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Final Report

Electromagnetic Backscattering from a Random Distribution of Lossy Dielectric Scatterers

by

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April 1980

Prepared for:

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, MD 20771

Grant NSG-5288

FOREWORD

This work was performed under NASA Goddard Space Flight Center Grant NGS-5288. The basic purpose of the research performed was to model subsurface media by employing stochastic techniques. A preliminary investigation indicated that modeling subsurface media by discrete particles having random position and orientation would be a viable procedure. The method has wide applicability since most subsurface media can be viewed as an aggregate of dielectric particles. In addition, several multiple scattering methods exist which enable one to calcualte the electromagnetic radiation from such a collection of particles for various parameter regimes.

To demonstrate the usefulness of the method, the electromagnetic backscattering from a half space of discrete lossy dielectric scatterers was analyzed. The method of Foldy was employed to find an equation for the mean field in the scattering region. From this equation, an effective permittivity was obtained. The effective permittivity was anisotropic reflecting the non-spherical nature of the particles being considered. Following this, the correlation of the scattered field was found by employing the distorted Born approximation. This method treats the scatterers as particles embedded in the effective medium. The backscattering coefficients were then computed.

Specialized results were obtained by letting the arbitrary particles take the form of discs. Since lossy dielectric discs closely resemble leaves, comparison of the calculated results was made with data obtained from microwave backscattering from deciduous trees. The calculated results agreed with the experiment data (in level). The most interesting fixture was, that the depolarized component of backscatter was accounted for correctly. This was due to the fact that the anisotropic nature of the scattering medium was correctly accounted for.

Finally, we would like to acknowledge the support and encouragement of Dr. D. LeVine and Mr. R. Meneghini of NASA. We would also like to thank Mr. Selim Seker of GWU for performing the numerical calculations.

INTRODUCTION

This paper studies electromagnetic backscattering from a sparse distribution of discrete lossy dielectric scatterers occupying a region V. The scatterers are assumed to have random position and orientation. Scattered fields are calculated by first finding the mean field and then by using it to define an equivalent medium within the volume V. The scatterers are then viewed as being embedded in the equivalent medium; the distorted Born approximation is then used to find the scattered fields. This technique represents an improvement over the standard Born approximation since it takes into account the attenuation of the incident and scattered waves in the equivalent medium.

In the past, electromagnetic scattering from a collection of discrete scatterers has been modeled by continuous and discrete random medium techniques. In the continuous case, the random medium is modeled by assuming that its permittivity $\varepsilon(\underline{x})$ is a random process whose moments are known. The average backscattering cross section is then calculated from a knowledge of the statistics of $\varepsilon(\underline{x})$. Usually it is only the mean and correlation of the permittivity that are required. The analysis of this problem can then proceed in a number of ways. One method involves calculating the mean field [Keller, 1962, Tatarskii and Gertsenshtein, 1963 and Keller and Karal 1966], using it to define an equivalent medium and then employing single scattering in the equivalent medium. In active remote sensing applications from terrain, the technique has been applied by a number of authors [Rosenbaun and Bowles, 1974;

Hevenor, 1976; Fung and Fung, 1977; Fung and Ulaby, 1978; Fung, 1979; and Zuniga, et al., 1979]. Another technique used to obtain the scattered fields from a continuous random medium is the radiative transport approach. Here the transport equations are obtained in terms of the statistics of $\varepsilon(\underline{x})$ [Tsang and Kong, 1978].

In this paper, we have adopted the alternative approach - modeling by discrete random media techniques. Here, the individual objects - such as leaves - are characterized by their scattering cross sections or dipole moments. Each object is then given a random placement and orientation. Techniques such as Born approximation; single scattering method and transport equations have been used to calculate scattered fields [Ishimaru, 1978]. In the area of active remote sensing from terrain Du and Peake [1969] have used the Born approximation to calculate the scattering from a layer of leaves. Lang [1979] employed the distorted Born approximation to calculate the backscatter from a half space of spherical scatterers.

In this paper we generalize the previous result of Lang [1979] to scatterers of arbitrary shape. The technique employed, as mentioned previously, is to find the mean field using a technique first developed by Foldy [1945] and later generalized by Lax [1951, 1952], Twersky [1962, 1978] and Keller [1964]. From this mean wave an equivalent medium is derived which,in general,is inhomogeneous, spacially dispersive and anisotropic. The scattered field is then obtained by employing single scattering in the equivalent medium.

The method is then applied to a half space of scatterers that are homogeneously distributed and have characteristic dimensions small compared to a wavelength. In this case the equivalent dielectric tensor is homogenous and nondispersive but is still anisotropic. Further simplifications are obtained when the orientation of the scatterers is assumed to be distributed uniformily in the azimuth direction. In this special case the medium becomes uniaxial. By using this equivalent medium, simple expressions are obtained for the horizontal, vertical and cross polarized backscattering coefficients. Finally, the method is used to model a leaf canopy when the leaves are modeled by lossy dielectric discs.

PROBLEM FORMULATION

Consider the problem of scattering of time harmonic electromagnetic waves from N discrete scatterers located in a volume V as is shown in Figure 1. The particles are all identical and each has volume V_p, relative dielectric constant ε_r and free space permeability μ_0. It is assumed that the background medium is free space having permittivity ε_0 and permeability μ_0.

The position of the i^{th} particle is specified by the vector $\underline{X}_i$ extending from an origin 0 to the center of that particle. The particle's center is located by the center of the smallest circumscribed sphere in which the particle can be placed. Although the particles are identical they have a rotation with respect to a fixed direction. The rotation for the i^{th} particle is specified by $\underline{\Omega}_i=(\Theta_i,\Phi_i)$ where Θ_i and Φ_i are polar and azimuth angles respectively with $0\leq\Theta_i\leq\pi$ and $0\leq\Phi_i\leq 2\pi$.

The electric field obeys the vector wave equation

$$\nabla x \nabla x \underline{E} - k_0^2 \varepsilon_r(\underline{x})\underline{E} = i\omega\mu_0 \underline{J} \qquad (1)$$

where a time dependence $e^{-i\omega t}$ has been assumed. In (1) $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the free space wavenumber and $\underline{J}$ is the current density of the source. The relative dielectric constant $\varepsilon_r(\underline{x})$ can be expressed in terms of individual particles by employing translations and rotations of the particle located at the origin. Let us assume that a particle located at the origin is characterized by the function $U(\underline{x})$ where

$$U(\underline{x}) = \begin{cases} 1 \ , & \underline{x}\varepsilon V_p \\ 0 \ , & x \not\varepsilon V_p \end{cases} \tag{2}$$

Using (2) we express $\varepsilon_r(\underline{x})$ as

$$\varepsilon_r(\underline{x}) = 1 + \Delta \sum_{i=1}^{N} U(\underline{x}-\underline{X}_i, \underline{\Omega}_i) \quad , \qquad \Delta = \varepsilon_r - 1 \tag{3}$$

where

$$U(\underline{x}, \underline{\Omega}) = U(\underline{\underline{R}}(\underline{\Omega}) \cdot \underline{x}) \tag{4}$$

Here $U(\underline{x}, \underline{\Omega})$ is the function $U(\underline{x})$ rotated by $\underline{\Omega}$ and $\underline{\underline{R}}(\Omega)$ is a rotation dyadic.

We will find it convenient to express (1) and (3) in a more abstract notation. We have

$$(\underline{\underline{L}} - \sum_{i=1}^{N} \underline{\underline{V}}_i) \cdot \underline{\underline{E}} = \underline{q} \tag{5}$$

where

$$\underline{\underline{L}} = \nabla x \nabla x \underline{\underline{I}} - k_0^2 \underline{\underline{I}} \tag{6}$$

$$\underline{\underline{V}}_i = k_0^2 \Delta U(\underline{x}-\underline{X}_i, \underline{\Omega}_i) \underline{\underline{I}} \ , \quad \underline{q} = i\omega\mu_0 \underline{J} \tag{7}$$

Here $\underline{\underline{I}}$ is the unit dyadic and $\underline{q}$ can be viewed as a normalized source term. At times it will be convenient to write

$$\underline{E}(\underline{x}) = \underline{E}_0(\underline{x}) + \underline{E}_s(\underline{x}) \tag{8}$$

where $\underline{E}_0(x)$ is the solution to (5) when no scatterers are present, i.e.,

$$\underline{\underline{L}} \cdot \underline{E}_0(\underline{x}) = \underline{q} \tag{9}$$

and $\underline{E}_s(\underline{x})$ is the scattered field from the particles.

SINGLE SCATTERER - TRANSITION OPERATOR

Before considering the N particle scattering problem, we will consider scattering from one particle located at the origin. Putting N=1 in (5) with $\underline{X}_1=0$ and $\underline{\Omega}_1=\underline{\Omega}$, we have

$$(\underline{\underline{L}} - \underline{\underline{V}})\cdot\underline{e} = \underline{g} \quad , \qquad \underline{\underline{V}} = \Delta k_0^2 U(\underline{x},\underline{\Omega}) \tag{10}$$

where

$$\underline{e} = \underline{e}_0 + \underline{e}_s \quad , \qquad \underline{\underline{L}}\cdot\underline{e}_0 = \underline{g} \tag{11}$$

and $\underline{e}_s$ is outgoing as $|\underline{x}|\to\infty$. We have used the small $\underline{e}$ notation for the field here to remind us that there is only one scatterer present.

If we use (11) in (10), we obtain

$$\underline{\underline{L}}\cdot\underline{e}_s = \underline{\underline{V}}\cdot\underline{e} \tag{12}$$

From (12) we see that the term on the left, $\underline{\underline{V}}\cdot\underline{e}$, can be viewed as the source of the scattered field. We write

$$\underline{g}_{eq} = \underline{\underline{V}}\cdot\underline{e} \tag{13}$$

where $\underline{g}_{eq}$ is an equivalent source term. Since $\underline{\underline{V}}=0$ when $\underline{x}\notin V_p$, the sources, $\underline{g}_{eq}$, exist inside the particle boundaries.

It is more natural to think of the equivalent sources as being caused by the incident field $\underline{e}_0$. Because Maxwell's equations are linear, we can write

$$\underline{g}_{eq} = \underline{\underline{T}}\cdot\underline{e}_0 \tag{14}$$

where the dyadic operator $\underline{\underline{T}}$ is known as the transition operator in the scattering literature [Lax, 1951]. Now using (13) and (14) in (12) and multiplying through by $\underline{\underline{L}}^{-1}$, we have

$$\underline{e}_s = \underline{\underline{L}}^{-1} \cdot \underline{\underline{T}} \cdot \underline{e}_0 \tag{15}$$

Thus the knowledge of $\underline{\underline{T}}$ completely characterizes the scattering properties of the particle. The operator $\underline{\underline{T}}$ is related to the dyadic scattering amplitude of the particle and for dipole scatterers $\underline{\underline{T}}$ can be determined from the polarizability of the particle. Thus $\underline{\underline{T}}$ is directly connected with quantities of physical interest.

The transition operator is a linear bounded operator and, as a result, can be expressed in integral form:

$$\underline{g}_{eq}(\underline{x}) = \underline{\underline{T}} \cdot \underline{e}_0 = \int d\underline{x}' \underline{\underline{t}}(\underline{x}, \underline{x}') \cdot \underline{e}_0(\underline{x}') \tag{16}$$

where the limits for the integral extend over all space. One can show that $\underline{\underline{t}}$ is 0 when $\underline{x}$ and $\underline{x}'$ are outside the particle [Frisch, 1968], i.e.,

$$\underline{\underline{t}}(\underline{x}, \underline{x}') = 0 \ , \quad x \notin V_p \quad \text{or} \quad \underline{x}' \notin V_p \tag{17}$$

The property follows directly from the fact that the equivalent sources for the scattered field are located within the particle boundaries.

We will now represent $\underline{\underline{t}}$ in terms of plane waves. The representation of $\underline{\underline{t}}$ can be directly related to the dyadic scattering amplitude. We proceed by representing $\underline{e}_0(\underline{x})$ by its Fourier transform, putting this in (16) and taking the Fourier transform of (16). We obtain

$$\tilde{\underline{g}}_{eq}(\underline{k}) = \int d\underline{k}' \tilde{\underline{\underline{t}}}(\underline{k}, \underline{k}') \cdot \tilde{\underline{e}}_0(\underline{k}') \tag{18}$$

where

$$\tilde{\underline{\underline{t}}}(\underline{k}, \underline{k}') = \frac{1}{(2\pi)^3} \int d\underline{x} d\underline{x}' \underline{\underline{t}}(\underline{x}, \underline{x}') e^{-i(\underline{k}\cdot\underline{x} - \underline{k}'\cdot\underline{x}')} \tag{19}$$

In (18) we have used the notation that $\tilde{\underline{h}}$ is the Fourier transform of $\underline{h}$. More specifically

$$\tilde{\underline{h}}(\underline{k}) = \int d\underline{x}\ \underline{h}(\underline{x})e^{-i\underline{k}\cdot\underline{x}} \tag{20}$$

Inverting (19) the transition kernel $\underline{\underline{t}}$ can be expressed in terms of its plane wave representation $\tilde{\underline{\underline{t}}}$:

$$\underline{\underline{t}}(\underline{x},\underline{x}') = \frac{1}{(2\pi)^3} \int d\underline{k}\ d\underline{k}'\ \tilde{\underline{\underline{t}}}(\underline{k},\underline{k}')e^{i(\underline{k}\cdot\underline{x}-\underline{k}'\cdot\underline{x}')} \tag{21}$$

The dyadic scattering amplitude will now be defined. Consider a plane wave incident upon a scatterer located at the origin. An arbitrary plane wave can be decomposed into two mutually orthogonal linearly polarized plane waves. The polarization directions are taken as $\underline{\alpha}°$ and $\underline{\beta}°$ where $\underline{\alpha}°$ and $\underline{\beta}°$ are orthogonal unit vectors with $\underline{\alpha}°$ and $\underline{\beta}°$ being perpendicular to the direction of propagation. The two incident waves are

$$\underline{e}_0(\underline{x},\underline{i};q) = \underline{q}°e^{-k_0\underline{i}\cdot\underline{x}} \quad , \qquad q\varepsilon\ \{\alpha,\beta\} \tag{22}$$

where $\underline{i}$ is a unit vector in the direction of incidence. It is more convenient to consider both polarizations simultaneously so we introduce the dyadic incident wave [Twersky, 1967]

$$\underline{\underline{e}}_0(\underline{x},\underline{i}) = \underline{e}_0(\underline{x},\underline{i};\alpha)\underline{\alpha}_0 + \underline{e}_0(\underline{x},\underline{i};\beta)\underline{\beta}_0 \tag{23}$$

$$= (\underline{\alpha}_0\underline{\alpha}_0 + \underline{\beta}_0\underline{\beta}_0)e^{ik_0\underline{i}\cdot\underline{x}} \tag{24}$$

$$= (\underline{\underline{I}} - \underline{i}\underline{i})e^{ik_0\underline{i}\cdot\underline{x}} \tag{25}$$

The dyadic scattered field from the particle is given by

$$\underline{\underline{e}}_s(\underline{x},\underline{i}) = \underline{e}_s(\underline{x},\underline{i};\alpha)\underline{\alpha}_0 + \underline{e}_s(\underline{x},\underline{i};\beta)\underline{\beta}_0 \tag{26}$$

where $\underline{e}_s(\underline{x},\underline{i};q)$ is the scattered field due to polarization q. The dyadic scattering amplitude, $\underline{\underline{f}}$, is defined in terms of the asymptotic expression for $\underline{\underline{e}}_s$ in the radiation zone. We have

$$\underline{\underline{e}}_s(\underline{x}\ \underline{i}) \sim \underline{\underline{f}}(\underline{0},\underline{i})\ \frac{e^{ik_0|\underline{x}|}}{|\underline{x}|}\ , \qquad |\underline{x}|\to\infty \tag{27}$$

where $\underline{0}$ is a unit vector in the $\underline{x}$ direction, $\underline{0}=\underline{x}/|\underline{x}|$.

The relationship between $\underline{\underline{f}}$ and $\tilde{\underline{\underline{t}}}$ can be found by employing (15) for large $|\underline{x}|$ (Appendix A). The result is

$$\underline{\underline{f}}(\underline{0},\underline{i}) = 2\pi^2(\underline{\underline{I}}-\underline{0}\ \underline{0})\cdot\tilde{\underline{\underline{t}}}(k_0\underline{0},k_0\underline{i})\cdot(\underline{\underline{I}}-\underline{i}\ \underline{i}) \tag{28}$$

From this relation, we see

$$\underline{0}\cdot\underline{\underline{f}} = 0\ , \quad \underline{\underline{f}}\cdot\underline{i} = 0 \tag{29}$$

Thus $\underline{\underline{f}}$ is a four component tensor - all combinations of two incident polarizations and two scattered polarizations. We also note that $\underline{\underline{f}}$ does not completely determine $\tilde{\underline{\underline{t}}}$ but only partially specifies it. In particular, a knowledge of $\underline{\underline{f}}$ for a free space wave number k_0 only determines $\tilde{\underline{\underline{t}}}(\underline{k},\underline{k}')$ at $|\underline{k}| = |\underline{k}'|=k_0$; also only four of the nine components of $\tilde{\underline{\underline{t}}}$ in the polarization directions are determined.

Before concluding this section, transition operators for particles not located at the origin will be needed. As before, the equivalent sources $\underline{g}_{eq}^{(i)}$ for a particle located at $\underline{X}_i$ can be related to the incident field. It follows that

$$\underline{g}_{eq}^{(i)}(\underline{x}) = \underline{\underline{T}}_i\cdot\underline{e}_0 = \int\underline{\underline{t}}_i(\underline{x},\underline{x}')\cdot\underline{e}_0(\underline{x}')d\underline{x}' \tag{30}$$

By shifting the sources and the incident field to the origin $\underline{\underline{t}}_i$ can be related to $\underline{\underline{t}}$. One finds

$$\underline{\underline{t}}_i(\underline{x},\underline{x}') = \underline{\underline{t}}(\underline{x}-\underline{X}_i,\underline{x}'-\underline{X}_i) \tag{31}$$

Note that throughout the discussion the dependence of $\underline{\underline{t}}$ on rotations has been suppressed for convenience of notation.

COHERENT FIELD

In this section we will develop an approximate equation for the coherent field by employing the Foldy approximation [Foldy, 1945]. The equation is in terms of the transition operator and thus, when the scattering amplitude is known, the equation is completely specified. After the equation has been derived it is pointed out that outside V the coherent fields obeys Maxwells' equations with free space permittivity and permeability. Inside V, the coherent fields obey Maxwells equations with free space permeability and a macroscopic permittivity that is inhomogeneous, anisotropic and spatially dispersive.

Before discussing the coherent field, the statistics that govern the particles position and rotation must be specified. It will be assumed that the position vectors $\underline{X}_i$, i=1...N and rotation vectors $\underline{\Omega}_i$, i=1..N are random variables that are specified by a 5N dimensional distribution function. In addition, it is assumed that interchanging particles leaves the distribution function unaffected. From this general distribution function we can obtain the probability density function for the i^{th} particle. It is

$$p_{\underline{X}_i\underline{\Omega}_i}(\underline{x},\underline{\omega}) = p_{\underline{X}\underline{\Omega}}(\underline{x},\underline{\omega}) \qquad i=1....N \tag{32}$$

where $\underline{\omega}=(\theta,\phi)$. In (32) we have explicitly noted the fact that the particles are identically distributed by omitting the index i on the left hand side of (32). We will assume that the particles location and rotation are independent, thus

$$p_{\underline{X}\underline{\Omega}}(\underline{x},\underline{\omega}) = p_{\underline{X}}(x)\,p_{\underline{\Omega}}(\underline{\omega}) \tag{33}$$

with the usual property:

$$\int_V p_{\underline{X}}(\underline{x})\,d\underline{x} = 1 \qquad \int_{4\pi} p_{\underline{\Omega}}(\underline{\omega})\,d\underline{\omega} = 1 \tag{34}$$

The particle density is defined by

$$\rho(\underline{x}) = N\, p_{\underline{X}}(\underline{x}) \tag{35}$$

so that

$$\int_V \rho(\underline{x})\,d\underline{x} = N\ . \tag{36}$$

In addition to the one particle density - when treating the correlation of the field - the two particle density will be required. We have

$$p_{\underline{X}_i\underline{X}_j\underline{\Omega}_i\underline{\Omega}_j}(\underline{x},\hat{\underline{x}},\underline{\omega},\hat{\underline{\omega}}) = p_{\underline{X}\hat{\underline{X}}\underline{\Omega}\hat{\underline{\Omega}}}(\underline{x},\hat{\underline{x}},\underline{\omega},\hat{\underline{\omega}}) = p_{\underline{X}\underline{\Omega}}(\underline{x},\underline{\omega})\,p_{\hat{\underline{X}}\hat{\underline{\Omega}}}(\hat{\underline{x}},\hat{\underline{\omega}}) \tag{37}$$

$$i,j = 1,\ldots N$$

In (37) we have assumed that the i^{th} and j^{th} particles are independent. The independence assumption is valid when the particles are sparsely distributed; the case we intend to treat.

We will now develop the approximate equation for the coherent field. We start by noting that the total field $\underline{E}$ can be thought of as a sum of the incident field $\underline{E}_0$ plus a sum of the fields scattered from each particle, $\underline{E}_s^{(i)}$. We have

$$\underline{E} = \underline{E}_0 + \sum_{i=1}^{N} \underline{E}_s^{(i)} \tag{38}$$

The total field incident on the i^{th} particle is called the effective field and is denoted by $\underline{E}^{(i)}$. Thus $\underline{\underline{T}}_i \cdot \underline{E}^{(i)}$ represents the equivalent sources generated by the incident field in the i^{th} particle and

$$\underline{E}_s^{(i)} = \underline{\underline{L}}^{-1}\cdot\underline{\underline{T}}_i\cdot\underline{E}^{(i)} \tag{39}$$

Using (39) in (38), we have

$$\underline{E} = \underline{E}_0 + \sum_{i=1}^{N} \underline{\underline{L}}^{-1}\cdot\underline{\underline{T}}_i\cdot\underline{E}^{(i)} \tag{40}$$

This is the equation that we wished to obtain.

Now we average this equation. The result is

$$\langle\underline{E}\rangle = \underline{E}_0 + \sum_{i=1}^{N} \underline{\underline{L}}^{-1}\cdot\langle\underline{\underline{T}}_i\cdot\underline{E}^{(i)}\rangle \tag{41}$$

To obtain an approximate equation for the mean we follow [Foldy, 1945] and assume

$$\underline{E}^{(i)} \simeq \langle\underline{E}\rangle \tag{42}$$

This means that the random quantity $\underline{E}^{(i)}$ is to first order equal to a deterministic quantity, i.e., to first order it is a ergodic quantity. Using (42) in (41) and noting that $\langle\underline{\underline{T}}_i\cdot\underline{E}^{(i)}\rangle\simeq\langle\underline{\underline{T}}_i\cdot\langle\underline{E}\rangle\rangle=\langle\underline{\underline{T}}_i\rangle\cdot\langle\underline{E}\rangle$ we have the approximate equation for the mean field

$$\langle\underline{E}\rangle = \underline{E}_0 + \sum_{i=1}^{N} \underline{\underline{L}}^{-1}\cdot\langle\underline{\underline{T}}_i\rangle\,\langle\underline{E}\rangle \tag{43}$$

Denoting explicitly the dependence of $\underline{\underline{T}}_i$ upon $\underline{X}_i$ and $\underline{\Omega}_i$, averaging and then using (33), we have

$$\begin{aligned}\langle\underline{\underline{T}}_i\rangle &= \langle\underline{\underline{T}}(\underline{X}_i,\underline{\Omega}_i)\rangle = \int_V d\underline{s}\int_{4\pi} d\underline{\omega}\; p_{\underline{X}\underline{\Omega}}(\underline{s},\underline{\omega})\,\underline{\underline{T}}(\underline{s},\underline{\omega}) \\ &= \int_V d\underline{s}\; p_{\underline{X}}(\underline{s})\,\overline{\underline{\underline{T}}}(\underline{s})\end{aligned} \tag{44}$$

where

$$\overline{\underline{\underline{T}}}(\underline{s}) = \int_{4\pi} d\underline{\omega}\; p_{\underline{\Omega}}(\underline{\omega})\,\underline{\underline{T}}(\underline{s},\underline{\omega}) \tag{45}$$

In (45) the bar over $\underline{\underline{T}}$ has been used to indicate an average of angular varies only. By putting (44) in (43), by noting that the scattered terms are identical and by introducing $\rho(\underline{s})$ via (35), we obtain

$$<\underline{E}> = \underline{E}_0 + \int_V d\underline{s}\ \rho(\underline{s})\underline{\underline{L}}^{-1}\cdot\overline{\underline{\underline{T}}}(\underline{s})\cdot<\underline{E}> \tag{46}$$

Multiplying from the left by $\underline{\underline{L}}$ and using (9), we get

$$\underline{\underline{\mathcal{L}}}\cdot<\underline{E}> = \underline{g} \tag{47}$$

where

$$\underline{\underline{\mathcal{L}}} = \underline{\underline{L}} - \int_V d\underline{s}\ \rho(\underline{s})\overline{\underline{\underline{T}}}(\underline{s}) \tag{48}$$

This is the equation for the coherent field.

The arguments that have led to the approximate equation (47) have been largely heuristic. The essential approximation is contained in (42) where the effective field is assumed approximately equal to the average field. Although we will not discuss the conditions under which (42) is valid, it will be shown elsewhere that the approximation is valid when the fraction of volume occupied by the particles is small compared to the total volume, i.e., $NV_p/V<<1$. We shall refer to a distribution of scatterers satisfying this condition as a sparse distribution.

Before proceeding we will write the equation for the mean in more concrete form. Using (30) and (31) in (47) and (48), we obtain

$$\underline{\underline{L}}\cdot<\underline{E}(\underline{x})> - \int_V d\underline{s}\int d\underline{x}'\ \rho(\underline{s})\overline{\underline{\underline{t}}}(\underline{x}-\underline{s},\underline{x}'-\underline{s})\cdot<\underline{E}(\underline{x}')> = g \tag{49}$$

where

$$\overline{\underline{\underline{t}}}(\underline{x},\underline{x}') = \int_{4\pi} d\underline{\omega}\ p_{\underline{\Omega}}(\omega)\underline{\underline{t}}(\underline{x},\underline{x}';\underline{\omega})\ . \tag{50}$$

Here the kernel $\underline{\underline{t}}(\underline{x},\underline{x}';\underline{\omega})$ is the same as given in (31), however we have explicitly shown its dependence on the angular coordinate $\underline{\omega}$.

We can now use (49) to obtain a macroscopic form of Maxwell's equations. First averaging the Faradary's law equation; we have

$$\nabla x \langle \underline{E}(\underline{x}) \rangle = i\omega\mu_0 \langle \underline{H}(\underline{x}) \rangle \ . \tag{51}$$

Then by using (51) in (49), we obtain the macroscopic Ampere's law equation.

$$\nabla x \langle \underline{H}(\underline{x}) \rangle = \underline{J} - i\omega \langle \underline{D} \rangle \ , \quad \langle \underline{D} \rangle = \varepsilon_0 \underline{\underline{\varepsilon}} \cdot \langle \underline{E} \rangle \tag{52}$$

when $\underline{\underline{\varepsilon}}$ is a macroscopic permittivity operator which describes the average behavior of the medium and $\underline{J}=\underline{g}/(i\omega\mu_0)$. It is

$$\underline{\underline{\varepsilon}} = \underline{\underline{I}} + \frac{1}{k_0^2} \int_V d\underline{s} \int d\underline{x}' \ \rho(\underline{s}) \overline{\underline{\underline{t}}}(\underline{x}-\underline{s},\underline{x}'-\underline{s}) \ . \tag{53}$$

This expression simplifies to $\underline{\underline{I}}$ (free space) when $\underline{x}\notin V$. To see this we note that when $\underline{x}\notin V$, we have $\underline{x}-\underline{s}\notin V_p$ since $\underline{s}\in V$. Now using (17) we have $\overline{\underline{\underline{t}}}=0$. When $\underline{x}\in V$ (47) does not simply in general. It describes an anisotropic, inhomogeneous, spatially dispersion medium.

Let us examine how (53) reduces to some more familiar expressions in some special situations. We will assume that V is infinite through the remainder of this section. First we will consider the case when the density is constant, i.e., $\rho(\underline{s})=\rho$. In this case the permittivity is translationally invariant or homogeneous. To see this we substitute (21) into (53) and we perform the integrations over $\underline{s}$ and $\underline{k}'$. We obtain

$$\underline{\underline{\varepsilon}} = \underline{\underline{I}} + \frac{\rho}{k_0^2} \int d\underline{x}' \int d\underline{k}\ e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\ \overline{\underline{\underline{\tilde{t}}}}(\underline{k},\underline{k}) \tag{54}$$

$$\text{when } \overline{\underline{\underline{\tilde{t}}}}(\underline{k},\underline{k}) = \int_{4\pi} d\underline{\omega}\ P_{\underline{\Omega}}(\underline{\omega})\underline{\underline{\tilde{t}}}(\underline{k},\underline{k}';\underline{\omega}) \ . \tag{55}$$

Since the integrand is a function of $\underline{x}-\underline{x}'$ the permittivity is translationally invariant however it is still anisotropic and spatially dispersive.

Another special case of interest is when $\underline{\underline{t}}$ is scalar, i.e. $\overline{\underline{\underline{t}}}(\underline{x},\underline{x}') = \overline{t}(\underline{x},\underline{x}')\underline{\underline{I}}$. This occurs when the scatterers are spherical. Then the permittivity is isotropic but inhomogeneous and spatially dispersive.

The last special case to be treated is when the wavelength is large compared to the size of a scatterer. Here the particle can be treated as an electric dipole. Its equivalent source distribution is given by

$$\underline{g}_{eq} = i\omega\mu_0\underline{J}_{eq} = i\omega\mu_0(-i\omega\underline{p}\delta(\underline{x})) \tag{56}$$

where ρ is the electric dipole moment of the scatterer and $\delta(\underline{x})$ is the Dirac delta function. The dipole moment is related to the incident field $\underline{e}_0$ by the polarizability tensor $\underline{\underline{\alpha}}$ [Jones, 1964]

$$\underline{p} = \varepsilon_0\ \underline{\underline{\alpha}}\cdot\underline{e}_0 \tag{57}$$

Using (57) in (56) and comparing it with (14), we find

$$\underline{\underline{T}} = k_0^2\underline{\underline{\alpha}}\delta(\underline{x}) \tag{58}$$

or

$$\underline{\underline{t}}(\underline{x},\underline{x}') = k_0^2\underline{\underline{\alpha}}\delta(\underline{x})\delta(\underline{x}') \tag{59}$$

Now putting (59) in (19), we obtain

$$\underline{\underline{\tilde{t}}}(\underline{k},\underline{k}') = k_0^2\underline{\underline{\alpha}}/(2\pi)^3 \tag{60}$$

Thus we see in the dipole limit $\tilde{\underline{\underline{t}}}$ is independent of $\underline{k}$ and $\underline{k}'$.

Since we have an expression for $\underline{\underline{t}}$ in the low frequency or dipole limit, the special form of $\underline{\underline{\varepsilon}}$ can be easily obtained. Using (59) in (54), we find

$$\underline{\underline{\varepsilon}} = \underline{\underline{I}} + \rho(\underline{x})\overline{\underline{\underline{\alpha}}} \tag{61}$$

Thus in the low frequency case, the permittivity is no longer spatially dispersive however it is still anisotropic and inhomogeneous.

CORRELATION

In this section we will calculate the correlation of the electric field. Rather than following procedures used to find the coherent wave, the distorted Born approximation will be employed. This is a single scattering approximation where the scatterers are assumed to be embedded in the equivalent medium which has been found in the previous section. The method is useful when the fractional volume is small ($NV_p/V \ll 1$) and the albedo of a single particle is small. The later condition implies that the energy absorbed by a particle must be much larger than the energy scattered by it.

We start by considering a volume V of equivalent medium surrounded by free space. There are N particles embedded in V as shown in Figure 1. The scattered field due to the i^{th} particle can be calculated by modifing (39). We assume that the incident field on the particle is the mean field $\langle \underline{E} \rangle$ and that the free space operator $\underline{\underline{L}}$ is replaced by the equivalent medium operator $\underline{\underline{\mathcal{L}}}$ as given in (48). We have

$$\underline{E}_s = \sum_{i=1}^{N} \underline{E}_s^{(i)} = \sum_{i=1}^{N} \underline{\underline{\mathcal{L}}}^{-1} \cdot \underline{\underline{T}}_i \cdot \langle \underline{E} \rangle \quad . \tag{62}$$

Before proceeding, we point out that our main interest in finding the correlation of the field is to use it to calculate the backscattering cross section. Since this cross section is related to the correlation of the field fluctuations, we now define

$$\underline{E}_f = \underline{E}_s - \langle \underline{E}_s \rangle \quad , \qquad \langle \underline{E}_f \rangle = 0 \tag{63}$$

Now computing the correlation of the fluctuating field, we obtain

$$<\underline{E}_f(\underline{x})\underline{E}_f^*(\hat{\underline{x}})> = <\underline{E}_s(\underline{x})\underline{E}_s^*(\hat{\underline{x}})> - <\underline{E}_s(\underline{x})><\underline{E}_s^*(\hat{\underline{x}})> \qquad (64)$$

where $\underline{z}^*$ is the conjugate of z. Putting (62) in (64) and noting that a portion of $<\underline{E}_s\underline{E}_s^*>$ cancels with $<\underline{E}_s><\underline{E}_s^*>$ if we use the fact that N>>1. We find

$$<\underline{E}_f(\underline{x})\underline{E}_f^*(\hat{\underline{x}})> = \int_{4\pi} d\underline{\omega}\; p_{\underline{\Omega}}(\underline{\omega}) <\underline{E}_f(\underline{x})\underline{E}_f^*(\hat{\underline{x}})>_{\underline{\omega}} \qquad (65)$$

where

$$<\underline{E}_f(\underline{x})\underline{E}_f^*(\hat{\underline{x}})>_{\underline{\omega}} = \int_V d\underline{s}\; \rho(\underline{s})\; \underline{\mathcal{E}}(\underline{x},\underline{s})\underline{\mathcal{E}}^*(\hat{\underline{x}},\underline{s}) \qquad (66)$$

with

$$\underline{\mathcal{E}}(\underline{x},\underline{s}) = \underline{\underline{\mathcal{L}}}^{-1}\cdot\underline{\underline{T}}(\underline{s})\cdot<\underline{E}> \qquad (67)$$

Here we have separated the average into rotation and coordinate space averages, thus introducing the conditional expectation, $<\underline{E}_f\underline{E}_f^*>_{\underline{\omega}}$, with respect to given $\underline{\omega}$.

To write (67) more explicitly, we introduce the dyadic Green's function $\underline{\underline{G}}(\underline{x},\underline{x}')$ for the operator $\underline{\underline{\mathcal{L}}}$. It satisfies

$$\underline{\underline{\mathcal{L}}}\cdot\underline{\underline{G}}(\underline{x},\underline{x}') = \underline{\underline{I}}\delta(\underline{x}-\underline{x}') \qquad (68)$$

$$+ \quad \underline{\underline{G}} - \text{outgoing as } |\underline{x}|\to\infty$$

where $\underline{\underline{\mathcal{L}}}$ is given in (48). Now (67) becomes

$$\underline{\mathcal{E}}(\underline{x},\underline{s}) = \int d\underline{x}'\underline{\underline{G}}(\underline{x},\underline{x}')\cdot\int d\underline{x}''\underline{\underline{t}}(\underline{x}'-\underline{s},\underline{x}''-\underline{s})\cdot<\underline{E}(\underline{x}'')> \qquad (69)$$

The expression simplifies greatly in the low frequency limit. Assuming that $\underline{\underline{t}}$ is given by (59) and using this in (69) gives

$$\underline{\mathcal{E}}(\underline{x},\underline{s}) = k_0^2\underline{\underline{G}}(\underline{x},\underline{s})\cdot\underline{\underline{\alpha}}\cdot<\underline{E}(\underline{s})> \qquad (70)$$

BACKSCATTERING COEFFICIENTS FOR A HALF SPACE OF DIPOLES

To illustrate the application of the methods developed in the previous sections, we will calculate the backscattering coefficients from a half space of scatterers that are small compared to wavelength. We will also assume that the density of scatterers ρ is constant. The physical configuration is shown in Figure 2. There, we have shown the direction of the incident wave and the polarization vectors $\underline{h}°$ and $\underline{v}°$ representing horizontal and vertical polarizations respectively.

To compute the scattered field using the distorted Born approximation, we must first calculate the mean field in the half space containing the particles. In the low frequency approximation the mean wave is computed by replacing the particles with an equivalent medium having relative permittivity tensor $\underline{\underline{\varepsilon}}=\underline{\underline{I}}+\rho\ \overline{\underline{\underline{\alpha}}}$ and free space permeability μ_0. The usual continuity conditions associated with macroscopic Maxwell's equations are assumed to hold at the interface z=0.

Before proceeding we would like to emphasize that the scatterers are sparsely distributed or that the fractional volume they occupy is small ($NV_p/V<<1$) - a condition necessasary for the validity of the mean equation. This restriction is reflected in the equivalent permittivity tensor. Small fractional volume requires that $|\rho\overline{\alpha}_{ij}|<<1$ where $\overline{\alpha}_{ij}$ are the components of $\underline{\underline{\alpha}}$. We can exhibit the dependence of this condition on the fractional volume $\delta\equiv NV_p/V=\rho V_p$ explicitly by introducing a normalized polarizability tensor $\underline{\underline{a}}$ as follows:

$$\underline{\underline{a}} = \underline{\underline{\alpha}}/V_p \tag{71}$$

where we can show that the components of $\underline{\underline{a}}$ remain bounded as $V_p \to 0$. Now the permittivity can be written as

$$\underline{\underline{\varepsilon}} \simeq \underline{\underline{I}} + \delta \overline{\underline{\underline{a}}} \tag{72}$$

and thus we have a small parameter for ordering purposes.

Although we are able to carry out the calcualtion of the mean wave for an arbitrary average polarizability tensor $\overline{\underline{\underline{a}}}$, it is convenient to partially specify the angular probability density, $p_{\underline{\Omega}}(\underline{\omega})$ in order to make $\overline{\underline{\underline{a}}}$ diagonal. First we choose a spherical coordinate system of mutually orthogonal unit vectors $\underline{r}_0$, $\underline{\theta}_0$ and $\underline{\phi}_0$. The position of these vectors is completely determined by the spherical angles θ and ϕ as shown in Figure 3. Now we align the principal axes of the scatterer along these unit vectors. Then we write

$$\underline{\underline{a}} = a_r \underline{r}^\circ \underline{r}^\circ + a_\theta \underline{\theta}^\circ \underline{\theta}^\circ + a_\phi \underline{\phi}^\circ \underline{\phi}^\circ \tag{73}$$

By using the usual transformation between spherical and cartesian coordinates, (73) becomes

$$\underline{\underline{a}} = \sum_{i=1}^{3} \sum_{j=1}^{3} a_{x_i x_j}(\theta,\phi) \underline{x}_i^\circ \underline{x}_j^\circ \tag{74}$$

where $x_1 = x$, $x_2 = y$ and $x_3 = z$ and $\underline{x}^\circ, \underline{y}^\circ$ and $\underline{z}^\circ$ are cartesian unit vectors. The relationship between the $a_{x_i x_j}$ and a_r, a_θ, a_ϕ are given in Appendix B.

Now assuming that the random variables Θ_i and Φ_i are independent, we write

$$p_{\underline{\Omega}}(\underline{\omega}) = p_\Theta(\theta) p_\Phi(\phi) \tag{75}$$

Averaging (74), we have

$$\overline{\underline{\underline{a}}} = \sum_{i=1}^{3} \sum_{j=1}^{3} \overline{a}_{x_i x_j} \underline{x}_i^\circ \underline{x}_j^\circ \tag{76}$$

where

$$\bar{a}_{x_i x_i} = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \; p_\Theta(\theta) p_\Phi(\phi) a_{x_i x_j}(\theta,\phi) \quad (77)$$

We will now assume that the particles are distributed uniformily in the ϕ variable, i.e., $p_\Phi(\phi)=1/2\pi$. We find

$$\bar{\bar{a}} = \bar{a}_{xx}\underline{x}^\circ\underline{x}^\circ + \bar{a}_{yy}\underline{y}^\circ\underline{y}^\circ + \bar{a}_{zz}\underline{z}^\circ\underline{z}^\circ \quad (78)$$

where

$$\bar{a}_{xx} = \bar{a}_{yy} = \frac{1}{2}[a_r \overline{\sin^2\theta} + a_\theta \overline{\cos^2\theta} + a_\phi] \quad (79)$$

$$\bar{a}_{zz} = a_r \overline{\cos^2\theta} + a_\theta \overline{\sin^2\theta} \quad (80)$$

Thus all off diagonal terms averaged to zero and two of the on diagonal terms are equal. Using this in (72), we see that the equivalent dielectric is uniaxial.

The mean wave in the equivalent medium will now be found for the case of particles uniformly distributed in the azimuth coordinate ϕ. The incident wave is given

$$\underline{E}_0(\underline{x},q) = \underline{q}^\circ e^{i\underline{k}\cdot\underline{x}} \qquad q\varepsilon\{h,v\} \quad (81)$$

where

$$\underline{k} = \underline{k}_{t_0} + k_{z_0}\underline{z}^\circ \quad , \quad (82)$$

with

$$\underline{k}_{t_0} = k_0 \sin\theta_0 \underline{x}^\circ \quad , \quad k_{z_0} = k_0\cos\theta_0 \quad (83)$$

and the polarization vectors are

$$\underline{h}^\circ = \underline{y}^\circ \quad , \quad \underline{v}^\circ = -\cos\theta_0\underline{x}^\circ + \sin\theta_0\underline{z}^\circ \quad (84)$$

The average electric field in the equivalent half space satisfies

$$[\nabla x(\nabla x\underline{\underline{I}}) - k_0^2(\underline{\underline{I}}+\delta\bar{\bar{a}})]\cdot\langle\underline{E}(\underline{x})\rangle = 0 \quad , \qquad z<0 \quad (85)$$

Let us assume a plane wave solution of the form

$$\langle \underline{E}(\underline{x}) \rangle = \underline{A}\, e^{i\underline{\kappa}\cdot\underline{x}} \tag{86}$$

where $\underline{\kappa} = \underline{\kappa}_t + \kappa_z \underline{z}^\circ$. In order to match fields at the interface, the transverse phase velocity of the incident and transmitted waves must be the same, thus $\underline{\kappa}_t = \underline{k}_t = k_0 \sin\theta_0 \underline{x}^\circ$. Putting (86) in (85), we have

$$\underline{\kappa}\times(\underline{\kappa}\times\underline{A}) + k_0^2\underline{A} + \delta k_0^2 \underline{\underline{\bar{a}}}\cdot\underline{A} = 0 \tag{87}$$

Representing $\underline{A}$ in cartesian components (87) can be written as

$$\begin{bmatrix} \kappa_z^2 - k_0^2\beta_{xx} & 0 & k_0\sin\theta_0\kappa_z \\ 0 & \kappa_z^2 - k_0^2\beta_{yy} & 0 \\ k_0\sin\theta_0\kappa_z & 0 & -k_0^2\beta_{zz} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = 0 \tag{88}$$

where

$$\beta_{xx} = 1 + \delta\bar{a}_{xx}, \quad \beta_{yy} = \cos^2\theta_0 + \delta\bar{a}_{yy}, \quad \beta_{zz} = \cos^2\theta_0 + \delta\bar{a}_{zz} \tag{89}$$

Since (88) is a homogeneous system, the determinant of coefficients must be zero for a solution to exist. This condition determines the allowable values of κ_z. We find

$$\kappa_z = \pm\kappa_z^{(h)} = \pm k_0\sqrt{\beta_{yy}} \tag{90}$$

$$\kappa_z = \pm\kappa_z^{(v)} = \pm k_0\left[\frac{\beta_{xx}\beta_{zz}}{\sin^2\theta_0 + \beta_{zz}}\right]^{1/2} \tag{91}$$

where the superscripts h and v have been used to designate the propagation constants associated with horizontal and vertical polarizations. For an incident wave that is not grazing, i.e. $\theta_0 \neq \pi/2$, expressions (90) and (91) can be simplified using the small δ parameter. We have

$$\kappa_z^{(h)} = k_0\left(\cos\theta_0 + \frac{\delta\bar{a}_{yy}}{2\cos\theta_0}\right) + \theta(\delta^2) \quad (92)$$

$$\kappa_z^{(v)} = k_0\left[\cos\theta_0 + \frac{\delta}{2}\left(\frac{\bar{a}_{zz}+\cos^2\theta_0(\bar{a}_{xx}-\bar{a}_{zz})}{\cos\theta_0}\right)\right] + 0(\delta^2) \quad (93)$$

We cannot calculate these propagation constants to higher accuracies than $0(\delta)$ since the original mean equation has only been found to this accuracy. We note that since we are considering lossy particles, the $\kappa_z^{(q)}$ are complex and thus the mean wave will decay away from the interface.

Next we calculate the amplitude coefficients for the mean wave of both polarizations. We have

$$<\underline{E}(\underline{x},h)> = \begin{cases} (e^{ik_{z_0}z}+\Gamma_h e^{-ik_{z_0}z})\, e^{i\underline{k}_{t_0}\cdot\underline{x}}\, \underline{y}^\circ , & z<0 \\ A_y e^{i\underline{\kappa}^{(h)}\cdot\underline{x}}\, \underline{y}^\circ , & z>0 \end{cases} \quad (94)$$

and

$$<\underline{E}(\underline{x},v)> = \begin{cases} (e^{ik_{z_0}z}+\Gamma_v e^{-ik_{z_0}z})\, e^{i\underline{k}_{t_0}\cdot\underline{x}}\, \underline{v}^\circ , & z<0 \\ (A_x\underline{x}^\circ+A_z\underline{z}^\circ)\, e^{i\underline{\kappa}^{(v)}\cdot\underline{x}} , & z>0 \end{cases} \quad (95)$$

where we have introduced a reflected wave in the free space medium at the specular angle. Now by using the fact that the tangential $<\underline{E}>$ and $<\underline{H}>$ must be continuous at the interface, the unknown reflection and transmission coefficients can be calculated. Since the major effect of the equivalent medium is to produce exponential decay, we next expand the coefficient for small δ and we keep only zero order terms. We find that $\Gamma_q=0(\delta)$ and thus it can be neglected. The transmitted mean fields are:

$$\langle \underline{E}(\underline{x},q)\rangle = \underline{q}^{\circ} e^{i(\kappa_z^{(q)} z+\underline{k}_{t_0}\cdot\underline{x})} + 0(\delta) \ , \quad z>0 \quad q\varepsilon\{h,v\} \tag{96}$$

Proceeding with our developemnt, we now relate the transverse Fourier transform of the correlation to the backscattering coefficients. This is done within the context of the distorted Born approximation developed in the previous section. We start by taking the transverse Fourier transform with respect to $\underline{x}$ and $\hat{\underline{x}}$ of (65), (66) and (70). We have

$$\langle \tilde{\underline{E}}_f(\underline{k}_t,z)\tilde{\underline{E}}_f^*(\hat{\underline{k}}_t,\hat{z})\rangle = \int_{4\pi} d\underline{\omega}\ p_{\underline{\Omega}}(\underline{\omega})\langle \tilde{\underline{E}}_f(\underline{k}_t,z)\tilde{\underline{E}}_f^*(\hat{\underline{k}}_t,\hat{z})\rangle_{\underline{\omega}} \tag{97}$$

$$\langle \tilde{\underline{E}}_f(\underline{k}_t,z)\tilde{\underline{E}}_f^*(\hat{\underline{k}}_t,\hat{z})\rangle_{\underline{\omega}} = \rho\int_V d\underline{s}\ \tilde{\underline{\mathcal{E}}}(\underline{k}_t,z,\underline{s})\cdot\tilde{\underline{\mathcal{E}}}^*(\hat{\underline{k}}_t,\hat{z},\underline{s}) \tag{98}$$

where

$$\tilde{\underline{\mathcal{E}}}(\underline{k}_t,z,\underline{s}) = k_0^2\tilde{\underline{\underline{G}}}(\underline{k}_t,z,\underline{s})\cdot\underline{\underline{\alpha}}\cdot\langle\underline{E}(\underline{s})\rangle \tag{99}$$

Since we will only require $\underline{\underline{G}}$ when $\underline{x}$ and $\underline{s}$ are in the equivalent medium and since the reflection at the interface is small we can replace $\underline{\underline{G}}$ by the dyadic Green's function for an infinite equivalent medium, i.e.,

$$\underline{\underline{G}}(\underline{x},\underline{s}) = \underline{\underline{G}}^{(\infty)}(\underline{x}-\underline{s}) + 0(\delta) \tag{100}$$

We have written the infinite space Green's function in terms of $\underline{x}-\underline{s}$ since it is translationally invariant. We then have

$$\tilde{\underline{\underline{G}}}^{(\infty)}(\underline{k}_t,z,\underline{s}) = \tilde{\underline{\underline{G}}}^{(\infty)}(\underline{k}_t,z-s)e^{-i\underline{k}_t\cdot\underline{s}} \tag{101}$$

Now by putting (96), (99), (100) and (101) in (98), by integrating over $\underline{s}_t$ and by setting $z=\hat{z}=0$, we have

$$\langle \tilde{\underline{E}}_f(\underline{k}_t,0)\tilde{\underline{E}}_f^*(\hat{\underline{k}}_t,0)\rangle_{\underline{\omega}} = \underline{\underline{S}}(\underline{k}_t,q|\underline{\omega})\,\delta(\underline{k}_t-\hat{\underline{k}}_t) \tag{102}$$

where

$$\underline{\underline{S}}(\underline{k}_t,q|\underline{\omega}) = (2\pi)^2 \delta k_0^4 V_p \int_0^\infty ds (\tilde{\underline{\underline{G}}}^{(\infty)}(k_t,-s)\cdot\underline{\underline{a}}\cdot\underline{q}^\circ)(\tilde{\underline{\underline{G}}}^{(\infty)*}(k_t,-s)$$

$$\cdot\underline{\underline{a}}^*\cdot\underline{q}^\circ)\,.e^{-2\mathrm{Im}\kappa_z^{(q)} s} \tag{103}$$

Here $\underline{\underline{S}}(\underline{k}_t,q|\underline{\omega})$ is the transverse dyadic spectral density at the interface assuming $\underline{\omega}$ is fixed. The normalized polarizability has been introduced by using (71).

By using the results of Appendix C and by noting that $\overline{\underline{\underline{S}}(\underline{k}_t,q|\underline{\omega})}$ we obtain the backscattering coefficients

$$\sigma^\circ_{pq} = \frac{k_0^2\cos^2\theta_0}{4\pi^3}\,\underline{p}^\circ\cdot\overline{\underline{\underline{S}}(-\underline{k}_{t_0},q|\omega)}\cdot\underline{p}^\circ \,, \qquad p,q\varepsilon\{h,v\} \tag{104}$$

To evaluate the integral of $\underline{\underline{S}}$ in (103), we will need the transformed Green's function. To obtain it, we first write the governing equation for $\underline{\underline{G}}^{(\infty)}$. It is

$$[\nabla\mathrm{x}(\nabla\mathrm{x}\underline{\underline{I}}) - k_0^2(\underline{\underline{I}}+\delta\underline{\underline{a}})]\cdot\underline{\underline{G}}^{(\infty)}(\underline{x}) = \underline{\underline{I}}\delta(\underline{x}) \,, \tag{105}$$

$$\underline{\underline{G}}^{(\infty)}(\underline{x}) \text{ - outgoing as } |\underline{x}|\to\infty$$

Using

$$\underline{\underline{G}}^{(\infty)}(\underline{x}) = \frac{1}{(2\pi)^3}\int d\underline{\kappa}\;\underline{\underline{g}}(\underline{\kappa})e^{i\underline{\kappa}\cdot\underline{x}} \tag{106}$$

in (105), we find

$$[(\underline{\kappa}\mathrm{x}\underline{\kappa}\mathrm{x}\underline{\underline{I}}) + k_0^2(\underline{\underline{I}}+\delta\underline{\underline{a}})]\cdot g(\underline{\kappa}) = -\underline{\underline{I}} \tag{107}$$

Then

$$\tilde{\underline{\underline{G}}}^{(\infty)}(\underline{\kappa}_t,z) = \frac{1}{2\pi}\int d\kappa_z\;\underline{\underline{g}}(\underline{\kappa})e^{i\underline{\kappa}\cdot\underline{x}} \tag{108}$$

To simplify the remaining computation for $\tilde{\underline{\underline{G}}}^{(\infty)}$ we note from (104) and (103) that $\tilde{\underline{\underline{G}}}^{(\infty)}$ will only be required for $\underline{\kappa}_t=-\underline{k}_{t_0} = -k_0\sin\theta_0\underline{x}^\circ$.

Inverting (107) and performing the integral in (107) by the method of residues, we obtain

$$\tilde{\underline{\underline{G}}}^{(\infty)}(-\underline{k}_{t_0},z) = \left\{\beta_{zz}\underline{x}^\circ\underline{x}^\circ + \frac{\sin\theta_0\kappa_z^{(v)}\sigma(z)}{k_0}(\underline{x}^\circ\underline{z}^\circ+\underline{z}^\circ\underline{x}^\circ)\right.$$

$$\left. - \frac{(\kappa_z^{(v)\,2}-k_0^2\beta_{xx})}{k_0^2}\underline{z}^\circ\underline{z}^\circ\right\}\frac{e^{i\kappa_z^{(v)}|z|}}{2i\kappa_z^{(v)}} + \frac{e^{i\kappa_z^{(h)}|z|}}{2i\kappa_z^{(h)}}\underline{y}^\circ\underline{y}^\circ \quad (109)$$

where

$$\sigma(z) = \begin{cases} 1\,, & z>0 \\ -1\,, & z<0 \end{cases} \quad (110)$$

and the β's are defined in (89). Approximating the coefficients to zeroth order in δ, we have the simplified expression

$$\tilde{\underline{\underline{G}}}^{(\infty)}(-\underline{k}_{t_0},z) = \underline{v}^\circ\underline{v}^\circ\frac{e^{i\kappa_z^{(v)}|z|}}{2ik_0\cos\theta_0} + \underline{h}^\circ\underline{h}^\circ\frac{e^{i\kappa_z^{(h)}|z|}}{2ik_0\cos\theta_0} \quad (111)$$

If we use this in (103), perform the integration and use the result in (104), we have our final form for the backscattering coefficients. It is

$$\sigma^\circ_{pq} = \frac{\delta k_0^4 v_p \overline{|a_{pq}|^2}}{8\pi(\mathrm{Im}\kappa_z^{(p)} + \mathrm{Im}\kappa_z^{(q)})} \quad (112)$$

where

$$\overline{|a_{pq}|^2} = \int_{4\pi} d\underline{\omega}\,|\underline{p}^\circ\cdot\underline{\underline{a}}\cdot\underline{q}^\circ|^2 \quad . \quad (113)$$

and $\kappa_z^{(s)}$, $s\epsilon\{h,v\}$ are given in (92) and (93). The dependence of $\overline{|a_{pq}|^2}$ on angle of incidence is worked out explicitly in Appendix B for scatterers that are distributed uniformly in the ϕ coordinate.

The final result given in (112) can be expressed in terms of scattering cross sections of individual particles. Using (28), (60) and (71), we have

$$f_{pq} = \underline{p}^{\circ} \cdot \underline{\underline{f}} \cdot q^{\circ} = 2\pi^2 \, S_{pq} = k_0^2 V_p a_{pq} / 4\pi \tag{114}$$

Now by recalling that the backscattering cross section from a particle $\sigma_{pq}^{(b)} = 4\pi |f_{pq}|^2$, (112) becomes

$$\sigma^{\circ}_{pq} = \frac{\rho\sigma_{pq}^{(b)}}{2\mathrm{Im}\kappa_z^{(p)} + 2\mathrm{Im}\kappa_z^{(q)}} \; , \qquad p,q \varepsilon \{h,v\} \tag{115}$$

Following Attema and Ulaby [1978] we can give a one dimensional interpretation of (115). If we rewrite (115) as

$$\sigma^{\circ}_{pq} = \int_{-\infty}^{0} dz \; \rho\sigma_{pq}^{(b)} e^{-2\mathrm{Im}\kappa_z^{(q)} |z|} e^{-2\mathrm{Im}\kappa_z^{(p)} |z|} \tag{116}$$

we can view the scattering as being decomposed into scattering from slabs of width dz. An intensity of $\exp(-2\mathrm{Im}\kappa_z^{(q)} |z|)$ is incident on the slab located at z. The incident intensity is backscattered with reflectivity factor $\rho\sigma_{pq}^{(b)}$. The backscattered wave then decays as $\exp(-2\mathrm{Im}\kappa_z^{(q)} |z|)$ until it reaches the interface.

DISCUSSION AND NUMERICAL EVALUATION

In this section, we will first discuss several general properties of σ°_{pq} that are independent of the particular scatterer chosen. Following this discussion, we use our method to model a forest canopy by a collection of lossy dielectric discs. The theoretical curves computed from this model are then compared with some experimental data.

Because of the simple dependence of σ°_{pq} on the medium properties and incidence angle certain general observations can be made that are independent of the particular nature of the scatterer. First, we note that σ°_{pq} as given by (115) is independent of the density of scatterers ρ. This follows directly from (92) and (93) where we see that the $\mathrm{Im}\,\kappa_z^{(s)}$, $s\varepsilon\{h,v\}$ are directly proportional to ρ. Thus the linear ρ dependence in the numerator of (115) is cancelled out by the denominator. Second, we note that $\sigma^\circ_{hh} \propto \cos\theta_0$. This is the same result as predicted by the scalar theory. Finally, we note that $\sigma^\circ_{hh}=\sigma^\circ_{vv}$ at normal incidence ($\theta^\circ=0$). This is an expected result. Since the scatterers are uniformly distributed in ϕ, the two polarizations see the same medium at normal incidence.

We now proceed to model a forest canopy by a collection of leaves. The leaves are in turn assumed to be lossy dielectric discs as mentioned previously. The discs have radius a and thickness h. Typical dimensions are radii of one to several centimeters and thicknesses of tenths of a millimeter. The electrical properties of discs can be characterized by their normalized polarizability tensor $\underline{\underline{a}}$ when the wavelength is large compared to the disc. From Jones [1964] and

Van de Hulst [1957] the polarizability of a disc along its principal axes is given by

$$a_r = \frac{\alpha_r}{V_p} = \frac{\Delta}{1+\Delta} \quad , \quad a_\theta = a_\phi = \Delta \quad , \quad \Delta = \varepsilon_r - 1 \tag{117}$$

when r, θ, ϕ are defined in Figure 3.

Because of the large volume of water present in vegetation, we can usually assume $|\varepsilon_r| >> 1$ in the microwave region. Using this assumption in (117), we find that $|a_\theta| = |a_\phi| >> |a_r|$. This inequality can now be used to simplify the scattering cross section of (115). We find that $\sigma^0_{pq} \propto |\varepsilon_r|^2 V_p / \mathrm{Im}\varepsilon_r$. Thus it follows that the magnitude of the backscattering cross sections are directly related to the volume and complex dielectric constant of the discs in a simple manner. Therefore, as leaves grow and as their moisture content changes these effects should be observable by measuring σ°_{pq} at different periods of the growing season.

Before computing the backscattering cross sections as a function of incidence angle, we will require the relative dielectric constant of the leaves and the angular distribution of leaves. First the relative dielectric constant is considered Our calculation follows that of Fung and Ulaby [1978] who in turn have based his results upon de Loor [1968] and Carlson [1967]. They model the leaves as a mixture of water and solid materials. For illustrative purposes we have chosen 50% water and 50% solid for our calculations. By using (3) and (4) of Fung and Ulaby [1978] at a frequency of 1.1 GHz we find that $\varepsilon_r = 30.8 + i1.8$. Our choice of frequency has been motivated by experimental results that appear in the literature. We have chosen to compare our results with those of Bush, et al [1976] who has measured σ°_{pq}, $p,q \epsilon \{h,v\}$ from forests for frequencies 1-18 GHz. Because of the dipole approximation made in our model, only the lowest frequency (1.1GHz) Ulaby measured was used for comparison purposes.

The angular distribution of leaves will now be considered. Field measurements of leave orientations have been made by Smith [1973] and others. It has been found that the leaves are distributed uniformly in the Φ coordinate (Figure 2). The distribution of leaves with respect to Θ is more vegetation type dependent. Several are given by Smith [1977]. Since no measurements of this type exist for the Ulaby data, we have assumed that Φ is uniformly distributed. For Θ we have considered the following two density functions:

$$p_\Theta(\theta) = \begin{cases} \dfrac{1}{\Delta\theta_{/\!/}} \ , & 0 \leq \theta \leq \Delta\theta_{/\!/} \\ 0 \ , & \Delta\theta_{/\!/} \leq \theta \leq \pi \end{cases} \tag{118}$$

or

$$p_\Theta(\theta) = \begin{cases} \dfrac{1}{2\Delta\theta_\perp} \ , & \pi - \Delta\theta_\perp \leq \theta \leq \pi + \Delta\theta_\perp \\ 0 \ , & \text{elsewhere} \end{cases} \tag{119}$$

In (118) when $\Delta\theta_{/\!/}$ is small, the leaves are approximately parallel to the interface (z=0); when $\Delta\theta_{/\!/} = \pi/2$, they are uniformly distributed in Θ. In (119) when $\Delta\theta_\perp$ is small, the leaves are perpendicular to the interface; when $\Delta\theta_\perp$ is $\pi/2$ they are uniformly distributed in Θ.

The numerical calcualtions are presented in Figures 4-9. In these figures the backscattering coefficient is plotted as a function of the angle of incidence θ_0. In Figures 4-7 we have used the angular distribution given in (118) while in Figures 7-9 we have used (119). Figure 7 corresponds to a uniform distribution over all θ and thus, for this case, (118) and (119) give the same results. One should note that since $|\epsilon_r|>>1$, a change in f, V_p or ϵ_r just shifts the level of the curves but it does not change their shape. Their shape only depends on $p_\theta(\Theta)$.

The following trends are observed in Figures 4-7: First, σ°_{hh} is always greater than σ°_{vv}. Their difference increases as $\Delta\theta_{/\!/}$ becomes smaller. Second, the cross polarized back-scatter becomes smaller as $\Delta\theta_{/\!/}$ becomes smaller. In Figure 7-9, we observe that: First, σ°_{vv} becomes greater than σ°_{hh} as $\Delta\theta_\perp$ is decreased; second, the curve for σ°_{hv} tends to flatten out as $\Delta\theta_\perp$ is increased; and third, the difference $\sigma_{hh}-\sigma_{hv}$ at $\theta_0=0$ becomes smaller as $\Delta\theta_\perp$ increases.

A comparison of our theory with the experimental results of Bush, et.al [1976] is made in Figure 10. There, we have plotted our Figure 8 along with his data for Kansas deciduous trees measured in the springtime at a frequency of 1.1 GHz. Figure 8 was chosen since it most clearly appears to follow the trends of the data i.e., flat cross polarization and $\sigma^\circ_{vv}>\sigma^\circ_{hh}$.

Although our theory follows the trends of the data, it is clear from the results that additions to our model should be made. The fact that $\sigma^\circ_{vv}>\sigma^\circ_{hh}$ is most likely due to the vertically oriented tree branches other than due to the leaves that tend to be parallel to the interface. In addition, an examination of the numerical results shows that the skin depth for the mean wave is large. Thus at a frequency of 1 GHz the underlying ground should be taken into account.

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APPENDIX A

Relationship Between the Transition Operator and the Scattering Amplitude

To find the relationship between $\underline{\underline{f}}$ and $\tilde{\underline{\underline{t}}}$, we start with (15). Using (23) and (26), we can write (15) in terms of dyadic incident and scattered wave

$$\underline{\underline{e}}_s = \underline{\underline{L}}^{-1} \cdot \underline{\underline{T}} \cdot \underline{\underline{e}}_0 \tag{1A}$$

Next we use (16) in (1A) along with the free space dyadic Green's function $\underline{\underline{\Gamma}}$ for $\underline{\underline{L}}^{-1}$. We have

$$\underline{\underline{e}}_s(\underline{x},\underline{i}) = \int d\underline{x}' \underline{\underline{\Gamma}}(\underline{x},\underline{x}') \cdot \int d\underline{x}'' \, \underline{\underline{t}}(\underline{x}',\underline{x}'') \cdot \underline{\underline{e}}_0(\underline{x}'',\underline{i}) \tag{2A}$$

Here the free space dyadic Green's function is given by

$$\underline{\underline{\Gamma}}(\underline{x},\underline{x}') = (\underline{\underline{I}} + \frac{\nabla\nabla}{k_0^2}) \frac{e^{ik_0|\underline{x}-\underline{x}'|}}{4\pi|\underline{x}-\underline{x}'|} \tag{3A}$$

To obtain $\underline{\underline{e}}_s$ in the radiation zone, the far field expression for $\underline{\underline{\Gamma}}$ will be required. It is [Twersky, 1967]

$$\underline{\underline{\Gamma}}(\underline{x},\underline{x}') \sim (\underline{\underline{I}} - \underline{0}\,\underline{0}) e^{-ik_0 \underline{0} \cdot \underline{x}'} \frac{e^{ik_0|\underline{x}|}}{4\pi|\underline{x}|}, \qquad |\underline{x}| \to \infty \tag{4A}$$

Now putting (4A) and (25) in (2A)

$$\underline{\underline{e}}_s(\underline{x},\underline{i}) \sim (\underline{\underline{I}} - \underline{0}\,\underline{0}) \frac{1}{4\pi} \int d\underline{x}' d\underline{x}'' \underline{\underline{t}}(\underline{x}',\underline{x}'') e^{ik_0(\underline{i}\cdot\underline{x}'' - \underline{0}\cdot\underline{x}')}$$

$$\cdot (\underline{\underline{I}} - \underline{i}\,\underline{i}) \frac{e^{ik_0|\underline{x}|}}{|\underline{x}|} \tag{5A}$$

Finally employing (19) in (5A), and comparing with (27) we obtain the required result

$$\underline{\underline{f}}(\underline{0},\underline{i}) = 2\pi^2 (\underline{\underline{I}} - \underline{0}\;\underline{0}) \cdot \tilde{\underline{\underline{t}}}(k_0\underline{0}, k_0\underline{i}) \cdot (\underline{\underline{I}} - \underline{i}\;\underline{i}) \tag{6A}$$

APPENDIX B - Polarizability Statistics

In this appendix calculate the mean square polarization statistics used in (112). The calcualtion will be performed with the assumption that the scatterers are uniformly distributed in Φ.

First we derive the components of the polarizability tensor in cartesian coordinates in terms of the principal axis components. The unit vectors $\underline{r}°$, $\underline{\theta}°$ and $\underline{\phi}°$ are related to $x°$, $\underline{y}°$ and $\underline{z}°$ as follows:

$$\begin{aligned} \underline{r}° &= \sin\theta\cos\phi\underline{x}° + \sin\theta\sin\phi\underline{y}° + \cos\theta\underline{z}° \\ \underline{\theta}° &= \cos\theta\cos\phi\underline{x}° + \cos\theta\sin\phi\underline{y}° - \sin\theta\underline{z}° \\ \underline{\phi}° &= -\sin\phi\underline{x}° + \cos\phi\underline{y}° \end{aligned} \tag{1B}$$

Using this in (73), the cartesian components of (74) are:

$$\begin{aligned} a_{xx} &= (a_r\sin^2\theta + a_\theta\cos^2\theta)\cos^2\phi + a_\phi\sin^2\phi \\ a_{yy} &= (a_r\sin^2\theta + a_\theta\cos^2\theta)\sin^2\phi + a_\phi\cos^2\phi \\ a_{zz} &= a_r\cos^2\theta + a_\theta\sin^2\theta \\ a_{xy} &= [(a_r\sin^2\theta + a_\theta\cos^2\theta) - a_\phi]\cos\phi\sin\phi \\ a_{xz} &= (a_r-a_\theta)\sin\theta\cos\theta\cos\phi \\ a_{yz} &= (a_r-a_\theta)\sin\theta\cos\theta\sin\phi \end{aligned} \tag{2B}$$

The other components are gotton from the fact that $\underline{\underline{a}}$ is a symmetric dyadic.

Now we will obtain the components of $\underline{\underline{a}}$ in the polarization directions in terms of the cartesian components. We have using (84)

$$
\begin{aligned}
a_{hh} &= \underline{h}^\circ \cdot \underline{\underline{a}} \cdot \underline{h}^\circ = a_{yy} \\
a_{hv} &= \underline{h}^\circ \cdot \underline{\underline{a}} \cdot \underline{v}^\circ = -\cos\theta_0 a_{yx} + \sin\theta_0 a_{yz} \\
a_{vh} &= \underline{v}^\circ \cdot \underline{\underline{a}} \cdot \underline{h}^\circ = a_{hv} \\
a_{vv} &= \underline{v}^\circ \cdot \underline{\underline{a}} \cdot \underline{v}^\circ = \cos^2\theta_0 a_{xx} - 2\cos\theta_0 \sin\theta_0 a_{xz} + \sin^2\theta_0 a_{zz}
\end{aligned}
\tag{3B}
$$

Following this, the mean square polarizabilities can be found. They are

$$
\begin{aligned}
\overline{|a_{hh}|^2} &= \overline{|a_{yy}|^2} \\
\overline{|a_{hv}|^2} &= \cos^2\theta_0 \overline{|a_{yx}|^2} + \sin^2\theta_0 \overline{|a_{yz}|^2} \\
\overline{|a_{vh}|^2} &= \overline{|a_{hv}|^2} \\
\overline{|a_{vv}|^2} &= \cos^4\theta_0 \overline{|a_{xx}|^2} + \sin^4\theta_0 \overline{|a_{zz}|^2} + \cos^2\theta_0 \sin^2\theta_0 (4\overline{|a_{xz}|^2} \\
&\quad + 2R_e \overline{a_{xx}a_{zz}^*})
\end{aligned}
$$

where the uniformity in ϕ has lead to

$$
\overline{a_{xx}a_{xz}^*} = \overline{a_{yy}a_{yz}^*} = \overline{a_{yx}a_{yz}^*} = \overline{a_{zz}a_{xz}^*} = 0 \ .
$$

The quantities $\overline{|a_{xx}|^2}$, $\overline{|a_{yy}|^2}$, $\overline{|a_{zz}|^2}$, $\overline{|a_{yz}|^2}$, $\overline{|a_{xz}|^2}$ and $\overline{a_{xx}a_{zz}^*}$ can be obtained easily from (2B) by using the known statistics of θ and Φ.

APPENDIX C - Relationship Between the Backscattering Coefficients and the Transverse Spectral Density

We start by considering the fluctuating portion of the scattered field, $\underline{E}_f$, as defined in (63) in the region $z \geq 0$. This field can be viewed as arising from sources on the interface. So that far field quantities can be found, we initially consider that portion of $\underline{E}_f$ that arises from sources contained within a finite region A on the interface. The radiated field from the region A will be denoted by $\underline{E}_{f_A}(\underline{x},q)$.

The field $\underline{E}_{f_A}(\underline{x},q)$ can be related to the interface fluctuations by employing a plane wave expansion in the region $z \leq 0$. We have

$$\underline{E}_{f_A}(\underline{x},q) = \frac{1}{(2\pi)^2}\int d\underline{k}_t \int_A d\underline{x}_t' \, \underline{E}_f(0,\underline{x}_t',q)\, e^{-ik_z z + i\underline{k}_t(\underline{x}_t - \underline{x}_t')}, \quad z \leq 0 \tag{1C}$$

$$k_z = \sqrt{k_0^2 - |\underline{k}_t|^2}, \qquad \mathrm{Im}\,k_z \geq 0$$

where $\underline{E}_f(0,\underline{x}_t,q)$ is the fluctuating field on the interface due to an incident wave of polarization q. The $\underline{k}_t$ integral in (1C) can be asymptotically evaluated for large $|\underline{x}|$ [Collin and Zucker, 1969]. We find

$$\underline{E}_{f_A}(\underline{x},q) \sim \frac{+ik_0\cos\theta_0}{2\pi|\underline{x}|}\, e^{+ik_0|\underline{x}|}\int_A d\underline{x}_t' \, \underline{E}_f(0,\underline{x}_t',q)\, e^{i\underline{k}_{t_0}\cdot\underline{x}_t'} \tag{2C}$$

where $\underline{x}$ has been specialized to the backscatter direction θ_0 and $\underline{k}_{t_0}$ is given by (83).

The backscattering coefficients are now defined. They are

$$\sigma^{\circ}_{pq} = \lim_{A\to\infty}\lim_{|\underline{x}|\to\infty} \frac{4\pi|\underline{x}|^2 I_s(p,q)}{A I_i(q)} \qquad q,p\in\{h,v\} \tag{3C}$$

where $I_i(q)$ is the incident intensity per unit area with polarization q, $I_s(p,q)$ is the average intensity at the observation point with polarization p due to an incident wave with polarization q. In view of (81), $I_i(q)=1$. We also have

$$I_s(p,q) = \langle | E_{f_A}(\underline{x},q)\cdot\underline{p}^\circ |^2 \rangle \tag{4C}$$

We can complete the development by first representing the field on the interface by its Fourier transform, $\tilde{\underline{E}}_f(\underline{k}_t,q)$:

$$\underline{E}_f(0,\underline{x}_t,q) = \frac{1}{(2\pi)^2}\int \tilde{\underline{E}}_f(k_t',q)\,e^{i\underline{k}_t\cdot\underline{x}_t}d\underline{k}_t' \tag{5C}$$

Now by using (2C), (4C) and (5C) in (3C), we have

$$\sigma^\circ_{pq} = \lim_{A\to\infty}\lim_{|\underline{x}|\to\infty}\frac{k_0^2\cos^2\theta_0}{2^4\pi^5 A}\int d\underline{k}_t' d\underline{k}_t'' \int_{A\times A} d\underline{x}_t' d\underline{x}_t'' \langle |\underline{p}^\circ\cdot\tilde{\underline{E}}_f(\underline{k}_t',q)$$

$$\tilde{\underline{E}}_f^*(\underline{k}_t'',q)\cdot\underline{p}^\circ| \rangle e^{i[\underline{k}_t'\cdot\underline{x}_t'-\underline{k}_t''\cdot\underline{x}_t''+\underline{k}_{t0}\cdot(\underline{x}_t''-\underline{x}_t')]} \tag{6C}$$

Next we introduce the transverse dyadic spectral density $\underline{\underline{S}}(\underline{k}_t,q)$ which is given by

$$\langle \tilde{\underline{E}}_f(\underline{k}_t',q)\tilde{E}_f^*(\underline{k}_t'',q)\rangle = \underline{\underline{S}}(\underline{k}_t',q)\,\delta(\underline{k}_t'-\underline{k}_t'') \tag{7C}$$

Note that this definition requires that the process $\tilde{\underline{E}}_f(\underline{k}_t,q)$ be homogeneous or stationary in $\underline{k}_t$.

By using (7C) in (6C) and carrying out the integrations, we obtain

$$\sigma^\circ_{pq} = \frac{k_0^2\cos^2\theta_0}{4\pi^3}\,S_p(-\underline{k}_{t_0},q) \tag{8C}$$

where

$$S_p(\underline{k}_t,q) = \underline{p}^\circ\cdot\underline{\underline{S}}(\underline{k}_t,q)\cdot\underline{p}^\circ \tag{9C}$$

List of Figures

Figure 1

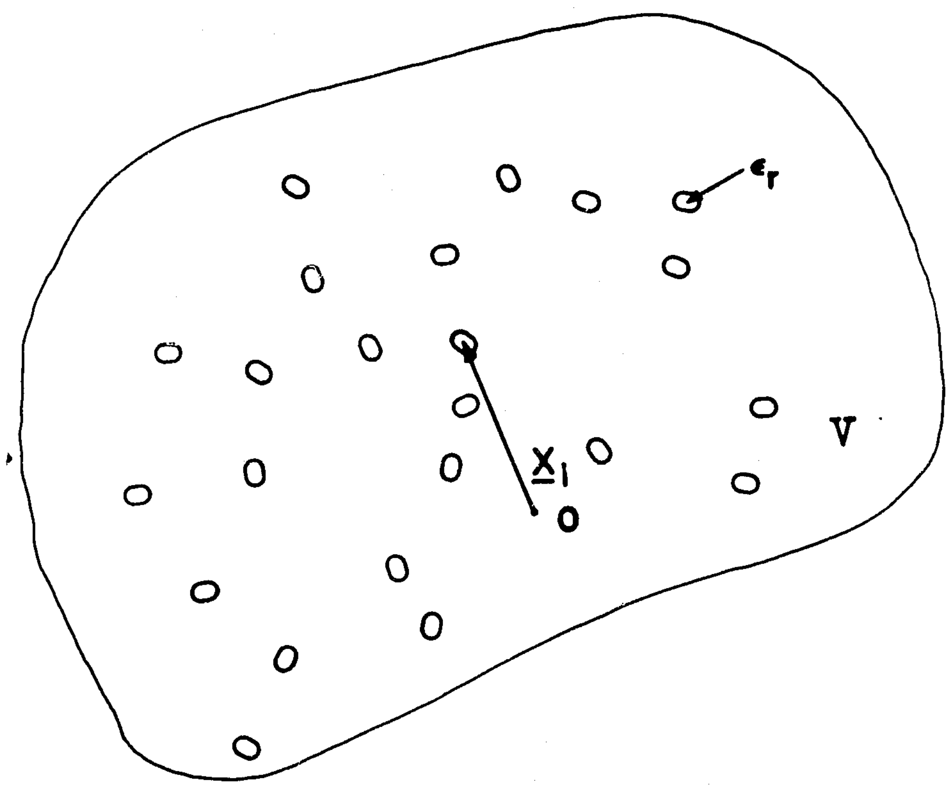


Figure 2

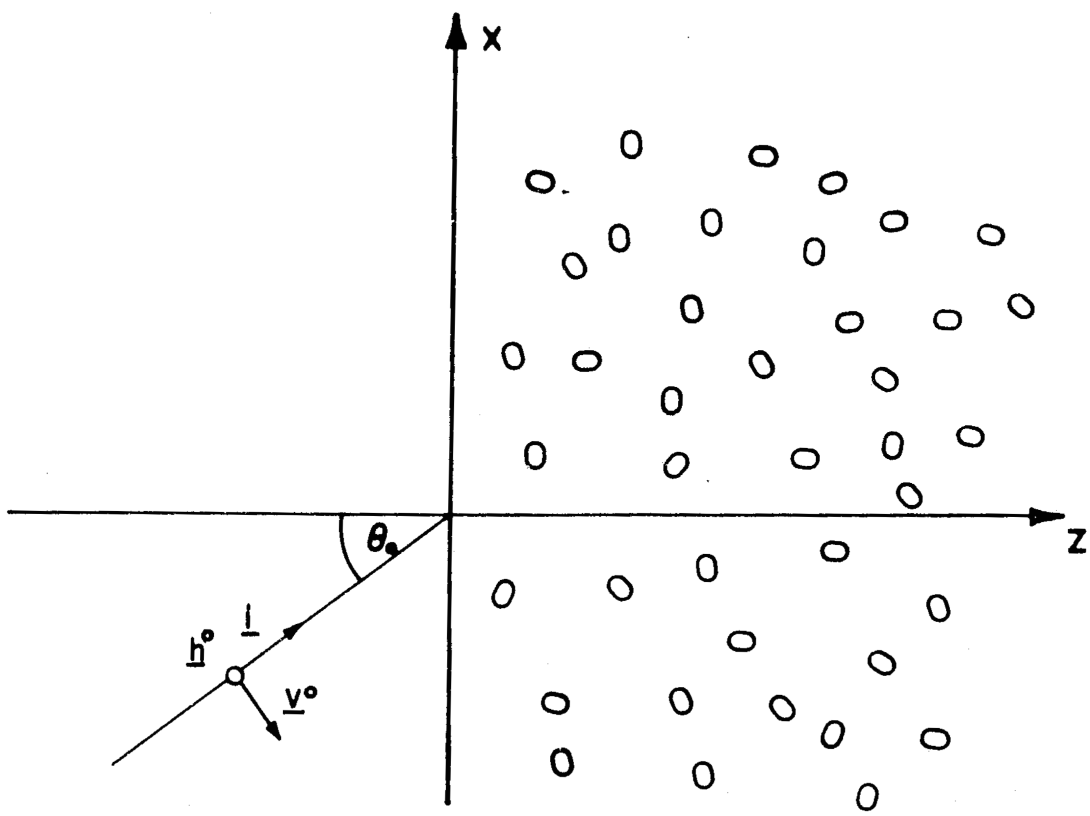
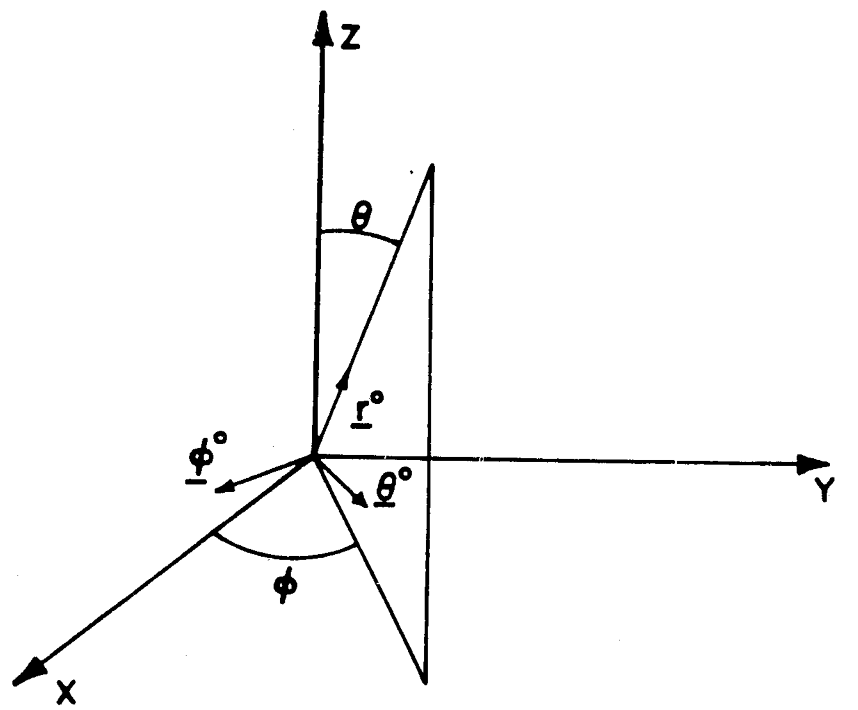


Figure 3



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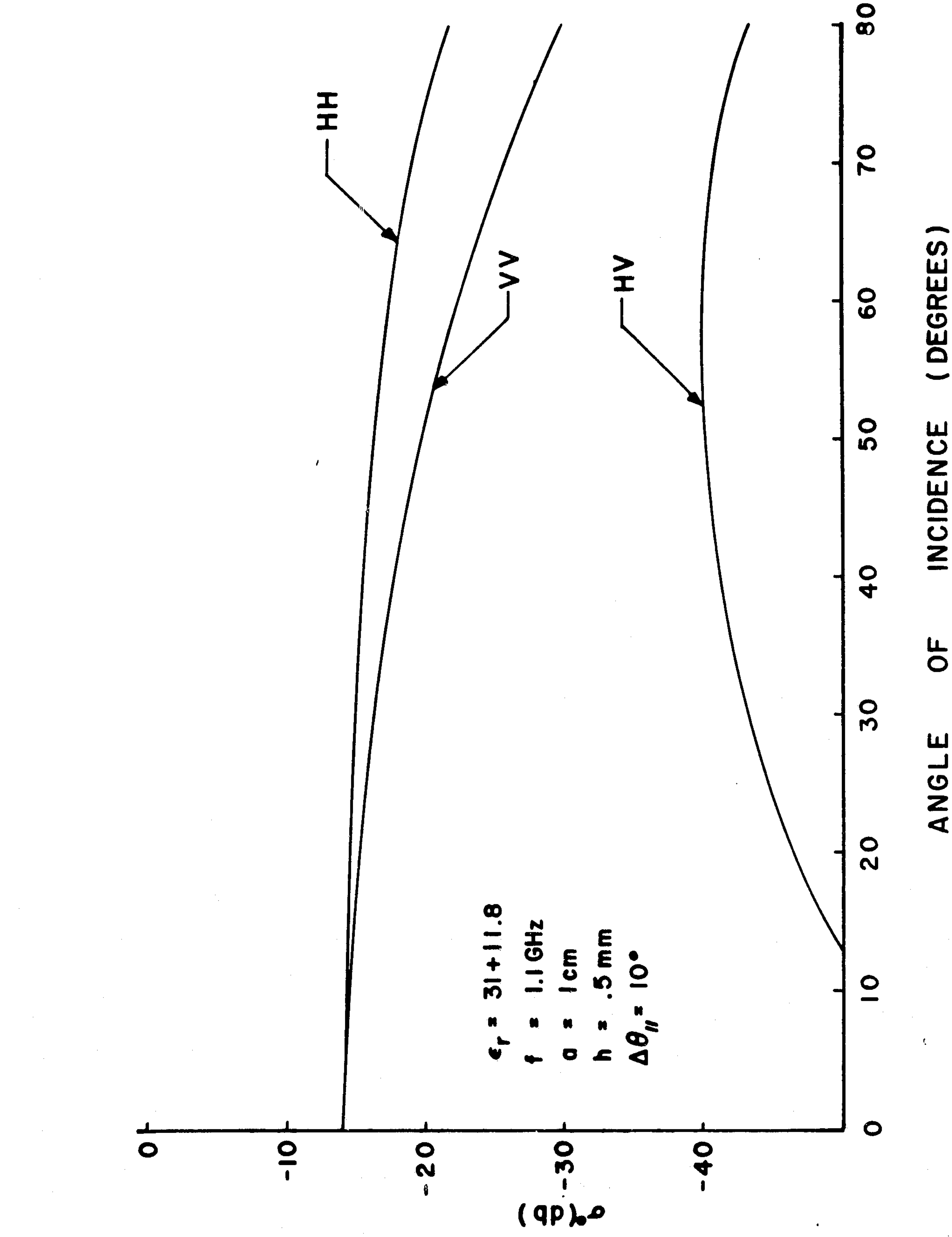
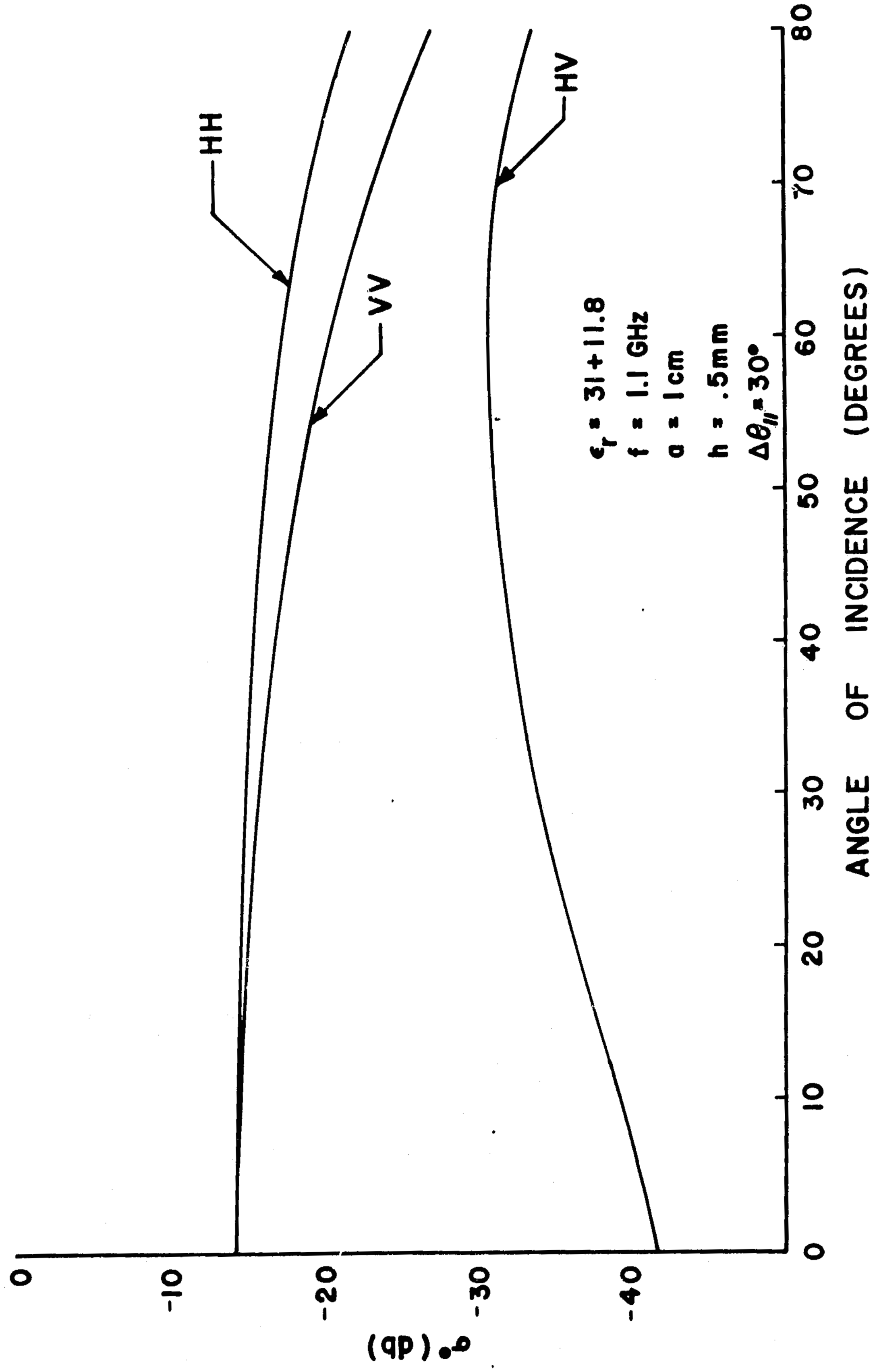


HH
VV
HV
ϵ_r = 31+11.8
f = 1.1GHz
a = 1cm
h = .5mm
$\Delta\theta_{II}$ = 10°
0
-10
-20
-30
-40
σ°(db)
0
10
20
30
40
50
60
70
80
ANGLE OF INCIDENCE (DEGREES)

Figure 5



HH
VV
HV
$\epsilon_r = 31 + i11.8$
f = 1.1 GHz
a = 1cm
h = .5mm
$\Delta\theta_{II} = 30°$
ANGLE OF INCIDENCE (DEGREES)
$\sigma°$ (db)
0
10
20
30
40
50
60
70
80
0
-10
-20
-30
-40

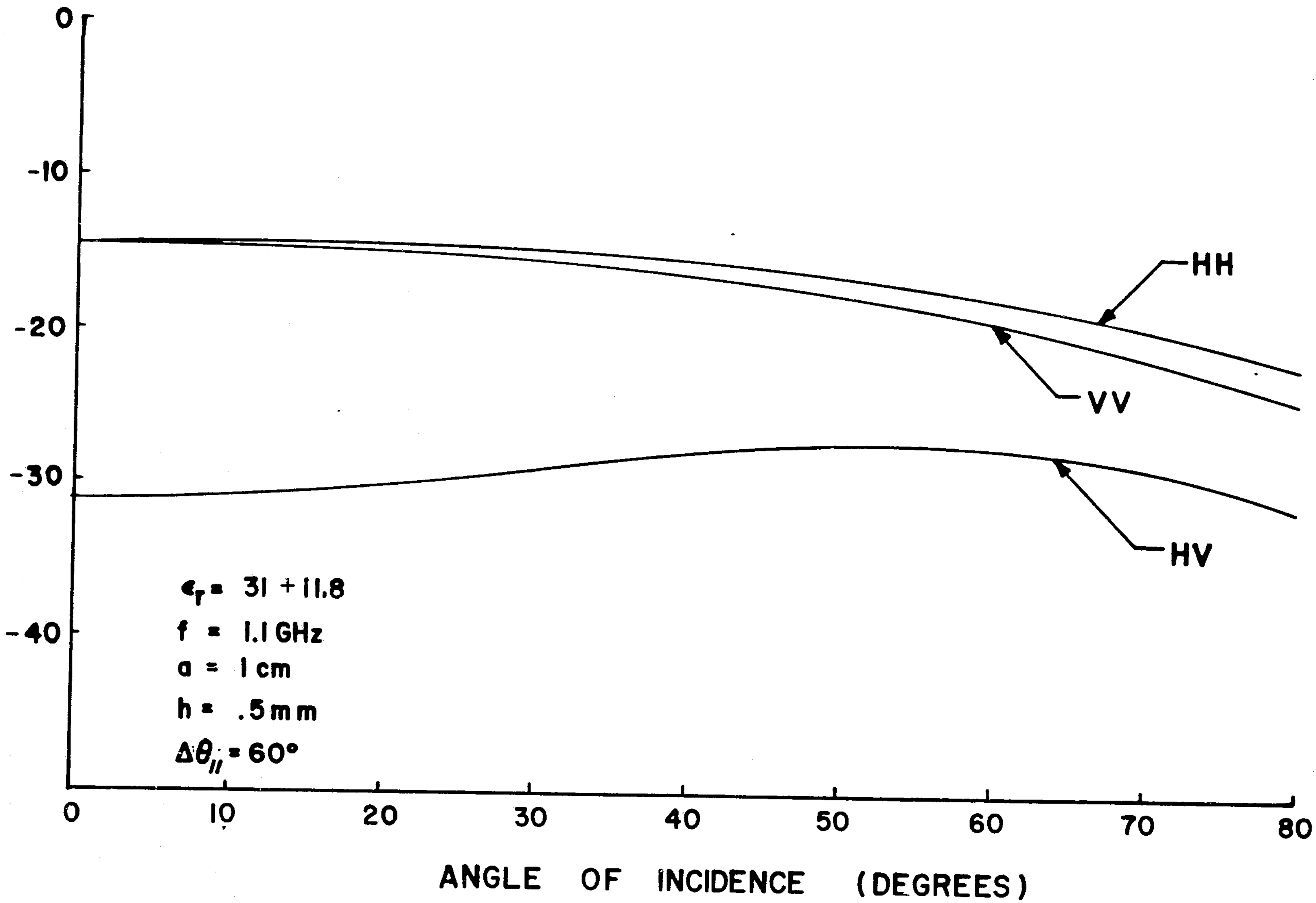
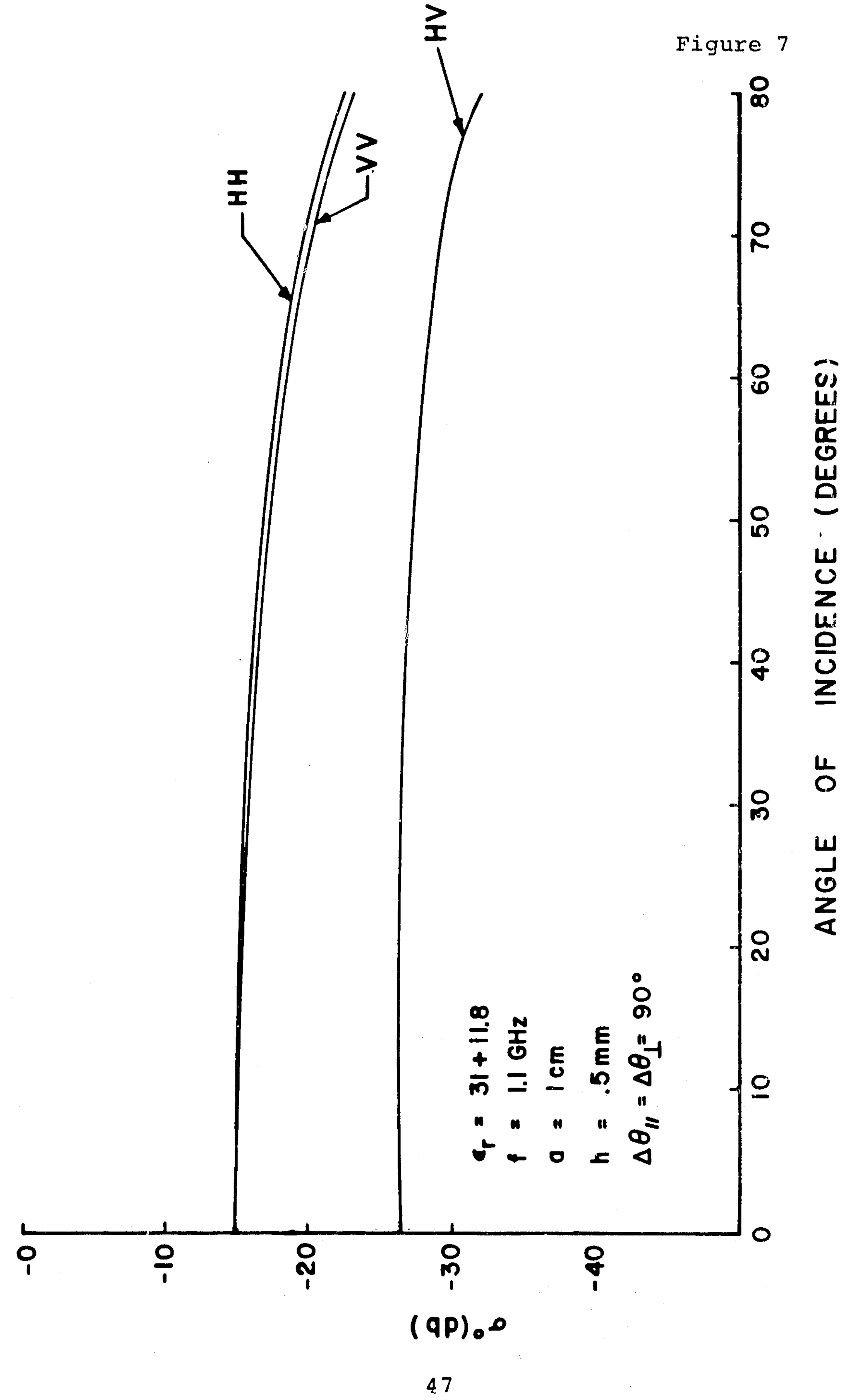

0
-10
-20
-30
-40
σ°(db)
HH
VV
HV
ε_r = 31 + 11.8
f = 1.1 GHz
a = 1 cm
h = .5 mm
Δθ_11 = 60°
0
10
20
30
40
50
60
70
80
ANGLE OF INCIDENCE (DEGREES)

Figure 6

Figure 7



HH
VV
HV
ϵ_r = 31 + 11.8
f = 1.1 GHz
a = 1 cm
h = .5 mm
$\Delta\theta_{//} = \Delta\theta_{\perp} = 90°$
-0
-10
-20
-30
-40
σ°(db)
0
10
20
30
40
50
60
70
80
ANGLE OF INCIDENCE (DEGREES)

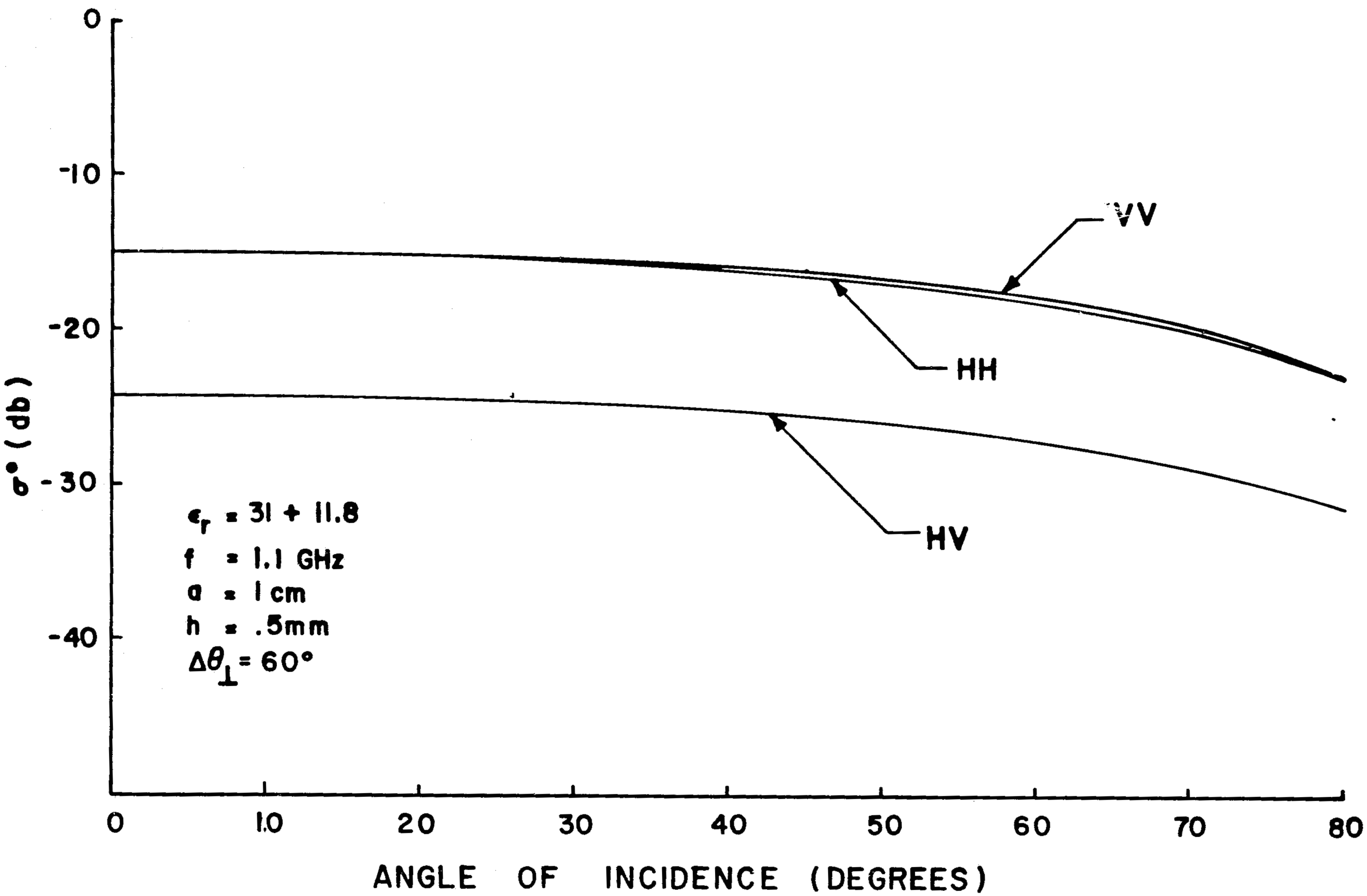
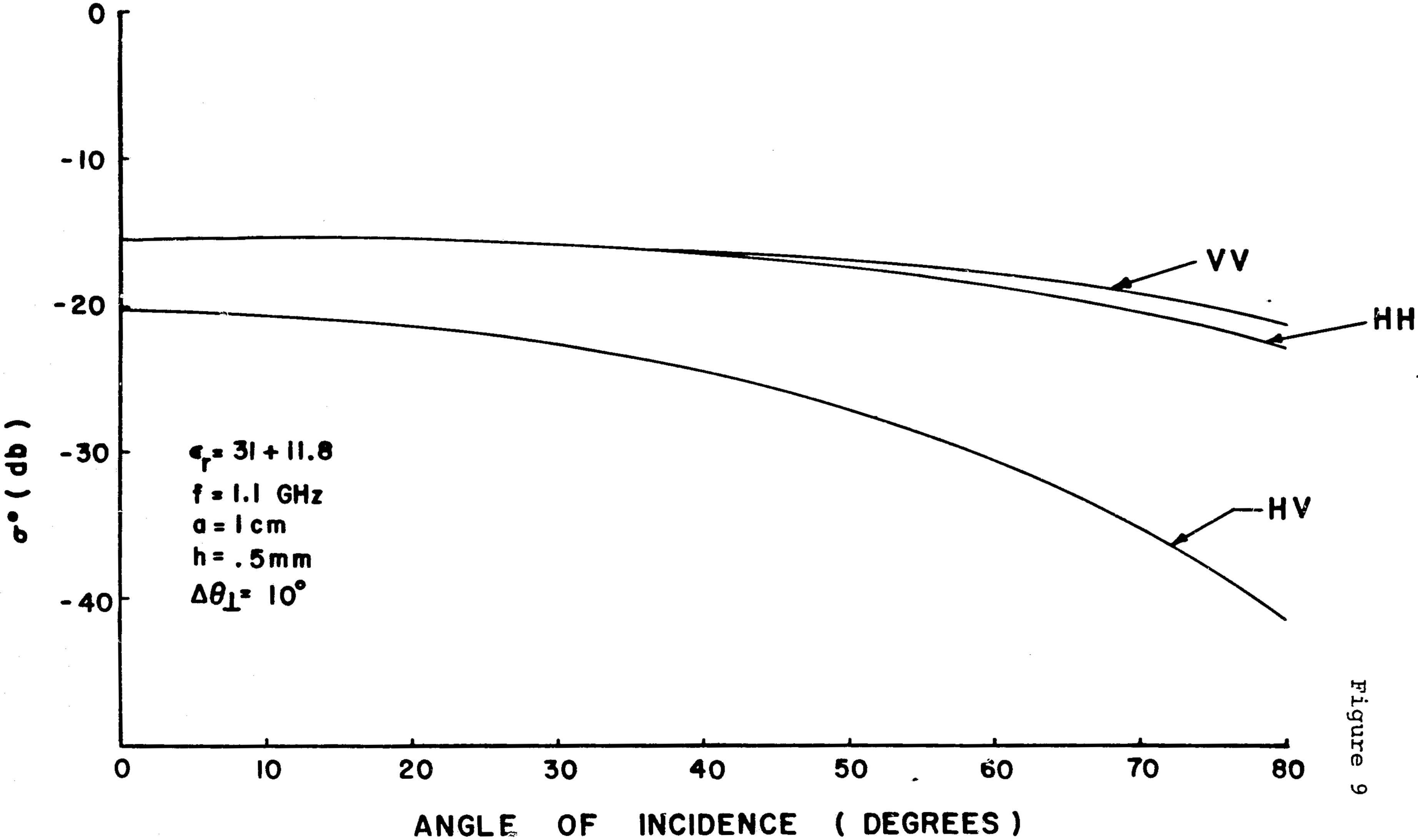

0
-10
-20
-30
-40
σ° (db)
VV
HH
HV
εr = 31 + 11.8
f = 1.1 GHz
a = 1 cm
h = .5mm
Δθ⊥ = 60°
0
10
20
30
40
50
60
70
80
ANGLE OF INCIDENCE (DEGREES)

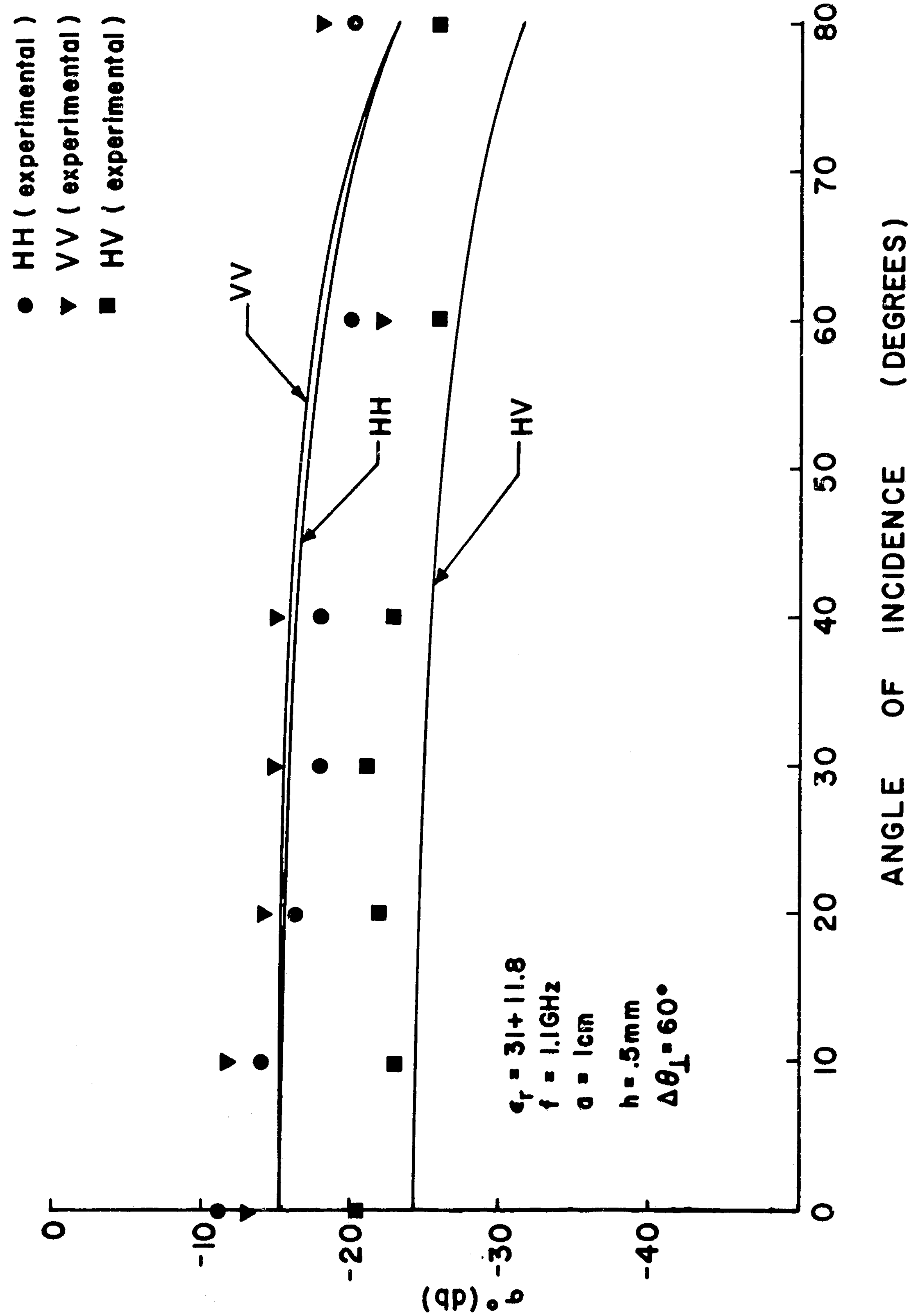
Figure 8



0
-10
-20
-30
-40
σ° (db)
VV
HH
HV
$\epsilon_r = 31 + 11.8$
f = 1.1 GHz
a = 1 cm
h = .5 mm
$\Delta\theta_\perp = 10°$
0
10
20
30
40
50
60
70
80
ANGLE OF INCIDENCE (DEGREES)

Figure 9

Figure 10



● HH (experimental)
▼ VV (experimental)
■ HV (experimental)
VV
HH
HV
ϵ_r = 31+i11.8
f = 1.1GHz
a = 1cm
h = .5mm
$\Delta\theta_\perp$ = 60°
0
-10
-20
-30
-40
$\sigma°$(db)
0
10
20
30
40
50
60
70
80
ANGLE OF INCIDENCE (DEGREES)