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PREDICTION OF FIBER COMPOSITE MECHANICAL BEHAVIOR MADE SIMPLE

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PREDICTION OF FIBER COMPOSITE MECHANICAL BEHAVIOR MADE SIMPLE

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ABSTRACT

A convenient procedure is described for the determination of the mechanical behavior (elastic properties and failure stresses) of angleplied fiber composite laminates using a pocket calculator. The procedure uses simple equations and appropriate graphs of elastic properties versus ply angles. The procedure can handle all types of fiber composites including hybrids. The versatility and generality of the procedure is illustrated using several step-by-step numerical examples.

INTRODUCTION

The determination of mechanical properties (elastic properties and failure stresses (strengths)) of angleplied laminates are required for the initial sizeing of structural components from fiber composites. These properties and strengths are determined using composite mechanics and laminate theory usually in a computer code (ref. 1). It is generally recognized that the use of a computer code is expedient and quite general. However, it does not provide the user with insight and instant feedback of the laminate behavior and capability as he proceeds with the design/analysis of the component.

A convenient procedure (method) is described in this paper which can be used to determine the elastic properties and strengths of angleplied laminates. The procedure is suitable for hand calculations using a pocket calculator. It consists of simple equations and appropriate graphs of ($\pm\theta$) ply combinations from the most frequently used composites. The procedure makes use of the well known transformation equations, the ply stress influence coefficients, and the ply uniaxial strengths. It can handle all types of composites including interply and intraply hybrids. The procedure is illustrated using several step-by-step numerical examples. The discussion in this paper is limited to mechanical loads and structures at normal atmospheric conditions. The procedure can readily be extended to handle hygrothermal environments following the methods described in reference 2.

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ELASTIC PROPERTIES

The laminate (composite) elastic properties for angleplied laminates with generic laminate configurations $[(\pm\theta)_n/O_m]_s$, $[O_n/(\pm\theta)_m]_s$, $[(\pm\theta)_n/(\pm\theta)_m]_s$, $[(\pm\theta_1)/O/(\pm\theta_2)/90/\ldots]_s$ including interply and intraply hybrids are determined as described below. The notation to be used is as follows: E, G, and ν denote normal modulus, shear modulus, and Poisson's ratio, respectively; subscripts ℓ and c denote ply and laminate (composite) property, respectively; subscripts 1, 2, and 3 denote ply coordinate axes directions (fig. 1); subscripts x, y, and z denote laminate coordinate axes directions (fig. 2); and θ denotes ply orientation angle measured from the x-axis to the 1-axis (fig. 2). The reduced ply stiffness properties Q_ℓ are given by

$$Q_{\ell 11} = \frac{E_{\ell 11}}{1 - \nu_{\ell 12}\nu_{\ell 21}}; \quad Q_{\ell 22} = \frac{E_{\ell 22}}{1 - \nu_{\ell 12}\nu_{\ell 21}}$$

$$Q_{\ell 12} = \frac{\nu_{\ell 12}E_{\ell 22}}{1 - \nu_{\ell 12}\nu_{\ell 21}} = \frac{\nu_{\ell 21}E_{\ell 11}}{1 - \nu_{\ell 12}\nu_{\ell 21}} = Q_{\ell 21} \tag{2.1}$$

$$G_{\ell 12} \equiv G_{\ell 12} \text{ (Identity given for completeness)}$$

The reduced stiffness elastic properties for a pair of plies $(\pm\theta)Q_\theta$ are given by

$$Q_{\theta 11} = \frac{E_{\theta 11}}{1 - \nu_{\theta 12}\nu_{\theta 21}}; \quad Q_{\theta 22} = \frac{E_{\theta 22}}{1 - \nu_{\theta 12}\nu_{\theta 21}}$$

$$Q_{\theta 12} = \frac{\nu_{\theta 12}E_{\theta 22}}{1 - \nu_{\theta 12}\nu_{\theta 21}} = \frac{\nu_{\theta 21}E_{\theta 11}}{1 - \nu_{\theta 12}\nu_{\theta 21}} = Q_{\theta 21} \tag{2.2}$$

$$G_{\theta 12} \equiv G_{\theta 12} \text{ (Identity given for completeness)}$$

Elastic properties for plies and for $(\pm\theta)$ combined plies are shown graphically in figures 3 to 7 for boron/epoxy (B/E), high modulus graphite/epoxy (HMG/E), AS graphite/epoxy (AS/E), S-glass/epoxy (S-G/E), and kevlar-49/epoxy (K/E), respectively. Similar properties for three typical intraply hybrids (80 HMG/E//20 S-G/E, 80 AS/E//20 S-G/E, and 80 AS/E//20 K/E, where the number denotes percent fiber) are shown in figures 8, 9, and 10, respectively. Unidirectional composite (ply) properties are obtained from these figures at $\theta = 0$. Corresponding curves for other composites may be generated using laminate theory. For intraply hybrids they may be generated using the procedures described in references 3 and 4 together with laminate theory.

The reduced stiffnesses (Q_c) for angleplied laminates $\left[(\pm\theta)_n/O_m\right]_s$ are given by

$$
\begin{aligned}
Q_{cxx} &= V_{P\theta}Q_{\theta 11} + V_{PO}Q_{\ell 11} \\
Q_{cyy} &= V_{P\theta}Q_{\theta 22} + V_{PO}Q_{\ell 22} \\
Q_{cxy} &= V_{P\theta}Q_{\theta 12} + V_{PO}Q_{\ell 12} = Q_{cxy} \\
G_{cxy} &= V_{P\theta}G_{\theta 12} + V_{PO}G_{\ell 12}
\end{aligned}
\tag{2.3}
$$

where $V_{P\theta}$ equals the volume ratio of the $\pm\theta$ combined plies and V_{PO} equals the volume ratio of the O plies. Equations (2.3) can be readily generalized for other laminate configurations. Either the O or the $\pm\theta$ combined plies can be from different composites or from the intraply hybrids.

The angleplied laminate elastic constants are given by

$$
\begin{aligned}
E_{cxx} &= Q_{cxx} - \frac{Q_{xcy}^2}{Q_{cyy}}; \quad E_{cyy} = Q_{cyy} - \frac{Q_{cxy}^2}{Q_{cxx}} \\
\nu_{cxy} &= \frac{Q_{cxy}}{Q_{cyy}}; \quad \nu_{cyx} = \frac{Q_{cxy}}{Q_{cxx}}
\end{aligned}
\tag{2.4}
$$

G_{cxy} is given by the last equation (2.3).

Equations (2.1), (2, 2), and (2.3) are relatively simple and can be easily programmed for programmable pocket calculators. Their use will now be illustrated as described in the following examples.

Example 2.1. - Determine the elastic constants of the angleplied laminate AS/E $\left[(\pm 45)/O_2\right]_s$. For this laminate: $\theta = 45^\circ$; $V_{PL} = 0.5 = V_{PO}$. From figure 5 the O ply elastic constants are (at $\theta = 0$)

$$
E_{\ell 11} = 18.2\times 10^6 \text{ psi}; \; E_{\ell 22} = 2.0\times 10^6 \text{ psi}; \; G_{\ell 12} = 0.60\times 10^6 \text{ psi}
$$

$$
\nu_{\ell 12} = 0.25; \; \nu_{\ell 21} = \nu_{\ell 12}E_{\ell 22}/E_{\ell 11} = 0.027
$$

Substituting these values in equations (2.1) yields the reduced ply stiffness Q_ℓ as follows:

$$Q_{\ell 11} = 18.32\times10^6 \text{ psi};\ Q_{\ell 22} = 2.01\times10^6 \text{ psi}$$

$$Q_{\ell 12} = Q_{\ell 21} = 0.50\times10^6 \text{ psi};\ G_{\ell 12} = 0.60\times10^6 \text{ psi}$$

Also from figure 5 the ±45 combined plies elastic constants ($\theta = \pm45$) are

$$E_{\theta 11} = E_{\theta 22} = 2.1\times10^6 \text{ psi};\ G_{\theta 12} = 4.9\times10^6 \text{ psi}$$

$$\nu_{\theta 12} = \nu_{\theta 21} = 0.81$$

Substituting these values in equations (2.2) yields the ±45 plies reduced stiffness Q_θ as follows:

$$Q_{\theta 11} = Q_{\theta 22} = 6.11\times10^6 \text{ psi}$$

$$Q_{\theta 12} = Q_{\theta 21} = 4.90\times10^6 \text{ psi}$$

$$G_{\theta 12} = 4.9\times10^6 \text{ psi}$$

Using the Q_ℓ and Q_θ values in equations (2.3) yields the angleplied laminate reduced stiffness Q_c as follows:

$$Q_{cxx} = 12.22\times10^6 \text{ psi};\ Q_{cyy} = 4.06\times10^6 \text{ psi}$$

$$Q_{cxy} = 2.72\times10^6 \text{ psi};\ G_{cxy} = 2.75\times10^6 \text{ psi}$$

The angleplied laminate elastic constants are now obtained by substituting the Q_c values in equations (2.4):

$$E_{cxx} = 10.4\times10^6 \text{ psi};\ E_{cyy} = 3.4\times10^6 \text{ psi};\ G_{cxy} = 2.8\times10^6 \text{ psi}$$

$$\nu_{cxy} = 0.67;\ \nu_{cyx} = 0.22$$

The accuracy of the calculations can be checked using the following well known relationship

$$\frac{\nu_{cxy}}{E_{cxx}} = \frac{\nu_{cxy}}{E_{cyy}} \tag{2.5}$$

Substituting numerical values in equation (2.5)

$$0.064\times10^{6} = 0.065\times10^{-6}$$

which is accurate to three significant figures.

Example 2.2. - Determine the elastic constants of the angleplied interply hybrid laminate $\left[(\pm45)/O_4/90_2\right]_S$ where the ±45 combined plies are from HMG/E, the O plies are from S-G/E and 90 plies are from (K/E). The ply ratios for this laminate are:

$$V_{P\theta} = 0.25; \; V_{PO} = 0.5; \text{ and } V_{P90} = 0.25$$

The elastic constants for the ±45 HMG/E plies from figure 4 are:

$$E_{\theta 11} = E_{\theta 22} = 2.9\times10^6 \text{ psi}; \; G_{\theta 12} = 7.7\times10^6 \text{ psi}; \; \nu_{\theta 12} = \nu_{\theta 21} = 0.83$$

The corresponding Q_θs, using these values in equations (2.2), are:

$$Q_{\theta 11} = Q_{\theta 22} = 9.32\times10^6 \text{ psi}; \; G_{\theta 12} = 7.7\times10^6 \text{ psi}$$

$$Q_{\theta 12} = Q_{\theta 21} = 8.23\times10^6 \text{ psi}$$

The elastic constants for the O S-G/E plies from figure 6 ($\theta = 0$) are:

$$E_{\ell 11} = 8.8\times10^6 \text{ psi}; \; E_{\ell 22} = 3.6\times10^6 \text{ psi}; \; G_{\ell 12} = 1.7\times10^6 \text{ psi}$$

$$\nu_{\ell 12} = 0.23; \; \nu_{\ell 21} = \nu_{\ell 12} R_{\ell 22}/E_{\ell 11} = 0.094$$

The corresponding Q_ℓs, using these values in equations (2.2), are:

$$Q_{\ell 11} = 8.99\times10^6 \text{ psi}; \; Q_{\ell 22} = 3.68\times10^6 \text{ psi}; \; G_{\ell 12} = 1.74\times10^6 \text{ psi}$$

$$Q_{\ell 12} = Q_{\ell 21} = 0.85\times10^6 \text{ psi}$$

The elastic constants for the 90 K/E plies from figure 7 ($\theta = 90$) are

$$E_{\ell 11} = 0.7\times10^6 \text{ psi; } E_{\ell 22} = 9.9\times10^6 \text{ psi; } G_{\ell 12} = 0.4\times10^6 \text{ psi}$$

$$\nu_{\ell 21} = 0.40; \ \nu_{\ell 12} = \nu_{\ell 21}E_{\ell 11}/E_{\ell 22} = 0.028$$

The corresponding Q_ℓs, using these values in equations (2.2) are:

$$Q_{\ell 11} = 0.71\times10^6 \text{ psi; } Q_{\ell 22} = 10.01\times10^6 \text{ psi; } G_{\ell 12} = 0.4\times10^6 \text{ psi}$$

$$Q_{\ell 12} = Q_{\ell 21} = 0.28\times10^6 \text{ psi}$$

The interply hybrid angleplied laminate Q_cs are obtained from equations (2.3) expanded to account for the 90 plies. In expanded form the first equation is

$$Q_{cxx} = V_{P\theta}Q_{\theta 11} + V_{PO}Q_{\theta 11} + V_{P90}Q_{9011}$$

and the corresponding equations for Q_{cyy}, Q_{cxy}, and G_{cxy}. Using the numerical values for the Q_θs, Q_0s, and Q_{90}s the Q_{cxx}, Q_{cyy}, Q_{csy}, and G_{cxy} are, respectively:

$$Q_{cxx} = 0.25\times9.32\times10^6 + 0.50\times8.99\times10^6 + 0.25\times0.71\times10^6 = 7.00\times10^6 \text{ psi}$$

$$Q_{cyy} = 0.25\times9.32\times10^6 + 0.50\times3.68\times10^6 + 0.25\times10.01\times10^6 = 6.67\times10^6 \text{ psi}$$

$$Q_{cxy} = 0.25\times8.32\times10^6 + 0.50\times0.85\times10^6 + 0.25\times0.28\times10^6 = 2.58\times10^6 \text{ psi}$$

$$G_{cxy} = 0.25\times7.7\times10^6 + 0.50\times1.74\times10^6 + 0.25\times0.40\times10^6 = 2.90\times10^6 \text{ psi}$$

The intraply angleplied laminate elastic constants are obtained by using these numerical values in equations (2.4). The results are:

$$E_{cxx} = 6.0\times10^6 \text{ psi; } E_{cyy} = 5.7\times10^6 \text{ psi; } G_{cxy} = 2.9\times10^6 \text{ psi}$$

$$\nu_{cxy} = 0.39; \ \nu_{cyx} = 0.37$$

$$\text{Check: } \frac{\nu_{cxy}}{E_{cxx}} = \frac{\nu_{cyx}}{E_{cyy}} \Longrightarrow 0.065\times10^{-6} = 0.065\times10^{-6} \text{ O.K.}$$

Example 2.3. - Determine the elastic constants of the angleplied interply-intraply hybrid laminate $[(\pm 30)/0_3]_S$ where the ±30 combined plies are from B/E and the 0 plies are from 80 AS/E//20 S-G intraply hybrid. The ply ratios for this laminate are:

$$V_{P\theta} = 0.40; \; V_{PO} = 0.60$$

The elastic constants for the ±30 B/E plies from figure 3 are:

$$E_{\theta 11} = 9.8\times10^6 \text{ psi}; \; E_{\theta 22} = 2.3\times10^6 \text{ psi}; \; G_{\theta 12} = 6.1\times10^6 \text{ psi}$$

$$\nu_{\theta 12} = 1.32; \; \nu_{\theta 21} = \nu_{\theta 12}E_{\theta 22}/E_{\theta 11} = 0.31$$

Substituting these values in equations (2.2) yields

$$Q_{\theta 11} = 16.59\times10^6 \text{ psi}; \; Q_{\theta 22} = 3.89\times10^6 \text{ psi}; \; G_{\theta 12} = 6.1\times10^6 \text{ psi}$$

$$Q_{\theta 12} = 5.14\times10^6 \text{ psi}; \; Q_{\theta 21} = 5.14\times10^6 \text{ psi}; \; (Q_{\theta 12} = Q_{\theta 21})$$

The elastic constants for the 0 intraply hybrid plies from figure 9 (θ = 0) are:

$$E_{\ell 11} = 16.0\times10^6 \text{ psi}; \; E_{\ell 22} = 2.2\times10^6 \text{ psi}; \; G_{\ell 12} = 0.72\times10^6 \text{ psi}$$

$$\nu_{\ell 12} - 0.25; \; \nu_{\ell 21} = \nu_{\ell 12}E_{\ell 22}/E_{\ell 11} = 0.034$$

The corresponding Q_ℓs, using equations (2.1), are

$$Q_{\ell 11} = 16.14\times10^6 \text{ psi}; \; Q_{\ell 22} = 2.22\times10^6 \text{ psi}; \; G_{\ell 12} = 0.72\times10^6 \text{ psi}$$

$$Q_{\ell 12} = Q_{\ell 21} = 0.56\times10^6 \text{ psi}$$

The laminate Q_cs are:

$$Q_{cxx} = 0.40\times16.59\times10^6 + 0.6\times16.14\times10^6 = 16.32\times10^6 \text{ psi}$$

$$Q_{cyy} = 0.40\times3.89\times10^6 + 0.6\times2.22\times10^6 = 2.89\times10^6 \text{ psi}$$

$$Q_{cxy} = 0.40 \times 5.14 \times 10^6 + 0.6 \times 0.56 \times 10^6 = 2.39 \times 10^6 \text{ psi}$$

$$Q_{cxy} = 0.40 \times 6.1 \times 10^6 + 0.6 \times 0.72 \times 10^6 = 2.87 \times 10^6 \text{ psi}$$

The angleplied interply-intraply hybrid laminate elastic constants are determined by using these Q_c values in equations (2.4). The results are:

$$E_{cxx} = 14.3 \times 10^6 \text{ psi}; \; E_{cyy} = 2.5 \times 10^6 \text{ psi}; \; G_{cxy} = 2.8 \times 10^6 \text{ psi}$$

$$\nu_{cxy} = 0.83; \; \nu_{cyx} = 0.15$$

$$\text{Check: } \frac{\nu_{cxy}}{E_{cxx}} = \frac{\nu_{cyx}}{E_{cyy}} \Longrightarrow 0.058 \; 10^{-4} = 0.06 \; 10^{-4} \text{ O.K.}$$

The three examples described above illustrate the versatility and generality of the procedure using the simple equations in conjunction with the accompanying figures.

PLY STRESS INFLUENCE COEFFICIENTS

The ply stress coefficients (PSICs) are defined as the ply stresses ($\sigma_{\ell 11}$, $\sigma_{\ell 22}$, and $\sigma_{\ell 12}$) due to a unit laminate stress (σ_{cxx}, σ_{cyy}, or σ_{cxy}). Using the notation $\mathscr{I}_{L/X}$ to denote ply longitudinal stress influence coefficient ($\sigma_{\ell 11}/\sigma_{cxx}$), when $\sigma_{cxx} \neq 0$ and $\sigma_{cyy} = \sigma_{cxy} = 0$, the governing equations are given, approximately (to within 1 percent), by

$$\mathscr{I}_{L/X} = \frac{E_{\ell 11}}{E_{cxx}} \left[\cos^2\theta - \nu_{cxy} \sin^2\theta\right]$$

$$\mathscr{I}_{T/X} = \frac{E_{\ell 22}}{E_{cxx}} \left[(\nu_{\ell 12} - \nu_{cxy}) \cos^2\theta + (1 - \nu_{cxy}\nu_{\ell 12}) \sin^2\theta\right] \qquad (3.1)$$

$$\mathscr{I}_{S/X} = -\frac{G_{\ell 12}}{E_{cxx}} (1 + \nu_{cxy}) \sin 2\theta$$

The PSICs due to σ_{cyy} stress only ($\sigma_{cyy} \neq 0$ and $\sigma_{cxx} = \sigma_{cxy} = 0$) are given by

$$\mathscr{I}_{L/Y} = \frac{E_{\ell 11}}{E_{cyy}}\left[\sin^2\theta - \nu_{cxy}\cos^2\theta\right]$$

$$\mathscr{I}_{T/Y} = \frac{E_{\ell 22}}{E_{cyy}}\left[(1 - \nu_{cxy}\nu_{\ell 12})\cos^2\theta + (\nu_{\ell 12} - \nu_{cxy})\sin^2\theta\right] \tag{3.2}$$

$$\mathscr{I}_{S/Y} = \frac{G_{\ell 12}}{E_{cyy}}(1 + \nu_{cxy})\sin 2\theta$$

The PSICs due to σ_{cxy} stress only ($\sigma_{cxy} \neq 0$ and $\sigma_{cyy} = \sigma_{cyy} = 0$) are given by

$$\mathscr{I}_{L/S} = \frac{1}{2}\frac{E_{\ell 11}}{G_{cxy}}(1 - \nu_{\ell 21})\sin 2\theta$$

$$\mathscr{I}_{T/S} = -\frac{1}{2}\frac{E_{\ell 22}}{G_{cxy}}(1 - \nu_{\ell 12})\sin 2\theta \tag{3.3}$$

$$\mathscr{I}_{S/S} = \frac{G_{\ell 12}}{G_{cxy}}\cos^2\theta$$

It can be seen in equations (3.1), (3.2), and (3.3) that the PSICs depend on:

1. Laminate properties (E_c, G_c, and ν_c)
2. Ply properties (E_ℓ, G_ℓ, and ν_ℓ)
3. Ply orientation angle (θ)

The graphical representation of the trigonometric functions used in the PSICs is shown in figure 11. The use of the PSICs for determining laminate fracture stresses to satisfy ply specified strengths are determined using the procedure described in the next section.

LAMINATE FAILURE STRESSES (STRENGTHS)

When the PSICs and the laminate stresses are known, the ply stresses are determined from equations (3.1), (3.2), and (3.3) as follows:

$$\sigma_{\ell 11} = \mathscr{S}_{L/X}\sigma_{cxx};\quad \sigma_{\ell 22} = \mathscr{S}_{T/X}\sigma_{cxx};\quad \sigma_{\ell 12} = \mathscr{S}_{S/X}\sigma_{cxx};$$

$$\sigma_{\ell 11} = \mathscr{S}_{L/Y}\sigma_{cyy};\quad \sigma_{\ell 22} = \mathscr{S}_{T/Y}\sigma_{cyy};\quad \sigma_{\ell 12} = \mathscr{S}_{S/Y}\sigma_{cyy}; \tag{4.1}$$

$$\sigma_{\ell 11} = \mathscr{S}_{L/S}\sigma_{cxy};\quad \sigma_{\ell 22} = \mathscr{S}_{T/Y}\sigma_{cxy};\quad \sigma_{\ell 12} = \mathscr{S}_{S/S}\sigma_{cxy};$$

Laminate failure stresses may be determined approximately from equations (4.1) by using the "maximum stress first ply failure criterion." According to this criterion, the laminate failure stress is that stress which causes any of the ply stresses to be equal to the corresponding ply uniaxial strengths. Letting S_ℓ with proper subscripts denote ply uniaxial strength and S_c denote laminate failure stress, the governing equations for laminate failure stresses are:

$$S_{cxxT,C} = \text{MINIMUM}\left[\frac{S_{\ell 11\alpha}}{\mathscr{S}_{L/X}}, \frac{S_{\ell 22\beta}}{\mathscr{S}_{L/X}}, \frac{S_{\ell 12S}}{\mathscr{S}_{L/X}}\right]$$

$$S_{cyyT,C} = \text{MINIMUM}\left[\frac{S_{\ell 11\alpha}}{\mathscr{S}_{L/Y}}, \frac{S_{\ell 22\beta}}{\mathscr{S}_{L/Y}}, \frac{S_{\ell 12S}}{\mathscr{S}_{L/Y}}\right] \tag{4.2}$$

$$S_{cxys} = \text{MINIMUM}\left[\frac{S_{\ell 11\alpha}}{\mathscr{S}_{L/S}}, \frac{S_{\ell 22\beta}}{\mathscr{S}_{L/S}}, \frac{S_{\ell 12S}}{\mathscr{S}_{L/S}}\right]$$

where the subscripts α and $\beta = T$ (tension) or C (compression) and S denotes shear. Equations (4.2) need to be checked for each ply. Laminate failure stresses are usually determined by the following procedure:

1. Assume one ply fails in one stress, say $S_{\ell 11T}$;
2. Calculate laminate failure stress (e.g., $S_{ccxT} = S_{\ell 11T}/\mathscr{S}_{L/X}$);
3. Use this S_{cxxT} and the PSICs to calculate; the other ply stresses in this ply and in each of the other plies;
4. Check with corresponding ply uniaxial strengths: if $\sigma_\ell < S_\ell$ O.K.: if $\sigma_\ell > S_\ell$, then reduce S_{cxxT} by (S_ℓ/σ_ℓ) and repeat the procedure.

This procedure is illustrated in the following example.

<u>Example 3.1</u>. - Determine the tensile failure stress S_{cxxT} at which the angleplied laminate $[\pm 45/O_2]$ AS/E in Example 2.1 will fail. This failure stress is determined by using the PSICs equations (3.1), the ply and laminate elastic properties from Example 2.1, and the uniaxial ply fracture stresses. The elastic

properties required in equations (3.1) and the corresponding values from Example 2.1 are:

$$E_{\ell 11} = 18.2\times10^6 \text{ psi};\ E_{\ell 22} = 2.0\times10^6 \text{ psi};\ G_{\ell 12} = 0.60\times10^6 \text{ psi}$$

$$\nu_{\ell 12} = 0.25;\ E_{cxx} = 10.4\times10^6 \text{ psi};\ \nu_{cxy} = 0.67$$

Typical uniaxial ply strengths for AS/E composites are:

Longitudinal tension	$S_{\ell 11T} = 220$ ksi
Longitudinal compression	$S_{\ell 11C} = 180$ ksi
Transverse tension	$S_{\ell 22T} = 8$ ksi
Transverse compression	$S_{\ell 22C} = 36$ ksi
Interlaminar shear	$S_{\ell 12S} = 10$ ksi

The PSICs for the 0 plies from equation (3.1) are:

$$\mathscr{I}_{L/X} = \frac{E_{\ell 11}}{E_{cxx}};\ \mathscr{I}_{T/X} = \frac{E_{\ell 22}}{E_{cxx}}(\nu_{\ell 12} - \nu_{cxy});\ \mathscr{I}_{T/X} = 0$$

Substituting the corresponding numerical values for the elastic constants yields

$$\mathscr{I}_{L/X} = \frac{18.2\times10^6}{10.4\times10^4} = 1.75;\ \mathscr{I}_{T/X} = \frac{2.0\times10^6}{10.4\times10^6}(0.25 - 0.67) = -0.081$$

Assume that the 0 plies fail by longitudinal tension ($\sigma_{\ell 11} = S_{\ell 11T}$). From equations (4.1) this condition is given by

$$S_{ccxT} = \frac{S_{\ell 11T}}{\mathscr{I}_{L/X}} \tag{4.3}$$

Substituting the numerical values for $S_{\ell 11T} = 220$ ksi and $\mathscr{I}_{L/X} = 1.75$

$$S_{ccxT} = 220/1.75 = 126 \text{ ksi}$$

The ply transverse stress due to a laminate stress of 126 ksi is

$$\sigma_{\ell 11} = \mathscr{I}_{T/X} S_{cxxT} = -0.081\times126 = -10.2 \text{ ksi}$$

which is about 30 percent of the ply transverse compression strength (36 ksi).

The ply stresses in the ±45 plies due to 126 ksi laminate stress are determined from equations (3.1) by letting $\theta = \pm 45$. The results are:

$$\mathscr{L}_{L/X} = \frac{18.2}{10.4}(0.50 - 0.67 \times 0.50) = 0.289$$

$$\mathscr{L}_{T/X} = \frac{2.0}{10.4}\left[(0.25 - 0.67) \times 0.50 + (1 - 0.67 \times 0.25) \times 0.50\right] = 0.040$$

$$\mathscr{L}_{S/X} = -\frac{0.6}{10.4}(1 + 0.67) \times 1.0 = -0.096 \text{ for the (+45 plies)}$$

$$\mathscr{L}_{S/X} = -\frac{0.6}{10.4}(1 + 0.67) \times (-1.0) = 0.096 \text{ for the (-45 plies)}$$

The ply stresses are now determined from

$$\sigma_{\ell 11} = \mathscr{L}_{L/X} S_{ccxT} = 0.289 \times 126 = 36.4 \text{ ksi}$$

$$\sigma_{\ell 22} = \mathscr{L}_{T/X} S_{ccxT} = 0.040 \times 126 = 5.0 \text{ ksi}$$

$$\sigma_{\ell 12} = \mathscr{L}_{S/X} S_{ccxT} = -0.096 \times 126 = -12.1 \text{ ksi for the (+45 plies)}$$

$$0.096 \times 126 = 12.1 \text{ ksi for the (-45 plies)}$$

Comparing these stresses to the corresponding ply uniaxial strengths it is seen that the intralaminar shear stress of 12.1 ksi is greater than 10.0 ksi and therefore a laminate stress of 126 ksi will cause failure in the +45 plies. To avoid this failure stress the laminate stress of 126 ksi must be reduced by the ratio (10.0/12.1) which yields 104 ksi. Therefore, the maximum laminate stresses which will cause initial failure in any of the plies is 104 ksi. The reader can obtain insight and practice by using the procedure to determine laminate failure stresses due to σ_{cyy} first and, then, due to σ_{cxy}. (The answer for S_{cyyT} is 16.3 ksi and for S_{cxys} is 46.7 ksi.) Typical properties for some other unidirectional composites are given in Table I.

CONCLUSIONS

A convenient procedure suitable for hand calculations is described for determining the elastic properties and failure strengths of angleplied laminates. The procedure consists of simple equations and appropriate graphs of elastic properties versus ply angles. The procedure can handle all types of symmetric laminates made from different composites including interply hybrids, intraply hybrids, and interply-intraply hybrids. Several examples are described in detail to illustrate the versatility and generality of the procedure.

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3. C. C. Chamis and J. H. Sinclair, "Prediction of Properties of Intraply Hybrid Composites," NASA TM 79087, 1979.

4. C. C. Chamis and J. H. Sinclair, "Micromechanics of Intraply Hybrid Composites: Elastic and Thermal Properties," NASA TM 79253, 1979.

TABLE I. - TYPICAL PROPERTIES OF UNIDIRECTIONAL FIBER COMPOSITES AT ROOM TEMPERATURE

Properties	Units	Boron/ epoxy AVCO5505	Boron/ polyimide WRD9371	Scotchply/ epoxy 1009-26-5901	Modmor I/ epoxy ERLA4617	Modmor I/ polyimide WRD 9371	Thornel 300/ epoxy NARMCO 5208	Kevlar 49/ epoxy CE-3305
1. Fiber volume ratio	-------	0.50	0.49	0.72	0.45	0.45	0.70	0.54
2. Density	lb/in^3	0.073	0.072	0.077	0.056	0.056	0.058	0.049
3. Longitudinal thermal coefficient	10^{-6} in/ in/°F	3.4	2.7	2.1	-------	0.0	0.01	-1.60
4. Transverse thermal coefficient	10^{-6} in/ in/°F	16.9	15.8	9.3	18.5	14.1	12.5	31.3
5. Longitudinal modulus	10^6 psi	29.2	32.1	8.8	27.5	31.3	26.3	12.2
6. Transverse modulus	10^6 psi	3.15	2.1	3.6	1.03	0.72	1.5	0.70
7. Shear modulus	10^6 psi	0.78	1.11	1.74	0.9	0.65	1.0	0.41
8. Major Poisson's ratio	-------	0.17	0.16	0.23	0.10	0.25	0.28	0.32
9. Minor Poisson's ratio	-------	0.02	0.02	0.09	-------	0.02	0.01	0.02
10. Longitudinal tensile strength	psi	199 000	151 000	187 000	122 000	117 000	218 000	172 000
11. Longitudinal compressive strength	psi	232 000	158 000	119 000	128 000	94 500	247 000	42 000
12. Transverse tensile strength	psi	8100	1600	6670	6070	2150	5850	1600
13. Transverse compressive strength	psi	17 900	9100	25 300	28 500	10 200	35 700	9400
14. Intralaminar shear strength	psi	9100	3750	6500	8900	3150	9800	4000

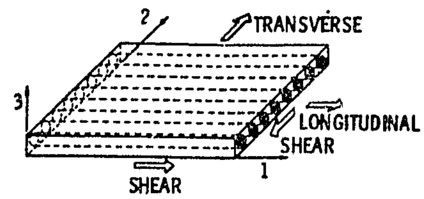

Figure 1. - Schematic of single ply.

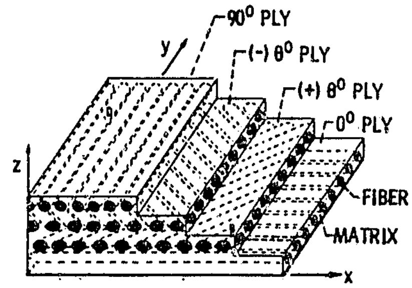

Figure 2. - Schematic of angleplied laminate.

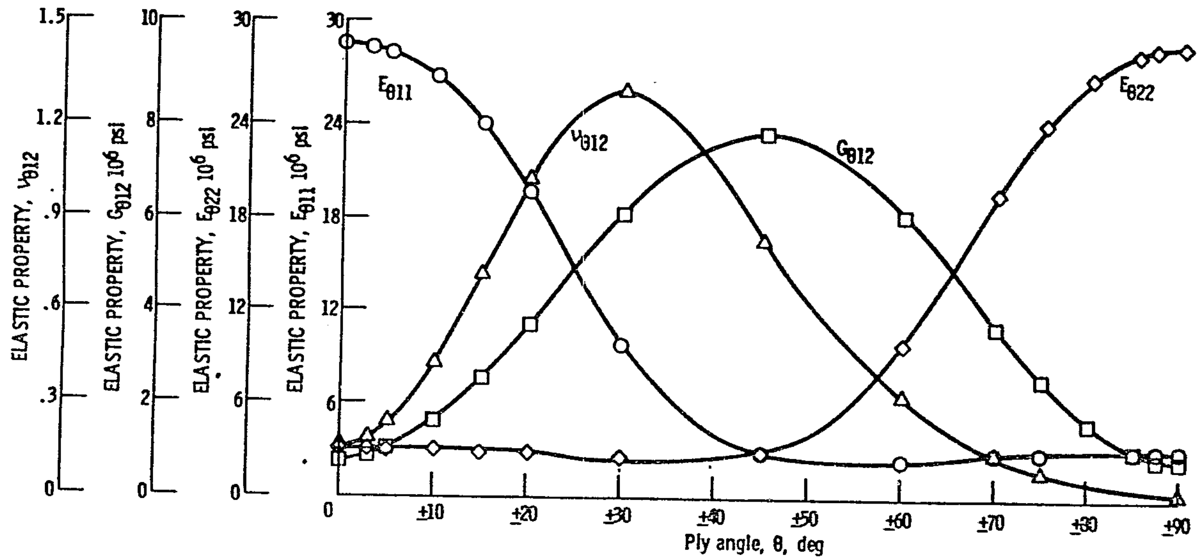

Figure 3. - Elastic properties of boron/epoxy (B/E) ±θ laminates.

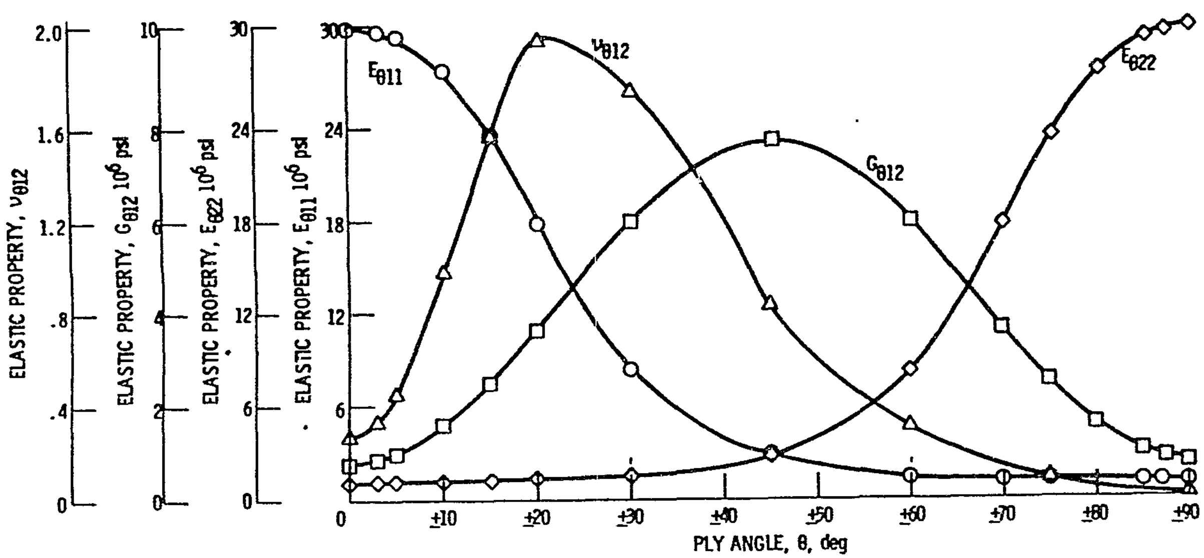

Figure 4. - Elastic properties of high-modulus-graphite fiber/epoxy (HMG/E) ±θ laminates.

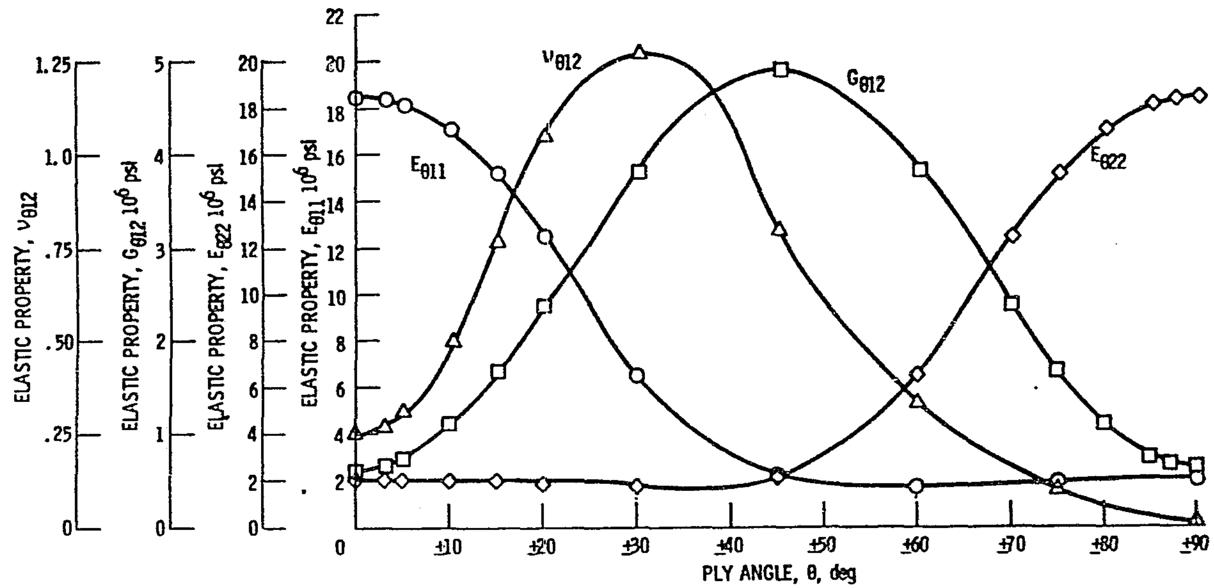

Figure 5. - Elastic properties of as-graphite-fiber/epoxy (AS/E) ±θ laminates.

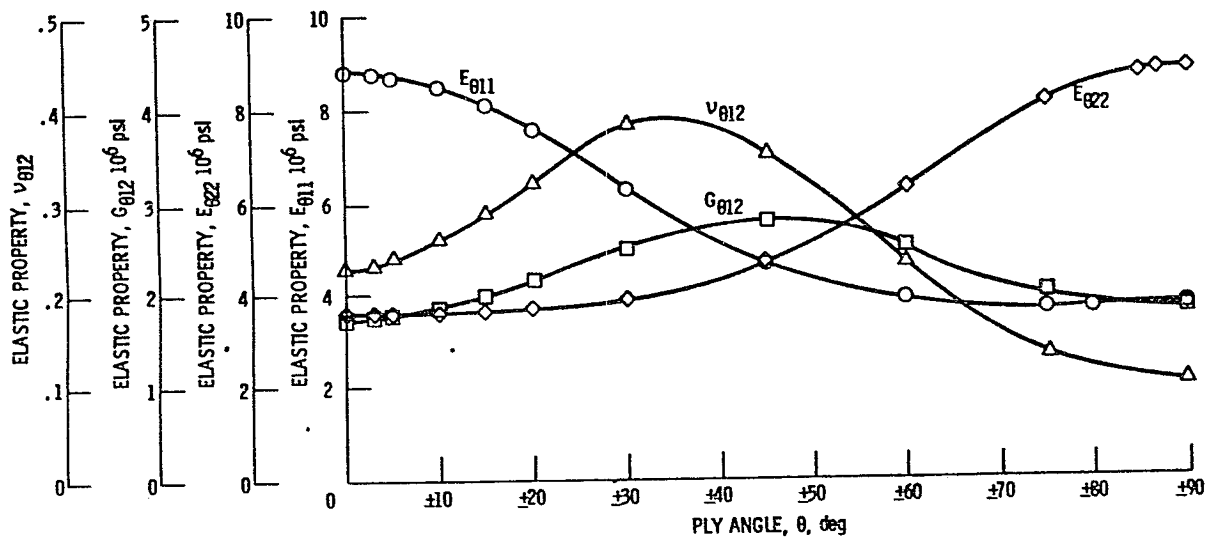

Figure 6. - Elastic properties of S-glass-fiber/epoxy (S-G/E) ±θ laminates.

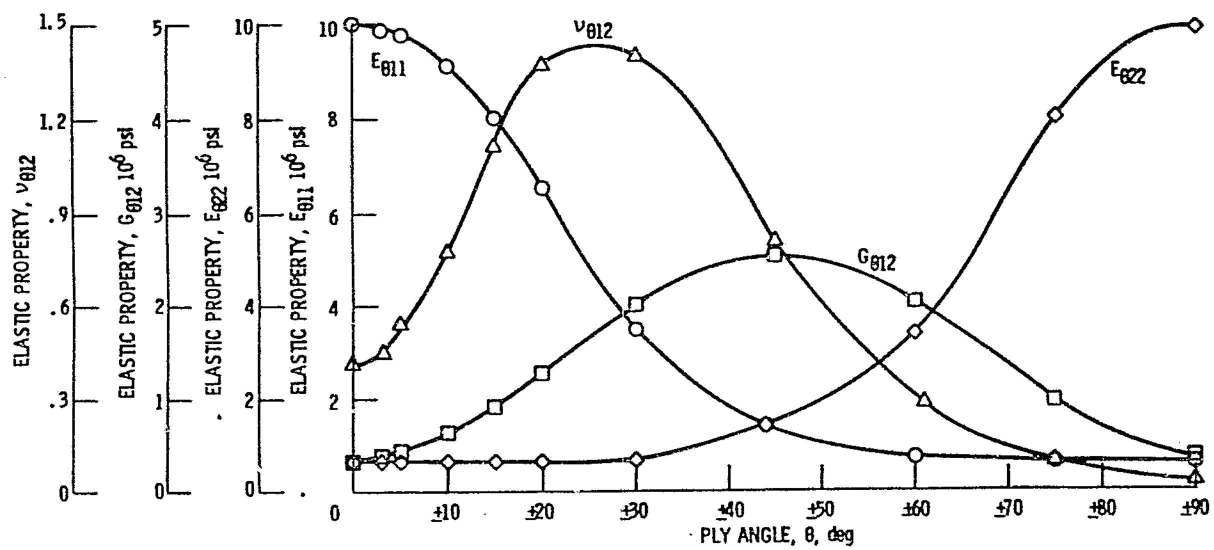

Figure 7. - Elastic properties of kevlar-fiber/epoxy (K/E) ±θ laminates.

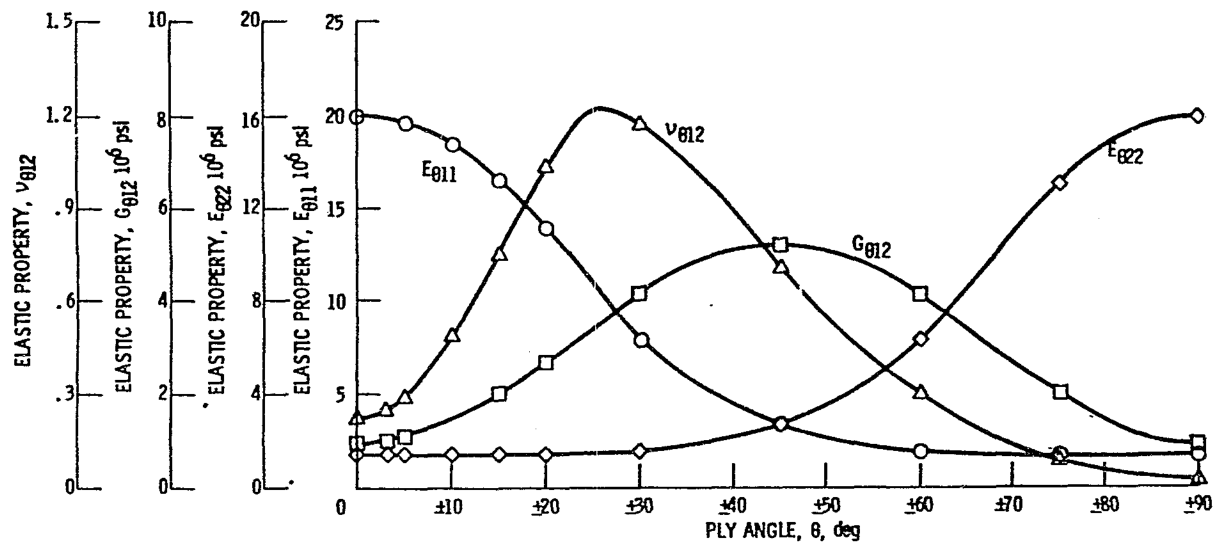

Figure 8. - Elastic properties of intraply hybrid (80% HMG/20% S-G/E) ±θ laminates.

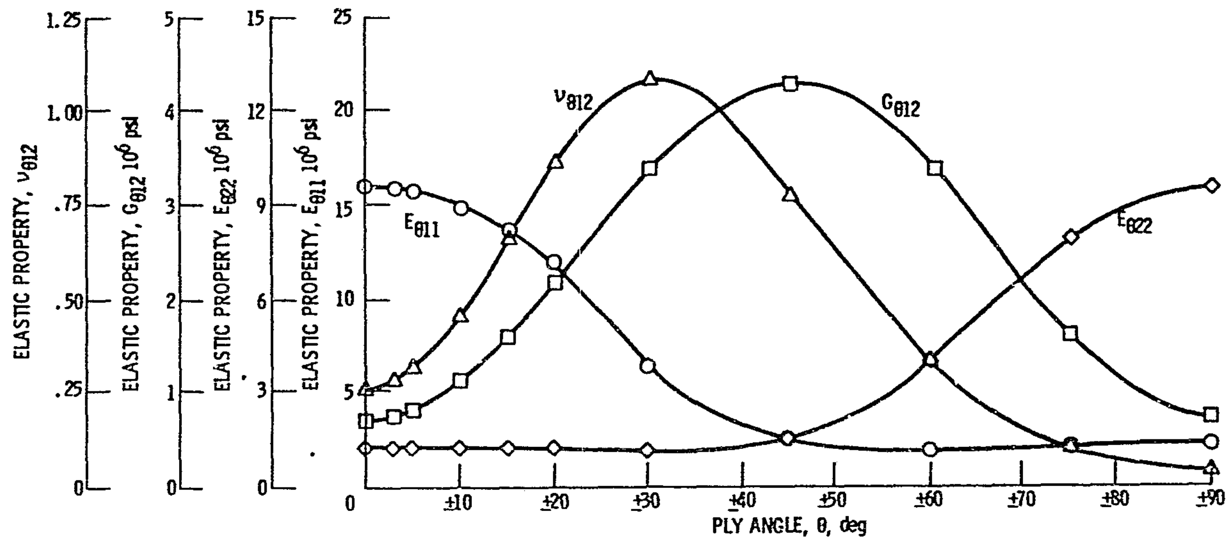

Figure 9. - Elastic properties of intraply hybrid (80% AS/E//20% S-G/E) ±θ laminates.

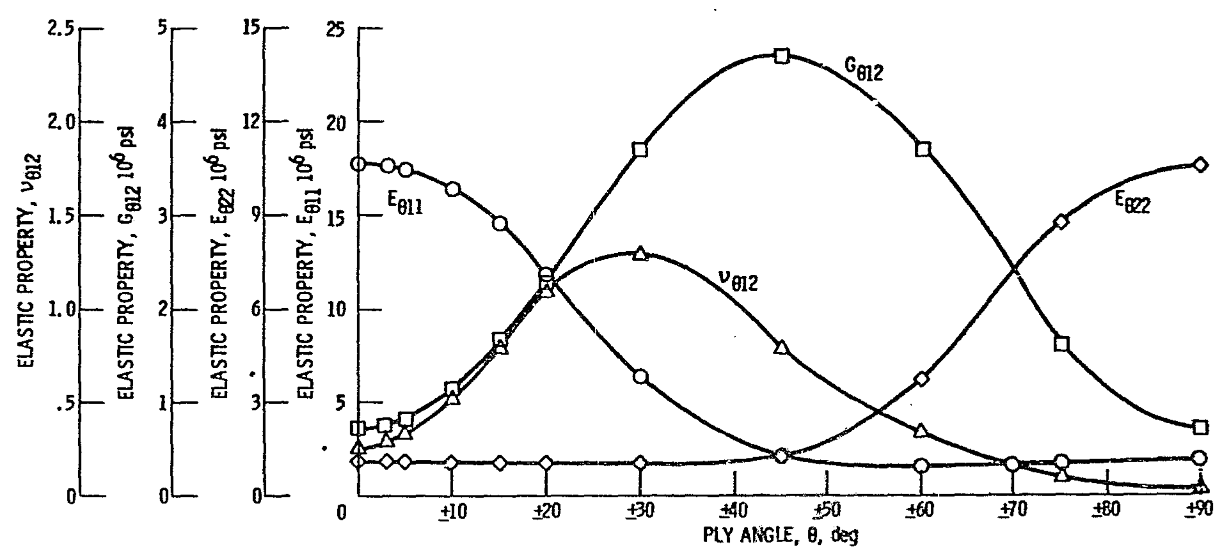

Figure 10. - Elastic properties of intraply hybrid (80% AS/E//20% K/E) ±θ laminates.

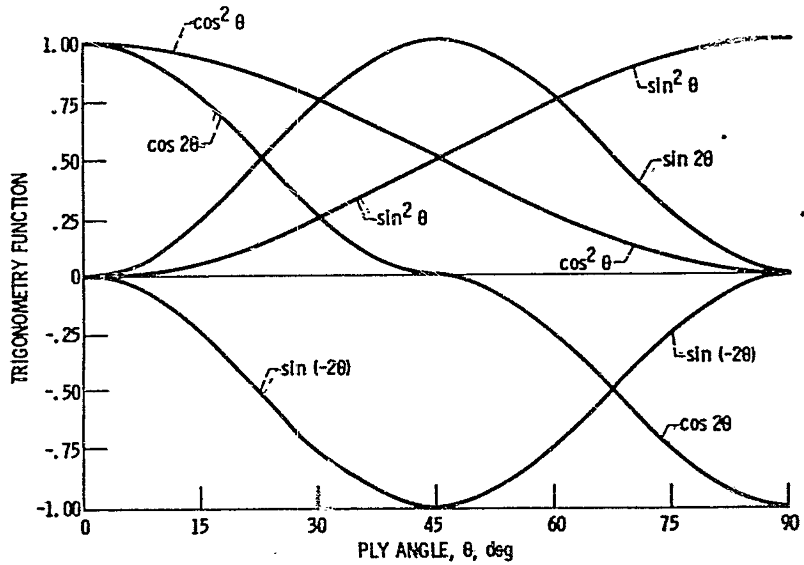

Figure 11. - Trigonometric functions for ply stress influence coefficients (PSICS).

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16. Abstract

A convenient procedure is described for the determination of the mechanical behavior (elastic properties and failure stresses) of angleplied fiber composite laminates using a pocket calculator. The procedure uses simple equations and appropriate graphs of elastic properties versus ply angles. The procedure can handle all types of fiber composites including hybrids. The versatility and generality of the procedure is illustrated using several step-by-step numerical examples.

17. Key Words (Suggested by Author(s)) Fiber composites; Mechanical behavior; Elastic properties; Failure stresses; Stress analysis; Boron/epoxy; Graphite-fibers/epoxy; S-glass/epoxy; Kevlar/epoxy; Hybrid composites; Interply hybrids; Intraply hybrids	18. Distribution Statement Unclassified - unlimited STAR Category 24		
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