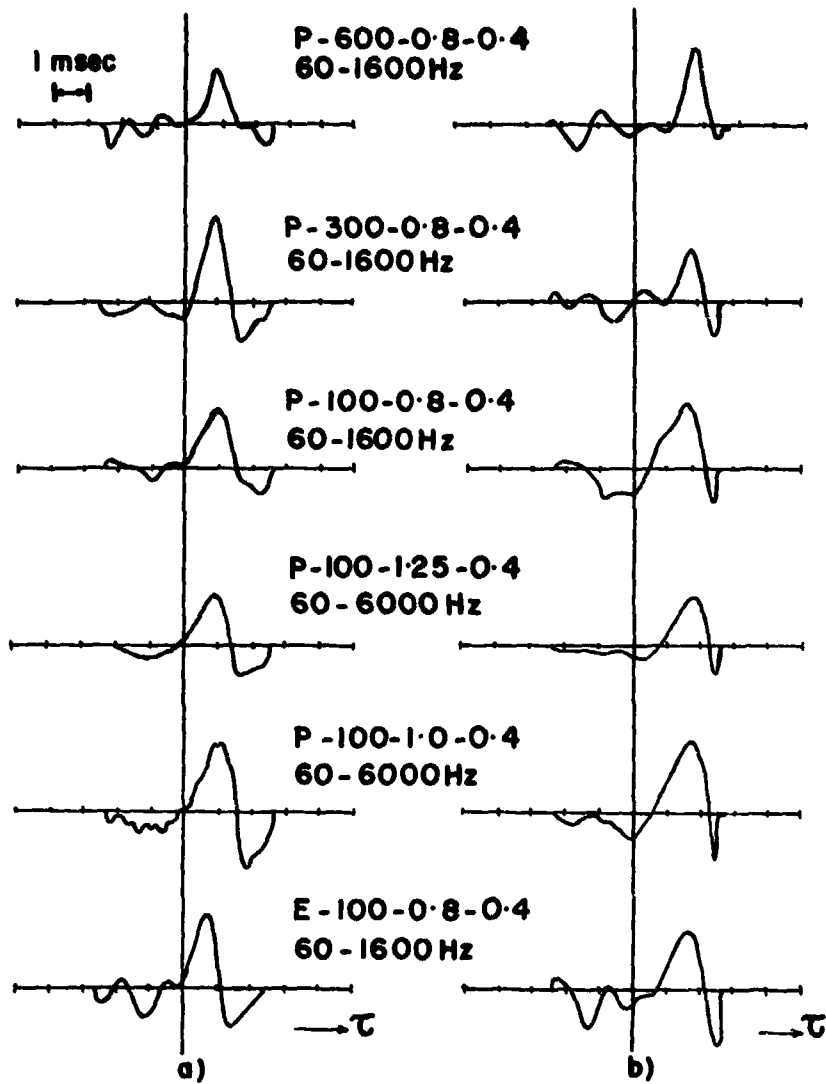


Figure 5. Cross correlations of the time derivative of the emission trace and the fluctuating pressure trace. Here E stands for ethylene-air and the other nomenclature is as shown on Figure 4. a and b are for two different microphones, with b being farther from the source.

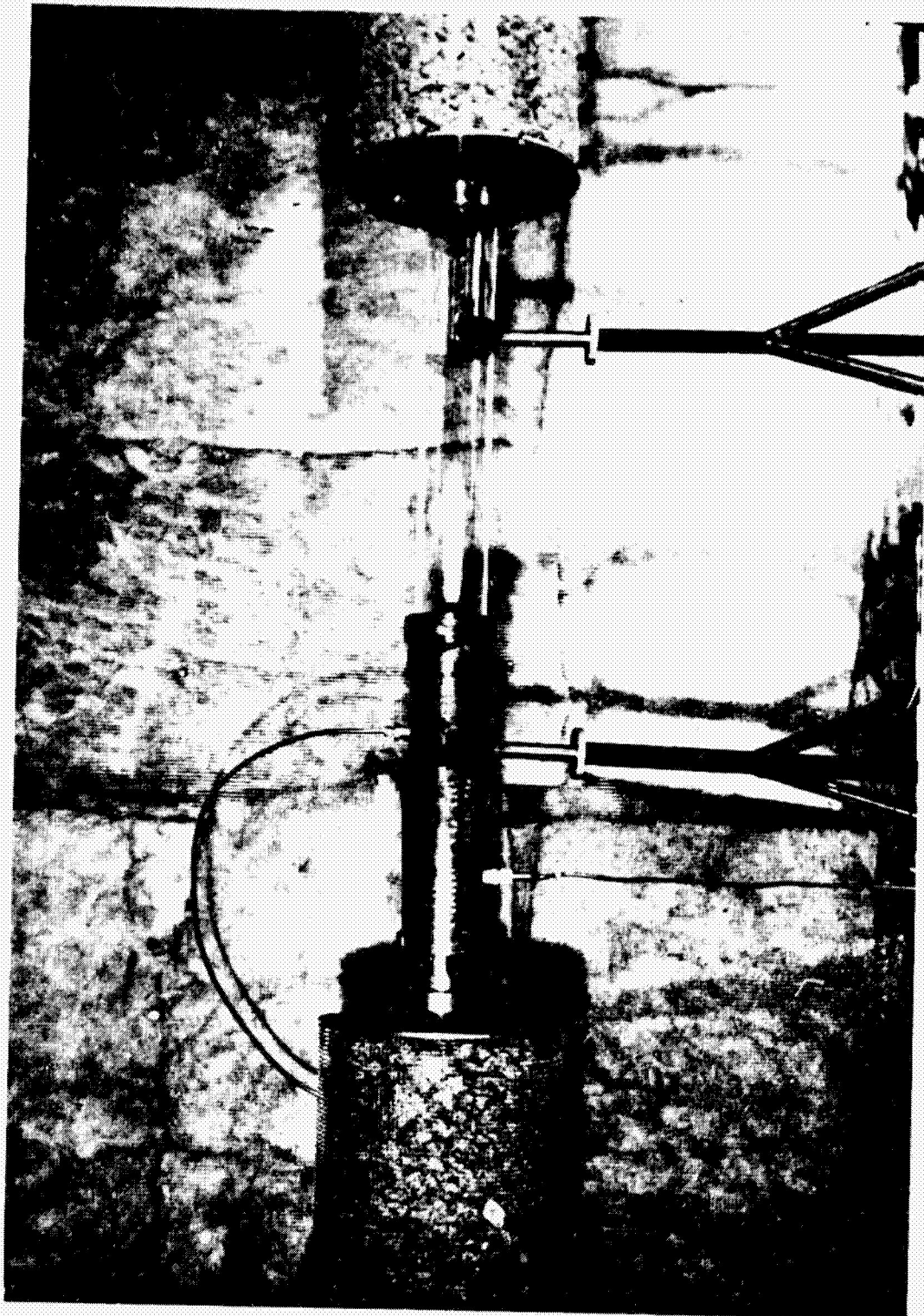


Figure 6. Photograph of the flame-in-tube experiment. The visible flame in the center of the photograph has the same visible shape as when burning in the open.

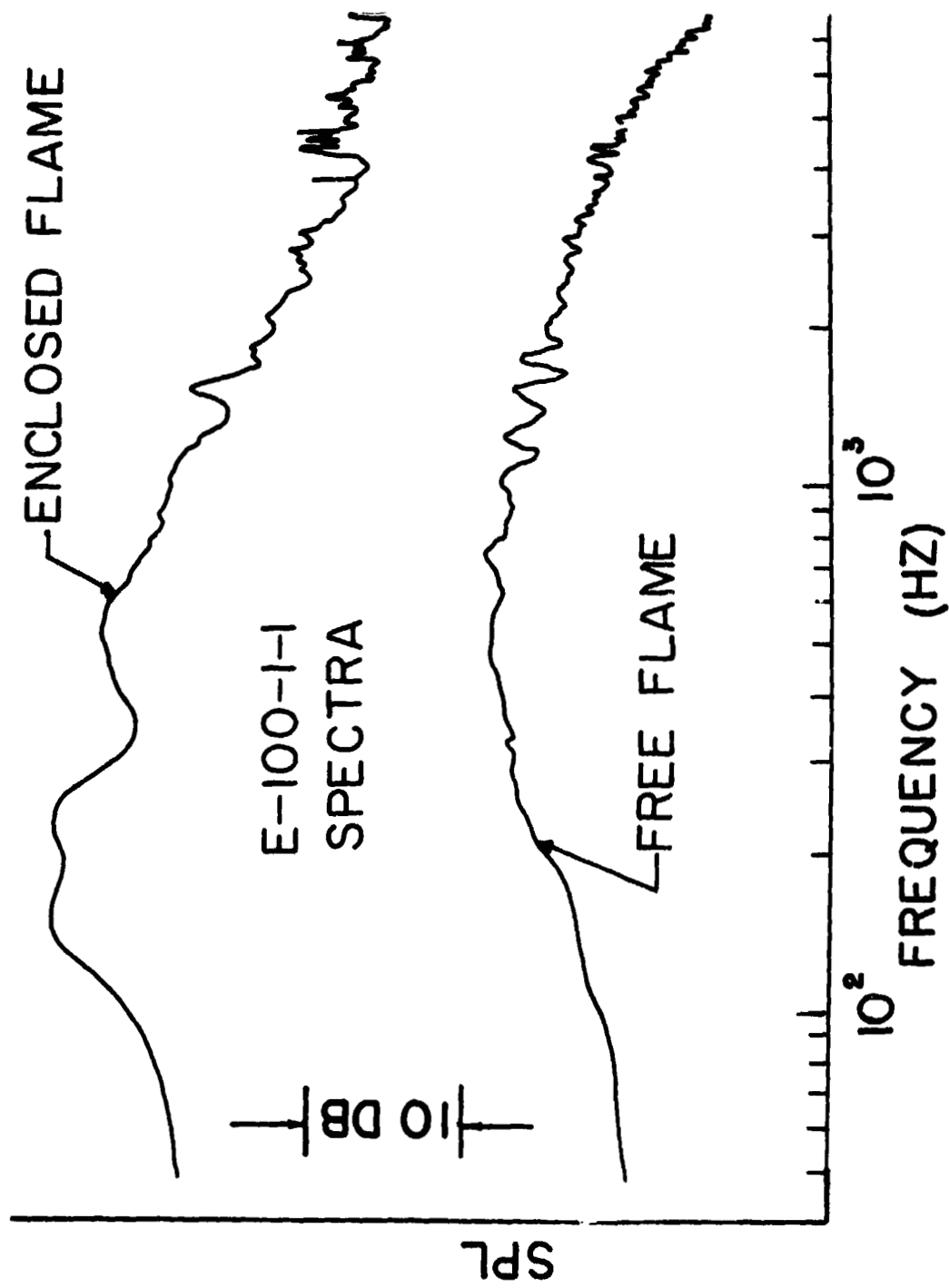


Figure 7. Spectral comparison of the sound pressure inside of the tube and that when the flame is burning in anechoic surroundings. These are compared here only on a relative, shape basis. The per flame spectrum is the same as on Figure 2. The spectral peaks above 4000 Hz for the enclosed flame correspond to cut-on of the transverse modes of the tube gases.

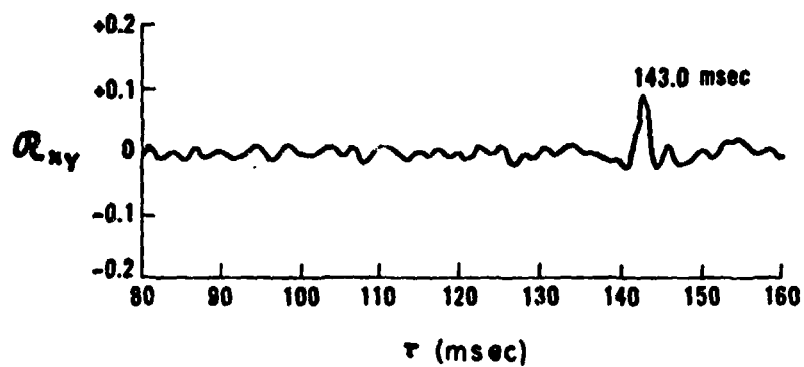


Figure 8. Cross correlation coefficient for the far field acoustic pressure and the burner interior fluctuating pressure on a Pratt and Whitney JT8D-109 engine. The peak occurs at a time corresponding to an acoustic travel time from the interior to the exterior microphone (after Mathews, Rikos and Nagel, Ref. 40).

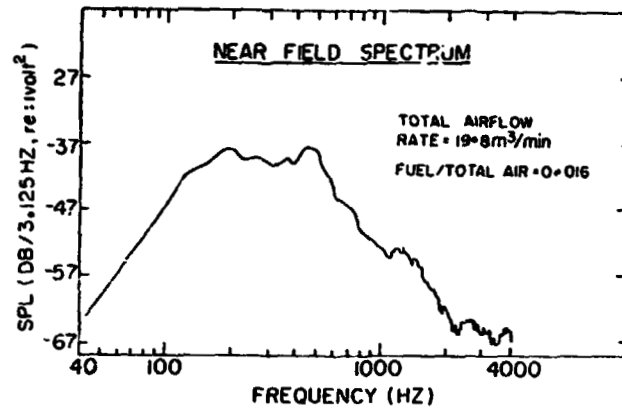


Figure 9. Exterior microphone spectrum for a can-type combustor exhausting directly to the atmosphere. The 1/4, 3/4, and 5/4 wave resonances are visible superimposed upon a broad band combustion noise which peaks around 200 Hz in this case.



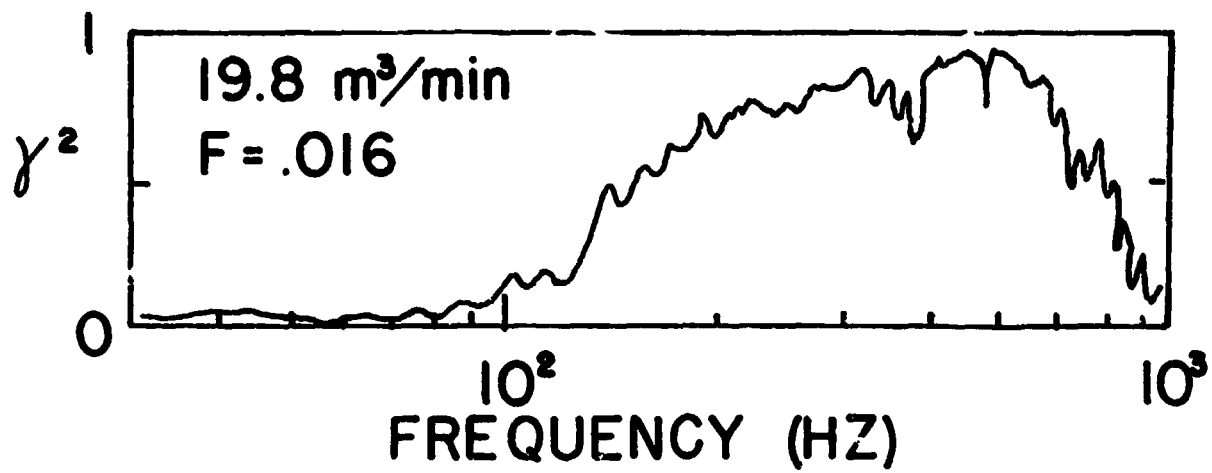


Figure 10. Coherence between the interior and exterior microphones  
for the run of Figure 9.

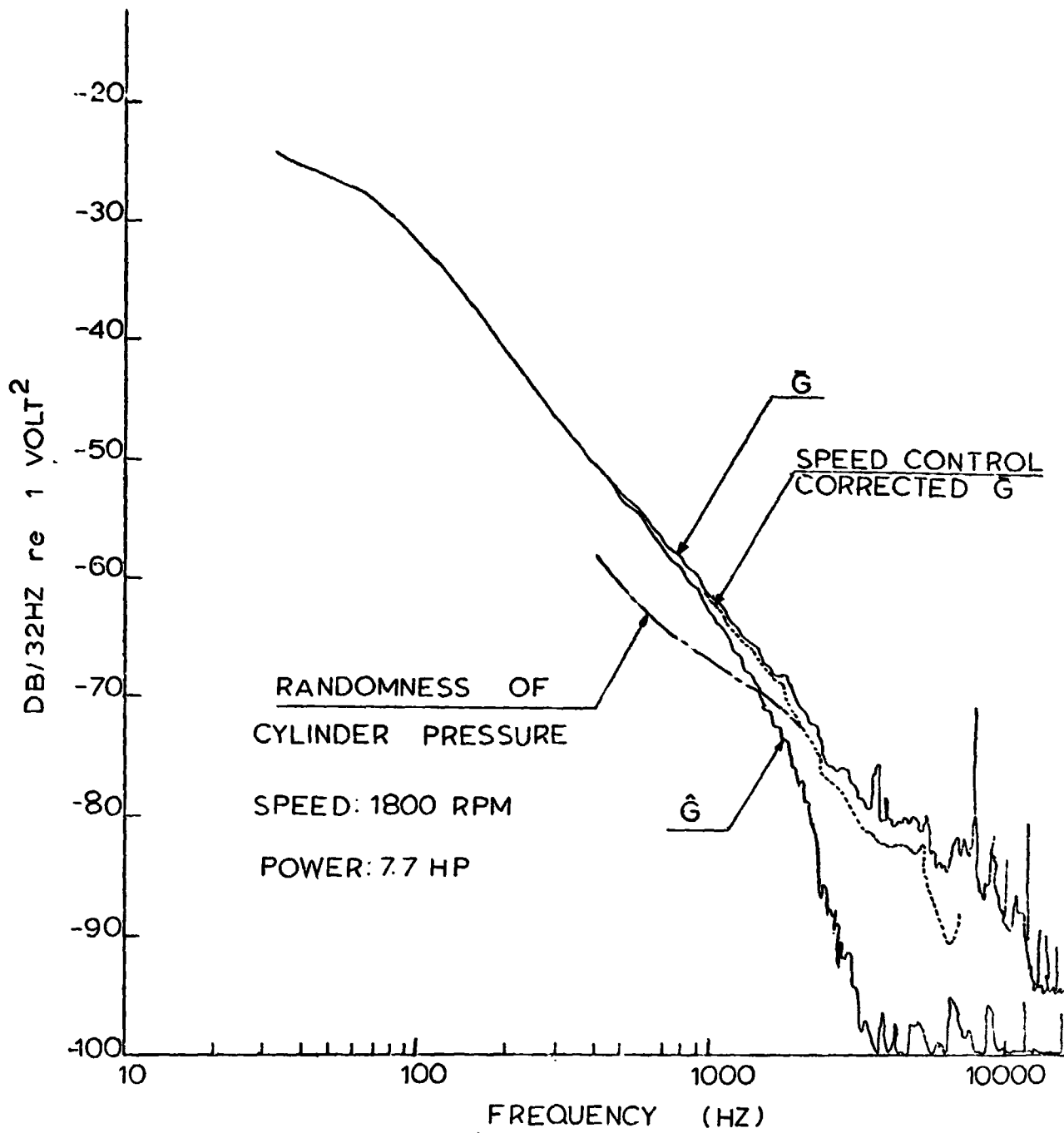


Figure 11. Spectra of Diesel engine cylinder pressure.  $\bar{G}$  is the spectral density,  $\hat{G}$  is the spectral density,  $\hat{G}$  is the spectral density of the average pressure-time diagram. The "randomness of cylinder pressure" is the difference between the two curves after  $\bar{G}$  is corrected for the "noise" introduced from lack of perfect speed control. The engine is a Deutz single cylinder, air cooled engine of 95 mm bore and stroke.

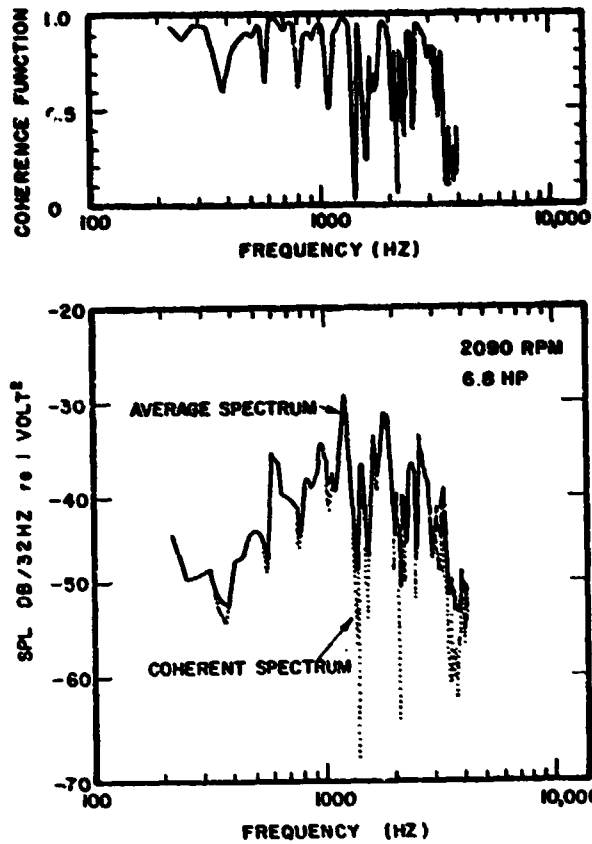


Figure 12. Coherence between the cylinder pressure and the radiated noise for the engine of Figure 11. Shown are the coherence function, the noise spectrum and the coherent noise spectrum, obtained by multiplication of the coherence function and the noise spectrum.

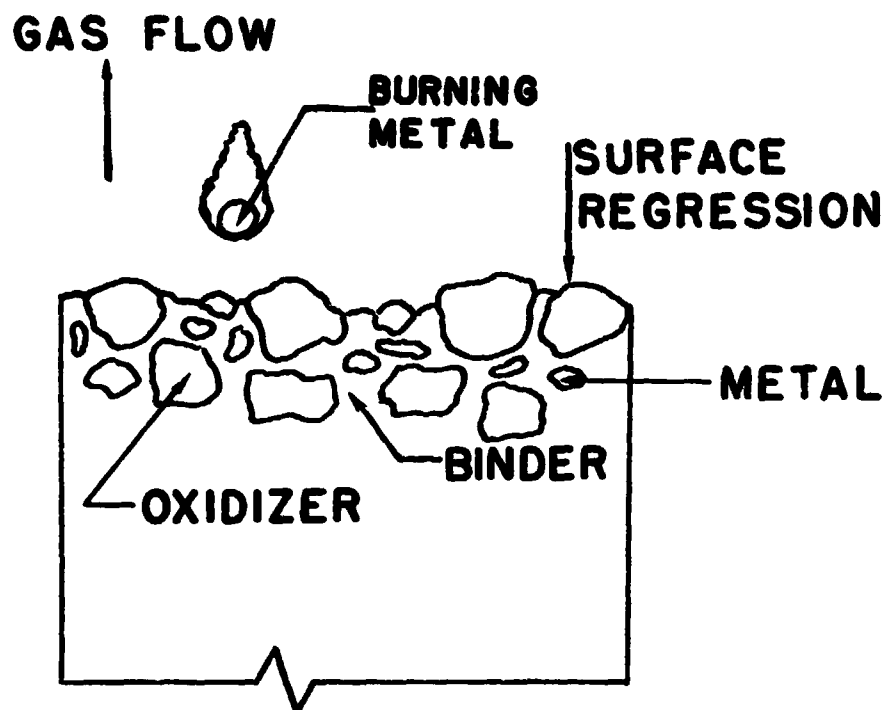


Figure 13. Schematic of the unsteadiness of flow above the surface of a composite solid propellant.

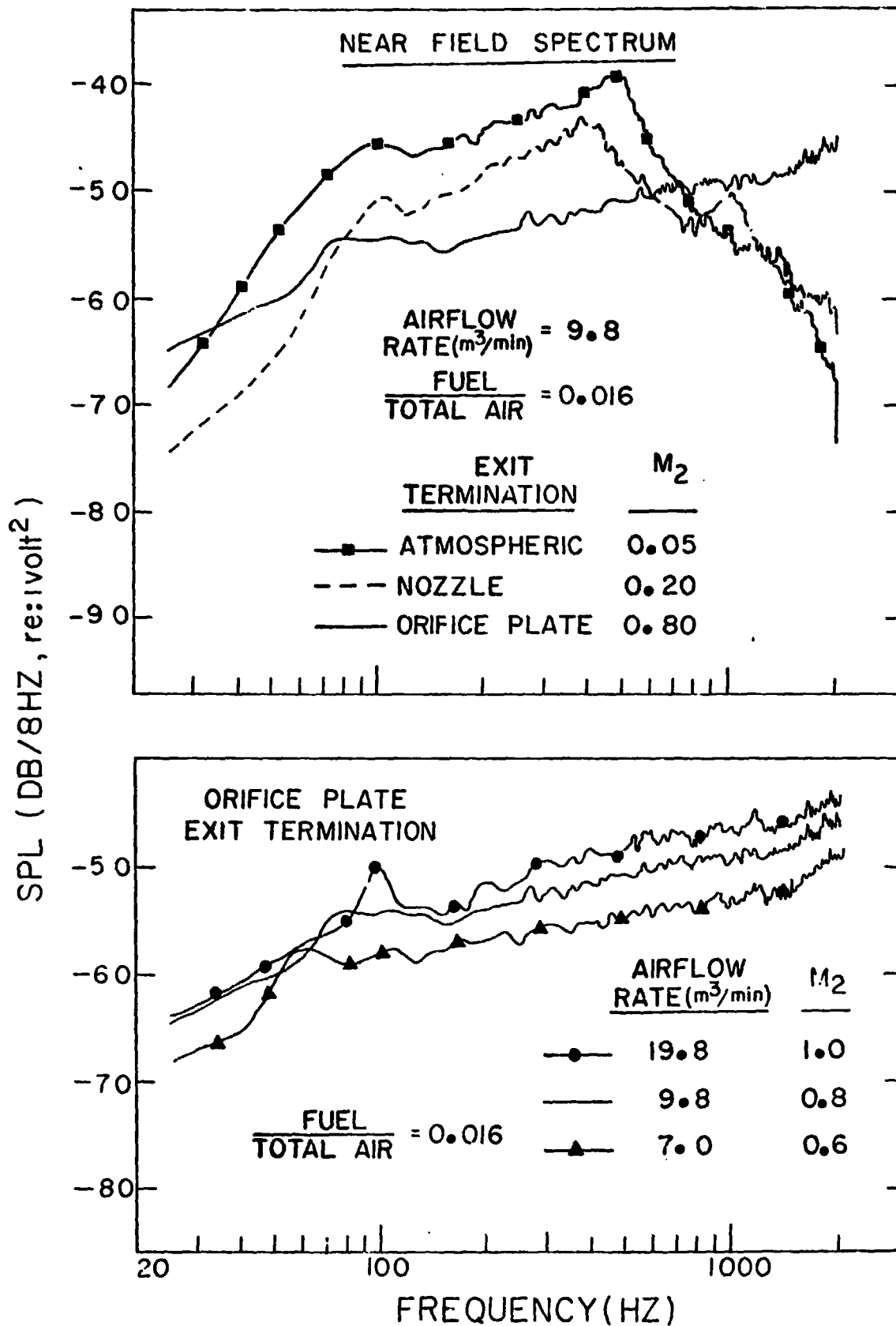


Figure 14. Exterior microphone spectra for a combustor can exhausting to the atmosphere with various mass flow and exit plane terminations.  $M_2$  is the exhaust exit plane Mach number.

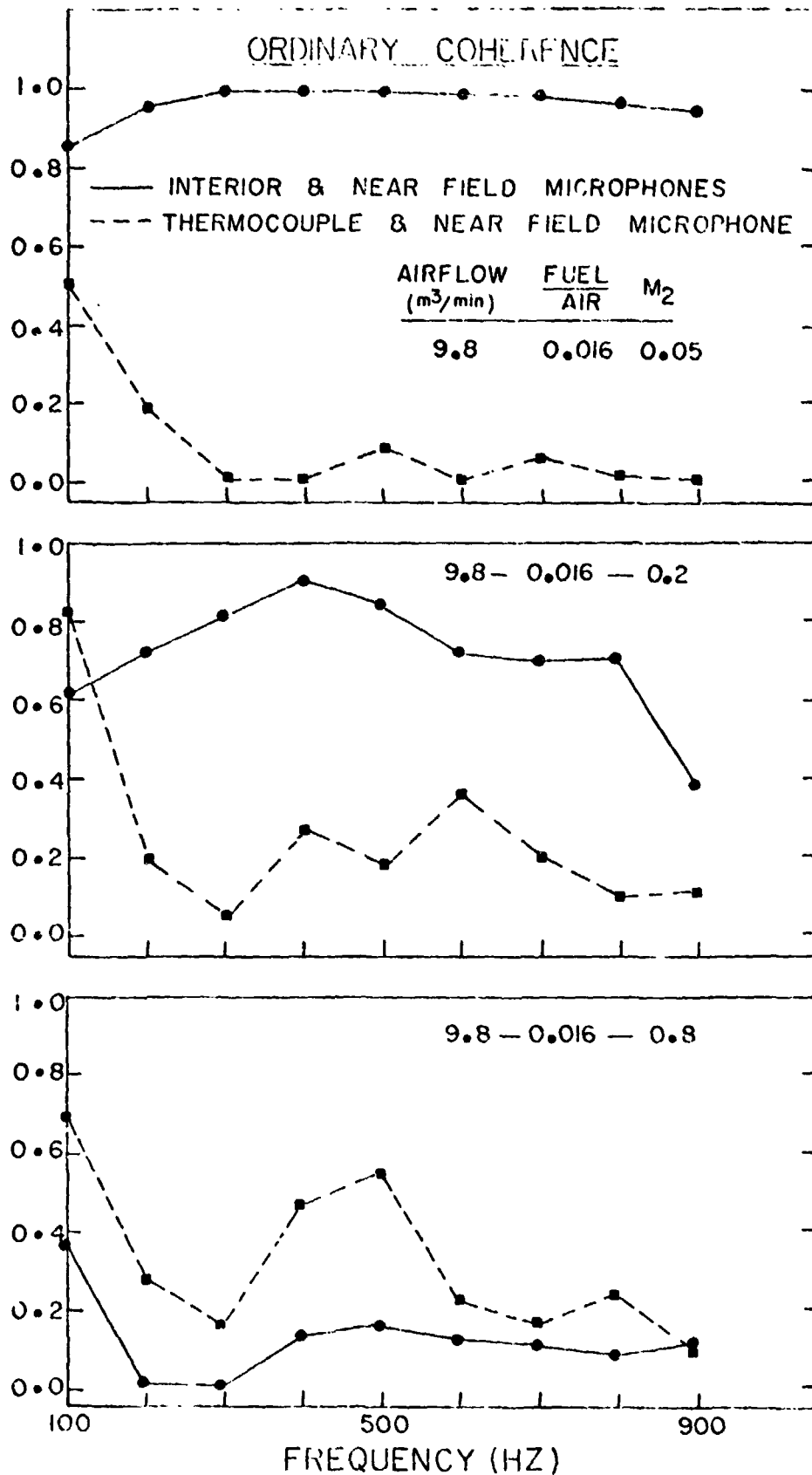


Figure 15. The ordinary coherence function for the (a) interior and exterior microphone and (b) the area weighted temperature fluctuation and the near field microphone.

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