Volume-Energy Parameters and Turbulent-Flow Density Fluctuations

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SUMMARY

Volume-energy relations that can be determined from an equation of state are used to group many sets of heat-transfer data, ranging from data for liquids to data for gases and including the near-critical region. The volume - Gibbs energy parameter (or Helmholtz energy parameter) grouped these data better than did such other parameters as enthalpy, temperature, or internal energy.

INTRODUCTION

In relating volume-energy, or density-energy, parameters to turbulent flows (ref. 1), consider the conservation equations with fluctuations (refs. 2 and 3). The turbulent momentum and energy fluxes for steady flow in a tube become

$$-\tau_t = \bar{\rho}\,\overline{u'v'} + \bar{v}\,\overline{\rho'u'} + \overline{\rho'u'v'} \tag{1}$$

$$-q_t = \bar{\rho}\,\overline{v'h'} + \bar{v}\,\overline{\rho'h'} + \overline{\rho'v'h'} \tag{2}$$

where u', v' are the velocity fluctuations and ρ', h' are the density and enthalpy fluctuations, respectively. Symbols are defined in appendix A. The density, or specific volume, can be related to the enthalpy through an equation of state:

$$\rho = \rho(h,\ p,\ x_i) \tag{3}$$

and the local density can be represented in terms of the average density by a Taylor series

$$\rho = \bar{\rho} + \overline{\frac{\partial \rho}{\partial h}}\,\Delta h + \overline{\frac{\partial \rho}{\partial P}}\,\Delta P + \overline{\frac{\partial \rho}{\partial x_i}}\,\Delta x_i + \ldots \tag{4}$$

or by using the volumetric parameters and fluctuating components (ref. 1)

$$\frac{\rho'}{\bar{\rho}} \simeq -\beta_2 h' + \kappa P' - \zeta_i x_i' \tag{5a}$$

For single-component fluids at nearly constant pressure, the density fluctuations can be written as

$$\rho' \simeq -\bar{\rho}\beta_2 h' \tag{5b}$$

where h' is the fluctuating turbulent energy and $\beta_2 = (-\partial \overline{\ln \rho / \partial h})_{P,b}$. The authors of reference 1 used these terms and the parameters Nu_R and ψ_1 or ψ_2 to group several heat-transfer data sets, ranging from low-temperature-liquid to high-temperature-gas data and including the near-critical region and high-pressure fluid states, where

$$\text{Nu}_R = \frac{\text{Nu}_{exp}}{\text{Nu}_{calc}} \tag{6}$$

$$\psi_1 = 1 + \beta_1 (T_w - T_b) \tag{7}$$

$$\psi_2 = 1 + \beta_2 (h_w - h_b) \tag{8}$$

$$\beta_1 = -\left(\frac{\partial \ln \rho}{\partial T}\right)_b \tag{9}$$

and Nu_{calc} is of the McAdams type. Herein

$$\text{Nu}_{calc} = 0.023 \text{ Re}_b^{0.8} \text{ Pr}_b^{0.4} \tag{10}$$

where properties are evaluated at the bulk temperature. The use of the Petukhov relation is equally valid:

$$\left.\begin{array}{l} \text{Nu} = \left(\frac{\xi}{8}\right)\dfrac{\text{Re Pr}}{\kappa_0} + 12.7\sqrt{\frac{\xi}{8}}\ (\text{Pr}^{2/3} - 1) \\[12pt] \xi = (1.82 \log_{10} \text{Re} - 1.64)^{-2} \\[12pt] \kappa_0 = 1 + \dfrac{900}{\text{Re}} \end{array}\right\} \tag{11}$$

The equation of state representing the PVT surface can also be written several different ways, such as

$$\rho = \rho(\varphi_k,\ P,\ x_i) \tag{12}$$

where

$$\left.\begin{array}{lll} \varphi_1 = T & \varphi_3 = A = E - TS & \varphi_5 = E = h - PV \\[12pt] \varphi_2 = h & \varphi_4 = G = h - TS & \varphi_6 = \mu_i = \dfrac{\partial G}{\partial x_i} \end{array}\right\} \tag{13}$$

Does it follow that ψ_k, where $\beta_k = -\left(\partial \ln \rho/\partial\varphi_k\right)_P$, will also group these data? In reference 1 it was shown that for $k = 1, 2$ the answer is yes; and in this report we investigate $k > 2$, with the thought that, for some processes, it may be more convenient to use energy potential functions rather than, for example, temperature.

THEORETICAL RELATIONS

From an equation of state, such as equation (12), the density can be written to a first approximation as

$$\Delta\rho \sim \frac{\partial\rho}{\partial\varphi_k} \Delta\varphi_k + \frac{\partial\rho}{\partial P} \Delta P + \frac{\partial\rho}{\partial x_i} \Delta x_i \tag{14}$$

(no sum on k) where the Helmholtz, Gibbs, and internal energies were defined in the preceding equations.[1]

In turbulent flow, since $\rho = \bar{\rho} + \rho'$, equation (14) represents the turbulent density fluctuation. For a process with a single-component fluid at nearly constant pressure,

$$\rho' = -\bar{\rho}\beta_k\varphi_k' \tag{15}$$

(no sum on k) where $\beta_k = -\left(\partial \ln \rho/\partial\varphi_k\right)_P$ and φ_k' represents the turbulent energy fluctuation. The particular form to be used would depend on the variables of the energy equation.

Thus we seek to determine the generalized parameter

$$\psi_k = 1 + \beta_k\left(\varphi_{k,w} - \varphi_{k,b}\right) \tag{16}$$

(no sum on k) that will group heat-transfer data sets for $k > 2$ as was done for $k \leq 2$. We will need the following relations, which can be developed by using the Bridgeman tables (ref. 4); for example, at constant pressure

$$\frac{\partial A}{\partial \ln V} = -\left(\frac{S}{\beta_1} + PV\right); \quad \frac{\partial G}{\partial \ln V} = -\frac{S}{\beta_1}; \quad \frac{\partial E}{\partial \ln V} = \frac{C_p}{\beta_1} - PV; \quad \frac{\partial h}{\partial \ln V} = \frac{C_p}{\beta_1} \tag{17}$$

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[1] $\varphi_6 = \varphi_4$ for the simple fluids considered herein.

RESULTS AND DISCUSSION

The figures presented herein are based on the data sets delineated in reference 1 and used to establish that Nu_R and ψ_1 or ψ_2 can be used to group heat-transfer data.

Gas-State Data

For gases, β_k to the first order is inversely proportional to temperature $\left(\text{i.e.,} \right.$ $\beta_k \sim T^{-1}\left. \right)$. In reference 1, it was shown that for $k \leq 2$

$$Nu_R = C\psi^{-n} \tag{18}$$

where $0.95 < C < 1.1$ and $0.45 < n < 0.6$. Representative values were given for C as 1 and for n as 0.55 for fluids and 0.50 for gases (ref. 1). As ψ approaches 1, the correction to the conventional McAdams equation becomes insignificant. (Note that for a gas, $\psi_1 \rightarrow T_w/T_b$.)

In figure 1, Nu_R is shown as a function of ψ_k, where $k = 1, 5$, for the data sets of references 5 and 10, which represent four gases - helium, hydrogen, air, and carbon dioxide. The grouping appears to follow equation (18). The figure presents 1345 data points that appear to be represented by $C \sim 1.05$ and $n \sim 0.5$ for $k = 1, 2, 5$ and $n \sim 0.4$ for $k = 3, 4$. The figure is intended to show that these data are grouped by ψ_k $(k = 1, 5)$ and not to develop specific values of C and n. The individual data sets are discussed in appendix B for $k = 1$.

Fluid-State Data

In the fluid state, which includes the liquid and dense-gas regimes, the thermophysical properties vary significantly and the situation is more complex. The data for fluid hydrogen are plotted in figure 2 for low-pressure hydrogen flows through a 0.478-centimeter- (0.188-in.-) diameter tube, where buoyancy effects were considered negligible (ref. 10). The high-pressure data sets of reference 11 are plotted in figure 3. The high-pressure data tend to follow equation (18); for the low-pressure data, ψ_1 and ψ_4 more closely follow equation (18), but ψ_3 appears to be cubic in $\log Nu_R - \log \psi$ for $\psi > 1.2$.

Fluid-nitrogen data (ref. 12) are plotted in figure 4. Fluid-oxygen data (refs. 13 and 14) for low pressure and high pressure are plotted in figures 5 and 6, respectively. The nitrogen and low-pressure oxygen data tend to follow equation (14), with the nitrogen data 30 percent low. The high-pressure oxygen data appear to be twice the values predicted by using equation (18) at low ψ. For both these data sets, it is felt that the

inside surface temperatures have not been properly corrected. Correction could result in a 25- to 30-percent increase in the nitrogen data and a lowering of the oxygen data.

Figure 7 (ref. 15) represents helium data for $k = 1$. No other comparisons were made.

Although each ψ_k groups these data sets, the Gibbs ψ_4 and Helmholtz ψ_3 functions group the data better than temperature ψ_1, enthalpy ψ_2, and internal energy ψ_5 in that order. This judgment is based on a comparative analysis for many plots; overlays of the various data sets; an analysis of data scatter; the fluid state or regime where this scatter occurred; and the relative steepness of the slope of the data grouping, all of which can contribute heavily to predictive errors.[2] Such a qualitative trend suggests that the turbulent energy equations should perhaps be developed in terms of the Gibbs or Helmholtz functions, depending on the process involved, for a better understanding of the dynamics.

The hydrogen, oxygen, and helium data of figures 2, 6, and 7, respectively, indicate an enhanced heat transfer at low ψ_k, more notably in the critical region. An increase in the deviations at high ψ_k is noted for hydrogen (fig. 2). The divergence of Nu_R at high ψ_k may be caused by flow disturbances (oscillations) for at high ψ_k the flow is quite sensitive to small disturbances and oscillations are commonplace (e.g., ref. 10). Therefore, it may become necessary to incorporate $\kappa = \partial \ln \rho / \partial P$ into ψ_k, that is,

$$\psi_k = 1 + \beta_k \, \Delta \varphi_k + \kappa \, \Delta P \tag{19}$$

These results again indicate that data in the near-critical region can be grouped better by using ψ_4 or ψ_3 and that high-pressure and gas-phase data can be satisfactorily grouped by using ψ_1.

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[2] For the data herein, L/D was not considered important; however, where L/D is important, two forms suggested in ref. 1 can be applied:

$$Nu_R = \psi_k^{-m}$$

or

$$Nu_R \left\{ \left[1 + (2D/L) \right] \right\} = \psi_k^{-n}$$

where

$$m = n - (1 + n) \, D/L.$$

SUMMARY OF RESULTS

An analysis of the turbulent transport fluxes, and heat-transfer data from several sources reveal the following results:

1. Volume-energy, or density-energy, parameters such as $\left(-\partial \ln \rho / \partial \varphi_k\right)_P$ that are readily obtained from an equation of state can be related to turbulent flow density fluctuations, where ρ is density and φ_k is an equation-of-state variable.

2. Volume-energy parameters can be used to group heat-transfer data sets ranging from liquid-state to gas-state data, including the near-critical region. The Gibbs or Helmholtz functions appear to group data better than other thermodynamic parameters; however, outside the near-critical region, volumetric expansion $(k = 1)$ can be used.

3. These results show that some insights might be gained by reformulating and solving the turbulent flow equations in terms of the Helmholtz or Gibbs functions.

Lewis Research Center,
    National Aeronautics and Space Administration,
        Cleveland, Ohio, July 31, 1979,
            506-16.

APPENDIX A

SYMBOLS

A	Helmholtz energy
C	constant
C_p	specific heat at constant pressure
D	diameter
E	internal energy
G	Gibbs energy
h	enthalpy
L	length
m, n	exponents
Nu	Nusselt number
P	pressure
Pr	Prandtl number
q_t	turbulent energy flux
Re	Reynolds number
S	entropy
T	temperature
u	axial velocity component
V	specific volume
v	radial velocity component
x_i	fraction of species i
β_i	isoparametric volumetric expansion, $-\left(\partial \ln \rho / \partial \varphi_i\right)_{\varphi_k}$
ξ_i	isoparametric composition coefficient, $-\left(\partial \ln \rho / \partial x_i\right)_{\varphi_k}$
κ	isoparametric compressibility, $\left(\partial \ln \rho / \partial P\right)_{\varphi_k}$
κ_0	defined by eq. (11)
μ	chemical potential
ξ	defined by eq. (11)

ρ density

τ_t turbulent momentum flux

φ equation-of-state or fundamental-equation variable

ψ defined by eq. (16)

Subscripts:

b   evaluated at bulk conditions

k   index

R   reduced, see eq. (6)

w   evaluated at wall conditions

Superscripts:

$(^-)$ average

$(\ )'$ fluctuating

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APPENDIX B

GAS-STATE DATA SETS

The data of individual investigators that constitute figure 1 require further analysis. These data sets appear to be self-consistent but exhibit significant deviations when compared by using such common parameters as those of equation (18) (i.e., Nu_R and ψ_1).

Gaseous Hydrogen

McCarthy (ref. 5) acquired an extensive data set for gaseous hydrogen flowing in tubes over a wide range of the control parameters, namely, the inlet conditions, mass flow, heat flux, and wall temperatures. The high thermal driving potential $(T_w - T_b)$ was in some cases achieved by precooling the gas before running it through the Joule-heated test section. Wall temperatures to 1260 K were achieved. These data, plotted as figure 8, are characterized by equation (18) where $n = 0.5$ and $C = 1.1$.

Weiland (ref. 7) also took an extensive set of data, although the range of heat fluxes was not as extensive as McCarthy's (ref. 5). The test-section L/D ranged to 237. Weiland also used a precooler to condition the gas. The axial temperature profiles showed significant departures from the conventional monotone turbulent-flow profile. The data are plotted as figure 9. For these data, $n = 0.6$ and $C = 1.0$.

Taylor (ref. 6) took an extensive set of data with very high wall temperatures (to 3000 K) by using a Joule-heated molybdenum tube. Under these conditions many questions as to accuracy in measurements and thermophysical properties enter. And even though no abnormalities or dissociation were reported, still some question exists at the higher values of ψ_1. These data are plotted as figure 10. Here the data follow one form of equation (18), namely, $n = 0.5$ and $C = 1.0$ for $\psi_1 < 5$, and appear to have little influence on ψ_1 for values greater than 5.

Hendricks, et al. (ref. 10) acquired a limited set of data for five tube diameters, ranging from 0.478 to 1.28 centimeters (0.188 to 0.504 in.). In general the driving potential is low, and the data should not be given the same weight as the data of references 5 to 7. These data are plotted as figure 11.

Gaseous Helium

To investigate the effects of an inert gas flowing in a Joule-heated tube, McCarthy (ref. 5), Weiland (ref. 7), and Taylor (ref. 6) also took extensive sets of helium data in their respective apparatus. These data are plotted as figures 12, 13, and 14, respec-

tively. The consistency of the helium and hydrogen data from each investigator is apparent and would appear to rule out dissociation effects in the data of Taylor (ref. 6).

It should also be pointed out that McCarthy (ref. 5) and Weiland (ref. 7) used log-average values and Taylor (ref. 6) used local values. Also suspect are the thermophysical properties routines, as many temperatures are well beyond the approved range (ref. 4).

Other Gases

Since the properties of gaseous air are quite similar to those of nitrogen, the properties of nitrogen were used to reduce the set of air data taken by Wolf (ref. 8). These data, although grouped by equation (18), appear to have some geometry effects not noted in the other data sets. These data are plotted as figure 15. Wolf (ref. 8) also took data for gaseous carbon dioxide. These data are plotted as figure 16. These results also show trends with geometry and may indicate these data to be apparatus dependent.

The gaseous air data set of Humble (ref. 9) was also reduced by using the properties of nitrogen. These data are plotted as figure 17 and show more scatter than the previous sets.

In general the gaseous data are grouped quite well by equation (18), an indication of the usefulness of the density-fluctuation terms (eq. (15)). And for gases $0.95 \le C \le 1.1$ and $0.5 \le n \le 0.6$.

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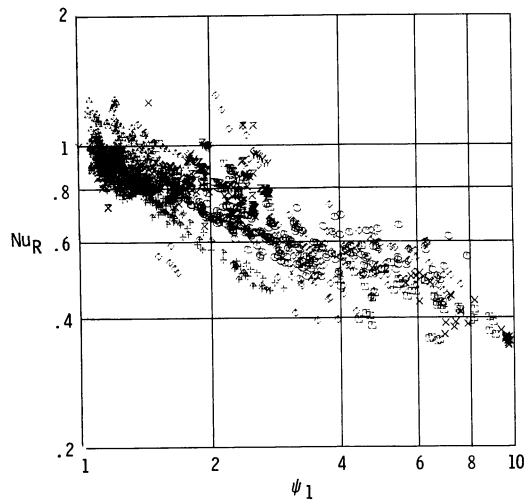
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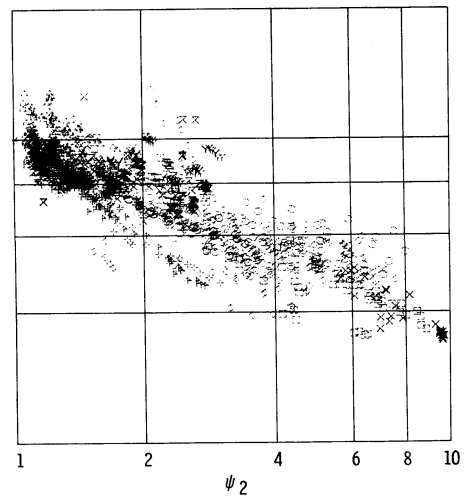
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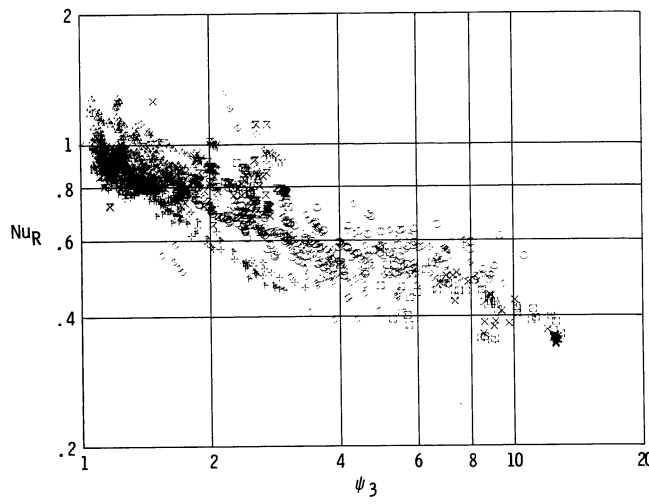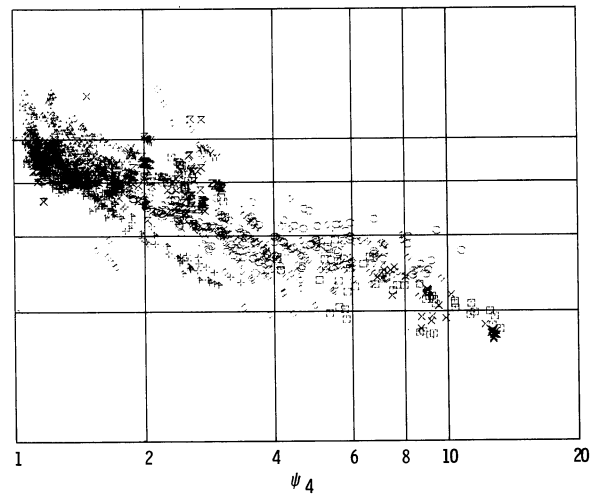
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(d) Gibbs energy.
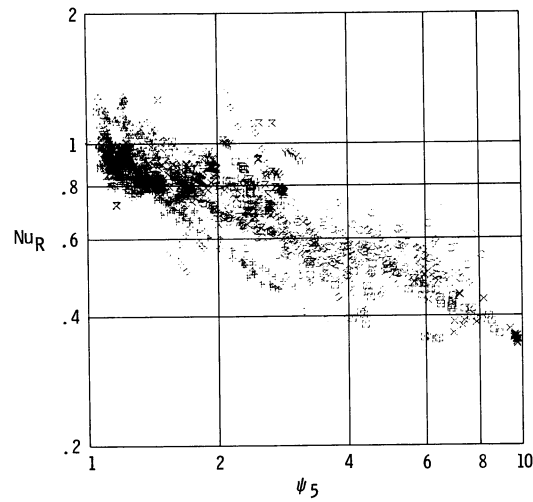
(e) Internal energy.

Figure 1. - Reduced Nusselt number for several gases as a function of volume-energy parameters.
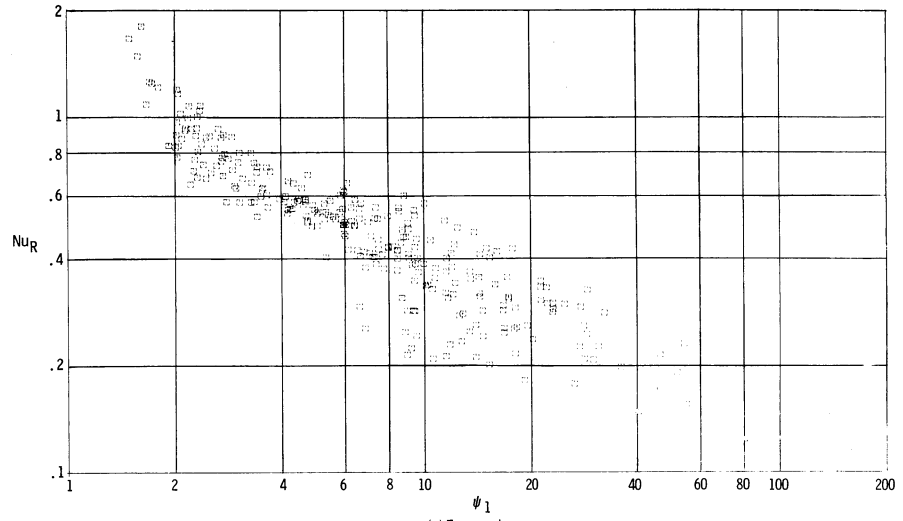
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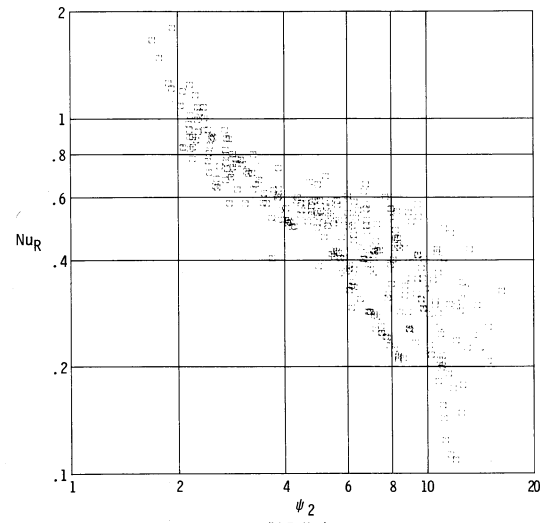
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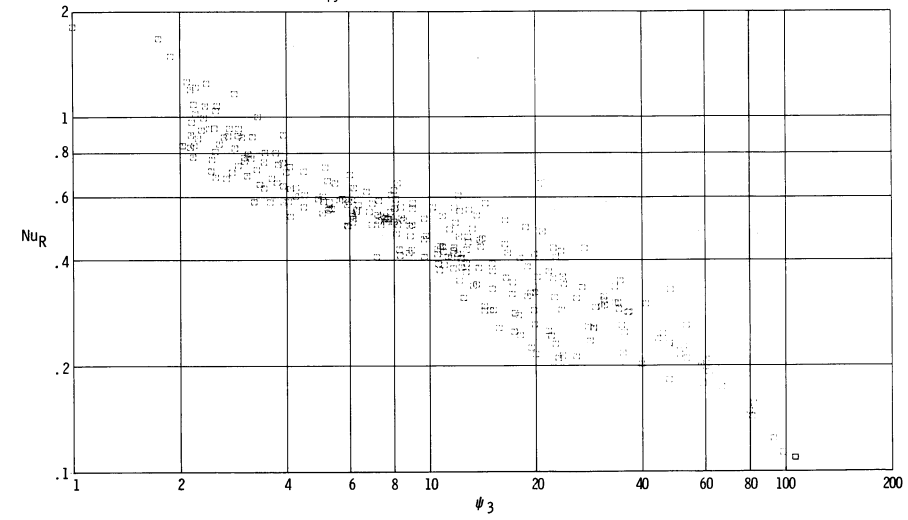
(c) Helmholtz energy.

Figure 2. – Reduced Nusselt number for fluid hydrogen to 5.5 MPa as a function of volume-energy parameters.
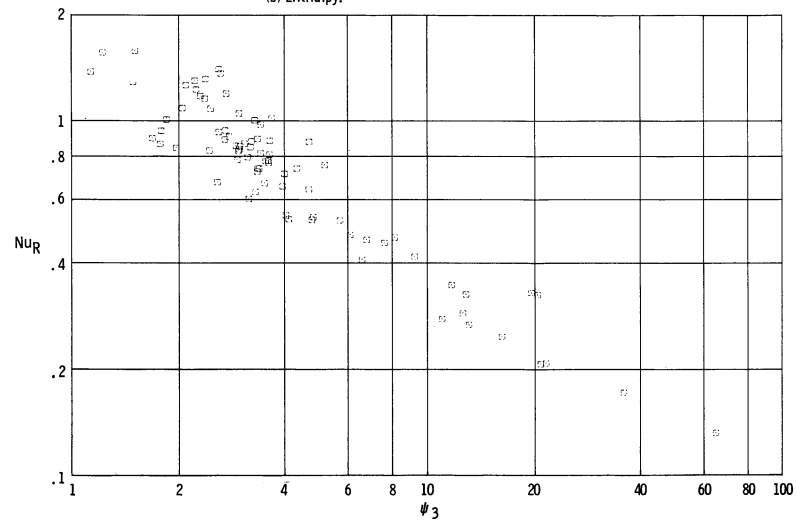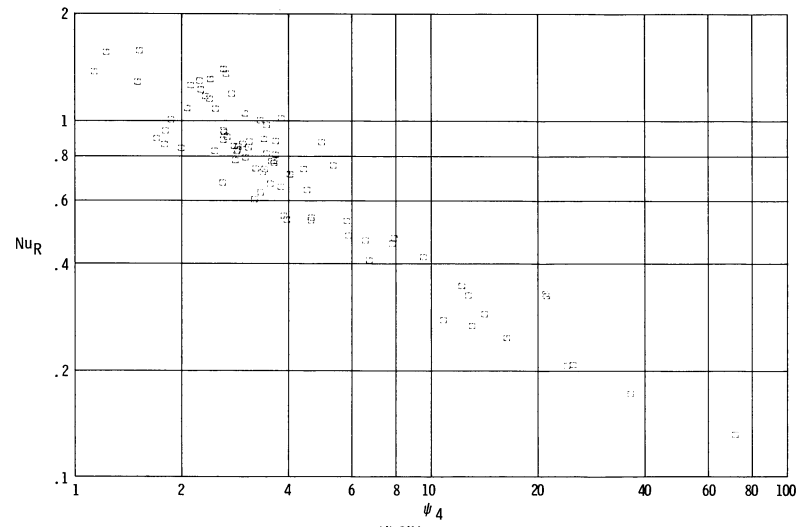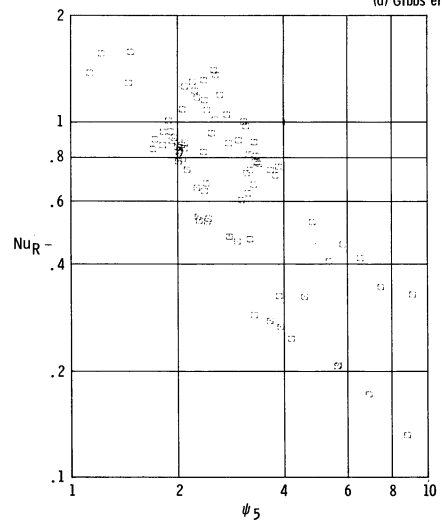
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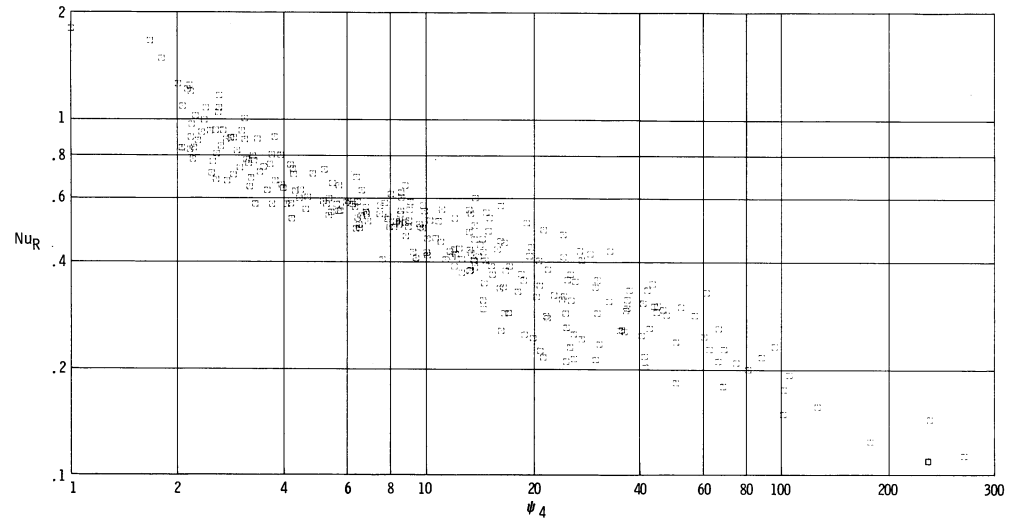
(d) Gibbs energy.

(e) Internal energy.

Figure 2. – Concluded.
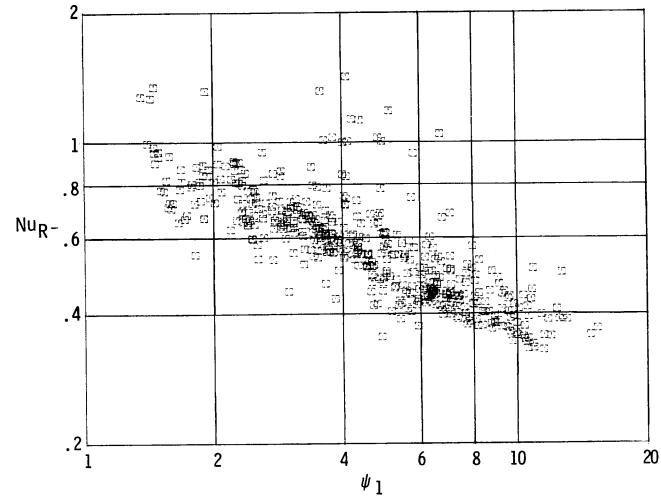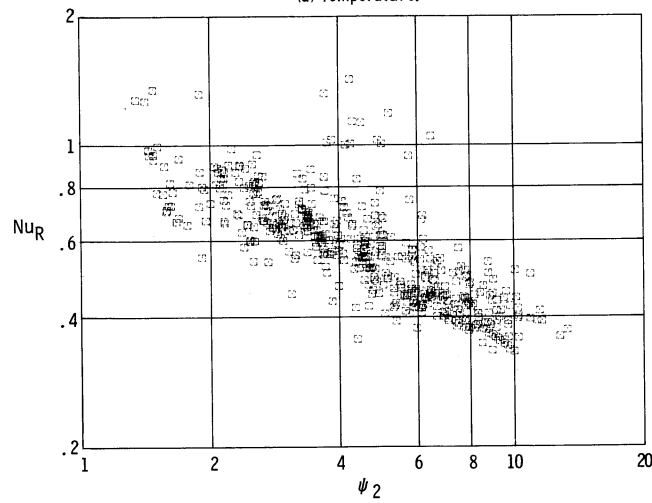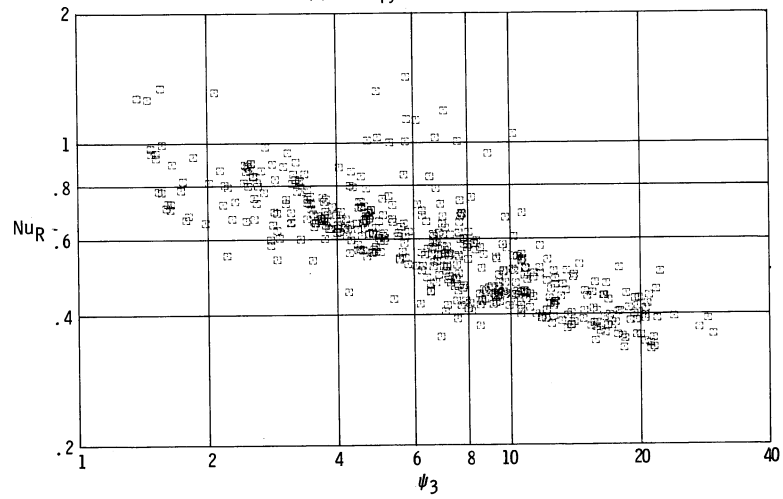
Figure 3. – Reduced Nusselt number for fluid hydrogen from 6.9 to 17.2 MPa as a function of volume-energy parameters.

16
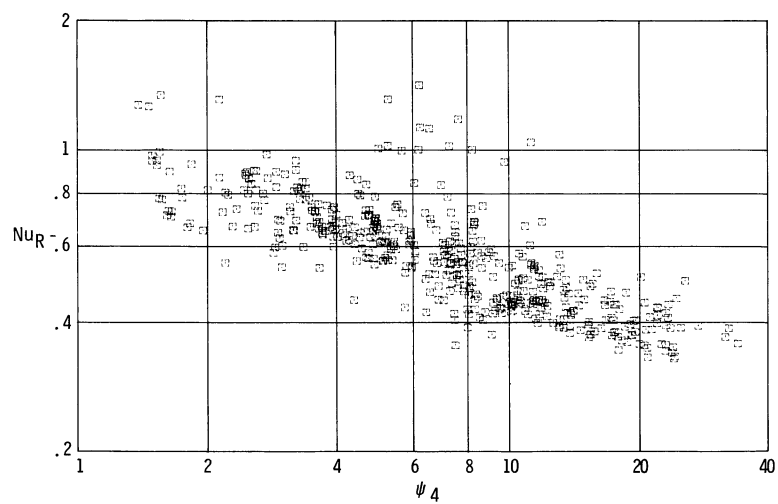
(d) Gibbs energy.

(e) Internal energy.

Figure 3. – Concluded.

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(b) Enthalpy.
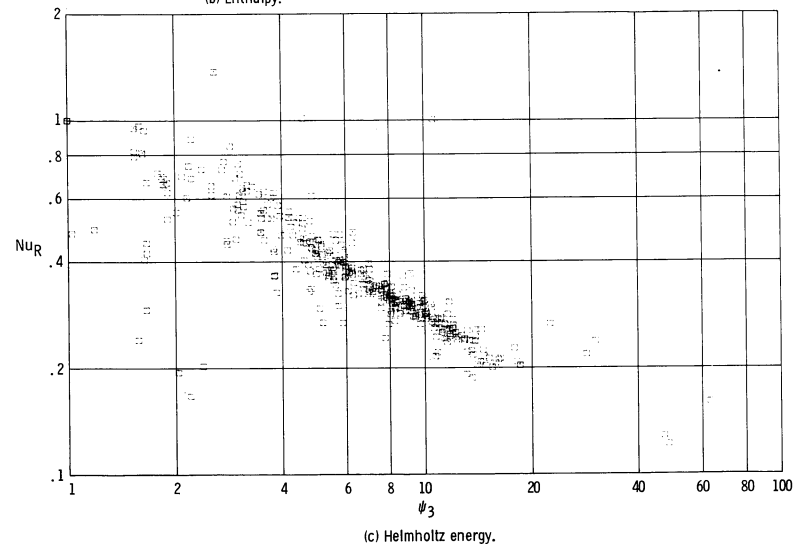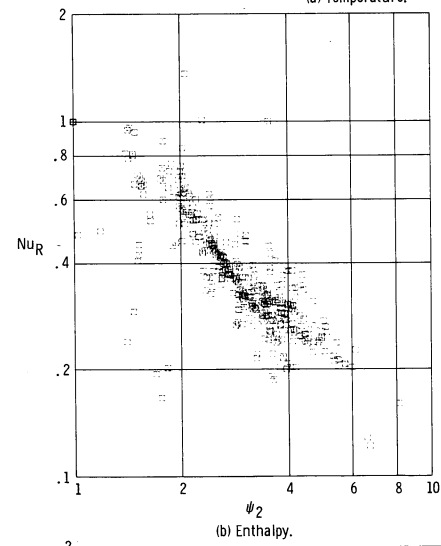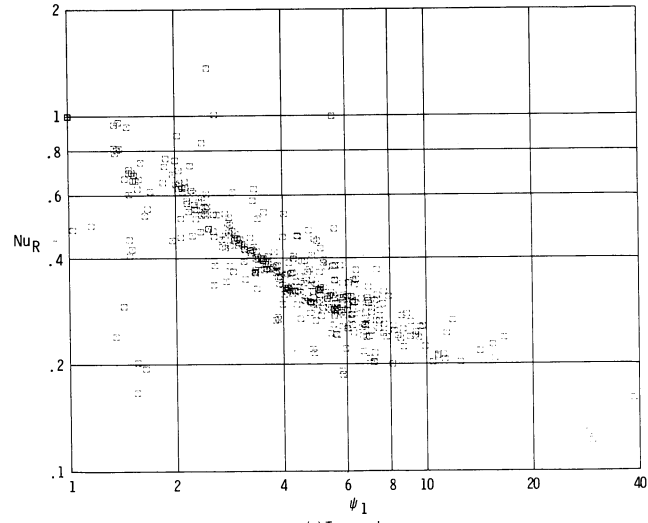
(c) Helmholtz energy.

Figure 4. – Reduced Nusselt number for fluid nitrogen as a function of volume-energy parameters.

18
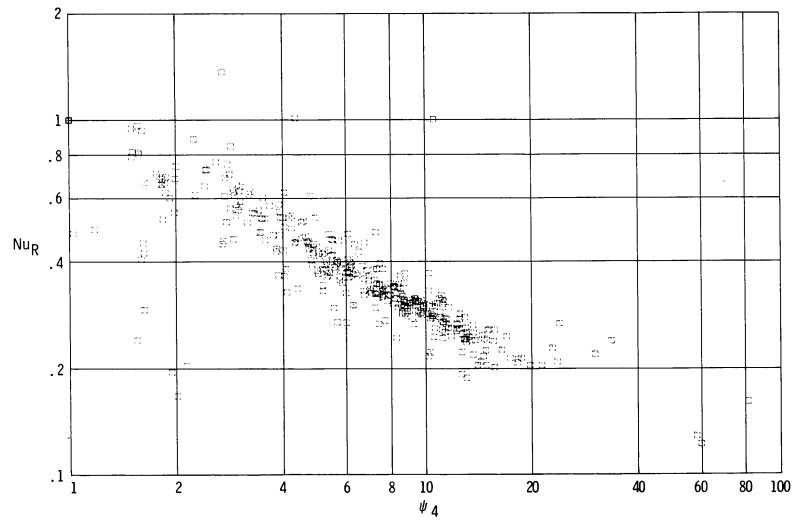
(d) Gibbs energy.

(e) Internal energy.

Figure 4. – Concluded.

19
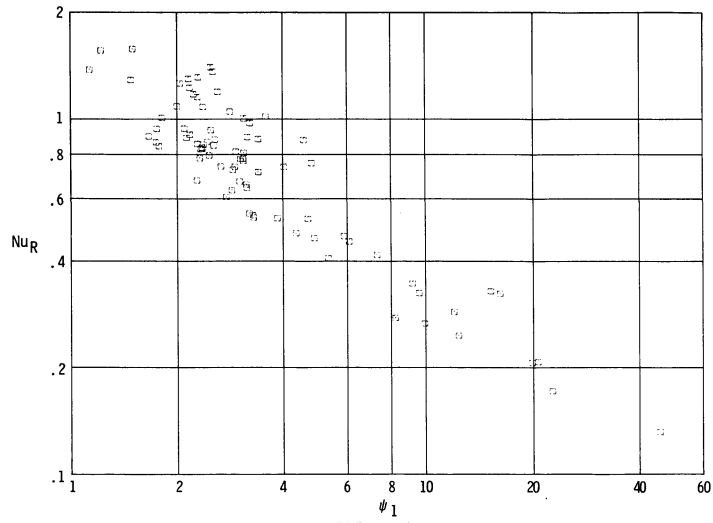
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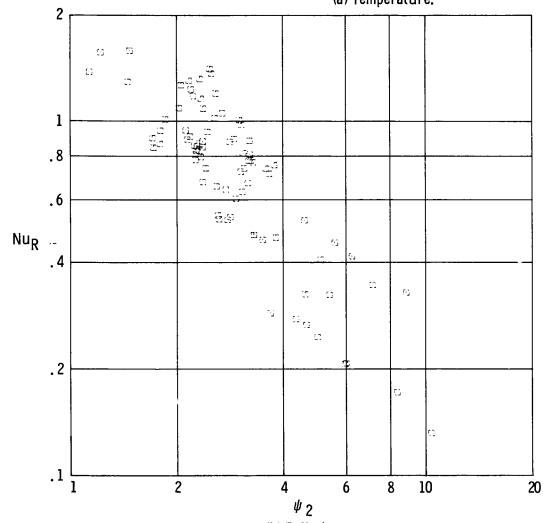
(b) Enthalpy.

(c) Helmholtz energy.

Figure 5. – Reduced Nusselt number for fluid oxygen (ref. 11) as a function of volume-energy parameters.

(d) Gibbs energy.

(e) Internal energy.

Figure 5. – Concluded.

(a) Temperature.

(b) Enthalpy.

(c) Helmholtz energy.

(d) Gibbs energy.

(e) Internal energy.
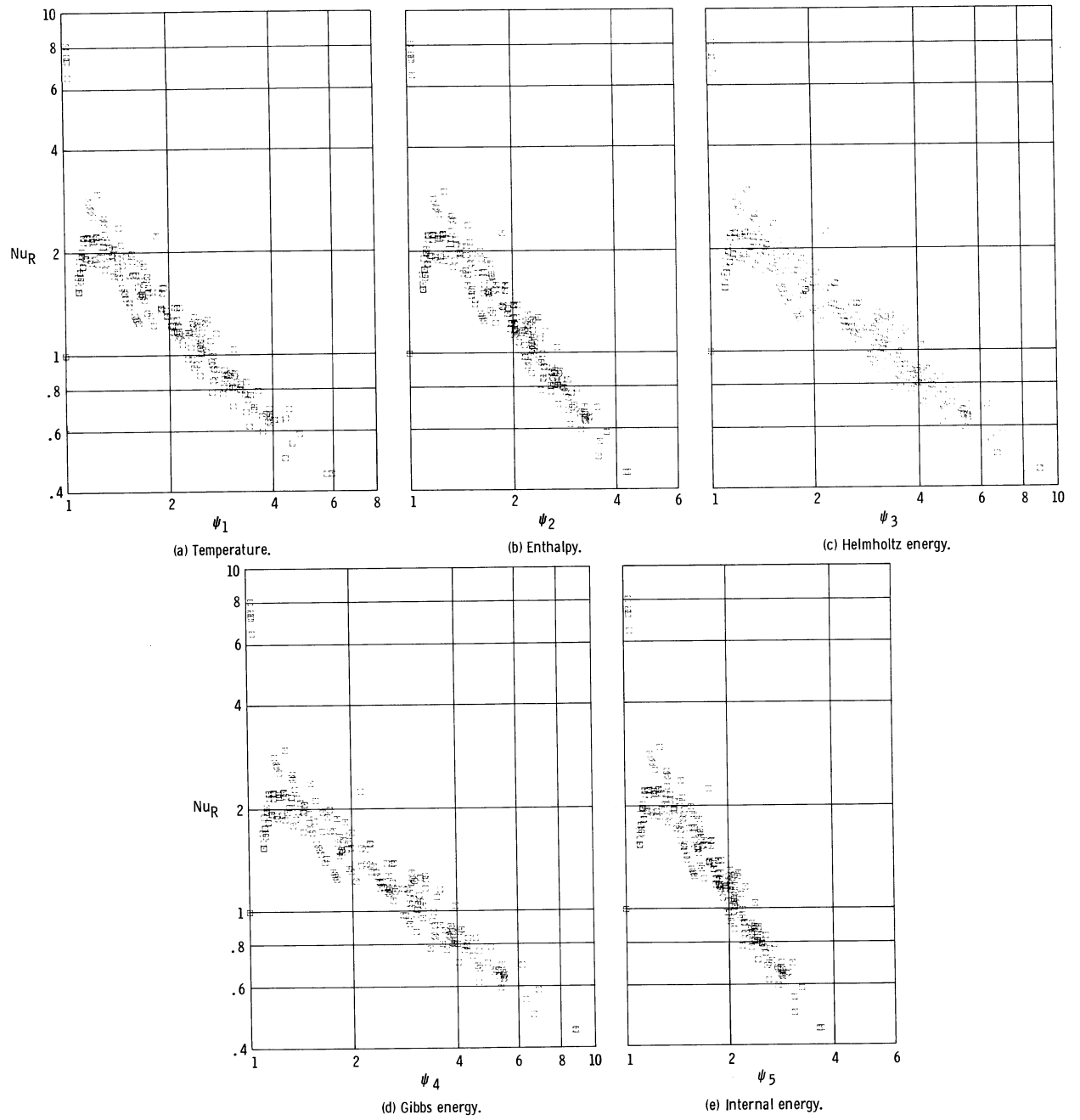
Figure 6. – Reduced Nusselt number for fluid oxygen (ref. 12) as a function of volume-energy parameters.

22
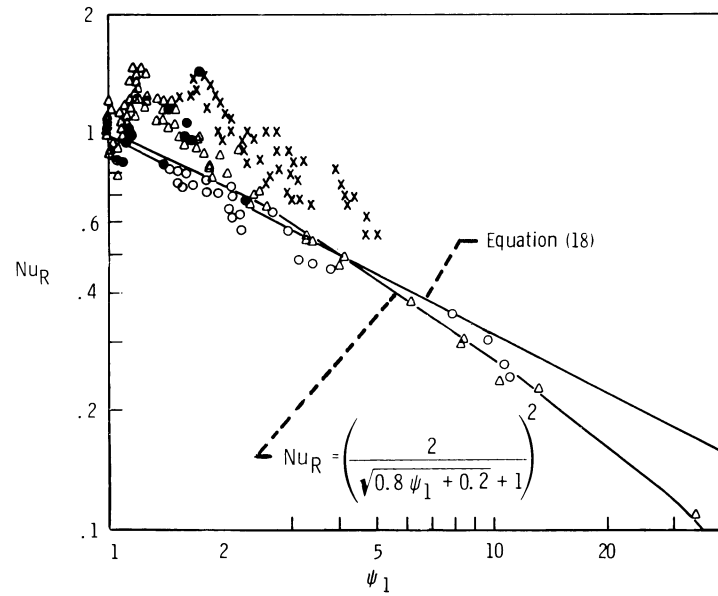
Figure 7. - Reduced Nusselt number as a function of ψ_1 for fluid helium (ref. 13).

$$Nu_R = \left(\dfrac{2}{\sqrt{0.8\,\psi_1 + 0.2} + 1}\right)^2$$
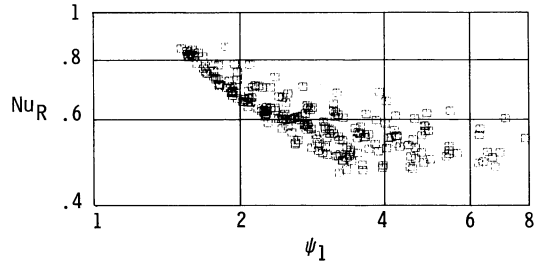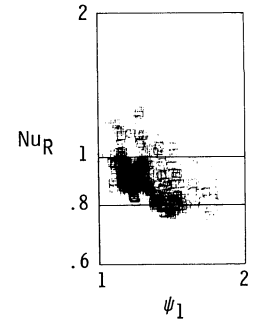
Equation (18)
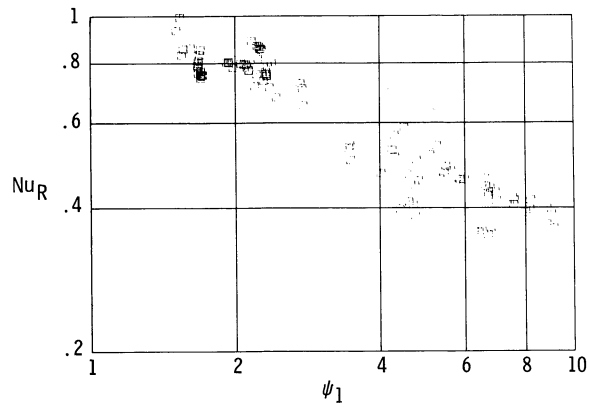
Figure 8. - Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state - hydrogen data from reference 5.
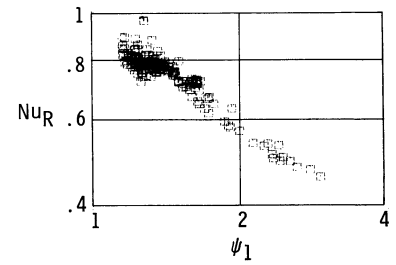
Figure 9. - Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state - hydrogen data from reference 7.

Figure 10. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – hydrogen data from reference 6.

Figure 11. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – hydrogen data from reference 10.
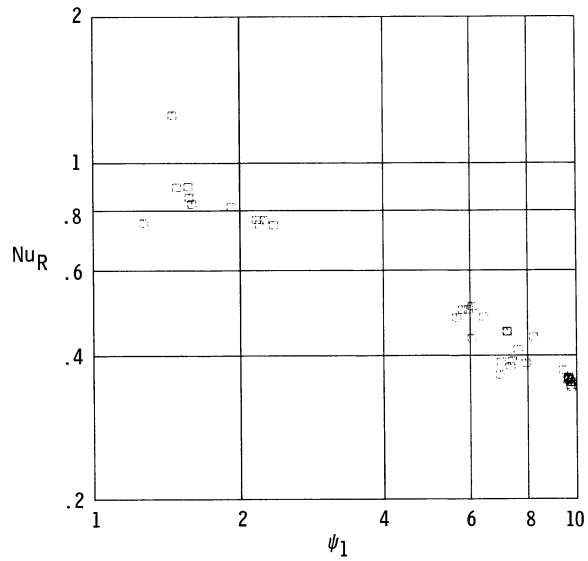

Figure 12. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – helium data from reference 5.
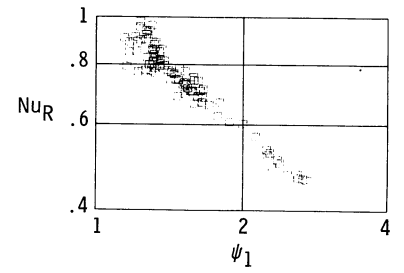

Figure 13. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – helium data from reference 7.
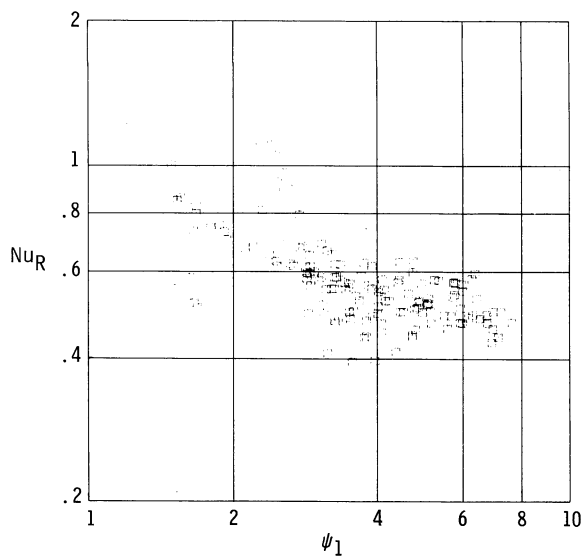
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Figure 14. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – helium data from reference 6.
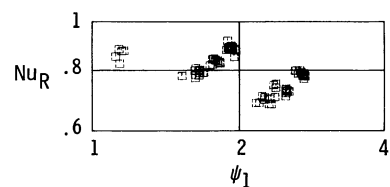

Figure 15. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – air data from reference 8.
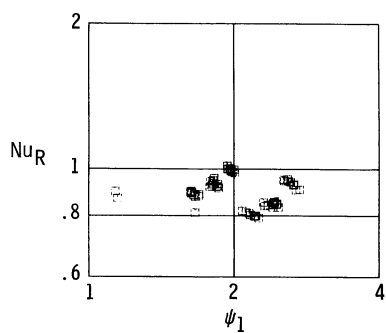

Figure 16. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – carbon dioxide data from reference 8.
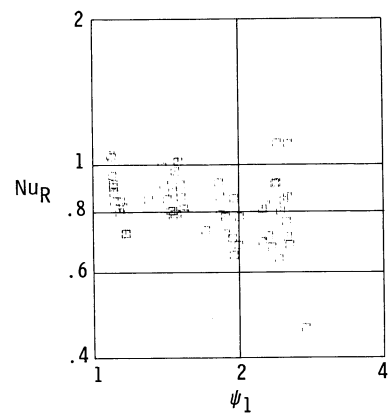

Figure 17. – Reduced Nusselt number as a function of the volumetric expansion parameter ψ_1 for gaseous state – air data from reference 9.

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15. Supplementary Notes

16. Abstract

Volume-energy relations that can be determined from an equation of state are used to group many sets of heat-transfer data, ranging from data for liquids to data for gases and including the near-critical region. The volume - Gibbs energy parameter (or Helmholtz energy parameter) grouped these data better than did such other parameters as enthalpy, temperature, or internal energy.

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