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On the Characteristics of Centrifugal-Reciprocating Machines

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ABSTRACT

A novel approach to help solve the problem of compressing helium gas for cryogenic coolers is presented in this paper. The innovative feature is the employment of centrifugal force to reduce the forces on the connecting rod and crankshaft in the usual reciprocating compressor. This is achieved by rotating the piston and cylinder assembly at sufficiently high speed so that the centrifugal force on the piston is sufficient to overcome the compressional force due to the working fluid.

A further innovative feature is a method for dynamically braking the rotating assembly in order to recharge the working space with new fluid in order to repeat the compression. Thus, the intake stroke consists of decelerating the rotating piston-cylinder assembly and the compression or exhaust stroke consists of accelerating the same assembly.

The advantages derived from the new technology are improved efficiency, and increased reliability.

ACKNOWLEDGMENT

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CONTENTS

SECTION I

INTRODUCTION

One of the objectives in developing a reliable compressor system is to use dry lubricated components; this in turn requires slowly moving parts. A difficulty in the usual reciprocating compressor operating at low speeds is that a given flow rate of working fluid may require a large piston-cylinder combination; this in turn requires brutally large driving forces, as shown in Figure 1-1. Not only would this necessitate heavy and rugged connecting rods and crankshafts, but the problem is compounded because large reduction gears and heavy flywheels are required to provide the large driving torques required. It is admitted that clever techniques may be employed to alleviate some of these problems; nonetheless, solutions are generally compromises, and an improvement in one aspect can be made only at the expense of some other feature. Figure 1-2 shows schematically a cross-sectional view of the standard reciprocating compressor.

Consider now the simple arrangement shown in Fig. 1-3a, which shows one of the arrangements of the present innovation. Two pistons are contained inside a cylinder and constrained by a spring of appropriate restoring force. The entire assembly is rotated at angular speed ω and centrifugal force is used to compress the working fluid. Figure 1-3b shows one possibility for the inlet and outlet of the working fluid. Figure 1-4 shows typical values of centrifugal force which are developed by pistons of different masses as a function of the rotation speed. The pistons are treated as point masses at a radius of 10 cm. Because of the linear dependence ($F = mR\omega^2$) of force on mass and radius, the curves in Figure 1-4 may also be interpreted as showing the dependence of force on radius for a 1-kg mass. The lowest curve, for example, is the force on a 1-kg mass for radius of 5 cm.

As an example, consider a 1-kg piston arranged to have a maximum radius of 10 cm at 2500 rpm and a minimum radius of 5 cm at 2000 rpm. We note from Figure 1-4 at point a that a spring with a force of 225 kg is required to pull in the mass at a speed of 2000 rpm. A linear spring will then have a restoring force of 450 kg when stretched to 10 cm. The centrifugal force as noted from point b in Figure 1-4 is around 700 kg; this leaves (700-450) 250 kg as the available force to compress the fluid. Hence it is noted from Figure 1-1 what piston diameter may be used for a given pressure differential Δp. The displacement of fluid per cycle is now determined, and the number of cycles per second is finally determined by the required flow rate of fluid. The centrifugal-reciprocating (CR) compressor is particularly attractive for applications which require relatively low flow rates so that 1 to 2 strokes or cycles per second is adequate. In the example above, the compressor would oscillate between 2000 and 2500 rpm once or twice per second as required by the flow rate of the fluid.

We now discuss the energy-efficient ways in which it might be possible to drive the compressor. We discuss the simplest application of dynamic or regenerative braking by employing a dc shunt motor as the prime mover.

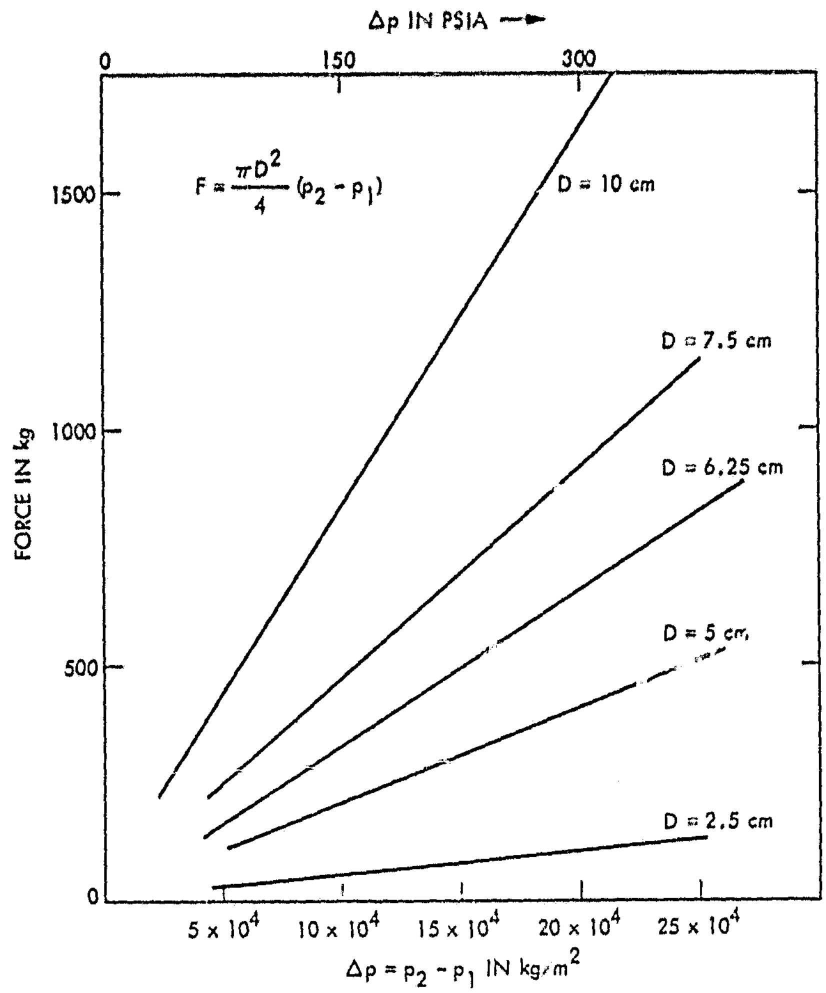


Figure 1-1. Compressional Force Versus Working Pressure in a Compressor

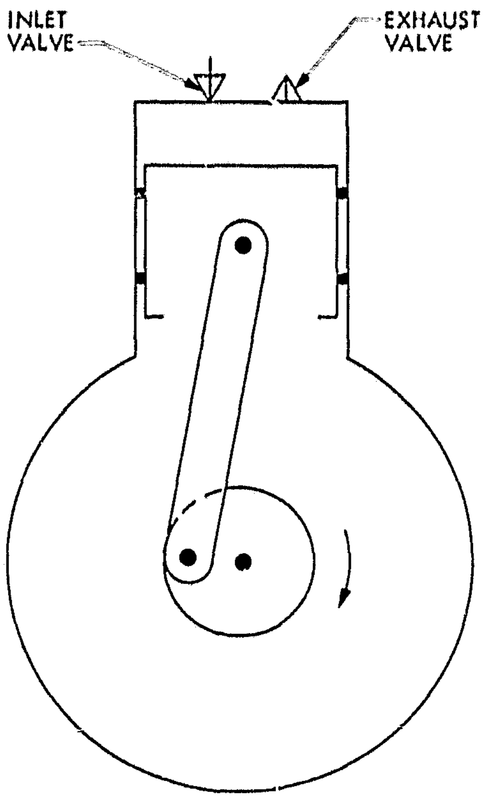
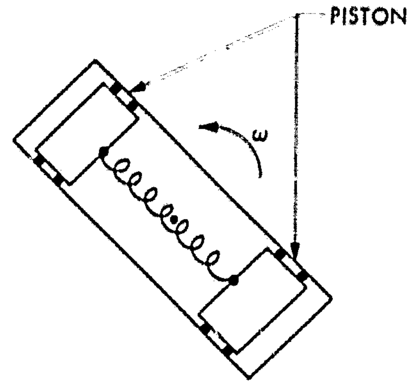
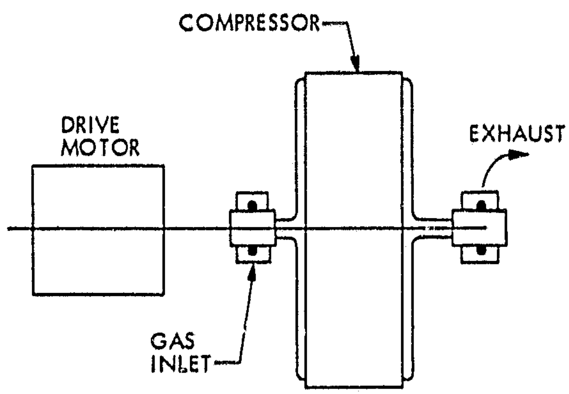


Figure 1-2. Standard Reciprocating Compressor



(a)

Figure 1-3a. Centrifugal-Reciprocating Compressor



(b)

Figure 1-3b. Gas Flow in Compressor

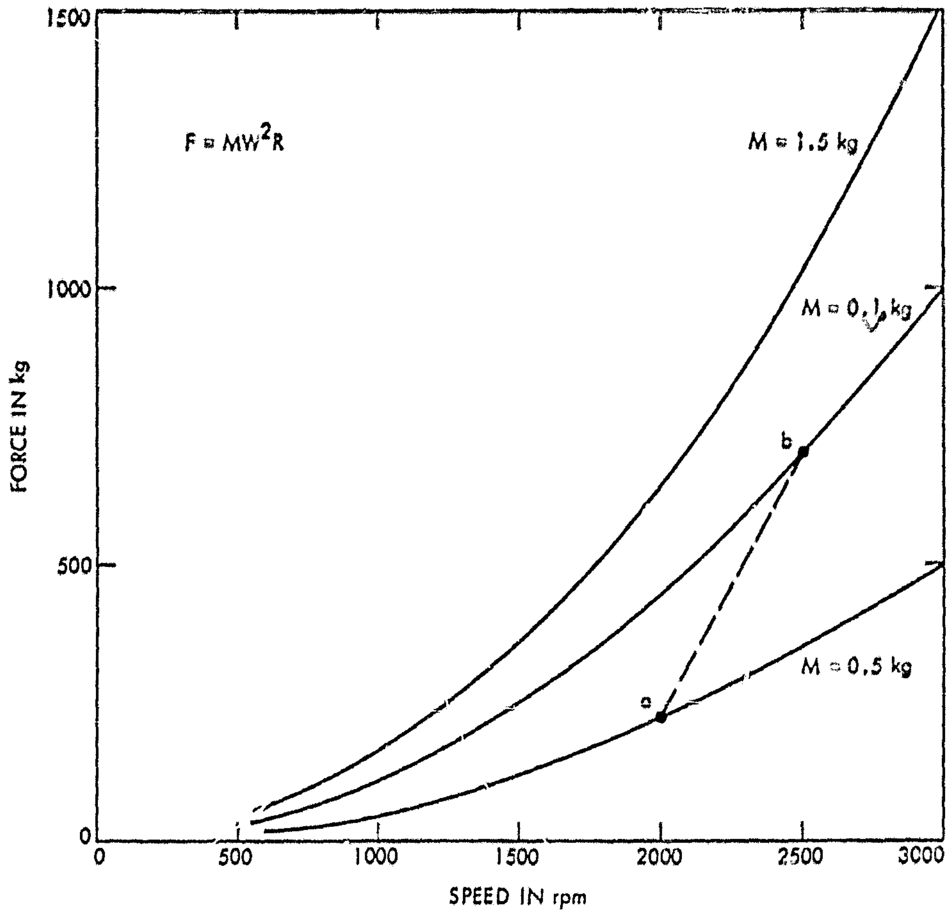


Figure 1-4. Centrifugal Force Versus Rotational Speed for Different Masses

Figure 1-5 shows a typical set of characteristics for a shunt motor running with fixed applied armature voltage and variable field current. Operating points a and b have been selected to match our example above. It is seen that by switching the field current between I_a and I_b it is possible to vary the armature speed between 2000 and 2500 rpm. Furthermore, we know from basic electrical machinery theory that during periods of accelerations the device acts like a motor and conversely like a generator during periods of decelerations. Thus, the design of an efficient system has been made possible. An obvious way to design a good system would be to build a pair of centrifugal-reciprocating compressors, which may then be synchronized so that when one machine is acting as a motor the other is functioning as a generator. Figure 1-6 shows schematically how this is done.

Finally, it is obvious that there are many variations of the technology described here; Figure 1-7 shows one such variation. Outwardly, the device looks exactly like a standard reciprocating compressor. The difference here is that means are provided to rotate both the piston and cylinder assemblies so that centrifugal force plays a dominant role in compressing the working fluid. The connecting rods, for example, need not be hefty or bulky.

A disadvantage is that special sealed-bearing assemblies are required to communicate the working fluid to and from the compressor. However, this problem already exists in automotive air-conditioning compressors and has been solved.

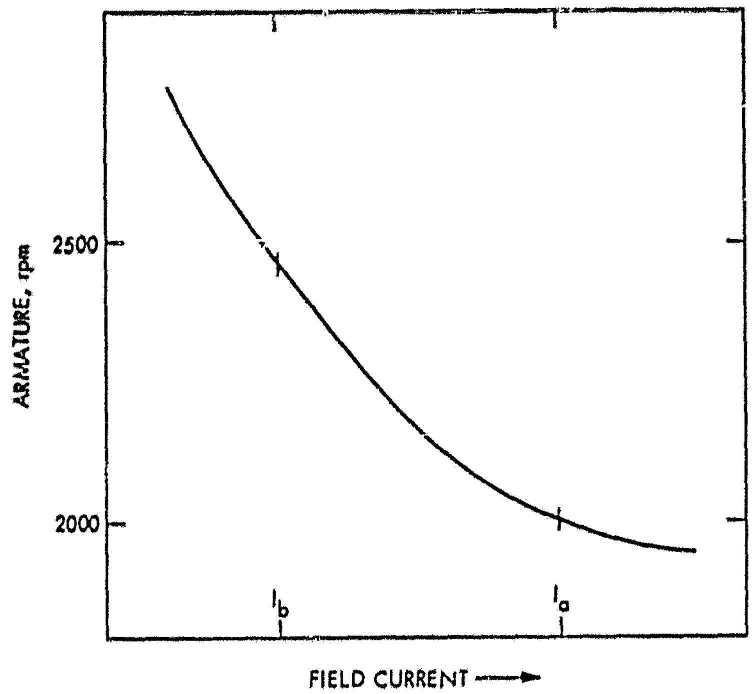


Figure 1-5. Typical Speed Versus Field-Current Characteristic for a dc Motor

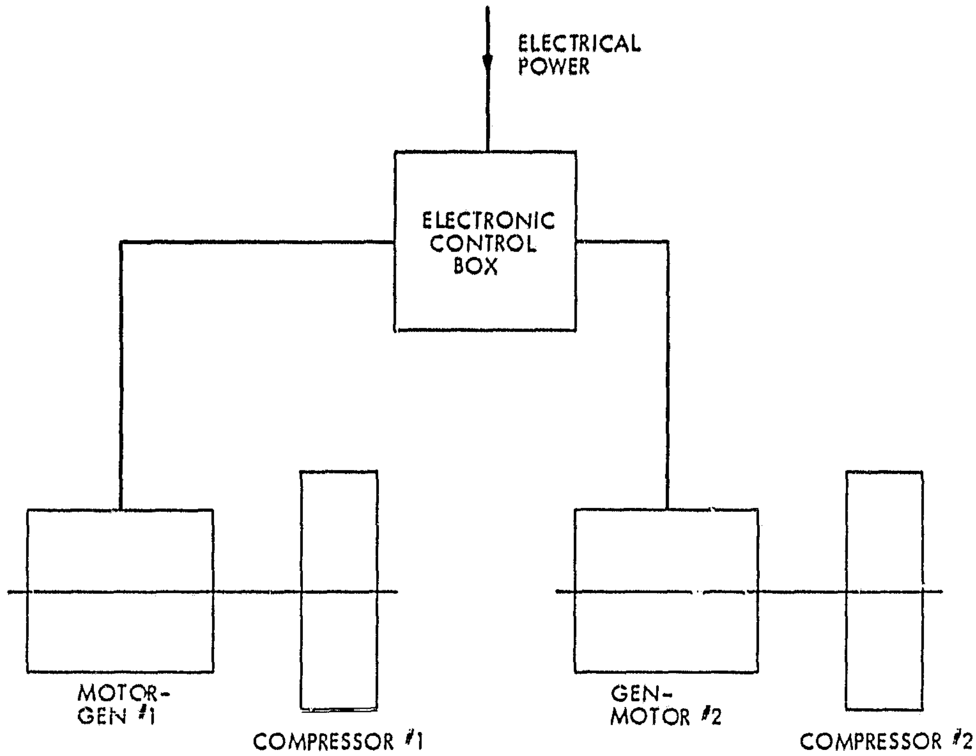


Figure 1-6. Matched Pair of C-R Compressors Operating with Dynamic Braking

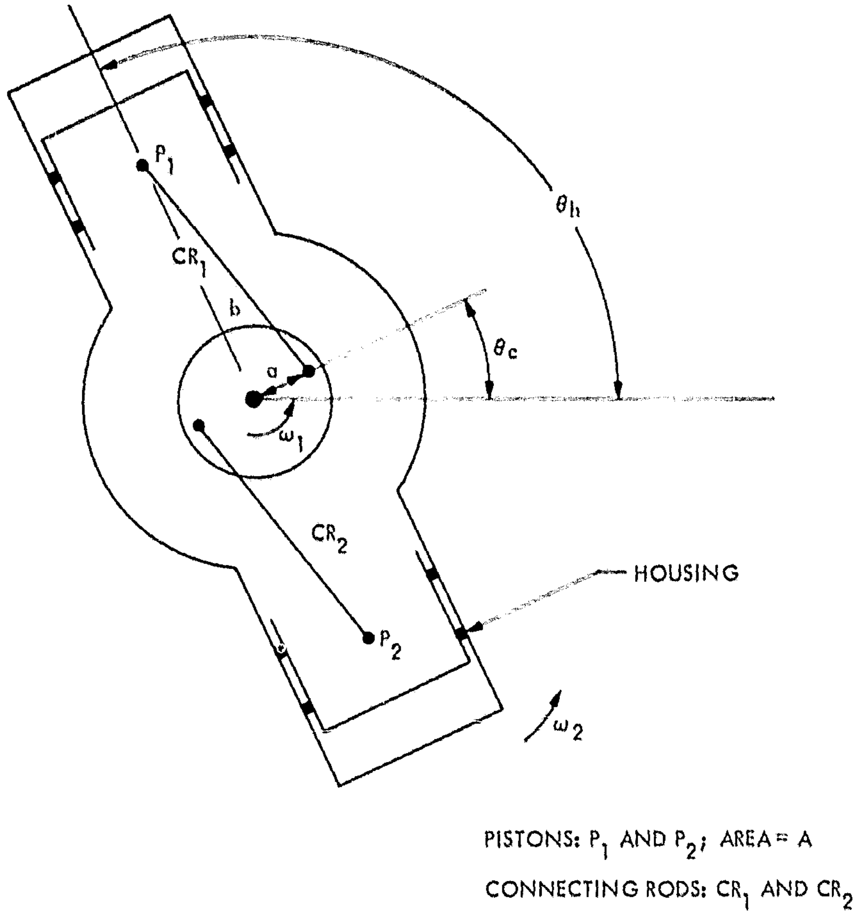


Figure 1-7. Cross Sectional View of C-R Machine with Dynamical Variables Employed

SECTION II

THEORY OF OPERATION

The qualitative descriptions of the centrifugal-reciprocating (C-R) machines have, up to this point, been given in terms of their applications to compressing a fluid. It is noted that in a quantitative analysis the C-R machine should be generalized to include the possibility of its use as a prime mover, or engine. Like most other fluid driven machines, the principle of reciprocity applies here and the C-R machine may be operated as a compressor or engine.

It will become clear in the ensuing discussion that the C-R machine will operate as an air engine, steam engine, gasoline I-C engine and diesel I-C engine. The specific application is left for future investigations.

The emphasis in this analysis shall be to study in detail the physical mechanisms which determine the characteristics of C-R machines. Accordingly, the approximations which are made for mathematical convenience must be done with care so as not to obscure or alter the physical basis of operation.

In order to derive generalized equations which are applicable to the various modes of operation of the C-R machine, it is best to use the Lagrangian method. The variables employed are shown in Figure 1-7. For simplicity, the mass of the piston will be assumed to be concentrated at radius R. The Lagrangian is then

$$L = (1/2)I_h\dot{\theta}_h^2 + (1/2)I_c\dot{\theta}_c^2 + MR^2\dot{\theta}_h^2 + M\dot{R}^2 - H \qquad (2\text{-}1)$$

where H is the enthalpy of the working fluid (to be discussed later). On examining Figure 1-7, we note that the crankshaft and connecting rods constrain the motion of the pistons in such a way that the radial distance R is an explicit function of the angular variables θ_h and θ_c. Furthermore, the analysis may be greatly simplified by assuming that the connecting rod is much longer than the crankshaft arm; i.e., b>>a. Then we have

$$R = b + a\cos(\theta_h - \theta_c) \qquad (2\text{-}2)$$

and

$$\dot{R} = -a(\dot{\theta}_h - \dot{\theta}_c)\sin(\dot{\theta}_h - \theta_c) \qquad (2\text{-}3)$$

Therefore, the Lagrangian is

$$L = (1/2)I_h\dot{\theta}_h^2 + (1/2)I_c\dot{\theta}_c^2 + Mb^2\dot{\theta}_h^2 + 2Mab\dot{\theta}_h^2\cos(\theta_h - \theta_c)$$

$$- p[V_0 - 2A(b + a\cos(\theta_h - \theta_c)] \tag{2-4}$$

where we have assumed isothermal conditions on the working fluid (for slow reciprocating motion) so that the enthalpy takes on the simple form

$$H = 2pV = p\left|V_0 - 2A[b + a\cos(\theta_h - \theta_c)]\right| \tag{2-5}$$

Here

P = pressure on piston face
V = total volume of fluid
V_o = total dead volume
A = cross-sectional area of piston

All the thermodynamic features of the system will now be contained in the pressure p, which will be a function of time dependent on the apparatus connected to the machine.

In Equation (2-4) we note the first three terms are the usual inertia terms and the fourth term is the Coriolis term, while the last term is, of course, the potential energy term due to the compressed fluid.

The equations of motion are derived in the usual way from the Lagrangian

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{2-6}$$

where

Q_i = sum of nonconservative forces (or torques) acting on coordinate q_i.

Thus, finally, we have

$$I_c\ddot{\theta}_c + 2a[Mb\dot{\theta}_h^2 - pA]\sin(\theta_h - \theta_c) = L_c \tag{2-7}$$

$$[I_0 + 4Mab\cos(\theta_h - \theta_c)]\ddot{\theta}_h$$

$$+ 2a[Mb\dot{\theta}_h^2 - 2Mb\dot{\theta}_c\dot{\theta}_h + pA]\sin(\theta_h - \theta_c) = L_h \tag{2-8}$$

where L_c and L_h are the torques applied to the crankshaft and housing, respectively, and $I_0 = (I_h + 2Mb^2)$, the total average moment of inertia of the housing and pistons.

Several features contained in the equations of motion are worth noting. In our qualitative discussion of the C-R compressor we stated that the centrifugal force $Mb\dot{\theta}_h^2$ must be larger than the force due to the fluid pA in order for there to be motion of the pistons.

In Equation (2-7) it is apparent that there can be no relative motion between crankshaft and housing until centrifugal force balances the fluid force, i.e. until the quantity in brackets vanishes. Then there shall be acceleration $\ddot{\theta}_c \neq 0$ to permit motion of the piston relative to the housing. In Equation (2-8) the same criterion is contained in the second bracketed quantity. This is apparent when it is realized that $\dot{\theta}_c \approx \dot{\theta}_h$ at the threshold of balanced forces.

The first bracketed quantity in Equation (2-8) is simply the total instantaneous moment of inertia of housing and pistons, and shows how this quantity is modulated by the motion of the pistons. The analogy to parametrically excited electrical networks wherein self-inductance is modulated is to be noted.

The equations of motion (2-7) and (2-8) are coupled nonlinear equations and cannot be linearized in the usual way since the nonlinear mechanisms are essential to the operation of the machine. On the other hand, the nonlinearities in (2-7) and (2-8) prohibit us from finding general solutions to these equations. What can be done is to find special solutions to the equations and the conditions under which the assumed solutions are valid.

In particular, we shall treat two special cases: (1) a periodic torque is applied to the crankshaft with no external torque applied to the housing; (2) the crankshaft and housing are separately driven at slightly different but constant speeds.

A. CASE (1): PERIODIC TORQUE APPLIED TO CRANKSHAFT

We assume, in spite of the nonlinear equations of motions, that our intuitive feeling is correct about a periodic torque producing a periodic response in the C-R machine. We start by assuming a simple periodic motion and calculate the torque required to produce it.

Referring to Figure 1-7, it can be shown that an acceptable form for the angular displacements as a function of time is given by

$$\theta_h = \omega_o t + \frac{\alpha}{\Delta} \sin\Delta t + \frac{\pi}{4}(1 - \cos \Delta t) \qquad (2\text{-}9)$$

$$\theta_c = \omega_o t + \frac{\alpha}{\Delta} \sin\Delta t - \frac{\pi}{4}(1 - \cos \Delta t) \qquad (2\text{-}10)$$

On taking the time derivatives, we have

$$\dot{\theta}_h = \omega_o + \alpha\cos\Delta t + \frac{\pi}{4} \Delta\sin \Delta t \qquad (2\text{-}11)$$

$$\dot{\theta}_c = \omega_o + \alpha\cos\Delta t - \frac{\pi}{4}\Delta\sin\Delta t \quad (2\text{-}12)$$

whence we see that ω_o is the mean angular velocity, α is amplitude of periodic angular velocity and Δ is the rate at which the angular velocity is modulated (also equal to 2π times the strokes per second).

In these equations we assume that the crankshaft makes a full 180 deg excursion when the pistons moves from TDC (top dead center) to BDC (bottom dead center). The coefficients of the last terms in Equations (2-9) and (2-10) were chosen to fulfill this assumption. To simplify this analysis we also assume that there is no dwell in any position of the pistons. We also need the expansions for

$$\sin(\theta_h - \theta_c) = \sin\frac{\pi}{2}(1 - \cos\Delta t)$$

$$= J_o\left(\frac{\pi}{2}\right) + 2J_2\left(\frac{\pi}{2}\right)\cos 2\Delta t + \ldots \quad (2\text{-}13)$$

and

$$\cos(\theta_h - \theta_c) = 2J_1\left(\frac{\pi}{2}\right)\sin\Delta t + 2J_3\left(\frac{\pi}{2}\right)\sin 3\Delta t + \ldots \quad (2\text{-}14)$$

where the J_n's are nth order Bessel functions of the first kind. These functions are shown in Fig. 2-1. Thus substituting all the foregoing quantities into the equations of motion we have, from (2-7),

$$L_c = \alpha\Big\{(4Mb\omega_o - 2p_oA\sin\theta)\cos\Delta t$$

$$+ (Mb\pi\Delta\omega_o - 2p_oA\cos\theta)\sin\Delta t\Big\}\left[J_0\frac{\pi}{2} + 2J_2\frac{\pi}{2}\cos 2\Delta t\right] \quad (2\text{-}15)$$

and from 2-8 (neglecting higher harmonics)

$$\left[\frac{\pi I_o\Delta^2}{4} - J_o\left(\frac{\pi}{2}\right)\left(2\alpha Mb\omega_o - p_oA\sin\theta\right)\right]\cos\Delta t$$

$$-\left[I_o\alpha\Delta - J_o\left(\frac{\pi}{2}\right)\left(\frac{\pi\Delta Mb\omega_o}{2} + p_oA\cos\theta\right)\right]\sin\Delta t = L_h = 0 \quad (2\text{-}16)$$

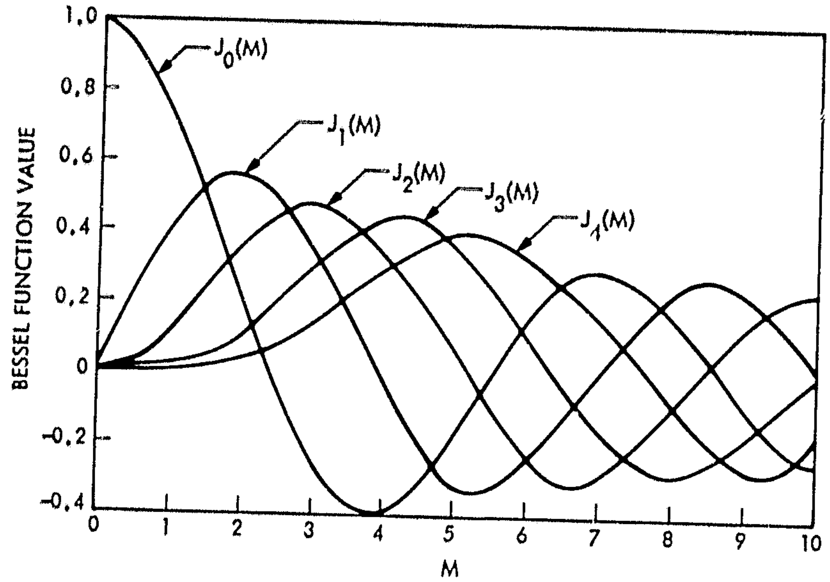


Figure 2-1. Bessel Functions of the First Kind

where we have used

$$p = P_o + p_o \sin(\Delta t + \sigma) \qquad (2\text{-}17)$$

and have neglected, for simplicity, the term due to the modulation of moment of inertia. As discussed in the introduction, the average gas pressure $P_o = Mb\omega_o{}^2$, the average centrifugal force for the dynamically balanced mode of operation.

Equation (2-15) gives the torque required to produce the assumed motion, and Equation (2-16) gives the dynamical constraints on α and Δ as a function of design parameters such as M, I_o, ω_o, and p_o. The equations of motion are inherently nonlinear, and we are allowed the assumed motions provided we can find real solutions (from Equation 2-16) for α and Δ from

$$\pi I_o \Delta^2 - J_0\left(\frac{\pi}{2}\right)\left[8\alpha Mb\omega_o - 4p_oA\sin\theta\right] = 0 \qquad (2\text{-}18)$$

and

$$2I_o\alpha\,\Delta - J_0\left(\frac{\pi}{2}\right)\left[\pi Mb\omega_o\Delta + 2p_oA\cos\theta\right] = 0 \qquad (2\text{-}19)$$

The crankshaft power required is given by

$$P_c = L_c \times \dot{\theta}_c = L_c \times (\omega_o\Delta + \alpha\cos\Delta t - \frac{\pi}{4}\Delta\sin\Delta t)$$

or

$$P_c \cong a\left[(4Mb\omega_o\alpha - 2p_oA\sin\theta)\cos\Delta t\right.$$

$$\left. + (Mb\pi\Delta\omega_o - 2p_oA\cos\theta)\sin\Delta t\right]\left[J_0\left(\frac{\pi}{2}\right)\right] \times (\omega_o + \alpha\cos\Delta t - \left(\frac{\pi}{4}\right)\Delta\sin\Delta t) \qquad (2\text{-}20)$$

It can be shown very easily that the constant part of (2-20) is given by:

$$P_c\ (\text{constant}) = a\left[(4Mb\omega_o\alpha - 2p_oA\sin\theta)\,\alpha\right.$$

$$\left. - (Mb\pi\Delta\omega_o - 2p_oA\cos\theta)\left(\frac{\pi}{4}\right)\Delta\right]\left[J_0\left(\frac{\pi}{2}\right)\right]$$

A little reflection shows that the time average value of P_c may be positive (compressor) or negative (engine) depending on the phase angle of the pressure wave. This ought not to be too surprising since the conventional air engine is simply a special case: $\dot{\theta}_h = 0$ of the C-R engine.

B. CASE (2): CRANKSHAFT AND HOUSING DRIVEN AT SLIGHTLY DIFFERENT BUT CONSTANT SPEEDS

This mode of operation, which may also be called the positive displacement mode, is primarily of academic interest since reducing the forces by a factor of 2 may not be appreciated by the designer. However, the equations reduce to such simple form that it is worthwhile digressing for a moment.

In Equations (2-7) and (2-8) we let

$$\dot{\theta}_h = \omega_h = \omega_o + \frac{\Delta}{2}$$

and

$$\dot{\theta}_c = \omega_c = \omega_o - \frac{\Delta}{2}$$

with

$$\ddot{\theta}_h = \ddot{\theta}_c = 0$$

Then

$$L_c = 2a[Mb(\omega_o^2 + \omega_o\Delta) - P_oA - p_oA \sin(\Delta t + \theta)] \times \sin \Delta t \qquad (2\text{-}21)$$

Again we let $Mb\omega_o^2 = P_oA$. Then

$$L_c = 2a[Mb\omega_o\Delta - p_oA \sin(\Delta t + \theta)] \sin \Delta t \qquad (2\text{-}22)$$

Similarly

$$L_h = 2a[Mb\omega_o\Delta + p_oA \sin(\Delta t + \theta)] \sin \Delta t \qquad (2\text{-}23)$$

Thus the total shaft power is

$$p = L_o \dot{\theta}_c + L_h \dot{\theta}_h$$

$$= (- ap_o A \cos \theta + 2aMb \Delta \sin \Delta t$$

$$+ ap_o A \cos 2\Delta t)(\omega_o - \frac{\Delta}{2})$$

$$+ [ap_o A \cos \theta + 2aMb\Delta \sin \Delta t$$

$$- ap_o A \cos (2\Delta t + \theta)](\omega_o + \frac{\Delta}{2}) \quad (2\text{-}24)$$

or

$$P = ap_o A\Delta \cos \theta + 2aMb\Delta\omega_o^2 \sin \Delta t - ap_o A\Delta \cos(2\Delta t + \theta) \quad (2\text{-}25)$$

Thus, the machine operates as an engine or compressor by proper choice of θ.

SECTION III

CONCLUSIONS

The C-R machine may be regarded as a velocity transformation device. When used as a compressor it produces slow reciprocating motion from high-speed rotational motion. In the conventional reciprocating compressor a large speed reduction gearbox and hefty crankshaft and connecting rods would be required. Even if the size and weight of the compressor did not matter, the reliability of gearboxes leaves much to be desired. Hence, the principal advantage of the C-R compressor is in the potentiality for small, lightweight compressors with high reliability.

When used as an internal combustion engine the C-R machine transforms slow reciprocating motion into high rotational speeds. The advantage here is that the engine can operate in a mode which comes closer to the Carnot cycle than in the standard configuration. In fact, the hypothetical Carnot cycle could really be executed in textbook fashion because the reciprocating motion may be made as slow as desired.

The standard reciprocating air engine may be regarded as a special case of the centrifugal-reciprocating machine. The performance characteristics of the C-R machine, however, are considerably different from the standard engine because of the nonlinearities introduced into the equations of motion by the rotating housing. It has been shown that the nonlinearities tend to limit the C-R machine to certain ranges of the parameters such as speed, working pressure, etc. However, there are many applications, such as electric power generation, where the restriction to a constant speed is not a problem.

Only two special cases have been considered in this study; the extension to other cases is expected to be straightforward.

An experimental machine shown in Figures 3-1a and 3-1b was constructed to demonstrate the fundamental concepts of this paper. The experimental model was designed to demonstrate feasibility of the concepts, and no attempt was made to compare quantitative performance with theory. The model was intended to be operable with the CTI 350 Cryodyne and was sized accordingly as shown in Table 3-1.

The large weight of the assembly made it difficult to obtain reciprocating speeds larger than 20 per minute, but the slow speed simplified the seal problem by permitting the use of lubricated rubber "O" rings for these tests. Only one test compressor was built, and it was necessary to use a flywheel on a second motor to demonstrate the regenerative braking concept as shown in Figure 3-2.

The preliminary test results were adequate to demonstrate the basic concepts of this paper. The maximum centrifugal force developed was 770 kgm, and the force due to the compressed gas was 710 kgm. The pressure ratio of 3:2 is encouraging, especially when compared with Vuilleumier systems. The slow reciprocating speeds achieved was disappointing, but not altogether unexpected since the structure was so massive. The important parameter which needs optimization is the ratio of piston mass to total structure mass. This ratio should be made as large as possible.

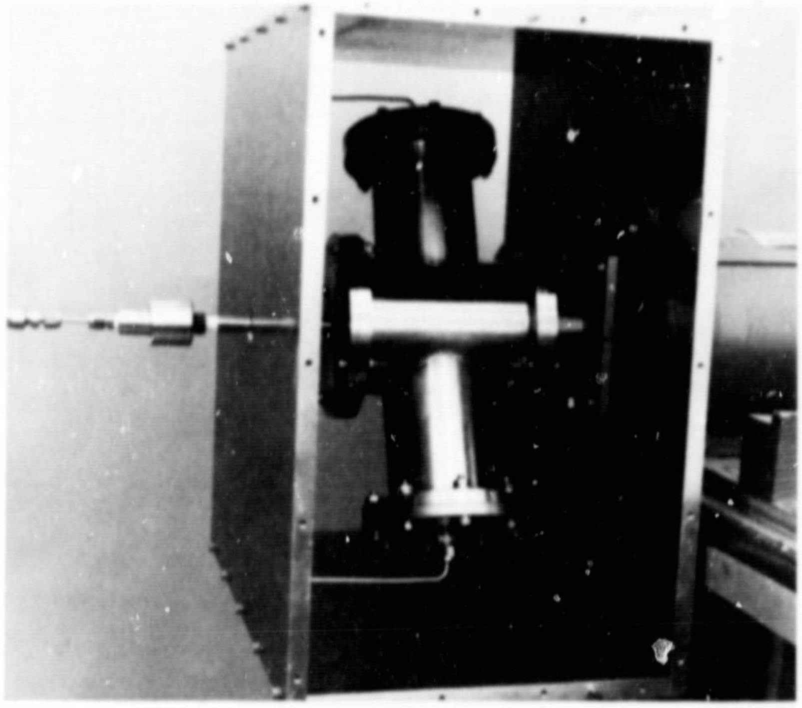


Figure 3-1a. Experimental C-R Machine

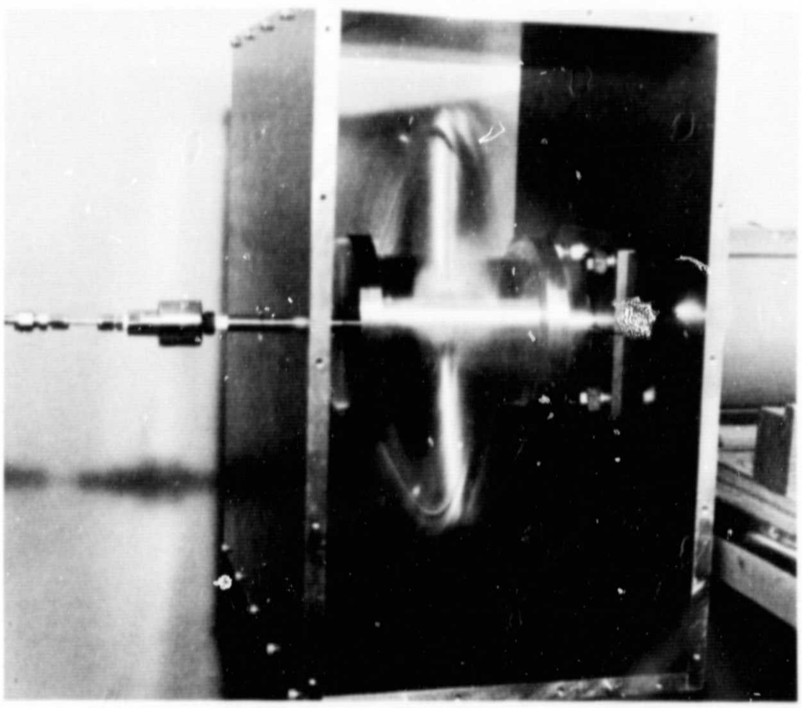


Figure 3-1b. Experimental Machine in Rotation

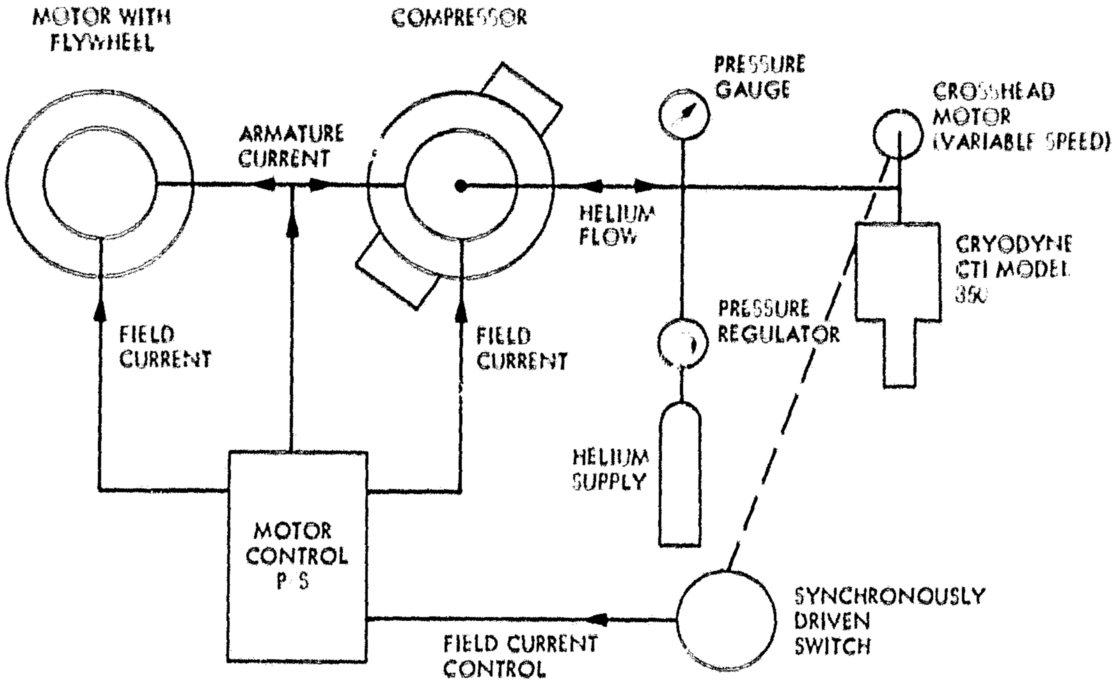


Figure 3-2. Experimental Arrangement to Demonstrate C-R Compressor and Concept of Regenerative Braking

Table 3-1. Data on Experimental Compressor

Piston diameter	7.5 cm
Piston weight	1.17 kgm
Stroke	7 cm
Maximum radius	15 cm
Upper piston ring	rubber "O" ring
Lower piston ring	Teflon
Total weight of compressor	13.6 kgm
Drive motor	Baldor 1 hp
DC shunt	1750 rpm

Typical Running Parameters:

Speed: 1,600 - 2,000 rpm
20 strokes per minute

Total average input power to motors: 900 watts

P_{max} = 225 psig

P_{min} = 150 psig