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MODEL TASK FOR THE DYNAMICS OF AN UNDERWATER TWO-LEGGED WALKER

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Recently investigations of the dynamics of walking of anthropomorphic apparatuses and man have been strongly developing. This direction is connected with the maturing possibilities of creating integral robots and anthropomorphic autonomic systems of a type of exoskeletons and pressure suits for operation in experimental conditions. /4*

The proposed work borders on the indicated investigations. A model task of two-legged underwater walking is examined. This task allows for the establishment of several characteristics of the walking of two-legged underwater apparatuses or pressure suits. The underwater walking device is represented by a substantial sphere, which moves on dual-member rigid legs under the action of momentums which are located in the joints of the legs. The legs of the apparatus are round cylinders, the joints are hinged between themselves and with the sphere.

The dynamics of this system are investigated with the calculation of the buoyancy of Archimedes, which acts on the sphere, and the force of hydrodynamic resistance, which acts on the sphere and legs. For the assigned walking velocity of the apparatus, the compensating vibration of the housing, momentums of force in the joints and the key reactions are determined. Several questions of stability are investigated. A comparison of underwater and terrestrial walking is given.

1. Definition of the Task. The Dynamic Model of a Walking Apparatus and Equalization of its Motion.

Plane-parallel motion of a walking apparatus in a liquid is /5

*Numbers in the margin indicate pagination in the original text.

examined. The apparatus consists of a rigid housing in the form of a sphere and a pair of legs which are mounted to the housing at one point of support П (Fig. 1).

For the description of this movement, let us introduce the stationary, right quadrate Descarte system of coordinates Nxyz, the plane Nyz which coincides with the plane of motion of the apparatus.

Let us make the following assumptions:

1. The legs of the apparatus are identical and each of them consists of two joints -- a thigh and shank. The thigh and shank are round cylinders, which have corresponding lengths a and b and diameters d_δ and d_e.

The legs are weightless and sufficiently thin, i.e.

$$m_\delta = m_e = 0, \qquad \frac{d_\delta}{a} \ll 1, \qquad \frac{d_e}{b} \ll 1$$

where m_δ and m_e are the masses of the thigh and shank.

3. The attachment of the shank to the thigh, and the thigh to the body (housing) is jointed with one degree of latitude.

4. The plane motion of the apparatus takes place in such a manner that the center of the sphere O, the point of support П and the legs are found in the plane Nyz.

5. The distribution of mass on the sphere and inside it is such that the plane Nyz is the plane of dynamic symmetry of the sphere (apparatus), and the center of mass C of the apparatus lies on a line which passes through the center of the sphere and the point of support.

6. The force of gravity is directed along z.

7. The forces of friction in the hinges (joints), the lifting forces of the leg joints and forces of viscid friction, which

act on the apparatus from the side of the liquid, are negligible.

8. The surface with which the legs of the apparatus contact is absolutely uneven.

9. The control by the action of the apparatus is realized with the aid of guiding momentums affixed in its joints.

10. Acceleration and the speed of any point of the apparatus are finite.

With the assumptuions made, let us determine the task of the motion of the walking apparatus in a liquid in the following manner. Let

1) the motion of the point of support be known and take place uniformly and rectilinearly along axis y (comfortable motion [1]):

2) the legs of the apparatus move along a given regular marked path, placed in the plane Nxy on the line Ny (in other words, the apparatus walks along plane Nxy along the line Ny with a constant pace):

3) the step of the apparatus is single-supported, periodic and repetitious [1,2]: at each point of time the apparatus is supported only on one leg; the movement of the legs of the apparatus take place periodically with a period of 2T (T is the length of one step); and one leg exactly repeats the motion of the other with a delay of T;

4) the motion of the end of the moving leg is specified in the form of a clear function of time.

The position of the body of the apparatus in space will be characterized by two Descartes points of support y_{Π}, z_{Π} and angle γ,

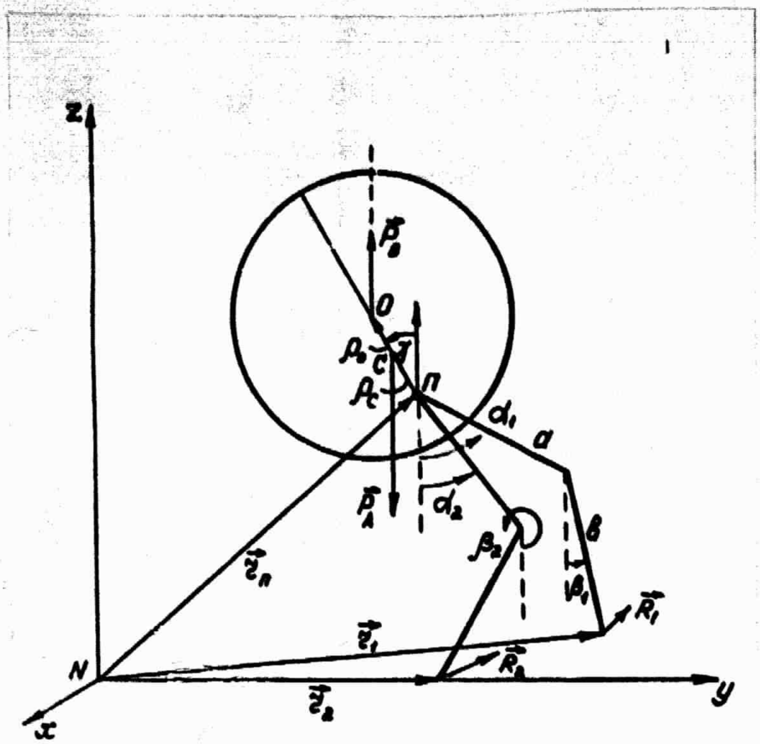

Figure 1

and the position of the legs by the angles α_ν, β_ν, α_π, β_π (Fig. 1).

Let us consider that the contact of each foot with the surface is a point and all the action of the surface on the foot is reduced to a single force (the force of reaction) $\vec{R}_i$ ($i = \nu, \pi$), applied to the foot at the point of support. /8

From here on, through $u_\nu^{(\Pi)}$, $u_\nu^{(K)}$, and $u_\pi^{(\Pi)}$, $u_\pi^{(K)}$ we will designate the momentums of the force of control, which acts from the side of the body on the thigh and from the side of the thigh on the knee respectively in the support and moving legs. By the establishment of the task, the coordinates of the point of support y_Π, z_Π and the angles α_ν, β_ν, α_π, β_π, which define the position of the leg in space, it is known to us as a function of time.

The forces of reaction $\vec{R}_i$ and also of control u_i appear as unknowns.

Applying to the walking apparatus the principle of the elimination of connections and using the theories concerning the change of the quality of the motion of the apparatus and its kinetic momentum relative to the point of support, we receive the equation for the definition of unknown forces of reaction and the angle γ. These equations are the actual development of the equation received in [1] and can be recorded in the form

$$
\begin{aligned}
&I_\Pi\dot{\vec\omega}=\vec\rho_c\times\vec P_A+\vec\rho_0\times\vec P_B+(\vec r_\nu-\vec r_\Pi)\times\vec R_\nu+(\vec r_\pi-\vec r_\Pi)\times\vec R_\pi+\\
&\quad+\vec M_K+\vec M_\nu+\vec M_\pi-\vec\omega\times I_\Pi^{(A)}\vec\omega-M_A(\vec\rho_c\times\dot{\vec V}_\Pi),\\
&\vec R=\vec R_\nu+\vec R_\pi=-\vec P-\vec F_K-\vec F_\nu-\vec F_\pi+M[\dot{\vec V}_\Pi+\dot{\vec\omega}\times\vec\rho_{np}+\vec\omega\times(\vec\omega\times\vec\rho_{np})],\\
&\vec P=\vec P_A+\vec P_B,\quad \vec\rho_{np}=\left(\frac{M_A}{M}\vec\rho_c+\frac{M_{np}}{M}\vec\rho_0\right),\\
&M=M_A+M_{np},\quad I_\Pi=I_\Pi^{(A)}+M_{np}\rho_0^2E,\quad \rho_0=|\vec\rho_0|,\\
&M_{np}=\frac{2\pi}{3}R_{c\phi}^3\rho,\quad E=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}.
\end{aligned}
\qquad (1.1)
$$

Here and henceforth the point denotes the differentialization in time. In (1.1) is indicated: $I_\Pi^{(A)}$ is the tesor of inertia of the apparatus at the point of support Π: $\vec\rho_0$, $\vec\rho_s$ are the radius-vectors leading from the point of support to the center of the sphere to the center of mass of the apparatus (they are, on the strength of assumption 5 on page 4, colinear); $\vec r_\Pi$ $\vec r_\nu$ $\vec r_\pi$ are the radius-vectors /9 which lead from the beginning of the selected system of coordinates respectively to the point of support and to the supporting points of the feet; M_A is the mass of the apparatus equal, on the strength of the weightlessness of the legs, to the mass of the body; $M_{\Pi P}$ is the combined mass of the body-sphere; $\vec\omega$ is the angular velocity of the body; $\vec V_\pi$ is the velocity of the point of support; $R_{c\phi}$ is the radius of the sphere: ρ is the density of the liquid; $\vec P_A$ and $\vec P_B$ are vectors of the buyoing force and force of the weight which acts on the body; $\vec F_K$, $\vec F_\nu$, $\vec F_\pi$ are the forces of hydrodynamic resistance of the liquid which acts respectively on the body (sphere) and the legs of the apparatus; $\vec M_K$, $\vec M_\nu$, $\vec M_\pi$ are the momentums of force of the hydrodynamic resistance relative to the point of support, which act respectively on the body and the legs of the apparatus.

In deriving equation (1.1) in a known approximation we considered the instability of the streamlining of the body-sphere of the walking apparatus in its motion in a liquid by means of the introduction of its combined masses [3] (the combined masses of the legs were assumed to equal zero, which on the strength of supposition 2 on page 4 is fully feasible).

From here on only the single support motion is examined. It is convenient to introduce the index "ν" for all values which pertain to the support to the leg and the index "π" for all values which relate to the moving leg (Fig. 2,3). Then on the strength of the establishment of the task and suppositions which have been made, we have (Fig. 1, 2, 3):

$$\begin{aligned}
&\vec{R}_\nu = \vec{R}, \qquad \vec{R}_\pi = 0, \\
&\vec{R} = R_y \vec{y}^\circ + R_z \vec{z}^\circ, \quad \vec{P}_A = -P_A \vec{z}^\circ, \quad \vec{P}_B = P_B \vec{z}^\circ, \\
&\vec{F}_i = F_{iy} \vec{y}^\circ + F_{iz} \vec{z}^\circ, \quad i = \kappa, \nu, \pi, \\
&\vec{N}_i = M_i \vec{x}^\circ, \quad i = \kappa, \nu, \pi,
\end{aligned}$$

$$\begin{aligned}
&\vec{\omega} = \dot{\gamma} \vec{x}^\circ, \quad \dot{\vec{\omega}} = \ddot{\gamma} \vec{x}^\circ, \quad I_n^{(A)} \vec{\omega} = J_n^{(A)} \dot{\gamma} \vec{x}^\circ, \\
&I_n \dot{\vec{\omega}} = J_n \ddot{\gamma} \vec{x}^\circ, \quad J_n = J_n^{(A)} + M_{np} \rho_0^2, \\
&\vec{r}_n = H \vec{z}^\circ + [V_n (t - t_0) + h_1] \vec{y}^\circ, \\
&\vec{V}_n = V_n \vec{y}^\circ, \quad \dot{\vec{V}}_n = 0, \\
&H = const, \quad V_n = const, \quad h_1 = const.
\end{aligned} \tag{1.2}$$

In addition /11

$$\begin{aligned}
&\vec{r}_\nu(t) = \left(\left[\frac{t - t_0}{T}\right] + 1\right) h \vec{y}^\circ, \; T = const, \; h = const, \; \vec{r}_\pi = y_\pi \vec{y}^\circ + z_\pi \vec{z}^\circ, \\
&\dot{\vec{\rho}}_i = -\rho_i \sin\gamma \vec{y}^\circ + \rho_i \cos\gamma \vec{z}^\circ, \; \rho_i = const, \; i = o, c, \; \rho_o = |\vec{\rho}_o| > 0, \\
&\rho_c = \begin{cases} |\vec{\rho}_c| > 0, & \text{if} \quad (\vec{\rho}_o, \vec{\rho}_c) > 0, \\ -|\vec{\rho}_c| < 0, & \text{if} \quad (\vec{\rho}_o, \vec{\rho}_c) < 0. \end{cases}
\end{aligned} \tag{1.3}$$

Here V_Π, P_A, P_B are the absolute values of the vectors $\vec{V}_\Pi$, $\vec{P}_A$, $\vec{P}_B$; $J_\Pi^{(A)}$ is the point of inertia of the apparatus relative to the axis which passes through the point of support and parallel coordinates of the axis x; H is the height of the point of support over the surface Nxy, h_1 is the projection of the radius of the vector $\vec{r}_\Pi$ on the axis y at the initial moment of time t_o; $\vec{r}_\nu$ is the radius-vector of the point of support; $\vec{r}_\pi$ is the radius-vector of the end of the moving foot; h is the value of the step of the apparatus; T is the earlier introduced time in which the apparatus makes one step; $\vec{x}^o$, $\vec{y}^o$, $\vec{z}^o$ are respectively the unit vectors of the coordinate axes x, y, z; $[t-t_o/T]$ is the entire part of the unit $t-t_o/T$.

With the written formula (1.2) it was assumed that at the initial moment of time one leg is found at the beginning of the system of coordinates N and the apparatus has ceased to be supported on this leg (the leg has started moving), and the other leg is found on the /12
axis y at a distance h from point N and the apparatus has just begun to be supported on this leg (the leg becomes supporting) (Fig. 3).

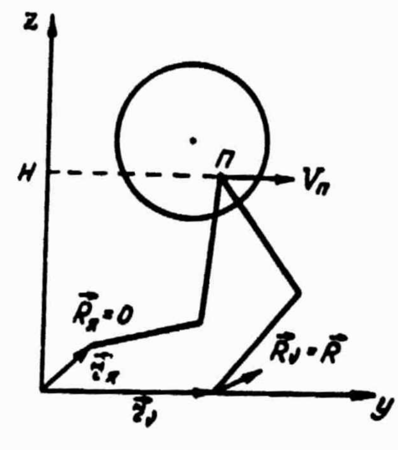

Figure 2

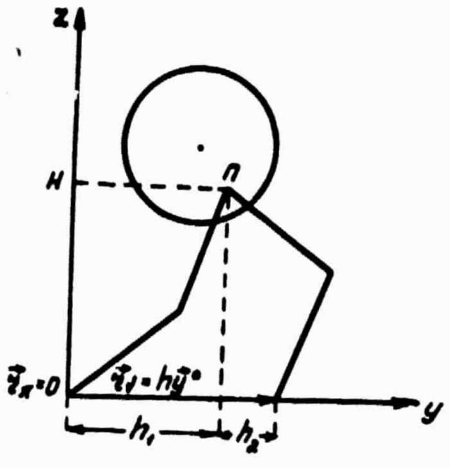

Figure 3

Substituting (1.2)-(1.3) into (1.1) and writing out the obtained vector of the equation on coordinate axes, we obtain the equation which describes the change of the angle γ with time, and the expression for the projection of the forces of reaction.

$$
\begin{aligned}
&\ddot{\gamma} = \frac{1}{J_{np}}\Big[(P_A\rho_c - P_B\rho_0)\sin\gamma + (P_A - P_B)\,\Delta y_\nu(t) + \\
&\quad + M\rho_{np}\dot{\gamma}^2(H\sin\gamma - \Delta y_\nu(t)\cos\gamma) + \\
&\quad + M_{\kappa F} + M_{\nu F} + M_{\pi F} + M_\kappa + M_\nu + M_\pi\Big], \\
&R_{\nu y} = R_y = -F_{\kappa y} - F_{\nu y} - F_{\pi y} - M\rho_{np}(\ddot{\gamma}\cos\gamma - \dot{\gamma}^2\sin\gamma), \\
&R_{\nu z} = R_z = P_A - P_B - F_{\kappa z} - F_{\nu z} - F_{\pi z} - M\rho_{np}(\ddot{\gamma}\sin\gamma + \dot{\gamma}^2\cos\gamma), \\
&R_{\pi y} = 0, \quad R_{\pi z} = 0. \\
&M_{iF}\vec{x}^0 = (\vec{r}_\nu - \vec{r}_n)\times\vec{F}_i, \qquad i = \kappa, \nu, \pi \qquad\qquad (1.4) \\
&J_{np} = J_n + M\rho_{np}[\Delta y_\nu(t)\sin\gamma + H\cos\gamma], \quad J_n = J_n^{(A)} + M_{np}\rho_0^2, \\
&\rho_{np} = \frac{M_A}{M}\rho_c + \frac{M_{np}}{M}\rho_0, \\
&\vec{r}_\nu - \vec{r}_n = \Delta y_\nu(t)\vec{y}^0 - H\vec{z}^0, \\
&\Delta y_\nu(t) = h\left(\left[\frac{t-t_0}{T}\right] - \frac{t-t_0}{T}\right) + h_2, \quad h_2 = h - h_1, \\
&\Delta y_\nu(t+T) = \Delta y_\nu(t).
\end{aligned}
$$

The four last formulas of (1.4) are written with the calculation of connections which exist between V_Π, T and h /13

$$h = V_\Pi T.$$

This connection is the consequence of the periodicity of motion of the walking apparatus.

With the absence of hydrodynamic forces and momentums the equations of (1.4) are converted to earlier knowns [1].

The first of the equations (1.4) has the structure

$$\ddot{\gamma} = f(\gamma, \dot{\gamma}, t). \tag{1.4'}$$

In the task which is examined in the present work, the right part of this equation depends on the clearly forward moving time <u>t</u> with a periodic pattern so that $f(\gamma, \dot{\gamma}, t + 2T) = f(\gamma, \dot{\gamma}, t)$. Let us designate the value of the phase changes at the initial moment t_0 through γ_0, $\dot{\gamma}_0$. Then the general solution of the equation (1.4') may be written in the form

$$\begin{aligned} &\gamma(t) = \gamma(\gamma_0, \dot{\gamma}_0, t_0, t); \\ &\gamma(\gamma_0, \dot{\gamma}_0, t_0, t_0) = \gamma_0, \quad \left.\frac{d\gamma(\gamma_0, \dot{\gamma}_0, t_0, t)}{dt}\right|_{t=t_0} = \dot{\gamma}_0. \end{aligned} \tag{1.5}$$

According to the establishment of the task $\gamma(t)$ must be a periodic function with a period of 2T. This indicates that from the series of solution of (1.5) we must choose only those solutions which meet the following condition of periodicity:

$$\begin{aligned} &\gamma(\gamma_0, \dot{\gamma}_0, t_0, t_0 + 2T) = \gamma_0 \\ &\dot{\gamma}(\gamma_0, \dot{\gamma}_0, t_0, t_0 + 2T) = \dot{\gamma}_0 \end{aligned} \tag{1.6}$$

Let us assume that the conditions of periodicity of (1.6) may be satisfied by the choice of initial givens γ, $\dot{\gamma}$. Then the relationships of (1.6) must be examined as a system of two transcendental equations with two unknowns γ_0, $\dot{\gamma}_0$. Finding the solution of the system of equations of (1.6), we find by that very solution the periodic solution of the differential equation in (1.4), and, as it has come /14
to be, the periodic motion of the walking apparatus. Thus the task of finding the periodic solutions of the equation for γ is reduced to the solution of the angular task of (1.4), (1.6). Below will be shown that this angular task actually has the solution.

2. Finding the Control Momentums in the Joints

For the realization of the indicated above motion of the walking apparatus, it is necessary to place corresponding momentums of control of the forces $u_i^{(n)}$, $u_i(K)$ ($i = \nu, \pi$) in its joints. Let us find these momentums. According to the assumption, they are located only in the hip and knee joints. Let us apply to the parts of the leg (from the gears to the point of support) the principle of the freedom of movement of connections and the theory concerning the change of kinetic momentum (respectively relative to the knee and the point of support).

Let $i = \nu, \pi$ is the index respectively of the support and the moving legs. Let us take the i-leg and having eliminated the body and the support surface, let us change the motion of the body on the thigh at the point П of support by the force of reaction $\vec{R}_i^{(n)}$ and momentum $u_i^{(n)}$ relative to П and the motion of the surface on the foot of the force of reaction R_i (Fig. 4). Applying the theory concerning the change of kinetic momentum of the leg relative to the point of support and using assumptions of the weightlessness of the leg and the finiteness of velocity and acceleration of any of its points, we obtain

$$u_i^{(n)}\vec{x}^0 + M_i\vec{x}^0 + (\vec{z}_i - \vec{z}_n) \times \vec{R}_i = 0, \quad i = \nu, \pi. \tag{2.1}$$

Here M_ν; M_π are the momentums of forces of hydrodynamic resistance, which act respectively on the support and moving legs.

Considering that

$$\vec{R}_\nu = \vec{R}, \quad \vec{R}_\pi = 0, \quad \vec{z}_\nu - \vec{z}_n = \Delta y_\nu(t)\vec{y}^0 - H\vec{z}^0$$

from (2.1) we find $u_\nu^{(n)}$ and $u_\pi^{(n)}$ consecutively assuming $i = \nu$ and $i = \pi$ /16

$$u_\nu^{(n)} = -M_\nu - \Delta y_\nu(t)R_z - HR_y, \quad u_\pi^{(n)} = -M_\pi. \tag{2.2}$$

Let us now apply the theory of kinetic momentum relative to the knee to the shank of the i-leg. Substituting the action of the thigh on the shank by the force of reaction $\vec{R}_i^{(K)}$ and momentum $u_i^{(K)}$, and the action of the support of the surface -- of the reactions at the point of support $\vec{R}_i$ and taking into consideration the weightlessness of the shank and the finiteness of velocity and acceleration of its points, we obtain (Fig. 5)

$$u_i^{(K)}\vec{x}^{(0)} + M_i^{(0)}\vec{x}^{(0)} + (\vec{z}_i - \vec{z}_i^{(K)}) \times \vec{R}_i = 0, \quad i = \nu, \pi$$

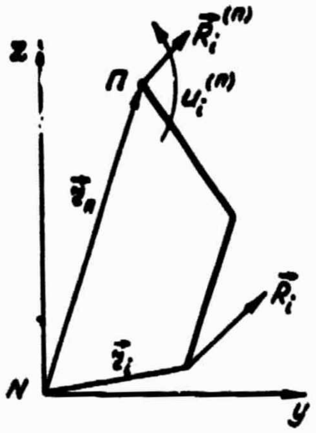

Figure 4

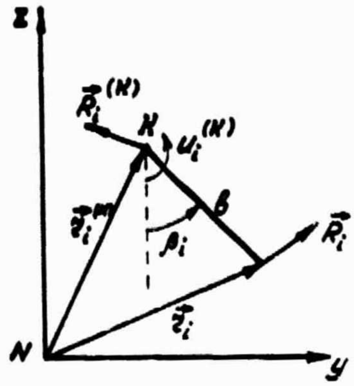

Figure 5

This gives, with $i = \nu$ and $i = \pi$ respectively

$$u_\nu^{(K)} = -M_\nu^{(2)} - b(R_y \cos\beta_\nu + R_z \sin\beta_\nu),$$
$$u_\pi^{(\Pi)} = -M_\pi^{(2)}.$$

Here $M_\nu^{(2)}$ and $M_\pi^{(2)}$ are the momentums of forces of hydrodynamic resistance which act respectively on the shank of the supporting and moving legs and are taken respective to the thigh; $\beta\nu$ is the angle β for the support leg. From the given formulas with the absence of hydrodynamic momentums the formulas of work are obtained [1]. The obtained formulas for the control in the joints (2.2, 2.3) hold for the single-support motion in a smooth motion of the walking apparatus in a liquid. From these formulas it is seen that the control in the joints of the moving leg in an examined case, despite its weightlessness, are not equal to zero, in contrast to the situation when the apparatus moves in a nonresisting environment [1]. In order to achieve movement of the leg in a resisting environment in the thigh and shank, it is necessary to place compensating controlling points. They are, according to the value, equal to the momentum of the forces of resistance which act respectively on the leg and the shank. After the solution of the angular task for γ and definition of reactions, /17
according to formulas (2.2), (2.3) the controls in the joints are

clearly computed (if trajectories are assigned of the motion of the points of support and foot of the moving leg, and also the path being followed).

3. Computation of Forces and Momentums of Forces of Hydrodynamic Resistance which Act on the Walking Apparatus

For the calculation of angle γ it is necessary to know how to compute the forces and momentums of forces of the hydrodynamic resistance which act on the walking apparatus. Let us assume that the fluid is incompressible; the forces of viscid friction are negligibly small in comparison with the forces of hydrodynamic pressure; the calculation of the hydrodynamic forces and momentums which act on the various parts of the apparatus may be arrived at independently, without calculation of the interference; the motion of the apparatus is quasistationary in a sense that for the calculation of the forces of hydrodynamic resistance it is possible to use the formulas which are correct in the case of stationary streamlining of the bodies (the unstationariness of the streamlining in a known approximation has already been calculated by us by means of introducing the combined masses of the body of the apparatus).

Using the assumptions made, let us calculate first the forces and momentums of hydrodynamic resistance which act on the body of the apparatus. With calculation of these forces and momentums let us consider the movement of the body-sphere as being set, and the velocity of this motion equal to the velocity of the center of the sphere. The hydrodynamic effects which are connected with the revolution of the oncoming flow of fluid on the sphere reduces to a single force which is applied to the center of the sphere. This force is given by the formula [4]

$$\vec{F}_{\kappa} = -c_{\kappa} V_0 \vec{V}_0 , \quad c_{\kappa} = \frac{1}{2} c_{c\varphi} (Re) \pi R_{c\varphi}^{2} \rho , \qquad Re = \frac{2 V_0 R_{c\varphi}}{\nu} , \quad V_0 = | \vec{V}_0 | . \tag{3.1}$$

where $C_{c\phi}(Re)$ is the coefficient of resistance which is dependent /18 on the Reynolds number Re; ρ and ν are respectively the density and coefficient of the kinematic viscosity of the fluid.

The dependence of the coefficient $C_{c\phi}$ on the Reynolds number is shown in the following table 4.

Table 4

Re	0.I	I	IO	IO^2	IO^3	IO^4	IO^5	IO^6
$C_{c\varphi}$	245	28	4.4	I.I	0,46	0,42	0,49	0,I4

In the work the motion of the apparatus is numerically investigated with $R_{c\phi}$ - 1 and $V_o \sim$ 1m/sec.

With these conditions, as the value shows

$$Re = \frac{2V_0 R_{c\varphi}}{\nu} = \frac{2\cdot 1\cdot 1}{0.0173\cdot 10^{-4}} \approx 10^6$$

the Reynolds number has a great value. Therefore the value of the coefficient $C_{c\phi}$ in the formula (3.1) in our case must be derived for great Reynolds numbers.

As has been noted, (3.1) was received as a result of the solution of the task of stationary streamlining of the sphere by a viscid, incompressible fluid; the hydrodynamic effects of the unstationary streamlining have been disregarded. However, these effects add to the simple value, for example, on the basis of the task concerning the periodic oscillatory movement of the sphere in a viscid fluid which was examined in the work [6]. In this work is noted the following expression for the momentum M_o of forces which act from the side of the fluid on the sphere which achieves a periodic oscillatory motion.

$$M_0 = \mathrm{Real}\,M, \quad M = \frac{4\sqrt{2}}{3}\pi R^4 \sqrt{\eta\rho\omega}\,(i-1)\Omega, \qquad (*)$$
$$\Omega = \Omega_0 e^{i\omega t},$$

where η is the viscosity, Ω is the angular velocity of the oscillations of the sphere, ω is the frequency of the change of the angular velocity, R is the radius of the sphere. The formula (*) is valid /19
in the hypothesis $\rho R^2 \omega >> \eta$, which in the given case is realized. The values show that the value M_0 according to (*) does not exceed 3% from the base momentum of hydrodynamic resistance which is calculated on the basis of formula (3.1). These formulas lead to the following expression for the momentum of the forces M_K and M_{KF}, which are defined by the motion of the forces of hydrodynamic resistance on the body of the apparatus

$$M_K = C_K V_0 \rho_0 (V_n \cos\gamma - \dot{\gamma}\rho_0)$$
$$M_{KF} = C_K V_0 [H V_n - \dot{\gamma}\rho_0 (\Delta y_1 \sin\gamma + H\cos\gamma)]. \tag{3.2}$$

Thus,

$$M_K + M_{KF} = C_K V_0 [V_n(\rho_0 \cos\gamma + H) - \dot{\gamma}\rho_0 (\Delta y_1 \sin\gamma + H\cos\gamma + \rho_0)]$$
$$V_0 = (V_n^2 + \dot{\gamma}^2 \rho_0^2 - 2 V_n \rho_0 \dot{\gamma} \cos\gamma)^{1/2}. \tag{3.2'}$$

Let us calculate now the forces and momentums of the hydrodynamic resistance which act on the legs of the apparatus. Let us take any leg and examine an arbitrary section of this leg (Fig. 6). An infinitely small element of this section of the leg $d\xi$ presents itself as a round thin cylinder.

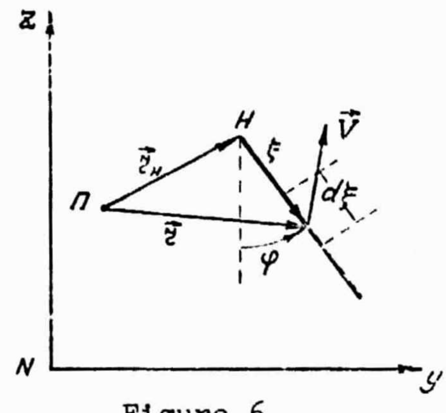

Figure 6

With motion of the legs of the apparatus the angles of incidence are close to 90°. Considering that it is possible to disregard the influence of viscosity on the streamlining of the cylinder under the angle of incidence, which is close to a right angle, we determine that hydrodynamic force acts along the normal to the axis of the cylinder. The force which acts on the element of the length of the leg $d\xi$, is described by the formula $dF_N = \overline{C}_p \rho (V_{\Pi}^2/2) d \cdot d\xi$, where $\overline{C}_p$ is the coefficient of any resistance of the cylinder at the angle of incidence $\alpha = 90^{\circ}$, V_{Π} is the component of velocity in the direction perpendicular to the axis of the cylinder. d is the diameter of the cross section of the section of the leg. In a vector form this may be recorded in the following fashion

$$
\begin{aligned}
&d\vec{F} = -\frac{1}{2}\bar{C}_p d\rho V_n |V_n| \vec{n}\, d\xi, \\
&\vec{\xi} = \xi \vec{\xi}_o, \quad \vec{\xi}_o = \vec{y}^{\circ} \sin\varphi - \vec{z}^{\circ} \cos\varphi, \\
&\vec{n} = \vec{x}^{\circ} \times \vec{\xi}_o = \vec{y}^{\circ} \cos\varphi + \vec{z}^{\circ} \sin\varphi, \\
&\vec{V} = \vec{V}_H + \dot{\varphi} \xi \vec{n}, \\
&V_n = (\vec{V}, \vec{n}) = (\vec{V}_H, \vec{n}) + \dot{\varphi} \xi .
\end{aligned}
\tag{3.3}
$$

Here ξ is the distance from the beginning of the section (point H) to the element $d\xi$; $\vec{V}$, $\vec{V}_H$ are the velocities respectively of the element $d\xi$ and the point H; $(\vec{V},\vec{n})$, $(\vec{V}_H,\vec{n})$ are the scalar products respectively of the vectors $\vec{V},\vec{n}$, and $\vec{V}_H$, $\vec{n}$. The coefficient of resistance $\vec{C}_p$ depends on the Reynolds number. /20

$$Re = \frac{dV}{\nu},$$

where V is the characteristic velocity of the motion of the element of the leg. In a characteristic situation with motion of the apparatus in water $V = V_o \sim 1$m/sec with $d \sim 0.09$ mm we will have

$$Re = \frac{dV}{\nu} \approx \frac{0.09 \cdot 1}{0.0173 \cdot 10^{-4}} \approx 0.5 \cdot 10^{5}$$

In conformance to the results of work [5] in a sufficiently large area of values of the Reynolds number being examined it is possible to accept $\overline{C}_p \approx 0.6$.

Considering that the streamlining of each element of the section move independantly, we receive the following expression for the full force which acts on the section

$$\vec{F} = -\frac{1}{2}\bar{c}_p d\rho \int_0^{\ell} V_n |V_n| \vec{n}\, d\xi \tag{3.4}$$

By an analogous method are computed the momentums of the forces of hydrodynamic resistance relative to the point of support (Fig. 6).

$$\begin{aligned} \vec{M} &= -\frac{1}{2}\bar{c}_p d\rho \int_0^{\ell} V_n |V_n| (\vec{z} \times \vec{n})\, d\xi = \\ &= -\frac{1}{2}\bar{c}_p d\rho (\vec{z}_H \times \vec{F}) - \frac{1}{2}\bar{c}_p d\rho \int_0^{\ell} V_n |V_n| (\vec{\xi} \times \vec{n})\, d\xi . \end{aligned} \tag{3.5}$$

Substituting $\vec{n}$, $\vec{\rho}$ and V_{Π} from (3.3) into (3.4) and (3.5) we find

$$\begin{aligned} &\vec{F} = F_y \vec{y}^{\circ} + F_z \vec{z}^{\circ} = -\frac{1}{2}\bar{c}_p \rho d f \vec{n}, \quad \vec{M} = M \vec{x}^{\circ}, \\ &F_y = -\frac{1}{2}\bar{c}_p \rho d f \cos\varphi, \quad F_z = -\frac{1}{2}\bar{c}_p \rho d f \sin\varphi, \\ &M = y_H F_z - z_H F_y - \frac{\bar{c}_p \rho d}{2} m = \\ &\quad = -\frac{1}{2}\bar{c}_p \rho d [f(y_H \sin\varphi - z_H \cos\varphi) + m], \\ &f = \int_0^{\ell} |(\vec{V}_H, \vec{n}) + \dot{\varphi}\xi| [(\vec{V}_H, \vec{n}) + \dot{\varphi}\xi]\, d\xi = \\ &\quad = \frac{\ell}{3} \frac{q+s}{|q|+|s|} (q^2 + s^2 + |q||s|), \\ &m = \frac{\ell}{4} f + \frac{\ell^2}{12} \left[q|q| + \frac{q}{|q|+|s|} \left(q^2 + s^2 + qs + q|s| \frac{q+s}{|q|+|s|} \right) \right], \\ &s = (\vec{V}_H, \vec{n}) = V_{Hy} \cos\varphi + V_{Hz} \sin\varphi, \quad q = s + \dot{\varphi}\ell, \\ &\vec{V}_H = V_{Hy} \vec{y}^{\circ} + V_{Hz} \vec{z}^{\circ}. \end{aligned} \tag{3.6}$$

From (3.6) we have

$$\begin{aligned} &\vec{M}_F = M_F \vec{x}^{\circ} = (\vec{z}_J - \vec{z}_n) \times \vec{F} \\ &M_F = \Delta y_J F_z + H F_y = -\frac{1}{2}\bar{c}_p \rho d f (\Delta y_J \sin\varphi + H \cos\varphi). \end{aligned} \tag{3.7}$$

Let us now apply formulas (3.6) and (3.7) for the calculation of forces and momentums of hydrodynamic resistance which act on the thigh and shanks of the walking apparatus.

For the thighs of the apparatus, assuming in formulas (3.6) and (3.7)

$$d = d_\delta, \quad \varphi = \alpha_i, \quad \dot\varphi = \dot\alpha_i, \quad l = a,$$
$$\vec V_H = \vec V_n = V_n \vec y^\circ,$$
$$y_H = 0, \quad z_H = 0$$

we receive the following formulas fo the forces $F_i^{(\delta)}$ and momentums $M_i^{(\delta)}$, $M_{iF}^{(\delta)}$ of hydrodynamic resistance, which act respectively on the thigh of the moving ($i = \pi$) and support ($i = \nu$) legs. /22

$$\vec F_i^{(\delta)} = F_{iy}^{(\delta)} \vec y^\circ + F_{iz}^{(\delta)} \vec z^{(o)}$$

$$\vec F_{iy}^{(\delta)} = -\frac{1}{2} \bar c_p \rho d_\delta f_i^{(\delta)} \cos\alpha_i, \quad F_{iz}^{(\delta)} = -\frac{1}{2} \bar c_p \rho d_\delta f_i^{(\delta)} \sin\alpha_i$$

$$M_i^{(\delta)} = -\frac{1}{2} \bar c_p \rho d_\delta m_i^{(\delta)}$$

$$M_{iF}^{(\delta)} = -\frac{1}{2} \bar c_p \rho d_\delta f_i^{(\delta)} (\Delta y_i \sin\alpha_i + H \cos\alpha_i)$$

$$f_i^{(\delta)} = \frac{a}{3} \frac{q_i^{(\delta)} + s_i^{(\delta)}}{|q_i^{(\delta)}| + |s_i^{(\delta)}|} (q_i^{(\delta)2} + s_i^{(\delta)2} + |q_i^{(\delta)}||s_i^{(\delta)}|) \tag{3.8}$$

$$m_i^{(\delta)} = \frac{a}{4} f_i^{(\delta)} + \frac{a^2}{12} \Big[q_i^{(\delta)} |q_i^{(\delta)}| + \frac{q_i^{(\delta)}}{|q_i^{(\delta)}| + |s_i^{(\delta)}|} (q_i^{(\delta)2} + s_i^{(\delta)2} +$$
$$+ q_i^{(\delta)} s_i^{(\delta)} + q_i^{(\delta)} |s_i^{(\delta)}| \frac{q_i^{(\delta)} + s_i^{(\delta)}}{|q_i^{(\delta)}| + |s_i^{(\delta)}|}) \Big]$$

$$s_i^{(\delta)} = V_n \cos\alpha_i, \quad q_i^{(\delta)} = s_i^{(\delta)} + \dot\alpha_i a, \quad i = \nu, \pi.$$

The formulas for the forces and momentums of hydrodynamic resistance which act on the shanks of the apparatus we receive from (3.6) and (3.7) by means of substitution

$$d = d_z, \quad \varphi = \beta_i, \quad \dot\varphi = \dot\beta_i, \quad l = b,$$
$$\vec V_H = \vec V_n + a\dot\alpha_i \vec n_i^{(\delta)}, \quad n_i^{(\delta)} = \vec y^\circ \cos\alpha_i + \vec z^\circ \sin\alpha_i,$$
$$\vec z_H = a \sin\alpha_i \vec y^\circ - a\cos\alpha_i \vec z^\circ.$$

As a result the following formulas are received for the forces $F_i^{(z)}$ and momentums $M_i^{(z)}$, $M_{iF}^{(z)}$ which act on the shank of the moving (i = π) and support legs (i = ν).

$$\vec{F}_i^{(г)} = F_{iy}^{(г)}\vec{y}^{\,0} + F_{iz}^{(г)}\vec{z}^{\,0},$$

$$F_{iy}^{(г)} = -\frac{1}{2}\bar{C}_p \rho d_z f_i^{(г)} \cos\beta_i, \quad F_{iz}^{(г)} = -\frac{1}{2}\bar{C}_p \rho d_z f_i^{(г)} \sin\beta_i,$$

$$M_i^{(г)} = -\frac{1}{2}\bar{C}_p \rho d_z [a f_i^{(г)} \cos(\alpha_i - \beta_i) + m_i^{(г)}], \qquad (3.9)$$

$$M_{iF}^{(г)} = -\frac{1}{2}\bar{C}_p \rho d_z f_i^{(г)} (\Delta y_\nu \sin\beta_i + H\cos\beta_i).$$

$$f_i^{(г)} = \frac{b}{3}\,\frac{q_i^{(г)} + s_i^{(г)}}{|q_i^{(г)}| + |s_i^{(г)}|}\left(q_i^{(г)2} + s_i^{(г)2} + |q_i^{(г)}||s_i^{(г)}|\right).$$

$$m_i^{(г)} = \frac{b}{4} f_i^{(г)} + \frac{b^2}{12}\Big[q_i^{(г)}|q_i^{(г)}| + \frac{q_i^{(г)}}{|q_i^{(г)}| + |s_i^{(г)}|}\Big(q_i^{(г)2} + s_i^{(г)2} +$$

$$+ q_i^{(г)} s_i^{(г)} + q_i^{(г)}|s_i^{(г)}|\,\frac{q_i^{(г)} + s_i^{(г)}}{|q_i^{(г)}| + |s_i^{(г)}|}\Big)\Big].$$

$$s_i^{(г)} = V_n \cos\beta_i + a\dot{\alpha}_i \cos(\alpha_i - \beta_i).$$

$$q_i^{(г)} = s_i^{(г)} + b\dot{\beta}_i,$$

$$i = \nu, \pi.$$

With the aid of formulas (3.8), (3.9) the forces of hydrodynamic resistance and their momentums which act on the legs of the apparatus are calculated conclusively in the following manner: /23

$$\vec{F}_\nu + \vec{F}_\pi = \vec{F}_\nu^{(б)} + \vec{F}_\nu^{(г)} + \vec{F}_\pi^{(б)} + \vec{F}_\pi^{(г)},$$

$$M_\nu + M_\pi = M_\nu^{(б)} + M_\nu^{(г)} + M_\pi^{(б)} + M_\pi^{(г)},$$

$$M_{\nu F} + M_{\pi F} = M_{\nu F}^{(б)} + M_{\nu F}^{(г)} + M_{\pi F}^{(б)} + M_{\pi F}^{(г)}.$$

Thus, the formulas received (3.1), (3.2), (3.2'), (3.8)-(3.10) which fully allow calculation of the forces and momentums of hydrodynamic resistance which act on the walking apparatus during its movement in a liquid. These formulas together with (1.4) allow the calculation of angle γ which describes the motion of the body of the apparatus relative to the point of support. /24

4. Numerical Method of Solving a Nonlinear Task of Walking

Solution of the task of walking is not reduced to solution of the angular task of (1.4), (1.6). The right parts of the differential equation (1.4) are described by the formulas (3.2), (3.2'), (3.8)-(3.10). The differential equation for γ is significantly nonlinear and complex. Therefore for the solution of the angular task (1.4), (1.6) it is necessary to use numerical methods. In the given work the numerical, iterative, generalized method of cords is used. Description of this method is given in the Appendix.

The value of the functions $\gamma(\gamma_0, \dot{\gamma}_0, t_0, t_0 + 2T)$ and $\gamma_0(\gamma_0, \dot{\gamma}_0, t_0, t_0 + 2T)$for the different values of γ_0 and γ_0, which must be known in the process of solving the angular task (1.4), (1.6) by the generalized method of cords, were received by a numerical integration of the differential equation (1.4) by the method of Runge-Kutt with a constant step. Here the necessary values of the angles $\alpha\nu(t)$,$\beta\nu$ (t), $\alpha\pi(t)$, $\beta\pi(t)$, which are formed by the thighs and the shanks of the legs with the axis z, were calculated along the assigned trajectories of the point of support and foot of the moving leg with the help of the algorithm used in work [2].

Like any local algorithm, the generalized method of cords requires knowledge of the zero approximation. As a zero approximation it is possible to use either the solution of the task of walking which is received in a linear approximation in the supposition of an insignificant amount γ(or $\gamma-\pi$), $\dot{\gamma}$ and $\ddot{\gamma}$ or the values $\gamma_0 = o(\gamma_0 = \pi)$, $\dot{\gamma}_0 = 0$. The indicated zero approximations are sufficiently good, in so far as calculations have shown, the oscillation of the body of the apparatus along γ and $\dot{\gamma}$ which are obtained as a result of solving the nonlinear task of walking is expressed in ananalytic form through quadratures. The solution of this task is given below. /25

5. Solution of the Task of Walking in a Linear Approximation

Let us assume that the equations of (1.4) have such solutions which have γ, $\dot{\gamma}$ and $\ddot{\gamma}$ sufficiently small. Then, applying the usual procedure of linearization to the formula for γ in (1.4), we receive the equation describing the small oscillations of the body of the apparatus:

$$\ddot{\gamma} + 2P_\gamma \dot{\gamma} + q_\gamma \gamma = m_K(t) + m_H(t).$$

Here

$$P_\gamma = \frac{C_K(\rho_0 + H)V_n \rho_0}{J_n^*}, \quad q_\gamma = \frac{P_u \rho_0 - \mu_A \rho_c}{J_n^*}, \tag{5.1}$$

$$m_K(t) = \frac{C_K(\rho_0 + H)V_n^2}{J_n^*} + \frac{P_A - P_B}{J_n^*} \Delta y_j(t).$$

$$m_H(t) = \frac{1}{J_n^*}\left[M_{xF}(t) + M_x(t) + M_{yF}(t) + M_y(t)\right],$$

$$J_n^* = J_n + M\rho_{np} H = J_{np}.$$

In the equation (5.1) the member $m_H(t)$ results from the action of the forces of hydrodynamic resistance on the legs of the apparatus, and the member $m_K(t)$ -- the action of the buoying force, the force of weight and the forces of hydrodynamic resistance on the body of the apparatus.

The general solution of the equation (5.1) may be obtained in the form

$$\gamma(t) = \gamma_K(t) + \gamma_H(t),$$

$$\gamma_H(t) = \gamma_H^{(n)}(t) + \gamma_H^{(a)}(t), \quad \gamma_K(t) = \gamma_K^{(n)}(t) + \gamma_K^{(a)}(t),$$

$$\gamma_K^{(a)}(t) = \frac{T\dot{\gamma}_0 - a_K}{\omega_\gamma T} e^{-P_\gamma \tau} sh(\omega_\gamma \tau) +$$

$$+ (\gamma_0 - B_K)\left[\frac{P_\gamma}{\omega_\gamma} sh(\omega_\gamma \tau) + ch(\omega_\gamma \tau)\right] e^{-P_\gamma \tau} -$$ /26

$$- \frac{a_K T}{1 - 2e^{-P_\gamma T} ch(\omega_\gamma T) + e^{-2P_\gamma T}} e^{-P_\gamma \tau}\left[\frac{P_\gamma}{\omega_\gamma} sh(\omega_\gamma \tau) + \right.$$

$$\left. + ch(\omega_\gamma \tau) + e^{-P_\gamma T}\left(\frac{P_\gamma}{\omega_\gamma} sh(\omega_\gamma (T - \tau)) - ch(\omega_\gamma (T - \tau))\right)\right],$$

$$\gamma_H^{(n)}(t) = a_H \frac{\theta(\tau)}{T} + b_H + \frac{a_H}{1-2e^{-p_\gamma T}ch(\omega_\gamma T)+e^{-2p_\gamma T}} \cdot$$
$$\cdot e^{-p_\gamma \theta(\tau)}\Big[\frac{p_\gamma}{\omega_\gamma} sh(\omega_\gamma \theta(\tau)) + ch(\omega_\gamma \theta(\tau)) +$$
$$+ e^{-p_\gamma T}\Big(\frac{p_\gamma}{\omega_\gamma} sh(\omega_\gamma (T-\theta(\tau))) - ch(\omega_\gamma (T-\theta(\tau)))\Big)\Big],$$

$$\gamma_H^{(a)}(t) = -\gamma_{H0}^{(n)} e^{-p_\gamma \tau} ch(\omega_\gamma \tau) - \Delta\dot{\gamma}_{H0}^{(n)} e^{-p_\gamma \tau}\frac{sh(\omega_\gamma \tau)}{\omega_\gamma},$$

$$\gamma_H^{(n)}(t) = \frac{1}{1-2e^{-p_\gamma T}ch(\omega_\gamma T)+e^{-2p_\gamma T}} \int_0^T e^{-p_\gamma (T+\theta(\tau)-\xi)} \cdot \tag{5.2}$$
$$\cdot\Big[\frac{sh(\omega_\gamma (T+\theta(\tau)-\xi))}{\omega_\gamma} - e^{-p_\gamma T}\frac{sh(\omega_\gamma(\theta(\tau)-\xi))}{\omega_\gamma}\Big] m_H(t_0+\xi)\,d\xi +$$
$$+ \int_0^{\theta(\tau)} e^{-p_\gamma(\theta(\tau)-\xi)} sh(\omega_\gamma(\theta(\tau)-\xi))\, m_H(t_0+\xi)\,d\xi,$$

$$\gamma_{H0}^{(n)} = \gamma_H^{(n)}(t_0) = \frac{1}{1-2e^{-p_\gamma T}ch(\omega_\gamma T)+e^{-2p_\gamma T}} \cdot$$
$$\cdot \int_0^T e^{-p_\gamma (T-\xi)}\Big[\frac{sh(\omega_\gamma (T-\xi))}{\omega_\gamma} + e^{-p_\gamma T}\frac{sh(\omega_\gamma \xi)}{\omega_\gamma}\Big] m_H(t_0+\xi)\,d\xi,$$

$$\Delta\dot{\gamma}_{H0}^{(n)} = \dot{\gamma}_H^{(n)}(t_0) + p_\gamma \gamma_H^{(n)}(t_0) =$$
$$= \frac{1}{1-2e^{-p_\gamma T}ch(\omega_\gamma T)+e^{-2p_\gamma T}} \int_0^T e^{-p_\gamma (T-\xi)}\big[ch(\omega_\gamma (T-\xi)) -$$
$$- e^{-p_\gamma T} ch(\omega_\gamma \xi)\big] m_H(t_0+\xi)\,d\xi,$$

$$\theta(\tau) = \tau - \Big[\frac{\tau}{T}\Big] T, \qquad \tau = t - t_0,$$

/27

$$\omega_\gamma = \begin{cases} |\omega_\gamma|, & \text{если } p_\gamma^2 - q_\gamma > 0 \\ i|\omega_\gamma|, & p_\gamma^2 - q_\gamma < 0, \end{cases} \qquad i^2 = -1,$$

$$|\omega_\gamma| = (|p_\gamma^2 - q_\gamma|)^{1/2},$$

$$b_H = 2\frac{p_\gamma}{h} a_H d_H - d_H - \frac{h_s}{h} a_H,$$

$$a_H = \frac{P_A - P_B}{P_A \rho_C - P_B \rho_0}\, h,$$

$$d_H = \frac{c_H(\rho_0 + H) V_H^2}{P_A \rho_C - P_B \rho_0}.$$

Here γ_0 and $\dot{\gamma}_o$ are the values of $\gamma(t)$ and $\dot{\gamma}(t)$ at the initial point of time t_o.

If $P_\gamma^2 - q_\gamma < 0$, then as is evident from formulas (5.2), $\omega\gamma$ is the virtual value. In this case $\gamma(t)$ is calculated according to those formulas of (5.2), only in them it is necessary to substitute $\omega\gamma$ for $|\omega\gamma|$, ch for cos and sh for sin. In addition, from the formulas of (5.2) it follows that in the linear approximation the solution of $\gamma(t)$ is presented in the form of the sums of two components, one of which $\gamma_K(t)$ results from the action of exterior forces on the body of the apparatus, and the second $\gamma_H(t)$ by the action of exterior forces on its legs.

In turn, each component is the sum of two parts -- the periodic $(\gamma_K^{(n)}(t),\ \gamma_H^{(n)}(t))$ with a period of T and a nonperiodic or aperiodic $(\gamma_K^{(a)}(t),\ \gamma_H^{(a)}(t)$.

In order that the solution of $\gamma(t)$ be periodic, it is necessary and sufficient that $\gamma_K^{(a)}(t) + \gamma_H^{(a)}(t) = 0$, which may be achieved by a selection of corresponding initial data γ_o and $\dot{\gamma}_o$. The initial data γ_o and $\dot{\gamma}_o$ which provides the periodicity of $\gamma(t)$ is defined clearly and equals

$$\gamma_0 = \gamma(t_0) = \gamma_K^{(n)}(t_0) + \gamma_H^{(n)}(t_0),$$

$$\gamma_K^{(n)}(t_0) = \beta_K + \frac{\alpha_K}{1 - 2e^{-p_\gamma T}ch(\omega_\gamma T) + e^{-2p_\gamma T}} \cdot \left[1 + e^{-p_\gamma T}\left(-ch(\omega_\gamma T) + \frac{p_\gamma}{\omega_\gamma} sh(\omega_\gamma T)\right)\right]$$ /28

$$\gamma_H^{(n)}(t_0) = \frac{1}{1 - 2e^{-p_\gamma T}ch(\omega_\gamma T) + e^{-2p_\gamma T}} \cdot \int_0^T e^{-p_\gamma (T-\xi)}\left[\frac{sh(\omega_\gamma (T-\xi))}{\omega_\gamma} + e^{-p_\gamma T}\frac{sh(\omega_\gamma \xi)}{\omega_\gamma}\right] m_H(t_0 + \xi)\, d\xi, \qquad (5.3)$$

$$\dot{\gamma}_0 = \dot{\gamma}(t_0) = \dot{\gamma}_K^{(n)}(t_0) + \dot{\gamma}_H^{(n)}(t_0).$$

$$\dot{\gamma}_K^{(\Pi)}(t_0) = \frac{1}{T} a_K + \frac{a_K(\omega_\gamma^2 - p_\gamma^2)\, e^{-p_\gamma T}}{1 - 2e^{-p_\gamma T} ch(\omega_\gamma T) + e^{-2p_\gamma T}} \cdot \frac{sh(\omega_\gamma T)}{\omega_\gamma} ,$$

$$\dot{\gamma}_H^{(\Pi)}(t_0) = -p_\gamma \gamma_H^{(\Pi)}(t_0) + \frac{1}{1 - 2e^{-p_\gamma T} ch(\omega_\gamma T) + e^{-2p_\gamma T}} \cdot$$

$$\cdot \int_0^T e^{-p_\gamma (T-\xi)} [ch(\omega_\gamma (T-\xi)) - e^{-p_\gamma T} ch(\omega_\gamma \xi)]\, m_H(t_0 + \xi)\, d\xi .$$

The periodic solution has the form

$$\gamma^{(\Pi)}(t) = \gamma_K^{(\Pi)} + \gamma_H^{(\Pi)}(t) \quad , \tag{5.4}$$

where $\gamma_K^{(\Pi)}(t) = \gamma_H^{(\Pi)}(t)$ are given by the formulas of (5.2).

Analysis of the formulas of (5.2) shows that if $q\gamma > 0$, then the solution of $\gamma(t)$ asymptotically approaches the periodic solution of $\gamma^{(\Pi)}(t)$. This indicates that in the case of $q\gamma > 0$ the periodic /29 oscillatory motion of the body of the apparatus is stable. If $q\gamma < 0$ then this motion is unstable.

Let us note that by "stability" and "instability" is understood here the usual stability and instability according to Lyapunov of the periodic solution of $\gamma(t)$ of equation (5.1) relative to the perturbation of $\Delta\gamma_0$, $\Delta\dot{\gamma}_0$ of the initial data γ_0, $\dot{\gamma}_0$. In terms of the initial dynamic task this means that the question lies in the conditional invariability of the assigned forward motion of the point of support of the legs and the legs themselves. The result of this invariability of the assigned forward motion (and motion of the legs) is the immobilization of the dynamic parameters which enter into equation (5.1). So, for example, the parameters considered invariable are V -- the velocity of the forward motion of the point of support of the legs of the apparatus, H -- the height of the point of support of the legs above the surface and so forth.

Establishment of the task of motion stability on the whole (relative to the entire series of perturbations) leads to the task of control -- the task of full walking stabilization of the apparatus. In the present work the task of full stabilization is not examined.

The described effect of stability does not have a place in the analogous situation with the motion of the apparatus in nonresisting surroundings [1]. The noted feature of apparatus motion in a liquid is explained by the simultaneous action on the body of the buoying force and the forces of hydrodynamic resistance (in the absence of a liquid, these forces are not present).

Because for the examined case of body position of the apparatus, close to the upper vertical, we have $J_n^* > 0$, then the conditon

$$q_\gamma = \frac{P_B \rho_0 - P_A \rho_c}{J_n^*} > 0 \tag{5.5}$$

indicates that the reducing momentum of the buoying force of the body relative to the point of support of the legs is greater than the momentum of its force of gravity. This condition provides the stability (in the sense indicated above) of the periodic motion of the body with the upper -- relative to the point of support of the legs -- position of the center of masses of the apparatus. The action on the body of the hydrodynamic forces of resistance, which are dissipative, exceed the stability at the symptotic. /30

If the momentums of the forces which act on the legs are considerably less in comparison with the momentums of the forces which act on the body of the apparatus, then it is then possible to set $m_H(t) \approx 0$ and for $\gamma(t)$ the following approximate value is received

$$\gamma(t) \approx \gamma_H^{(0)}(t) + \gamma_H^{(n)}(t),$$

which in an explicit manner is expressed through the elementary functions. In this case, as analysis of the formulas for $\gamma(t)$ show, in order for the procedure of linearization to be valid, it is necessary to require that

$$\begin{gathered}|\gamma_0| \ll 1, \quad |\dot{\gamma}_0 T| \ll 1, \\ \left|\frac{(P_A - P_0)h}{(P_0\rho_0 - P_A\rho_c)}\right| \ll 1, \quad \left|\frac{C_N(\rho_0 + H)V_n^2}{P_0\rho_0 - P_A\rho_c}\right| \ll 1\end{gathered} \tag{5.6}$$

In an analogous manner, in a linear approximation it is possible to examine the motion in a case when the body of the apparatus is close to the lower vertical position, i.e. receive and examine the solution of the equation for γ(1.4) in the area $\gamma = \pi$, $\dot{\gamma} = 0$, considering $\Delta\gamma = \gamma - \pi$, $\Delta\dot{\gamma}$ and $\Delta\ddot{\gamma}$ with small values. In this case, the equation and formulas describing the change of $\Delta\gamma$ with time, are received from the equations and formulas for γ when $\gamma \simeq 0$ (formulas (5.1)-(5.4), if set in them

$$\gamma = \Delta\gamma, \quad J_n^* = J_n - M\rho_{np}H, \quad q_\gamma = -(P_0\rho_0 - P_A\rho_c)/J_n^*$$

The conditions for the applicability of the linear theory in /31
the examined case have that form (5.6), only γ_0 must be substituted for $\Delta\gamma_0 = \gamma_0 - \pi$. The qualitative derivation about the character of body motion relative to the point of support of the legs in the area $\gamma = \pi$, $\dot{\gamma} = 0$ remains the same as in the case of $\gamma = 0$, $\dot{\gamma} = 0$. Only the conditions of the asymptotic stability of the oscillations of angle γ change. In this case, it turns out that the oscillations will be asymptotically stable if

$$(P_0\rho_0 - P_A\rho_c)/(J_n - M\rho_{np}H) < 0 \tag{5.7}$$

and unstable if

$$(P_0\rho_0 - P_A\rho_c)/(J_n - M\rho_{np}H) > 0 \tag{5.8}$$

Here it is relevant to note that according to the base equation of motion (1.1), the momentum of the forces of control act on the apparatus, which --through the support reaction -- depends on the angular acceleration of the apparatus. Thus the angular acceleration enters into the equation of motion not only through the change of kinetic momentum of the apparatus, but also directly -- through the controls. In the examined planar task this is equivalent to the introduction into the examination of any effective "introduced" momentum of inertia -- of the coefficient entering into the equation of motion by a multiple factor with the ang''lar acceleration. The effective momentum of inertia, generally speaking, depends on the angular position of the apparatus, that is, it is not constant. It can even change sign. In the examined case of small oscillations the effective momentum of inertia J_{Π}^{*} is a constant value. But the sign of this value -- dependent on the value of the parameters -- may be positive or negative, which in turn, acts on the stability of the body motion of the apparatus in the sense earlier indicated according to the condition introduced above. And namely, in the case of oscillations of the body near the upper vertical position, the value of J_{Π}^{*} is positive: $J_{\Pi}^{*} = J_{\Pi} + M\rho_{\Pi p}H > 0$. In this case, as the condition /32 of stability was indicated (5.5) reduces to the condition of positiveness of the numerator in (5.5).

With oscillation of the body near the lower vertical position we have $J_{\Pi}^{*} = J_{\Pi}^{*} + M\rho_{\Pi p}H$, so that $J_{\Pi}^{*} > 0$ with $J_{\Pi} > M\rho_{\Pi p}H$ and $J_{\Pi}^{*} < 0$ with $J_{\Pi} < M\rho_{\Pi p}H$. The condition of stability for such a case has the form of (5.7) and is fulfulled if the numerator and denominator in (5.7) have different signs.

It is necessary to underline that the conditional stability defined above of the periodic oscillations of the body of the apparatus, as also all motion of the body, implies calculation of the controlling momentum in the hip joint of the apparatus. The great number of possible controlling momentums answer the assigned motion, being calculated by the formulas of (2.2) and are distinguished one from the other only through the various motions $\gamma(t)$ of the body.

According to (1.1) and (2.2) with the assigned motion of the legs of the apparatus, the controlling momentum $u^{(\Pi)}(t, \gamma, \dot{\gamma}, \ddot{\gamma})$ may be varied only on account of the variation of $\Delta\gamma$, $\Delta\dot{\gamma}$, $\Delta\ddot{\gamma}$ of the oscillation of the body which is also calculated with examination of the stability of the periodic oscillation $\gamma(t)$ of the body of the apparatus. The corresponding variation of control is defined by the formula

$$\Delta u_{K}^{(n)} = \Delta(J_{np}\ddot{\gamma}) = \pm M\rho_{np} H \Delta\ddot{\gamma}$$

where the sign "plus" answers the variation near the value $\gamma = 0$, the sign "minus" -- the variation near $\gamma = \pi$.

In concluding this paragraph let us examine one significantly nonlinear effect, important for the understanding of the mechanism of underwater walking. The periodic solution of equation (1.4) could be sought in the form of a Foure series, preliminarily having expanded to series the periodic coefficients of this equation. Realization of the procedure is hardly efficient in view of the great amount of computation; but the main, constant member of the periodic solution which gives a median position of the body, with a good approximation received from equation (1.4) by the establishment of $\gamma = \gamma_0 =$ const /33 and with the retention of only the main, constant, members in the expansion of the coefficients leads to the formula

$$\sin\gamma_0 = \frac{(P_A - P_B)(h_2 - 2h) + \langle M_\Sigma \rangle}{P_A\rho_c - P_B\rho_0} . \qquad (5.9)$$

Here $\langle M_\Sigma \rangle$ is the median value of the summated momentum of the forces of hydrodynamic resistance.

The condition $|\sin\gamma_0| \leq 1$, which is

$$-1 \leq \frac{(P_A - P_B)(h_2 - 2h) + \langle M_\Sigma \rangle}{P_A\rho_c - P_B\rho_0} \leq 1 \qquad (5.10)$$

presents a necessary condition of the existence of the sought periodic, compensating motion of the body, which is the condition of the occurrence of the walking being examined. In the absence of hydrodynamic resistance $\langle M_\Sigma \rangle = 0$ and $P_B = 0$, and the condition (5.10) gives

$$-\rho_c+2h \le h_2 \le \rho_c+2h.$$

(5.11)

(here it is accepted $\rho_c > 0$). The inequalities of (5.11) define the series of possible configurations of the walking of the apparatus; these configurations with the assigned height H of the point of support above the surface and the assigned length h of the step, are clearly defined by the support section h_2.

The condition (5.10) may be recorded so:

$$|h_2-2h+\langle M_\Sigma\rangle \frac{1}{P_{эф}}| \le |\rho_c+\frac{P_B}{P_{эф}}(\rho_c-\rho_0)|,$$

$$P_{эф}=P_A-P_B>0. \quad (5.12)$$

As it is possible to understand from the formulas written out earlier for the momentum of the forces of resistance, we have $\langle M_\Sigma \rangle > 0$; accepting for the values $\rho_c \sim \rho_0$, we receive from (5.12)

$$-\rho_c+2h-\frac{\langle M_\Sigma\rangle}{P_{эф}} \le h_2 \le \rho_c+2h-\frac{\langle M_\Sigma\rangle}{P_{эф}} < \rho_c+2h.$$

(5.13)

This means that the diapason of possible values of the support section h_2 in underwater walking displaces to the side lesser values in com- /34 parison with the diaposon in terrestrial walking. From Fig. 3 it is possible to understand that this corresponds, in turn, to those walking configurations in which the point of support of the legs is shifted farther forward in motion, so that the body would be "placing its chest on the water," moving somewhat forward relative to the legs. This fact is accounted for in numerical calculations where value $h_2 = 0$ is accepted. With $h_2 = 0$ the point of support of the legs at the moment of change of the support leg is projected precisely on the point of support. Presentation of the result of calculation is given in the following paragraph.

6. Results of Calculations

Let us describe several results of calculations. The motion of a walking apparatus in water was examined. For the base variation, the variant with the following values of parameters was chosen:

Radius of the sphere of the apparatus	$R_{c\phi} = 0.9$ m
Distance from the point of support to the center of the sphere	$\rho_o = 0.6$ m
Distance from the point of support to the center of masses of the sphere	$\rho_c = 0.3$ m
Weight of the apparatus	$P_A = 3180$ kg
Buoying force	$P_B = 3130$ kg
Effective weight of the apparatus	$P_A - P_B = 50$ kg
Velocity of the point of support of the legs	$V_\Pi = 0.5$ m/sec
Height of the point of support above the supporting surface	$H = 1.5$ m
Length of step	$h = 1$ m
Length of time of step	$T = h/V_\Pi = 2$ sec
Density of water	$\rho = 104$ kg sec^2/m^4

In addition, in the base variant, the configuration of motion is defined by the fact that the point of support of the legs at the moment of change projects to the point of support ($h_1 = h$, $h_2 = 0$). /35

In Fig. 7 results are shown of calculation of the base variant. In this figure behavior with time of angle γ is shown -- the digression of the axis of the apparatus from the vertical, the components R_y, R_z of the reaction of support, the controls u_j^i in the thighs and shanks of the supporting and moving legs. Hereafter these functions as a set will be called the "dynamic characteristics" of the apparatus. Behavior of the apparatus which follows after the results shown in Fig. 7, seemed to little resemble the behavior of an analogous apparatus in the absence of resisting surroundings.

Comparison of the given dynamic characteristics with the dynamic characteristics of [1] shows a qualitative and quantitative difference-- in the behavior of the horizontal component R_y of the reaction of support. For a terrestrial apparatus the function $R_y(t)$ is almost linear and changes sign in the middle of the step. For an underwater device this function is significantly not monotonic and in almost all parts of motion is positive. The maximum value of it ~ 30 kg with an effective weight of 50 kg (by the effective weight of the apparatus is understood the difference between its weight and the buoying force of the body). This is a factor of 10 greater than for an equivalent terrestrial apparatus. Such behavior of the horizontal component of the support reaction is explained by the fact that it compensates the resistance of the environment. The vertical component of reaction is close to a constant value equal to the effective weight (50 kg). The controlling momentums in the joints of the support leg are significantly nonmonotonic, in contrast to the terrestrial case. Controls in the thigh and shank of the support leg attain a maximum (by the module) ~ 40 kg m. Controls in the moving leg are much less (of an order of 5 kg m); they are directed only on the passage of water resistance. /37 Controls in the joints are comparable in value with the control in a terrestrial apparatus. Finally, the amplitude of oscillations of the body is not great (~ 3°).

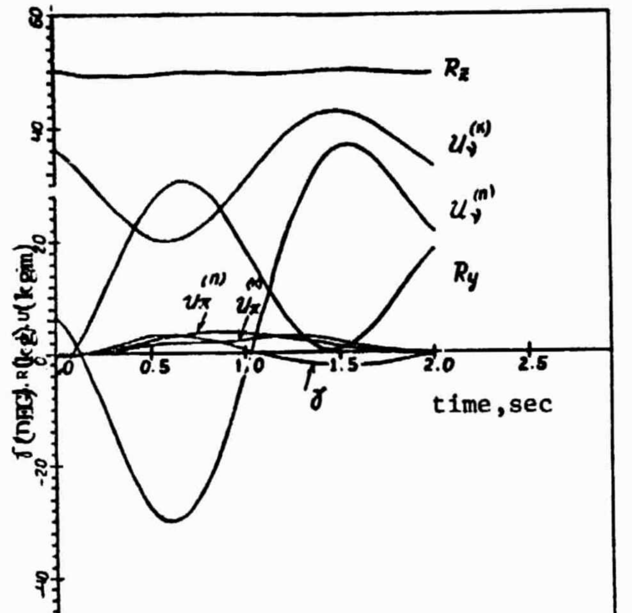

Figure 7. Dynamic characteristics of the apparatus in its motion on one step. P_A = 3180 kg, P_B = 3130 kg, V_{Π} = 0.5 m/sec, $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.3 m.

The noted difference in the dynamics of an underwater and terrestrial apparatus does not always take place. The conditions in which the dynamic characteristics of the underwater apparatus bear a significantly oscillatory nature, and also the mechanisms of these oscillations will be described below.

As calculations show, the decrease of the distance ρ_c between the point of support and the center of masses leads to an increase of the frequency of oscillations of the dynamic characteristics: in that temporary section appear a great quantity of minimums and maximums of functions. This tendency of increase of the frequency of oscillations with a decrease of ρ_c is seen in the picture of Fig. 8. In this figure, results are shown of calculation for a very small value $\rho_c = 10^{-5}$m (all remaining parameters are as in the base variant). It is possible to explain the noted effect theoretically also (in a linear approximation). From the formulas for the frequency in the natural oscillations of the body $\omega\gamma$ in (5.2) we find:

$$\omega_\gamma = (C_0 - C_I\rho_c)^{1/2},$$

where C_0, C_I are positive, independent of ρ_c, constants. From here it follows that with a decrease of ρ_c the frequency of natural oscillations of the body of the apparatus increases, and this means that the frequency of oscillations of the dynamic characteristics caused by it also increase.

In Fig. 9 the results are shown of calculation of a variant, the initial parameters of which are distinguished from the parameters of the base variant only by the velocity of motion: $V_\Pi = 0.2$ m/sec. The change of velocity of motion acts little on the frequency of natural oscillations, which is evident from comparison of Fig. 9 with Fig. 7: for one and the same interval an identical number of extremes of functions take place. Increase of the number of oscillations in a period of the length of time of a step is explained by the increase of this very period. /40

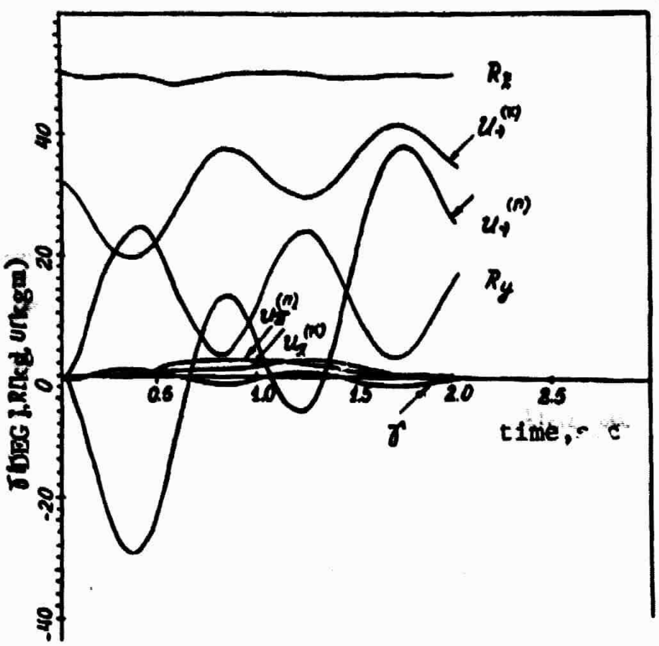

Figure 8. The dynamic characteristics of the apparatus in its motion on one step. P_A = 3180 kg, P_B = 3130 kg, V_{Π} = 0.5 m/sec, $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.0[4]1 m.

The amplitude of oscillations of the body somewhat increases, but remains small: the initial value γ_o = $-0^o.6$, and the amplitude of oscillations near this value have an order ~40.

With an increase of the weight of the apparatus, the maximum values of the forces of reaction and the amplitude of oscillations of the body arise, which Fig. 10 illustrates (this supports also the qualitative analysis of the motion of the apparatus carried out in the boundaries of linear theory). In this figure the initial parameters

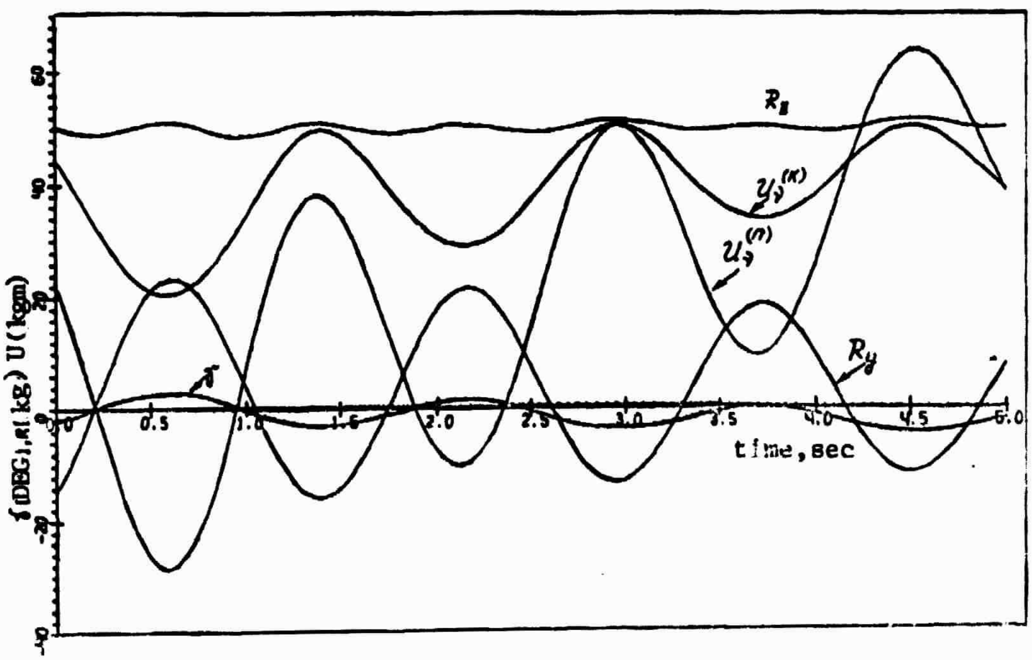

Figure 9. Dynamic characteristics of the apparatus in its motion on one leg. P_A = 3180 kg, P_B = 3130 kg, V_{Π} = 0.2 m/sec, $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.3 m.

are distinguished from the parameters of the base variant only by the weight of the apparatus P_A = 3330 kg, such that there is a difference of weight and buoying force $P_A - P_B$ = 200 kg. The maximum value of the horizontal component of reaction here becomes ~ 100 kg, the maximum amplitudes of oscillations of the body ~ 20°.

The oscillatory nature of the dynamic characteristics is induced by the fact that in the examined case

$$P_B \rho_0 - P_A \rho_c > 0 \qquad (6.1)$$

and the summated momentum of the force of gravity and Archimedean force tends to put the body of the apparatus in the position $\gamma = 0$. Oscillation of all the charcteristics is a reflection of the natural oscillations of the body near the position $\gamma = 0$.

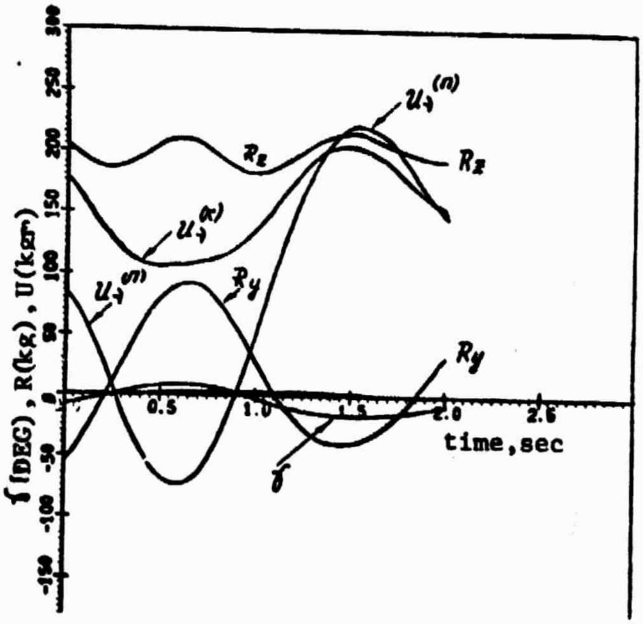

Figure 10. Dynamic characteristics of the apparatus in its motion in one step. P_A = 3330 kg, P_B = 3130 kg, V_Π = 0.5 m/sec, $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.3 m.

It is possible to try to attain a planar, almost nonoscillatory change of reactions and controls in time T of the length of one step, having selected this length sufficiently small in comparison with the period Tc of the natural oscillations of the system. So, for example, having imposed the condition

$$T \leq \frac{1}{4} T_c , \qquad (6.2)$$

it is possible to expect that the dynamic characteristics in time T will not be able to complete a great number of oscillations and will not have more than one or two extremes.

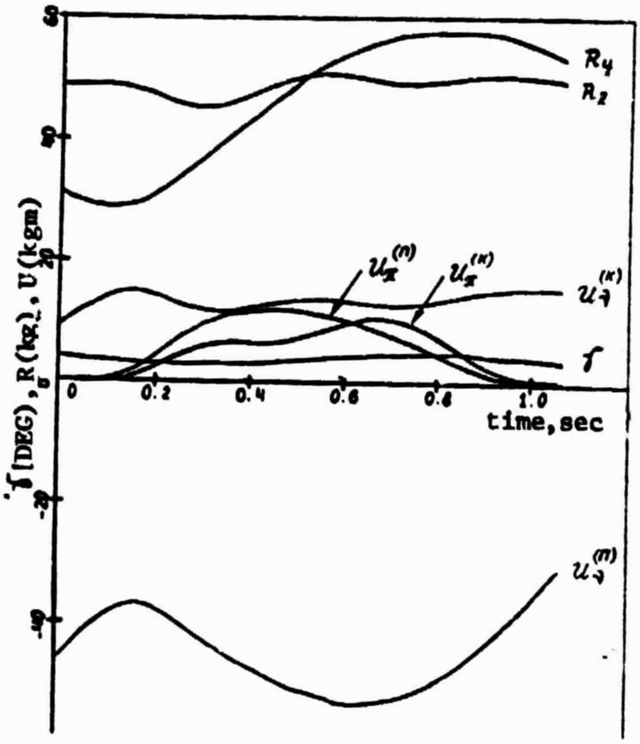

Figure 11. Dynamic characteristics of the apparatus in its motion in step. P_A = 3180 kg, P_B = 3130 kg, V_Π = 0.97 m/sec, $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.3 m.

In terms of the parameters of control of small oscillations (5.1) the condition (6.2) is recorded in the following form:

$$V_n \geq \frac{2h}{\pi} \frac{\sqrt{(P_B\rho_0 - P_A\rho_c)/J_n^*}}{\sqrt{[1+2hc_\kappa\rho_0(\rho_0+H)/\pi J_n^*]^2}} \qquad (6.3)$$

The characteristics of motion of the apparatus with a velocity V_Π = 0.97 m/sec, which corresponds to the sign of the equality in (6.3) (with values of the remaining parameters of the base variant) is shown in Fig. 11. It is obvious that the dynamic characteristics are, in this case, actually weakly oscillating.

The variants examined above answer the case of (6.1). If the parameters are selected in such a manner that (6.1) is not fulfilled, then the picture of change of the function characteristics will be such as in Fig. 12. (ρ_o = 0.6 m, ρ_c = 0.5995 m). Here, as in the base variant, at the moment of change of the step, the point of support projects on the support point. It is obvious that the characteristics change very smoothly in contrast to the case of fulfilling the conditions of (6.1). Let us note, however, in the examined case, the solution found is unstable, as in the tasks of terestrial walking. (Instability may have a positive consequence for the controllability of the apparatus.)

As has been shown earlier, at least two periodic modes of operation take place: in the area $\gamma = 0$ and in the area $\gamma = \pi$. The latter mode for those values of parameters, which are also in Fig.12

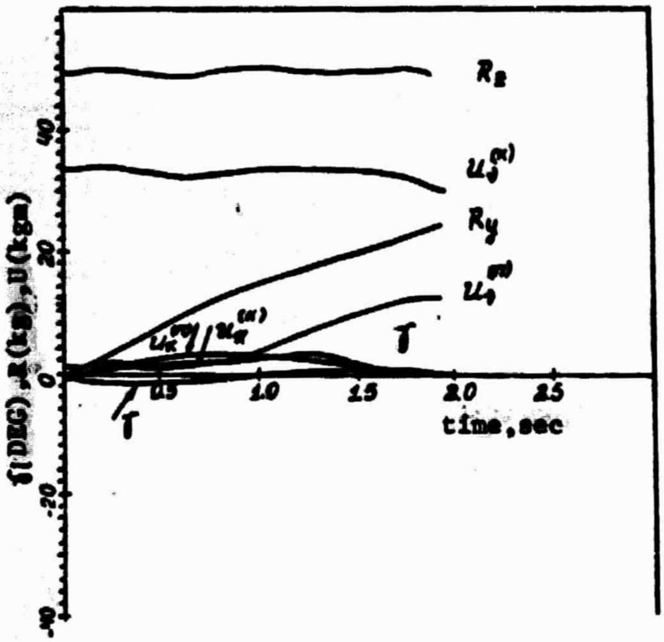

Figure 12. Dynamic characteristics of the apparatus in its motion on one step. P_A = 3180 kg, P_B = 3130 kg, V_{Π} = 0.5 m/sec, $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.5995 m.

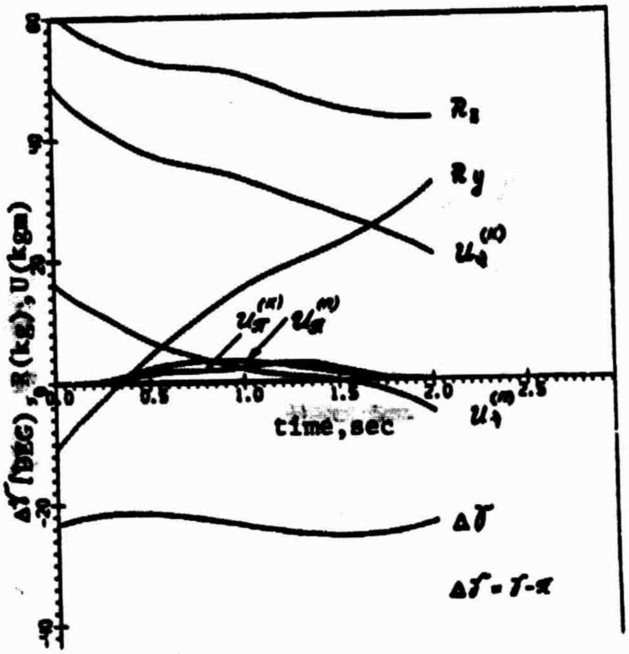

Figure 13. Dynamic characteristics of the apparatus in its motion on one step. P_A = 3180 kg, P_B = 3130 kg, V_{Π} = 0.5 m/sec. $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.5995 m (solution in the area $\gamma = \pi$, $\dot{\gamma} = 0$)

are shown in Fig. 13. Let us suggest that stability in the area of γ = 180° depends on the sign of the value $(P_B\rho_o - P_A\rho_c)/(J_{\Pi} - M\rho_{\Pi p}H)$, and stability in the area γ = 0 only on the sign of the value $P_B\rho_o - P_A\rho_c$. Therefore, instability in the area of γ = 0 does not certainly answer the stability in the area γ = 180°. Both modes may appear unstable (such as takes place in the cases shown in Fig. 12, 13).

In Fig. 14 the case of a stable mode is shown in the area γ = 180°. The parameters are taken from the base variant, which give a stable motion in the area γ = 0. Stability is kept also in the area γ = 180° owing to the fact that $J^*_{\Pi} = J_{\Pi} - M\rho_{\Pi\rho}H < 0$, as this was explained in the /47

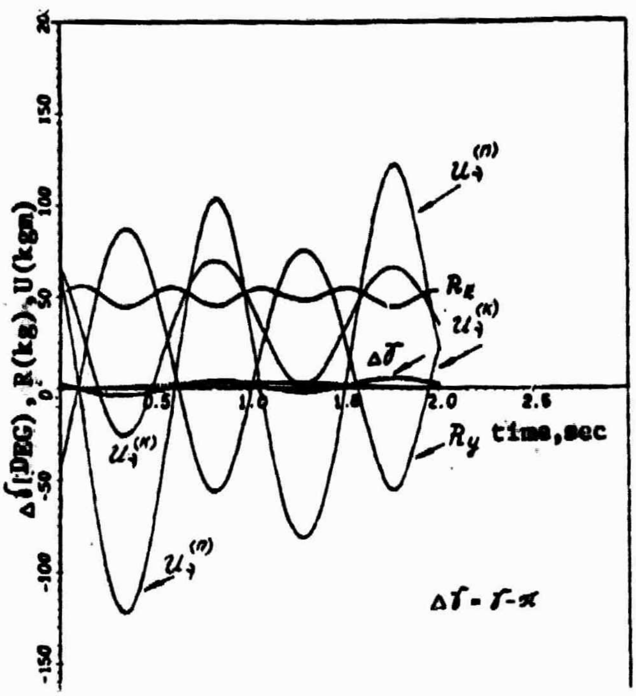

Figure 14. Dynamic characteristics of the apparatus in its motion on one step. $P_A = 3180$ kg, $P_B = 3130$ kg, $V_\Pi = 0.5$ m/sec, $R_{c\phi} = 0.9$ m, $\rho_o = 0.6$ m, $\rho_c = 0.3$ m (solution in the area $\gamma = \pi$, $\dot{\gamma} = 0$)

preceding paragraph. Very strong oscillations of all characteristics are noted -- the consequences of a small (by the module effective, "introduced") momentum of inertia J^*_Π).

It is useful to recall that stability (instability) here is understood in the sense indicated above, in Section 5, -- as stability of the angular motion of the body with invariability of the forward motion and motion of the legs.

In conclusion it is necessary to note that the disregard of the combined masses qualitatively changes nothing (all rules remain in force). Only quantitative relations are able to change. What has been said directly follows from the equation of motion of the apparatus. This is obvious from comparison of Fig. 7 and 15, where the results of calculations are shown of one and the same variant with regard (Fig. 7) and without regard (Fig. 15) of combined masses.

Conclusion

The main quantitative results of the calculations done are given in two consecutive tables. The maximum values, rounded off, of the values of reaction and controls are combined in them, and also

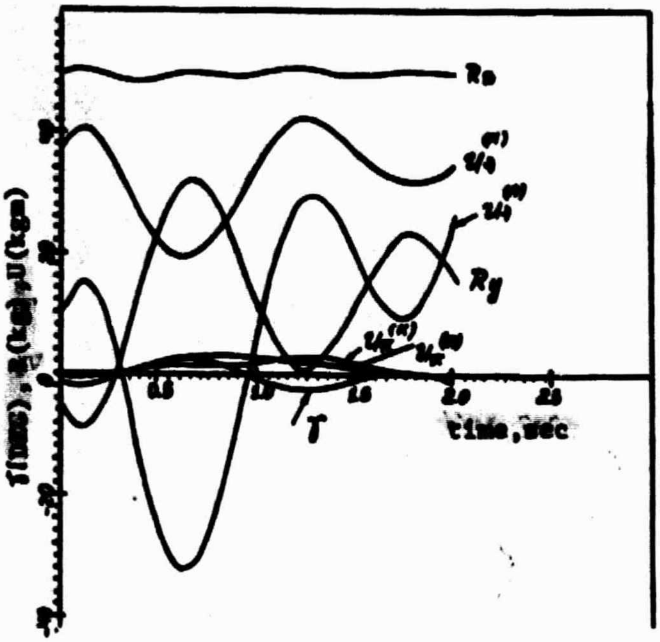

Figure 15. Dynamic characteristics of the apparatus in its motion on one foot. P_A = 3180 kg, P_B = 3130 kg, V_{Π} = 0.5 m/sec, $R_{c\phi}$ = 0.9 m, ρ_o = 0.6 m, ρ_c = 0.3 m. (Solution without calculation of combined masses)

the median values and amplitude of oscillation of the angle of inclination of the body of the apparatus. Shown also is the stable or unstable angular motion of the body in that limited sense which was examined in the work (with an invariable forward motion). The last graph of the table contains the conditional number "П" characterizing the smoothness of the dynamics of the apparatus: this is the "number of peaks" (maximums and minimums) in the controls and the horizontal component in the time of the length of one step.

Table 1 contains the indicated values for the apparatus

TABLE 1

P_a =50 kg / V, m/sec	stability устойчивость	$\lvert R_z\rvert_{max}$, kg	$\lvert R_y\rvert_{max}$, kg	$\lvert U\rvert_{max}$, kgm	$\gamma \pm \Delta\gamma$, deg	n
0,2	yes	50	25	50	$0^0 \pm 4^0$	6
0,5	yes	50	25÷ 30	40	$0^0 \pm 3^0$	2 ÷ 4
	no	50	25	35	$0^0 \pm 2^0$	0
	yes	55	90	120	$180^0 \pm 5^0$	4
	no	60	40	50	$155^0 \pm 2^0$	0
0,97	yes	50	60	50	$0^0 \pm 3^0$	1÷2

with an effective weight P_e = 50 kg. with various values of velocity of the apparatus. In Table 2 with a fixed value of velocity (V_{Π} = 0.5 m/sec) the effective weight of the apparatus is varied. /50

TABLE 2

Таблица 2

P_e \ V=0.5 m/sec	stability устойчивость	$\|R_z\|_{max}$, kg	$\|R_y\|_{max}$ kg	$\|U\|_{max}$, kgm	$\gamma \pm \Delta\gamma$	n
50	ДА yes	50	30	40	$0^\circ \pm 3^\circ$	2
200	по no	220	100	250	$0^\circ \pm 20^\circ$	2

It seems natural to consider such motion "good", which, with the other equal conditions provides a) a minimum of maximum values of the dynamic characteristics and b) a smoothness of change with time of these dynamic characteristics. Actually fulfullment of condition a) is advantageous energetically, and fulfillment of condition b) represents a simplicity of control by the apparatus in a nominal mode of operation. In aggregate fulfillment of conditions a) and b) make it possible to call it "good controllability" of the apparatus.

From the given tables it is obvious that the motions with a moderate speed meet these conditions, with an upper position of the center of masses relative to the point of support of the legs and moreover <u>unstable</u> in the sense introduced above. Stability mars the power engineering, particularly with a lower ($\gamma \sim 180^\circ$) position of the center of masses. In addition stability leads to a strong oscillation of the dynamic characteristics, whereas the unstable modes give smooth, monotonic functions.

Instability of the angular motion in the sense examined here does not present any danger. It is known that adequate changes of the law of control are able to stabilize similar instabilities. Stabilization is achieved for example, by a small systematic change of the length and length of time of the step [7].

From the second table it is also obvious that an increase of apparatus weight, naturally, leads to a corresponding increase of maximum values of the dynamic characteristics.

The conducted research of a model task of the dynamics of underwater two-legged walking allows the following conclusions to be made. /51

1. Modes of operation of motion exist which provide its conformability with periodic compensating oscillations of the body of the apparatus.

2. At least two classes of periodic oscillations of the body exist: with an upper and lower position of the center of masses of the body relative to the point of support of the legs.

3. The conditional stability of the angular motion of the body is understood in invariable forward motion of the point of support of the legs. If the maximum momentum of the Archimedean buoying force is greater than the maximum force of gravity, then the reducing momentum is positive and the "upper" mode of motion in the indicated sense is asymptotically stable. In an opposite case -- unstable.

4. Stability or instability of the "lower" mode is defined not only by the sign of the reducing momentum, but also by the sign of the introduced momentum of inertia. The asymptotic stability is achieved when these values have different signs.

5. The oscillatory nature of change of the dynamic characteristics of walking is accompanied by the stable mode of motion. These oscillations take place with the frequency of the natural oscillations of the apparatus. A smooth (often monotonic) nature of change of the dynamic characteristics is accompanied by an unstable mode of motion.

6. With the change of legs the values of the support reaction and of controls undergo a disruption which is a consequence of single-support movement.

7. The vertical component of the support reaction at the point of support is little distinguished from the difference between the force of the weight and the buoying force of the body. The horizontal component of the support reaction has this order: it is less than the vertical component only by one and one half to two times. Almost henceforth the horizontal component of reaction is positive, there /52
being a strong qualitative and quantitative difference in the behavior of the horizontal component of the underwater and equivalent terrestrial apparatus. This is explained by the fact that in underwater walking the horizontal component of reaction must compensate for the resistance of the enviornment.

8. The controlling momentums in the joints of the legs in order of value are such as the controlling momentums for an equivalent terrestrial apparatus. In the support leg the momentums in the hip joint are approximately one factor of that of the momentums in the shank. The momentums in the moving leg are small in comparison with the momentums in the support leg. In addition, the momentum in the hip joint on one leg, as a rule, changes sign while the momentum in the knee joint ~~in basis of sign~~ does not change sign and is positive.

9. The unstable modes require a lesser maximum value of controlling momentums in comparison with stable.

10. The unstable mode with an upper position of the center of masses of the body by the minimum of maximum values of the dynamic characteristics and by the smoothness of their behavior is most preferable. The worst in this sense is the stable mode with a lower position of the center of masses of the body.

In the present work, for the solution of the boundary task (1.4)-(1.6), the numerical, iterative, generalized method of cords was used. The basic idea of this method is included in the following. Let it be necessary for us to find the solution of a system of transcendental equations

$$f(x)=0\,, \tag{P1}$$

where $f'(x) = f_1(x),\dots,f_n(x)$ is the vector-function of the vector argument $x'=(x_1,\dots,x_n)$ of the dimension n (here and henceforth by the prime is meant the operation of transposition). Let us assume that we know several approximation $x^{(n)}$ of the solution of the system (P1). In the population $x^{(n)}$ let us approximate the nonlinear statement f(x) by the linear

$$f(x)\simeq A(x-x^{(n)})+B\,. \tag{P2}$$

Here A is (nxn) -- the matrix, and B is n -- the vector. Let us find the unknown matrix A and the unknown vector B from the following conditions.

Let us assign in the population $x^{(n)}$ the point $x^{(i)}(i = 0,1,\dots,n)$and compute the value $f(x^{(i)})$ $(i = 0,1,\dots,n-1)$. Then let us require that at the points $x^{(i)}(i = 0,1,\dots,n)$ the values of the nonlinear and linear statements coincided

$$f(x^{(i)})=A(x^{(i)}-x^{(n)})+B\,,\qquad i=0,1,\dots,n\,.$$

This is equivalent to the matrix equations

$$\Delta F=A\Delta X\,,\qquad B=f(x^{(n)})\,, \tag{P3}$$

where ΔF and Δx are (nxn) -- the matrixes of the form

$$\Delta F=\begin{pmatrix}\Delta f_1^{(0)} & \Delta f_1^{(1)} & \dots & \Delta f_1^{(n-1)}\\ \Delta f_2^{(0)} & \Delta f_2^{(1)} & \dots & \Delta f_2^{(n-1)}\\ \dots & \dots & \dots & \dots\\ \Delta f_n^{(0)} & \Delta f_n^{(1)} & \dots & \Delta f_n^{(n-1)}\end{pmatrix}$$

$$\Delta X = \begin{pmatrix} \Delta x_1^{(0)} & \Delta x_1^{(1)} & \dots & \Delta x_1^{(n-1)} \\ \Delta x_2^{(0)} & \Delta x_2^{(1)} & \dots & \Delta x_2^{(n-1)} \\ \dots & \dots & \dots & \dots \\ \Delta x_n^{(0)} & \Delta x_n^{(1)} & \dots & \Delta x_n^{(n-1)} \end{pmatrix}$$ /54

$$\Delta f_j^{(i)} = f_j(x^{(i)}) - f_j(x^{(n)}), \quad \Delta x_j^{(i)} = x_j^{(i)} - x_j^{(n)}.$$

Let us assume that the points $x^{(i)}$ are selected such that

$$\det \Delta x \neq 0.$$

Then from (P3) we find

$$A = \Delta F(\Delta X)^{-1}$$

which together with (P2) and (P3) gives

$$f(x) \approx \Delta F(\Delta X)^{-1}(x - x^{(n)}) + f(x^{(n)})$$

The following approximation $x^{(n+1)}$ is received in the usual manner by means of solution of the system of linear equations

$$\Delta F(\Delta X)^{-1}(x^{(n+1)} - x^{(n)}) + f(x^{(n)}) = 0$$

Supposing that

$$\det \Delta F \neq 0$$

we find

$$x^{(n+1)} = x^{(n)} - \Delta X(\Delta F)^{-1} f(x^{(n)}).$$

Having received $x^{(n+1)}$, let us compute $f(x^{(n+1)})$. Then among the points $x^{(0)}, x^{(1)}, \dots, x^{(n)}$ let us eliminate the most remote from $x^{(n+1)}$ points. The distance between points is considered in several assigned metrics. In the given work the distance $\rho(x^{(i)}, x^{(K)})$ between points $x^{(i)}$ and $x^{(K)}$ were computed by the formula

$$\rho(x^{(i)}, x^{(K)}) = \sum_{j=1}^{n} P_j (x_j^{(i)} - x_j^{(K)})^2,$$

where P_j are the assigned nonnegative weight coefficients. The number of the eliminated point i_* is defined by the condition

$$\rho(x^{(i_*)}, x^{(n+1)}) = \max_{0 \le i \le n} \rho(x^{(i)}, x^{(n+1)})$$

The following approximation of $x^{(n+2)}$ is calculated by the same formulas that were used for $x^{(n+1)}$, but only in these formulas it is necessary to substitute $x^{(i_*)}$ for $x^{(n)}$ (concurrently $f(x^{(i_*)})$ for $f(x^{(n)})$, if $i_* \neq n$, $x^{(n)}$ for $x^{(n+1)}$ (respectively $f(x^{(n)})$ for $f(x^{n+1})$). Having /55 obtained $x^{(n+2)}$ in an analogous manner let us calculate $x^{(n+3)}$, then $x^{(n+4)}$ and so forth. The iterative process ends if at the s iteration the conditions are fulfulled

$$|x_j^{(n+s)} - x_j^{(n+s-1)}| \leq \varepsilon_j \; ; \quad s \geq 1, \; j=1,2,\ldots,n \, ,$$

where ε_j are the assigned positive numbers which characterize the precision with which it is necessary to know the solution.

The weight coefficients P_j are selected from the considerations of dimensions. In particular it is possible to set

$$P_j = \frac{1}{\varepsilon_j^2} \qquad (j=1,\ldots,n),$$

if x_j is dissimilar, or

$$P_j = 1 \quad (j=1,\ldots,n)$$

if x_j is similar.

It is possible to show that the generalized method of cords converges if the zero approximation (points $x^{(0)}$, $x^{(1)}$,..., $x^{(n)}$ are sufficiently close to the solution. Here the speed of convergence of the method in proportion to the approximation to the solution strives to the speed of convergence of the method of Newton. Practical use of the generalized method of cords showed that the general volume of computation which must be done in order to receive a solution with the assigned precision is less than with the use of the method of Newton, despite the fact that the number of iterations which must be done is larger. The generalized method of cords by speed of convergence being no worse than the method of Newton, has one significant advantage: with its use it is not necessary to compute the matrix of partial products

$$\frac{\partial f(x)}{\partial x} = \left\{ \frac{\partial f_i}{\partial x_j} \right\}_{i,j=1}^{n}$$

which for many tasks has a considerable value. In addition, in contrast to the method of Newton, when for each iteration it is necessary to convert the matrix of n factor $\frac{\partial f}{\partial x}$, the iteration process in the generalized method of cords can be so limited, that on the first iteration the matrix will be converted by an n factor, and in all the following -- the matrixes only of the second order, which is very /56 important if n is important. The latter is achieved in the following manner. Let us designate through $\Delta F^{(1)}$ the matrix ΔF, which is computed in the first iteration by the points $x^{(0)}, x^{(1)}, \ldots, x^{(n-1)}$ see formula (P4). In the first iteration in the process of obtaining the first approximation $x^{(n+1)}$ it is necessary for us to convert the matrix of n factor $\Delta F^{(1)}$. According to the generalized method of cords, after the first approximation of $x^{(n+1)}$ is obtained, one of the points $x^{(0)}, x^{(1)}, \ldots, x^{(n-1)}$ is eliminated. Let the eliminated point prove to be the point with the number i. Then in the following iteration it will be necessary for us to convert the matrix

$$\Delta F^{(2)} = \begin{array}{ccccc} (f^{(0)}-f^{(n+1)} & f^{(1)}-f^{(n+1)} \ldots & f^{(n)}-f^{(n+1)} \ldots & f^{(n-1)}-f^{(n+1)}) \\ 1 & 2 & i & n \end{array}$$

where $f^{(K)}=f(x^{(K)}),\ f^{T}(x^{(K)})=(f_1(x^{(K)})\ldots f_n(x^{(K)})),\ (K=0,1,\ldots,n+1)\quad (f^{(j)}-f^{(n+1)})$

is the j column of the matrix $\Delta F^{(2)}$. The matrix $\Delta F^{(2)}$ may be recorded in the form

Here

$$B^{(i)}=(f^{(n)}-f^{(i)}\quad f^{(n)}-f^{(n+1)})$$

$$C^{(i)}=\begin{array}{c} \qquad\qquad i \\ \begin{pmatrix} \underbrace{\begin{matrix} 0\ 0 \ldots 0\ 1\ 0 \ldots 0 \\ 1\ 1 \ldots 1\ 1\ 1 \ldots 1 \end{matrix}} \end{pmatrix} \end{array}$$

For the conversion of matrix $\Delta F^{(2)}$ we use the formula

$$(A+BDC)^{-1}=A^{-1}-A^{-1}B(D^{-1}+CA^{-1}B)^{-1}CA^{-1}. \tag{P5}$$

This formula takes place for any matrix A, B, C, D for which all operations are fulfilled, which are figured in (P5) (it is easy to be assured of the accuracy of (P5) by direct checking). Supposing in (P5)

$$A=\Delta F^{(1)},\ B=B^{(i)},\ C=C^{(i)},\ D=E_2=\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

we obtain

$$\Delta F^{(2)^{-1}}=\Delta F^{(1)^{-1}}-\Delta F^{(1)^{-1}}B^{(i)}(E_2+C^{(i)}\Delta F^{(1)^{-1}}B^{(i)})^{-1}C^{(i)}\Delta F^{(1)^{-1}}.$$

From this formula it is obvious that conversion of the matrix $\Delta F^{(2)}$ is reduced to conversion of the matrix of the second order

$E_2 + c^{(i)} \Delta F^{(1)-1} B^{(i)}$, in as much as the matrix $\Delta F^{(1)-1}$ has already been computed in the preceding iteration. In an analogous manner in /57
each final iteration, using the results of the preceding iteration, the conversion of matrix ΔF will reduce to conversion of the matrix of the second order.

In the particular case when $i = n$, that is when point $x^{(n)}$ is eliminated, conversion of the matrix $\Delta F^{(2)}$ may be reduced to division by the number. Actually in this case

$$\Delta F^{(2)} = \Delta F^{(1)} + BC$$
$$B = f^{(n)} - f^{(n+1)}, \quad C = (\underbrace{1\ 1 \ldots 1}_{n})$$

from this it follows

$$\Delta F^{(2)^{-1}} = \Delta F^{(1)^{-1}} - \frac{\Delta F^{(1)^{-1}} B C \Delta F^{(1)^{-1}}}{1 + C \Delta F^{(1)^{-1}} B}.$$

Thus when the point which answers the approximation obtained in the preceding iteration does not take part in the construction of the given approximation conversion of the matrix ΔF reduces to multiplication of the knowns of the matrix and division by the number.

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