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Analytical and Design Techniques for Drag Reduction Studies on Wavy Surfaces

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SUMMARY

Numerical models for two-dimensional turbulent boundary layers over wavy surfaces have been investigated. Computations for wavy wall boundary layers indicate possibilities of overall drag reduction in a parameter range of the geometry of the wall. The correction technique using integral methods for analyzing arbitrary surfaces has been found to be unsuitable for some cases of interest in drag reduction: a Navier-Stokes solver for wavy walls has, hence, been built to test these problems. Test results of the Navier-Stokes solver indicate that the solution techniques are accurate enough to handle complex geometries and steep variations in fluid properties.

LIST OF SYMBOLS

a	amplitude of the wavy surface
K	curvature of the wavy surface
k	turbulent kinetic energy
p	pressure
R, r_w	Radius of curvature of the wavy surface
t	time
u	velocity in the longitudinal direction
v	velocity in the normal direction
x	distance along the longitudinal direction
y	normal distance
δ	boundary layer thickness
δ^*	momentum deficit in the boundary layer
δ_{ij}	Kronecker delta
λ	wave length
μ, μ_t	dynamic viscosity in laminar and turbulent cases
σ_x', σ_y'	apparent normal stresses due to turbulent flow
τ	apparent shear stress due to turbulent flow
∇^2	Laplacian

1. INTRODUCTION

The energy crisis of recent years has brought to focus the need for testing both conventional and unconventional ideas of turbulence control. Some candidate concepts of this type are being studied at NASA Langley Research Center and include compliant surfaces, wavy geometries and turbulent eddy breakup devices. The present contract work was initiated to provide logistical support for the research in turbulence control and drag reduction at the Fluid Mechanics Branch, LaRC, NASA.

The current state of experimental and theoretical research on wavy geometries has been reviewed in ref. 1. Briefly stated there are two condidate experiments (i.e. references 2 and 3) that show substantial skin friction drag reduction in flows over wavy surfaces. However the total drag for these surfaces is above the flat plate value due to the enormous pressure drag contribution.

In reference 1 details of a procedure for predicting skin friction drag and the pressure drag over wavy surfaces, have been presented. The present research effort, based on this work was directed to develop algorithms for the design of wavy wall experiments and to predict the performance of these geometries. Details of this effort are documented in Section 2.

In section 3 details of the effort to develop a two-dimensional Navier-Stokes solver for wavy geometries are presented. In section 4 a review is presented of the feasibility study of candidate concepts and directions for future research.

2. TURBULENT FLOW OVER WAVY SURFACES

Turbulent flows over wavy geometries undergo two major effects which are absent in flat geometries, (a) oscillatory curvature effects, and (b) oscillatory pressure gradient effects. For waves of small curvature in thick turbulent boundary layer the wavy surface may be viewed as a perturbation from flat geometry and estimates on flow modifications are sufficiently easy to predict. However where certain parameters ($\frac{a}{\lambda}$) (i.e. wave number) and (δ/R) (i.e. nondimensional curvature) are moderate the problem can no longer be viewed as a small perturbation of the flat boundary layer case. The mean boundary layer equations for two-dimensional incompressible turbulent flows of non-negligible surface curvature in body-fitted coordinates are given below:

Continuity:

$$\frac{\partial}{\partial x}\left\{r_w\,\bar{\rho}\,\bar{u}\right\} + \frac{\partial}{\partial y}\left\{r_w\,(1+Ky)\,(\bar{\rho}\,\bar{v} + \overline{\rho'\,v'}\,)\right\} = 0 \qquad \text{(Ia)}$$

Mean Momentum Equations:

$$\frac{1}{(1+Ky)}\,\bar{\rho}\,\bar{u}\,\frac{\partial\bar{u}}{\partial x} + (\bar{\rho}\,\bar{v} + \overline{\rho'\,v'})\,\frac{\partial\bar{u}}{\partial y} + (\bar{\rho}\,\bar{v} + \overline{\rho'\,v'})* \; * \frac{\bar{u}K}{(Ky+1)}$$

$$= -\frac{1}{1+Ky}\,\frac{\partial\bar{p}}{\partial x} + \frac{\partial\tau}{\partial y} + \frac{2K}{(1+Ky)}\tau$$

$$-\frac{K}{1+Ky}\,\frac{\partial}{\partial y}\,(\bar{\mu}\,\bar{u}) - \bar{\mu}\,\bar{u}\left(\frac{K}{1+Ky}\right)^2 \qquad \text{(Ib)}$$

$$\frac{K}{1+Ky}\,\bar{\rho}\,\bar{u}^2 = \frac{\partial\bar{p}}{\partial y} \qquad \text{(Ic)}$$

The presence of equation Ic is due to non-negligible gradients of pressure introduced by the curvature. The conventional boundary layer assumption ($\frac{\partial p}{\partial y} = 0$) is thus inappropriate. The equation system I given above is difficult to solve due to the presence of equation Ic. The closure model for turbulence must include the non-equilibrium nature of the flow over the waves and their relaxation to equilibrium when the curvature and pressure gradient effects are applied to the flow.

In reference 1 a procedure for solving the equations of motion for the turbulent flow over wavy surfaces was presented. The procedure adopted there is to use an existing two-dimensional boundary layer code (i.e. the Beckwith-Bushnell compressible boundary layer program of ref. 4) with a specified input pressure field and suitable modifications to length scales due to curvature and pressure gradient. The prescription of P(y) means that to first order the vertical momentum equation can be ignored. Thus one is left with solving a boundary layer program to obtain the required results such as skin friction, velocity profiles, etc.

The approach to wavy wall turbulent flows therefore involves:

(i) evaluation of pressure distribution for input to the program, and

(ii) use of the modified Beckwith-Bushnell program for obtaining the required solution.

The pressure distribution over the wavy surface is evaluated using an inviscid perturbation method of the Miles type discussed in ref. 5. An algorithm which is third order accurate in space has been developed to obtain the pressure distribution for this case. Unfortunately, this algorithm is of the inviscid type and hence no information of the pressure phase shift can be obtained by this method. The phase shift of the pressure is currently evaluated using empirical data as shown in fig. 1.

A procedure for designing wavy wall surfaces with total drag reduction is given in ref. 1. However, extensive testing of this approach on moderately curved wavy surfaces indicates that the δ^* correction approach does not work for these problems. The failure of this first order approach suggests that the correct way to approach the problem should be by solving the full Navier-Stokes equations on the computer. In the next section details of such a procedure are described.

3. NAVIER-STOKES ALGORITHM FOR WAVY SURFACES

The governing equations for a two-dimensional flow over a wavy surface in a Cartesian x-y coordinate system are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2}\right]$$

$$+ \frac{1}{\rho}\left(\frac{\partial \sigma_x{}'}{\partial x} + \frac{\partial \tau'}{\partial y}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]$$

$$+ \frac{1}{\rho}\left(\frac{\partial \tau'}{\partial x} + \frac{\partial \sigma_x{}'}{\partial y}\right) \qquad \text{(II)}$$

The modelling of viscous terms due to turbulence, (σ_x', τ, σ_y') is done by using eddy vicosity (i.e. through ad hoc length scales which seem to work very well for moderate [(a/λ), (δ/R) cases]), obtained from the 2D modified Beckwith-Bushnell code (ref. 4). Thus,

$$\left.\begin{aligned} \sigma_x{}' &\equiv -\rho\,\overline{u'^2} \\ \tau' &\equiv -\rho\,\overline{u'\,v'} \\ \sigma_y{}' &\equiv -\rho\,\overline{v'^2} \end{aligned}\right\} \qquad \text{(III)}$$

and $-\rho\,\overline{u_i\,u_j} = \mu_t\left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\rho k\,\delta_{ij}$

where μ_t is turbulent eddy viscosity. The relation for k is provided by empirical data as:

$$\frac{\overline{u' \ v'}}{k} = 0.15 \tag{IV}$$

Since μ_t is a function of both x and y, equation system II has to be solved very carefully to obtain the required accuracy.

Proposed Method

The method for solving the Navier-Stokes equations is by converting the semi-infinite domain of analysis into a rectangular box by using a conformal mapping program which uses FFTs and is hence very inexpensive. In fig. 2 the mapping is shown schematically. The contra and covariant derivatives connecting the derivatives in the mapped plane and in the untransformed plane are evaluated very cheaply and stored in a preprocessing routine. The dependent variables are expanded in Fourier-Chebycheff modes. The solution technique used guarantees spectral accuracies in space thereby limiting the core storage to some manageable level. Velocity splitting is used. A schematic of the computation is shown in fig. 3. A Richardson extrapolation technique is used to obtain second order accurate solution in time. The spatial accuracy is N fold where N is the number of grids used for solution. At present the analysis is confined to over only one wavelength to keep the expenses of running the code minimal.

Initial test runs using the above spectral code indicate spectral accuracy and that the method is second order accurate in time.

4. DISCUSSION OF RESULTS

Wavy Wall Drag Reduction

Figures 4 and 5 are the cumulative findings of a large number of test runs using the modified Beckwith-Bushnell program. The pressure phase shift is input by relying on fig. 1; the pressure decay by calculations using the algorithms discussed in Section 2. The results of fig. 4 indicate that (a/λ) of 0.004 gives the largest reduction in drag. Figure 5 indicates that again for this waveshape the equilibrium value of total drag is attained after about 40 wavelengths and the net drag reduction is not improved any further farther downstream.

Navier-Stokes Test Results

The Navier-Stokes code that has been developed is in modular form. The various modules and their interconnection are shown in figure 6. The Navier-Stokes algorithm was initially tested to check for any bugs in the various models. Two of the important modules that needed to be tested thoroughly were the Poisson solver and viscous stabilization solvers. Below the results of these tests are presented.

Test No. 1:

Stokes Equation

The equation system given below was solved:

$$\begin{aligned} u_t &= -p_x + u_{xx} + u_{yy} \\ v_t &= -p_y + v_{xx} + v_{yy} \\ u_x + v_y &= 0 \end{aligned} \qquad \text{(V)}$$

with $u = \sin 2\, x\, e^{-y}$

$v = +\, 2 \operatorname{Cos} 2\, x\, e^{-y}$, as the initial velocity field. (VI)

The time evolution indicated decay of the velocity profile as e^{-3t} and the scheme was found to be second order accurate in time. The code was run with meshes of 8x8, 6x16, 32x32 and 64x64 and also 8x16, 8x32 and 8x64. It was found that if the Fourier content involves only the first four wave numbers only 8 points are required in the x directions and with 32 and 64 modes in the Tchebycheff direction spectral accuracies can be obtained. This test gives confidence that the heart of the Navier-Stokes solver, i.e. the Poisson solver, is working very well.

<u>Test No. 2</u>:

$$
\begin{aligned}
& u_t = \mu \nabla^2 u \\
\text{with} \quad & u = \sin x \, e^{-2y} \\
\text{and} \quad & u(x,o) = e^{3t} \sin x
\end{aligned}
\qquad \text{(VII)}
$$

Again the test results indicate that the viscous updates are solved exactly and that the viscous stabilization module is working well.

CONCLUSIONS

A Navier-Stokes solver for flow over two-dimensional wavy surfaces has been constructed. The Navier-Stokes solver is extensively debugged and can be used for solving flow over arbitrarily wavy surfaces. The Navier-Stokes code can use time steps of about four orders of magnitudes higher than the explicit CFL limit, it is second order accurate in time and infinite accurate in space.

The computation of wavy flow over moderate amplitude waves indicates some total drag reduction in a parameter range previously left untouched by experimenters which opens up exciting possibilities for future research.

The author would like to thank Mr. A. M. Cary for his contribution to the work on wavy wall computations. The author would also like to thank Prof. S. A. Orszag for providing guidance in setting up the Navier-Stokes code.

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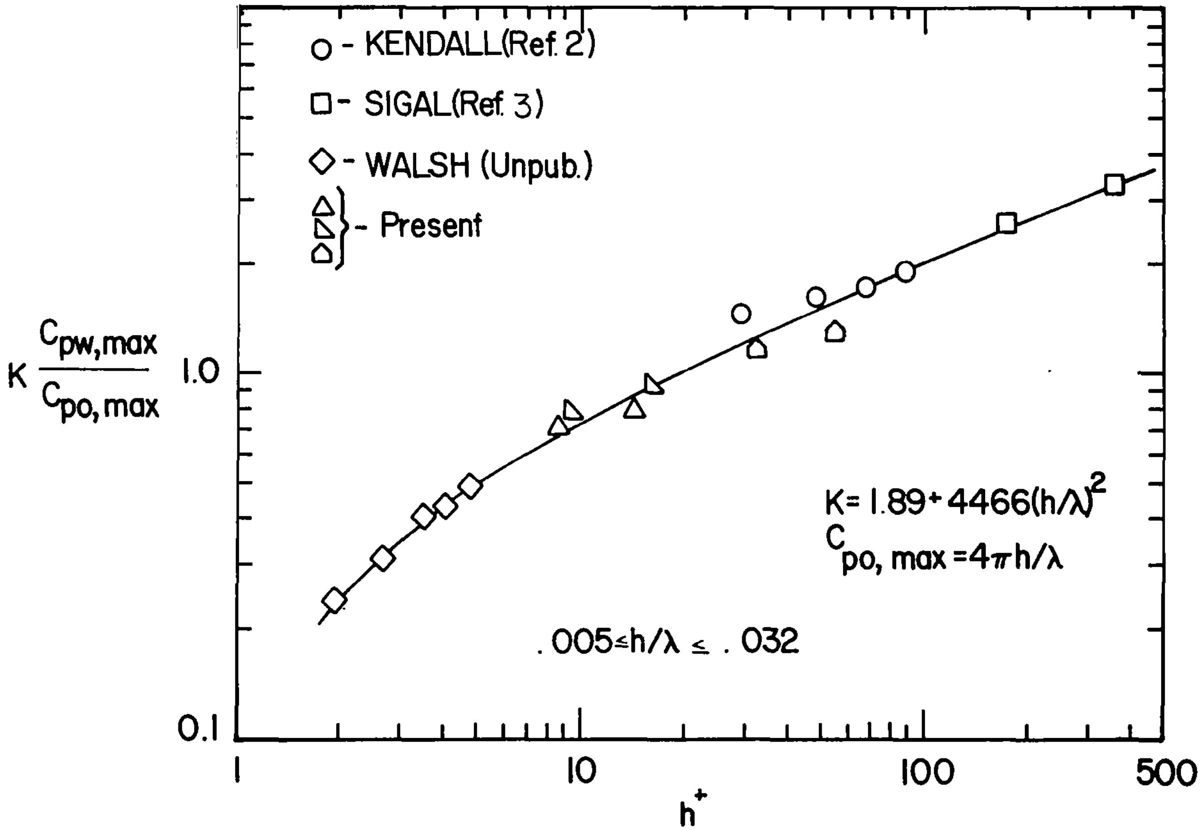
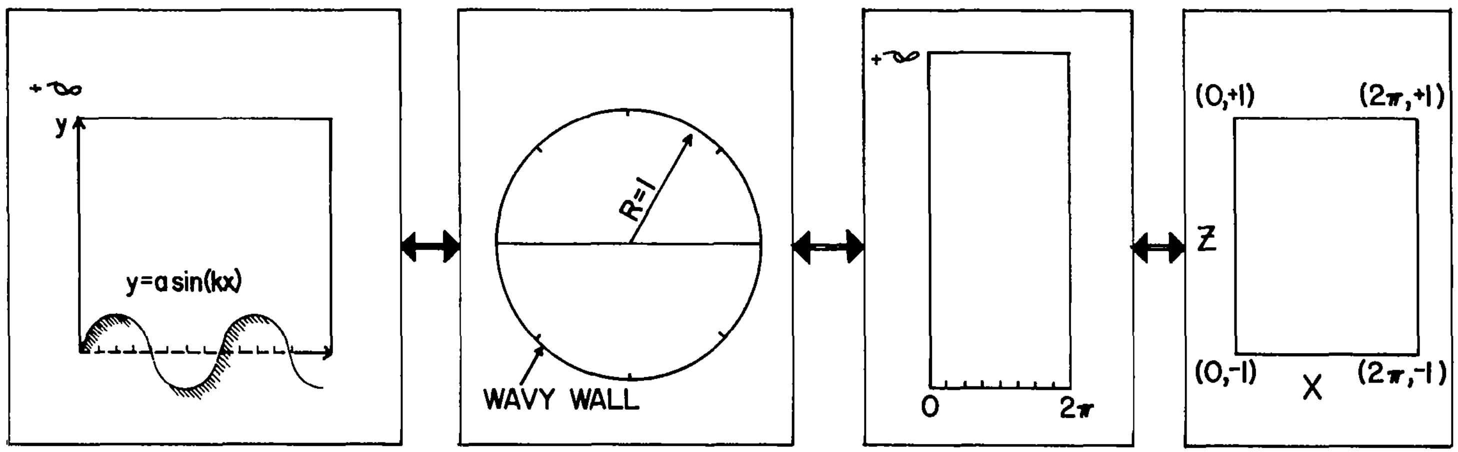


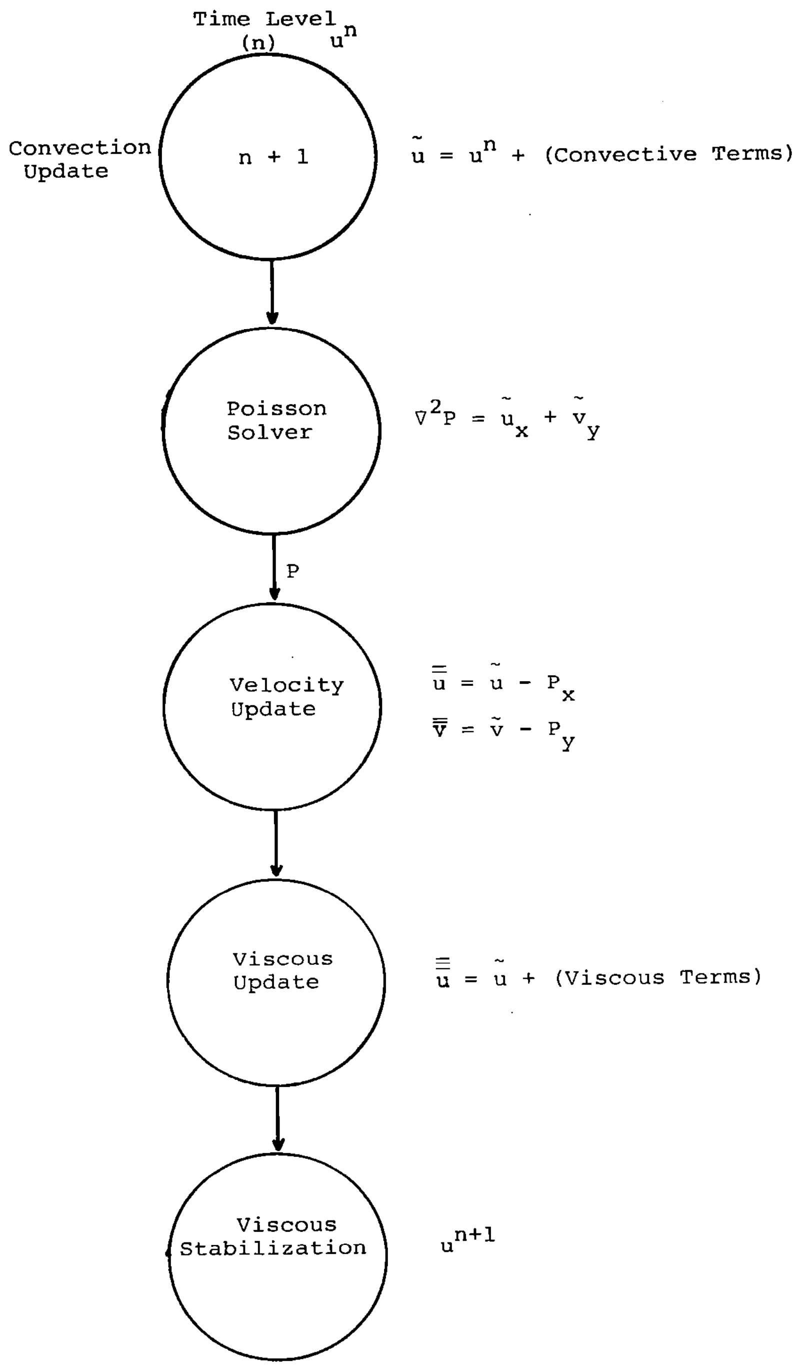
Correlation of the maximum pressure coefficient over rigid sine-wave walls with small a/λ - low speed flow.

Figure 1



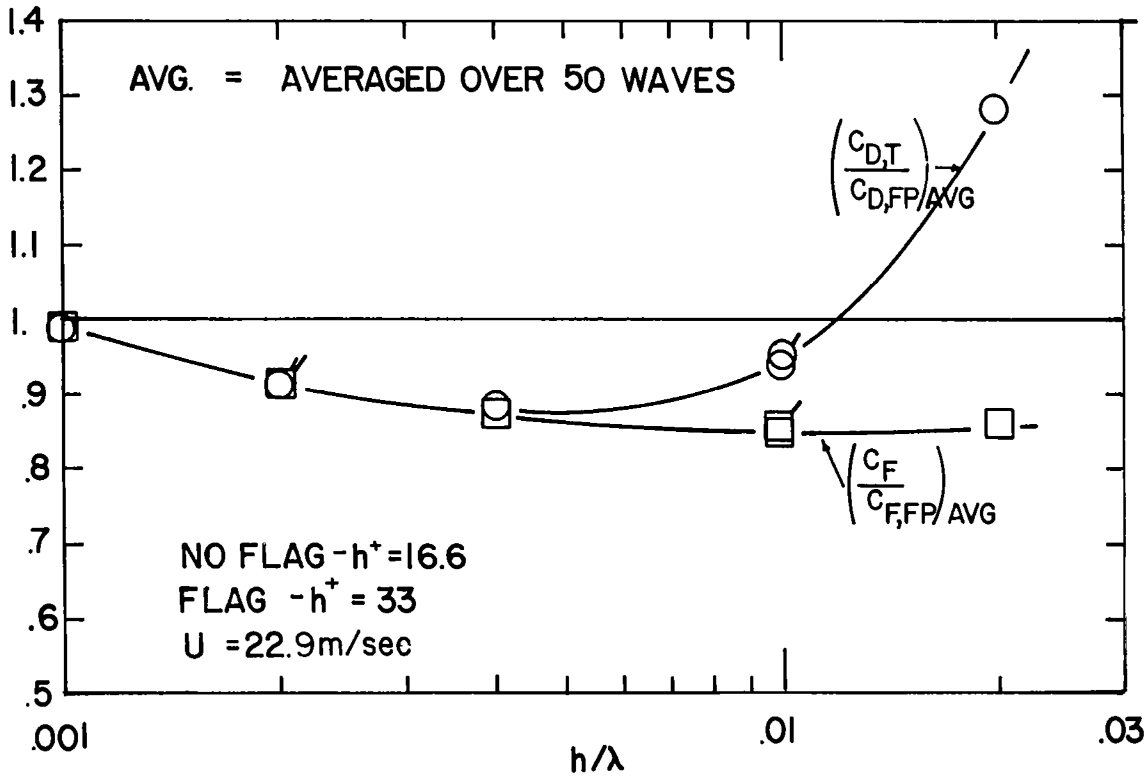
Schematic of Mappings

Figure 2



Schematic of Calculation
Procedure for Navier Stokes Problem

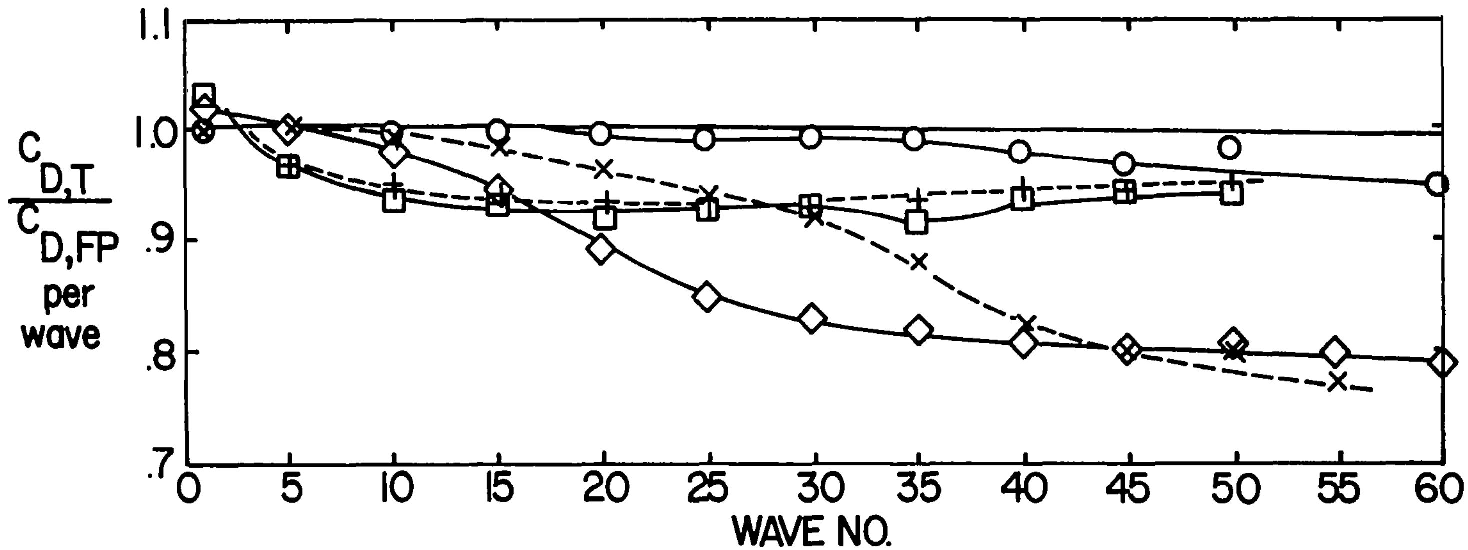
Figure 3



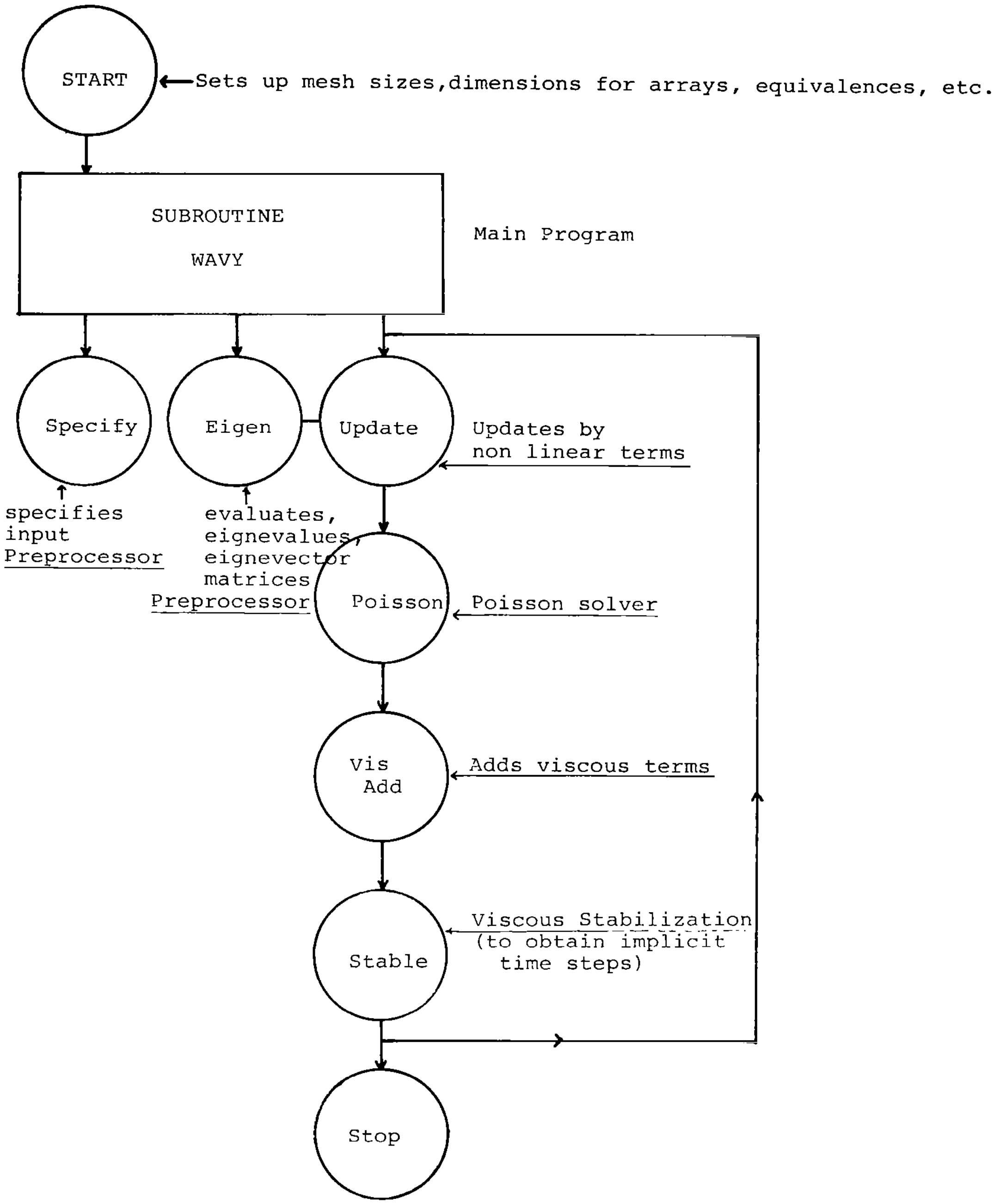
Analytical prediction of the drag over 50 waves for several shallow sine-wave walls.

Figure 4

	h	λ	h/λ	h^+
○	.01	10	.001	16.6
◇	.01	2.5	.004	16.6
□	.01	1.0	.01	16.6
×	.02	10	.002	33
+	.02	2.0	.01	33



Analytical predictions of the average drag per wave
for several shallow sine-wave walls.
Figure 5



Schematic of N-S Algorithm

Figure 6

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15. Supplementary Notes

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Final Report

16. Abstract

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