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RECENT IMPROVEMENTS TO THE SPINNING BODY VERSION OF THE "EDDYBL" COMPUTER PROGRAM

by

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ABSTRACT

Two important capabilities have been added to the spinning-body version of the EDDYBL computer program currently operational at the NASA Ames Research Center. First, the conventional mixing-length model, specialized for thick boundary layers, has been added to the program's array of possible turbulence models. Second, provision has been added for using a more general model for pressure-strain correlation terms in the Reynolds-stress-model option.

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NOTATION

Symbol	Definition
A	Van Driest damping coefficient, Equation (42)
c_f	Skin friction
C	Constant in the law of the wall, Equation (30)
C_1, C_2, C_3	Closure coefficients
C_p	Specific heat at constant pressure
D_{ij}	Production rate tensor, Equation (6)
e	Turbulent kinetic energy, $-\frac{1}{2}\tau_{ii}$
h	Specific mean enthalpy
$H(\beta)$	Heaviside stepfunction
J	Absolute value of mean strain-rate tensor
P_{ij}	Production rate tensor, Equation (6)
P	Half the trace of production rate tensors, Equation (7)
Pr_L, Pr_T	Laminar, turbulent Prandtl numbers
q_i	Turbulent heat flux vector
q	Component of q_i normal to surface
r	Radial coordinate
R_e,R_ω	Closure coefficients in viscous modifications
Re_{θ_k}	Reynolds number based on θ_k
s	Arclength
s_f, s_t	Value of s at the end of, beginning of transition
S_{ij}	Mean strain-rate tensor
t	Time

Symbol	Definition
T	Mean temperature
T_t	Mean total temperature
u_i	Mean velocity vector
u,v,w	x,y,z components of u_i
u^+	Dimensionless sublayer-scaled velocity, u/u_τ
u_τ	Friction velocity, $\sqrt{\tau_w/\rho_w}$
U_e	Velocity at boundary-layer edge
V	Dimensionless transformed vertical velocity
x_i	Position vector
x,y,z	Streamwise, normal, lateral components of x_i
y^+	Dimensionless sublayer-scaled normal distance, $u_\tau y/\nu$
α	Eddy-viscosity coefficient, Equation (44)
$\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$	Launder-Reece-Rodi closure coefficients
β,β^*,β^{**}	Closure coefficients in dissipation terms
$\tilde{\beta}$	Eddy-viscosity coefficient, Equation (49)
γ,γ^*	Closure coefficients controlling production
$\tilde{\gamma}$	Intermittancy in boundary layer, Equation (45)
$\Gamma(s)$	Intermittancy in streamwise direction, Equation (51)
δ	Boundary layer thickness
δ_k^*	Kinetic displacement thickness, Equation (46)
δ_{ij}	Kronecker delta
Δs_t	Width of transition region
ε	Eddy diffusivity
ε^+	Dimensionless eddy diffusivity, ε/ν
$\varepsilon_i,\varepsilon_o$	Inner, outer layer eddy diffusivity

Symbol	Definition
θ_k	Kinetic momentum thickness, Equation (47)
κ	Karman's constant
λ	Closure coefficient in viscous modifications
$\hat{\lambda}$	Intermittancy coefficient, Equation (52)
μ	Molecular viscosity
ν	Kinematic molecular viscosity, μ/ρ
π	Eddy viscosity parameter, Equation (48)
ρ	Mean density
$\sigma, \sigma^*, \sigma^{**}$	Turbulent "Prandtl" numbers
τ_{ij}	Reynolds stress tensor
τ	Reynolds shear stress, $<-u'v'>$
χ	Strain-rate parameter, $\sqrt{S_{nm} S_{mn}/\beta \omega^2}$
ω	Turbulent dissipation rate
ℓ	Turbulent length scale; also mixing length
ℓ_i	Mixing length in inner region

Subscripts

e	Boundary-layer edge
o, w	Surface

Other Notation

For a given variable ψ:

$<\psi>$, $\bar{\psi}$	Long-time-mass-averaged value of ψ
ψ'	Fluctuating part of ψ

RECENT IMPROVEMENTS TO THE SPINNING BODY VERSION OF THE "EDDYBL" COMPUTER PROGRAM

1. INTRODUCTION

In order to analyze effects of three-dimensionality on turbulent boundary layers, reliable experimental data are needed. In order to help provide such data, the NASA Ames Research Center has been developing an experimental facility for measuring properties of swirling axisymmetric boundary layers. While such boundary layers are not fully three dimensional, they at once (a) display coupling amongst all six Reynolds stresses similar to the 3-D situation and (b) can be computed with 2-D solution methods.

Concurrent with development of the experimental facility, the NASA Ames Research Center has sponsored development[1-3] of a swirling version of DCW Industries' EDDYBL computer program.[4] Prior to this research project, the program contained the two equation and Reynolds stress models devised by Wilcox and Rubesin.[3]

Since the time of the program's original development, developments in turbulence modeling have spurred interest in expanding the capabilities of the spinning version of EDDYBL, viz:

1. The pressure-strain-correlation closure approximations of Launder, Reece and Rodi[5] have been found to be more physically sound than those used by Wilcox and Rubesin;

2. General overall success with refined versions of the mixing-length model such as that proposed by Aguilar[6] provides justification for including mixing-length predictions in any meaningful boundary-layer study.

The purpose of this project has been to include the Launder-Reece-Rodi (LRR) pressure-strain-correlation terms and the Aguilar mixing-length model in the program's cadre of turbulence models and closure approximations.

Section 2 presents details of analysis based on the LRR pressure-strain-correlation terms while Section 3 focuses on the mixing-length model. Section 4 summarizes results of this study. The Appendix presents modifications to code input and output.

2. REVISED PRESSURE-STRAIN-CORRELATION CLOSURE APPROXIMATIONS

To enhance the generality of the spinning-body version of the EDDYBL computer program (hereafter referred to as EDDYBL for brevity), one objective of this project has been to include provision for using the Launder, Reece and Rodi[5] (LRR) pressure-strain-correlation closure approximations. Several steps are involved in accomplishing this objective, viz:

1. Stating the complete model;
2. Establishing values of the closure coefficients;
3. Developing viscous modifications; and
4. Incorporating revisions in the program.

Complete details of Steps 1-3 are given in this section. As for incorporating the modifications (Step 4), the most significant point is the manner in which input and output are affected. We now have several new input variables and certain default values have been changed. The Appendix summarizes revised input and output.

2.1 THE MODEL EQUATIONS

For a compressible fluid of density ρ moving with mass-averaged velocity u_i, the Reynolds-stress tensor, τ_{ij}, and turbulent heat-flux vector, q_i, are computed from the following equations:

$$\frac{\partial}{\partial t}\left(\rho\tau_{ij}\right) + \frac{\partial}{\partial x_k}\left(\rho u_k \tau_{ij}\right) = -\rho P_{ij} + \frac{2}{3}\beta^* \rho\omega e \delta_{ij} - C_1\beta^*\left[\tau_{ij} + \frac{2}{3}e\delta_{ij}\right]\rho\omega$$

$$+ \hat{\alpha}\rho\left[P_{ij} - \frac{2}{3}P\delta_{ij}\right] + \hat{\beta}\rho\left[D_{ij} - \frac{2}{3}P\delta_{ij}\right]$$

$$+ \hat{\alpha}\rho\left[S_{ij} - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right] + \frac{\partial}{\partial x_k}\left[(\mu + \sigma^*\rho\varepsilon)\frac{\partial\tau_{ij}}{\partial x_k}\right] \tag{1}$$

$$\frac{\partial}{\partial t}(\rho q_i) + \frac{\partial}{\partial x_k}(\rho u_k q_i) = \rho\tau_{ij}\frac{\partial h}{\partial x_j} - \rho q_j\frac{\partial u_i}{\partial x_j} - \beta^{**}\rho\omega q_i$$

$$+ \frac{\partial}{\partial x_k}\left[\left(\frac{\mu}{Pr_L} + \sigma^{**}\rho\varepsilon\right)\frac{\partial q_i}{\partial x_k}\right] \tag{2}$$

$$\frac{\partial}{\partial t}(\rho\omega^2) + \frac{\partial}{\partial x_k}(\rho u_k\omega^2) = \gamma\frac{\omega^2}{e}\rho\tau_{ij}\frac{\partial u_i}{\partial x_j} - \left[\beta + 2\sigma\left(\frac{\partial \ell}{\partial x_k}\right)\right]^2\rho\omega^3$$

$$+ \frac{\partial}{\partial x_k}\left[(\mu + \sigma\rho e)\frac{\partial\omega^2}{\partial x_k}\right] \tag{3}$$

where h is mass averaged enthalpy, μ is molecular viscosity, Pr_L is laminar Prandtl number, t is time, x_i is position vector and ε is eddy diffusivity defined as the ratio of turbulent energy, $e = -\tau_{ii}/2$, to turbulent dissipation rate, ω, i.e.,

$$\varepsilon = e/\omega \tag{4}$$

Similarly, ℓ is the turbulent length scale defined in terms of e and ω as

$$\ell = e^{1/2}/\omega \tag{5}$$

The tensors P_{ij}, D_{ij} and S_{ij} are defined as

$$\left.\begin{aligned} P_{ij} &= \tau_{im} \frac{\partial u_j}{\partial x_m} + \tau_{jm} \frac{\partial u_i}{\partial x_m} \\ D_{ij} &= \tau_{im} \frac{\partial u_m}{\partial x_j} + \tau_{jm} \frac{\partial u_m}{\partial x_i} \\ S_{ij} &= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \end{aligned}\right\} \qquad (6)$$

while the quantity P is half the trace of P_{ij} (and of D_{ij}), viz

$$P = \frac{1}{2} P_{kk} = \frac{1}{2} D_{kk} = \tau_{mn} S_{nm} \qquad (7)$$

Finally, the quantities $\hat{\alpha}$, β, $\hat{\beta}$, β^*, β^{**}, γ, $\hat{\gamma}$, σ, σ^*, σ^{**} and C_1 are closure coefficients whose values are established in the following section.

2.2 VALUES OF THE CLOSURE COEFFICIENTS

2.2.1 Setting the Values of $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$.

As shown by Launder, Reece and Rodi, symmetry properties of the exact pressure-strain-correlation term imply that $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ can be expressed in terms of a single unknown coefficient, C_2, as follows:

$$\left.\begin{aligned} \hat{\alpha} &= \frac{1}{11}(C_2 + 8) \\ \hat{\beta} &= \frac{1}{11}(8C_2 - 2) \\ \hat{\gamma} &= \frac{4}{55}(15C_2 - 1) \end{aligned}\right\} \qquad (8)$$

In the LRR model, a value $C_2 = 0.4$ appears most appropriate which corresponds to

$$\hat{\alpha} = 0.76, \hat{\beta} = 0.11, \hat{\gamma} = 0.36; \text{ WR* Model} \tag{9}$$

Note that we denote the new model as the WR* model. In the WR model there is no single value for C_2 so that some of the symmetry properties of the pressure-strain-correlation tensor are violated. The values for $\hat{\alpha}$, etc., are

$$\hat{\alpha} = \hat{\beta} = 1/2, \hat{\gamma} = 4/3; \text{ WR Model} \tag{10}$$

2.2.2 Setting the Value of C_1.

Turning to the coefficient C_1, study of the decay of anisotropic turbulence and its asymptotic return to isotropy implies a value for C_1 which, in the WR model, depends upon the strain rate parameter $\chi = \sqrt{S_{nm} S_{mn}/\beta^* \omega^2}$ according to

$$C_1 = 4.5 - 2.5 \exp(-5\chi); \text{ WR Model} \tag{11}$$

By contrast, C_1 is constant and equal to 1.5 in the LRR model. We assume this value applies for the WR* model.

$$C_1 = 1.5 ; \qquad \text{WR* Model} \tag{12}$$

2.2.3 Setting the Values of σ, σ^* and σ^{**}.

For the turbulent Prandtl numbers σ, σ^*, σ^{**} we elect to use the same values as those in the WR model, wherefore we have

$$\sigma = \sigma^* = 1/2, \qquad \sigma^{**} = 2; \text{ Both Models} \tag{13}$$

2.2.4 Setting the Value of β.

Accurate simulation of the decay of homogeneous, isotropic turbulence means we must require β and β^* to stand in the following ratio:

$$\beta/\beta^* = 5/3 \text{ ; Both Models} \tag{14}$$

2.2.5 Setting the Values of β^* and γ.

Analysis of the incompressible wall layer shows that the coefficient γ must satisfy

$$\gamma = \beta/\beta^* - \frac{2\alpha\kappa^2}{\sqrt{\beta^*\gamma^*}}\gamma^* \tag{15}$$

where γ^* is the (constant) ratio of shear stress, τ , to $\varepsilon\partial u/\partial y$, i.e.,

$$\tau = \gamma^* \frac{e}{\omega} \frac{\partial u}{\partial y} \tag{16}$$

and is given by the following equation:

$$\gamma^* = \frac{1}{C_1\beta^*}\left\{\frac{\hat{\gamma}}{2} + \frac{2}{3C_1}\left[(1-\hat{\alpha})(C_1-1+\hat{\alpha}-2\hat{\beta})-\hat{\beta}(C_1+2-2\hat{\alpha}+\hat{\beta})\right]\right\} \tag{17}$$

Also, in the wall layer,

$$\tau/e \to \sqrt{\gamma^*\beta^*} \tag{18}$$

while the normal stresses are given by

$$\left.\begin{aligned}
\langle u'^2\rangle/e &= \frac{2}{3C_1}\left[C_1 + 2-2\hat{\alpha} + \hat{\beta}\right] \\
\langle v'^2\rangle/e &= \frac{2}{3C_1}\left[C_1 - 1+ \hat{\alpha} -2\hat{\beta}\right] \\
\langle w'^2\rangle/e &= \frac{2}{3C_1}\left[C_1 - 1+ \hat{\alpha} + \hat{\beta}\right]
\end{aligned}\right\} \tag{19}$$

If, as in the WR model, we select

$$\beta^* = 9/100 \tag{20}$$

then it follows that

$$\gamma^* = \begin{cases} 1.097 & ; \quad \text{WR Model} \\ 1.413 & ; \quad \text{WR* Model} \end{cases} \tag{21}$$

and γ is given by

$$\gamma \doteq \begin{cases} 13/11 & ; \quad \text{WR Model} \\ 4/3 & ; \quad \text{WR* Model} \end{cases} \tag{22}$$

Note that in the wall layer we now have:

$$\tau/e \rightarrow \begin{cases} 0.31 & ; \quad \text{WR Model} \\ 0.36 & ; \quad \text{WR* Model} \end{cases} \tag{23}$$

and

$$\langle u'^2\rangle : \langle v'^2\rangle : \langle w'^2\rangle = \begin{cases} 4 : 2 : 3 & ; \quad \text{WR Model} \\ 4 : 2 : 2.6 & ; \quad \text{WR* Model} \end{cases} \tag{24}$$

2.2.6 Setting the Value of β^{**}.

Finally, to specify β^{**} we note that for an incompressible boundary layer the radial heat flux equation simplifies, in the wall layer, to a balance between production and dissipation, i.e.,

$$0 = -\rho\langle v'^2\rangle \frac{\partial C_p T}{\partial y} - \beta^{**}\rho\omega q \qquad (25)$$

so that q follows a gradient diffusion law defined by

$$q = -\frac{1}{\beta^{**}} \frac{\langle v'^2\rangle}{\omega} \frac{\partial (C_p T)}{\partial y} \qquad (26)$$

Now, since Equation (16) tells us the eddy diffusivity in the momentum equation is $\bar{\varepsilon} = \gamma^* e/\omega$, the classical analogy between momentum and heat transfer encourages us to write

$$q = -\frac{\bar{\varepsilon}}{Pr_T} \frac{\partial (C_p T)}{\partial y} \qquad (27)$$

which implies [upon inspection of Equations (16), (19), (26) and (27)] that β^{**} and Pr_T are related by the following equation:

$$\beta^{**} = \frac{2Pr_T}{3C_1\gamma^*} \left[C_1 - 1 + \hat{\alpha} - 2\hat{\beta}\right] \qquad (28)$$

Hence, for the two models we have (demanding that $Pr_T = 8/9$ in the wall layer):

$$\beta^{**} = \begin{cases} 0.36 & ; \quad \text{WR Model} \\ 0.29 & ; \quad \text{WR* Model} \end{cases} \qquad (29)$$

2.3 VISCOUS MODIFICATIONS

Just as with the WR model, we must devise viscous modifications for the new model. Integration through the sublayer with no viscous modifications shows that, in terms of standard sublayer coordinates, the new model yields

$$u^+ \to \frac{1}{\kappa} \ln y^+ + C \tag{30}$$

where C = 8.6 for a perfectly smooth wall. As with the WR model we introduce the following viscous modifications:

$$C_1 = \frac{3}{2}\left[1 - (1-\lambda^2)\exp\left(-\varepsilon^+/R_e\right)\right]^{-1} \tag{31}$$

$$\frac{\gamma}{C_1} = \frac{\gamma_\infty}{C_{1_\infty}}\left[1 - (1-\lambda^2)\exp\left(-\varepsilon^+/R_\omega\right)\right] \tag{32}$$

where $\varepsilon^+ = \rho e/\omega\mu$, subscript ∞ denotes value for high Reynolds number turbulence, and R_e, R_ω, λ are coefficients to be determined.

The value of λ is set by demanding that turbulence dissipation exceed turbulence production below the minimum critical Reynolds number for the Blasius Boundary layer. This condition is satisfied provided

$$\lambda^2 = \frac{.0045 C_{1\infty}}{3\hat{\gamma}+4(1-\hat{\alpha}-\hat{\beta})} \tag{33}$$

Hence, we obtain

$$\lambda \doteq \begin{cases} 1/14 & ; \quad \text{WR Model} \\ 2/31 & ; \quad \text{WR* Model} \end{cases} \tag{34}$$

To set the values of R_e and R_ω we proceed as in previous analyses noting that there is a locus of pairs of values (R_e, R_ω) which yield $C = 5.5$ for a perfectly smooth wall (Figure 1). The optimum pair appears to be

$$R_e = 1, \; R_\omega = 2.67 \quad ; \quad \text{WR* Model} \tag{35}$$

which is close to the pair of values used in the WR model, viz,

$$R_e = 1, \; R_\omega = 3 \quad ; \quad \text{WR Model} \tag{36}$$

Figures 2 and 3 compare predicted results for $(R_e, R_\omega) = (1,2.67)$ and $(R_e, R_\omega) = (0,0.93)$. Clearly, the larger value of R_e is preferable. Further increase in R_e tends to drive the turbulence production away from the experimental data.

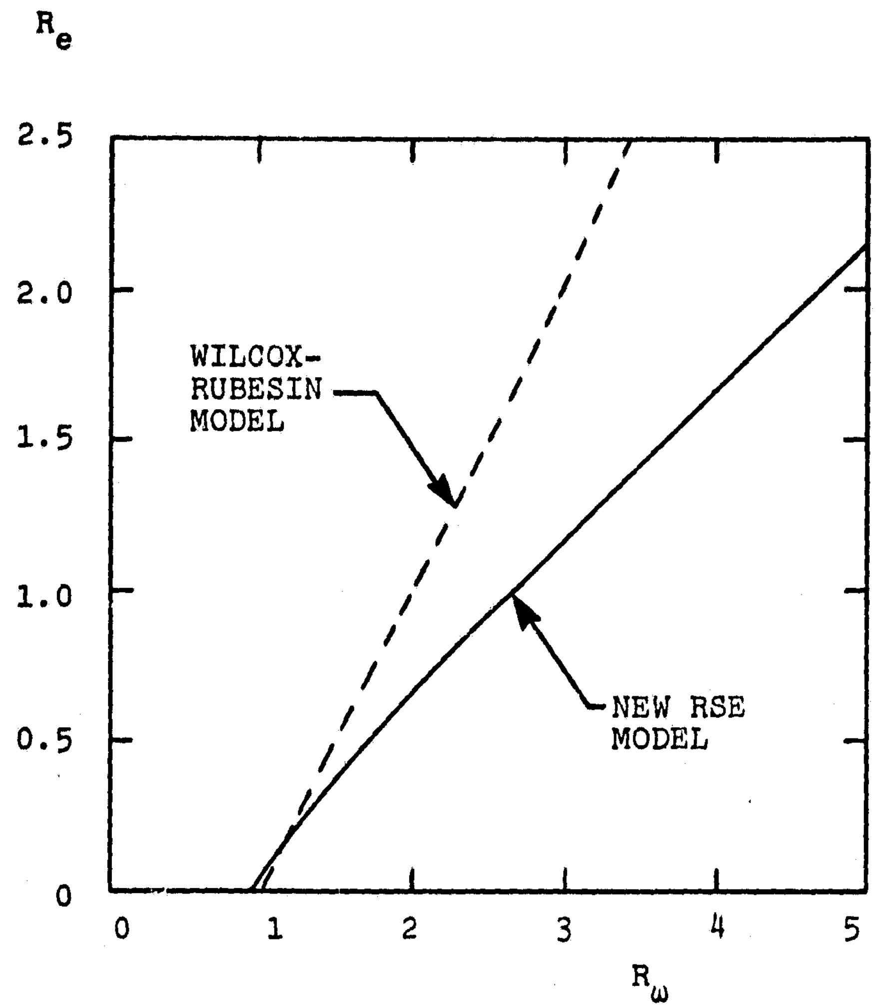

Figure 1. Loci of values of R_e and R_ω which yield a smooth-wall constant in the law of the wall of 5.5.

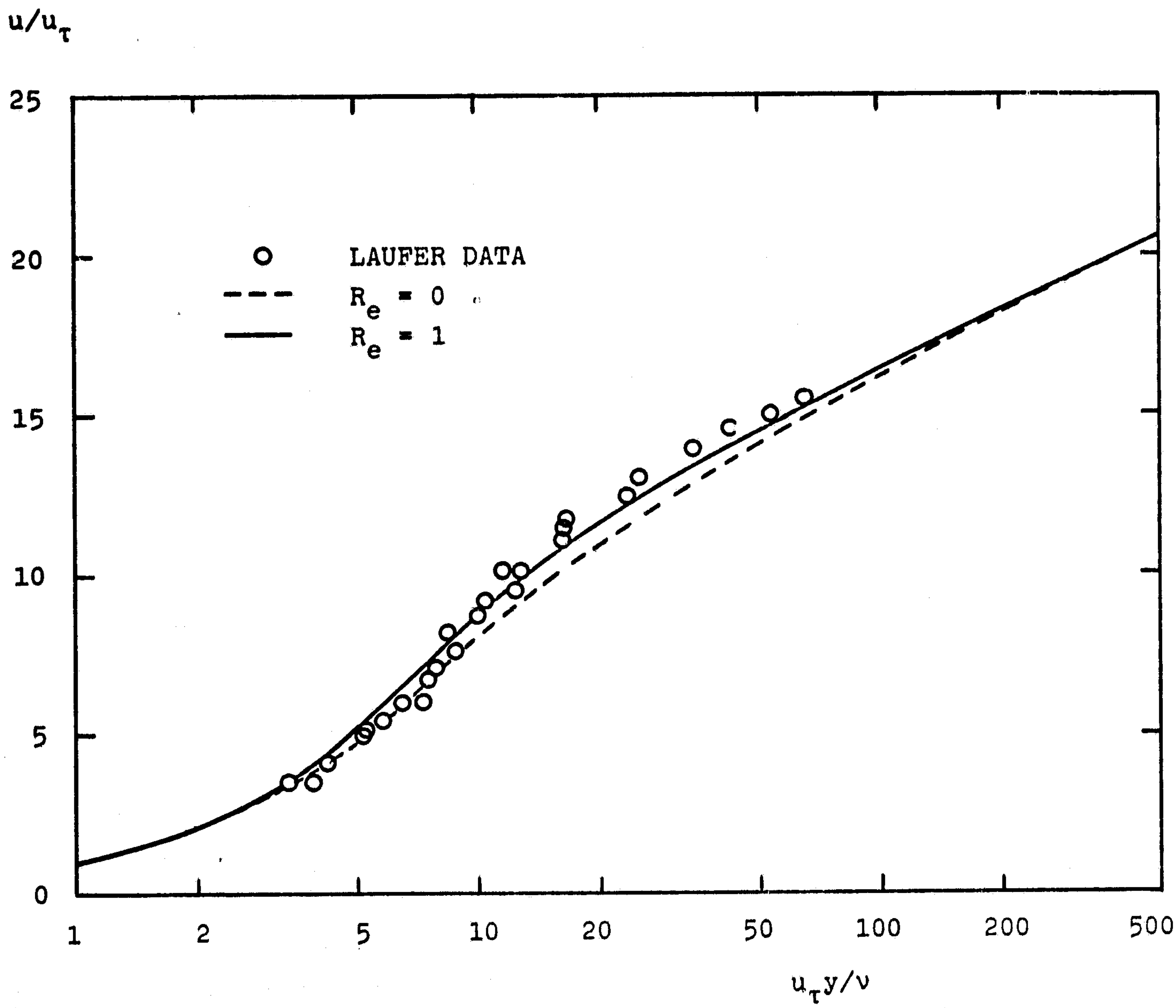

Figure 2. Velocity profiles in the sublayer.

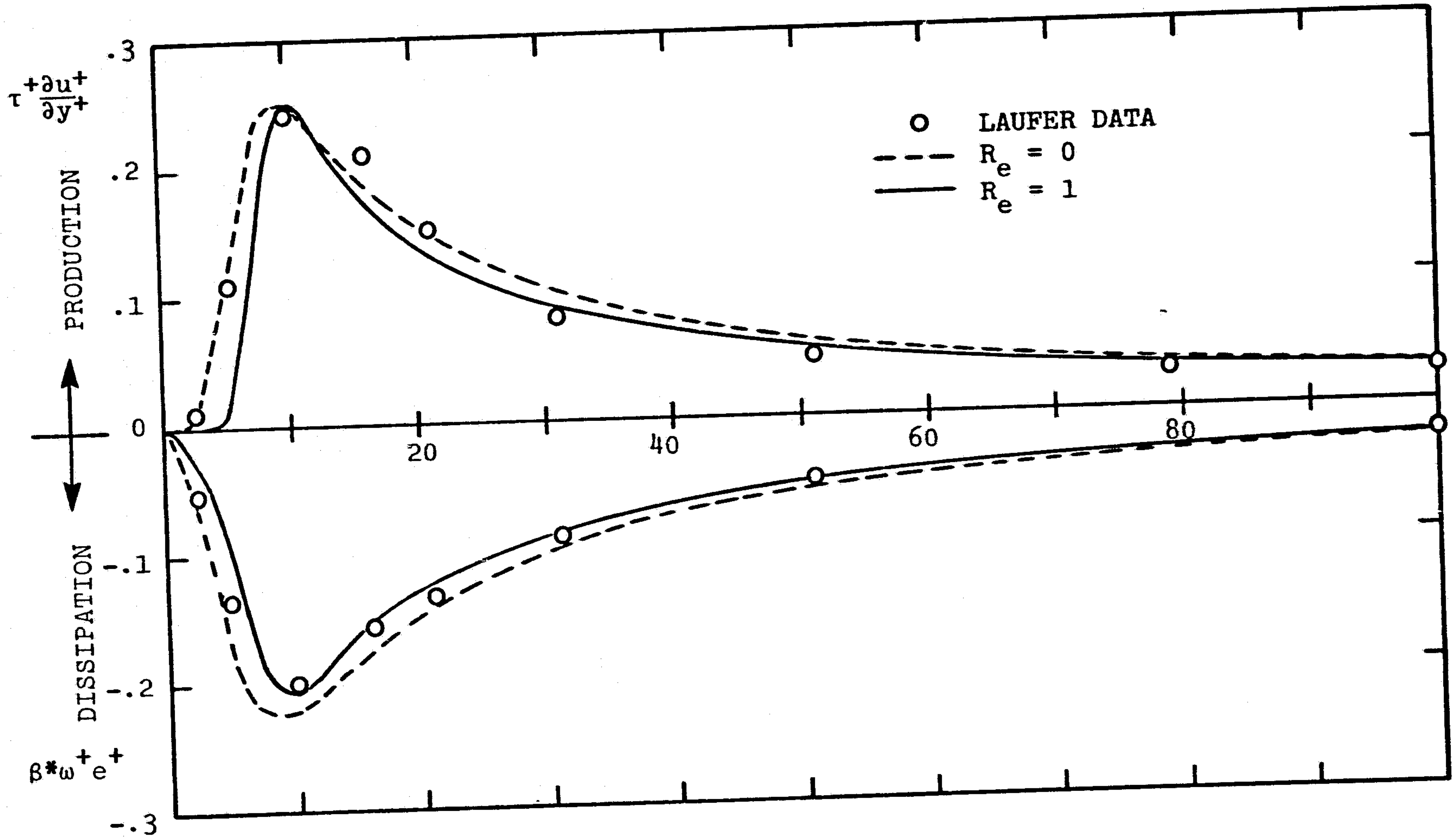

Figure 3. Turbulence energy balance in the sublayer.

3. MIXING-LENGTH MODEL

The second objective of this project has been to include the Aguilar[7] mixing-length model in the program. Three steps are involved in accomplishing this objective, viz:

1. Stating the complete model;
2. Specifying the transition point and width of the transition region;
3. Incorporating revisions in the program.

As with the modifications discussed in Section 2, the Appendix summarizes changes in program input and output. Complete details of Steps 1 and 2 are given in this section which concludes with results of a test case.

3.1 THE MODEL

This subsection summarizes the Aguilar mixing-length model for a spinning, thick boundary layer. The Reynolds stress tensor and turbulent heat-flux vector are written as

$$\tau_{ij} = 2\varepsilon\left(S_{ij} - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right) \tag{37}$$

$$q_i = -\frac{\varepsilon}{Pr_T}\frac{\partial(C_p T)}{\partial x_i} \tag{38}$$

where ε is the eddy diffusivity.

To compute ε, the model divides the boundary layer into an inner and an outer region. The eddy diffusivity in the inner region,

ε_i, is

$$\varepsilon_i = \ell_i^2 |J| \tag{39}$$

where $|J|$ is the magnitude of the mean strain rate and ℓ_i is the inner region mixing length. For flow over an axisymmetric body of radius r_o, these quantities are given by

$$J^2 = \left(\frac{\partial w}{\partial r} - \frac{w}{r}\right)^2 + \left(\frac{\partial u}{\partial r}\right)^2 \tag{40}$$

and

$$\ell_i = \kappa r_o \ln\left(\frac{r}{r_o}\right) \left[1 - \exp\left(\frac{-r_o \ln(r/r_o)}{A}\right)\right]\left(\frac{r}{r_o}\right)^{\frac{1}{2}} \tag{41}$$

where u denotes streamwise velocity, w is swirl velocity, r is radial distance, and κ is Karman's constant. The quantity A is the van Driest damping coefficient defined by

$$A = \frac{26\ \nu}{\sqrt{\nu_w\ J_o}} \tag{42}$$

where J_o is the value of J at $r = r_o$ and ν denotes molecular diffusivity.

In the outer region, the eddy diffusivity, ε_o, is given by

$$\varepsilon_o = \alpha\ U_e\ \delta_k^*\ \tilde{\gamma} \tag{43}$$

where U_e is freestream velocity and the quantity α is given by

$$\alpha = \begin{cases} .0168 & , Re_{\theta_k} \geq 5000 \\ .0168\left(\frac{1.55}{1+\pi}\right) & , Re_{\theta_k} < 5000 \end{cases} \tag{44}$$

Also, in Equation (43) the quantity $\tilde{\gamma}$ is the intermittancy defined by

$$\tilde{\gamma} = [1 + 5.5\ (r - r_o)/\delta]^{-1} \tag{45}$$

In Equations (43) and (44), δ_k^* and θ_k are the kinetic displacement and momentum thicknesses defined as follows:

$$\delta_k^* \equiv \int_{r_o}^{\infty} (1 - u/U_e)\ dr \tag{46}$$

$$\theta_k \equiv \int_{r_o}^{\infty} \frac{u}{U_e} (1 - u/U_e)\ dr \tag{47}$$

Finally, the quantity π is given by the following formula:

$$\pi = 0.55 \left[1-\exp(-0.243\tilde{\beta}^{1/2} - 0.298\tilde{\beta})\right] H(\tilde{\beta}) \tag{48}$$

where $H(\tilde{\beta})$ is the Heaviside stepfunction and $\tilde{\beta}$ is defined by

$$\tilde{\beta} \equiv Re_{\theta_k}/425 - 1 \tag{49}$$

The two regions are joined by the requirement that the eddy viscosity be continuous across the boundary layer, which is insured by requiring

$$\varepsilon = \min\ (\varepsilon_i, \varepsilon_o) \tag{50}$$

3.2 TRANSITION POINT AND WIDTH OF TRANSITION REGION

To complete specification of the mixing-length model we must establish the location of transition, the width of the transition region, and the streamwise intermittancy on the transition region. This must be done because, unlike the two-equation and RSE models, the mixing-length model has no natural way of predicting transition location or width.

Following Dhawan and Narasimha,[8] we introduce a streamwise intermittancy, $\Gamma(s)$, defined as follows:

$$\Gamma(s) \equiv 1 - \exp\left[-0.412\left(\frac{s-s_t}{\hat{\lambda}}\right)^2\right] \tag{51}$$

where s is streamwise distance, s_t is the value of s at the transition point, and $\hat{\lambda}$ is defined by

$$\hat{\lambda} = (s_f - s_t)\sqrt{\frac{\ln 50}{0.412}} \tag{52}$$

with s_f denoting the value of s at the end of transition (at which point $\Gamma = 0.98$).

Finally, to determine the width of the transition region we can either specify it empirically or we can use the correlation devised by Dhawan and Narasimha, viz,

$$Re_{\Delta s_t} = 5 Re_{s_t}^{0.8} \tag{53}$$

where

$$\Delta s_t = s_f - s_t \tag{54}$$

This completes specification of the mixing-length model. In the following subsection we present a simple test case.

3.3 SAMPLE COMPUTATION

As a preliminary test case, we present results for an incompressible ($M_e = 0.13$) boundary layer on a cylinder of sufficient radius for the boundary layer to be essentially two-dimensional ($\delta/r_o \sim 1/10$). Figure 4 compares skin friction, c_f, (a) computed with the mixing-length model, (b) computed with the Wilcox-Rubesin RSE model and (c) as correlated by von Karman.[9] As shown, except very near transition, the computations and the correlation are within 10%.

The only significant observation about the program's operation is that, in order to achieve an accurate solution with the mixing-length model, the solution should be required to differ from iteration to iteration by a tenth of a percent as opposed to one percent for the more advanced models.

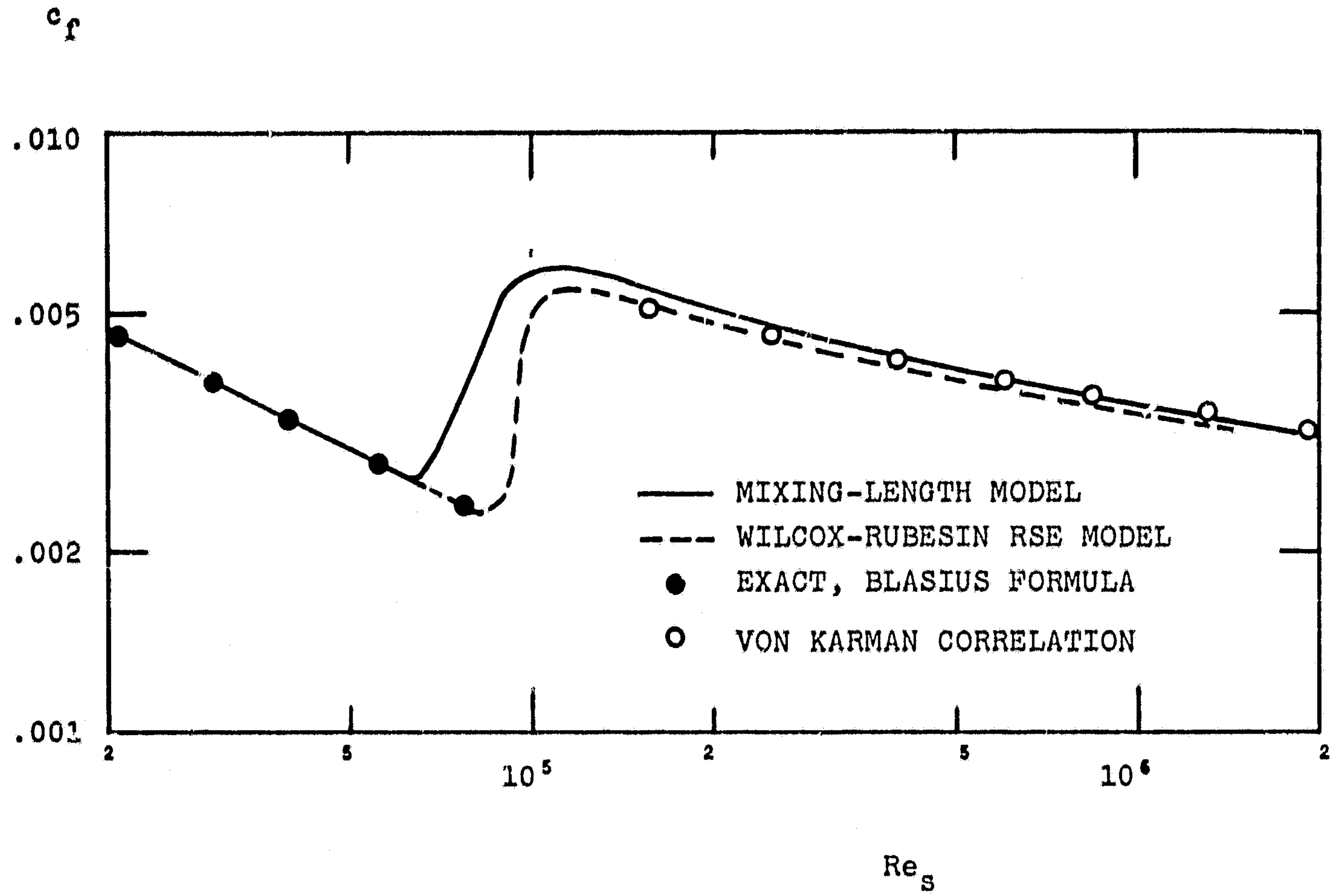

Figure 4. Comparison of computed and measured skin friction for an incompressible, axisymmetric, thin boundary layer in the absence of pressure gradient.

4. DISCUSSION

The modifications described in Sections 2 and 3 have been made to the spinning-body version of the EDDYBL computer program. Consequently, the program now has provision for computing boundary-layer development on segmented spinning bodies using the following turbulence models:

1. Aguilar mixing-length model;
2. Wilcox-Rubesin two-equation model;
3. Wilcox-Rubesin RSE model.

In the latter option either the original pressure-strain-correlation closure approximations or those devised by Launder, Reece and Rodi can be used. The program is now ready for general applications.

APPENDIX

MODIFICATIONS TO INPUT AND OUTPUT

In this Appendix we summarize all changes to program input and output which have been made in this research study.

A.1 INPUT MODIFICATIONS

All modifications to input are in the input NAMELIST's. The following input quantities have been added.

NAMELIST SAFCØN†

VARIABLE NAME	SYMBOL	DEFINITION	DEFAULT VALUE
AHAT	$\hat{\alpha}$	LRR Closure Coefficient	0.76
BHAT	$\hat{\beta}$	LRR Closure Coefficient	0.11
GHAT	$\hat{\gamma}$	LRR Closure Coefficient	0.36
RMIO	$(r_o)_i$	Radius of cylindrical body at which computation begins; to be used when CØNE $<10^{-6}$	1.00
RSUBE	R_e	Viscous Modification Coefficient	1.00
RSUBW	R_ω	Viscous Modification Coefficient	2.67
ZC1	C_1	LRR Closure Coefficient	1.50
ZC3	C_3	Special Wilcox-Rubesin Closure Coefficient	0.00

†For clarity, the letter O is denoted by Ø in all FØRTRAN names while zero is denoted by 0.

NAMELIST NAM2

VARIABLE NAME	SYMBOL		DEFAULT VALUE
KØDPRT	-	Turbulent Prandtl number flag. KØDPRT = 1 gives $Pr_T = 0.89$ and KØDPRT = 2 gives $Pr_T = 0.95 - 0.45\ (y/\delta)^2$	1
KTCØD	-	Transition width flag. KTCØD = 1 if TLNGTH input and KTCØD = 2 for TLNGTH computed from Equation (53).	2
SST	s_t	Transition-point location.	1.E8
TLNGTH	$(s_f/s_t - 1)$	Transition width	2.

Using the default values gives the hybrid Wilcox-Rubesin and LRR model (WR*) described in Section 2. If the original WR model is desired, one uses the following values:

$$\hat{\alpha} = \hat{\beta} = 1/2, \ \hat{\gamma} = 4/3$$

$$R_e = 1, \ R_\omega = 2, \ C_1 = 9/2, \ C_3 = 5/2$$

Note also that QBETA (β**) and SLAMDA (γ) are no longer input but are computed according to formulae given in Section 2, viz, Equations (28) and (33). Also, note that in NAMELIST SAFCØN, the input variable NFLAG now has the following meaning:

$$\text{NFLAG} = \begin{cases} 0 \ , & \text{RSE Model} \\ 1 \ , & \text{Two Equation Model} \\ 2 \ , & \text{Mixing Length Model} \end{cases}$$

Finally, note that for mixing-length runs (NFLAG = 2), it is necessary to impose a tighter convergence criterion than with advanced models. The following values are recommended:

$$\text{ERRLIM} = \begin{cases} 0.001 & , \text{NFLAG} = 2 \\ 0.010 & , \text{NFLAG} = 0 \text{ or } 1 \end{cases}$$

A.2 OUTPUT MODIFICATIONS

The only output modification is to the printed output which has been added for the mixing-length model. The printout at each station of skin friction, momentum thickness, etc., is as with the other two models with the following alternations:

1. The quantities DSMXØ, JPRINT and KPRINT are always zero;
2. The quantity ES is the streamwise intermittancy, $\Gamma(s)$.

The first two sets of printed profiles are slightly different from those given for the advanced turbulence models. First the program prints η, y/δ, u/U_e, y^+, u^+, (T/T_e-1), T_t/T_{t_e}, $(1 + \varepsilon/\nu)$. Then, the program prints η, y/δ, V, w/U_e, $<-\rho u'v'>/\bar{\rho}$, $<-\rho v'w'>/\bar{\rho}$, $q/C_pT_eU_e$ and ℓ/δ where ℓ denotes mixing length and V is the dimensionless transformed vertical velocity.[4]

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